Noise Analysis
Optomechanical Cavity

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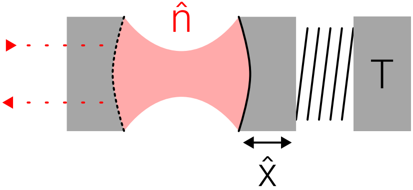
Modeling Quantum Hardware: open dynamics and control
Universität Konstanz

Problem Statement



"Cavity optomechanics", Aspelmeyer et al. 2014

Quantum Optomechanics, Bowen et al. 2015

Hamiltonian

Optical Cavity $\hat{a}$, $\omega_o(\hat{x}_{\text{mech}}) = \omega_o + \frac{g}{\omega_o}\hat{x}_{\text{mech}}$; mechanical oscillations $\hat{b}, \omega_m$; coupling g; Drive E, ω_L

$$H = \underbrace{\omega_o\ a^\dagger a}_{\text{Cavity}} + \underbrace{\omega_m\ b^\dagger b}_{\text{Mechanical}} - \underbrace{g\ a^\dagger a\,(b + b^\dagger)}_{\text{Interaction}} + \underbrace{E(ae^{i\omega_L t} + a^\dagger e^{-i\omega_L t})}_{\text{Drive}}$$

Rotating Wave Approximation at ω_L with $\Delta = \omega_o - \omega_L$, $a \to ae^{i\omega_L t}$:

$$H_{\text{RWA}} = \Delta\ a^\dagger a + \omega_m\ b^\dagger b - g\ a^\dagger a\,(b^\dagger + b) + E(a + a^\dagger)$$

$\hbar = 1$

Quantum Optomechanics, Bowen et al. 2015 (2.3)

QuantumOptics.jl, Krämer et al. 2024 (Optomechanical Cavity)

Hamiltonian Linearization (Currently not used)

$$H_{\text{RWA}} = \Delta\ a^\dagger a + \omega_m\ b^\dagger b - g\ a^\dagger a\,(b^\dagger + b) + E(a + a^\dagger)$$

Linearize $a = \alpha + \delta a$, $b = \beta + \delta b$; with α, β steady state.

$$\begin{aligned} H_{\text{Interaction}} &= -g\ a^\dagger a\,(b^\dagger + b) \\ &\approx -\underbrace{g|\alpha|}_{G}\left(\delta a + \delta a^\dagger + \mathcal{O}(a^2 + \delta a \delta a^\dagger)\right)\left(\delta b + \delta b^\dagger + 2\beta\right) \end{aligned}$$

$$a + a^\dagger = |\alpha| + \delta a + \delta a^\dagger \sim \delta a + \delta a^\dagger$$

Therefore for small G:

$$\begin{aligned} H_{\text{lin}} &= \Delta\ \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - G(\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger) + E(a + a^\dagger) \\ &\sim \frac{\Delta}{2}(\hat{X}^2 + \hat{Y}^2) + \frac{\omega}{2}(\hat{Q}^2 + \hat{P}^2) - G\hat{X}\hat{Q} + E\hat{X} \end{aligned}$$

Linearization in Quadratures

$X = \delta a + \delta a^\dagger \quad Q = \delta b + \delta b^\dagger$
$Y = i(\delta a^\dagger - \delta a) \quad P = i(\delta b^\dagger - \delta b)$
$n = \delta a^\dagger a \quad m = \delta b^\dagger b$

$$H_{\mathrm{RWA}} = \Delta n + \omega m - gnQ + EX$$
$$H_{\mathrm{lin}} = \Delta n + \omega m - GXQ + EX$$

The drive EX is not getting lost in linearization.
There is no point in the linearization if solved numerically.

Do I understand it correctly?

