

# Ideal Chomp Game on K-Algebras.sagews

Author        Leopold Karl

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1 # This worksheet was converted from a notebook running Jupyter kernel
2 # version sage-10.6.

3 # Sage Worksheet: Check whether the reduction steps in table 2 are correct.
4 from sage.all import LatexExpr
5
6 # Function checking the equality of given ideals.
7 def check_ideals(tuples, M):
8     # tuples: list of tuples (n, PR, I, a), n in N
9     # (the index of the ring PR/I in table 1),
10    # PR a polynomial ring over K, I an ideal of PR,
11    # a ∈ PR an element that is added to I.
12
13    # M: list of tuples (m, PRc, J), m ∈ N
14    # (the index of the ring PRc/J in {R_1, R_4, R_12, R_13, R_17}),
15    # the set of local K-algebras up to dim_K R = 6
16    # where player B wins from table 1,
17    # PRc a polynomial ring over K, J an ideal of PRc.
18
19    # return: list of tuples (n, m, match), where n is the index of ring R_n
20    # that is reduced to R_m and match is true if there exists such an index m,
21    # i.e. if PR/I+(a) = PRc/J for some index m in M.
22
23    results = []
24    for (n, PR, I, a) in tuples:
25
26        # Search in M
27        match = False
28        for (m, PRc, J) in M:
29            if PRc == PR:
30                S = I + PR.ideal(a)
31                if J == S:
32                    match = True
33                    results.append((n,m,match))
34                    break
35
36        # no index in M found to which the ring reduces
37        if match == False:
38            results.append((n,0,match))
39
40    return results
41
42 # Field we work over
43 K = QQ
44
45 # Polynomial rings in various variables
46 PR1.<x> = PolynomialRing(K)
47 PR2.<x,y> = PolynomialRing(K,2)
48 PR3.<x,y,z> = PolynomialRing(K,3)
49 PR4.<x,y,z,w> = PolynomialRing(K,4)

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50 PR5.<x,y,z,w,v> = PolynomialRing(K,5)
51 PR6.<x,y,z,w,v,u> = PolynomialRing(K,6)
52
53
54 # Create M (rings where player B wins in table 1 and their isomorphic analogues with more variables)
55 I1_1 = PR1.ideal(x)
56 I1_2 = PR2.ideal(x,y)
57 I1_3 = PR3.ideal(x,y,z)
58 I1_4 = PR4.ideal(x,y,z,w)
59 I1_5 = PR5.ideal(x,y,z,w,v)
60 I1_6 = PR6.ideal(x,y,z,w,v,u)
61
62 I4_2 = PR2.ideal(x^2, x*y,y^2)
63 I4_3_1 = PR3.ideal(x^2,x*y,y^2,z)
64 I4_3_2 = PR3.ideal(y^2,z^2,y*z,x)
65 I4_3_3 = PR3.ideal(x^2, y^2, z^2, x*y, x*z, y*z, x+y)
66 I4_4 = PR4.ideal(x^2,x*y,y^2,z,w)
67 I4_5 = PR5.ideal(x^2,x*y,y^2,z,w,v)
68 I4_6 = PR6.ideal(x^2,x*y,y^2,z,w,v,u)
69
70 I12_2 = PR2.ideal(x^3,x*y,y^3)
71 I12_3_1 = PR3.ideal(x^3,x*y,y^3,z)
72 I12_3_2 = PR3.ideal(y^3,z^3,y*z,x)
73 I12_4 = PR4.ideal(x^3,x*y,y^3,z,w)
74 I12_5 = PR5.ideal(x^3,x*y,y^3,z,w,v)
75 I12_6 = PR6.ideal(x^3,x*y,y^3,z,w,v,u)
76
77 I13_2 = PR2.ideal(x^2,x*y^2,y^3)
78 I13_3 = PR3.ideal(x^2,x*y^2,y^3,z)
79 I13_4 = PR4.ideal(x^2,x*y^2,y^3,z,w)
80 I13_5 = PR5.ideal(x^2,x*y^2,y^3,z,w,v)
81 I13_6 = PR6.ideal(x^2,x*y^2,y^3,z,w,v,u)
82
83 I17_4 = PR4.ideal(x^2,y^2,z^2,w^2,x*y,x*z,x*w,y*z,y*w,z*w)
84 I17_5 = PR5.ideal(x^2,y^2,z^2,w^2,x*y,x*z,x*w,y*z,y*w,z*w,v)
85 I17_6 = PR6.ideal(x^2,y^2,z^2,w^2,x*y,x*z,x*w,y*z,y*w,z*w,v,u)
86
87 M = [(1,PR1,I1_1), (1,PR2,I1_2), (1,PR3,I1_3), (1,PR4,I1_4), (1,PR5,I1_5), (1,PR6,I1_6),
88      (4,PR2,I4_2), (4,PR3,I4_3_1), (4,PR3,I4_3_2), (4,PR3,I4_3_3), (4,PR4,I4_4), (4,PR5,I4_5), (4,PR6,I4_6),
89      (12,PR2,I12_2), (12,PR3,I12_3_1), (12,PR3,I12_3_2), (12,PR4,I12_4), (12,PR5,I12_5), (12,PR6,I12_6),
90      (13, PR2, I13_2), (13,PR3,I13_3), (13,PR4,I13_4), (13,PR5,I13_5), (13,PR6,I13_6),
91      (17, PR4, I17_4), (17, PR5, I17_5), (17,PR6,I17_6)]
92
93 # Rings where we check if they reduce to one of the rings in M
94
95 # R2:  $K[x]/(x^2)$ 
96 I2 = PR1.ideal(x^2)
97 t2 = (2,PR1,I2,x)
98
99 # R3:  $K[x]/(x^3)$ 
100 I3 = PR1.ideal(x^3)
101 t3 = (3,PR1,I3,x)
102
103 # R5:  $K[x]/(x^4)$ 
104 I5 = PR1.ideal(x^4)
105 t5 = (5, PR1, I5, x)
106
107 # R6:  $K[x,y]/(x^2, x*y, y^3)$ 
108 I6 = PR2.ideal(x^2, x*y, y^3)
109 t6 = (6, PR2, I6, y^2)
110
111 # R7:  $K[x,y]/(x^2, y^2)$ 
112 I7 = PR2.ideal(x^2, y^2)
113 t7 = (7, PR2, I7, x*y)
114

