Noise Analysis
Optomechanical Cavity

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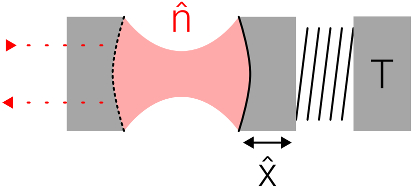
Modeling Quantum Hardware: open dynamics and control
Universität Konstanz

Problem Statement



"Cavity optomechanics", Aspelmeyer et al. 2014

Quantum Optomechanics, Bowen et al. 2015

Hamiltonian

Optical Cavity $\hat{a}$, $\omega_o(\hat{x}_{\text{mech}}) = \omega_o + \frac{g}{\omega_o}\hat{x}_{\text{mech}}$; mechanical oscillations $\hat{b}, \omega_m$; coupling g; Drive E, ω_L

$$H = \underbrace{\omega_o \; a^\dagger a}_{\text{Cavity}} + \underbrace{\omega_m \; b^\dagger b}_{\text{Mechanical}} - \underbrace{g \; a^\dagger a \, (b + b^\dagger)}_{\text{Interaction}} + \underbrace{E(a e^{i\omega_L t} + a^\dagger e^{-i\omega_L t})}_{\text{Drive}}$$

$\hbar = 1$

Quantum Optomechanics, Bowen et al. 2015 (2.3)

QuantumOptics.jl, (Optomechanical Cavity)

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Rotating Wave Approximation at ω_L with $\Delta = \omega_o - \omega_L$, $a \to ae^{i\omega_L t}$:

$$H_{\text{RWA}} = \Delta\ a^\dagger a + \omega_m\ b^\dagger b - g\ a^\dagger a\,(b^\dagger + b) + E(a + a^\dagger)$$

$\hbar = 1$

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QuantumOptics.jl, (Optomechanical Cavity)

Hamiltonian Linearization

$$H_{\text{RWA}} = \Delta\ a^\dagger a + \omega_m\ b^\dagger b - g\ a^\dagger a\,(b^\dagger + b) + E(a + a^\dagger)$$

Linearize $a = \alpha + \delta a$, $b = \beta + \delta b$; with α, β steady state.

$$\begin{aligned} H_{\text{Interaction}} &= -g\ a^\dagger a\,(b^\dagger + b) \\ &\approx -\underbrace{g|\alpha|}_{G}\left(\delta a + \delta a^\dagger + \mathcal{O}(a^2 + \delta a \delta a^\dagger)\right)\left(\delta b + \delta b^\dagger + 2\beta\right) \end{aligned}$$

Quantum Optomechanics, Bowen et al. 2015 (2.7)

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Therefore for small G:

$$H \approx \Delta\ \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - G(\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger)$$

Quantum Optomechanics, Bowen et al. 2015 (2.7)

Dissipation

Optical decay κ:

$$L_O = \sqrt{\kappa}\ \delta a$$

Mechanical resonator with γ and a thermal bath at the n-th thermal state:

$$L_M = \sqrt{\gamma(n+1)}\ \delta b + \sqrt{\gamma n}\ \delta b^{\dagger}$$

Quantum Optomechanics, Bowen et al. 2015 (2.8)

