Quantum Measurement
Zeno Effect
Standard Quantum Limit

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01.02.2025

Modeling Quantum Hardware: open dynamics and control
Universität Konstanz

No phenomenon is a real until it is observed.

– John A. Wheeler 1970

Historical Note

1900 Plank & Einstein: Blackbody Radiation

1920 Bohr, Heisenberg: Copenhagen interpretation
Born: Probabilistic interpretation $P(m) = |\langle m|\psi\rangle|^2$
Schrödinger: Measurement Problem

1930 EPR Paradox

1932 von Neumann: *Mathematical Foundations of Quantum Mechanics*

1970 Decoherence Theory

present Experimental Interest

Projective (von Neumann) Measurement

Measurement Operator $\hat{A} = \sum m|m\rangle$ on ψ:

$$p(m) = |\langle m|\psi\rangle|^2$$

$$\psi \xrightarrow{\text{Measuring } m} |m\rangle$$

More formally with projector $\hat{M} \sim |m\rangle\langle m|$:

$$\rho' \propto \hat{M}\rho\hat{M}$$

Neglegting Normalization and Degenercy: POVM Measurement

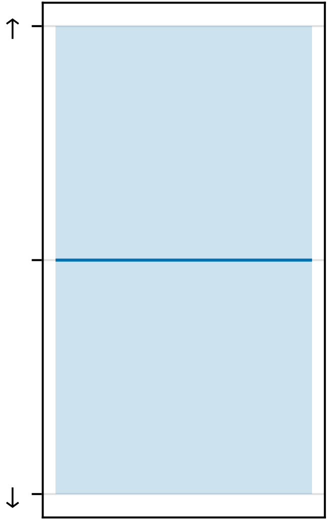
Quantum Computation and Quantum Information, Nielsen et al. 2010 (2.2.3)

Quantum Measurement and Control, Wiseman et al. 2010 (1)

Example: Superposition

$$H = \sigma_z$$
$$|\psi > \propto |\uparrow\rangle + |\downarrow\rangle$$
$$i\partial_t \psi = H\psi$$

$\Rightarrow$ Superposition is stable



Example: Superposition

$$H = \sigma_z$$
$$|\psi > \propto |\uparrow\rangle + |\downarrow\rangle$$
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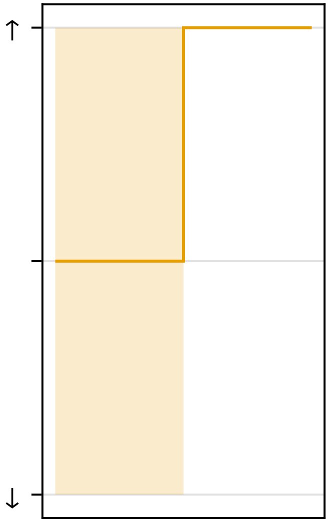
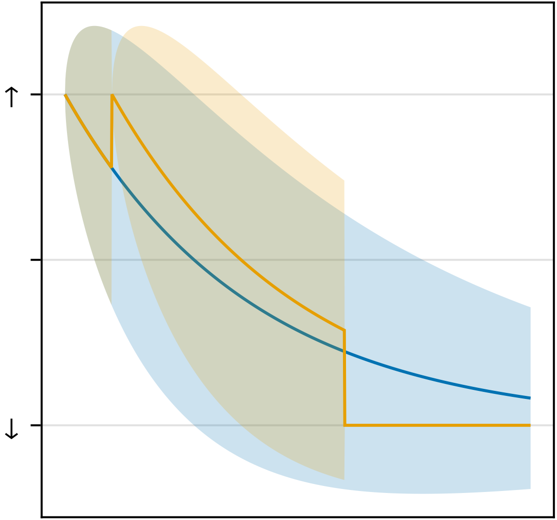
$$M = \sigma_z$$
$$p(\updownarrow) = |\langle \updownarrow | \psi \rangle|^2 = \frac{1}{2}$$

