Numerical Methods For Dynamical System : Assignement 10

Leo-Paul BAUDET

1er décembre 2023

Table des matières

1 RTBP

In that assignement we are interesting in the restricted three body problem for $\mu \in [0, \frac{1}{2}]$, an two body placed in μ and $1-\mu$. The position of the third body is denoted by (x,y). Here we want to study this third body. Here a description of the situation :

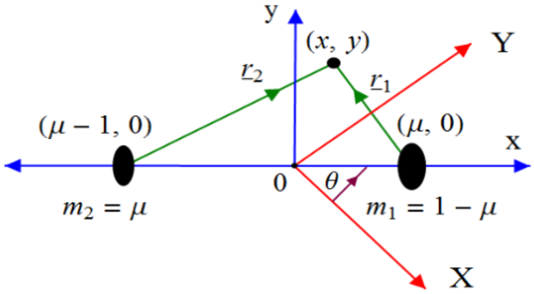

FIGURE 1 – RTBP schematic

This problem can be modelised by the following system of order 2 :

$$\begin{cases} x'' - 2y' = \Omega_x \\ y'' + 2x' = \Omega_y \end{cases} \tag{1}$$

Where

$$\Omega_x(x,y) = x - \frac{(1-\mu)(x-\mu)}{r_1(x,y)^3} - \frac{\mu(x-\mu+1)}{r_2(x,y)^3} \tag{2}$$

$$\Omega_y(x,y) = y - \frac{\mu * y}{r_2(x,y)^3} - \frac{(1-\mu)y}{r_1(x,y)^3} \tag{3}$$

with (looking at the previous schematic)

$$r_1(x,y) = \sqrt{(x-\mu)^2 + y^2} \tag{4}$$

$$r_2(x,y) = \sqrt{(x-\mu+1)^2 + y^2} \tag{5}$$

We can rewrite this system of order 2 into a system of order 1. Indeed by setting $x_1 = x$, $x_2 = y$, $x_3 = x'$ and $x_4 = y'$ we get :

$$\begin{cases} x_1' = x_3 \\ x_2' = x_4 \\ x_3' = 2x_4 + \Omega_{x_1} \\ x_4' = -2x_3 + \Omega_{x_2} \end{cases} \tag{6}$$

1.1 A symetry of the RTBP

Let's considering the following solution of the RTBP : (t,x_1,x_2,x_3,x_4). We have that it verifies (6).

$$\begin{cases} x_1'(t) = x_3(t) \\ x_2'(t) = x_4(t) \\ x_3'(t) = 2x_4(t) + \Omega_{x_1(t)} \\ x_4'(t) = -2x_3(t) + \Omega_{x_2(t)} \end{cases}$$

This is why we have :

$$\begin{cases} x_1'(-t) = -x_3(-t) \\ x_2'(-t) = -x_4(-t) \\ x_3'(-t) = -2x_4(-t) - \Omega_{x_1(-t)} \\ x_4'(-t) = 2x_3(-t) - \Omega_{x_2(-t)} \end{cases} \tag{7}$$

Now if we consider this : (-t,x_1,$-x_2$,$-x_3$,x_4), with (7) we have :

$$\begin{cases} x_1'(-t) = x_3(-t) \\ x_2'(-t) = x_4(-t) \\ x_3'(-t) = 2x_4(-t) + \Omega_{x_1(-t)} \\ x_4'(-t) = -2x_3(-t) - \Omega_{-x_2(-t)} = -2x_3(-t) + \Omega_{x_2(-t)} \text{ because } \Omega_y = -\Omega_{-y} \end{cases} \tag{8}$$

This is why (-t,x,-y,-x',y') is also a solution if (t,x,y,x',y') is solution.

