Noise Analysis
Optomechanical Cavity

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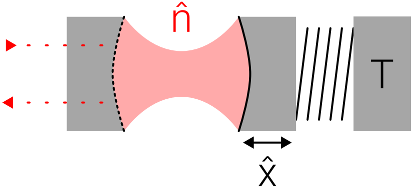
Modeling Quantum Hardware: open dynamics and control
Universität Konstanz

Problem Statement



"Cavity optomechanics", Aspelmeyer et al. 2014

Quantum Optomechanics, Bowen et al. 2015

Hamiltonian

Optical Cavity $\hat{a}$, $\omega_o(\hat{x}_{\text{mech}}) = \omega_o + \frac{g}{\omega_o}\hat{x}_{\text{mech}}$; mechanical oscillations $\hat{b}, \omega_m$; coupling g; Drive E, ω_L

$$H = \underbrace{\omega_o \; a^\dagger a}_{\text{Cavity}} + \underbrace{\omega_m \; b^\dagger b}_{\text{Mechanical}} - \underbrace{g \; a^\dagger a \, (b + b^\dagger)}_{\text{Interaction}} + \underbrace{E(a e^{i\omega_L t} + a^\dagger e^{-i\omega_L t})}_{\text{Drive}}$$

Rotating Wave Approximation at ω_L with $\Delta = \omega_o - \omega_L$, $a \to a e^{i\omega_L t}$:

$$H_{\text{RWA}} = \Delta \; a^\dagger a + \omega_m \; b^\dagger b - g \; a^\dagger a \, (b^\dagger + b) + E(a + a^\dagger)$$

$\hbar = 1$

Quantum Optomechanics, Bowen et al. 2015 (2.3)

QuantumOptics.jl, Krämer et al. 2024 (Optomechanical Cavity)

Hamiltonian Linearization (Currently not used)

$$H_{\text{RWA}} = \Delta\ a^{\dagger} a + \omega_m\ b^{\dagger} b - g\ a^{\dagger} a\ (b^{\dagger} + b) + E(a + a^{\dagger})$$

Linearize $a = \alpha + \delta a$, $b = \beta + \delta b$; with α, β steady state.

$$\begin{aligned} H_{\text{Interaction}} &= -g\ a^{\dagger} a\ (b^{\dagger} + b) \\ &\approx -\underbrace{g|\alpha|}_{G}\left(\delta a + \delta a^{\dagger} + \mathcal{O}(a^2 + \delta a \delta a^{\dagger})\right)\left(\delta b + \delta b^{\dagger} + 2\beta\right) \end{aligned}$$

$$a + a^{\dagger} = |\alpha| + \delta a + \delta a^{\dagger} \sim \delta a + \delta a^{\dagger}$$

Therefore for small G:

$$\begin{aligned} H_{\text{lin}} &= \Delta\ \delta a^{\dagger} \delta a + \omega_m \delta b^{\dagger} \delta b - G(\delta a + \delta a^{\dagger})(\delta b + \delta b^{\dagger}) + E(a + a^{\dagger}) \\ &\sim \frac{\Delta}{2}(\hat{X}^2 + \hat{Y}^2) + \frac{\omega}{2}(\hat{Q}^2 + \hat{P}^2) - G\hat{X}\hat{Q} + E\hat{X} \end{aligned}$$

Linearization in Quadratures

$$X = \delta a + \delta a^\dagger \quad Q = \delta b + \delta b^\dagger$$
$$Y = i(\delta a^\dagger - \delta a) \quad P = i(\delta b^\dagger - \delta b)$$
$$n = \delta a^\dagger a \quad m = \delta b^\dagger b$$

$$H_{\text{RWA}} = \Delta n + \omega m - g n Q + E X$$
$$H_{\text{lin}} = \Delta n + \omega m - G X Q + E X$$

The drive EX is not getting lost in linearization.
There is no point in the linearization if solved numerically.

Do I understand it correctly?

