CASE STUDY BASED ON GUROBI MODELS, IN PARTNERSHIP WITH PFIZER INC.

Assigning Regions to Sales Representatives at Pfizer Turkey

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November 30, 2022

1 Introduction

Created in 1849, Pfizer is today one of the largest companies in the world, especially in the pharmaceutical sector. To achieve our ambition of making prevention, wellness and treatments accessible to the world we are localised in many countries over the world. Turkey, which is 12th largest pharmaceutical market in the world is a complicated and disputed market where we manage to be the 4th largest company. This result is possible thanks to the excellent profiles in each team. Nevertheless as time passes, we seem to drop behind our competitors these last years, in fact this last year (2005) we lost a few million dollars in sales. If this dynamic seems to also affect other companies, we still can found solutions to better optimize our sector, especially in our Sales Representatives (SR) distribution. These experts are key to our actual success as they visit Medical Doctors (MD) in order to provide information such as indications and adverse effects of drugs, supply samples, discuss and obtain feedback, and keep a close relationship. Each SR is assigned a working region, where he visits all the MDs. These territories are formed by combining smaller geographical units, called “bricks”. Each SR has an office located at a certain brick, called a "center brick".

Due to the dynamic structure of the market bricks and the number of customer have changed overtime and sales territories have never been updated. Updating the bricks distribution with respect to precise predetermined precise criteria would allow our company to be more effective and relevant for the next years.

In this document, we will present a model that optimizes the distribution of bricks and center bricks to our SRs. To do so we will in a first place present the tools we used to model the situation. This will include a precise description of the data and optimization framework we used. Then, in the second part we will determine the criterias used to model the environment we work in. These criterias will then help us creating a mathematical formulation of the problem. The two next parts will then be used to present the basic and more advanced models we created. Each one answers a specific question and will help the company make the best choice. Finally we will present some limitations we found with our model.

Contents

1 Introduction **1**

2 Tools used to model the situation **3**
- 2.1 Data . . . 3
  - 2.1.1 Current distribution of bricks . . . 3
  - 2.1.2 Workload . . . 3
  - 2.1.3 Distance between bricks . . . 3
- 2.2 Optimizing framework used . . . 4

3 Choices and Criterias **4**
- 3.1 Minimizing the traveled distance . . . 4
- 3.2 Keeping the workload balanced . . . 4
- 3.3 Minimizing the disruption . . . 4

4 Mathematical Formulation **5**
- 4.1 List of Variables used . . . 5
- 4.2 Mathematical expressions . . . 5

5 Basic models and objectives **6**
- 5.1 Basic models . . . 6
- 5.2 Dual objectives models . . . 7
- 5.3 Flexibility of the model . . . 7

6 Advanced models **9**
- 6.1 Assigning multiple SR to the same brick . . . 9
- 6.2 Recruiting a new sales representative if workload suddenly increases by 20% in all districts . . . 9
- 6.3 Reassigning center brick and districts . . . 11
- 6.4 Preferences . . . 11

7 Finals Recommendations **12**

8 Limits of our case study **13**

9 Appendix python code **13**

2 Tools used to model the situation

At the beginning of the project, we only had three datasets to model our situation. These datasets were the current distribution of bricks, the workload of each brick and finally the distance between each brick.

2.1 Data

2.1.1 Current distribution of bricks

The current structure is the following: we have four SR that each have a center brick and 4 to 7 bricks assigned. This table is very important as it gives the structure of many of our results. Table 1

SR#	Center Brick	Bricks assigned
1	4	4,5,6,7,8,9,19,20
2	14	11,13,14,18
3	16	10,15,16,17
4	22	1,2,3,21,22

Table 1: Bricks distribution dataset

2.1.2 Workload

The workload represents the amount of work needed for each brick. One of our goals will be to keep it similar between all salesman. Table 2

Brick#	Index Value		
1	0.1609	12	0.0828
2	0.1164	13	0.0975
3	0.1026	14	0.8177
4	0.1516	15	0.4115
5	0.0939	16	0.3795
6	0.1320	17	0.0710
7	0.0687	18	0.0427
8	0.0930	19	0.1043
9	0.2116	20	0.0997
10	0.2529	21	0.1698
11	0.0868	22	0.2531

