

Auto correlation of Gaussian signal

$\eta(t) = a \mathcal{N}(0)$ for every t independent

$$\langle \eta_i(t) \eta_j(t') \rangle = b \delta_{ij} \delta(t-t')$$

how does b depend on a, σ^2 ?

$$\eta(t) \eta^*(t-\tau) =$$

$$\int_{-\infty}^{\infty} \eta_i(t) \eta_i^*(t) dt$$

$$= \int_{-\infty}^{\infty} \eta_i(t)^2 dt = a \langle \mathcal{N}(\sigma^2)^2 \rangle$$

$\stackrel{= a \sigma^2}{\text{see def. of variance}}$ expected value

Langevin equation: $m \frac{d\vec{v}}{dt} = -A \vec{v} + \vec{\eta}(t)$

with $\eta_i(t) = A \mathcal{N}(\sigma^2)$

and $\langle \eta_i(t) \eta_j(t+\tau) \rangle$

$$= 2 \lambda k_B T \delta_{ij} \delta(\tau) = A \sigma^2$$