The Double Slit Experiment Performed on a Quantum Computer

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This paper presents a method of conducting the famous double-slit experiment using a hybrid algorithm. We first describe the double slit experiment before analyzing it geometrically and solving for the phase difference. We then describe the hybrid algorithm that calculates the phase difference and creates a quantum circuit to simulate the double-slit experiment. Next, we run the algorithm using IBMQ's hardware and use a previously tested experimental setup, demonstrating consistency between the results. We finally discuss further applications of the algorithm and pose the question of whether or not the algorithm is truly representative of the quantum interference pattern as opposed to being an abstracted version.

1 The double-slit experiment

1.1 An overview

The double-slit experiment looks at the interference behavior of a straight wavefront that passes through two slits within a barrier. The wavefront undergoes diffraction as it passes through the slits, changing it from a straight wavefront to a circular one. The two slits act as independent sources for these circular waves, which constructively and destructively interfere with each other depending on location. An interference pattern is shown when measuring the effect of this interference across an arbitrary two-dimensional plane parallel to the barrier.

1.2 The setup

Figure 1 shows the basic geometry of the double-slit experiment. The barrier on the left has two slits separated by length d. As mentioned before, the two slits can be represented as independent sources S_1 and S_2, which produce circular waves when a straight wavefront approaches the barrier. On the right is a detection screen, D units from the barrier, that shows the square amplitude of the wave at Δx units from the center line. The two lines extending from

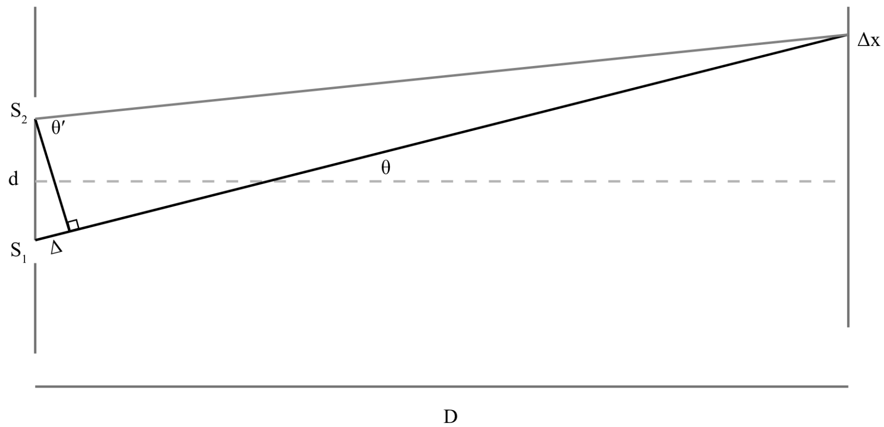

Figure 1: Double-slit experiment setup

S_1 and S_2 represent the one-dimensional slices of the circular wavefronts that reach Δx on the barrier. As a wave passes through the slits with wavelength λ, the waves coming through the slits will travel different lengths to reach point Δx. This path difference, Δ, means that the waves are out of phase with a difference of ϕ by the time they reach the detection screen. We look to abstract this experiment into an algorithm, and use the phase difference between the waves as the parameter. Thus we want to solve the experiment for the phase difference given the physical setup so that we can apply the algorithm.

1.3 Solving for ϕ

This phase difference ϕ must be an integer multiple of 2π for there to be constructive interference at that point. In other words,

$$\phi = n \cdot 2\pi$$
$$\implies n = \frac{\phi}{2\pi}$$

where n is an integer. Equivalently, the path difference must be an integer multiple of the wavelength, or

$$\Delta = n \cdot \lambda.$$

Combining these two equations, we have

$$\Delta = \frac{\phi\lambda}{2\pi}.$$

Now, $\frac{\phi}{2\pi}$ determines the interference pattern; there will be maximum constructive interference when it is an integer. Since $\sin\theta' = \frac{\Delta}{d}$ from the figure, we have

