Noise Analysis
Optomechanical Cavity

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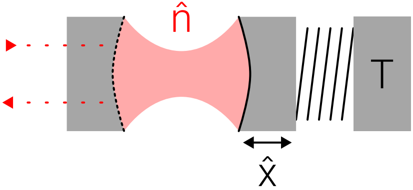
Modeling Quantum Hardware: open dynamics and control
Universität Konstanz

Problem Statement



"Cavity optomechanics", Aspelmeyer et al. 2014

Quantum Optomechanics, Bowen et al. 2015

Hamiltonian

Optical Cavity $\hat{a}$, $\omega_o(\hat{x}_{\text{mech}}) = \omega_o + \frac{g}{\omega_o}\hat{x}_{\text{mech}}$; mechanical oscillations $\hat{b}, \omega_m$; coupling g; Drive E, ω_L

$$H = \underbrace{\omega_o \ a^\dagger a}_{\text{Cavity}} + \underbrace{\omega_m \ b^\dagger b}_{\text{Mechanical}} - \underbrace{g \ a^\dagger a \, (b + b^\dagger)}_{\text{Interaction}} + \underbrace{E(a e^{i\omega_L t} + a^\dagger e^{-i\omega_L t})}_{\text{Drive}}$$

Rotating Wave Approximation at ω_L with $\Delta = \omega_o - \omega_L$, $a \rightarrow a e^{i\omega_L t}$:

$$H_{\text{RWA}} = \Delta \ a^\dagger a + \omega_m \ b^\dagger b - g \ a^\dagger a \, (b^\dagger + b) + E(a + a^\dagger)$$

$\hbar = 1$

Quantum Optomechanics, Bowen et al. 2015 (2.3)

QuantumOptics.jl, Krämer et al. 2024 (Optomechanical Cavity)

Hamiltonian Linearization (Currently not used)

$$H_{\text{RWA}} = \Delta\ a^\dagger a + \omega_m\ b^\dagger b - g\ a^\dagger a\,(b^\dagger + b) + E(a + a^\dagger)$$

Linearize $a = \alpha + \delta a$, $b = \beta + \delta b$; with α, β steady state.

$$\begin{aligned} H_{\text{Interaction}} &= -g\ a^\dagger a\,(b^\dagger + b) \\ &\approx -\underbrace{g|\alpha|}_{G}\left(\delta a + \delta a^\dagger + \mathcal{O}(a^2 + \delta a \delta a^\dagger)\right)\left(\delta b + \delta b^\dagger + 2\beta\right) \end{aligned}$$

$$a + a^\dagger = |\alpha| + \delta a + \delta a^\dagger \sim \delta a + \delta a^\dagger$$

Therefore for small G:

$$\begin{aligned} H_{\text{lin}} &= \Delta\ \delta a^\dagger \delta a + \omega_m \delta b^\dagger \delta b - G(\delta a + \delta a^\dagger)(\delta b + \delta b^\dagger) + E(a + a^\dagger) \\ &\sim \frac{\Delta}{2}(\hat{X}^2 + \hat{Y}^2) + \frac{\omega}{2}(\hat{Q}^2 + \hat{P}^2) - G\hat{X}\hat{Q} + E\hat{X} \end{aligned}$$

Linearization in Quadratures

$$X = \delta a + \delta a^\dagger \quad Q = \delta b + \delta b^\dagger$$
$$Y = i(\delta a^\dagger - \delta a) \quad P = i(\delta b^\dagger - \delta b)$$
$$n = \delta a^\dagger a \quad m = \delta b^\dagger b$$

$$H_{\text{RWA}} = \Delta n + \omega m - gnQ + EX$$
$$H_{\text{lin}} = \Delta n + \omega m - GXQ + EX$$

The drive EX is not getting lost in linearization.
There is no point in the simplification if solved numerically.

