

# Functional Dependencies

Designing Schemas

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

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# Learning Objectives

By the end of this video, you will be able to:

- Define what functional dependency means.
- Give examples of functional dependencies.

# Functional Dependencies

- A form of constraint, and therefore part of a schema.
- A key factor for determining and organizing good schemas.

# About Major and Birthday

- How many majors do you have?
  - None, one, or multiple
  - **id → major?**
- How many birthdays do you have?
  - One
  - **id → birthday?**

<b>id</b>	<b>name</b>	<b>major</b>	<b>birthday</b>
1	Bugs Bunny	CS	2004-11-06
1	Bugs Bunny	Music	2004-11-06
2	Donald Duck	Bio	1997-02-01
3	Peter Pan	Econ	1998-10-01
3	Peter Pan	Social	1998-10-01
3	Peter Pan	ME	1998-10-01
4	Mickey Mouse	CS	1995-04-01

Example Students table

# Functional Dependency (FD)

- Notation:  $A_1, \dots, A_m \rightarrow B_1, \dots, B_n$
- We say:  $A_1, \dots, A_m$  **functionally determines**  $B_1, \dots, B_n$ .
- Meaning:
  - If any tuples agree on  $A_1, \dots, A_m$  values, then they must also agree on  $B_1, \dots, B_n$ .
  - I.e., the mapping from  $A_1, \dots, A_m$  to  $B_1, \dots, B_n$  is **functional** (many-one).
- Whether FDs hold is your **knowledge/assumption** of the domain.
  - $\text{id} \rightarrow \text{birthday}$
  - $\text{id, course} \rightarrow \text{grade}$

# A ” Functionally Determines” B

- Given a value of A, there exists at most one value of B-- no ambiguity.
- It does not mean: B can be computed from A by a formula.
  - For  $\text{id} \rightarrow \text{birthday}$ , you cannot compute birthday from id.
- It does not mean: B can be easily found by A.
  - For  $\text{id} \rightarrow \text{birthday}$ , you may not be able to identify the birthday if it is not disclosed to you.

# FD: Your Domain Knowledge/Assumption

- Understand the domain and make assumptions accordingly.
- $\text{id} \rightarrow \text{major}$ 
  - Possibly true, if a student can have only one major
- $\text{id, course} \rightarrow \text{grade}$ 
  - Possibly false, if a student can take a course multiple times

# Keys

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# Learning Objectives

By the end of this video, you will be able to:

- Define key in terms of functional dependencies.
- Distinguish the notions of key and superkey.

# What Is a Key?

- After defining FDs, we can now define keys formally.
- **Key** of a relation  $R(A_1, \dots, A_n)$  is a set of attributes  $K$  that
  - Functionally determines all attributes of  $R$ ,  $K \rightarrow A_1, \dots, A_n$ , and
  - None of its subsets does
- Superkey
  - A set of attributes that contains a key

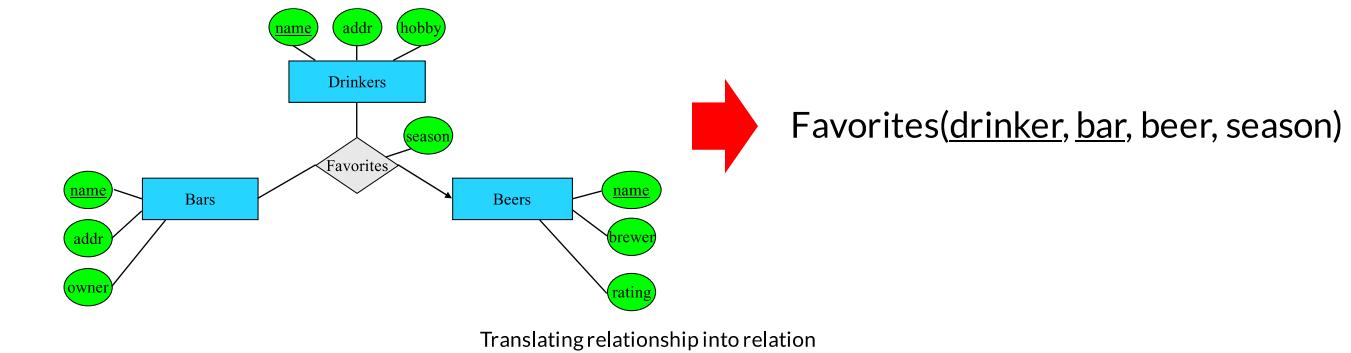
# Explaining the Key Rule in ER Translation

- Recall this rule:

## Relationship → Relation

Rule: Translating relationship  $X$  of  $E_1, \dots, E_n$  to relation  $R$

- Attributes of  $R$  = key attributes of  $E_1, \dots, E_n$  plus attributes of  $X$
- Key of  $R$  = key of  $E_1, \dots, E_n$  except those “arrowed” entities



- Why?

