Quantum Measurement
Zeno Effect

Leon Oleschko
30.01.2025

Modeling Quantum Hardware: open dynamics and control
Universität Konstanz

No phenomenon is a real until it is observed.

– John A. Wheeler 1970

Historical Note

1900	Plank & Einstein: Blackbody Radiation
1920	Bohr, Heisenberg: Copenhagen interpretation Born: Probabilistic interpretation $P(m) = \|\langle m\|\psi\rangle\|^2$ Schrödinger: Measurement Problem
1930	EPR Paradox
1932	von Neumann: *Mathematical Foundations of Quantum Mechanics*
1970	Decoherence Theory
present	Experimental Interest

Projective (von Neumann) Measurement

Measurement Operator $\hat{A} = \sum m|m\rangle$ on ψ:

$$p(m) = |\langle m|\psi\rangle|^2$$

$$\psi \xrightarrow{\text{Measuring } m} |m\rangle$$

More formally with projector $\hat{M} \sim |m\rangle\langle m|$:

$$\rho' \propto \hat{M}\rho\hat{M}$$

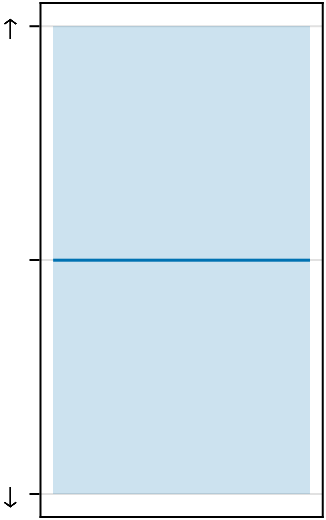
Neglegting Normalization and Degenercy: POVM Measurement

Quantum Computation and Quantum Information, Nielsen et al. 2010 (2.2.3)
Quantum Measurement and Control, Wiseman et al. 2010 (1)

Example: Superposition

$$H = \sigma_z$$
$$|\psi > \propto |\uparrow\rangle + |\downarrow\rangle$$
$$i\partial_t \psi = H\psi$$

$\Rightarrow$ Superposition is stable



Example: Superposition

$$H = \sigma_z$$

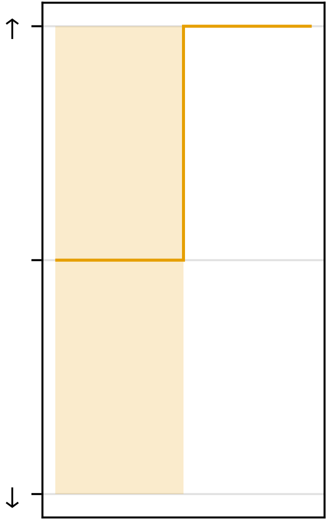
$$|\psi > \propto |\uparrow\rangle + |\downarrow\rangle$$

