

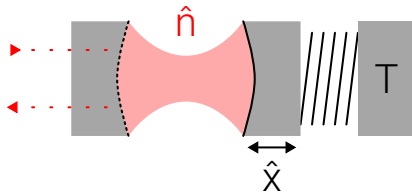
Noise Analysis

Optomechanical Cavity

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02.01.2025

Modeling Quantum Hardware: open dynamics and control
Universität Konstanz

Problem Statement



"Cavity optomechanics", Aspelmeyer et al. 2014

Quantum Optomechanics, Bowen et al. 2015

Hamiltonian

Optical Cavity \hat{a} , $\omega_o(\hat{x}_{\text{mech}}) = \omega_o + \frac{g}{\omega_o} \hat{x}_{\text{mech}}$; mechanical oscillations \hat{b} , ω_m ; coupling g ; Drive E , ω_L

$$H = \underbrace{\omega_o a^\dagger a}_{\text{Cavity}} + \underbrace{\omega_m b^\dagger b}_{\text{Mechanical}} - \underbrace{g a^\dagger a (b + b^\dagger)}_{\text{Interaction}} + \underbrace{E(a e^{i\omega_L t} + a^\dagger e^{-i\omega_L t})}_{\text{Drive}}$$

$$\hbar = 1$$

Quantum Optomechanics, Bowen et al. 2015 (2.3)

QuantumOptics.jl, (Optomechanical Cavity)

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Rotating Wave Approximation at ω_L with $\Delta = \omega_o - \omega_L$, $a \rightarrow ae^{i\omega_L t}$:

$$H_{\text{RWA}} = \Delta a^\dagger a + \omega_m b^\dagger b - g a^\dagger a (b^\dagger + b) + E(a + a^\dagger)$$

$$\hbar = 1$$

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QuantumOptics.jl, (Optomechanical Cavity)

Hamiltonian Linearization

$$H_{\text{RWA}} = \Delta a^\dagger a + \omega_m b^\dagger b - g a^\dagger a (b^\dagger + b) + E(a + a^\dagger)$$

Linearize $a = \alpha + \delta a$, $b = \beta + \delta b$; with α, β steady state.

$$\begin{aligned} H_{\text{Interaction}} &= -g a^\dagger a (b^\dagger + b) \\ &\approx -\underbrace{g|\alpha|}_G (\delta a + \delta a^\dagger + \mathcal{O}(a^2 + \delta a \delta a^\dagger)) (\delta b + \delta b^\dagger + 2\beta) \end{aligned}$$

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Therefore for small G :

$$H \approx \Delta \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - G(\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger)$$

Dissipation

Optical decay κ :

$$L_O = \sqrt{\kappa} \delta a$$

Mechanical resonator with γ and a thermal bath at the n -th thermal state:

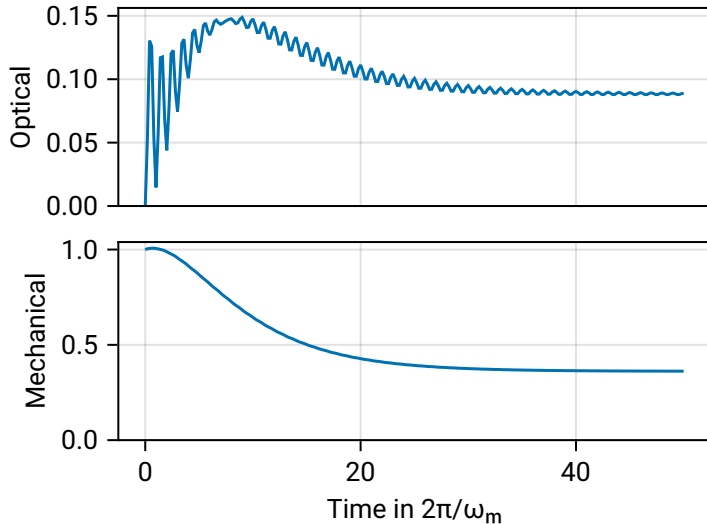
$$L_M = \sqrt{\gamma(n+1)} \delta b + \sqrt{\gamma n} \delta b^\dagger$$

Implementation

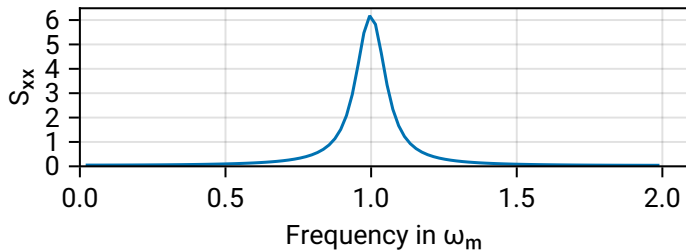
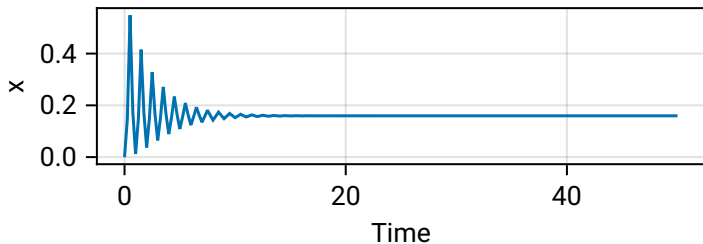
1. truncated Fock Basis: $F_{\text{optical}} \otimes F_{\text{mechanical}}$
2. definition of H, J with $\delta a \otimes 1$
3. Time Evolution using the *Lindblad equation*:

$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

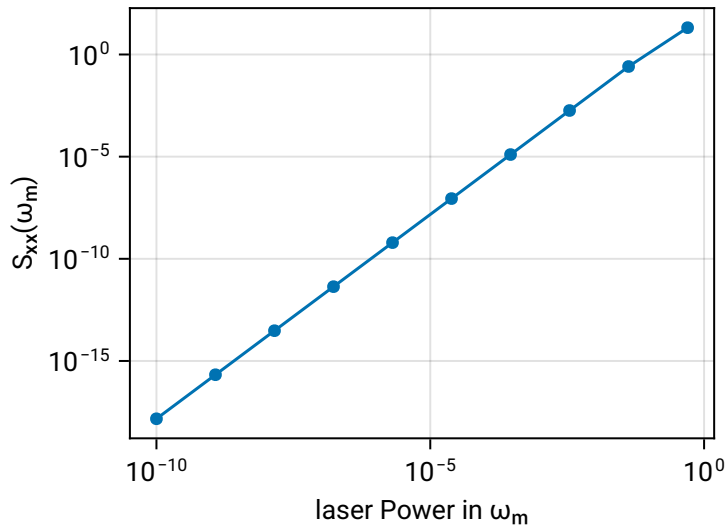
Time evolution of n



Spectrum



Power Scaling



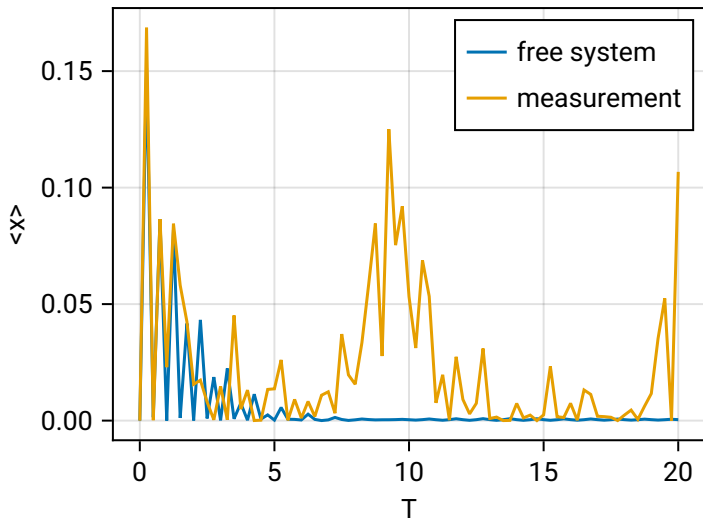
Stochastic Master Equation

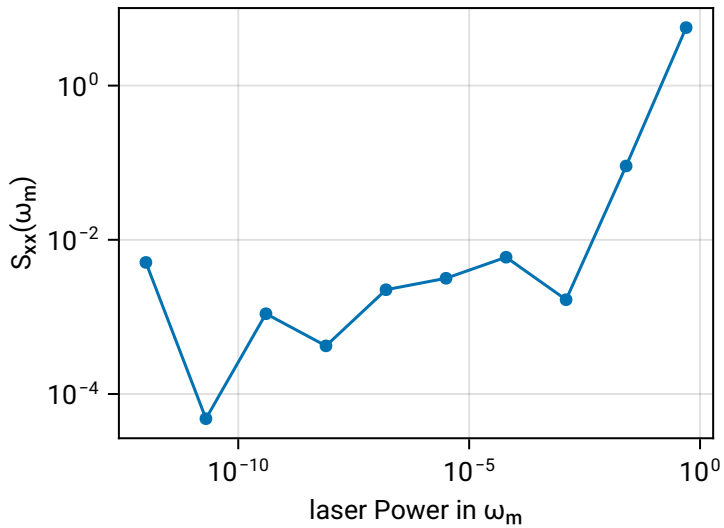
Lindblad Master Equation:

$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

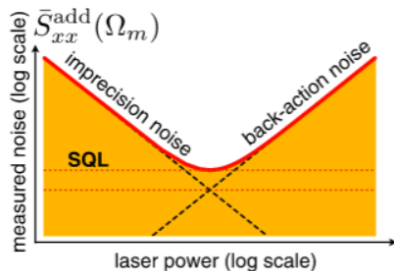
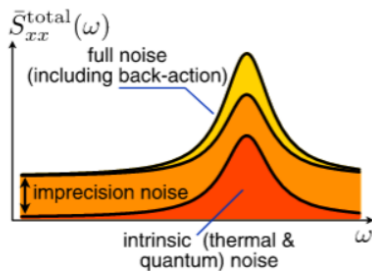
$$\dot{\rho} = -i[H, \rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\} + (C\rho + \rho C^\dagger - \text{Tr}(C\rho + \rho C^\dagger)) \xi(t)$$

Let's look at the Quadrature $C = \eta\sqrt{\kappa} (\delta a + \delta a^\dagger)$





Expectation



How to add the imprecision noise?