- {drinker, bar, beer} is not a key, since {drinker, bar} already is.
- {drinker, bar, beer} is a superkey.

# Reasoning with FDs

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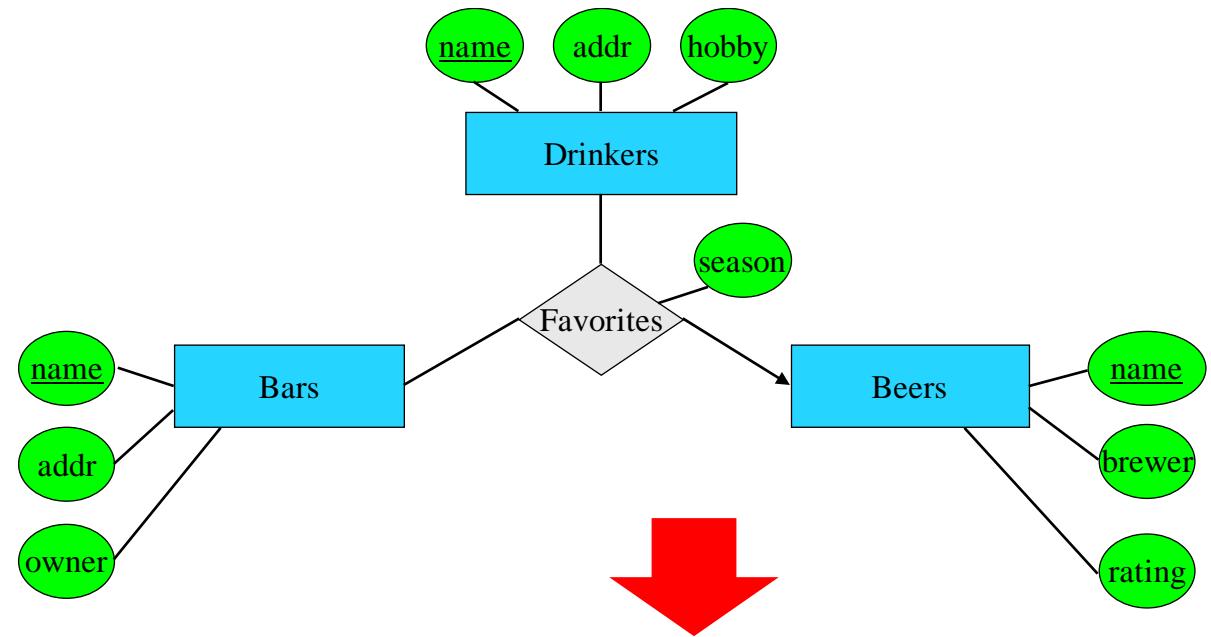
# Learning Objectives

By the end of this video, you will be able to:

- Describe why reasoning with FDs is useful.
- State Armstrong's Axioms and their derived rules.
- Determine if an FD holds based on a given set of FDs.

# Why Do I Need to Reason?

- Recall the Favorites relation:
  - How to know {drinker, bar} is a key?
  - We are given
    - drinker, bar, beer → season
    - drinker, bar → beer
  - Can we reason that
    - drinker, bar → drinker, beer, ba
    - And {drinker} or {bar} does not.
  - If so, then we know {drinker, bar} is



## Favorites(drinker, bar, beer, season)

Translating the Favorites relationship to a relation

# Basic Rules: Armstrong's Axioms

- Reflexivity rule
  - If  $B \subseteq A$ , then  $A \rightarrow B$ .
- Augmentation rule
  - If  $A \rightarrow B$ , then  $AC \rightarrow BC$ .
- Transitivity rule
  - If  $A \rightarrow B$  and  $B \rightarrow C$ , then  $A \rightarrow C$ .

# Deriving More Rules

- Splitting rule
  - If  $A \rightarrow BC$ , then  $A \rightarrow B$ , and  $A \rightarrow C$ .
  - Derivation
    - $\because A \rightarrow BC$  and  $BC \rightarrow B$  (by reflexivity)
    - $\therefore A \rightarrow B$  (by transitivity)
    - Similarly for  $A \rightarrow C$
- Combining rule
  - If  $A \rightarrow B$ , and  $A \rightarrow C$ , then  $A \rightarrow BC$ .
  - Derivation
    - $\because A \rightarrow AB$  (by augmentation) and  $AB \rightarrow BC$  (by augmentation)
    - $\therefore A \rightarrow BC$  (by transitivity)

# Reasoning: Is {drinker, bar} a Superkey?

- Suppose Favorites(drinker, bar, beer, season, price)
- **Given:**
  - drinker, bar, beer → season
  - drinker, bar → beer
  - bar, beer → price
- **Decide: Is {drinker, bar} a superkey?**
- To determine, can we reason:
  - Is **drinker, bar → drinker, bar, beer, season, price** an FD?
  - If it is an FD, then YES.