$$i\partial_t \psi = H\psi$$

$\Rightarrow$ Superposition is stable

$$M = \sigma_z$$

$$p(\updownarrow) = |\langle \updownarrow | \psi \rangle|^2 = \frac{1}{2}$$



Example: Decay

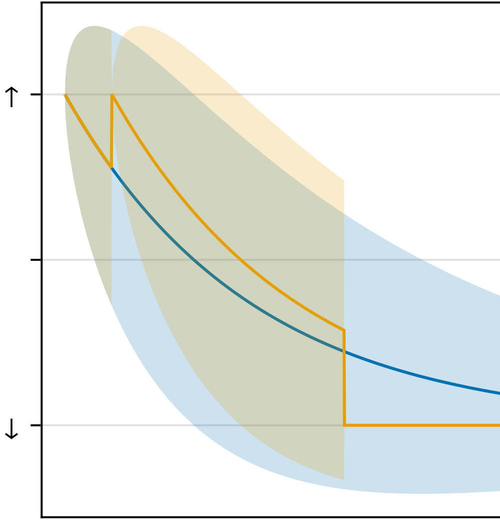
$$H = \sigma_z$$

$$|\psi > \propto |\uparrow\rangle$$

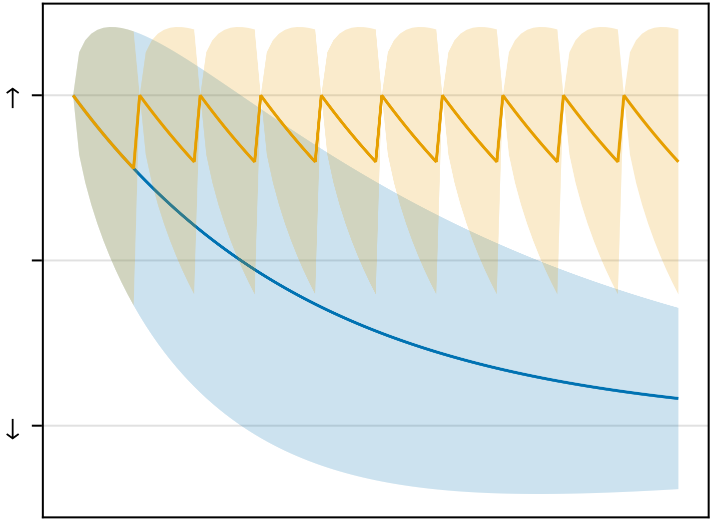
$$M = \sigma_z$$

$$J = \kappa \sigma_-$$

$$\partial_t \rho = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$



Example: Zeno



Zeno Effect

- Zeno of Elea (460 BCE): Arrow paradox
- Misra and Sudarshan (1977): “The Zeno’s paradox in quantum theory”

Zeno Effect

- Zeno of Elea (460 BCE): Arrow paradox
- Misra and Sudarshan (1977): “The Zeno’s paradox in quantum theory”
- Experimentally demonstrated (1990) with 5000 $^9Be^+$ ions at 250 mK

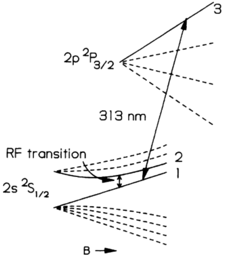


FIG. 2. Diagram of the energy levels of $^9Be^+$ in a magnetic field B. The states labeled 1, 2, and 3 correspond to those in Fig. 1 .

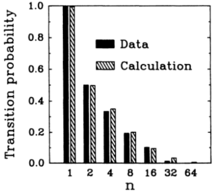


FIG. 3. Graph of the experimental and calculated 1 → 2 transition probabilities as a function of the number of measurement pulses n. The decrease of the transition probabilities with increasing n demonstrates the quantum Zeno effect.

“Quantum Zeno effect”, Itano et al. 1990

Zeno Effect

- Zeno of Elea (460 BCE): Arrow paradox
- Misra and Sudarshan (1977): “The Zeno’s paradox in quantum theory”
- Experimentally demonstrated (1990) with 5000 $^9Be^+$ ions at 250 mK
- Used in Magnetometers and possibly birds

“Quantum Zeno effect explains magnetic-sensitive radical-ion-pair reactions”, Kominis 2009

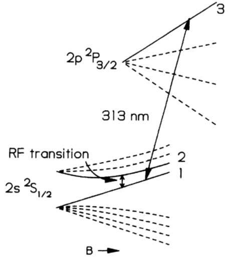


FIG. 2. Diagram of the energy levels of $^9Be^+$ in a magnetic field B. The states labeled 1, 2, and 3 correspond to those in Fig. 1 .

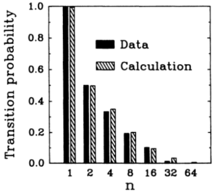


FIG. 3. Graph of the experimental and calculated $1 \rightarrow 2$ transition probabilities as a function of the number of measurement pulses n. The decrease of the transition probabilities with increasing n demonstrates the quantum Zeno effect.

“Quantum Zeno effect”, Itano et al. 1990

Practical Examples of strong measurements

- Trapped Ions

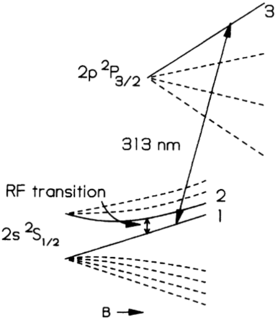
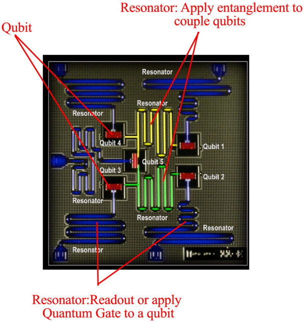


FIG. 2. Diagram of the energy levels of $^9Be^+$ in a magnetic field B. The states labeled 1, 2, and 3 correspond to those in Fig. 1 .

