

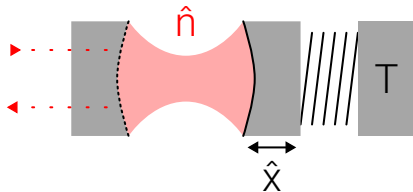
# Noise Analysis

# Optomechanical Cavity

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*Modeling Quantum Hardware: open dynamics and control*  
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# Problem Statement



*"Cavity optomechanics", Aspelmeyer et al. 2014*

*Quantum Optomechanics, Bowen et al. 2015*

# Hamiltonian

Optical Cavity  $\hat{a}$ ,  $\omega_o(\hat{x}_{\text{mech}}) = \omega_o + \frac{g}{\omega_o} \hat{x}_{\text{mech}}$ ; mechanical oscillations  $\hat{b}$ ,  $\omega_m$ ; coupling  $g$ ; Drive  $E$ ,  $\omega_L$

$$H = \underbrace{\omega_o a^\dagger a}_{\text{Cavity}} + \underbrace{\omega_m b^\dagger b}_{\text{Mechanical}} - \underbrace{g a^\dagger a (b + b^\dagger)}_{\text{Interaction}} + \underbrace{E(ae^{i\omega_L t} + a^\dagger e^{-i\omega_L t})}_{\text{Drive}}$$

*Rotating Wave Approximation* at  $\omega_L$  with  $\Delta = \omega_o - \omega_L$ ,  $a \rightarrow ae^{i\omega_L t}$ :

$$H_{\text{RWA}} = \Delta a^\dagger a + \omega_m b^\dagger b - g a^\dagger a (b^\dagger + b) + E(a + a^\dagger)$$

$\hbar = 1$

*Quantum Optomechanics*, Bowen et al. 2015 (2.3)

*QuantumOptics.jl*, Krämer et al. 2024 (Optomechanical Cavity)

# Hamiltonian Linearization (Currently not used)

$$H_{\text{RWA}} = \Delta a^\dagger a + \omega_m b^\dagger b - g a^\dagger a (b^\dagger + b) + E(a + a^\dagger)$$

Linearize  $a = \alpha + \delta a$ ,  $b = \beta + \delta b$ ; with  $\alpha, \beta$  steady state.

$$\begin{aligned} H_{\text{Interaction}} &= -g a^\dagger a (b^\dagger + b) \\ &\approx -\underbrace{g|\alpha|}_G (\delta a + \delta a^\dagger + \mathcal{O}(a^2 + \delta a \delta a^\dagger)) (\delta b + \delta b^\dagger + 2\beta) \\ a + a^\dagger &= |\alpha| + \delta a + \delta a^\dagger \sim \delta a + \delta a^\dagger \end{aligned}$$

Therefore for small  $G$ :

$$\begin{aligned} H_{\text{lin}} &= \Delta \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - G(\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger) + E(a + a^\dagger) \\ &\sim \frac{\Delta}{2}(\hat{X}^2 + \hat{Y}^2) + \frac{\omega}{2}(\hat{Q}^2 + \hat{P}^2) - G\hat{X}\hat{Q} + E\hat{X} \end{aligned}$$

# Linearization in Quadratures

$$\begin{aligned}X &= \delta a + \delta a^\dagger & Q &= \delta b + \delta b^\dagger \\Y &= i(\delta a^\dagger - \delta a) & P &= i(\delta b^\dagger - \delta b) \\n &= \delta a^\dagger a & m &= \delta b^\dagger b\end{aligned}$$

$$H_{\text{RWA}} = \Delta n + \omega m - gnQ + EX$$

$$H_{\text{lin}} = \Delta n + \omega m - GXQ + EX$$

The drive  $EX$  is not getting lost in linearization.  
There is no point in the linearization if solved numerically.

Do I understand it correctly?

# Dissipation

As Lindblad jump operator  $L$  with a coupling to the  $n_T, m_T$  thermal mode with coupling strengths  $\kappa, \gamma$

Optical:

$$L = \sqrt{\kappa(n_T + 1)} \delta a + \sqrt{\kappa n_T} \delta a^\dagger$$

Mechanical:

$$+ \sqrt{\gamma(m_T + 1)} \delta b + \sqrt{\gamma m_T} \delta b^\dagger$$

# Continuous measurement

Lindblad Master Equation:

$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

Stochastic Master Equation:

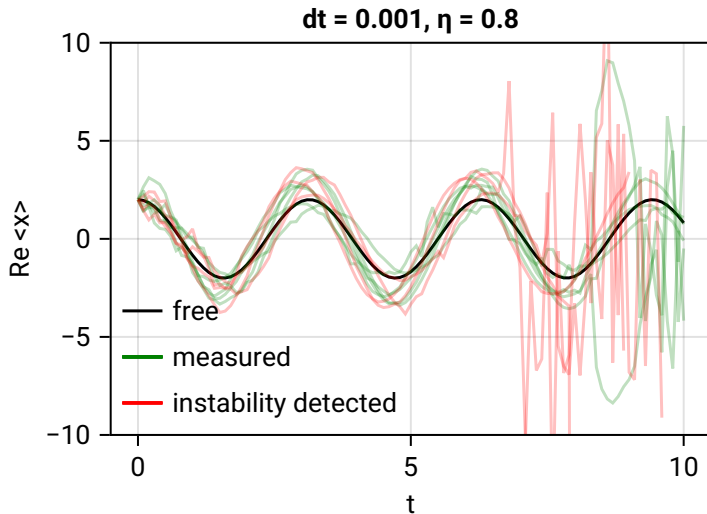
$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\} + (C\rho + \rho C^\dagger - \text{Tr}(C\rho + \rho C^\dagger)) \xi(t)$$

Let's measure the Quadrature  $C = \eta\sqrt{\kappa} (\delta a + \delta a^\dagger)$

Or the non hermitian  $C = \eta\sqrt{\kappa} \delta a$

# Harmonic Oscillator

For stability  $dt \ll 1/\omega$ , therefore much longer compute time (2s).





# Implementation

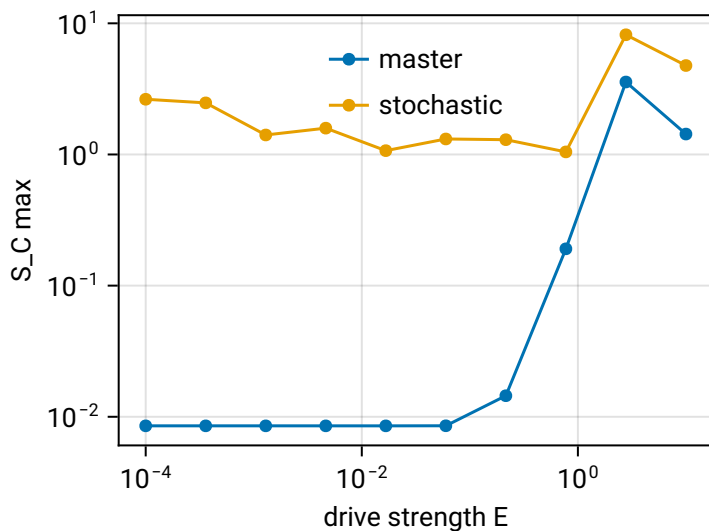
truncated Fock Basis:  $F_{\text{optical}} \otimes F_{\text{mechanical}}$

definition of  $H, J$  with  $\delta a \otimes 1$

$$\psi(0) = |0\rangle \otimes |0\rangle$$

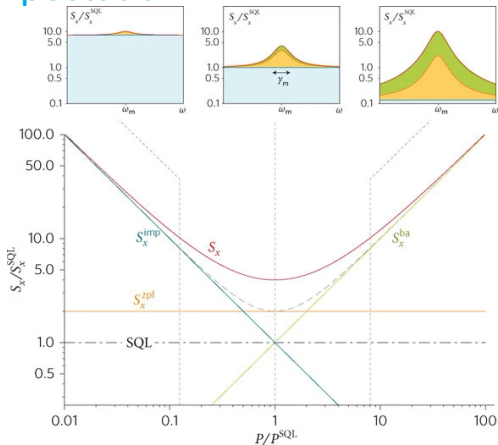
Time Evolution using the *Stochastic Master Equation*

## Power Dependence $\sim G$

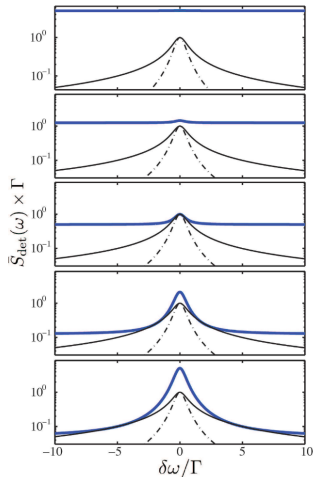


Took **3h** of compute time to be stable!

# Expectation



*"Nanomechanical motion measured with an imprecision below that at the standard quantum limit", Teufel et al. 2009 (Fig. 1)*



$$P/P^{\text{SQL}} = \{0.1, 0.4, 1, 4, 10\}$$

*Quantum Optomechanics, Bowen et al. 2015 (Fig. 3.5)*

# How to procede?

## Pivot to Quantum Zeno Effect

Krämer et al., *QuantumOptics.jl*, Quantum Zeno Effect

## Reduce the complexity?

Carmichael, An open systems approach to quantum optics: Lectures presented at the Université Libre de Bruxelles, 2009

Krämer et al., *QuantumOptics.jl*, Stochastic Schrödinger equation

## Go into depth with the SME

## Adding Poisson Process to detection

Jacobs and Steck, "A Straightforward Introduction to Continuous Quantum Measurement", 10.2