Dissipation

As Lindblad jump operator L with a coupling to the n_T, m_T thermal mode with coupling strengths κ, γ

Optical:

$$L = \sqrt{\kappa(n_T + 1)}\,\delta a + \sqrt{\kappa n_T}\,\delta a^\dagger$$

Mechanical:

$$+\sqrt{\gamma(m_T + 1)}\,\delta b + \sqrt{\gamma m_T}\,\delta b^\dagger$$

Quantum Optomechanics, Bowen et al. 2015 (2.8)

Continous measurement

Lindblad Master Equation:

$$\dot{\rho} = -i[H,\rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

Stochastic Master Equation:

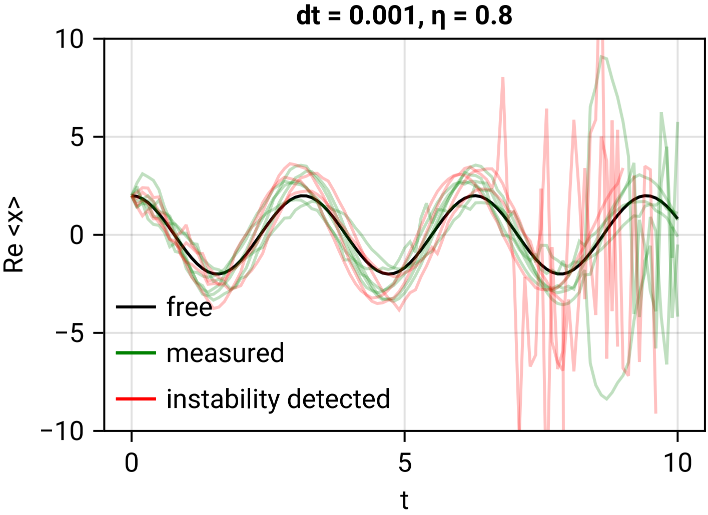
$$\dot{\rho} = -i[H,\rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\} + \left(C\rho + \rho C^\dagger - \mathrm{Tr}(C\rho + \rho C^\dagger)\right)\xi(t)$$

Let's measure the Quadrature $C = \eta\sqrt{\kappa}\,(\delta a + \delta a^\dagger)$
Or the non hermitian $C = \eta\sqrt{\kappa}\ \delta a$

QuantumOptics.jl, Krämer et al. 2024 (Stochastic Master equation, Quantum Zeno Effect)
"A Straightforward Introduction to Continuous Quantum Measurement", Jacobs et al. 2006

Harmonic Osciallator

For stability $dt \ll 1/\omega$, therefore much longer compute time (2s).



Implementation

truncated Fock Basis: $F_{\text{optical}} \otimes F_{\text{mechanical}}$

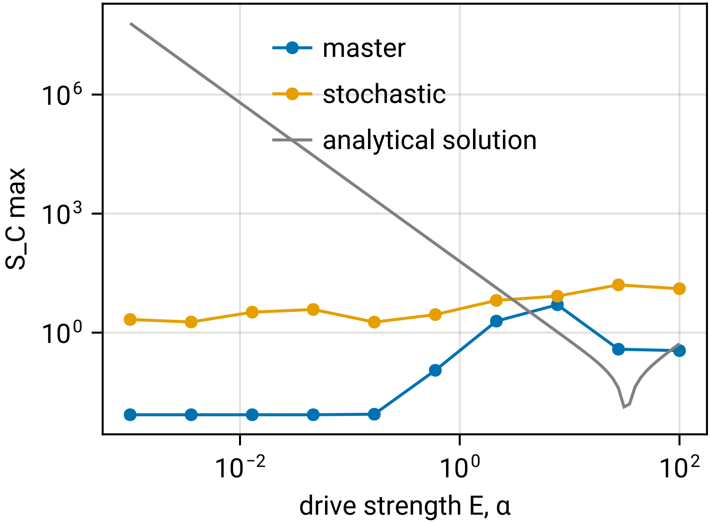
definition of H, J with $\delta a \otimes 1$

