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DUE 11-5-2023
CSCE625

Problem 1:
Step 1: first order logic
1. $\forall x\ (Crow(x) \land \neg White(x) \rightarrow \exists y\ (Jewel(y) \land Loves(x, y)))$
2. $\forall x\ (Talk(x) \rightarrow Intel(x))$
3. $\forall x\ (Intel(x) \rightarrow \forall y\ (Book(y) \rightarrow Loves(x, y)))$
4. $\neg \exists x\ (\forall y\ (Book(y) \rightarrow Loves(x, y)) \land \exists z\ (Jewel(z) \land Loves(x, z)))$
5. $Crow(Jack) \land Talk(Jack)$

Step 2: convert to canonical form:

For 1: $\forall x\ (Crow(x) \land \neg White(x) \rightarrow \exists y\ (Jewel(y) \land Loves(x, y)))$
Using Skolemization for the existential quantifier: f(x) is a Skolem function,gives a jewel that x loves.
the formula becomes:
$\forall x\ (Crow(x) \land \neg White(x) \rightarrow (Jewel(f(x)) \land Loves(x, f(x))))$
To get the CNF, using the formula $\neg P \lor Q$. So, the formula becomes:
$\forall x\ (\neg Crow(x) \lor White(x) \lor (Jewel(f(x)) \land Loves(x, f(x))))$

This gives us two clauses:
a) $\forall x \neg Crow(x) \lor White(x) \lor Jewel(f(x))$
b) $\forall x \neg Crow(x) \lor White(x) \lor Loves(x, f(x))$

For 2: $\forall x\ (Talk(x) \rightarrow Intel(x))$
the formula becomes: $\forall x\ (\neg Talk(x) \lor Intel(x))$

For 3: $\forall x\ (Intel(x) \rightarrow \forall y\ (Book(y) \rightarrow Loves(x, y)))$
the formula becomes: $\forall x\ \forall y\ (\neg Intel(x) \lor \neg Book(y) \lor Loves(x, y))$

For 4: $\neg \exists x\ (\forall y\ (Book(y) \rightarrow Loves(x, y)) \land \exists z\ (Jewel(z) \land Loves(x, z)))$
Do the negation, and the formula becomes:
$\forall x(\exists y(Book(y) \land \neg Loves(x,y)) \lor \neg \exists z(Jewel(z) \land Loves(x,z)))$
After negating:
$\forall x(\exists y(Book(y) \land \neg Loves(x,y)) \lor \forall z(\neg Jewel(z) \lor \neg Loves(x,z)))$
And then do the skolemization for the existential quantifier y, g(x) is a skolem function, specific book that x does not love.
$\forall x((Book(g(x)) \land \neg Loves(x,g(x))) \lor \forall z(\neg Jewel(z) \lor \neg Loves(x,z)))$

For 5, $Crow(Jack) \land Talk(Jack)$
This gives us two clauses:
a)Crow(Jack)
b)Talk(Jack)

Step 3: Proof by resolution:
List of clauses:
For 1: two clauses:
a)$\neg Crow(x) \lor White(x) \lor Jewel(f(x))$
b)$\neg Crow(x) \lor White(x) \lor Loves(x, f(x))$

For 2: $(\neg Talk(x) \lor Intel(x))$

For 3: $(\neg Intel(x) \lor \neg Book(y) \lor Loves(x, y))$

For 4: $((Book(g(x)) \land \neg Loves(x,g(x))) \lor (\neg Jewel(z) \lor \neg Loves(x,z)))$

For 5, two clauses:
a) Crow(Jack)
b) Talk(Jack)

Conclusion: $\exists x(Crow(x) \land White(x))$
Negated conclusion: $\neg \exists x(Crow(x) \land White(x))$
For 6: After negating: $\forall x(\neg Crow(x) \lor \neg White(x))$

Resolution step:
For 7: From 5b and 2: Intel(Jack)
For 8: From 5a and 6: $\neg$White(Jack)
For 9: From 1a and 5a: $White(Jack) \lor Loves(Jack, f(Jack))$
For 10: From 8 and 9: Loves(x, f(x))
For 11: From 7 and 3: $\neg Book(y) \lor Loves(Jack, y)$ —> false, conflict with premise 4
Here is the premise 4: $((Book(g(x)) \land \neg Loves(x,g(x))) \lor (\neg Jewel(z) \lor \neg Loves(x,z)))$
Take the first part: $Book(g(x)) \land \neg Loves(x,g(x))$

This directly contradicts the derived statement that Jack loves all books, and hence we've reached the required contradiction which shows that our initial negation of the conclusion was false which there must be at least one white crow. Therefore, the proof by resolution shows that there exists a white crow is a logical consequence of the given premises.