$$Y_e\, G = (\lambda f.\ (\lambda x.\ (x\,x))[\lambda y.\ f\ (\lambda v.\ ((y\,y)\,v))])\ G$$

β

$$= \lambda x.\ (x\,x)\,[\lambda y.\ G\ (\lambda v.\ ((y\,y)\,v))]$$

β

$$= [\lambda y.\ G\ (\lambda v.\ ((y\,y)\,v)]$$

β

$$[\lambda y.\ G\ (\lambda v.\ ((y\,y)\,v)] \quad - (*)$$

$$= G\ (\lambda v.\ ((\ Y_e\ G\ )\ v\ )$$

Because note $y\,y$

is $[\lambda y.\ G\ (\lambda v.\ ((y\,y)\,v)]$

$[\lambda y.\ G\ (\lambda v.\ ((y\,y)\,v)]$

shown to be $Y_e\, G$ on Step $(*)$

$$= G\ (\ (Y_e\ G\ )\ )$$