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115 # R7*: K[x,y]/(x^2 + y^2, x*y)
116 I7s = PR2.ideal(x^2 + y^2, x*y)
117 t7s = (71, PR2, I7s, x^2)
118
119 # R8: K[x,y,z]/(x,y,z)^2
120 I8 = PR3.ideal(x, y, z)^2
121 t8 = (8, PR3, I8, z)
122
123 # R9: K[x]/(x^5)
124 I9 = PR1.ideal(x^5)
125 t9 = (9, PR1, I9, x)
126
127 # R10: K[x,y]/(x^2, x*y, y^4)
128 I10 = PR2.ideal(x^2, x*y, y^4)
129 t10 = (10, PR2, I10, y^2)
130
131 # R11: K[x,y]/(x^2 + y^3, x*y)
132 I11 = PR2.ideal(x^2 + y^3, x*y)
133 t11 = (11, PR2, I11, y^2)
134
135 # R14: K[x,y,z]/(x^2, y^2, x*y, x*z, y*z, z^3)
136 I14 = PR3.ideal(x^2, y^2, x*y, x*z, y*z, z^3)
137 t14 = (14, PR3, I14, z)
138
139 # R15: K[x,y,z]/(x^2, y^2, z^2, x*y, x*z)
140 I15 = PR3.ideal(x^2, y^2, z^2, x*y, x*z)
141 t15 = (15, PR3, I15, z)
142
143 # R15*: K[x,y,z]/(x^2, x*y, x*z, y*z, y^2 + z^2)
144 I15s = PR3.ideal(x^2, x*y, x*z, y*z, y^2 + z^2)
145 t15s = (151, PR3, I15s, z)
146
147 # R16: K[x,y,z]/(x*y, x*z, y*z, x^2 + y^2, x^2 + z^2)
148 I16 = PR3.ideal(x*y, x*z, y*z, x^2 + y^2, x^2 + z^2)
149 t16 = (16, PR3, I16, x)
150
151 # R17: K[x,y,z,w]/(x,y,z,w)^2
152 I17 = PR4.ideal(x, y, z, w)^2
153 t17 = (17, PR4, I17, x)
154
155 # R18: K[x]/(x^6)
156 I18 = PR1.ideal(x^6)
157 t18 = (18, PR1, I18, x)
158
159 # R19: K[x,y]/(x^2, x*y, y^5)
160 I19 = PR2.ideal(x^2, x*y, y^5)
161 t19 = (19, PR2, I19, y^2)
162
163 # R20: K[x,y]/(x^2 + y^4, x*y)
164 I20 = PR2.ideal(x^2 + y^4, x*y)
165 t20 = (20, PR2, I20, y^2)
166
167 # R21: K[x,y]/(x*y, x^3, y^4)
168 I21 = PR2.ideal(x*y, x^3, y^4)
169 t21 = (21, PR2, I21, y^3)
170
171 # R22: K[x,y]/(x*y, x^3 + y^3)
172 I22 = PR2.ideal(x*y, x^3 + y^3)
173 t22 = (22, PR2, I22, x^3)
174
175 # R23: K[x,y]/(x^2, x*y^2, y^4)
176 I23 = PR2.ideal(x^2, x*y^2, y^4)
177 t23 = (23, PR2, I23, y^3)
178
179 # R24: K[x,y]/(x^2 + y^3, x*y^2, y^4)

