

Signatures in Cryptography

Cryptography: $\kappa\rho\nu\pi\tau\delta\varsigma$ = hidden secret and $\gamma\rho\delta\varphi\epsilon\iota\nu$ = to write

- Cryptography exists decades before cryptocurrencies existed
- First cipher: the scytale
- First steganography app: tattoo on shaved heads
- Second world war: Enigma cipher broke helped allies won the war
- Modern cryptography: ~1970-now

Prime in cryptography

- Encryption protects privacy/confidentiality of information (plaintext)
 - Only the owner of the secret key can decrypt (owners of secret keys are trusted)
 - Symmetric: Encryption and Decryption keys are the same and should be kept secret, e.g. AES, (X)ChaCha2020
 - Assymetric: Encryption key is publicly available to everyone and knowledge thereof does not harm security of the encryption scheme. Secret decryption key should be kept secret (RSA Encryption, Elgamal, Paillier)
 - In practise:
 - Symmetric key cryptography is very fast compared with Assymetric.
 - But how parties in symmetric cryptography exchange their secret key in public channels?
 - Hybrid setup(e.g: TLS): Use Assymetric cryptography to agree on a secret key, use that secret key to encrypt with symmetric cryptography all the entire communication

What does it mean encryption X is secure?

- We have to define our security model:
 - What does the adversary know (attack model)?

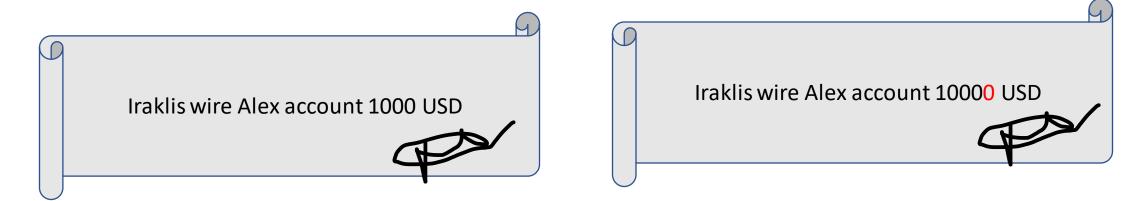
• What the adversary is trying to break in a system, or what we consider as a

successfull attack

Attack model/Attack	Ciphertext only	Known plaintext	Chosen plaintext	Chosen ciphertext
Key extraction	Insecure			
Plaintext decipher				
Some information about plaintext (e.g: 1st bit)				
Distinguish ciphertexts				Secure

Signatures in the physical world

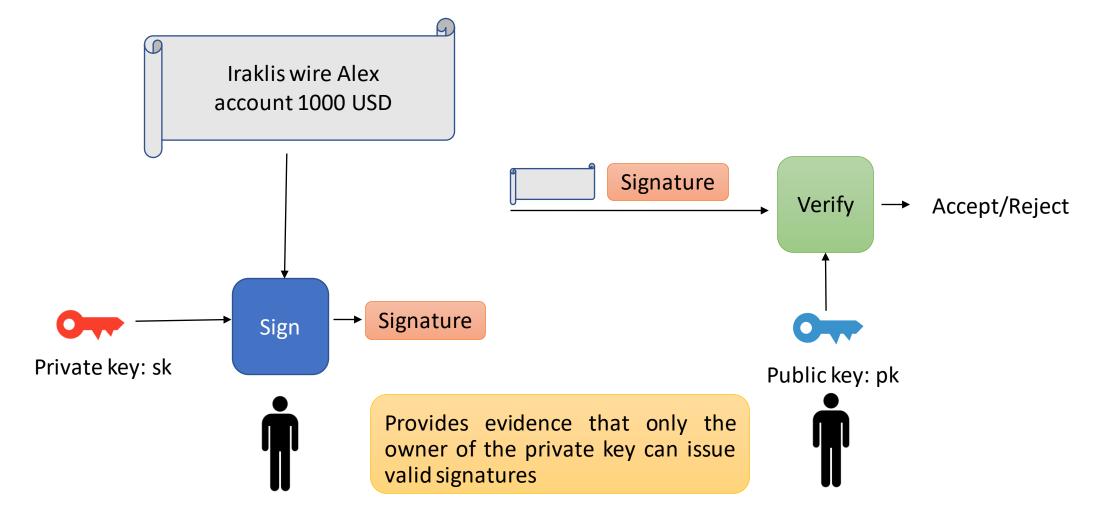
• Problem: How to bind a physical person with a document



