Quantum Measurement
Zeno Effect

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Modeling Quantum Hardware: open dynamics and control
Universität Konstanz

No phenomenon is a real until it is observed.

– John Archibald Wheeler 1970

Historical Note

1900 Plank & Einstein: Blackbody Radiation

1920 Bohr, Heisenberg: Copenhagen interpretation
Born: Probabilistic interpretation $P(m) = |\langle m|\psi\rangle|^2$
Schrödinger: Measurement Problem

1930 EPR Paradox

1932 von Neumann: *Mathematical Foundations of Quantum Mechanics*

1970 Decoherence Theory

Experimental Interest

Projective (von Neumann) Measurement

Measurement Operator $\hat{A} = \sum m|m\rangle$ on ψ:

$$p(m) = |\langle m|\psi\rangle|^2$$

$$\psi \xrightarrow{\text{Measuring } m} |m\rangle$$

More formally with projector $\hat{M} \sim |m\rangle\langle m|$:

$$\rho' \propto \hat{M}\rho\hat{M}$$

Neglegting Normalization and Degenercy: POVM Measurement

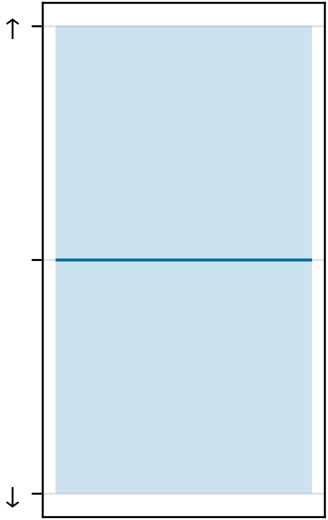
Quantum Computation and Quantum Information, M. A. Nielsen et al. 2010 (2.2.3)

Quantum Measurement and Control, Wiseman et al. 2010 (1)

Example: Superposition

$$H = \sigma_z$$
$$|\psi > \propto |\uparrow\rangle + |\downarrow\rangle$$
$$i\partial_t \psi = H\psi$$

$\Rightarrow$ Superposition is stable



Example: Superposition

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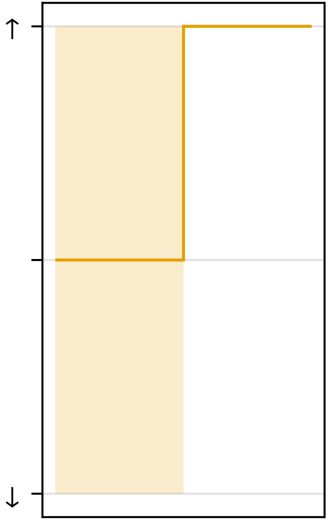
$$M = \sigma_z$$
$$p(\updownarrow) = |\langle \updownarrow |\psi\rangle|^2 = \frac{1}{2}$$