Table 2: Workload dataset

2.1.3 Distance between bricks

Finally the last dataset gives us the distance between one bricks to another. This will be useful during the entire project to found the shortest distance to travel for each SR. Table 3

	brick1	brick2	brick3	brick4	brick5	brick6	brick7	brick9	...
brick1	0.00	7.35	13.21	16.16	18.22	16.68	14.00	17.45	...
brick2	7.35	0.00	6.58	19.00	20.06	19.06	16.30	21.45	...
brick3	13.21	6.58	0.00	25.29	26.05	25.24	22.54	27.93	...
brick4	16.16	19.00	25.29	0.00	3.07	1.22	2.80	3.80	...
brick5	18.22	20.06	26.05	3.07	0.00	1.92	4.22	6.19	...
brick6	16.68	19.06	25.24	1.22	1.92	0.00	2.79	4.83	...
brick7	14.00	16.30	22.54	2.80	4.22	2.79	0.00	6.20	...
...	...	...	...	...	...	...	...	...	...

Table 3: Distance between each brick

2.2 Optimizing framework used

For this optimization problem we decided to use Gurobi, a state-of-the-art solver for mathematical programming. We more precisely used Gurobipy, the python framework of the solver.

3 Choices and Criterias

Modelling a real situation mathematically isn't easy, often some choices are made to model the complex world the model takes place. In our situation, we defined three different criteria that are for us the most relevant ones for both the company's efficiency and the Sales Representatives work.

3.1 Minimizing the traveled distance

As we said earlier, one of the main issues we have today is the time we loose when SR go from a brick to another. Therefore the primary criterion is the distance between a SR and a brick which is the distance between a center brick and the other bricks. To optimize this parameter we will use the dataset in Table 3.

3.2 Keeping the workload balanced

The workload is the second criterion. It corresponds to the amount of work needed for each brick. Its distribution between SR must be done meticulously to keep balance and equality between Sales Representatives. A bad distribution would lead to an important loss in our efficiency and would affect us a lot in the long run.

3.3 Minimizing the disruption

The disruption can be seen as the inclusion of new bricks in the territories of SRs. Disruption is undesirable as some of the established relationships between SRs and MDs cannot be utilized any more. To optimize the efficiency and the moral of our SR we must minimize it.

These three criteria are at the chore of our study. Threw this document we will modify them and add some new in order to give relevant results.

4 Mathematical Formulation

This real life situation needs to be modeled first into a mathematical problem to be able to optimize it, and hence provide a solution. Many aspects of this situation can be replaced as variables, constraints, objectives, matrices etc. Here is a list and explanations of each mathematical attributes that will be used in our model to represent and optimize our case study.

4.1 List of Variables used

First of all, the most important variable that will be used over the whole report is the V_{ij} matrix. It is a 22×4 binary matrix that represents the affectation of each Sales Representatives to a specific district. The 22 rows represent the 22 districts, and the 4 columns represent the 4 sales rep. Moreover, V_{ij} is made of binary cells, therefore a cell equals 1 if the district i is affected to the sales rep j, or 0 if it's not. The content of this matrix will be chosen according to our gurobi's optimizer model to affect optimally each sales representatives to a district, depending on the objectives and the constraints wanted.
The main constraints chosen i our model are the workload and the disruption. These two constraints are modelled as a vector W_i for the workload. This vector contains 22 values representing the workload associated with each district. Therefore a sales rep total workload will be the sum of the workload associated with each of his assigned districts. Since we look for a balance of the workload among the four sales representatives, we will constraint the sum of workload for each SR to be within 0.8 and 1.2 (this balance is subject to changes).
The disruption constraint was modelled by the number of bricks assigned to a different SR. We modified this constraint to include the workload of the new brick assigned. The more workload a brick contain, the more disruption it will cause if it is assigned to a new SR. Therefore the disruption is modelled as the workload of the bricks assigned to a new sales rep.
Other characteristics have been implemented such as a 22 by 4 matrix representing the preferences of each salesman for the 22 possible districts.
The main objective of this model is to assign optimally all 22 districts to the sales representatives to reduce the distances travelled from their offices to their assigned districts. The distances are defined as a 22×22 matrix filled with the distances from each district to the other 22 districts, called D_{ij}.