$$\sin\theta' = \frac{\phi\lambda}{2\pi d}.$$

Solving for ϕ, we get

$$\phi = \frac{2\pi d \sin\theta'}{\lambda}.$$

If the distance from the barrier to the screen is significantly larger than the distance between the slits at the barrier, then the lines from S_1 and S_2 will be almost parallel, so $\theta \approx \theta'$. Thus, we have

$$\phi = \frac{2\pi d \sin\theta}{\lambda}. \tag{1}$$

At this point we have abstracted the double slit experiment into a single variable ϕ based on constant physical parameters d and λ, as well as the variable angle θ (which is based on Δx and D). This gives us the power to use the wave from S_2 as the basis for the wave for S_1 (or vice versa), which we will then transform by the phase difference ϕ.

1.4 The double slit experiment with electrons

The profound result of the double slit experiment comes from using specifically photons (or other small particles like electrons) for the experiment. When a light beam is fired at the barrier, the light produces dark and light bands on the screen behind the barrier. This is unexpected in the classical realm; if the light behaved solely like a particle, one would expect it to form two shapes directly behind the slits that are the same size as the slits. This demonstrates that in some cases, light behaves with a wave-like nature. It was later confirmed in the 1923-27 Davisson-Germer experiment that electrons produced an interference pattern as well.

2 The algorithm

2.1 An overview

The algorithm is a hybrid algorithm, using both a classical computer to perform calculations, and a quantum computer to actually run the experiment several times. The classical computer sets the initial parameters d, λ, and D, and iterates through a range of numbers for Δx. It then calculates ϕ using Equation 1 before it constructs the quantum circuit. The quantum circuit is composed of one quantum bit and one classical bit. We first apply a Hadamard gate onto the qubit, followed by a rotation around the Z axis by ϕ, before the qubit is finally measured in the $\{+, -\}$ basis.

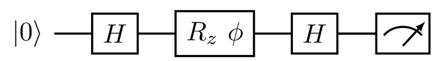

Figure 2: Quantum circuit for the double slit experiment with both slits open

The initial Hadamard gate is analogous to saying that the probability of the particle passing through both waves is 50/50. Then, we rotate the state by ϕ to "move" the wave across the screen, adjusting the probability of the electron actually hitting that part of the screen. Finally we measure in the $\{+, -\}$ basis so that the difference in phase affects the probability outcome as opposed to measuring in the computational basis. By working in the language of probability, we can manipulate the experiment in different ways, like closing a slit entirely or even having one slit partially closed. We can achieve this by first removing the Hadamard gate and then either adding an X gate or not depending on whether we want the left or right slit open. In these two cases, the R_z gate will not have an effect on the measurement outcome when we measure in the $\{+, -\}$ basis, so the resulting amplitude will remain constant. To simulate partially closing the slit, we can measure in the same $\{+, -\}$ basis but rotated over the y-axis by some angle according the the "amount" that the slit would be closed.

2.2 Results

Figure 3 shows the interference pattern described for phases of $[-180°, 180°]$ in intervals of $10°$. The circuit was run with 128 shots for all phase values. The circuit was run on `ibmq_quito` for the case with both slits open, and `ibmq_jakarta` for the single slit cases. When either slit is closed, we see a straight line across the graph.

3 Experimental testing

3.1 The setup

We will now verify the algorithm with Claus Jönsson's experimental setup published in *Zeitschrift für Physik* in 1961. In their setup, the distance from between the barrier and the detection screen (D) is 0.35 meters. The electrons fired at the barrier are accelerated at 50 kV, which have a relativistic wavelength (λ) of 5.3408×10^{-12} meters [Appendix A]. Additionally, the distance between the slits (d) is 2×10^{-6} meters. The width of the observation screen is 1×10^{-5} meters (so we define $\Delta x \in [-5 \times 10^{-6},\ 5 \times 10^{-6}]$ meters).

