Week 5 Designing Schemas

Assignment Solutions

1. Consider the set of FDs
$F = \{B \rightarrow A, C \rightarrow D, E \rightarrow B, BC \rightarrow D, AC \rightarrow B, AD \rightarrow E, BC \rightarrow D, ACD \rightarrow BE\}$.

   Which of the following sets of attributes has the closure as $\{A, B, C, D, E\}$?

   - $\{A, B, D\}$
   - $\boxed{\{A, C\}}$
   - $\{A, E\}$
   - $\{E\}$
   - $\{A, D, E\}$

Answer: B

Explanation: We can solve this by finding the closure of each of the different options.

$\{A, C\}^+ = \{A, C\}$ Due to trivial dependencies.
$\{A, C\}^+ = \{A, C, D\}$ From $C \rightarrow D$.
$\{A, C\}+ = \{A, C, D, E\}$ From $AD \rightarrow E$.
$\{A, C\}+ = \{A, B, C, D, E\}$ From $AC \rightarrow B$.

$\{E\}+ = \{E\}$ From trivial dependencies.
$\{E\}+ = \{B, E\}$ From $E \rightarrow B$.
$\{E\}+ = \{A, B, E\}$ From $B \rightarrow A$.

$\{A, B, D\}+ = \{A, B, D\}$ From trivial dependencies.
$\{A, B, D\}+ = \{A, B, D, E\}$ From $AD \rightarrow E$.

$\{A, E\}+ = \{A, E\}$ From trivial dependencies.
$\{A, E\}+ = \{A, B, E\}$ From $E \rightarrow B$.

$\{A, D, E\}+ = \{A, D, E\}$ From trivial dependencies.
$\{A, D, E\}+ = \{A, B, D, E\}$ From $E \rightarrow B$.

2. Consider a relation $R(A, B, C, D, E)$.

You are told that

$A^+ = \{A, D\}$,

$B^+ = \{B, C, D\}$ and

$C^+ = \{C, D\}$.

In addition, the following FDs are **a subset of the FDs** that hold in R,

$$F = \{AC \rightarrow E, ACD \rightarrow B, ABC \rightarrow DE, BE \rightarrow AD\}.$$

Which of the following is a superkey of R?

- ○ $\{B, C, D\}$
- ○ $\{C, E\}$
- ○ $\boxed{\{A, B\}}$
- ○ $\{B, C\}$
- ○ $\{A, E\}$

Answer: C

Explanation: Note that the given closures also represent FDs. In particular, the three closures are equivalent to:

A → AD, B → BCD and C → CD

To find the superkey we have to check that an option gives us a closure with all the attributes {A, B, C, D, E}. Just like in Question 1, we can find the closure for each of the options.

{B, C, D}+ = {B, C, D}
{C, E}+ = {C, D, E}
{A, B}+ = {A, B, C, D, E}
{B, C}+ = {B, C, D}
{A, E}+ = {A, D, E}

3. If F is a set of FDs, then its closure, F^+, is the set of all the FDs that can be derived from the FDs in F.

Suppose

$$F = \{C \to D, E \to C, A \to D, AC \to E, CD \to B, BC \to A\}.$$

Which of the following FD is not in F^+?

- ○ $C \to A$
- ○ $BC \to AED$
- ○ $\boxed{A \to DE}$
- ○ $E \to BC$
- ○ $C \to E$

Answer: C

Explanation: To check if each of the functional dependencies is in F+ we can get the closures of the items in the LHS of each option. Then we can cross check that the RHS is in the closure.

{A}+ = {A, D}
{C}+ = {C, D, B, A, E}
{B, C}+ = {C, D, B, A, E}
{E}+ = {E, C, D, B, A}

4. A relation $R(A, B, C, D, E)$ has the following set of FDs:

$$F = \{A \rightarrow D, B \rightarrow AD, C \rightarrow B, D \rightarrow C, AC \rightarrow E, AD \rightarrow BCE, CD \rightarrow A\}$$

The projection of F to a set of attributes L, denoted $\pi_L F$, is the set of all FDs that follow from F and only involve the attributes in L.

What is $\pi_{\{A,C,D\}} F$? To simplify matters, consider only FDs with single attribute on the right-hand side and non-trivial FDs.

- ○ $\{A \rightarrow D, B \rightarrow AD, C \rightarrow B, D \rightarrow C, AC \rightarrow E, AD \rightarrow BCE, CD \rightarrow A\}$
- ○ $\{D \rightarrow C, A \rightarrow D\}$
- ○ $\{A \rightarrow D, A \rightarrow C, D \rightarrow C, CD \rightarrow A, AD \rightarrow C, AC \rightarrow E\}$
- ○ $\boxed{\{A \rightarrow D, A \rightarrow C, D \rightarrow C, D \rightarrow A, C \rightarrow A, C \rightarrow D, CD \rightarrow A, AC \rightarrow D, AD \rightarrow C\}}$
- ○ $\{A \rightarrow D, CD \rightarrow A\}$

Answer: D

Explanation: To solve this we look only at the answers that contain A, C, D due to the projection. Therefore option {A → D, A → C, D → C, CD → A, AD → C, AC → E} and {A → D, B → AD, C → B, D → C, AC → E, AD → BCE, CD → A} are incorrect.

We are left with options {A → D, A → C, D → C, D → A, C → A, C → D, CD → A, AC → D, AD → C}, {A → D, CD → A} and {D → C, A → D}.

