

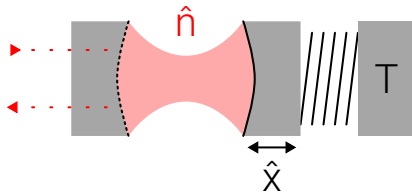
# Noise Analysis

# Optomechanical Cavity

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*Modeling Quantum Hardware: open dynamics and control*  
Universität Konstanz

# Problem Statement



*"Cavity optomechanics", Aspelmeyer et al. 2014*

*Quantum Optomechanics, Bowen et al. 2015*

# Hamiltonian

Optical Cavity  $\hat{a}$ ,  $\omega_o(\hat{x}_{\text{mech}}) = \omega_o + \frac{g}{\omega_o} \hat{x}_{\text{mech}}$ ; mechanical oscillations  $\hat{b}$ ,  $\omega_m$ ; coupling  $g$ ; Drive  $E$ ,  $\omega_L$

$$H = \underbrace{\omega_o a^\dagger a}_{\text{Cavity}} + \underbrace{\omega_m b^\dagger b}_{\text{Mechanical}} - \underbrace{g a^\dagger a (b + b^\dagger)}_{\text{Interaction}} + \underbrace{E(a e^{i\omega_L t} + a^\dagger e^{-i\omega_L t})}_{\text{Drive}}$$

$$\hbar = 1$$

*Quantum Optomechanics*, Bowen et al. 2015 (2.3)

*QuantumOptics.jl*, (Optomechanical Cavity)

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*Rotating Wave Approximation* at  $\omega_L$  with  $\Delta = \omega_o - \omega_L$ ,  $a \rightarrow ae^{i\omega_L t}$ :

$$H_{\text{RWA}} = \Delta a^\dagger a + \omega_m b^\dagger b - g a^\dagger a (b^\dagger + b) + E(a + a^\dagger)$$

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# Hamiltonian Linearization

$$H_{\text{RWA}} = \Delta a^\dagger a + \omega_m b^\dagger b - g a^\dagger a (b^\dagger + b) + E(a + a^\dagger)$$

Linearize  $a = \alpha + \delta a$ ,  $b = \beta + \delta b$ ; with  $\alpha, \beta$  steady state.

$$\begin{aligned} H_{\text{Interaction}} &= -g a^\dagger a (b^\dagger + b) \\ &\approx -\underbrace{g|\alpha|}_G (\delta a + \delta a^\dagger + \mathcal{O}(a^2 + \delta a \delta a^\dagger)) (\delta b + \delta b^\dagger + 2\beta) \end{aligned}$$

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Therefore for small  $G$ :

$$H \approx \Delta \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - G(\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger)$$

# Dissipation

Optical decay  $\kappa$ :

$$J_O = \sqrt{\kappa} \delta a$$

Mechanical resonator with  $\gamma$  and a thermal bath at the  $n$ -th thermal state:

$$J_M = \sqrt{\gamma(n+1)} \delta b + \sqrt{\gamma n} \delta b^\dagger$$

# Implementation

1. truncated Fock Basis:  $F_{\text{optical}} \otimes F_{\text{mechanical}}$
2. definition of  $H, J$  with  $\delta a \otimes 1$
3. Master Time Evolution:

$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}J^\dagger J\rho - \frac{1}{2}\rho J^\dagger J$$



## Time evolution