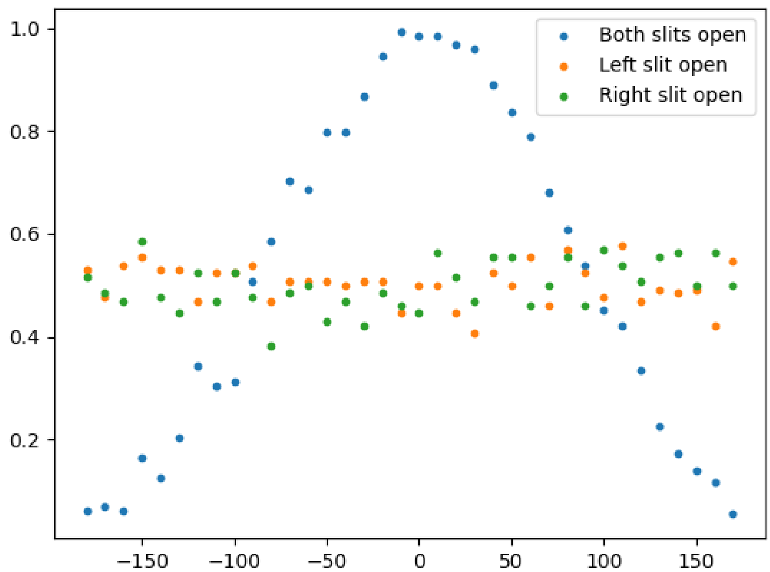

Figure 3: Scatterplot of the results of the quantum circuit run on IBMQ hardware

3.2 Simulation results

Figure 4 shows the results of the algorithm being run on `qasm_simulator` with 2048 shots for each of the 100 points using the physical parameters from before. The same interference pattern is shown but across several cycles instead of one. Note that at $\Delta x = 0$ there is maximum constructive interference since the phase difference between the waves is zero and thus both of the waves' crests meet at that point on the detection screen.

3.3 Verification

The intensity function for $N = 2$ slits as described by Jönsson's paper (adapted to this paper's notation) is

$$I = 4\cos^2\left(\frac{\Delta x \pi d}{\lambda D}\right) \tag{2}$$

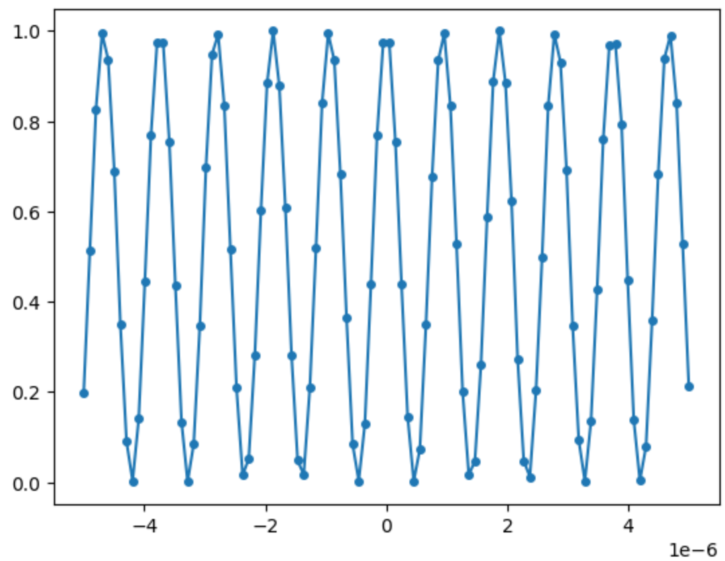

Figure 4: Interference pattern as a function of Δx, using physical parameters (lines added for clarity)

Using Equation 2 we can solve for the points of maximum destructive interference ($I = 0$):

$$
\begin{aligned}
0 &= 4\cos^2\left(\frac{\Delta x \pi d}{\lambda D}\right) \\
\Rightarrow \frac{\Delta x \pi d}{\lambda D} &= \frac{\pi}{2} + \pi k,\ k \in \mathbb{Z} \\
\Rightarrow \Delta x \pi d &= \frac{\lambda D \pi}{2} + \pi k \lambda D \\
\Rightarrow \Delta x &= \frac{\lambda D}{2d} + \frac{k \lambda D}{d}
\end{aligned}
$$

Plugging in the constants for λ, D, and d, we get the following for the set of destruction points:

$$\Delta x \approx (4.6795 + 9.359k) \times 10^{-7} \text{ meters}$$

This is clearly consistent with the simulated results, and a similar process can be followed to show consistency with maximum constructive interference.