$\psi(0) = |0\rangle \otimes |0\rangle$

Time Evolution using the *Stochastic Master Equation*

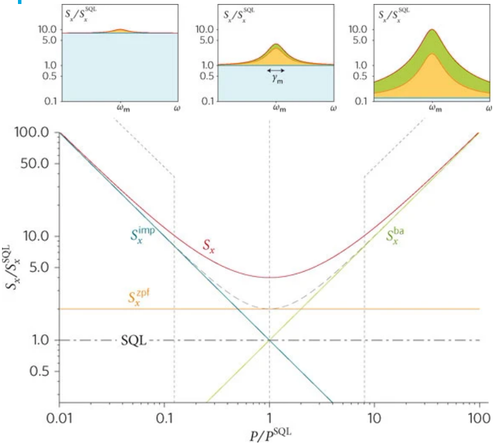
QuantumOptics.jl, Krämer et al. 2024

Power Dependence ~ G

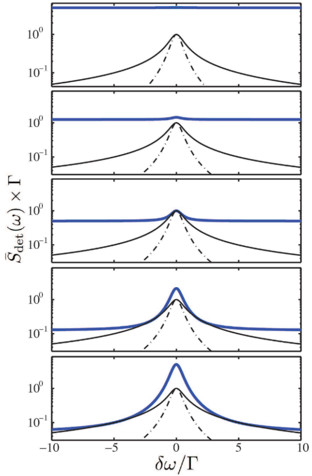


Took **3h** of compute time to be stable!

Expectation



"Nanomechanical motion measured with an imprecision below that at the standard quantum limit", Teufel et al. 2009 (Fig. 1)



$P/P^{\mathsf{SQL}} = \{0.1, 0.4, 1, 4, 10\}$

Quantum Optomechanics, Bowen et al. 2015 (Fig. 3.5)

Analytical Solution

$$\bar{S}_{\text{det}}(\omega) = \frac{1}{8\eta\Gamma|C_{\text{eff}}|} + 2\Gamma|\chi(\omega)|^2|C_{\text{eff}}|$$

$C_{\text{eff}}(\omega) = \frac{4G^2}{\kappa\Gamma(1-2i\omega/\kappa)^2}$

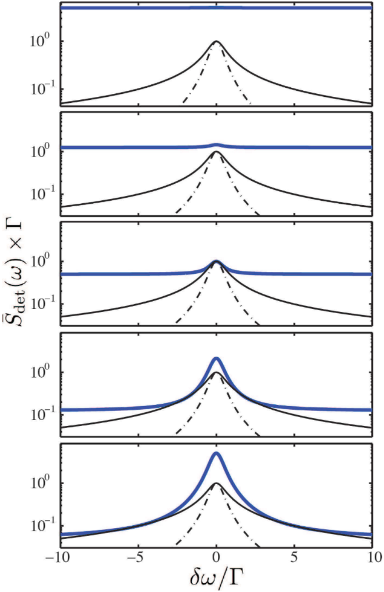
$\chi(\omega) = \frac{\Omega}{\Omega^2-\omega^2-i\omega\gamma}$

η: Detection efficiency

$\Gamma = \gamma$: Damping of oscillator

$$\bar{S}_{\text{det}}(\omega) = \frac{\kappa\Gamma|1-2i\omega/\kappa|}{8\eta\Gamma 4g^2|\alpha|^2} + 2\Gamma\frac{\Omega^2}{|\Omega^2-\omega^2-i\omega\gamma|^2}\frac{4g^2|\alpha|^2}{\kappa\Gamma(1-2i\omega/\kappa)^2}$$

Quantum Optomechanics, Bowen et al. 2015 (eq. 3.51)



$P/P^{\text{SQL}} = \{0.1, 0.4, 1, 4, 10\}$

Quantum Optomechanics, Bowen et al. 2015 (Fig. 3.5)

Parameter Space

Parameter	Searched Values
κ	$0.1 \ldots 1$
γ	$0.1 \ldots 1$
g	$0.1 \ldots 1$
n_T	$0.001 \ldots 1$
m_T	$0.001 \ldots 1$
Δ	$-\omega, \omega$
C	$Q, \delta a$
$S_\square$	Q, C

With multiple compute hours per run this space is too large.

QuantumOptics.jl, Krämer et al. 2024 (Optomechanical Cavity)

How to procede?

Pivot to Quantum Zeno Effect

Krämer et al., *QuantumOptics.jl*, Quantum Zeno Effect

Reduce the complexity?

Carmichael, An open systems approach to quantum optics: Lectures presented at the Université Libre de Bruxelles, 2009

Krämer et al., *QuantumOptics.jl*, Stochastic Schrödinger equation

Go into depth with the SME

Adding Poisson Process to detection

Jacobs and Steck, “A Straightforward Introduction to Continuous Quantum Measurement”, 10.2