Noise Analysis
Optomechanical Cavity

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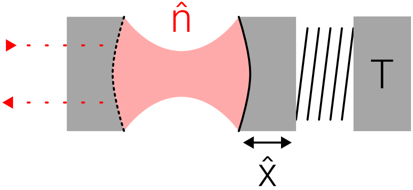
Modeling Quantum Hardware: open dynamics and control
Universität Konstanz

Problem Statement



"Cavity optomechanics", Aspelmeyer et al. 2014

Quantum Optomechanics, Bowen et al. 2015

Hamiltonian

Optical Cavity $\hat{a}$, $\omega_o(\hat{x}_{\text{mech}}) = \omega_o + \frac{g}{\omega_o}\hat{x}_{\text{mech}}$; mechanical oscillations $\hat{b}$, ω_m; coupling g; Drive E, ω_L

$$H = \underbrace{\omega_o \; a^\dagger a}_{\text{Cavity}} + \underbrace{\omega_m \; b^\dagger b}_{\text{Mechanical}} - \underbrace{g \; a^\dagger a \,(b + b^\dagger)}_{\text{Interaction}} + \underbrace{E(a e^{i\omega_L t} + a^\dagger e^{-i\omega_L t})}_{\text{Drive}}$$

Rotating Wave Approximation at ω_L with $\Delta = \omega_o - \omega_L$, $a \rightarrow a e^{i\omega_L t}$:

$$H_{\text{RWA}} = \Delta \; a^\dagger a + \omega_m \; b^\dagger b - g \; a^\dagger a \,(b^\dagger + b) + E(a + a^\dagger)$$

$\hbar = 1$

Quantum Optomechanics, Bowen et al. 2015 (2.3)

QuantumOptics.jl, Krämer et al. 2024 (Optomechanical Cavity)

Hamiltonian Linearization

$$H_{\text{RWA}} = \Delta\ a^\dagger a + \omega_m\ b^\dagger b - g\ a^\dagger a\,(b^\dagger + b) + E(a + a^\dagger)$$

Linearize $a = \alpha + \delta a$, $b = \beta + \delta b$; with α, β steady state.

$$\begin{aligned} H_{\text{Interaction}} &= -g\ a^\dagger a\,(b^\dagger + b) \\ &\approx -\underbrace{g|\alpha|}_{G}\left(\delta a + \delta a^\dagger + \mathcal{O}(a^2 + \delta a \delta a^\dagger)\right)\left(\delta b + \delta b^\dagger + 2\beta\right) \end{aligned}$$

Therefore for small G:

$$\begin{aligned} H &\approx \Delta\ \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - G(\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger) + E(a + a^\dagger) \\ &\sim \frac{\Delta}{2}(\hat{X}^2 + \hat{Y}^2) + \frac{\omega}{2}(\hat{Q}^2 + \hat{P}^2) - G\hat{X}\hat{Q} + E\hat{X} \end{aligned}$$

Quantum Optomechanics, Bowen et al. 2015 (2.7)

Dissipation

Optical decay κ:

$$L_O = \sqrt{\kappa}\ \delta a$$

Mechanical resonator with γ and a thermal bath at the n-th thermal state:

$$L_M = \sqrt{\gamma(n+1)}\ \delta b + \sqrt{\gamma n}\ \delta b^\dagger$$

Quantum Optomechanics, Bowen et al. 2015 (2.8)

