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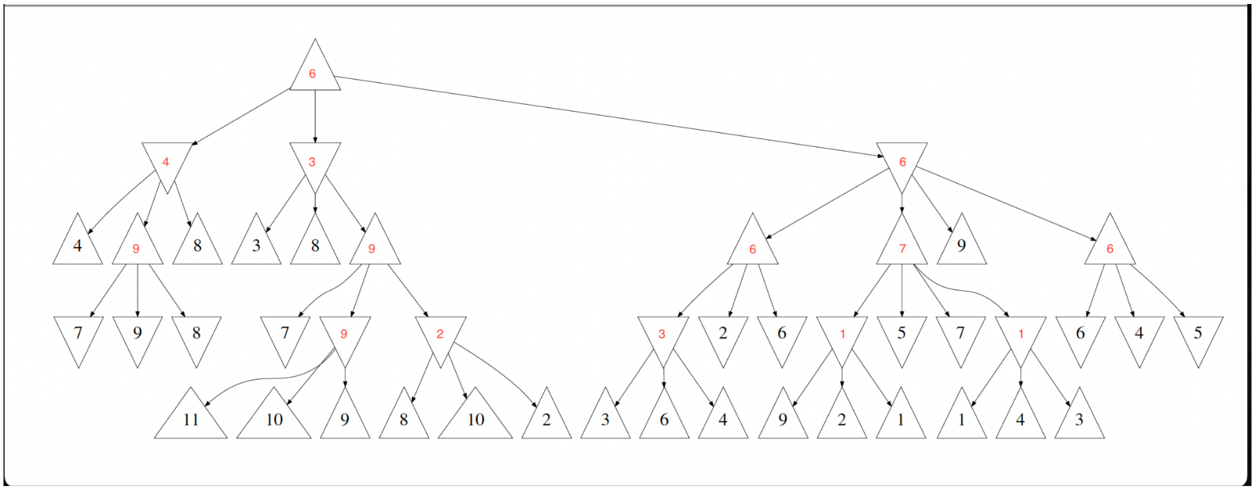
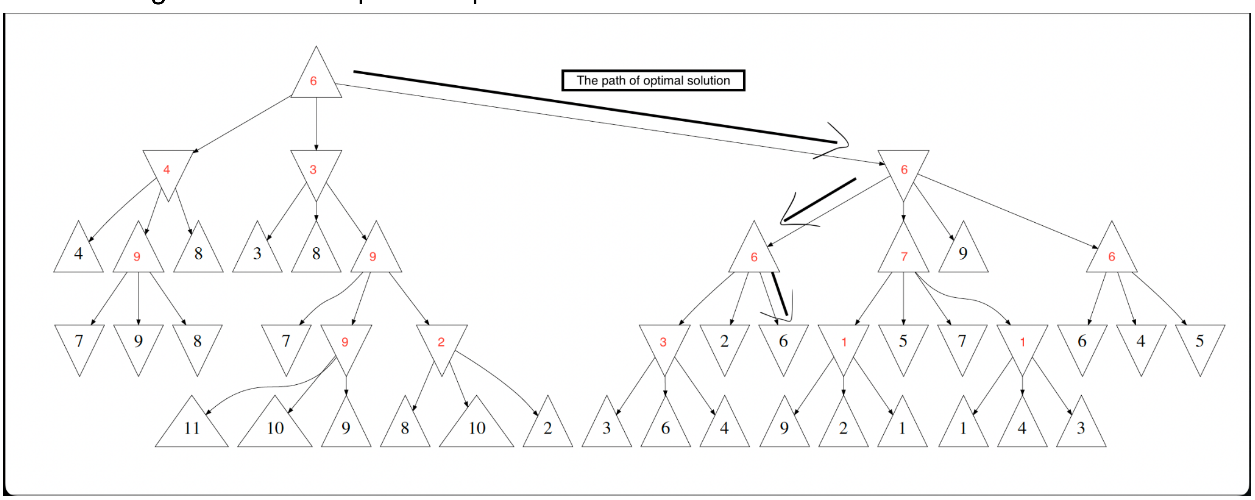
CSCE625