# Reasoning: Is drinker, bar → all an FD?

- 1. drinker, bar → drinker, bar (reflexivity)
- 2. drinker, bar → beer (given)
- 3. drinker, bar → season
  - drinker, bar → drinker, bar, beer (augmenting 2)
  - drinker, bar, beer → season (given)
  - drinker, bar → season (transitivity)
- 4. drinker, bar → price
  - How to derive?
- 5. drinker, bar → drinker, bar, beer, season, price (combining all above)

- Given:
  - drinker, bar, beer → season
  - drinker, bar → beer
  - bar, beer → price
- Decide: Is {drinker, bar} a Superkey?

# Attribute Closures

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# Learning Objectives

By the end of this video, you will be able to:

- Define the concept of attribute closure.
- Describe how and why attribute closure is useful.
- Find the closure of a set of attributes given a set of FDs.
- Determine keys of a relation using attribute closures.

# Reasoning: Is {drinker, bar} a Superkey?

- We are asking:
    - Can {drinker, bar} determine all attributes?
  - We may ask, more fundamentally:
    - What attributes can {drinker, bar} determine?
    - This is called the **closure** of {drinker, bar}.
- **Given:**
    - drinker, bar, beer → season
    - drinker, bar → beer
    - bar, beer → price
  - **Decide: Is {drinker, bar} a Superkey?**

# Closure of Attributes

- Problem: *What can these attributes determine?*
  - Given a set of attributes  $\{A_1, \dots, A_n\}$  and a set of dependencies  $F$
  - Find all attributes  $B_1, \dots, B_m$  such that any relation that satisfies  $F$  also satisfies:
$$A_1, \dots, A_n \rightarrow B_1, \dots, B_m$$
  - The closure of  $\{A_1, \dots, A_n\}$  is  $B_1, \dots, B_m$ , i.e.,
$$\{A_1, \dots, A_n\}^+ = \{B_1, \dots, B_m\}$$
- Ex: What can  $\{\text{drinker, bar}\}$  determine with the given FDs?
  - $\{\text{drinker, bar}\}^+$   
=  $\{\text{drinker, beer, beer, season, price}\}$

- Given:
    - drinker, bar, beer  $\rightarrow$  season
    - drinker, bar  $\rightarrow$  beer
    - bar, beer  $\rightarrow$  price
  - Decide: Is  $\{\text{drinker, bar}\}$  a Superkey?

# Finding Attribute Closures

- Given a set of attributes  $\{A_1, \dots, A_n\}$  and a set of dependencies  $F$
- $C = \{A_1, \dots, A_n\}$
- Repeat until  $C$  does not change:
  - If  $X_1, \dots, X_m \rightarrow Y$  is in  $F$ , and  $X_1, \dots, X_m$  are all in  $C$ , and  $Y$  not in  $C$ :
    - $C := C + Y$
- Ex:  $\{\text{drinker, bar}\}^+ = ?$ 
  - $C = \{\text{drinker, bar}\}$
  - Add beer,  $\because \text{drinker, bar} \rightarrow \text{beer}$
  - Add season,  $\because \text{drinker, bar, beer} \rightarrow \text{season}$
  - Add price,  $\because \text{bar, beer} \rightarrow \text{price}$
  - $C = \{\text{drinker, bar, beer, season, price}\}$

- **Given:**
  - drinker, bar, beer  $\rightarrow$  season
  - drinker, bar  $\rightarrow$  beer
  - bar, beer  $\rightarrow$  price
- **Decide: Is {drinker, bar} a Superkey?**

# Reasoning: {drinker, bar} Is a Key

- $\{\text{drinker}, \text{bar}\}^+ = \{\text{drinker}, \text{bar}, \text{beer}, \text{season}, \text{price}\}$ 
  - So, {drinker, bar} is a superkey.
- $\{\text{drinker}\}^+ = \{\text{drinker}\}$ 
  - So, {drinker} is not a superkey.
- $\{\text{bar}\}^+ = \{\text{bar}\}$ 
  - So, {bar} is not a superkey.
- So, {drinker, bar} is a key!

**Food for Thought**

*Can you use attribute closure to determine if an FD  $A \rightarrow B$  holds? How?*

# Normal Forms

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# Learning Objectives

By the end of this video, you will be able to:

- Describe what normal forms mean.
- Explain why the concept of normal forms is useful for designing schemas.

# Recall the Problematic Schema

- Why is it bad?
- A student's name and birthday are repeated for each major.
- So? We should not store name and birthday with major.

<b>id</b>	<b>name</b>	<b>major</b>	<b>birthday</b>
1	Bugs Bunny	CS	2004-11-06
1	Bugs Bunny	Music	2004-11-06
2	Donald Duck	Bio	1997-02-01
3	Peter Pan	Econ	1998-10-01
3	Peter Pan	Social	1998-10-01
3	Peter Pan	ME	1998-10-01
4	Mickey Mouse	CS	1995-04-01

Example Students table

# A Better Design

**Students**

<b>id</b>	<b>name</b>	<b>birthday</b>
1	Bugs Bunny	2004-11-06
2	Donald Duck	1997-02-01
3	Peter Pan	1998-10-01
4	Mickey Mouse	1995-04-01

- Redundancy?
- Update anomaly?
- Delete anomaly?