Dissipation

Optical decay κ:

$$L = \sqrt{\kappa(n_T + 1)}\, \delta a + \sqrt{\kappa n_T}\, \delta a^\dagger$$

Mechanical resonator with γ and a thermal bath at the n-th thermal state:

$$+\sqrt{\gamma(m_T + 1)}\, \delta b + \sqrt{\gamma m_T}\, \delta b^\dagger$$

Quantum Optomechanics, Bowen et al. 2015 (2.8)

Implementation

truncated Fock Basis: $F_{\text{optical}} \otimes F_{\text{mechanical}}$

definition of H, J with $\delta a \otimes 1$

$\psi(0) = |0\rangle \otimes |0\rangle$

Time Evolution using the *Lindblad equation*:

$$\dot{\rho} = -i[H, \rho] + J\rho J^{\dagger} - \frac{1}{2}\{J^{\dagger}J, \rho\}$$

QuantumOptics.jl, Krämer et al. 2024

Continous measurement

Lindblad Master Equation:

$$\dot{\rho} = -i[H,\rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\}$$

Stochastic Master Equation:

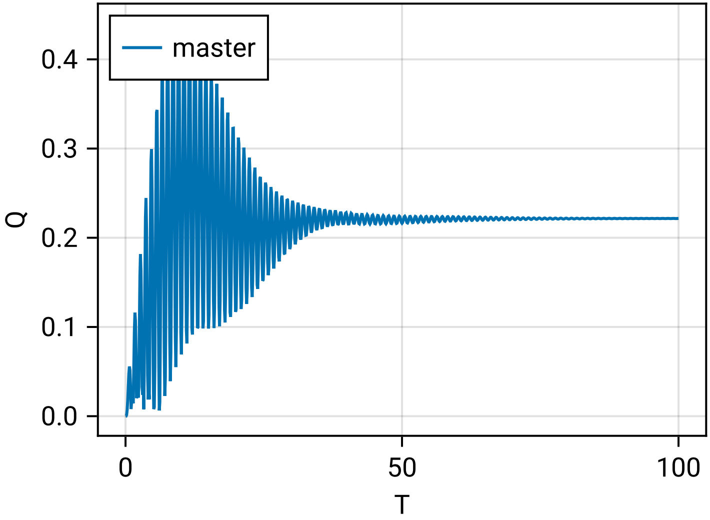
$$\dot{\rho} = -i[H,\rho] + J\rho J^\dagger - \frac{1}{2}\{J^\dagger J, \rho\} + \left(C\rho + \rho C^\dagger - \mathrm{Tr}(C\rho + \rho C^\dagger)\right) \xi(t)$$

Let's look at the Quadrature $C = \eta \sqrt{\kappa}\,(\delta a + \delta a^\dagger)$

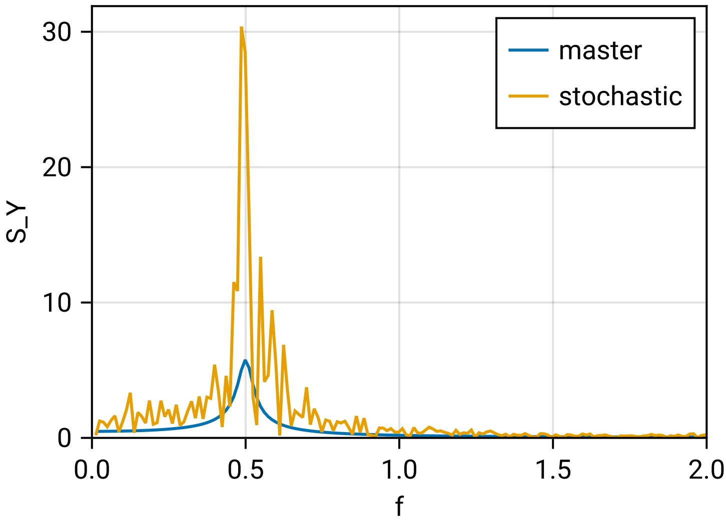
QuantumOptics.jl, Krämer et al. 2024 (Stochastic Master equation, Quantum Zeno Effect)

"A Straightforward Introduction to Continuous Quantum Measurement", Jacobs et al. 2006