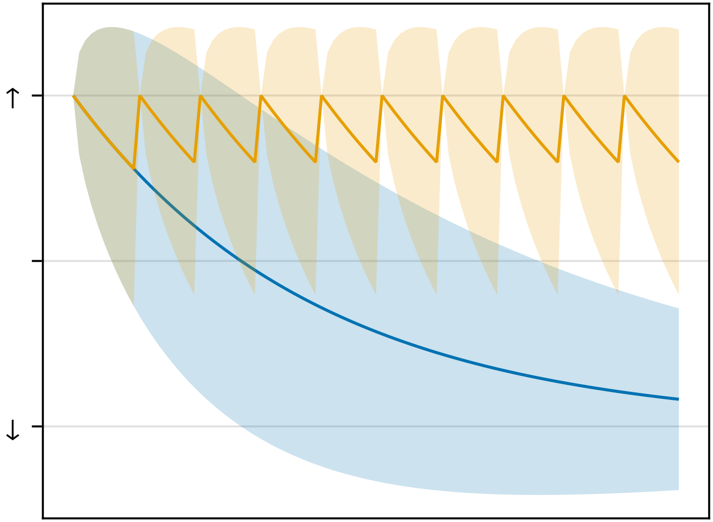


Example: Decay

$$H = \sigma_z$$
$$|\psi > \propto |\uparrow\rangle$$
$$M = \sigma_z$$
$$J = \kappa \sigma_-$$
$$\partial_t \rho = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$



Example: Zeno



Zeno Effect

- Zeno of Elea (460 BCE): Arrow paradox
- Misra and Sudarshan (1977): “The Zeno’s paradox in quantum theory”

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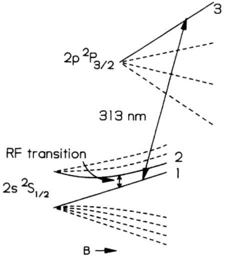


FIG. 2. Diagram of the energy levels of $^{9}Be^{+}$ in a magnetic field B. The states labeled 1, 2, and 3 correspond to those in Fig. 1 .

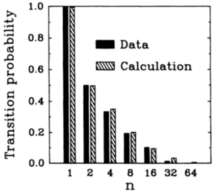


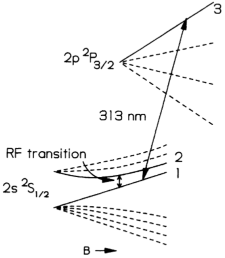
FIG. 3. Graph of the experimental and calculated 1 → 2 transition probabilities as a function of the number of measurement pulses n. The decrease of the transition probabilities with increasing n demonstrates the quantum Zeno effect.

“Quantum Zeno effect”, Itano et al. 1990

Zeno Effect

- Zeno of Elea (460 BCE): Arrow paradox
- Misra and Sudarshan (1977): “The Zeno’s paradox in quantum theory”
- Experimentally demonstrated (1990) with 5000 $^9Be^+$ ions at 250 mK
- Used in Magnetometers and possibly birds

“Quantum Zeno effect explains magnetic-sensitive radical-ion-pair reactions”, Kominis 2009



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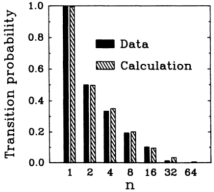

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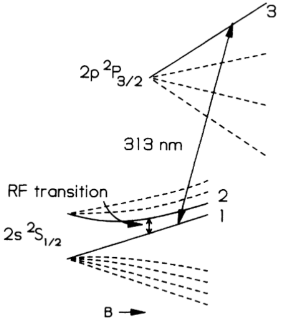
Practical Examples of strong measurements

- Trapped Ions



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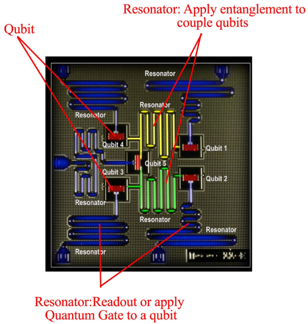
Practical Examples of strong measurements

- Trapped Ions
- Superconducting Qubits



Practical Examples of strong measurements

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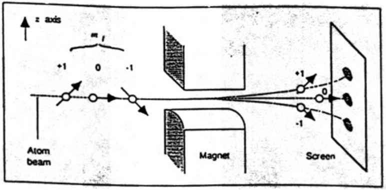
- Stern-Gerlach Experiment



