Attribute Closures

Designing Schemas

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Kevin C.C. Chang, Professor
Computer Science @ Illinois

Learning Objectives

By the end of this video, you will be able to:

- Define the concept of attribute closure.
- Describe how and why attribute closure is useful.
- Find the closure of a set of attributes given a set of FDs.
- Determine keys of a relation using attribute closures.

Reasoning: Is {drinker, bar} a Superkey?

- We are asking:
  - Can {drinker, bar} determine all attributes?
- We may ask, more fundamentally:
  - What attributes can {drinker, bar} determine?
  - This is called the **closure** of {drinker, bar}.

- **Given:**
  - **drinker, bar, beer $\longrightarrow$ season**
  - **drinker, bar $\longrightarrow$ beer**
  - **bar, beer $\longrightarrow$ price**
- **Decide: Is {drinker, bar} a Superkey?**

Closure of Attributes

- Problem: *What can these attributes determine?*
  - Given a set of attributes $\{A_1, \ldots, A_n\}$ and a set of dependencies F
  - Find all attributes $B_1, \ldots, B_m$ such that any relation that satisfies F also satisfies:

    $$A_1, \ldots, A_n \longrightarrow B_1, \ldots, B_m$$

  - The closure of $\{A_1, \ldots, A_n\}$ is $B_1, \ldots, B_m$, i.e.,

    $$\{A_1, \ldots, A_n\}^+ = \{B_1, \ldots, B_m\}$$

- **Given:**
  - **drinker, bar, beer ⟶ season**
  - **drinker, bar ⟶ beer**
  - **bar, beer ⟶ price**
- **Decide: Is {drinker, bar} a Superkey?**

- Ex: What can {drinker, bar} determine with the given FDs?
  - $\{\text{drinker, bar}\}^+$

    $= \{\text{drinker, beer, beer, season, price}\}$

Finding Attribute Closures

- Given a set of attributes $\{A_1, \ldots, A_n\}$ and a set of dependencies F
- $C = \{A_1, \ldots, A_n\}$
- Repeat until C does not change:
  - If $X_1, \ldots, X_m \longrightarrow Y$ is in F, and $X_1, \ldots, X_m$ are all in C, and Y not in C:
    - $C := C + Y$

- Ex: {drinker, bar }$^+$ = ?
  - C = {drinker, bar}
  - Add beer, ∵ drinker, bar ⟶ beer
  - Add season, ∵ drinker, bar, beer ⟶ season
  - Add price, ∵ bar, beer ⟶ price
  - C = {drinker, bar, beer, season, price}

- **Given:**
  - **drinker, bar, beer ⟶ season**
  - **drinker, bar ⟶ beer**
  - **bar, beer ⟶ price**
- **Decide: Is {drinker, bar} a Superkey?**

Reasoning: {drinker, bar} Is a Key

- $\{\text{drinker, bar}\}^+ = \{\text{drinker, bar, beer, season, price}\}$
  - So, {drinker, bar} is a superkey.

- $\{\text{drinker}\}^+ = \{\text{drinker}\}$
  - So, {drinker} is not a superkey.

- $\{\text{bar}\}^+ = \{\text{bar}\}$
  - So, {bar} is not a superkey.

- So, {drinker, bar} is a key!

Food for Thought

Can you use attribute closure to determine if an FD A → B holds? How?