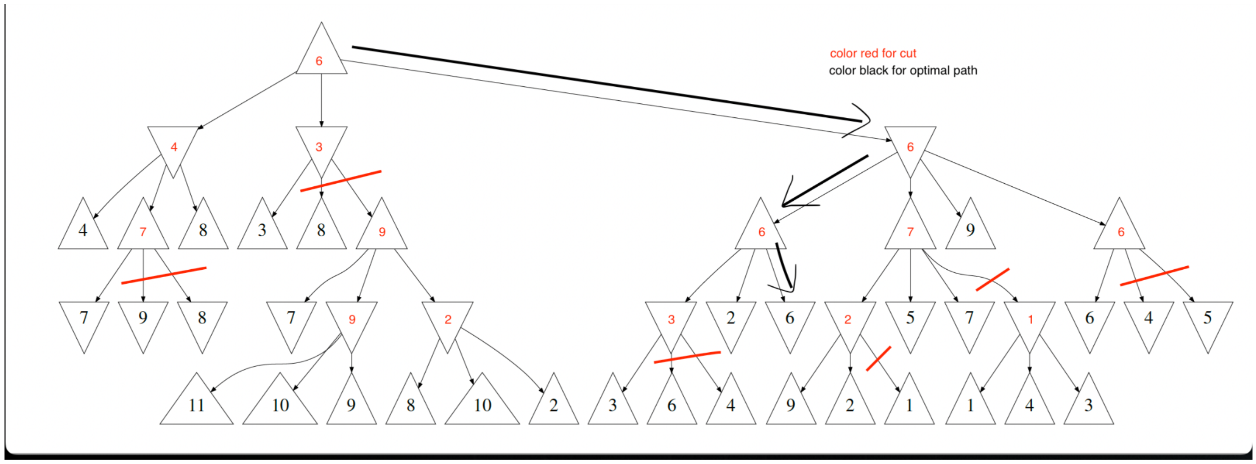
Problem 1
Minmax Search
This figure shows all internal node values.



Below the figure shows the path of optimal solution.



Problem 2)
alpha-beta pruning
α = 6
β = 6



Problem 3)

Minimax gives the optimal solution. First, it generates the whole tree, and apply util function the leaves. Second, go back upward assigning utility value to each node. It can assgin min() function for MIN node, and max() function for MAX node. Therefore, the opponent will act optimally which gives us optimal solutions.

Problem 5)

P	Q	R	$(P \vee Q)$	$(\neg Q \vee R)$	$(P \vee Q) \wedge (\neg Q \vee R)$	$(P \vee R)$	$((P \vee Q) \wedge (\neg Q \vee R)) \rightarrow (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	T
F	F	T	F	T	F	T	T
F	F	F	F	T	F	F	T

Problem 6)

1. Convert P ⋁ ¬(¬Q ⋁ R) into conjunctive normal form.
   = P ⋁ (Q ⋀ ¬R)
   = (P ⋁ Q) ⋀ (P ⋁ ¬R)

2. Convert P ⋁ (R ⋀ ¬S) ⋁ (R ⋀ Q) into conjunctive normal form.

2) $A \lor (B \land C) \equiv (A \lor B) \land (A \lor C)$

$P \lor (R \land \neg S) \lor (R \land Q)$

$\cancel{P \lor ((R \land \neg S) \lor (R \land Q))}$

$P \lor (R \land \neg S) \lor (R \land Q)$

$= (P \lor (R \land \neg S)) \lor (R \land Q)$

$= ((P \lor R) \land (P \lor \neg S)) \lor (R \land Q)$

$= ((P \lor R) \lor (R \land Q)) \land ((P \lor \neg S) \lor (R \land Q))$

$= ((P \lor R \lor R) \land (P \lor R \lor Q)) \land ((P \lor \neg S) \lor (R \land Q))$

$= (P \lor R) \land (P \lor R \lor Q) \land ((P \lor \neg S \lor R) \land (P \lor \neg S \lor Q))$

$= (P \lor R) \land (P \lor R \lor Q) \land (P \lor \neg S \lor R) \land (P \lor \neg S \lor Q)$

3. Convert ¬((P ∧ Q) ∧ (R → S)) into disjunctive normal form.

③ $\neg((P \land Q) \land (R \to S))$

$= \neg((P \land Q) \land (\neg R \lor S))$ $\quad A \to B \equiv \neg A \lor B$

$= \neg(P \land Q) \lor \neg(\neg R \lor S)$

$= (\neg P \lor \neg Q) \lor (R \land \neg S)$

$= ((\neg P \lor \neg Q) \lor R) \land ((\neg P \lor \neg Q) \lor \neg S)$

$= (\neg P \lor \neg Q \lor R) \land (\neg P \lor \neg Q \lor \neg S)$

Problem 7)
C1: S ∨ P
C2: ¬Q ∨ R ∨ S
C3: ¬R ∨ T ∨ ¬P
C4: ¬S
C5: ¬T
C6: Q

Problem 8)
Resolution 1: C1: S ∨ P and C4: ¬S which will get C7: P
C1: S ∨ P
C2: ¬Q ∨ R ∨ S
C3: ¬R ∨ T ∨ ¬P
C4: ¬S
C5: ¬T
C6: Q
C7: P

Resolution 2: C3: ¬R ∨ T ∨ ¬P and C7: P which will get C8: ¬R ∨ T
C1: S ∨ P
C2: ¬Q ∨ R ∨ S
C3: ¬R ∨ T ∨ ¬P
C4: ¬S
C5: ¬T
C6: Q
C7: P
C8: ¬R ∨ T