Implementation

truncated Fock Basis: $F_{\text{optical}} \otimes F_{\text{mechanical}}$

definition of H, J with $\delta a \otimes 1$

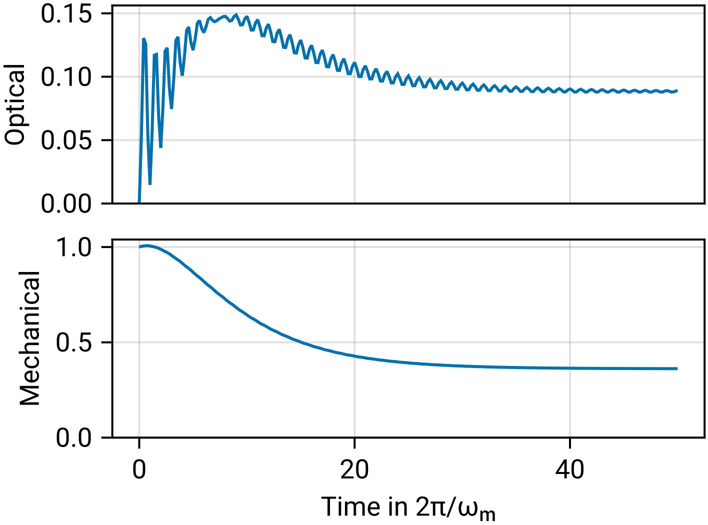
$\psi(0) = |0\rangle \otimes |0\rangle$

Time Evolution using the *Lindblad equation*:

$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

QuantumOptics.jl, Krämer et al. 2024

Time evolution of n



QuantumOptics.jl, Krämer et al. 2024 (Optomechanical Cavity)

Continous measurement

Lindblad Master Equation:

$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

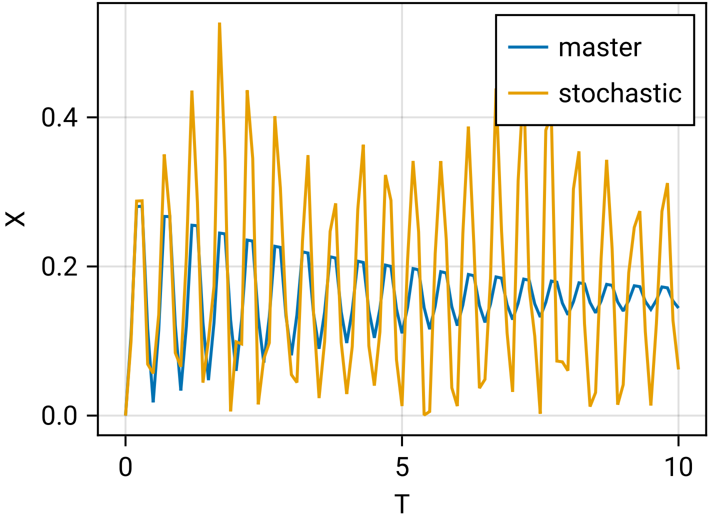
Stochastic Master Equation:

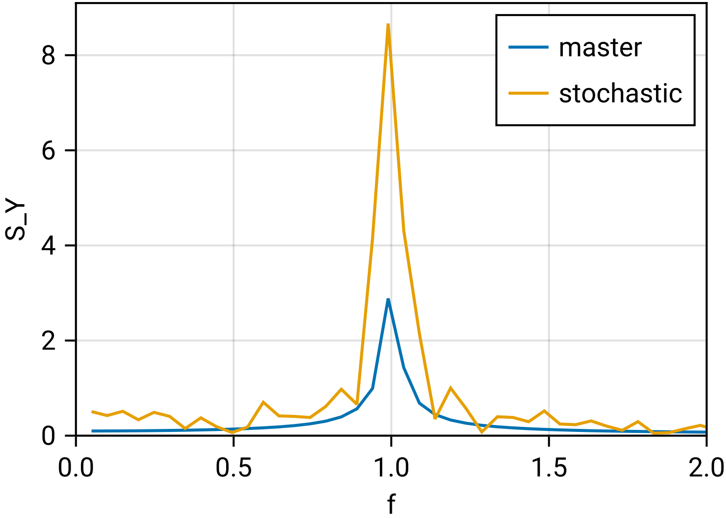
$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\} + \left(C\rho + \rho C^\dagger - \mathrm{Tr}(C\rho + \rho C^\dagger)\right)\xi(t)$$

Let's look at the Quadrature $C = \eta\sqrt{\kappa}\,(\delta a + \delta a^\dagger)$

