Noise Analysis
Optomechanical Cavity

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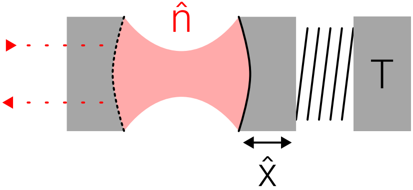
Modeling Quantum Hardware: open dynamics and control
Universität Konstanz

Problem Statement



"Cavity optomechanics", Aspelmeyer et al. 2014

Quantum Optomechanics, Bowen et al. 2015

Hamiltonian

Optical Cavity $\hat{a}$, $\omega_o(\hat{x}_{\text{mech}}) = \omega_o + \frac{g}{\omega_o}\hat{x}_{\text{mech}}$; mechanical oscillations $\hat{b}, \omega_m$; coupling g; Drive E, ω_L

$$H = \underbrace{\omega_o \; a^\dagger a}_{\text{Cavity}} + \underbrace{\omega_m \; b^\dagger b}_{\text{Mechanical}} - \underbrace{g \; a^\dagger a \,(b + b^\dagger)}_{\text{Interaction}} + \underbrace{i(E a^\dagger e^{-i\omega_L t} - E^* a e^{i\omega_L t})}_{\text{Drive}}$$

$\hbar = 1$

Quantum Optomechanics, Bowen et al. 2015 (2.3)

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Rotating Wave Approximation at ω_L with $\Delta = \omega_o - \omega_L$, $a \rightarrow a e^{i\omega_L t}$:

$$H_{\text{RWA}} = \Delta \ a^\dagger a + \omega_m \ b^\dagger b - g \ a^\dagger a \ (b^\dagger + b) + i(E a^\dagger - E^* a)$$

$\hbar = 1$

Quantum Optomechanics, Bowen et al. 2015 (2.3)

Hamiltonian Linearization

$$H_{\text{RWA}} = \Delta\, a^\dagger a + \omega_m\, b^\dagger b - g\, a^\dagger a\,(b^\dagger + b) + i(E a^\dagger - E^* a)$$

Linearize $a = \alpha + \delta a$, $b = \beta + \delta b$; with α, β steady state.

$$\begin{aligned} H_{\text{Interaction}} &= -g\, a^\dagger a\,(b^\dagger + b) \\ &\approx -\underbrace{g|\alpha|}_{G}\left(\delta a + \delta a^\dagger + \mathcal{O}(a^2 + \delta a \delta a^\dagger)\right)\left(\delta b + \delta b^\dagger + 2\beta\right) \end{aligned}$$

Quantum Optomechanics, Bowen et al. 2015 (2.7)

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Therefore for small G:

$$H \approx \Delta\ \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - G(\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger)$$

Quantum Optomechanics, Bowen et al. 2015 (2.7)

Dissipation

Optical decay κ:

$$J_O = \sqrt{\kappa}\ \delta a$$

Mechanical resonator with γ and a thermal bath at the n-th thermal state:

$$J_M = \sqrt{\gamma(n+1)}\ \delta b + \sqrt{\gamma n}\ \delta b^\dagger$$

Quantum Optomechanics, Bowen et al. 2015 (2.8)

Implementation

1. truncated Fock Basis: $F_{\text{optical}} \otimes F_{\text{mechanical}}$
2. definition of H, J with $\delta a \otimes 1$
3. Master Time Evolution:

$$\dot{\rho} = -i[H, \rho] + J\rho J^{\dagger} - \frac{1}{2}J^{\dagger}J\rho - \frac{1}{2}\rho J^{\dagger}J$$

QuantumOptics.jl,

Time evolution