Practical Examples of strong measurements

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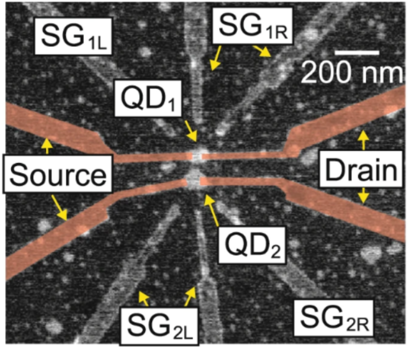
- Quantum Dot Charge Readout



Practical Examples of strong measurements

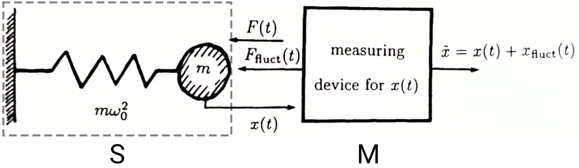
- Trapped Ions
- Superconducting Qubits
- Stern-Gerlach Experiment
- Quantum Dot Charge Readout

$\Rightarrow$ Used when the quantum wave function is collapsed into a classical result

- measurement of conjugate variables
- optical measurements
- Continuous measurement not possible

Quantum Measurement and Control, Wiseman et al. 2010

Weak Measurement



$$H = H_S \otimes 1 + 1 \otimes H_M + g(t)\ C_S \otimes P_M$$

measuring $\hat{X}$ in M: $\langle C \rangle + \xi_g$ and partial collapse in S

Continuous application leads to *SME* for S:

$$\partial_t \rho = -i[H,\rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\} + \left(C\rho + \rho C^\dagger - \mathrm{Tr}(C\rho + \rho C^\dagger)\right) \xi(t)$$

Quantum measurement, Vladimir Braginsky et al. 1992 (Fig 8.4 modified)

"A Straightforward Introduction to Continuous Quantum Measurement", Jacobs et al. 2006

