

# CS 3100, Models of Computation, Spring 20, Lec 10

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[bit.ly/3100s20Syllabus](https://bit.ly/3100s20Syllabus)

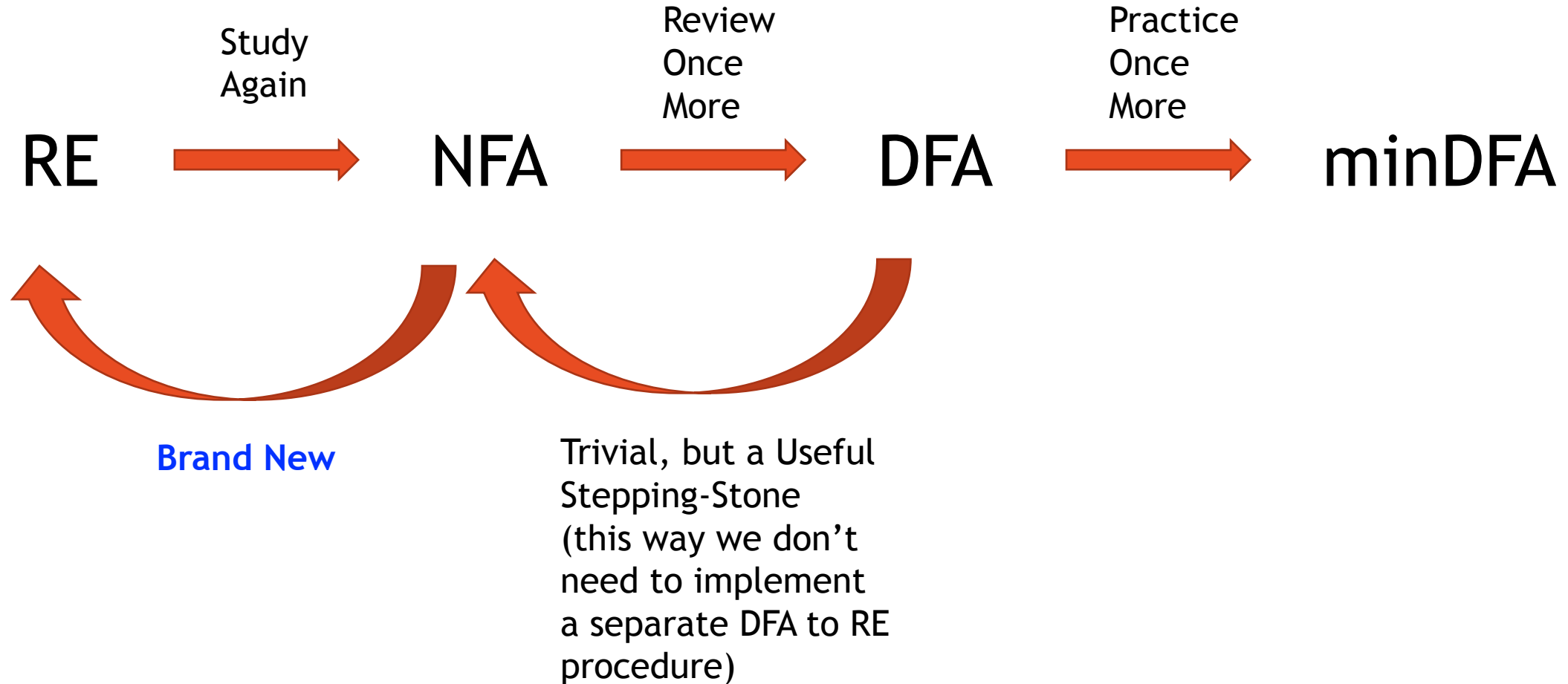


# Lecture 10, covering Ch 1-9 😊



- Walk the Kleene-Pipeline
- The Postage-Stamp Problem

# Walk the Kleene-Pipeline



# The Postage-Stamp Problem

Wolfram MathWorld™  
*Built with Mathematica Technology*

the web's most extensive mathematics resource

Algebra

Applied Mathematics

Calculus and Analysis

Discrete Mathematics

Foundations of Mathematics

Geometry

History and Terminology

Number Theory

Number Theory > Integer Relations >

Number Theory > Diophantine Equations >

Discrete Mathematics > Combinatorics > Partitions >

## Frobenius Postage Stamp Problem

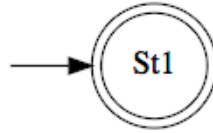
**SEE:**

Coin Problem, McNugget Number, Postage Stamp Problem

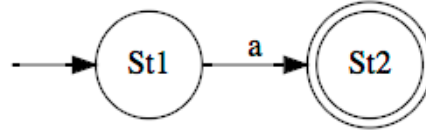
# RE → NFA examples

```
1 dotObj_nfa(re2nfa("'))
```