Example Students and Majors tables

**Majors**

<b>id</b>	<b>major</b>
1	CS
1	Music
2	Bio
3	Econ
3	Social
3	ME
4	CS

# Schema Refinement: Desired Properties

- Minimize redundancy.
- Avoid info loss.
- Preserve dependency.
- Ensure good query performance.

# First Normal Form

- From the very beginning, when relational model was defined
- **First Normal Form (1NF):** Each attribute contains only single atomic values.
- As proposed by E. F. Codd:  
E. F. Codd (1970). A Relational Model of Data for Large Shared Data Banks. *Communications of the ACM*, 13(6): 377-387

employee (*man#*, name, birthdate, jobhistory, children)  
jobhistory (*jobdate*, title, salaryhistory)  
salaryhistory (*salarydate*, salary)  
children (*childname*, birthyear)

FIG. 3(a). Unnormalized set

employee' (*man#*, name, birthdate)  
jobhistory' (*man#*, *jobdate*, title)  
salaryhistory' (*man#*, *jobdate*, *salarydate*, salary)  
children' (*man#*, *childname*, birthyear)

FIG. 3(b). Normalized set

Examples used in Codd's paper on relational model

# Normal Forms

- **First Normal Form (1NF):** Each attribute contains only single atomic values.
- Many normal forms were proposed, as more desired properties were discovered.
  - 2NF, 3NF, BCNF, 4NF, ...
- We will study BCNF – Boyce Codd Normal Forms (and 3NF, 4NF).

# **Boyce-Codd Normal Form**

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# Learning Objectives

By the end of this video, you will be able to:

- Define what BCNF is.
- Determine whether a given relation is in BCNF.
- Transform a relation into BCNF with BCNF decomposition.

# Towards BCNF: Boyce and Codd

- **Codd** invented the relational model, created the era of declarative (in contrast to procedural) data management.
- **Boyce** -- in addition to BCNF, he has a major contribution to make declarative data management a reality. *Stay tuned!!*

Boyce–Codd normal form

Boyce–Codd normal form is a normal form used in database normalization. It is a slightly stronger version of the third normal form. [Wikipedia](#)

**Abbreviation:** BCNF  
**Year introduced:** 1974

Developed by

Edgar F. Codd      Raymond F. Boyce

Raymond F. Boyce

Computer scientist



Raymond 'Ray' Boyce was an American computer scientist who was known for his research in relational databases. He is most known for his work co-developing the SQL database language and Boyce-Codd normal form. [Wikipedia](#)

**Born:** 1947  
**Died:** 1974

Google search result of  
"Boyce Codd Normal Form"

Google search result of  
"Raymond F. Boyce"

# Recall the Problematic Schema

- Why is it bad?
- A student's name and birthday are repeated for each major.
- So? We should not store name and birthday with major.

<b>id</b>	<b>name</b>	<b>major</b>	<b>birthday</b>
1	Bugs Bunny	CS	2004-11-06
1	Bugs Bunny	Music	2004-11-06
2	Donald Duck	Bio	1997-02-01
3	Peter Pan	Econ	1998-10-01
3	Peter Pan	Social	1998-10-01
3	Peter Pan	ME	1998-10-01
4	Mickey Mouse	CS	1995-04-01

Example Students table

# What Is the Culprit?

- Why would name and birthday repeat, but major does not?
- id → name, birthday
- id ↗ major
- id determines some attributes A.
- id does not determine other attributes; i.e., it is not a superkey.
- Thus, A will repeat!
- Lesson: *An FD by non-key attributes can cause redundancy.*

<b>id</b>	<b>name</b>	<b>major</b>	<b>birthday</b>
1	Bugs Bunny	CS	2004-11-06
1	Bugs Bunny	Music	2004-11-06
2	Donald Duck	Bio	1997-02-01
3	Peter Pan	Econ	1998-10-01
3	Peter Pan	Social	1998-10-01
3	Peter Pan	ME	1998-10-01
4	Mickey Mouse	CS	1995-04-01

Example Students table

# Boyce-Codd Normal Form

- A relation R is in BCNF if and only if:

Whenever there is a **nontrivial FD** for  $R$ ,

$$A \rightarrow B$$

then  $A$  is a superkey for  $R$ .

- Whenever a set of attributes of  $R$  determines another attribute, it should determine all attributes of  $R$ .
- That is, no bad FDs!

# BCNF or Not?

- Likes(name, addr, likeBeer)
  - name → addr
  - name ↗ likeBeer
- BCNF?

