

Homework 2

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Problem 1: Information Gain

1. Calculate the conditional entropy for both X_1 and X_2 .

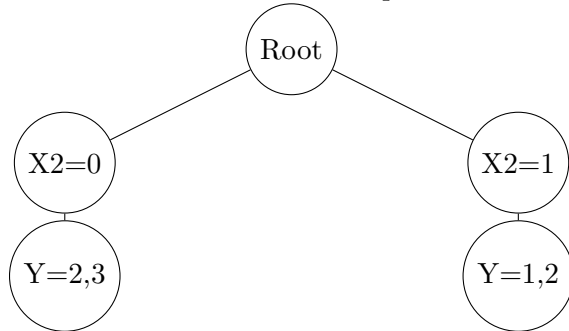
$$H(Y | X_1) = - \left[\frac{2}{6} \left(\frac{0}{2} \log \left(\frac{0}{2} \right) + \frac{1}{2} \log \left(\frac{1}{2} \right) + \frac{1}{2} \log \left(\frac{1}{2} \right) \right) + \frac{4}{6} \left(\frac{2}{4} \log \left(\frac{2}{4} \right) + \frac{1}{2} \log \left(\frac{1}{2} \right) + \frac{1}{2} \log \left(\frac{1}{2} \right) \right) \right] = - \left[-\frac{2}{6} - 1 \right] = \frac{4}{3}$$

$$H(Y | X_2) = - \left[\frac{3}{6} \left(\frac{2}{3} \log \left(\frac{2}{3} \right) + \frac{1}{3} \log \left(\frac{1}{3} \right) + \frac{0}{3} \log \left(\frac{0}{3} \right) \right) + \frac{3}{6} \left(\frac{0}{3} \log \left(\frac{3}{3} \right) + \frac{1}{3} \log \left(\frac{1}{3} \right) + \frac{2}{3} \log \left(\frac{2}{3} \right) \right) \right] = 0.918$$

2. Calculate the information gain if we split based on X_1 or X_2 .

$$\begin{aligned} H(Y) &= 3 * \frac{1}{3} \log \left(\frac{1}{3} \right) = 1.585 \\ IG(X_1) &= 1.585 - 1.333 = 0.252 \\ IG(X_2) &= 1.585 - 0.918 = 0.667 \end{aligned}$$

3. Report which attribute is used for the first split. Draw the decision tree using this split.
 X_2 will be used for the first split because it has higher information gain than X_1



4. $X_1 = 0$ and $X_2 = 1$ which will be classified as $Y=1$ on this tree since $X_2=1$.