Swimmer Environment - RLGlue Implementation

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1 State

In the current implementation of the environment, the state is stored in a double array u of size $2n + 4$, where n is the number of segments.

- A_0, position of the swimmer head, is stored in u_0, u_1.
- $\dot{A}_0$, velocity of the swimmer head, is stored in u_2, u_3.
- θ_i, the angle i at the beginning of the segment i, is stored in u_{3+i}, for $i \in \{1, ..., n\}$.
- $\dot{\theta}_i$, the first derivative of the angle i at the beginning of the segment i, is stored in u_{3+n+i}, for $i \in \{1, ..., n\}$.

2 Solving dynamics equations

2.1 Variables

- n is the number of segments.
- l_i is the length of segment i.
- θ_i is the angle i at the beginning of the segment i, for $i \in \{1, ..., n\}$. See the figure in Remy's paper, Annex B.4 [1].
- G_i is the mass center of the segment i, for $i \in \{1, ..., n\}$.

2.2 Unknowns

The goal is to compute the following unknowns:

- n unknowns of dimension 1: $\ddot{\theta}_1, \ddot{\theta}_2, ..., \ddot{\theta}_n \in \mathbb{R}$
- $n + 1$ unknowns of dimension 2: $\vec{f_0}, \vec{f_1}, ..., \vec{f_n} \in \mathbb{R}^2$
- n unknowns of dimension 2: $\ddot{\vec{G}}_1, \ddot{\vec{G}}_2, ..., \ddot{\vec{G}}_n \in \mathbb{R}^2$

Let $X \in \mathbb{R}^{5n+2}$ be the unknown vector encoding all the unknown variables. We index the components of this vector from 0 to $5n+1$:

- $\ddot{\theta}_i$ corresponds to $X[i-1]$ for $i \in \{1, ..., n\}$.
- $\vec{f_i} = \begin{pmatrix} f_i^x \\ f_i^y \end{pmatrix}$ corresponds to $\begin{pmatrix} X[n+2i] \\ X[n+2i+1] \end{pmatrix}$ for $i \in \{0, ..., n\}$.
- $\ddot{G}_i$ corresponds to $\begin{pmatrix} X[3n+2i] \\ X[3n+2i+1] \end{pmatrix}$ for $i \in \{1, ..., n\}$.

2.3 Equations

Since $X \in \mathbb{R}^{5n+2}$, we need to write $5n+2$ equations. From Remy's model [1] in Annex B.4.3, we have the following equations.

- Equations 0 to $n-1$, for $i \in \{1, ..., n\}$:

$$m_i \frac{l_i}{12} \ddot{\theta}_i - \frac{l_i}{2} \left(\cos\theta_i (f_i^y + f_{i+1}^y) - \sin\theta_i (f_i^x + f_{i+1}^x) \right) = \mathcal{M}_i - u_i + u_{i-1} \quad (1)$$

- Equations n to $3n+3$, for $i \in \{1, ..., n\}$:

$$\vec{f_0} = 0 \quad (2)$$

$$\vec{f_{i-1}} - \vec{f_i} + m_i \ddot{G}_i = \vec{F_i} \quad (3)$$

$$\vec{f_n} = 0 \quad (4)$$

- Equations $3n+4$ to $5n+1$, for $i \in \{1, ..., n-1\}$:

$$-\ddot{G}_{i+1} + \ddot{G}_i + \frac{l_i}{2} \ddot{\theta}_i \begin{pmatrix} -\sin\theta_i \\ \cos\theta_i \end{pmatrix} + \frac{l_{i+1}}{2} \ddot{\theta}_{i+1} \begin{pmatrix} -\sin\theta_{i+1} \\ \cos\theta_{i+1} \end{pmatrix} = \frac{l_i}{2} \dot{\theta}_i^2 \begin{pmatrix} \cos\theta_i \\ \sin\theta_i \end{pmatrix} + \frac{l_{i+1}}{2} \dot{\theta}_{i+1}^2 \begin{pmatrix} \cos\theta_{i+1} \\ \sin\theta_{i+1} \end{pmatrix} \quad (5)$$

  which is the second derivative of the following equations:

$$G_{i+1} = G_i + \frac{l_i}{2} \begin{pmatrix} \cos\theta_i \\ \sin\theta_i \end{pmatrix} + \frac{l_i}{2} \begin{pmatrix} \cos\theta_{i+1} \\ \sin\theta_{i+1} \end{pmatrix} \quad (6)$$

Using the same indices of the unknowns as X, left members of the previous equations are stored in $A \in \mathbb{R}^{(5n+2)\times(5n+2)}$, right members of the previous equations are stored in $B \in \mathbb{R}^{5n+2}$.

Thus, we have the relation $AX = B$, and solving the linear system is done by using $X = A^{-1}B$.

References

[1] Rémi Coulom. *Reinforcement Learning Using Neural Networks, with Applications to Motor Control*. PhD thesis, Institut National Polytechnique de Grenoble, 2002.