Example: Decay

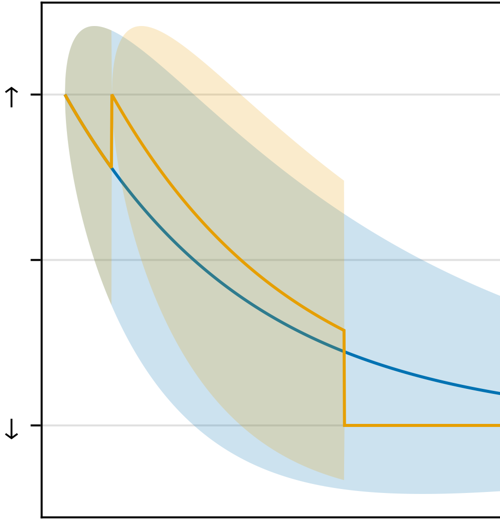
$$H = \sigma_z$$

$$|\psi > \propto |\uparrow\rangle$$

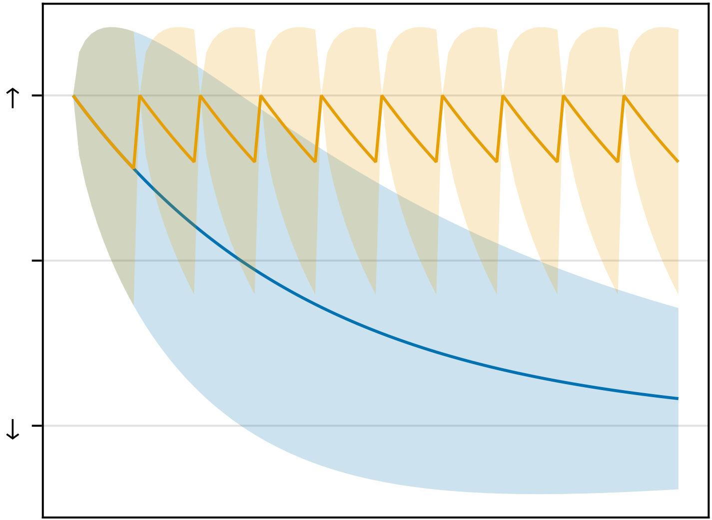
$$M = \sigma_z$$

$$J = \kappa\sigma_-$$

$$\partial_t\rho = -i[H,\rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$



Example: Zeno



Zeno Effect

- Zeno of Elea (460 BCE): Arrow paradox
- Misra and Sudarshan (1977): "The Zeno's paradox in quantum theory"

"Quantum Zeno effect explains magnetic-sensitive radical-ion-pair reactions", Kominis 2009

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"Quantum Zeno effect explains magnetic-sensitive radical-ion-pair reactions", Kominis 2009

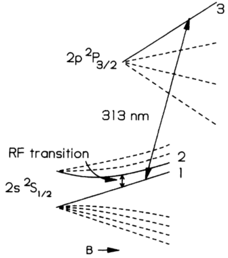


FIG. 2. Diagram of the energy levels of $^9Be^+$ in a magnetic field B. The states labeled 1, 2, and 3 correspond to those in Fig. 1 .

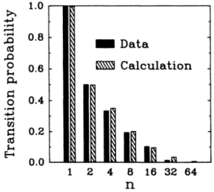


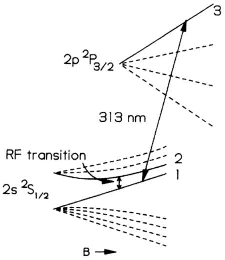
FIG. 3. Graph of the experimental and calculated 1 → 2 transition probabilities as a function of the number of measurement pulses n. The decrease of the transition probabilities with increasing n demonstrates the quantum Zeno effect.

"Quantum Zeno effect", Itano et al. 1990

Zeno Effect

- Zeno of Elea (460 BCE): Arrow paradox
- Misra and Sudarshan (1977): "The Zeno's paradox in quantum theory"
- Experimentally demonstrated (1990) with 5000 $^9Be^+$ ions at 250 mK
- Used in Magnetometers and possibly birds

"Quantum Zeno effect explains magnetic-sensitive radical-ion-pair reactions", Kominis 2009



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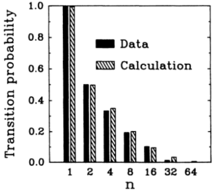

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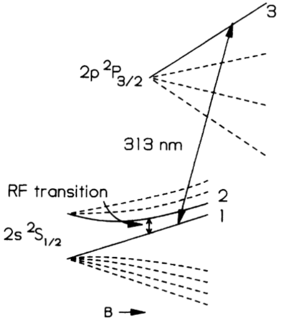
Practical Examples of strong measurements

- Trapped Ions



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Practical Examples of strong measurements

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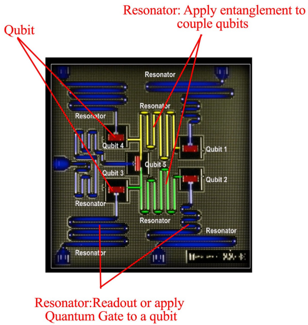
- Superconducting Qubits