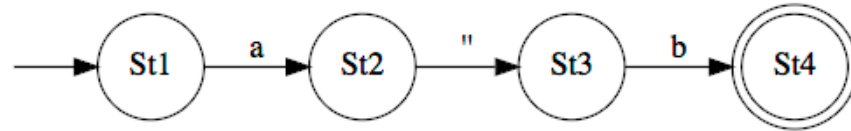
Generating LALR tables



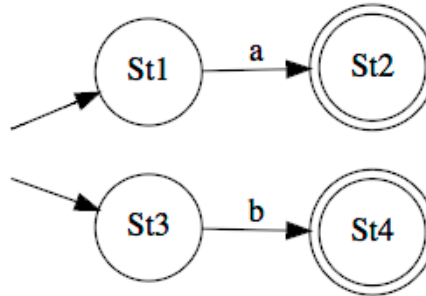
```
1 dotObj_nfa(re2nfa("a"))
```



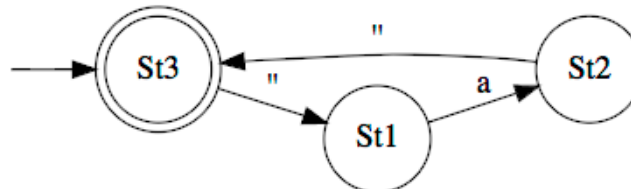
```
1 dotObj_nfa(re2nfa("ab"))
```



```
1 dotObj_nfa(re2nfa("a+b"))
```



```
1 dotObj_nfa(re2nfa("a*"))
```



# What are RE?

- ☐ Epsilon
- ☐  $a$  in Sigma
- ☐ If  $R_1$  and  $R_2$  are RE, then  $R_1 + R_2$  is an RE
- ☐ If  $R_1$  and  $R_2$  are RE, then  $R_1 R_2$  is an RE
- ☐ If  $R$  is an RE, then  $( R )$  is an RE
- ☐ If  $R$  is an RE, then  $R^*$  is an RE
- ☐ Nothing else is an RE

# Cover RE $\rightarrow$ NFA for each case

- ❑ Epsilon

- ❑  $a$  in  $\Sigma$

- ❑ If  $R_1$  and  $R_2$  are RE, then  $R_1 + R_2$  is an RE

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❑  $a$  in  $\Sigma$

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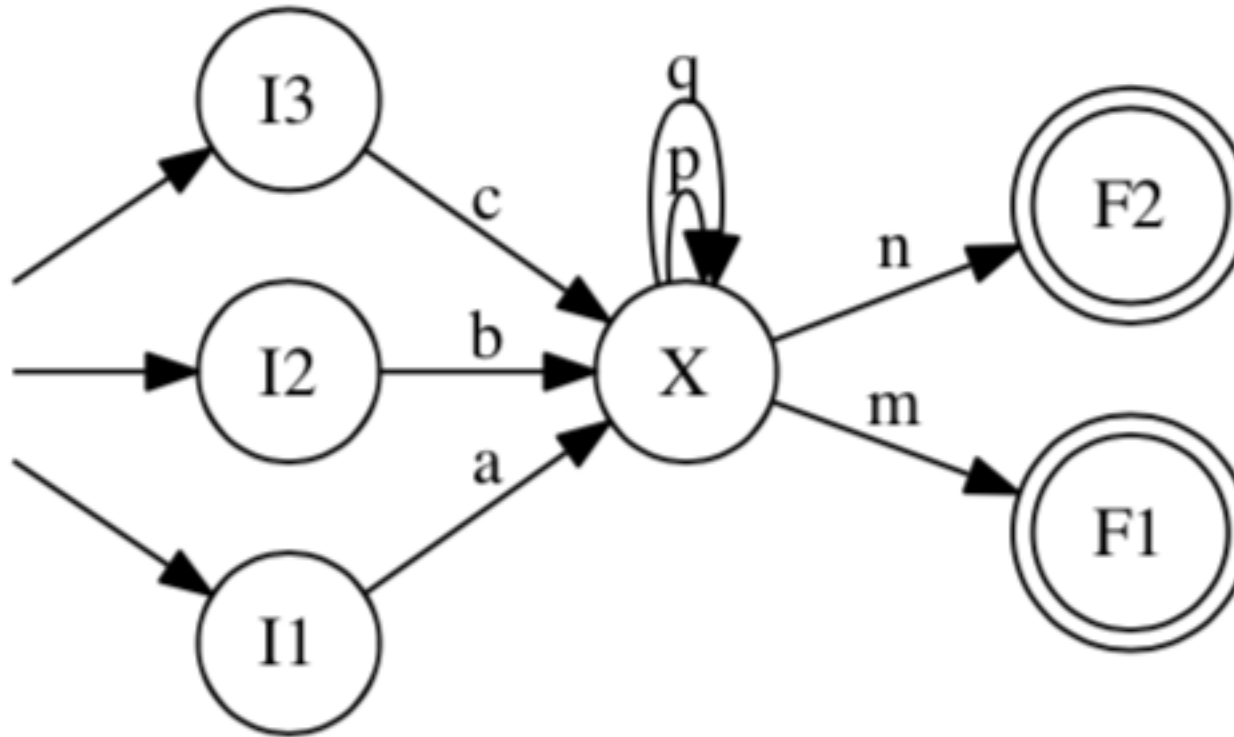
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# NFA to RE conversion