2 Symetric Homoclinic of L3

We have seen in class that the RTBP has 5 equilibrium point. Here we are interesting to the third one L3. We have shown that the the cordinates of L3 are : L3($\mu + \xi$,0,0,0) where ξ is the root of the Eleur quitinc equation. Here an sechematic with the equilibrium points :

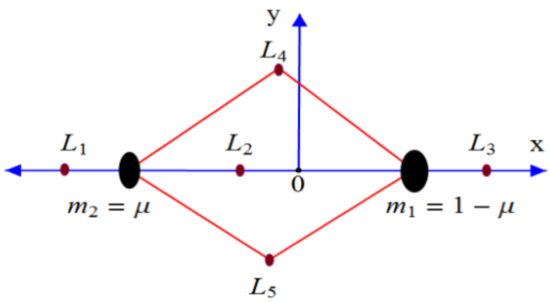

FIGURE 2 – Equilibirum point of the RTBP

Now we compute, for a given $\mu = 0.008$ the first crossing with the Poincaré section $\Sigma : y = 0$ of the negative branch of the unstable manifold for L3 (using the linear approximation)

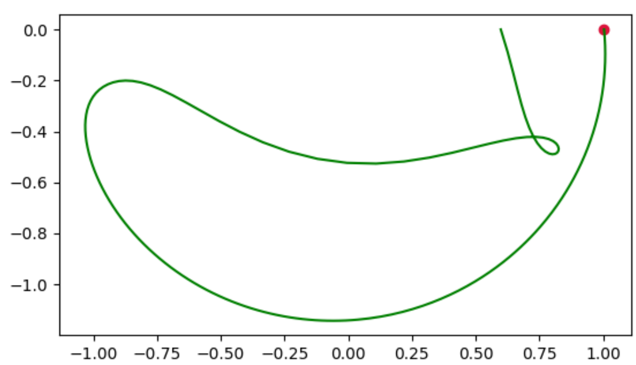

FIGURE 3 – Firt crossing oh the poincaré section for $\mu = 0.008$

2.1 Research of symetric homoclinics orbits

As we have discuss in class, because of the symetry shown in the first part, we have an homoclinic symetric orbit if x'=0 on the Poincaré section. Thus, we are going for different values of μ to compute the first crossing of the negative branch of the unstable manifold with the Poincaré section and plot mu in function of x'. Where x'=0, this means that we have a symetric homoclinic orbit :

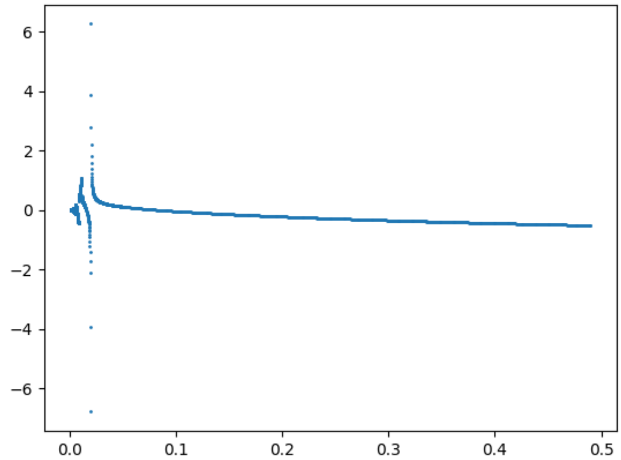

FIGURE 4 – μ in function of x'

For this plot, we have taken three intervals of μ : one from 0.001 to 0.015 (with increment=0.00001), an other from 0.015 to 0.05 (with increment 0.0001 and an othet from 0.05 to 0.49 (with increment 0.001)

We can see that there are discontinuities, let's try to understand these discontinuities. First let's zoom on the plot for values fo μ between 0.001 and 0.1 :