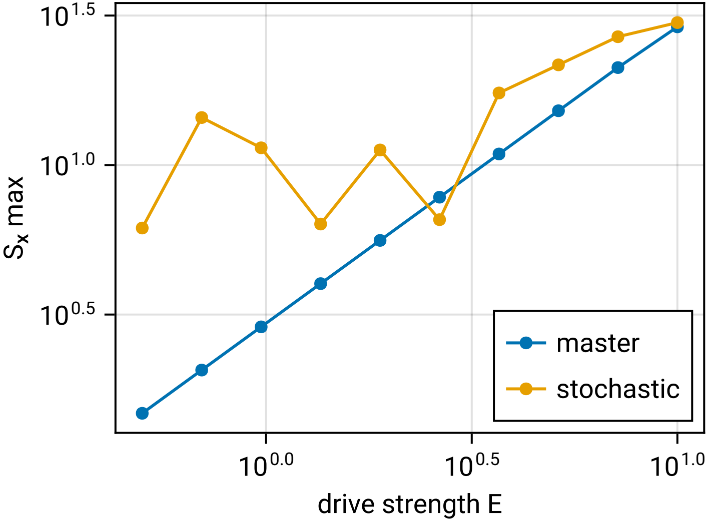
QuantumOptics.jl, Krämer et al. 2024 (Stochastic Master equation, Quantum Zeno Effect)

"A Straightforward Introduction to Continuous Quantum Measurement", Jacobs et al. 2006


master
stochastic
0.4
0.2
0.0
X
0
5
10
T

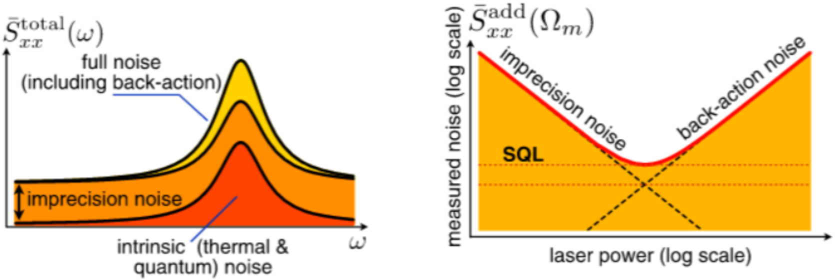


master
stochastic
S_Y
8
6
4
2
0
0.0
0.5
1.0
1.5
2.0
f



$10^{1.5}$
$10^{1.0}$
$10^{0.5}$
S_x max
$10^{0.0}$
$10^{0.5}$
$10^{1.0}$
drive strength E
master
stochastic

Expectation



"Cavity optomechanics", Aspelmeyer et al. 2014 (FIG. 22)

Uncerstanding $S_{\mathrm{det}}(\omega, P_{\mathrm{in}})$

Where is the power dependence?

$$\bar{S}_{\mathrm{det}}(\omega) = \frac{1}{8\eta\Gamma|C_{\mathrm{eff}}|} + 2\Gamma|\chi(\omega)|^2|C_{\mathrm{eff}}|$$

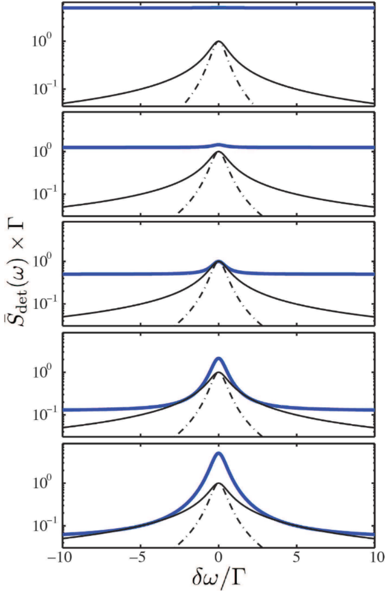
$$C_{\mathrm{eff}}(\omega) = \frac{4g^2}{\kappa\Gamma(1-2i\omega/\kappa)^2}$$

$$\chi(\omega) = \frac{\Omega}{\Omega^2-\omega^2-i\omega\gamma}$$

η: Detection efficiency

$\Gamma = \gamma$: Damping of oscillator

Quantum Optomechanics, Bowen et al. 2015 (eq. 3.51)



$P/P^{\mathrm{SQL}} = \{0.1, 0.4, 1, 4, 10\}$

Quantum Optomechanics, Bowen et al. 2015 (Fig. 3.5)