<b>name</b>	<b>addr</b>	<b>likeBeer</b>
Alex	100 Green St	Sam Adams
Bob	300 Purple St	Sam Adams
Carissa	200 Green St	Bud Light
Alex	100 Green St	Coors

Example Likes table

- Favorites(name, addr, favoriteBeer)
  - name → addr
  - name → favoriteBeer
- BCNF?

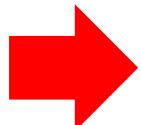
<b>name</b>	<b>addr</b>	<b>favoriteBeer</b>
Alex	100 Green St	Sam Adams
Carissa	200 Green St	Bud Light
Alex	100 Green St	Coors

Example Favorites table

# BCNF Decomposition

- **Algorithm BCNF**
- Input: Relation  $R$ , FDs  $F$
- If (exists an FD  $A \rightarrow B$  that violates the BCNF condition)
  - Decompose  $R(A, B, C)$  into  $R_1(A, B)$  and  $R_2(A, C)$ .
  - Compute FDs for  $R_1$  and  $R_2$  as  $F_1$  and  $F_2$ .
  - Return  $\text{BCNF}(R_1, F_1) \cup \text{BCNF}(R_2, F_2)$ .
- Else
  - Return  $R$ .

A	B	C
...	...	...



A	B
...	...

A	C
...	...

BCNF decomposition

# Decomposing into BCNF

- $R = (\text{id}, \text{name}, \text{major}, \text{birthday})$
- $F = \{\text{id} \rightarrow \text{name}, \text{id} \rightarrow \text{birthday}\}$
- FD  $f: \text{id} \rightarrow \text{name}, \text{birthday}$  violates BCNF
- Decompose into:
  - $R_1 = (\text{id}, \text{name}, \text{birthday}), F_1 = \{\text{id} \rightarrow \text{name}, \text{id} \rightarrow \text{birthday}\}$
  - $R_2 = (\text{id}, \text{major}), F_2 = \emptyset$
- Done?

<b>id</b>	<b>name</b>	<b>major</b>	<b>birthday</b>
1	Bugs Bunny	CS	2004-11-06
1	Bugs Bunny	Music	2004-11-06
2	Donald Duck	Bio	1997-02-01
3	Peter Pan	Econ	1998-10-01
3	Peter Pan	Social	1998-10-01
3	Peter Pan	ME	1998-10-01
4	Mickey Mouse	CS	1995-04-01

Example Students table

# Decomposing into BCNF

- $R = (\text{id}, \text{name}, \text{major}, \text{birthday}, \text{adviser})$
- $F = \{\text{id} \rightarrow \text{name}, \text{id} \rightarrow \text{birthday}, \text{major} \rightarrow \text{adviser}\}$
- FD  $f: \text{id} \rightarrow \text{name}, \text{birthday}$  violates BCNF
- Decompose into:
  - $R_1 = (\text{id}, \text{name}, \text{birthday}), F_1 = \{\text{id} \rightarrow \text{name}, \text{id} \rightarrow \text{birthday}\}$
  - $R_2 = (\text{id}, \text{major}, \text{adviser}), F_2 = \{\text{major} \rightarrow \text{adviser}\}$
- Done?

*Is BCNF unique?*

*i.e., given a relation R and FDs F, does BCNF decomposition results in a unique BCNF?*

- Students(id, name, phone)
- Suppose:  $\text{id} \rightarrow \text{name}$ ,  $\text{name} \rightarrow \text{id}$
- What BCNF exists?

# **Lossless Decomposition**

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# Learning Objectives

By the end of this video, you will be able to:

- Define lossless decomposition, and explain why it is desired.
- Explain why BCNF decomposition is lossless.

# A Decomposition Can Be Lossy

- Favorites(drinker, bar, beer)

drinker	bar	beer
Alex	John Bar	Sam Adams
Carissa	Green Bar	Bud Light
Alex	Purple Bar	Coors

Example Favorites relation

- Decomposition:  $R_1 =$

drinker	bar
Alex	John Bar
Carissa	Green Bar
Alex	Purple Bar

drinker	beer
Alex	Sam Adams
Carissa	Bud Light
Alex	Coors

- Favorites =  $R_1 \bowtie R_2$ ?

