CS4186 ASM2

Question 1:

You are using **RANSAC** to fit a **linear regression model** to a dataset that contains **70% inliers** and **30% outliers**. The minimal sample size required to fit a line is **2 points**.

1. **Probability of a Good Sample:**

What is the probability that a randomly selected minimal sample (2 points) consists of **only inliers**?

2. **Iterations Needed for Reliable Fitting:**

How many RANSAC iterations are needed to ensure **at least 95% confidence** that at least one of the selected samples has no outliers?

Ans:

1. Since the required minimum sample size is 2 points, the probability that both points are inliers is equal to 0.49 (0.7×0.7).

2. Probability of a good sample with both inliers:

   $P(\text{good}) = 0.49$

   Probability of a bad sample any outliers:

   $P(\text{bad}) = 1 - P(good) = 0.51$

   Probability that all n samples are bad:

   $P(\text{all bad}) = (0.51)^n$

   Probability that at least one good sample in all n samples:

   $P(\text{at least one good}) = 1 - P(\text{all bad}) = 1 - (0.51)^n$

   To ensure that in all n samples, at least 95% confidence that at least one of the selected samples has no outliers, we have $P(\text{at least one good}) \geq 0.95$. Thus:

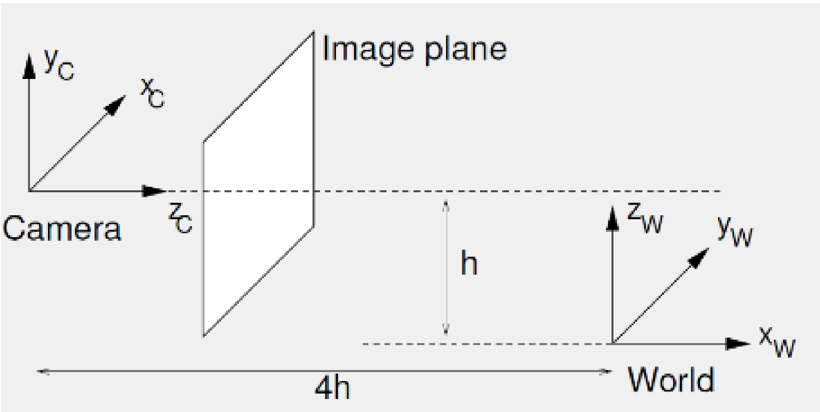
$$
\begin{gathered}
1-(0.51)^n \geq 0.95 \\
(0.51)^n \leq 0.05 \\
n \cdot log(0.51) \leq log(0.05) \\
n \geq \frac{log(0.05)}{log(0.51)} \\
n \geq 4.45
\end{gathered}
$$

Since n must be an integer, $n = 5$.

Therefore, 5 iterations are needed to ensure at least 95% confidence that at least one sample has no outliers.

Question 2:

The camera shown in the following figure has its x, y and z axes aligned with the world's y, z and x axes respectively. The world frame's origin is at (0, -h, 4h) in the camera's frame.



1. Find the camera's extrinsic camera calibration matrix [R T], such that

$$X_c = \begin{bmatrix} R & T \\ 0^T & 1 \end{bmatrix} X_W$$

2. Assuming that intrinsic camera matrix K is just a 3 x 3 identity matrix. Derive the image coordinates of the vanishing point of the family of lines parallel to the following line, expressed parametrically as:

$$(X_W, Y_W, Z_W) = (2 + 4t, 3 + 2t, 4 + 3t)$$

Ans:

1. The world origin in the camera frame $= (0, -h, 4h)$, we have:

$$T = \begin{bmatrix} 0 \\ -h \\ 4h \end{bmatrix}$$

Since the camera's x, y, and z axes are aligned with the world's y, z, and x axes respectively, we have:

$$R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

The extrinsic matrix with $T = \begin{bmatrix} 0 \\ -h \\ 4h \end{bmatrix}$ and $R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$:

$$\begin{bmatrix} R & T \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -h \\ 1 & 0 & 0 & 4h \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. We have $X_W = \begin{bmatrix} 2+4t \\ 3+2t \\ 4+3t \\ 1 \end{bmatrix}$.

$$X_c = \begin{bmatrix} R & T \\ 0^T & 1 \end{bmatrix} X_W$$

$$X_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -h \\ 1 & 0 & 0 & 4h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2+4t \\ 3+2t \\ 4+3t \\ 1 \end{bmatrix}$$

$$X_c = \begin{bmatrix} 3+2t \\ 4+3t-h \\ 2+4t+4h \\ 1 \end{bmatrix}$$

Therefore, $X_c = (x_c, y_c, z_c) = (3+2t, 4+3t-h, 2+4t+4h)$.

Since the vanishing point corresponds to $t \to \infty$, we rewrite X_c into:

$$X_c = \begin{bmatrix} 3+2t \\ 4+3t-h \\ 2+4t+4h \end{bmatrix} = t \begin{bmatrix} \frac{3}{t}+2 \\ \frac{4}{t}+3-\frac{h}{t} \\ \frac{2}{t}+4+\frac{4h}{t} \end{bmatrix}$$

For $t \to \infty$,

$$X_c \approx t \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

Since the the intrinsic matrix K is identity, the vanishing point X in homogeneous coordinates:

$$X = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

The vanishing point X_{img} in image plane coordinates:

$$X_{\text{img}} = \left(\frac{x_c}{z_c}, \frac{y_c}{z_c}\right) = \left(\frac{2}{4}, \frac{3}{4}\right) = (0.5, 0.75)$$