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180 I24 = PR2.ideal(x^2 + y^3, x*y^2, y^4)
181 t24 = (24, PR2, I24, y^3)
182
183 # R25: K[x,y]/(x^2, y^3)
184 I25 = PR2.ideal(x^2, y^3)
185 t25 = (25, PR2, I25, x*y^2)
186
187 # R25*: K[x,y]/(x^2 + x*y^2, y^3)
188 I25s = PR2.ideal(x^2 + x*y^2, y^3)
189 t25s = (251, PR2, I25s, x^2)
190
191 # R25**: K[x,y]/(x^2, x*y^2 + y^3)
192 I25ss = PR2.ideal(x^2, x*y^2 + y^3)
193 t25ss = (252, PR2, I25ss, y^3)
194
195 # R26: K[x,y]/(x,y)^3
196 I26 = PR2.ideal(x, y)^3
197 t26 = (26, PR2, I26, x*y)
198
199 # R27: K[x,y,z]/(x^2, x*y, y^2, x*z, y*z, z^4)
200 I27 = PR3.ideal(x^2, x*y, y^2, x*z, y*z, z^4)
201 t27 = (27, PR3, I27, z)
202
203 # R28: K[x,y,z]/(x^2, x*y, y^2 + z^3, x*z, y*z, z^4)
204 I28 = PR3.ideal(x^2, x*y, y^2 + z^3, x*z, y*z, z^4)
205 t28 = (28, PR3, I28, z)
206
207 # R29: K[x,y,z]/(x^2, x*y + z^3, y^2, x*z, y*z, z^4)
208 I29 = PR3.ideal(x^2, x*y + z^3, y^2, x*z, y*z, z^4)
209 t29 = (29, PR3, I29, z)
210
211 # R29*: K[x,y,z]/(x^2 + z^3, x*y, y^2 + z^3, x*z, y*z, z^4)
212 I29s = PR3.ideal(x^2 + z^3, x*y, y^2 + z^3, x*z, y*z, z^4)
213 t29s = (291, PR3, I29s, z)
214
215 # R30: K[x,y,z]/(x*y, y*z, z^2, y^2 - x*z, x^3)
216 I30 = PR3.ideal(x*y, y*z, z^2, y^2 - x*z, x^3)
217 t30 = (30, PR3, I30, x)
218
219 # R31: K[x,y,z]/(x*y, z^2, x*z - y*z, x^2 + y^2 - x*z)
220 I31 = PR3.ideal(x*y, z^2, x*z - y*z, x^2 + y^2 - x*z)
221 t31 = (31, PR3, I31, x)
222
223 # R31*: K[x,y,z]/(x^2, z^2, y^2 - x*z, y*z)
224 I31s = PR3.ideal(x^2, z^2, y^2 - x*z, y*z)
225 t31s = (311, PR3, I31s, x)
226
227 # R32: K[x,y,z]/(x^2, x*y, x*z, y^2, y*z^2, z^3)
228 I32 = PR3.ideal(x^2, x*y, x*z, y^2, y*z^2, z^3)
229 t32 = (32, PR3, I32, z)
230
231 # R33: K[x,y,z]/(x^2, x*y, x*z, y*z, y^3, z^3)
232 I33 = PR3.ideal(x^2, x*y, x*z, y*z, y^3, z^3)
233 t33 = (33, PR3, I33, x)
234
235 # R34: K[x,y,z]/(x*y, x*z, y^2, z^2, x^3)
236 I34 = PR3.ideal(x*y, x*z, y^2, z^2, x^3)
237 t34 = (34, PR3, I34, x + y)
238
239 # R34*: K[x,y,z]/(x*y, x*z, y*z, y^2 - z^2, x^3)
240 I34s = PR3.ideal(x*y, x*z, y*z, y^2 - z^2, x^3)
241 t34s = (341, PR3, I34s, x + y)
242
243 # R35: K[x,y,z]/(x*y, x*z, y*z, x^2 + y^2 - z^2)
244 I35 = PR3.ideal(x*y, x*z, y*z, x^2 + y^2 - z^2)

