

# A Short Summary of my prospective Thesis

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My master thesis focuses on the development and application of the étale homotopy type of higher stacks, as presented by D. Carchedi [Car16b]. First, I start with a historical overview of the (pro) étale homotopy type constructed by B. Mazur and M. Artin in [Art69] and Friedlander in [Fri82]. Then, I provide a concise introduction to (structured)  $\infty$ -topoi. Building on this framework, I introduce étale morphisms in the context of  $\infty$ -topoi, and discuss the perspective of étale blossoms, as developed by Carchedi in [Car16a]. I then define the étale homotopy type of higher stacks, and show how this generalizes classical notions of étale homotopy types. Currently, I am studying Bhatt and Scholze's pro-étale topology from [BS14], to work out whether the pro-étale homotopy type is also generalized by Carchedi's definition. Afterwards, I plan to explore Noohi's analytification functor in further detail and present Carchedi's version of the comparison theorem, which by the above generalizes the usual comparison theorem. To illustrate the theoretical developments, I provide several commonly known examples, such as the étale homotopy type of smooth curves of fixed genus and marked points and elliptic curves. I also aim to find new examples that connect to existing literature. Depending on my progress, I would be glad to also present the connection to  $\mathbb{A}^1$  homotopy theory in my thesis, as studied by Isaksen [Isa03] or the application of the étale homotopy type in log geometry, as discussed in [Car+19].

## References

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