From AD → BCE we can see that both options {A → D, CD → A} and {D → C, A → D} are missing AD → C. Therefore, the answer is the remaining option by the process of elimination.

5. Consider the relation $R(A, B, C, D, E)$ and the following set of FDs that hold over R,

$$F = \{BD \rightarrow C, A \rightarrow BC, ABC \rightarrow E, E \rightarrow D\}.$$

Let T be the sum of the number of attributes in the relations obtained from a BCNF decomposition.

What is the smallest possible value of T that can be obtained from decomposing R into BCNF?

Is there a unique BCNF decomposition that corresponds to this value in this case?

- $\boxed{\text{6, yes}}$
- 7, no
- 7, yes
- 8, no
- 6, no
- 5, yes

Answer: A

Explanation: From the FDs we can see that {A}+ = {A, B, C, D, E}, therefore we know that A is a key to this relation. This means that A, ABC are both superkeys.

From the definition of BCNF, we have to make sure that items on the LHS of a functional dependency are superkeys. Hence we are left with {BD → C, E → D} which violate this rule.

Using the process of separating relations to comply with BCNF, we split the relation on E → D as that would lead to the smallest possible value of T.

Our relations would be R1(A, B, C, E) with F D1 = {A → BC, ABC → E} and R2(E, D) with F D2 = {E → D}. This is a unique BCNF decomposition as splitting with respect to {BD → C} would give us two different relations.

6. Consider a relation $R(A, B, C, D, E)$ that is decomposed into relations $R_1(A, C, D)$ and $R_2(B, C, D, E)$
.

The FDs $F = \{B \rightarrow C, BD \rightarrow E, BC \rightarrow D, CD \rightarrow A\}$ hold over R.

Is the above decomposition in BCNF?

- ○ No.
- ○ Yes.

Answer: B

Explanation: We can notice that the first relation only has the functional dependency $\{CD \rightarrow A\}$. This covers all the attributes in R1, therefore we can see that $\{C, D\}$ is a superkey.

We can see the same thing for R2, since B is a key to the relation.

7. Let $attr(f)$ be the set of attributes that make up the FD f. E.g., $attr\{AB \rightarrow C\} = \{A, B, C\}$

Suppose a set of FDs, F, holds over a relation R such that, $\forall f_i, f_j \in F \text{ and } i \neq j, attr(f_i) \cap attr(f_j) = \emptyset$

.

Every possible BCNF decomposition of R will be _____

- ◯ lossless and not dependency-preserving.
- ◯ lossless and dependency-preserving.
- ◯ lossy and not dependency-preserving.
- ◯ lossy and dependency-preserving.

Answer: B

Explanation: All BCNF decompositions are lossless.

Also, as no two FDs have the same attributes, whenever we decompose a relation using a FD f that violates the BCNF condition, no FD will have its attributes split across relations.

This ensures that all FDs will belong to a single relation and all of them can be checked without joining relations.

8. Consider a relation $R(A, B, C, D, E)$ that is decomposed into relations $R_1(A, B, C)$ and $R_2(A, D, E)$.

The FDs $F = \{A \to BC, AD \to E, E \to D\}$ hold over R.

Is the above decomposition in 3NF?

- ○ Yes.
- ○ No.

Answer: A

Explanation: Attribute A on the left-hand side of A → BC is a key for R1.

The other FDs do not hold for R1 so we are done with R1. Consider R2. AD → E implies {A, D} is a key for R2.

The attribute on the right-hand side of E → D is a member of the key for R2 so it is in 3NF.

Therefore the conditions for 3NF are satisfied.

9. Consider the following instance of a relation $R(A, B, C, D)$. Which of the following multivalued dependencies does R not satisfy?

A	B	C	D
1	2	3	4
1	1	3	4
1	2	4	5

- ○ $CD \twoheadrightarrow AB$
- ○ $\boxed{AB \twoheadrightarrow C}$
- ○ $AB \twoheadrightarrow CD$
- ○ $C \twoheadrightarrow D$

Answer: B

Explanation: We can test out each option to see if the MVD is satisfied.

Options b and d are trival and always satisfied as they contain all the attributes.

Option c is satisfied because the value in D is the same for both tuples 1 and 2.

Swapping the values in A, B won't cause a difference.

AB ↠ C is missing tuples (1, 2, 3, 5) and (1, 2, 4, 4).

10. The relation $R(A, B, C, D, E)$ has MVDs $\{AB \twoheadrightarrow C, C \twoheadrightarrow D\}$ and is known to contain tuples $(1, 2, 3, 4, 5)$ and $(1, 2, 6, 7, 8)$.

Which of the following tuple must also be in R?

- ○ $(1, 2, 5, 6, 7)$
- ○ $(1, 2, 3, 6, 7)$
- ○ $(1, 2, 4, 5, 7)$
- ○ $(1, 2, 4, 5, 6)$
- ○ $\boxed{(1, 2, 3, 7, 5)}$

Answer: E

Explanation: One simple way to find which other tuple is in R is by creating the tuples in R using the FDs.

From AB ↠ C we can see that (1, 2, 3, 7, 8) and (1, 2, 6, 4, 5) is also in R.

Then from C ↠ D we can see that we are still missing (1, 2, 3, 7, 5), which is option E, (1, 2, 3, 4, 8), (1, 2, 6, 4, 8), (1, 2, 6, 7, 5).