Example: Weak measurement

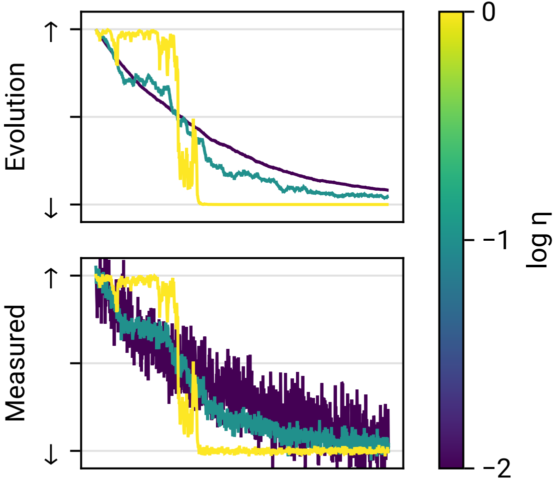
$H = \sigma_z$

$\psi = \uparrow$

$M = \sigma_z$

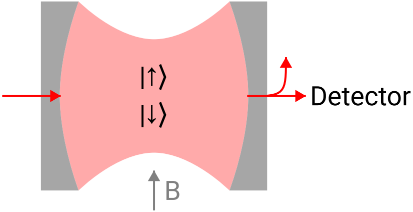
$J = \kappa \sigma_-$

$C = \eta \sigma_z$



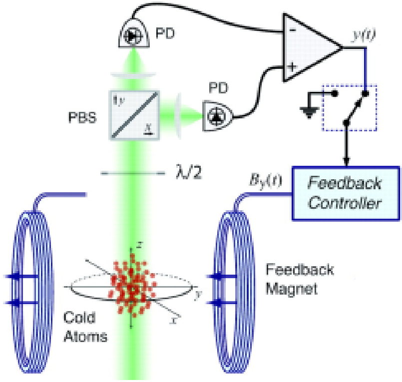
Practical Examples of weak measurements

- continuous measurements



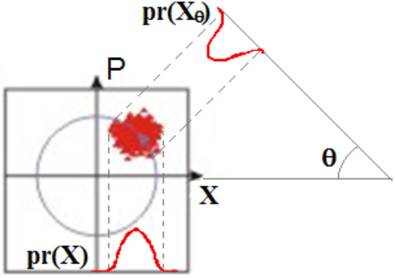
Practical Examples of weak measurements

- continuous measurements
- feedback control

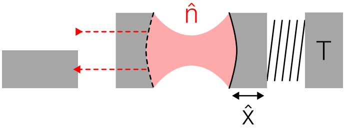


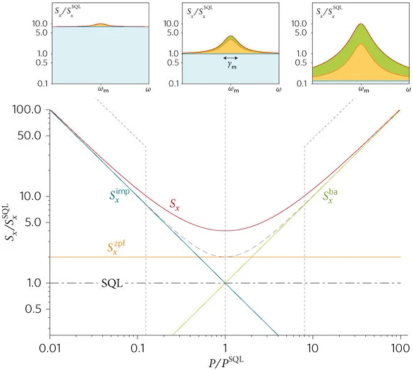
Practical Examples of weak measurements

- continuous measurements
- feedback control
- state tomography



Interferometer and SQL





“Quantum-mechanical limitations in macroscopic experiments and modern experimental technique”, BraginskiT et al. 1974

“Nanomechanical motion measured with an imprecision below that at the standard quantum limit”, Teufel et al. 2009

“Quantum-Enhanced Advanced LIGO Detectors in the Era of Gravitational-Wave Astronomy”, Tse et al. 2019

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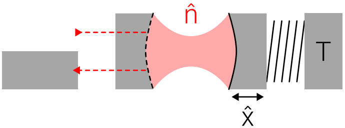

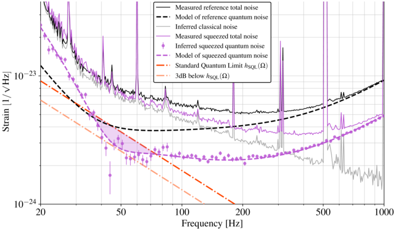

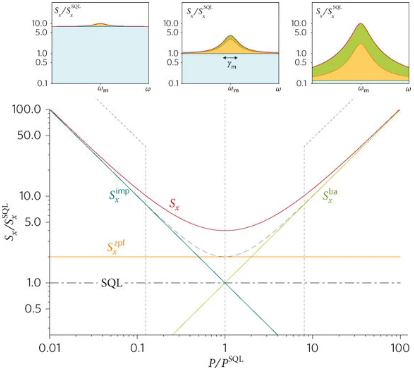

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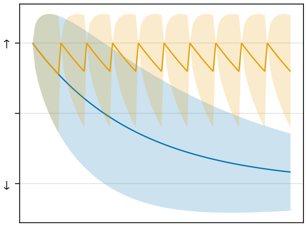
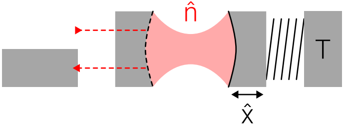
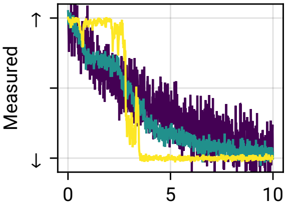
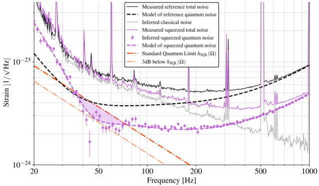

↑
↓

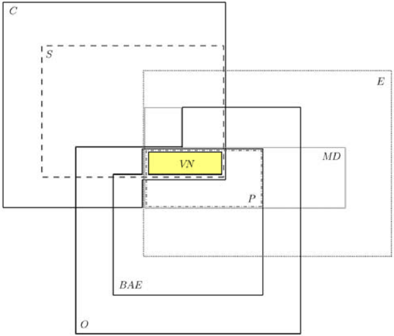

n̂
T
X̂


↑
↓
Measured
0
5
10


Measured reference total noise
Model of reference quantum noise
Inferred classical noise
Measured squeezed total noise
Inferred squeezed quantum noise
Model of squeezed quantum noise
Standard Quantum Limit $h_{SQL}(\Omega)$
3dB below $h_{SQL}(\Omega)$
Strain [1/√Hz]
10^{-23}
10^{-24}
20
50
100
200
500
1000
Frequency [Hz]