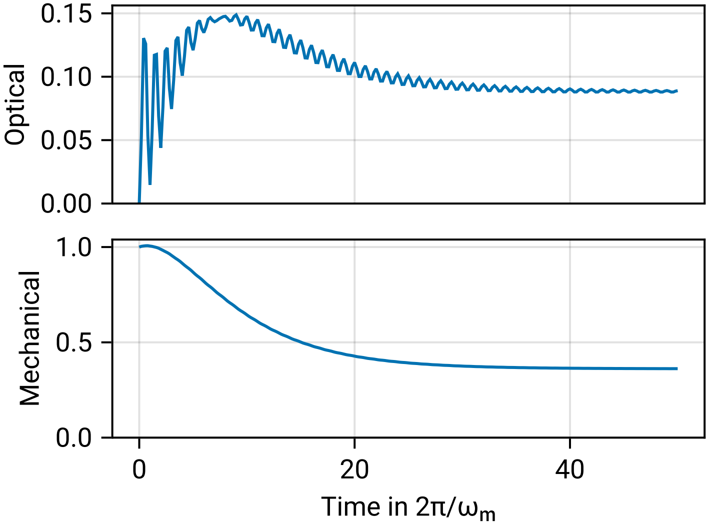
Implementation

1. truncated Fock Basis: $F_{\text{optical}} \otimes F_{\text{mechanical}}$
2. definition of H, J with $\delta a \otimes 1$
3. Time Evolution using the *Lindblad equation*:

$$\dot{\rho} = -i[H, \rho] + J\rho J^{\dagger} - \frac{1}{2}\{J^{\dagger}J, \rho\}$$

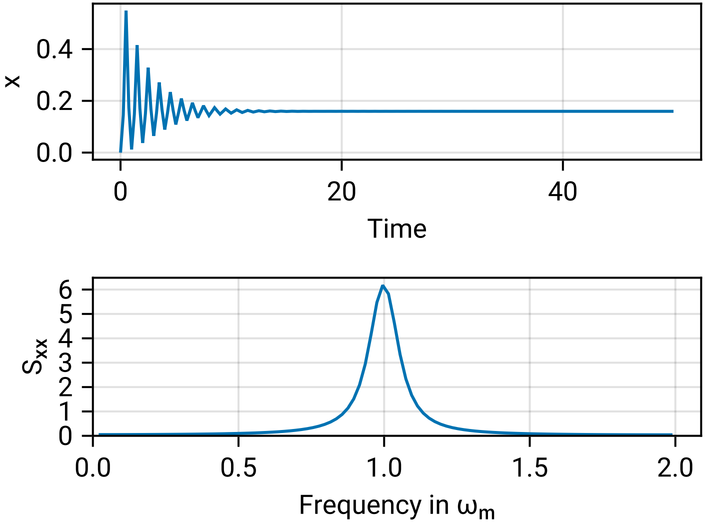
QuantumOptics.jl,

Time evolution of n



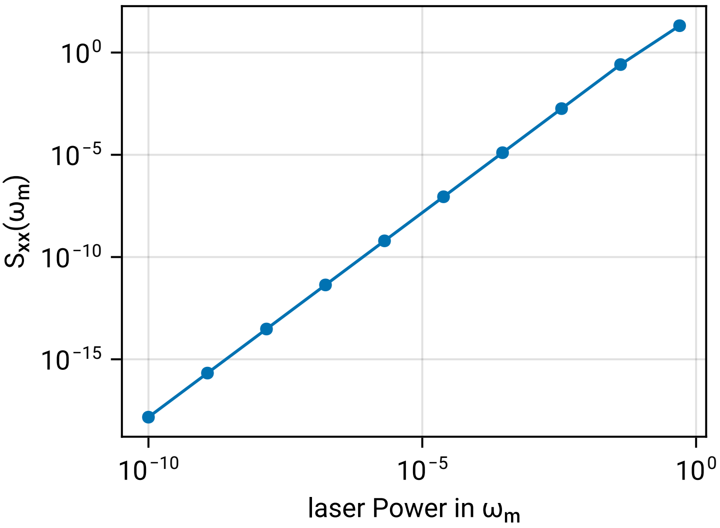
QuantumOptics.jl, (Optomechanical Cavity)

Spectrum



x
0.0
0.2
0.4
0
20
40
Time
S_{xx}
0
1
2
3
4
5
6
0.0
0.5
1.0
1.5
2.0
Frequency in ω_m

Power Scaling



Stochastic Master Equation

Lindblad Master Equation:

$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\} + \left(C\rho + \rho C^\dagger - \mathrm{Tr}(C\rho + \rho C^\dagger)\right) \xi(t)$$

Let's look at the Quadrature $C = \eta \sqrt{\kappa}\,(\delta a + \delta a^\dagger)$

QuantumOptics.jl, (Stochastic Master equation, Quantum Zeno Effect)