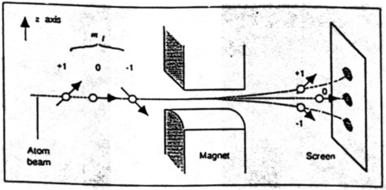
Practical Examples of strong measurements

- Trapped Ions
- Superconducting Qubits



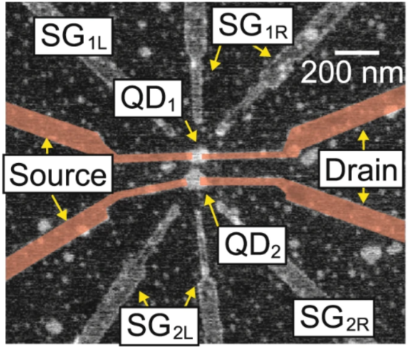
Practical Examples of strong measurements

- Trapped Ions
- Superconducting Qubits
- Stern-Gerlach Experiment



Practical Examples of strong measurements

- Trapped Ions
- Superconducting Qubits
- Stern-Gerlach Experiment
- Quantum Dot Charge Readout



Practical Examples of strong measurements

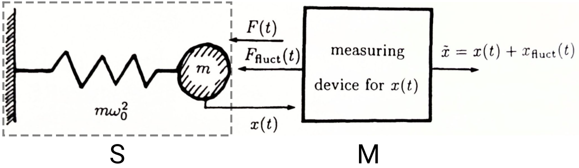
- Trapped Ions
- Superconducting Qubits
- Stern-Gerlach Experiment
- Quantum Dot Charge Readout

$\Rightarrow$ Used when the quantum wave function is collapsed into a classical result

- measurement of conjugate variables
- optical measurements
- Continuous measurement not possible

Quantum Measurement and Control, Wiseman et al. 2010

Weak Measurement



$$H = H_S \otimes 1 + 1 \otimes H_M + g(t)\; C_S \otimes P_M$$

measuring $\hat{X}$ in M: $\langle C \rangle + \xi_g$ and partial collapse in S

Continuous application leads to *SME* for S:

$$\partial_t \rho = -i[H,\rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J,\rho\} + \left(C\rho + \rho C^\dagger - \mathrm{Tr}(C\rho + \rho C^\dagger)\right)\xi(t)$$

Quantum measurement, Vladimir Braginsky et al. 1992 (Fig 8.4 modified)

"A Straightforward Introduction to Continuous Quantum Measurement", Jacobs et al. 2006