Can be easily forged

Digital signatures (no privacy on the message)

How to prove ownership/integrity on a bytestream



Signatures Security

- Adversary cannot tamper/change not even a single bit from the message by having access to the message, its signature and the public key
- Signatures SHOULD NOT be used if confidentiality/privacy of message needs to be protected.
- Signatures are not designed to protect the privacy of the message: It is ok the adversary to know the message in plaintext and still have secure signatures
- Signatures protect integrity of the message: Receiver is getting assurances that the message has not and cannot been changed by intermediate entities without knowledge of the secret signing key after it has been signed by the sender.

Signatures Security (contd.)

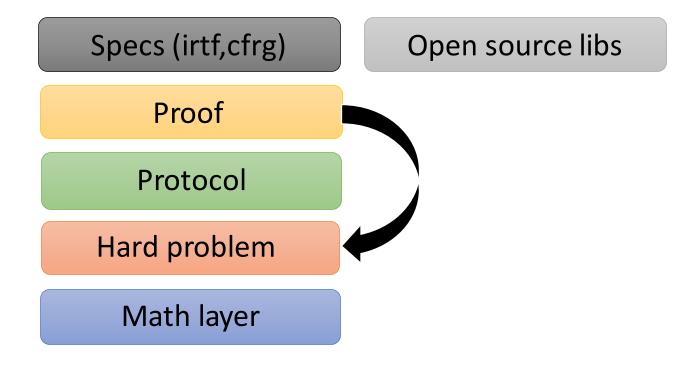
Attack model/Attack	Key only	Known message	Adaptive chosen message
Total break	Insecure		
Universal forgery			
Selective forgery			
Existential forgery			Secure

A Real World example

- Director of engineering: We need to install that library from xyz.com
- Engineer: Ok, let's download it from xyz.com
- Security engineer: How we know that the software is reliably issued from xyz.com?
- Xyz.com: All packages are signed with our private key, for which we have a certificate from PKI X.
- Security engineer: Ok their public key is signed with the PKI X private key, certifying that the claimed pk of xyz.com is owned by them.

CAs have been compromised many times, e.g. Diginotar, Monpass

Cryptographic primitive Layers



Prime in basic number theory

- Groups
- Hard problems
 - DL
- EC in cryptography

Group

- A Group **G** is a finite set of elements with a binary operation op (e.g: +, *) such that:
- [closure] for all a,b in G we have a op b is also in G
- there exists an identity id such that for all x in G we have x op id = x
- for every x there exists an inverse thereof x.inv, such that x op x.inv = id (0 for +, 1 for *)
- [associativity] for all a,b,c in G we have (a+b)+c =a+(b+c)
- [commutativity] for all a,b in G we have a+b =b+a (if this holds, the group is called abelian)

Examples

- R (reals) is not a group with multiplication.
- $\mathbb{R} \setminus \{0\}$ is a group with multiplication.
- **Z** (integers):
 - is a group under addition (identity element: 0),
 - is not a group under multiplication.
- Z_N = {0,...,N-1} (integers modulo N) is a group under addition modulo
 N (identity element: 0)
- If p is a prime then $Z_p^* = \{1,..., p-1\}$ is a group under multiplication modulo p

Hard problems

- Discrete logarithm (in certain groups):
 - Given $g^x = y$ to find x it is believed hard $(x = log_g y)$
 - in Z_n it is easy because $g^x = g \cdot x \mod n$
 - In Z_p^* (where p is prime) it is believed to be hard.

Extremely useful group since all elements can produce all other elements! That uniform structure makes **DL problem** hard in real world cryptographic protocols

Elliptic curves

- An **EC** is the set of points {x,y} \in R that validate the EC equation
- Together the point O of infinity
- (EC,+) is a group:
 - Closed under addition : (x1,y1)+(x2,y2) = (x3,y3) also in **EC**
 - Identity element = **O**
 - Inverse (x,y) = (x,-y)
 - Associativity holds