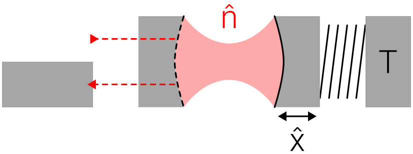
Types of Measurements

Symbol	Name	Definition
E	Efficient	$\forall r, \exists \hat{M}_r,\ \mathcal{O}_r = \mathcal{J}[\hat{M}_r]$
C	Complete	$\forall \rho, \forall r,\ \mathcal{O}_r \rho \propto \mathcal{O}_r \hat{1}$
S	Sharp	$\forall r,\ \mathrm{rank}(\hat{E}_r) = 1$
O	Of an observable X	$\forall r,\ \hat{E}_r = E_r(\hat{X})$
BAE	Back-action-evading	O with $\forall \rho, \forall x \in \lambda(\hat{X}),\ \mathrm{Tr}[\hat{\Pi}_x \rho] = \mathrm{Tr}[\hat{\Pi}_x \mathcal{O} \rho]$
MD	Minimally disturbing	E with $\forall r,\ \hat{M}_r = \hat{M}_r^\dagger$
P	Projective	MD and O
VN	von Neumann	P and S



Quantum Measurement and Control, Wiseman et al. 2010

Problem Statement



"Cavity optomechanics", Aspelmeyer et al. 2014

Quantum Optomechanics, Bowen et al. 2015

Hamiltonian

Optical Cavity $\hat{a}$, $\omega_o(\hat{x}_{\text{mech}}) = \omega_o + \frac{g}{\omega_o}\hat{x}_{\text{mech}}$; mechanical oscillations $\hat{b}, \omega_m$; coupling g; Drive E, ω_L

$$H = \underbrace{\omega_o \; a^\dagger a}_{\text{Cavity}} + \underbrace{\omega_m \; b^\dagger b}_{\text{Mechanical}} - \underbrace{g \; a^\dagger a \, (b + b^\dagger)}_{\text{Interaction}} + \underbrace{E(a e^{i\omega_L t} + a^\dagger e^{-i\omega_L t})}_{\text{Drive}}$$

Rotating Wave Approximation at ω_L with $\Delta = \omega_o - \omega_L$, $a \to a e^{i\omega_L t}$:

$$H_{\text{RWA}} = \Delta \; a^\dagger a + \omega_m \; b^\dagger b - g \; a^\dagger a \, (b^\dagger + b) + E(a + a^\dagger)$$

$\hbar = 1$

Quantum Optomechanics, Bowen et al. 2015 (2.3)

QuantumOptics.jl, Krämer et al. 2024 (Optomechanical Cavity)

Hamiltonian Linearization (Currently not used)

$$H_{\mathrm{RWA}} = \Delta\ a^\dagger a + \omega_m\ b^\dagger b - g\ a^\dagger a\,(b^\dagger + b) + E(a + a^\dagger)$$

Linearize $a = \alpha + \delta a$, $b = \beta + \delta b$; with α, β steady state.

$$\begin{aligned} H_{\mathrm{Interaction}} &= -g\ a^\dagger a\,(b^\dagger + b) \\ &\approx -\underbrace{g|\alpha|}_{G}\left(\delta a + \delta a^\dagger + \mathcal{O}(a^2 + \delta a \delta a^\dagger)\right)\left(\delta b + \delta b^\dagger + 2\beta\right) \end{aligned}$$

$$a + a^\dagger = |\alpha| + \delta a + \delta a^\dagger \sim \delta a + \delta a^\dagger$$

Therefore for small G:

$$\begin{aligned} H_{\mathrm{lin}} &= \Delta\ \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - G(\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger) + E(a + a^\dagger) \\ &\sim \frac{\Delta}{2}(\hat{X}^2 + \hat{Y}^2) + \frac{\omega}{2}(\hat{Q}^2 + \hat{P}^2) - G\hat{X}\hat{Q} + E\hat{X} \end{aligned}$$

Linearization in Quadratures

$$X = \delta a + \delta a^\dagger \quad Q = \delta b + \delta b^\dagger$$
$$Y = i(\delta a^\dagger - \delta a) \quad P = i(\delta b^\dagger - \delta b)$$
$$n = \delta a^\dagger a \quad m = \delta b^\dagger b$$

$$H_{\text{RWA}} = \Delta n + \omega m - gnQ + EX$$
$$H_{\text{lin}} = \Delta n + \omega m - GXQ + EX$$

The drive EX is not getting lost in linearization.
There is no point in the simplification if solved numerically.

