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COM232

ASSIGNMENT #3
CCMACLRL

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An online store wants to predict whether a visitor will purchase a product ($y=1$) or leave without buying ($y=0$). We collect two features about each customer:

- x_1 = time spent browsing the website (minutes)
- x_2 = number of product pages viewed
- y = purchase outcome (1= purchase, 0= no purchase)

DATASET

CUSTOMER	TIME ON SITE (x_1)	PAGES VIEWED (x_2)	PURCHASE (y)
A	1	4	0
B	2	3	0
C	3	7	1
D	5	2	1
E	6	6	1

Initial model parameters:

- $m_1 = 0.8$
- $m_2 = 0.4$
- $b = -4.0$

TASKS

- Compute probabilities
- Compute average loss
- Update the slope and intercept using Gradient Descent
- Compute new probabilities using the new slopes and intercept
- Compute new average loss

COMPUTE PROBABILITIES

CUSTOMER	x_1	x_2	y	$\hat{y}$
A	1	4	0	0.168
B	2	3	0	0.231
C	3	7	1	0.769
D	5	2	1	0.690
E	6	6	1	0.961

$$y = \frac{1}{1 + e^{-(m_1(x_1) + (m_2)(x_2) + b}}$$

$e = 2.718$

A $z = 0.8(1) + 0.4(4) + (-4.0) = -1.6$ $\quad \hat{y} = \frac{1}{1 + e^{-(-1.6)}} = 0.168$

B $z = 0.8(2) + 0.4(3) + (-4.0) = -1.2$ $\quad \hat{y} = \frac{1}{1 + e^{-(-1.2)}} = 0.231$

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C $z = 0.8(3) + 0.4(7) + (-4.0) = 1.2$ $\hat{y} = \frac{1}{1+e^{-1.2}} = 0.769$

D $z = 0.8(5) + 0.4(2) + (-4.0) = 0.8$ $\hat{y} = \frac{1}{1+e^{-0.8}} = 0.690$

E $z = 0.8(6) + 0.4(6) + (-4.0) = 3.2$ $\hat{y} = \frac{1}{1+e^{-3.2}} = 0.961$

COMPUTE AVERAGE LOSS

CUSTOMER	x_1	x_2	y	$\hat{y}$	loss
A	1	4	0	0.168	0.1839
B	2	3	0	0.231	0.2627
C	3	7	1	0.769	0.2627
D	5	2	1	0.690	0.3711
E	6	6	1	0.961	0.0398

$$loss = -(y_i \times \ln(\hat{y}_i) + (1-y_i) \times \ln(1-\hat{y}_i))$$

A $loss = -(0 \times \ln(0.168) + (1-0) \times \ln(1-0.168))$
$loss = 0.1839$

B $loss = -(0 \times \ln(0.231) + (1-0) \times \ln(1-0.231))$
$loss = 0.2627$

C $loss = -(1 \times \ln(0.769) + (1-1) \times \ln(1-0.769))$
$loss = 0.2627$

D $loss = -(1 \times \ln(0.690) + (1-1) \times \ln(1-0.690))$
$loss = 0.3711$

E $loss = -(1 \times \ln(0.961) + (1-1) \times \ln(1-0.961))$
$loss = 0.0398$

AVERAGE LOSS: 0.2240

UPDATE THE SLOPE AND INTERCEPT USING GRADIENT DESCENT

CUSTOMER	x_1	x_2	y	$\hat{y}$	$\hat{y}-y$
A	1	4	0	0.168	0.168
B	2	3	0	0.231	0.231
C	3	7	1	0.769	-0.231
D	5	2	1	0.690	-0.310
E	6	6	1	0.961	-0.039
					-0.181

CUSTOMER	x_1	x_2	y	$\hat{y}$	$(\hat{y}-y)*x_1$
A	1	4	0	0.168	0.168
B	2	3	0	0.231	0.462
C	3	7	1	0.769	-0.693
D	5	2	1	0.690	-1.55
E	6	6	1	0.961	-0.234
					-1.847

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CUSTOMER	X_1	X_2	y	$\hat{y}$	$(\hat{y}-y)*X_2$
A	1	4	0	0.168	0.672
B	2	3	0	0.231	0.693
C	3	7	1	0.769	-1.617
D	5	2	1	0.690	-0.620
E	6	6	1	0.961	-0.234
					-1.106

$$\frac{d\ loss}{d\ b} = \frac{1}{5}(-0.181) = -0.0362$$

$$\frac{d\ loss}{d\ m_1} = \frac{1}{5}(-1.847) = -0.3694$$

$$\frac{d\ loss}{d\ m_2} = \frac{1}{5}(-1.106) = -0.2212$$

new $b = -4.0 - 0.1 \times (-0.0362) = -3.996$

new $m_1 = 0.8 - 0.1 \times (-0.3694) = 0.8369$

new $m_2 = 0.4 - 0.1 \times (-0.2212) = 0.4221$

- $b = -3.996$
- $m_1 = 0.8369$
- $m_2 = 0.4221$

COMPUTE NEW PROBABILITIES USING THE NEW SLOPES AND INTERCEPT

CUSTOMER	X_1	X_2	y	new $\hat{y}$
A	1	4	0	0.187
B	2	3	0	0.258
C	3	7	1	0.813
D	5	2	1	0.737
E	6	6	1	0.972

$$y = \frac{1}{1 + e^{-(m_1(x_1) + (m_2)(x_2) + b}}$$

$e = 2.718$

A $z = 0.8369(1) + 0.4221(4) + (-3.996) = -1.4707$

$$\hat{y} = \frac{1}{1 + e^{-(-1.4707)}} = 0.187$$

B $z = 0.8369(2) + 0.4221(3) + (-3.996) = -1.0559$

$$\hat{y} = \frac{1}{1 + e^{-(-1.0559)}} = 0.258$$

C $z = 0.8369(3) + 0.4221(7) + (-3.996) = 1.4694$

$$\hat{y} = \frac{1}{1 + e^{-1.4694}} = 0.813$$

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D $z = 0.8369(5) + 0.4221(2) + (-3.996) = 1.0327$

$$\hat{y} = \frac{1}{1+e^{-1.0327}} = 0.737$$

E $z = 0.8369(6) + 0.4221(6) + (-3.996) = 3.558$

$$\hat{y} = \frac{1}{1+e^{-3.558}} = 0.972$$

COMPUTE NEW AVERAGE LOSS

CUSTOMER	X_1	X_2	y	new $\hat{y}$	new loss
A	1	4	0	0.187	0.207
B	2	3	0	0.258	0.298
C	3	7	1	0.813	0.207
D	5	2	1	0.737	0.305
E	6	6	1	0.972	0.028

$$\text{loss} = -(y_i \times \ln(\hat{y}_i) + (1-y_i) \times \ln(1-\hat{y}_i))$$

A $\text{loss} = -(0 \times \ln(0.187) + (1-0) \times \ln(1-0.187))$

$\text{loss} = 0.207$

B $\text{loss} = -(0 \times \ln(0.258) + (1-0) \times \ln(1-0.258))$

$\text{loss} = 0.298$

C $\text{loss} = -(1 \times \ln(0.813) + (1-1) \times \ln(1-0.813))$

$\text{loss} = 0.207$

D $\text{loss} = -(1 \times \ln(0.737) + (1-1) \times \ln(1-0.737))$

$\text{loss} = 0.305$

E $\text{loss} = -(1 \times \ln(0.972) + (1-1) \times \ln(1-0.972))$

$\text{loss} = 0.028$

AVERAGE LOSS: 0.209