Dissipation

As Lindblad jump operator L with a coupling to the n_T, m_T thermal mode with coupling strengths κ, γ

Optical:

$$L = \sqrt{\kappa(n_T + 1)}\,\delta a + \sqrt{\kappa n_T}\,\delta a^\dagger$$

Mechanical:

$$+\sqrt{\gamma(m_T + 1)}\,\delta b + \sqrt{\gamma m_T}\,\delta b^\dagger$$

Quantum Optomechanics, Bowen et al. 2015 (2.8)

Continous measurement

Lindblad Master Equation:

$$\dot{\rho} = -i[H,\rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

Stochastic Master Equation:

$$\dot{\rho} = -i[H,\rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\} + \left(C\rho + \rho C^\dagger - \mathrm{Tr}(C\rho + \rho C^\dagger)\right)\xi(t)$$

Let's measure the Quadrature $C = \eta\sqrt{\kappa}\,(\delta a + \delta a^\dagger)$
Or the non hermitian $C = \eta\sqrt{\kappa}\;\delta a$

QuantumOptics.jl, Krämer et al. 2024 (Stochastic Master equation, Quantum Zeno Effect)
"A Straightforward Introduction to Continuous Quantum Measurement", Jacobs et al. 2006

Stochastic Schrödinger Equation

Is this faster?

Allows this to model measurements?

QuantumOptics.jl, Krämer et al. 2024 (Stochastic Schrödinger Equation)

"A Straightforward Introduction to Continuous Quantum Measurement", Jacobs et al. 2006 (10.2)

Implementation

truncated Fock Basis: $F_{\text{optical}} \otimes F_{\text{mechanical}}$

definition of H, J with $\delta a \otimes 1$

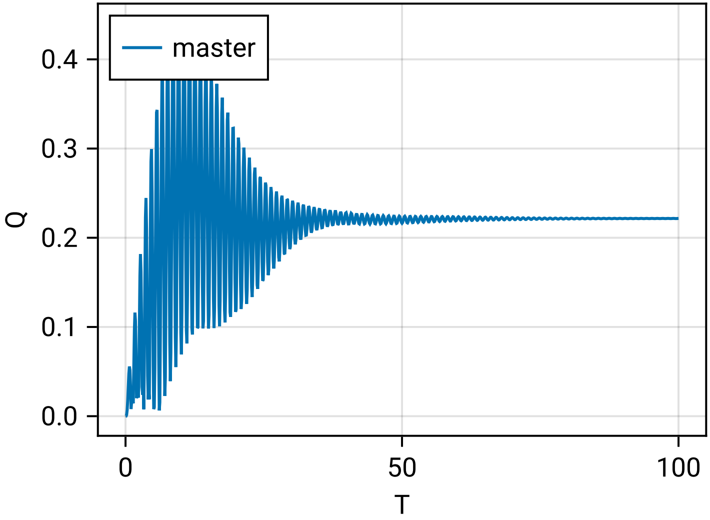
$\psi(0) = |0\rangle \otimes |0\rangle$

Time Evolution using the *Stochastic Master Equation*:

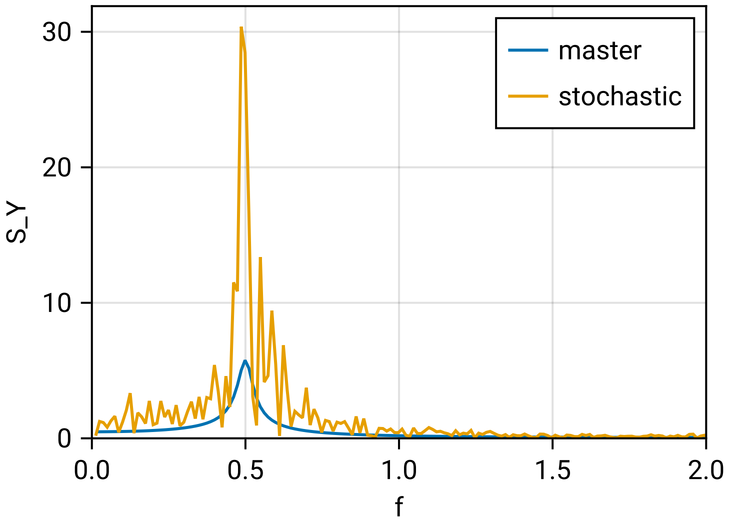
$$\dot{\rho} = -i[H, \rho] + J\rho J^{\dagger} - \frac{1}{2}\{J^{\dagger}J, \rho\}$$