Dissipation

Optical decay κ:

$$L = \sqrt{\kappa(n_T + 1)}\,\delta a + \sqrt{\kappa n_T}\,\delta a^\dagger$$

Mechanical resonator with γ and a thermal bath at the n-th thermal state:

$$+\sqrt{\gamma(m_T + 1)}\,\delta b + \sqrt{\gamma m_T}\,\delta b^\dagger$$

Quantum Optomechanics, Bowen et al. 2015 (2.8)

Implementation

truncated Fock Basis: $F_{\text{optical}} \otimes F_{\text{mechanical}}$

definition of H, J with $\delta a \otimes 1$

$\psi(0) = |0\rangle \otimes |0\rangle$

Time Evolution using the *Lindblad equation*:

$$\dot{\rho} = -i[H, \rho] + J\rho J^{\dagger} - \frac{1}{2}\{J^{\dagger}J, \rho\}$$

QuantumOptics.jl, Krämer et al. 2024

Continous measurement

Lindblad Master Equation:

$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

Stochastic Master Equation:

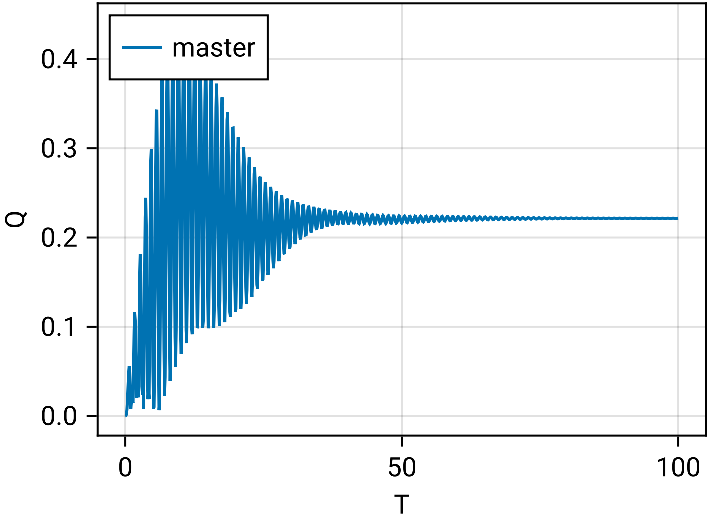
$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\} + \left(C\rho + \rho C^\dagger - \mathrm{Tr}(C\rho + \rho C^\dagger)\right) \xi(t)$$

Let's look at the Quadrature $C = \eta \sqrt{\kappa}\, (\delta a + \delta a^\dagger)$

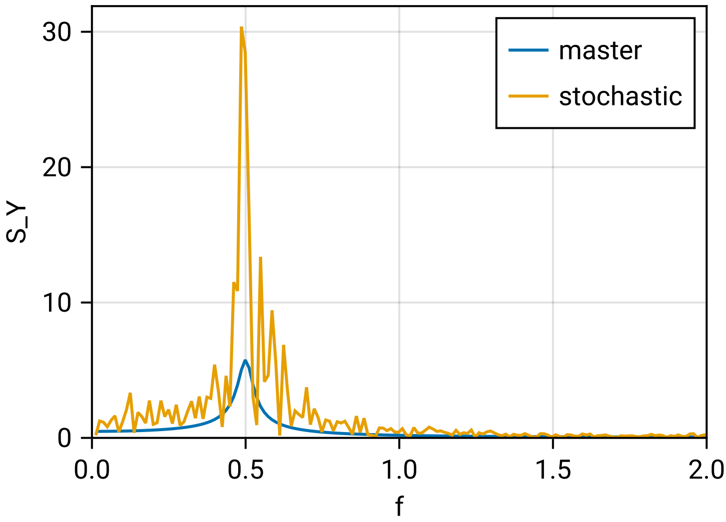
QuantumOptics.jl, Krämer et al. 2024 (Stochastic Master equation, Quantum Zeno Effect)

"A Straightforward Introduction to Continuous Quantum Measurement", Jacobs et al. 2006