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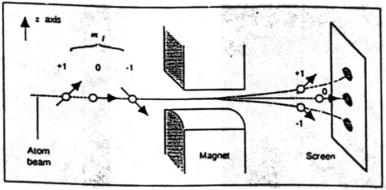
- Stern-Gerlach Experiment



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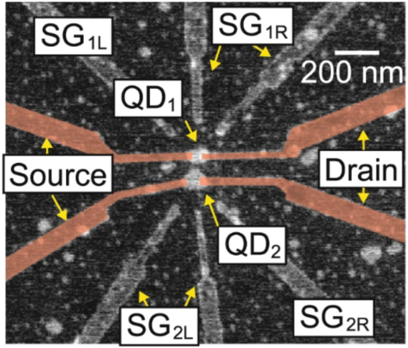
- Superconducting Qubits
- Stern-Gerlach Experiment
- Quantum Dot Charge Readout



Practical Examples of strong measurements

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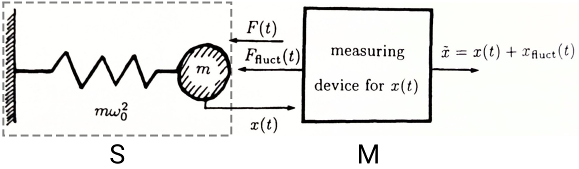
$\Rightarrow$ Used when the quantum wave function is collapsed into a classical result

But breaks on:

- simultaneous measurement of conjugate variables
- optical measurements

Quantum Measurement and Control, Wiseman et al. 2010

Weak Measurement



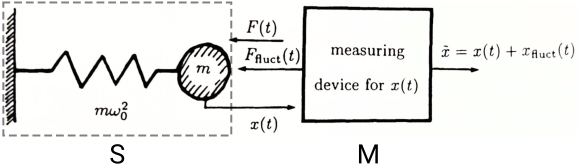
Measured system S and Measurement Device S combined to:

$$H = H_S \otimes 1 + 1 \otimes H_M + g(t)\; C_S \otimes P_M$$

Then a strong measurement X is performed on the M
this yields $\langle C \rangle + \xi$ and partial collapse in S

Quantum measurement, Vladimir Braginsky et al. 1992 (Fig 8.4 modified)
"Introduction to Weak Measurements and Weak Values", Tamir et al. 2013

Weak Measurement



Continuous application leads to *SME* for S:

$$\partial_t \rho = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\} + \left(C\rho + \rho C^\dagger - \mathrm{Tr}(C\rho + \rho C^\dagger)\right) \xi(t)$$

Quantum measurement, Vladimir Braginsky et al. 1992 (Fig 8.4 modified)
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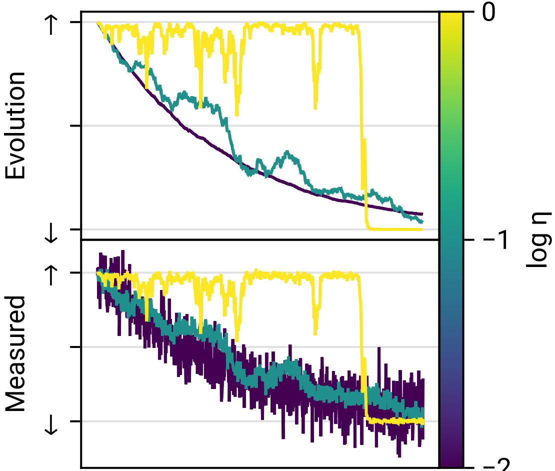
Example: Weak measurement

$$H = \sigma_z$$
$$\psi = \uparrow$$
$$M = \sigma_z$$
$$J = \kappa\sigma_-$$
$$C = \eta\sigma_z$$