Example: Weak measurement

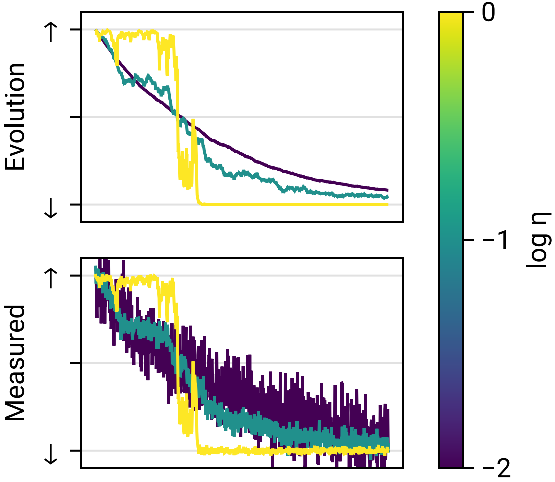
$H = \sigma_z$

$\psi = \uparrow$

$M = \sigma_z$

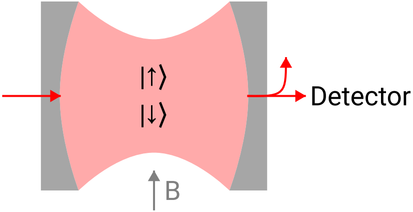
$J = \kappa \sigma_-$

$C = \eta \sigma_z$



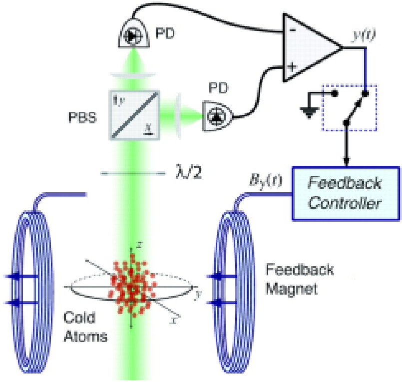
Practical Examples of weak measurements

- continuous measurements



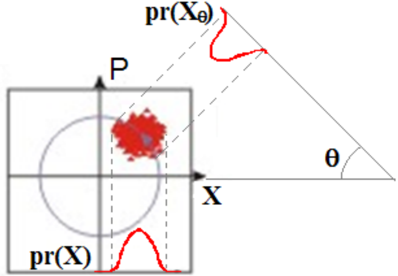
Practical Examples of weak measurements

- continuous measurements
- feedback control

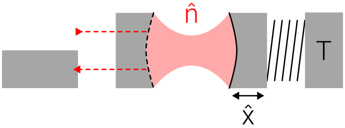


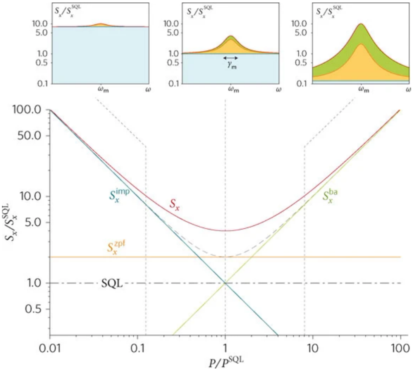
Practical Examples of weak measurements

- continuous measurements
- feedback control
- state tomography



Interferometer and SQL



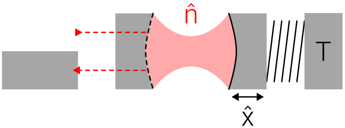


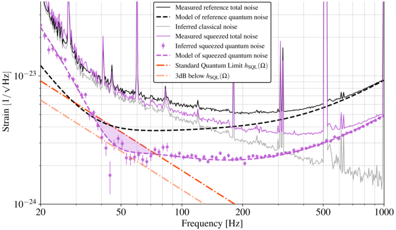
"Quantum-mechanical limitations in macroscopic experiments and modern experimental technique", BraginskiT et al. 1974

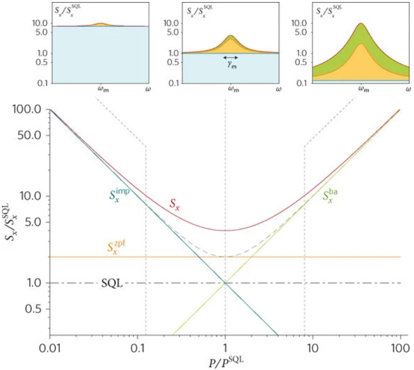
"Nanomechanical motion measured with an imprecision below that at the standard quantum limit", Teufel et al. 2009

"Quantum-Enhanced Advanced LIGO Detectors in the Era of Gravitational-Wave Astronomy", Tse et al. 2019

Interferometer and SQL



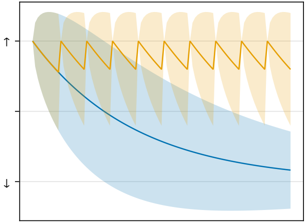


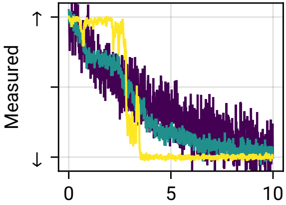


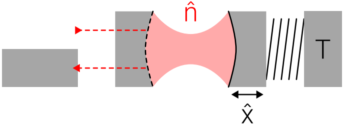
"Quantum-mechanical limitations in macroscopic experiments and modern experimental technique", BraginskiT et al. 1974

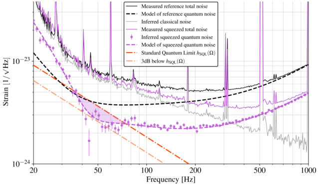
"Nanomechanical motion measured with an imprecision below that at the standard quantum limit", Teufel et al. 2009

"Quantum-Enhanced Advanced LIGO Detectors in the Era of Gravitational-Wave Astronomy", Tse et al. 2019


↑
↓


Measured
↑
↓
0
5
10


$\hat{n}$
T
$\hat{X}$


Measured reference total noise
Model of reference quantum noise
Inferred classical noise
Measured squeezed total noise
Inferred squeezed quantum noise
Model of squeezed quantum noise
Standard Quantum Limit $h_{SQL}(\Omega)$
3dB below $h_{SQL}(\Omega)$
Strain [$1/\sqrt{Hz}$]
10^{-23}
10^{-24}
20
50
100
200
500
1000
Frequency [Hz]