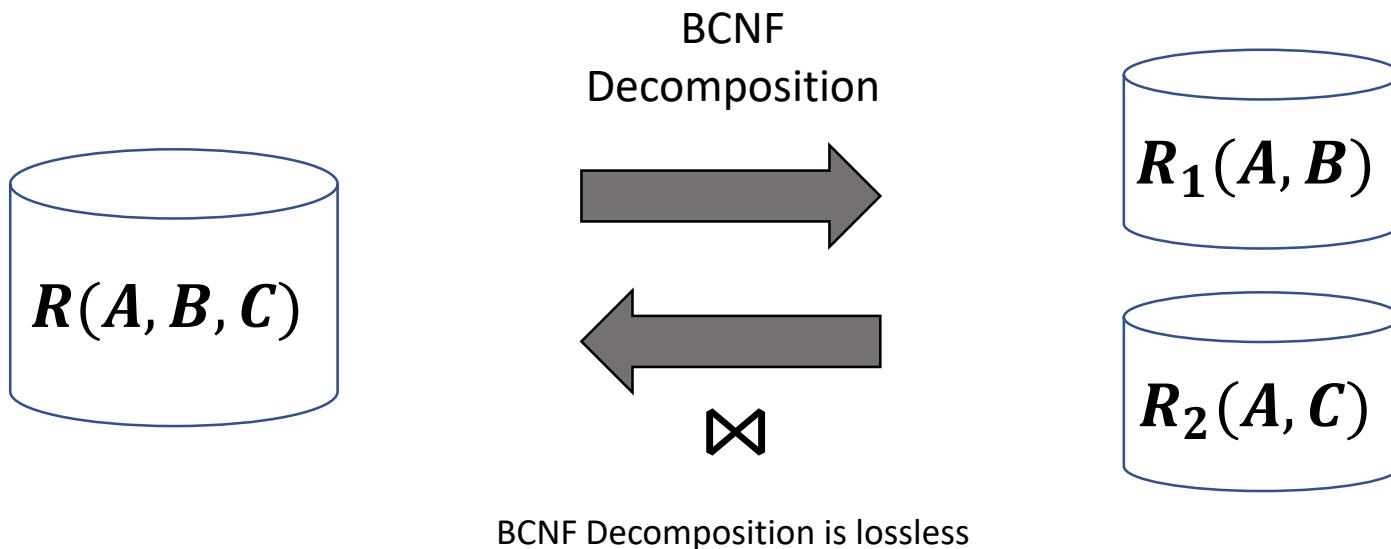
- No! We got extra tuples:

- (Alex, John Bar, Coors)
- (Alex, Purple Bar, Sam Adams)

Example decomposition of the Favorites relation

# BCNF: Lossless Decomposition

- That is, we reconstruct the original relation.
- If we decompose  $R(A, B, C)$  into  $R_1(A, B)$  and  $R_2(A, C)$ , due to  $A \rightarrow B$ , then  $R = R_1 \bowtie R_2$ .



# BCNF Decomposition Is Lossless

- Reconsider the lossy example: (drinker, bar, beer).
- Will BCNF decompose to (drinker, bar), (drinker, beer)?
  - Yes, but only when  $\text{drinker} \rightarrow \text{bar}$  or  $\text{drinker} \rightarrow \text{beer}$ .
- Suppose  $\text{drinker} \rightarrow \text{bar}$ ?
- Suppose  $\text{drinker} \rightarrow \text{beer}$ ?

- Favorites(drinker, bar, beer)

drinker	bar	beer
Alex	John Bar	Sam Adams
Carissa	Green Bar	Bud Light
Alex	Purple Bar	Coors

- Decomposition:  $R_1 =$

drinker	bar
Alex	John Bar
Carissa	Green Bar
Alex	Purple Bar

drinker	beer
Alex	Sam Adams
Carissa	Bud Light
Alex	Coors

- Favorites =  $R_1 \bowtie R_2$ ?

- No! We got extra tuples:

- (Alex, John Bar, Coors)
- (Alex, Purple Bar, Sam Adams)

Example decomposition of the Favorites relation

# Dependency-Preserving Decomposition

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# Learning Objectives

By the end of this video, you will be able to:

- Define dependency preserving, and explain why it is desired.
- Explain why BCNF may not be dependency preserving.

# A Decomposition May Not Preserve Dependency

- A schema  $S$  is dependency preserving if, for every FD  $f$  of it:
  - $f$  can be checked in a table  $T$  in  $S$ , or
  - $f$  can be implied by those FDs that can be checked in single tables.

- Favorites(drinker, bar, beer)
- $f_1$ : beer  $\rightarrow$  bar,  $f_2$ : drinker, bar  $\rightarrow$  beer
- Decomposition:  
 $R_1 = (\text{beer}, \text{bar})$ ,  $R_2 = (\text{beer}, \text{drinker})$
- Preserve dependencies?
  - $f_1$ : beer  $\rightarrow$  bar
  - $f_2$ : drinker, bar  $\rightarrow$  beer

drinker	bar	beer
Alex	John Bar	Sam Adams
Carissa	Green Bar	Bud Light
Alex	Purple Bar	Coors



beer	bar
Sam Adams	John Bar
Bud Light	Green Bar
Coors	Purple Bar

beer	drinker
Sam Adams	Alex
Bud Light	Carissa
Coors	Alex

Example decomposition

# BCNF May Not Preserve Dependency

- Favorites(drinker, bar, beer)
- $f_1$ : beer  $\rightarrow$  bar,  $f_2$ : drinker, bar  $\rightarrow$  beer
- Decomposition:

$$R_1 = (\text{beer}, \text{bar}), R_2 = (\text{beer}, \text{drinker})$$

drinker	bar	beer
Alex	John Bar	Sam Adams
Carissa	Green Bar	Bud Light
Alex	Purple Bar	Coors



beer	bar
Sam Adams	John Bar
Bud Light	Green Bar
Coors	Purple Bar

beer	drinker
Sam Adams	Alex
Bud Light	Carissa
Coors	Alex

Example decomposition

- Yes, this is lossless.
- But it does not preserve dependency!

