CSC421: Written Homework 1

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Problem 1

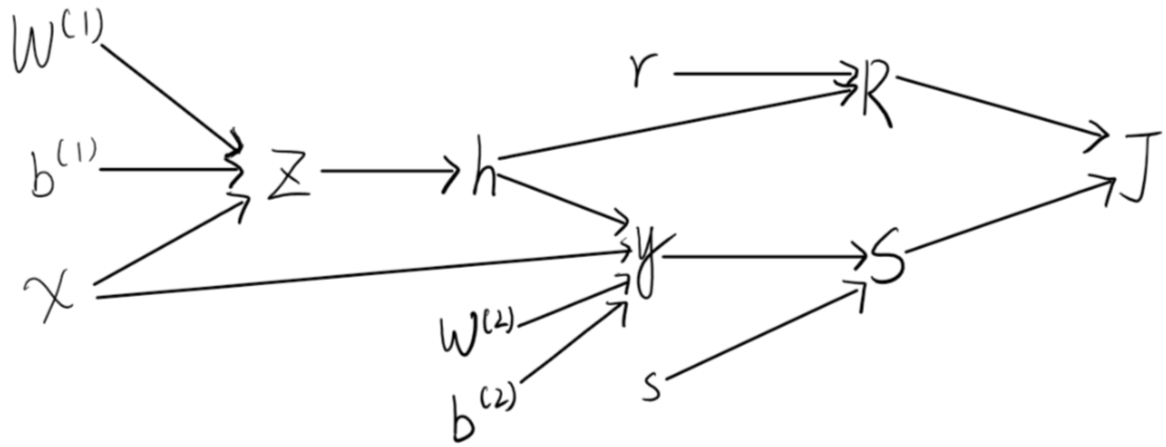
Hard-Coding a Network

$$W^{(1)} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}, b^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, W^{(2)} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}, b^{(2)} = 0$$

Problem 2

Backprop

a)



b)

$$\bar{R} = \frac{\partial J}{\partial R} = 1,\ \bar{S} = \frac{\partial J}{\partial S} = 1,\ \bar{\mathbf{y}} = \bar{S}\frac{\partial S}{\partial \mathbf{y}} = 1 * \begin{bmatrix} \frac{\partial S}{\partial y_1} \\ \vdots \\ \frac{\partial S}{\partial y_n} \end{bmatrix} = \begin{bmatrix} y_1 - s_1 \\ \vdots \\ y_n - s_n \end{bmatrix} = \mathbf{y} - \mathbf{s}$$

$$\bar{\mathbf{h}} = \frac{\partial J}{\partial \mathbf{h}} = \frac{\partial R}{\partial \mathbf{h}}\bar{R} + \frac{\partial \mathbf{y}}{\partial \mathbf{h}}^T \bar{\mathbf{y}} = \begin{bmatrix} \frac{\partial R}{\partial h_1} \\ \vdots \\ \frac{\partial R}{\partial h_k} \end{bmatrix} * 1 + \begin{bmatrix} \frac{\partial y_1}{\partial h_1} & \cdots & \frac{\partial y_1}{\partial h_k} \\ \vdots & & \vdots \\ \frac{\partial y_n}{\partial h_1} & \cdots & \frac{\partial y_n}{\partial h_k} \end{bmatrix}^T \bar{\mathbf{y}} = \begin{bmatrix} r_1 \\ \vdots \\ r_k \end{bmatrix} + \begin{bmatrix} w_{11}^{(2)} & \cdots & w_{1k}^{(2)} \\ \vdots & & \vdots \\ w_{n1}^{(2)} & \cdots & w_{nk}^{(2)} \end{bmatrix}^T \bar{\mathbf{y}} = \mathbf{r} + \mathbf{W}^{(2)T}\bar{\mathbf{y}}$$