Weierstras

• $y^2 = x^3 + ax + b$

Mongomery

• By 2 = x^3 +A x^2 +x

Edwards

• $x^2+y^2 = 1 + dx^2y^2$

Elliptic Curve Cryptography

- In some EC there is 1-1 map with a finite field
- Instead of working in R in cryptography we use EC under a field (Zp)
- Same security guarantees with smaller parameters!
- EC(p,c,G,n,N,h) parameters:
 - The **prime p** that specifies the size of the finite field.
 - The **coefficients** c of the elliptic curve equation.
 - The **base point G** that generates EC points
 - The order n of the subgroup
 - The order N of the EC.
 - The cofactor h of the subgroup: h = N/n (extra care)

ECDSA

- Descendant of DSA(FIPS 186-4, ANSi X9.62, IEEE P 1363)
- Not patended (that is why probably has been chosen by bitcoin in 2008)
- Weak security proof
- Implementation can easily go wrong
- PK can be recovered from signature
- First versions malleable





ECDSA Description (informal)

- KGen()->sk,pk
 - sk a random scalar
 - pk =sk *G, where G is the base point
- Sign(sk,m)
 - Pick uniform at random k (integer)
 - Compute R = k * G (ec point)
 - Let R=(rx,ry) (ec point)
 - s = (H(m) + sk *rx)/k (integer)
 - Output (rx,s)
- •Verify(σ,pk,m):
 - •a=H(m)/s, b=r/s (integers)
 - •u = G * a + pk * b (ec point)
 - •u=(u x,u y) (ec point)
 - •rv' 11 v

the Most of from reusing k in

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U=G*a+pk*b=G*H(m)/s +
(sk*G*r)/s= ((H(m)+sk*rx)/s)*G = k*G = R
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Schnorr Signature

- Simpler (linear->multisig optimality)
- Formal proofs from standard assumptions. They were under patent law of Claus P. Schnorr till 2008
- Non-malleable
- Lack of public key recovery

Schnorr Signature(informal, no group notation)

- KGen()->sk,pk
 - sk a random integer
 - pk =sk *G, where G is the base point
- Sign(sk,m)
 - Pick uniform at random k (integer)
 - Compute R = k * G (ec point)
 - Let R=(rx,ry) (ec point)
 - s = k + Hash(rx||pk||m)*sk (integer)
 - Output (rx,s)
- •Verify(σ,pkσ,pk):
 - •If s*G == R + Hash(rx||pk||m)*pk accept, otherwise reject.

EdDSA (Ed25519)

- Dual standarization efforts: NIST-FIPS and IETF
- It avoids side channel attacks
- No need for nonces
- Faster compared with Schnorr and ECDSA
- TLS 1.3, SSH, Tor, GnuPGP, Signal
- Corda, Tezos, Stellar, Libra

EdDSA(informal)

- KGen()->sk,pk
 - sk a random scalar
 - pk =sk *G, where G is the base point
- Sign(sk,m)
 - r = hash(hash(sk) + m) (integer)
 - R = r * G (ec point)
 - h = hash(R + pk + m) (integer)
 - s = (r + h * sk) (integer)
 - $\sigma = (R, s)$
- •Verify(σ,pkσ,pk):
 - h = hash(R + pk + m)
 - P1 = s * G
 - P2 = R + h * pk

Curves in signatures

- ECDSA: secp curves and other in Weierstras form
- Schnorr: ristretto25519, secp curves
- EdDSA: ristretto25519, Edwards: Curve25519

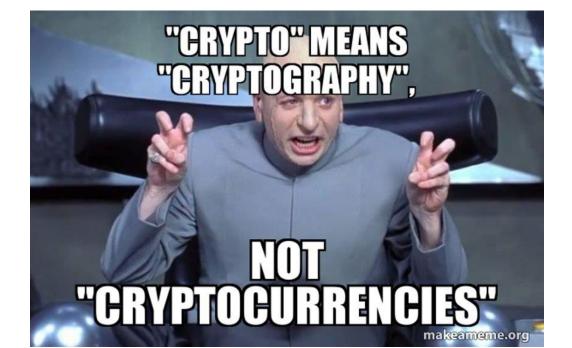
- Edward curves outperform ECDSA common curves:
 - Faster
 - Easier to program (no extra cases in addition law)
 - Safer to avoid timing attacks

Final Takeaways

 Security of the cryptographic scheme relies on the entire system and in not in an isolated "paper" world

• In the end no matter of what cryptographic secure signature is used, the scheme is as secure as the secrecy of the secret signing key in the

system...



ECDSA Challenge

https://hackmd.io/kCZFAk4nRPuTzUibyfBrDw