## Food for Thought

*For a relation  $R$  and FDs  $F$ , a dependency-preserving BCNF may not exist.*

*Agree?*

drinker	bar	beer
Alex	John Bar	Sam Adams
Carissa	Green Bar	Bud Light
Alex	Purple Bar	Coors

Any BCNF that would  
preserve all dependencies?



beer	bar
Sam Adams	John Bar
Bud Light	Green Bar
Coors	Purple Bar

beer	drinker
Sam Adams	Alex
Bud Light	Carissa
Coors	Alex

$f_1: \text{beer} \rightarrow \text{bar}$ ,  
 $f_2: \text{drinker}, \text{bar} \rightarrow \text{beer}$

Example decomposition

# Third Normal Form

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# Learning Objectives

By the end of this video, you will be able to:

- Define third normal form and describe when and why it is desired.
- Determine if a given relation is in 3NF.
- Settle for 3NF-- happily-- if your BCNF decomposition does not give you a dependency-preserving schema.

# Since BCNF May Not Preserve FDs

- What happens if we cannot find a BCNF that preserves FDs?

*Food for Thought*

*For a relation  $R$  and FDs  $F$ , a dependency-preserving BCNF may not exist.  
Agree?*

drinker	bar	beer
Alex	John Bar	Sam Adams
Carissa	Green Bar	Bud Light
Alex	Purple Bar	Coors



beer	bar
Sam Adams	John Bar
Bud Light	Green Bar
Coors	Purple Bar

beer	drinker
Sam Adams	Alex
Bud Light	Carissa
Coors	Alex

Example decomposition

Any BCNF that would preserve all dependencies?

$f_1: \text{beer} \rightarrow \text{bar}$ ,  
 $f_2: \text{drinker}, \text{bar} \rightarrow \text{beer}$

# 3NF: Making a Compromise from BCNF

- A relation R is in 3NF if and only if:

Whenever there is a **nontrivial FD** for  $R$ ,

$$A \rightarrow B$$

then  $A$  is a superkey for  $R$ ,

**or  $B$  is a prime attribute (i.e., a member of a key) for  $R$ .**

# Settling for 3NF

- Favorites(drinker, bar, beer)
- $f_1$ : beer  $\rightarrow$  bar,  $f_2$ : drinker, bar  $\rightarrow$  beer
- It is in 3NF.
  - Key is {drinker, bar}.
  - $f_1$ : beer  $\rightarrow$  bar – Ok, because bar is part of a key.
  - $f_2$ : drinker, bar  $\rightarrow$  beer – Ok, because drinker, bar is a superkey.
- It is not in BCNF, though.
  - Since BCNF would not preserve FD, we will settle for 3NF.

# Why Is 3NF Acceptable?

- 3NF decomposition is both:
  - Lossless decomposition
  - Dependency preserving
- It removes “bad FDs” mostly
  - Except those involving key attributes
  - I.e., will not split a key in two relations
- Favorites(drinker, bar, beer)
- $f_1$ : beer  $\rightarrow$  bar,  $f_2$ : drinker, bar  $\rightarrow$  beer

drinker	bar	beer
Alex	Joe's Bar	Sam Adams
Carissa	Green Bar	Bud Light
Alex	Fancy Bar	Coors
Bob	Green Bar	Bud Light

Example Favorites relation

# Multivalued Dependencies

Designing Schemas

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

Kevin C.C. Chang, Professor  
Computer Science @ Illinois

# Learning Objectives

By the end of this video, you will be able to:

- Define multi-valued dependencies.
- Describe why MVD may cause redundancies.
- Explain why FD is a special case of MVD.

# More Than Functional Dependencies

- There are other kinds of dependencies than FDs.
  - Multivalued dependencies (MVD)
  - Inclusion dependencies (IND)
  - Join dependencies (JD)
- We will take a look at MVD.
  - So you can say you know about (multiple kinds of) dependencies!

# Consider Our Academic World

- We know that id determines birthday,  
 $\text{id} \rightarrow \text{birthday}$ .
- But, doesn't id also determine majors?
  - Donald Duck? Majors = {Bio}.
  - Bugs Bunny? Majors = {CS, Music}.
- So, id determines major – as **a set of** majors instead of a unique value.
- We call this determination a multivalued dependency:  
 $\text{id} \twoheadrightarrow \text{major}$ .