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245 t35 = (35, PR3, I35, x + y)
246
247 # R36:  $K[x,y,z]/(x^2, x*y, y*z, y^2 - z^2)$ 
248 I36 = PR3.ideal(x^2, x*y, y*z, y^2 - z^2)
249 t36 = (36, PR3, I36, z)
250
251 # R36*:  $K[x,y,z]/(x^2, x*y, y*z, x*z + y^2 - z^2)$ 
252 I36s = PR3.ideal(x^2, x*y, y*z, x*z + y^2 - z^2)
253 t36s = (361, PR3, I36s, z)
254
255 # R37:  $K[x,y,z]/(x^2, x*y, y^2, z^2)$ 
256 I37 = PR3.ideal(x^2, x*y, y^2, z^2)
257 t37 = (37, PR3, I37, z)
258
259 # R37*:  $K[x,y,z]/(x^2, x*y, y^2, z^2 - x*z)$ 
260 I37s = PR3.ideal(x^2, x*y, y^2, z^2 - x*z)
261 t37s = (371, PR3, I37s, z)
262
263 # R38:  $K[x,y,z,w]/(x^2, y^2, z^2, x*y, x*z, x*w, y*z, y*w, z*w, w^3)$ 
264 I38 = PR4.ideal(x^2, y^2, z^2, x*y, x*z, x*w, y*z, y*w, z*w, w^3)
265 t38 = (38, PR4, I38, w^2)
266
267 # R39:  $K[x,y,z,w]/(x^2, y^2, z^2, w^2, x*y, x*z, x*w, y*z, y*w)$ 
268 I39 = PR4.ideal(x^2, y^2, z^2, w^2, x*y, x*z, x*w, y*z, y*w)
269 t39 = (39, PR4, I39, z*w)
270
271 # R39*:  $K[x,y,z,w]/(x^2, y^2, z^2 + w^2, x*y, x*z, x*w, y*z, y*w, z*w)$ 
272 I39s = PR4.ideal(x^2, y^2, z^2 + w^2, x*y, x*z, x*w, y*z, y*w, z*w)
273 t39s = (391, PR4, I39s, z^2)
274
275 # R40:  $K[x,y,z,w]/(x^2, y^2 + z^2, y^2 + w^2, x*y, x*z, x*w, y*z, y*w, z*w)$ 
276 I40 = PR4.ideal(x^2, y^2 + z^2, y^2 + w^2, x*y, x*z, x*w, y*z, y*w, z*w)
277 t40 = (40, PR4, I40, y^2)
278
279 # R41:  $K[x,y,z,w]/(x^2, y^2, z^2, w^2, x*y - z*w, x*z, x*w, y*z, y*w)$ 
280 I41 = PR4.ideal(x^2, y^2, z^2, w^2, x*y - z*w, x*z, x*w, y*z, y*w)
281 t41 = (41, PR4, I41, x*y)
282
283 # R41*:  $K[x,y,z,w]/(x^2 + y^2, x^2 + z^2, x^2 + w^2, x*y, x*z, x*w, y*z, y*w, z*w)$ 
284 I41s = PR4.ideal(x^2 + y^2, x^2 + z^2, x^2 + w^2, x*y, x*z, x*w, y*z, y*w, z*w)
285 t41s = (411, PR4, I41s, x^2)
286
287 # R42:  $K[x,y,z,w,v]/(x,y,z,w,v)^2$ 
288 I42 = PR5.ideal(x, y, z, w, v)^2
289 t42 = (42, PR5, I42, v)
290
291 tuples = [t2,t3,t5,t6,t7,t7s,t8,t9,t10,
292           t11,t14,t15,t15s,t16,t18,t19,t20,
293           t21,t22,t23,t24,t25,t25s, t25ss,t26,t27,t28,t29,t29s,t30,
294           t31,t31s,t32,t33,t34,t34s,t35,t36,t36s,t37,t37s,t38,t39,t39s,t40,t41,t41s,t42]
295
296 # run checker function
297 results = check_ideals(tuples,M)
298
299 # print
300 for (n,m,match) in results:
301     if n < 50:
302         show(LatexExpr(f"R_{{{n}}}"), " reduces to ", LatexExpr(f"R_{{{m}}}"), ":", match)
303     elif n % 10 == 1:
304         N = int(n/10)
305         show(LatexExpr(f"R_{{{N}},*}"), " reduces to ", LatexExpr(f"R_{{{m}}}"), ":", match)
306     elif n % 10 == 2:
307         N = int(n/10)
308         show(LatexExpr(f"R_{{{N}},**}"), " reduces to ", LatexExpr(f"R_{{{m}}}"), ":", match)

```