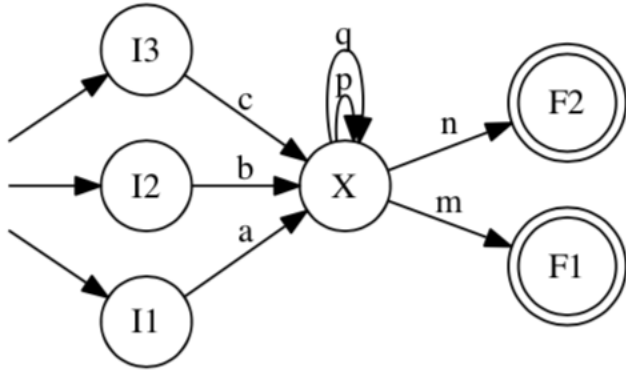


What is the language of this NFA?

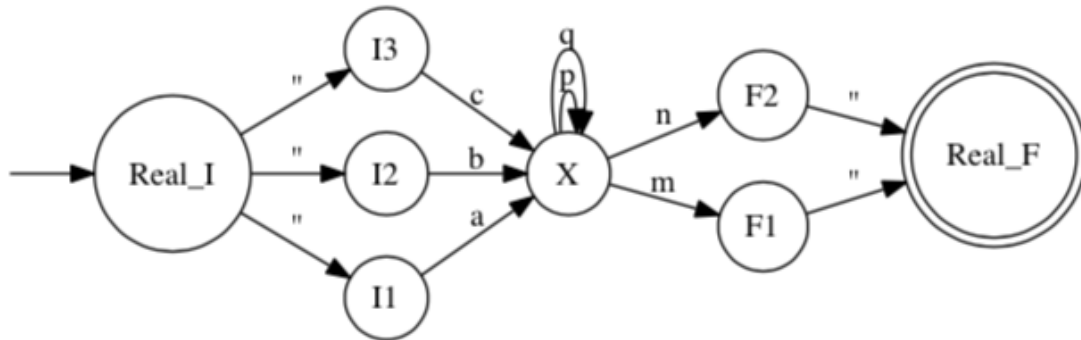
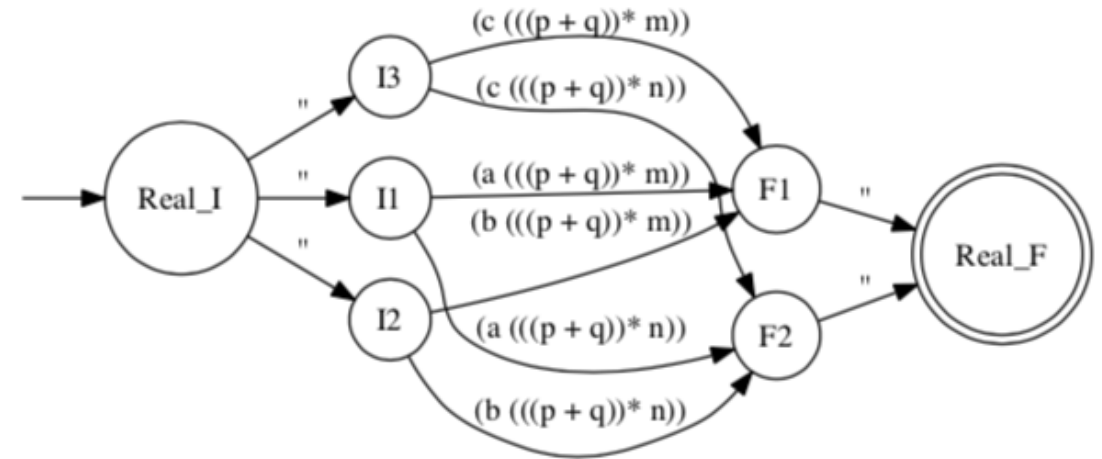
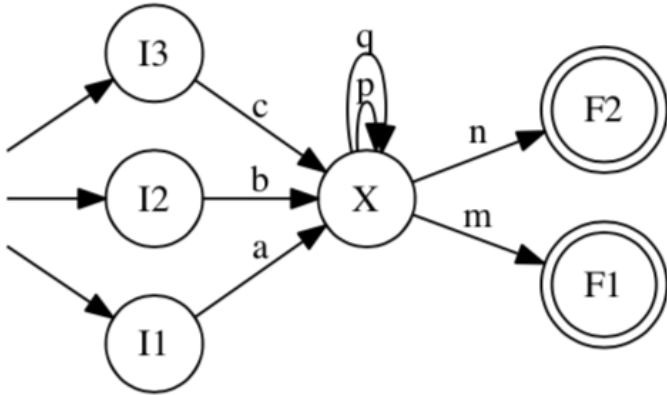
Let's convert this to an RE and check our work against this RE