Resolution 3: C8: ¬R ∨ T and C5: ¬T which will get C9: ¬R
C1: S ∨ P
C2: ¬Q ∨ R ∨ S
C3: ¬R ∨ T ∨ ¬P
C4: ¬S
C5: ¬T
C6: Q
C7: P
C8: ¬R ∨ T
C9: ¬R

Resolution 4: C2: ¬Q ∨ R ∨ S and C9: ¬R which will get C10: ¬Q ∨ S
C1: S ∨ P
C2: ¬Q ∨ R ∨ S
C3: ¬R ∨ T ∨ ¬P
C4: ¬S
C5: ¬T
C6: Q
C7: P
C8: ¬R ∨ T
C9: ¬R
C10: ¬Q ∨ S

Resolution 5: C10: ¬Q ∨ S and C4: ¬S which will get C11: ¬Q

C1: $S \vee P$
C2: $\neg Q \vee R \vee S$
C3: $\neg R \vee T \vee \neg P$
C4: $\neg S$
C5: $\neg T$
C6: Q
C7: P
C8: $\neg R \vee T$
C9: $\neg R$
C10: $\neg Q \vee S$
C11: $\neg Q$

Resoluion 6: C6: Q and C6: Q will get the empty clause -> False

Problem 9)

1. $\forall x \neg(\exists y\, P(x, y) \wedge \neg Q(x))$
   $= \forall x \forall y(\neg P(x,y) \vee Q(x))$

2. $\neg \exists x\, (P(x) \rightarrow \neg(\exists y\, Q(x, y)))$
   $= \forall x\, (P(x) \rightarrow \neg(\exists y\, Q(x, y)))$
   $= \forall x\, (\neg P(x) \vee \neg(\exists y\, Q(x, y)))$
   $= \forall x\, (\neg P(x) \vee \forall y\, \neg Q(x, y)))$

3. $\neg \exists x\, (\neg(\forall y\, Q(x, y)) \vee \neg P(x))$
   $= \forall x\, (\exists y\, \neg Q(x, y) \vee \neg P(x))$

Problem 10)

1) $\exists x P(x) = P(a)$, a is constant

2) $\forall x\ \exists y\, P(x, y) = \forall x\, P(x, f(x))$

3) $\exists x\ \exists y\ \forall z\, P(x, y) \wedge Q(y, z) = \forall z\, P(a, b) \wedge Q(b, z)$, a and b are constant

4) $\forall x\ \exists y\ \exists z\, P(x, y, z) \wedge Q(y, z) = \forall z\, P(x, f(x), g(x)) \wedge Q(f(x), g(x))$

5) $\forall x\ \forall y\ \exists z\, P(x, y) \wedge Q(x, y, z) = \forall x\ \forall y\, P(x, y) \wedge Q(x, y, f(x,y))$

Problem 11)

$\forall x\ [\neg(\exists z(\neg Q(z) \land R(x, z))) \rightarrow \exists y P(x, y)]$

Prenex:

$\forall x\ [\neg(\exists z(\neg Q(z) \land R(x, z))) \rightarrow \exists y\ P(x, y)]$

$= \forall x\ [(\exists z(\neg Q(z) \land R(x, z))) \lor \exists y\ P(x, y)]$

$= \forall x\ \exists z\ \exists y\ (\neg Q(z) \land R(x, z)) \lor P(x, y)$

Skolemization:

$\forall x\ (\neg Q(f(x)) \land R(x, f(x))) \lor P(x, g(x))$

CNF:

$\forall x[(\neg Q(f(x)) \lor P(x,g(x))) \land (R(x,f(x)) \lor P(x,g(x)))]$

Problem 12)

1. Apply {x/f(A)} to $P(x, y) \lor Q(x)$.

   $P(f(A), y) \lor Q(f(A))$

2. Apply {x/A, y/f(z)} to $P(x, y) \lor Q(x)$.

   $P(A, f(z)) \lor Q(A)$

3. Apply {y/x} to $P(x, y) \lor Q(x)$.

   $P(x, x) \lor Q(x)$

Problem 13)

1. P(x, f(x)) and P(A, f(B))
   Unifer: {x/A, B/x}
   Unifer expression: P(A, f(A))

2. P(x,A) and P(y, y)
   Unifer: {x/A, y/A}
   Unifer expression: P(A,A)

3. P(x, f(x), y) and P(A, f(g(w)), z)
   Unifer: {x/A, y/z,x/g(w)} which is not valid
   Reason: there are two x which assign for different terms.

4. P(A, f(y), y, y) and P(x, f(g(x)), g(A),w)
   Unifer: {x/A, y/g(A), w/y}
   Unifer expression: P(A,f(g(A)),g(A),g(A)}