

A Short Summary of my prospective Thesis

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My master thesis focuses on the development and application of the (étale) homotopy type of higher stacks, as presented by D. Carchedi [**carType**]. First, I start with a historical overview of the (pro) étale homotopy type constructed by B. Mazur and M. Artin in [**mazArEt**] and Friedlander in [**friedlEt**]. Then, I provide a concise introduction to ∞ -topoi, in particular ∞ -sheaves and sites, the symmetric monoidal structure on presentable ∞ -categories and hypercompleteness. Afterwards I define the fundamental ∞ -groupoid of a sheaf over any site as the shape of its corresponding slice- ∞ -topos. To relate this construction with more familiar examples, I show how this generalizes the classical notions of étale homotopy types of [**mazArEt**] and [**friedlEt**], in the case of the étale site. Currently, I am working on an outline of how Bhatt and Scholze's pro-étale fundamental group [**bhattscholzeProEt**] can be obtained from the version of the pro-étale site. In the next section, I prove a slightly more general version of the profinite comparism theorem of [**carType**], which uses sheaves instead of hypersheaves. This theorem is obtained by carefully lifting the GAGA anlytification theorem to étale higher stacks and proving a statement similar to [**ha**]. I'm currently reading [**carRelEt**], which is a generalizaton of [**isak**], to understand the role of above constructions for motivic spaces. This will form another section of my thesis.