$$\bar{\mathbf{z}} = \frac{\partial J}{\partial \mathbf{z}} = \bar{\mathbf{h}}\frac{\partial \mathbf{h}}{\partial \mathbf{z}} = \bar{\mathbf{h}} \begin{bmatrix} \frac{\partial h_1}{\partial z_1} & \cdots & \frac{\partial h_1}{\partial z_k} \\ \vdots & & \vdots \\ \frac{\partial h_k}{\partial z_1} & \cdots & \frac{\partial h_k}{\partial z_k} \end{bmatrix} = \bar{\mathbf{h}} \begin{bmatrix} \sigma^{'}(z_1) & & 0 \\ & \ddots & \\ 0 & & \sigma^{'}(z_k) \end{bmatrix} = \bar{\mathbf{h}} \circ \sigma^{'}(\mathbf{z})$$

$$\bar{\mathbf{x}} = \frac{\partial J}{\partial \mathbf{x}} = \frac{\partial \mathbf{z}}{\partial \mathbf{x}}^T \bar{\mathbf{z}} + \bar{\mathbf{y}}\frac{\partial \mathbf{y}}{\partial \mathbf{x}} = \mathbf{W}^{(1)T}\bar{\mathbf{z}} + \bar{\mathbf{y}} \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial y_n}{\partial x_1} & \cdots & \frac{\partial y_n}{\partial x_n} \end{bmatrix} = \mathbf{W}^{(1)T}\bar{\mathbf{z}} + \bar{\mathbf{y}} \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix} = \mathbf{W}^{(1)T}\bar{\mathbf{z}} + \bar{\mathbf{y}}$$

$$\bar{x_i} = (\sum_{j=1}^{k} W_{ji}^{(1)}\bar{z_j}) + \bar{y_i}$$

Problem 3

Sparsifying Activation Functions

$\frac{\partial L}{\partial w_1}$: YES

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial y}\frac{\partial y}{\partial w_1} = \frac{\partial L}{\partial y} \times h_1 = \frac{\partial L}{\partial y} \times 0 = 0$$

$\frac{\partial L}{\partial w_2}$: YES
$h_1 = ReLU(z_1)$ where z_1 is the input of h_1 and $z_1 = -1$. $\partial h_1/\partial z_1 = ReLU'(z_1) = ReLU'(-1) = 0$

$$\frac{\partial L}{\partial w_2} = \frac{\partial L}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial w_2} = \frac{\partial L}{\partial h_1} \times 0 \times h_3 = 0$$

$\frac{\partial L}{\partial w_3}$: No

$$\frac{\partial L}{\partial w_3} = \frac{\partial L}{\partial h_3}\frac{\partial h_3}{\partial z_3}\frac{\partial z_3}{\partial w_3}$$

$h_3 = ReLU(z_3)$ where z_3 is the input of h_3 and z_3 can be greater than zero, so $\partial h_3/\partial z_3 = ReLU'(z_3)$ could be 1. $\partial z_3/\partial w_3 = x_1$ could also be non-zero.

$$\frac{\partial L}{\partial h_3} = \frac{\partial L}{\partial y}\frac{\partial y}{\partial h_3} = \frac{\partial L}{\partial y}(\frac{\partial y}{\partial h_1}\frac{\partial h_1}{\partial z_1}\frac{\partial z_1}{\partial h_3}+\frac{\partial y}{\partial h_2}\frac{\partial h_2}{\partial z_2}\frac{\partial z_2}{\partial h_3}) = \frac{\partial L}{\partial y}(w_1\times ReLU'(z_1)\times w_2+w_5\times ReLU'(z_2)\times w_4)$$

As stated in the question, $z_1 = -1, ReLU'(z_1) = 0$. However, z_2 could be above zero. In that case, $ReLU'(z_2) = 1$

$$\frac{\partial L}{\partial h_3} = \frac{\partial L}{\partial y}(w_1 \times 0 \times w_2 + w_5 \times 1 \times w_4) = \frac{\partial L}{\partial y} \times w_5 \times w_4$$

Since $\partial L/\partial h_3, \partial h_3/\partial z_3, \partial z_3/\partial w_3$ can all be non-zero, $\partial L/\partial w_3$ is not guaranteed to be zero.