QuantumOptics.jl, Krämer et al. 2024

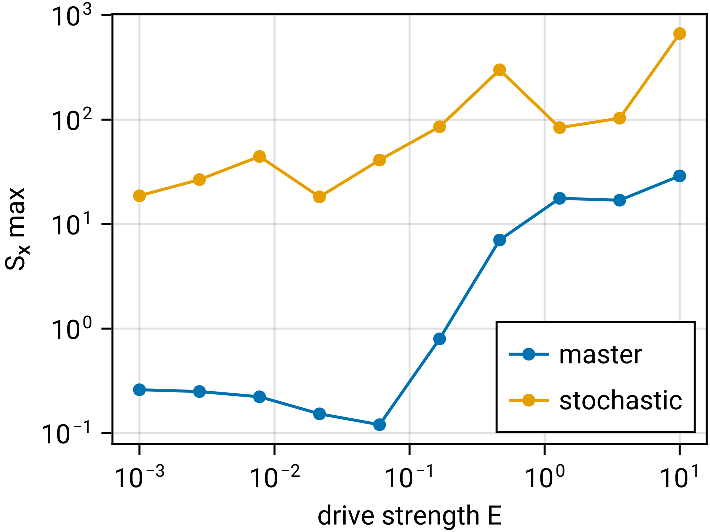
Time Evolution



Spectrum

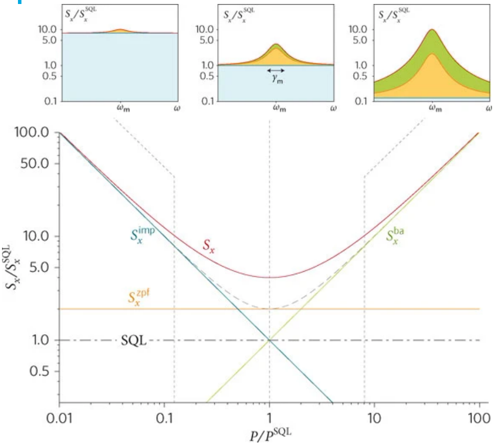


Power Dependence ~ G

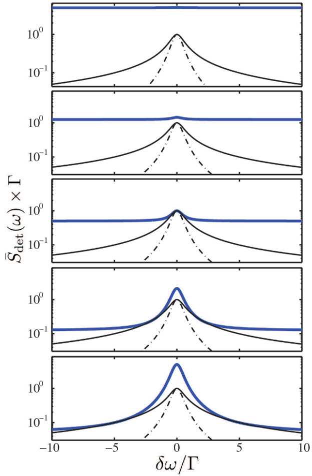


Took **2h** of compute time and is still unstable. Why is the SME so much slower?

Expectation



"Nanomechanical motion measured with an imprecision below that at the standard quantum limit", Teufel et al. 2009 (Fig. 1)



$$P/P^{\text{SQL}} = \{0.1, 0.4, 1, 4, 10\}$$

Quantum Optomechanics, Bowen et al. 2015 (Fig. 3.5)

How to procede?

Pivot to Quantum Zeno Effect

Krämer et al., *QuantumOptics.jl*, Quantum Zeno Effect

Reduce the complexity

Carmichael, An open systems approach to quantum optics: Lectures presented at the Université Libre de Bruxelles, 2009

Krämer et al., *QuantumOptics.jl*, Stochastic Schrödinger equation

Go into depth with the SME

Adding Poisson Process to detection

Jacobs and Steck, “A Straightforward Introduction to Continuous Quantum Measurement”, 10.2