4 Discussion and conclusion

The approach to the algorithm can be further utilized to explore other setups, like adding a third slit to the barrier. However, the question still remains whether the algorithm presented is truly representative of the double slit experiment. The rotation and measurement of the state across any axis and in the corresponding basis will always produce a sinusoidal curve, since that is how the quantum state and rotation unitary is defined. Essentially the algorithm is just using a classical computer to compute the angle by which to rotate the state, and applying a quantum circuit to perform the rotation and measurement. On one hand, the interference patterns could easily have been generated more efficiently classically with the intensity function. On the other hand, one could argue that the quantum computer is actually doing the interference that is fundamental to the experiment, utilizing quantum properties otherwise not available. By applying the phase shift to the qubit and measuring its state against a non-shifted basis, we are essentially measuring the difference in phase of the wave that corresponds to the interference pattern. The classical computer would be applying a *model* of the interference behavior, rather than performing the interference itself, which is what the quantum computer would do. The question still remains, however, whether this application of the phase shift and measurement truly represents the interference in the double-slit experiment, or if it is just a simulation of it, where the rotation operator performed on the physical qubit is sinusoidal in nature. Further research would be required to determine this.

A Calculation of 50kV electron's relativistic de Broglie wavelength

The equation of the de Broglie wavelength of a particle with relativistic momentum is

$$\lambda = \frac{hc}{\sqrt{K(K + 2E_0)}}$$

where h is Planck's constant, c is the speed of light, K is the kinetic energy of the particle, and E_0 is the rest energy of the particle [4]. In the case of an electron with mass m_e and charge e that is accelerated through a voltage V_a, we can substitute using $K = e \cdot V_a$ and $E_0 = m_e \cdot c^2$ to find the wavelength of an electron traveling at high speeds:

$$\lambda = \frac{hc}{\sqrt{(e \cdot V_a)^2 + 2 \cdot (e \cdot V_a) \cdot (m_e \cdot c^2)}}$$

Using this equation we calculate the relativistic de Broglie wavelength of an electron accelerated through a 50 kV potential difference to be 5.3408×10^{-12} meters.

B Python code for the algorithm

The following code was used to run the algorithm on IBMQ's `qasm_simulator`. It uses the Qiskit library to create and execute the quantum circuit. Make sure to set up your IBMQ provider at the top of the script.

```python
from qiskit import QuantumCircuit, Aer, IBMQ, execute
import numpy as np
import math

# Set up IBMQ provider here

def run_experiment():
    NUM_PLOT_POINTS = 100
    DISTANCE_FROM_SLIT = 0.35 # meters
    WAVELENGTH = 5.3408e-12 # meters
    DISTANCE_BETWEEN_SLITS = 2e-6 # meters
    SHOTS=2048

    X_RANGE = (-5e-6, 5e-6)

    xs = list(np.linspace(*X_RANGE, NUM_PLOT_POINTS))
    y_values = []

    for x in xs:
        theta = math.atan(x / DISTANCE_FROM_SLIT)
        phi = (2 * math.pi * DISTANCE_BETWEEN_SLITS * math.sin(theta)) / WAVELENGTH

        qc = QuantumCircuit(1, 1)

```

```
        qc.h(0)
        qc.rz(phi, 0)
        qc.h(0)
        qc.measure(0, 0)

        job = execute(qc, backend=Aer.get_backend('qasm_simulator'),shots=SHOTS)
        result = job.result()
        counts = result.get_counts()

        if "0" in counts:
            y_values.append(counts["0"] / SHOTS)
        else:
            y_values.append(0)

    return xs, y_values
```

References

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