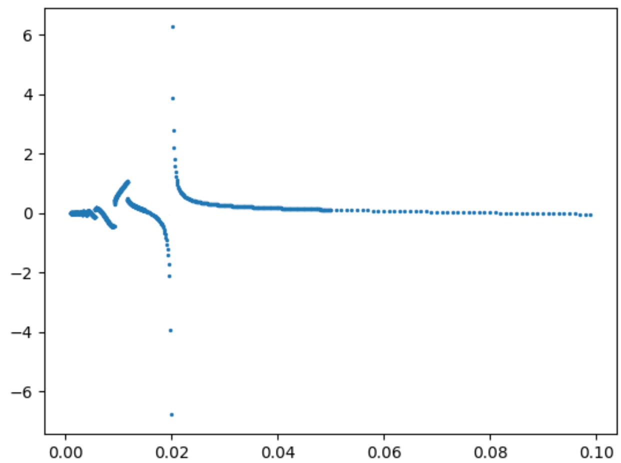

FIGURE 5 – μ in function of x', for $\mu \in [0.001, 0.1]$

2.1.1 Anlysis of the firts discontinuities

Here we are going to study all the discontinuity except the on in 0.02. We can see for example, for $mu = 0.00925$ we have a discontinuity. Because we have stored the value of x and y, we can see that there is an impotant variation on x. Thi why we plot the orbits for μ around 0.00925 to see what is happening.

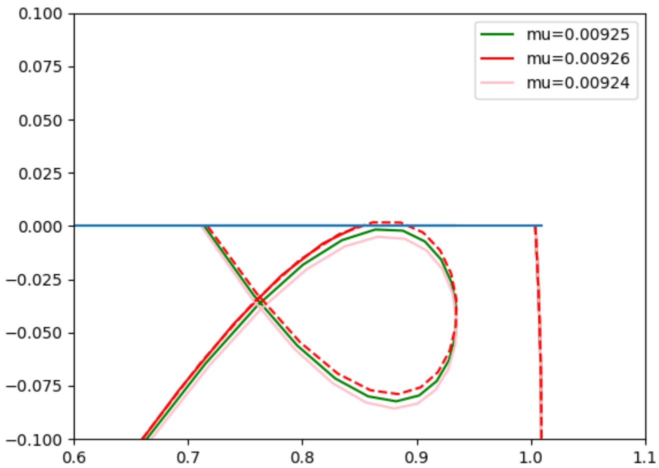

FIGURE 6 – Negative branch of the unstable manifold around the Poincaré section for μ around 0.00925

In dash point this is the orbits for $mu = 0.00926$ after the first crossing of the Poincaré section.

First of all, we can see that there is a loop on the orbits. We can also see that increasing μ approaches the poincaré section.

For $\mu = 0.00926$, we can see that the point where the solution crossed the Poincaré section is very different than for the other orbits. This is why x' is completely different than in the two others cases. This is why we have a discontinuity around $\mu = 0.00925$

2.1.2 Analysis of the last discontinuity

Here we will try to understand the discontinuity around $\mu = 0.02$. For the same reason as before, we plot the orbits arounf $\mu = 0.02$

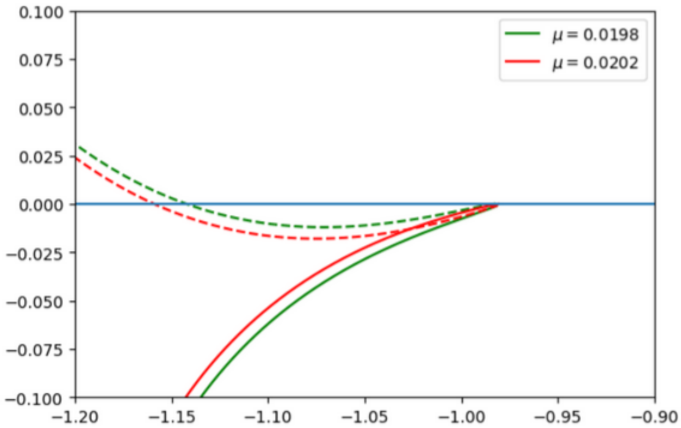

FIGURE 7 – Negative branch of the unstable manifold arounf the Poincaré section for μ around 0.02

In the plot, we see that the solution has an abruptly change of direction and that this happens (if we check x) before the crossing for $\mu = 0.0198$ and after for $\mu = 0.0202$. This change of direction means that the derivative has an asymptotic point that just falls over the x-axis on $\mu = 0.02$. For this reason, we have a ,discontinuity at the point in our plot.

A Link to the code

All the code is available here : `https://github.com/leopaulbis/numerical_methods_dynamical_system_assignement10`