# NFA to RE conversion

(space for work)

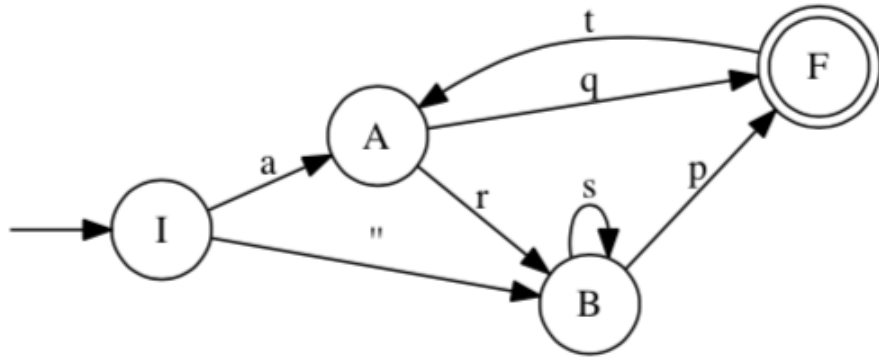


# NFA to RE conversion: Results



# Another example of NFA to RE

(space for work)





# One more example of RE $\rightarrow$ NFA $\rightarrow$ DFA



$ac^* + acd^*$ , simplified as  $a(c^* + cd^*)$

# Postage-stamp problems

- For relatively prime numbers  $a, b$  what is the largest integer not expressible as a linear combination of  $a, b$  ?
  - Let  $a = 2, b = 3$
  - (for larger numbers, let's use a tool)
- How about for more pairwise relatively prime numbers  $a, b, c$
- How about non relatively-prime numbers?

# Review

# Find the strings in the language of these RE

- $(00^*1 + 11^*01)^*$
- $((00^*1)^* + 11^*01)^*$
- $(00^*1 + (11^*01)^*)^*$

```
: 1 iso_dfa(  
2     min_dfa(  
3         nfa2dfa(  
4             re2nfa( " (00*1 + 11*01)* " )),  
5         min_dfa(  
6             nfa2dfa(  
7                 re2nfa( " ( (00*1)* + 11*01)* "  )))  
8     )
```

Generating LALR tables  
Generating LALR tables

```
: True
```

- Find out by developing a min DFA
  - Use `iso_dfa`

# Compare these RE pairwise

- $(00^*1)^*$
- $(0(0+1)^*1)^*$

# Compare these RE pairwise

- $(00^*1 + 11^*01)^*$
- $(0(0+1)^*1 + 11^*01)^*$