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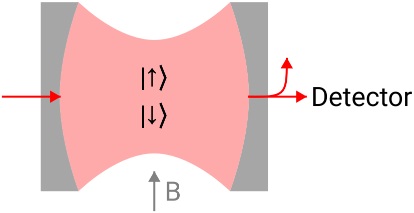


$$H = \sigma_z$$ Atom

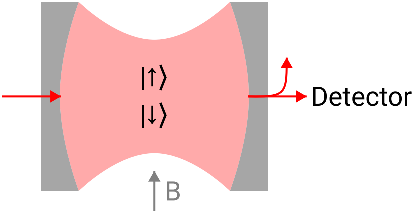
$$+ a^\dagger a + g\sigma_z a^\dagger a$$ M

$$J = \kappa \ a$$ Dissipation

$$C = \kappa\eta \ a$$ Measurement

"Stochastic master equation for a probed system in a cavity", A. E. B. Nielsen et al. 2008

Rabi Oscillations Setup



$$H = g\,(a^\dagger a)(\sigma^+\sigma^-)$$ Coupling

$$+\, g_s\,(\sigma^+ + \sigma^-)$$ Magnetic

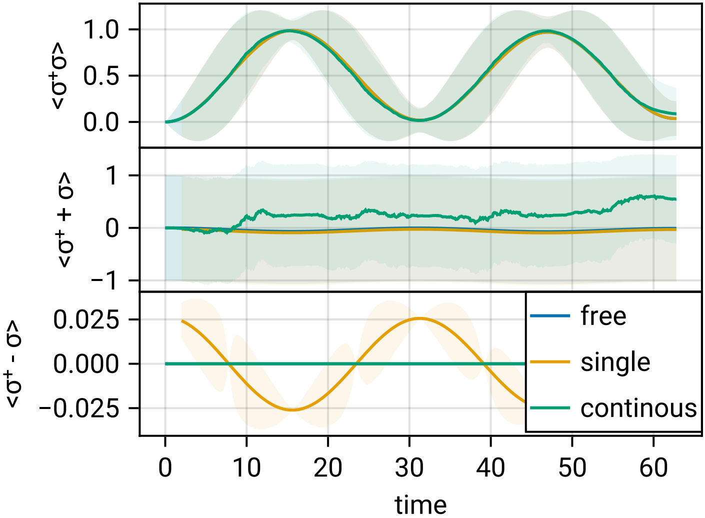
$$-\, i\beta(a^\dagger - a)$$ Optic

$$J = \kappa\ a$$ Dissipation

$$C = \sqrt{\kappa\eta}\ a$$ Measurement

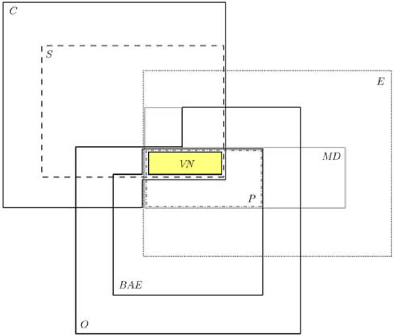
"Stochastic master equation for a probed system in a cavity", A. E. B. Nielsen et al. 2008

Time evolution



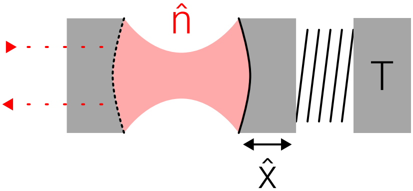
Types of Measurements

Symbol	Name	Definition
E	Efficient	$\forall r, \exists \hat{M}_r,\ \mathcal{O}_r = \mathcal{J}[\hat{M}_r]$
C	Complete	$\forall \rho, \forall r,\ \mathcal{O}_r \rho \propto \mathcal{O}_r \hat{1}$
S	Sharp	$\forall r,\ \mathrm{rank}(\hat{E}_r) = 1$
O	Of an observable X	$\forall r,\ \hat{E}_r = E_r(\hat{X})$
BAE	Back-action-evading	O with $\forall \rho, \forall x \in \lambda(\hat{X}),\ \mathrm{Tr}[\hat{\Pi}_x \rho] = \mathrm{Tr}[\hat{\Pi}_x \mathcal{O} \rho]$
MD	Minimally disturbing	E with $\forall r,\ \hat{M}_r = \hat{M}_r^\dagger$
P	Projective	MD and O
VN	von Neumann	P and S