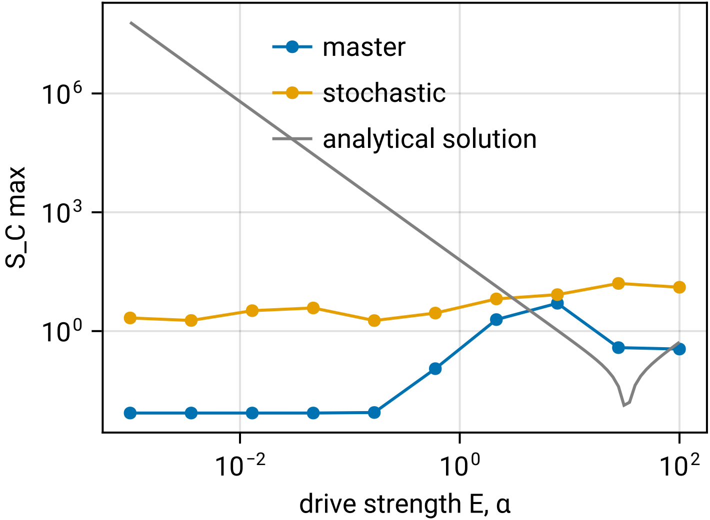
Time Evolution



Spectrum



Power Dependence ~ G

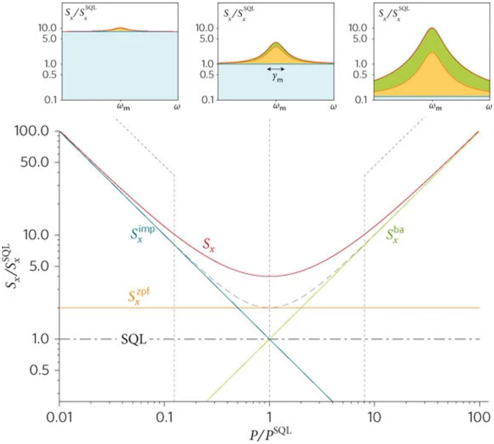


Took ≈ 1 min of compute time. Why is the SME so much slower? True Random Values?

Expectation

$$S_x = S_x^{\text{imp}} + S_x^{\text{ba}} + S_x^{\text{zpf}}$$

$S_x^{\text{imp}} \propto P^{-1}\omega^0$: Imprecision / Shot noise
$S_x^{\text{ba}} \propto P$: Back action
S_x^{zpf}: Zero Point Fluctuation



"Nanomechanical motion measured with an imprecision below that at the standard quantum limit", Teufel et al. 2009 (Fig. 1)

Uncerstanding $S_{\text{det}}(\omega, P_{\text{in}})$

Where is the power dependence?

$$\bar{S}_{\text{det}}(\omega) = \frac{1}{8\eta\Gamma|C_{\text{eff}}|} + 2\Gamma|\chi(\omega)|^2|C_{\text{eff}}|$$

$C_{\text{eff}}(\omega) = \frac{4g^2}{\kappa\Gamma(1-2i\omega/\kappa)^2}$

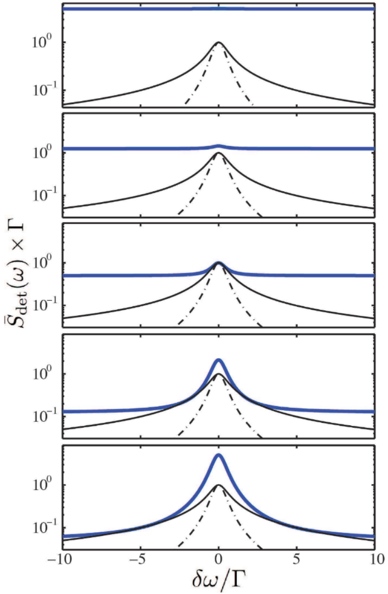
$\chi(\omega) = \frac{\Omega}{\Omega^2-\omega^2-i\omega\gamma}$

η: Detection efficiency

$\Gamma = \gamma$: Damping of oscillator

$$\bar{S}_{\text{det}}(\omega) = \frac{\kappa\Gamma|1-2i\omega/\kappa|}{8\eta\Gamma 4g^2} + 2\Gamma\frac{\Omega^2}{|\Omega^2-\omega^2-i\omega\gamma|^2}\frac{4g^2}{\kappa\Gamma(1-2i\omega/\kappa)^2}$$

Quantum Optomechanics, Bowen et al. 2015 (eq. 3.51)



$P/P^{\text{SQL}} = \{0.1, 0.4, 1, 4, 10\}$

Quantum Optomechanics, Bowen et al. 2015 (Fig. 3.5)

Looking for a source that derives $S_{\text{det}}(\omega, P)$?