<b>id</b>	<b>name</b>	<b>major</b>	<b>birthday</b>
1	Bugs Bunny	CS	2004-11-06
1	Bugs Bunny	Music	2004-11-06
2	Donald Duck	Bio	1997-02-01
3	Peter Pan	Econ	1998-10-01
3	Peter Pan	Social	1998-10-01
3	Peter Pan	ME	1998-10-01
4	Mickey Mouse	CS	1995-04-01

Example Students relation

# MVD Is “Tuple Generating”

- Students(id, name, major, gpa, hobby, level)
- Suppose id → major, gpa: Do you think this table is “complete”?

<b>id</b>	<b>name</b>	<b>major</b>	<b>gpa</b>	<b>hobby</b>	<b>level</b>
1	Bugs Bunny	CS	3.0	Tennis	Beginner
1	Bugs Bunny	Music	3.5	Tennis	Beginner
1	Bugs Bunny	CS	3.0	Chess	Advanced
2	Donald Duck	Bio	3.2	Basketball	Intermediate
3	Peter Pan	Econ	2.8	Piano	Beginner
3	Peter Pan	Social	3.0	Reading	Advanced
3	Peter Pan	ME	3.6	Swimming	Advanced
...	...	...	...	...	...

Example Students relation

# Multivalued Dependency (MVD)

- Notation:  $A_1, \dots, A_m \twoheadrightarrow B_1, \dots, B_n$
- We say:  $A_1, \dots, A_m$  **multidetermines**  $B_1, \dots, B_n$
- Meaning:
  - If two tuples agree on  $A_1, \dots, A_m$  values, then swapping their  $B_1, \dots, B_n$  values will result in two tuples that are also in the relation.
  - I.e.,  $B$  depends only on  $A$ , and is independent of the remaining attributes.

- Ex:  $\text{id} \twoheadrightarrow \text{major, gpa}$

Example Students relation

<b>id</b>	<b>name</b>	<b>major</b>	<b>gpa</b>	<b>hobby</b>	<b>level</b>
1	Bugs Bunny	CS	3.0	Tennis	Beginner
1	Bugs Bunny	Music	3.5	Tennis	Beginner
1	Bugs Bunny	CS	3.0	Chess	Advanced
1	Bugs Bunny	Music	3.5	Chess	Advanced
2	Donald Duck	Bio	3.2	Basketball	Intermediate
3	Peter Pan	Econ	2.8	Piano	Beginner
3	Peter Pan	Social	3.0	Reading	Advanced
3	Peter Pan	ME	3.6	Swimming	Advanced
...	...	...	...	...	...

# FD: Special Case of MVD

- FD is a special case of MVD.
- $\text{id} \rightarrow \text{birthday} \implies \text{id} \rightarrow \text{birthday}$

<b>id</b>	<b>name</b>	<b>major</b>	<b>birthday</b>
1	Bugs Bunny	CS	2004-11-06
1	Bugs Bunny	Music	2004-11-06
2	Donald Duck	Bio	1997-02-01
3	Peter Pan	Econ	1998-10-01
3	Peter Pan	Social	1998-10-01
3	Peter Pan	ME	1998-10-01
4	Mickey Mouse	CS	1995-04-01

Example Students relation

# FD/MVD Are Domain Knowledge

- What FDs/MVDs hold is your **knowledge** of the domain.
- $\text{id} \rightarrow \text{birthday}$ ?  $\text{id} \twoheadrightarrow \text{birthday}$ ?
- $\text{id} \rightarrow \text{major}$ ?  $\text{id} \twoheadrightarrow \text{major}$ ?
- $\text{age} \rightarrow \text{major}$ ?  $\text{age} \twoheadrightarrow \text{major}$ ?

*How do you suggest to normalize the problematic Students relation to eliminate MVDs?*

*Yes, you probably have (re-)invented 4NF!*

**Fourth normal form**

Fourth normal form is a normal form used in database normalization. Introduced by Ronald Fagin in 1977, 4NF is the next level of normalization after Boyce–Codd normal form. [Wikipedia](#)

**Abbreviation:** 4NF  
**Developed by:** Ronald Fagin  
**Year introduced:** 1977

Google search result of 4NF

You need a magnifier to view it  
(Pixabay, 2017)



<b>id</b>	<b>name</b>	<b>major</b>	<b>gpa</b>	<b>hobby</b>	<b>level</b>
1	Ron Burgundy	Mass	3.5	Tennis	Beginner
1	Ron Burgundy	CJ	3.0	Chess	Advanced
1	Ron Burgundy	Math	3.8	Gaming	Intermediate
2	Donald Duck	Bio	3.2	Basketball	Intermediate
3	Peter Pan	Social	3.0	Reading	Advanced
3	Peter Pan	ME	3.6	Scuba diving	Advanced



Sample answer for this FoT

# References

- Pixabay, 2017. *Image of a magnifier*[Online image]. Retrieved from <https://pixabay.com/en/inspector-man-detective-male-160143/>.