Quantum Measurement and Control, Wiseman et al. 2010

Problem Statement



"Cavity optomechanics", Aspelmeyer et al. 2014

Quantum Optomechanics, Bowen et al. 2015

Hamiltonian

Optical Cavity $\hat{a}$, $\omega_o(\hat{x}_{\text{mech}}) = \omega_o + \frac{g}{\omega_o}\hat{x}_{\text{mech}}$; mechanical oscillations $\hat{b}, \omega_m$; coupling g; Drive E, ω_L

$$H = \underbrace{\omega_o\ a^\dagger a}_{\text{Cavity}} + \underbrace{\omega_m\ b^\dagger b}_{\text{Mechanical}} - \underbrace{g\ a^\dagger a\,(b + b^\dagger)}_{\text{Interaction}} + \underbrace{E(ae^{i\omega_L t} + a^\dagger e^{-i\omega_L t})}_{\text{Drive}}$$

Rotating Wave Approximation at ω_L with $\Delta = \omega_o - \omega_L$, $a \to ae^{i\omega_L t}$:

$$H_{\text{RWA}} = \Delta\ a^\dagger a + \omega_m\ b^\dagger b - g\ a^\dagger a\,(b^\dagger + b) + E(a + a^\dagger)$$

$\hbar = 1$

Quantum Optomechanics, Bowen et al. 2015 (2.3)

QuantumOptics.jl, Krämer et al. 2024 (Optomechanical Cavity)

Hamiltonian Linearization (Currently not used)

$$H_{\text{RWA}} = \Delta\ a^\dagger a + \omega_m\ b^\dagger b - g\ a^\dagger a\,(b^\dagger + b) + E(a + a^\dagger)$$

Linearize $a = \alpha + \delta a$, $b = \beta + \delta b$; with α, β steady state.

$$\begin{aligned} H_{\text{Interaction}} &= -g\ a^\dagger a\,(b^\dagger + b) \\ &\approx -\underbrace{g|\alpha|}_{G}\left(\delta a + \delta a^\dagger + \mathcal{O}(a^2 + \delta a \delta a^\dagger)\right)\left(\delta b + \delta b^\dagger + 2\beta\right) \end{aligned}$$

$$a + a^\dagger = |\alpha| + \delta a + \delta a^\dagger \sim \delta a + \delta a^\dagger$$

Therefore for small G:

$$\begin{aligned} H_{\text{lin}} &= \Delta\ \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - G(\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger) + E(a + a^\dagger) \\ &\sim \frac{\Delta}{2}(\hat{X}^2 + \hat{Y}^2) + \frac{\omega}{2}(\hat{Q}^2 + \hat{P}^2) - G\hat{X}\hat{Q} + E\hat{X} \end{aligned}$$

Linearization in Quadratures

$$X = \delta a + \delta a^\dagger \quad Q = \delta b + \delta b^\dagger$$
$$Y = i(\delta a^\dagger - \delta a) \quad P = i(\delta b^\dagger - \delta b)$$
$$n = \delta a^\dagger a \quad m = \delta b^\dagger b$$

$$H_{\mathrm{RWA}} = \Delta n + \omega m - gnQ + EX$$
$$H_{\mathrm{lin}} = \Delta n + \omega m - GXQ + EX$$

The drive EX is not getting lost in linearization.
There is no point in the simplification if solved numerically.

Dissipation

Optical decay κ:

$$L = \sqrt{\kappa(n_T + 1)}\ \delta a + \sqrt{\kappa n_T}\ \delta a^\dagger$$

Mechanical resonator with γ and a thermal bath at the n-th thermal state:

$$+\sqrt{\gamma(m_T + 1)}\ \delta b + \sqrt{\gamma m_T}\ \delta b^\dagger$$

Quantum Optomechanics, Bowen et al. 2015 (2.8)

Implementation

truncated Fock Basis: $F_{\text{optical}} \otimes F_{\text{mechanical}}$

definition of H, J with $\delta a \otimes 1$

$\psi(0) = |0\rangle \otimes |0\rangle$

Time Evolution using the *Lindblad equation*:

$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

QuantumOptics.jl, Krämer et al. 2024

Continous measurement

Lindblad Master Equation:

$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

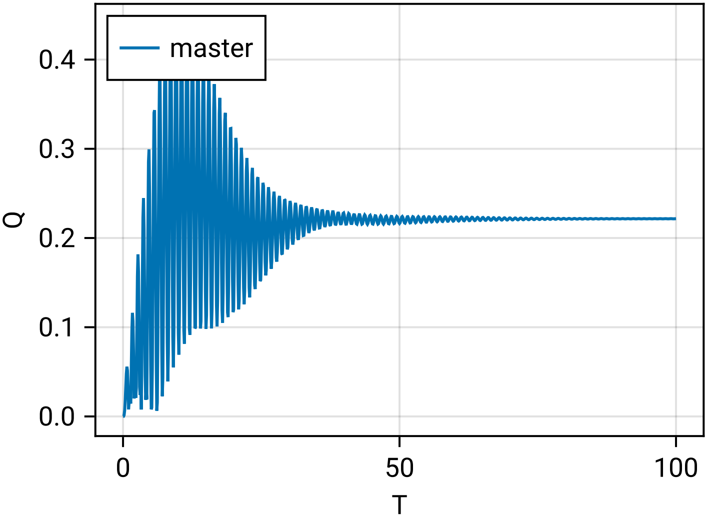
Stochastic Master Equation:

$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\} + \left(C\rho + \rho C^\dagger - \mathrm{Tr}(C\rho + \rho C^\dagger)\right) \xi(t)$$

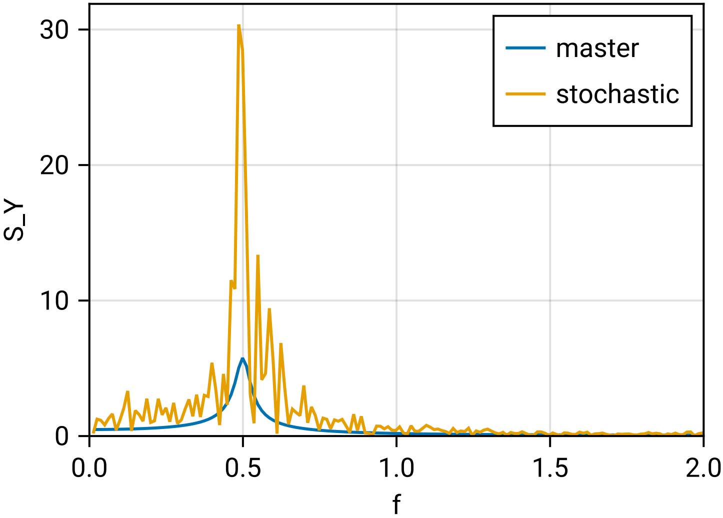
Let's look at the Quadrature $C = \eta \sqrt{\kappa}\,(\delta a + \delta a^\dagger)$

QuantumOptics.jl, Krämer et al. 2024 (Stochastic Master equation, Quantum Zeno Effect)
"A Straightforward Introduction to Continuous Quantum Measurement", Jacobs et al. 2006