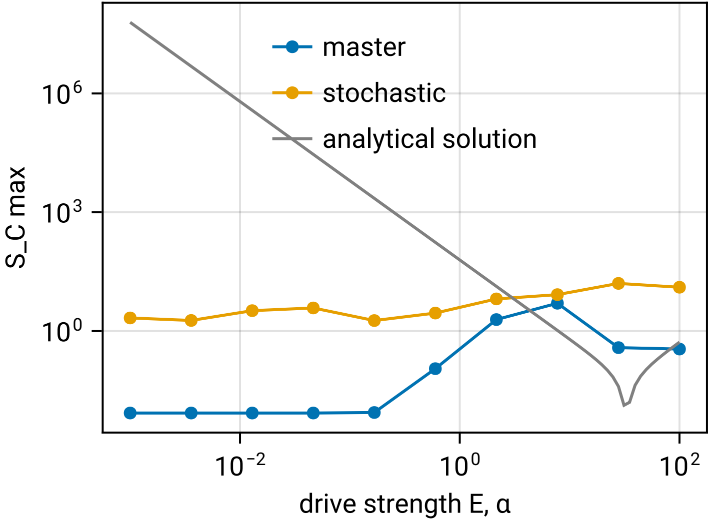
Time Evolution



Spectrum



Power Dependence ~ G

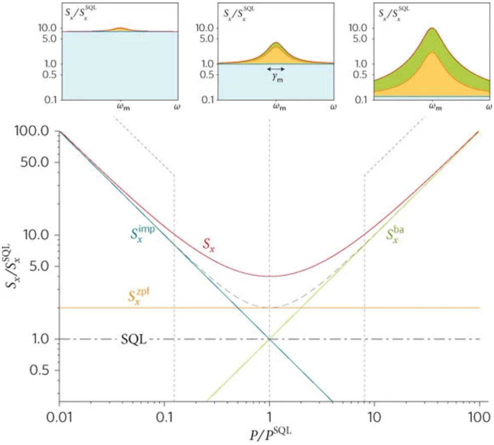


Took ≈ 1 min of compute time. Why is the SME so much slower? True Random Values?

Expectation

$$S_x = S_x^{\text{imp}} + S_x^{\text{ba}} + S_x^{\text{zpf}}$$

$S_x^{\text{imp}} \propto P^{-1}\omega^0$: Imprecision / Shot noise
$S_x^{\text{ba}} \propto P$: Back action
S_x^{zpf}: Zero Point Fluctuation



"Nanomechanical motion measured with an imprecision below that at the standard quantum limit", Teufel et al. 2009 (Fig. 1)

Uncerstanding $S_{\text{det}}(\omega, P_{\text{in}})$

Where is the power dependence?

$$\bar{S}_{\text{det}}(\omega) = \frac{1}{8\eta\Gamma|C_{\text{eff}}|} + 2\Gamma|\chi(\omega)|^2|C_{\text{eff}}|$$

$$C_{\text{eff}}(\omega) = \frac{4g^2}{\kappa\Gamma(1-2i\omega/\kappa)^2}$$

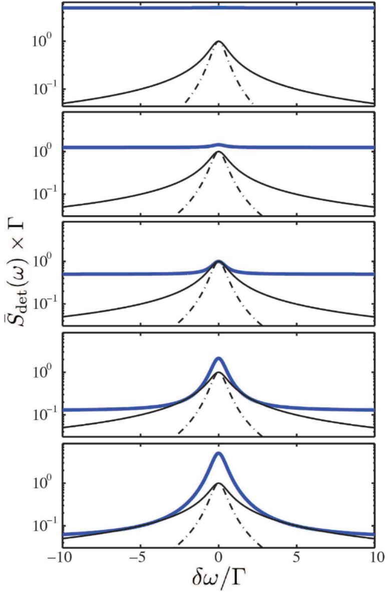
$$\chi(\omega) = \frac{\Omega}{\Omega^2-\omega^2-i\omega\gamma}$$

η: Detection efficiency

$\Gamma = \gamma$: Damping of oscillator

$$\bar{S}_{\text{det}}(\omega) = \frac{\kappa\Gamma|1-2i\omega/\kappa|}{8\eta\Gamma 4g^2} + 2\Gamma\frac{\Omega^2}{|\Omega^2-\omega^2-i\omega\gamma|^2}\frac{4g^2}{\kappa\Gamma(1-2i\omega/\kappa)^2}$$

Quantum Optomechanics, Bowen et al. 2015 (eq. 3.51)



$P/P^{\text{SQL}} = \{0.1, 0.4, 1, 4, 10\}$

Quantum Optomechanics, Bowen et al. 2015 (Fig. 3.5)

Looking for a source that derives $S_{\text{det}}(\omega, P)$?
Or even states $S_{\text{det}}(\omega, P)$.

Or, selecting a simpler system?

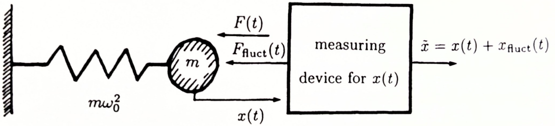


Fig. 8.4 Detection of a classical force by monitoring the coordinate of an oscillator on which it acts.

Not simulating the detector, just the oscillator and adding the Backaction in the measurement operator . . .

Quantum measurment, Vladimir Braginsky et al. 1992 (Fig. 8.4)