```

R2 reduces to R1:True
R2 reduces to R1:True
R3 reduces to R1:True
R3 reduces to R1:True
R5 reduces to R1:True
R5 reduces to R1:True
R6 reduces to R4:True
R6 reduces to R4:True
R7 reduces to R4:True
R7 reduces to R4:True
R7,* reduces to R4:True
R7,* reduces to R4:True
R8 reduces to R4:True
R8 reduces to R4:True
R9 reduces to R1:True
R9 reduces to R1:True
R10 reduces to R4:True
R10 reduces to R4:True
R11 reduces to R4:True
R11 reduces to R4:True
R14 reduces to R4:True
R14 reduces to R4:True
R15 reduces to R4:True
R15 reduces to R4:True
R15,* reduces to R4:True
R15,* reduces to R4:True
R16 reduces to R4:True
R16 reduces to R4:True
R18 reduces to R1:True
R18 reduces to R1:True
R19 reduces to R4:True
R19 reduces to R4:True
R20 reduces to R4:True
R20 reduces to R4:True
R21 reduces to R12:True
R21 reduces to R12:True
R22 reduces to R12:True
R22 reduces to R12:True
R23 reduces to R13:True
R23 reduces to R13:True
R24 reduces to R13:True
R24 reduces to R13:True
R25 reduces to R13:True
R25 reduces to R13:True
R25,* reduces to R13:True
R25,* reduces to R13:True
R25,** reduces to R13:True
R25,** reduces to R13:True
R26 reduces to R12:True
R26 reduces to R12:True
R27 reduces to R4:True
R27 reduces to R4:True
R28 reduces to R4:True
R28 reduces to R4:True
R29 reduces to R4:True
R29 reduces to R4:True
R29,* reduces to R4:True

```

```

R29,* reduces to R4:True
R30 reduces to R4:True
R30 reduces to R4:True
R31 reduces to R4:True
R31 reduces to R4:True
R31,* reduces to R4:True
R31,* reduces to R4:True
R32 reduces to R4:True
R32 reduces to R4:True
R33 reduces to R12:True
R33 reduces to R12:True
R34 reduces to R4:True
R34 reduces to R4:True
R34,* reduces to R4:True
R34,* reduces to R4:True
R35 reduces to R4:True
R35 reduces to R4:True
R36 reduces to R4:True
R36 reduces to R4:True
R36,* reduces to R4:True
R36,* reduces to R4:True
R37 reduces to R4:True
R37 reduces to R4:True
R37,* reduces to R4:True
R37,* reduces to R4:True
R38 reduces to R17:True
R38 reduces to R17:True
R39 reduces to R17:True
R39 reduces to R17:True
R39,* reduces to R17:True
R39,* reduces to R17:True
R40 reduces to R17:True
R40 reduces to R17:True
R41 reduces to R17:True
R41 reduces to R17:True
R41,* reduces to R17:True
R41,* reduces to R17:True
R42 reduces to R17:True
R42 reduces to R17:True

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