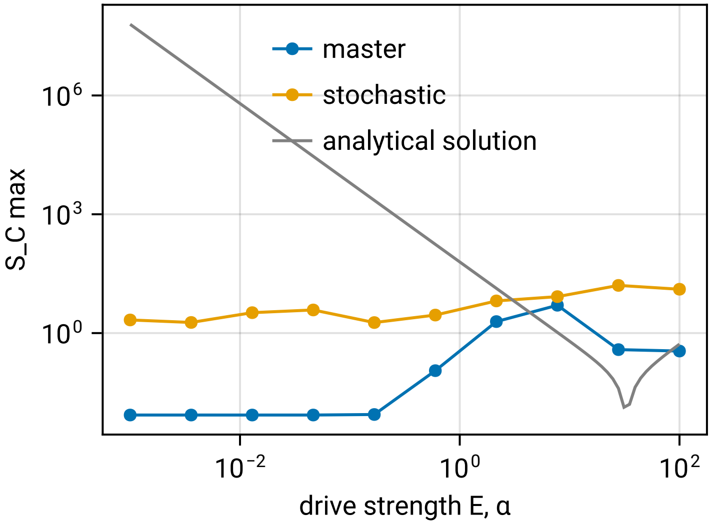
Time Evolution



Spectrum



Power Dependence ∼ G

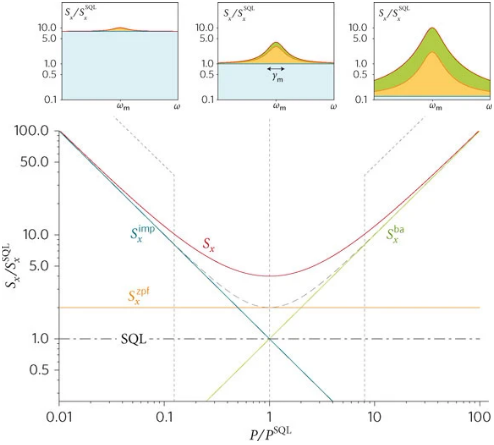


Took ≈ 1 min of compute time. Why is the SME so much slower? True Random Values?

Expectation

$$S_x = S_x^{\text{imp}} + S_x^{\text{ba}} + S_x^{\text{zpf}}$$

$S_x^{\text{imp}} \propto P^{-1}\omega^0$: Imprecision / Shot noise
$S_x^{\text{ba}} \propto P$: Back action
S_x^{zpf}: Zero Point Fluctuation



"Nanomechanical motion measured with an imprecision below that at the standard quantum limit", Teufel et al. 2009 (Fig. 1)

Uncerstanding $S_{\text{det}}(\omega, P_{\text{in}})$

Where is the power dependence?

$$\bar{S}_{\text{det}}(\omega) = \frac{1}{8\eta\Gamma|C_{\text{eff}}|} + 2\Gamma|\chi(\omega)|^2|C_{\text{eff}}|$$

$$C_{\text{eff}}(\omega) = \frac{4g^2}{\kappa\Gamma(1-2i\omega/\kappa)^2}$$

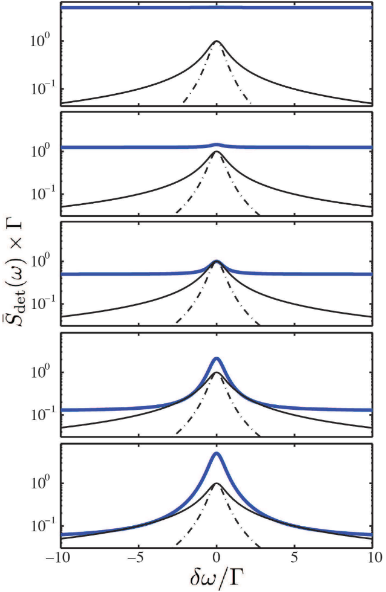
$$\chi(\omega) = \frac{\Omega}{\Omega^2-\omega^2-i\omega\gamma}$$

η: Detection efficiency

$\Gamma = \gamma$: Damping of oscillator

$$\bar{S}_{\text{det}}(\omega) = \frac{\kappa\Gamma|1-2i\omega/\kappa|}{8\eta\Gamma 4g^2} + 2\Gamma\frac{\Omega^2}{|\Omega^2-\omega^2-i\omega\gamma|^2}\frac{4g^2}{\kappa\Gamma(1-2i\omega/\kappa)^2}$$

Quantum Optomechanics, Bowen et al. 2015 (eq. 3.51)



$P/P^{\text{SQL}} = \{0.1, 0.4, 1, 4, 10\}$

Quantum Optomechanics, Bowen et al. 2015 (Fig. 3.5)

Looking for a source that derives $S_{\mathrm{det}}(\omega, P)$?