4.2 Mathematical expressions

The distance to be minimized is the sum of distances from each sales representatives' offices to their assigned districts. This sum of distances can be computed as the sum over all sales rep and all districts of $V_{ij} \times D_{ij}$. Since the V_{ij} matrix is made of zeros and ones, multiplying it to the distances matrix will output only the distances from the districts assigned to the SR to his office. Therefore we have:

$$\text{distances to be minimized: } \sum_j \sum_i D_{ij} V_{ij}$$
$$\sum_j \sum_i D_{brick_i, centerbrick_j} V_{brick_i, salesrep_j}$$

Similarly, the workload of each sales rep is the sum of workload computed by the matrix multiplication of the V_{ij} affectation matrix to the W_i district workload vector:

$$0.8 \leq \sum_{i,j} V_{i,j} W_i \geq 1.2$$

Finally the disruption is also defined in a similar way:

$$\text{minimize disruption: } \sum_{i,j} W_i |V_{ij}^{original} - V_{ij}^{new}|$$

Since the matrix V_{ij} is made of binary variables, the absolute value can be computed using by raising the expression to the power of 2.

5 Basic models and objectives

To compare the results of our model to the actual situation, let's compute the current total distance from each sales rep offices to their districts given this assignment:

SR#	Center Brick	Bricks assigned
1	4	4,5,6,7,8,15
2	14	10,11,13,14
3	16	9,16,17,18
4	22	1,2,3,19,20,21,22

Table 4: Initial brick distribution

The total distance is 187.34km, and the workload for each SR is respectively 0.95, 1.34, 0.7, 1.0.

5.1 Basic models

Two basics mono objective models have been implemented to have a quick overview of the problem.
The first designates SR to their respective districts to minimizes the distances from their offices to their districts. Running our model optimizer we get the following attribution of districts for each sales representatives:
The minimum distance found is 154.62km, the workload for each SR is respectively 1.04, 1.05, 1.12, 0.8, and the disruption caused by this new assignment of districts is 1.2 units of workload.
The second affects sales representatives in order to minimize disruption. This affectation

SR#	Center Brick	Bricks assigned
1	4	4,5,6,7,8,9,19,20
2	14	11,13,14,18
3	16	10,15,16,17
4	22	1,2,3,21,22

Table 5: Basic model brick distribution

SR#	Center Brick	Bricks assigned
1	4	4,5,6,7,8,15
2	14	10,13,14,18
3	16	9,11,12,16,17,18
4	22	1,2,3,19,20,21,22

Table 6: Basic model brick distribution with workload balance

returns the status quo as the minimum disruption happens when no sales rep are assigned new districts. However, if the workload balance constraints is applied, the minimum disruption affectation is the following:
The amount of disruption as a workload unit is 0.169, and the total distance is 188.89km. Therefore, using only these two simple mono objectives models we found a way to reduce by 32 kilometers the total travelled by the sales rep, and while keeping their workload in a good balance.

5.2 Dual objectives models

Other more complicated models have been implemented to take both the distances and disruption in account in the minimization objective: Using two different techniques, weighted sum and epsilon constraint, we plotted the distances against the disruption:
The weighted sum method will add a weight associated with the disruption to the objective function. The bigger the weight, the bigger is the importance of the disruption in the objective. The epsilon constraint set a new constraint such as the disruption is less than epsilon value. From figure 1 we notice that the weighted sum method gives only 6 possible possible distribution of districts to sales rep, where as the epsilon constraints gives 17. Figure 2 gives a good appreciation of the relation between the distance and the disruption. The decision maker can choose effectively the amount of disruption while maintaining a relatively low distance between SR offices and districts.

5.3 Flexibility of the model

Moreover, we can have different results by modifying slightly the constraints. For example, let's change the workload balance. Figure 3 and 4 shows the relationship between disruption and distances when workload varies from a large range(0.7 to 1.3) and a tight range (0.9 to 1.1). The smaller the range of workload that sales rep can take, the less possibilities of assignments of districts to SR are available. For example, if we want all sales representatives to have a workload value of one, no solutions are possible, meaning that no assignment of districts can give a perfectly equitable workload balance between SRs.

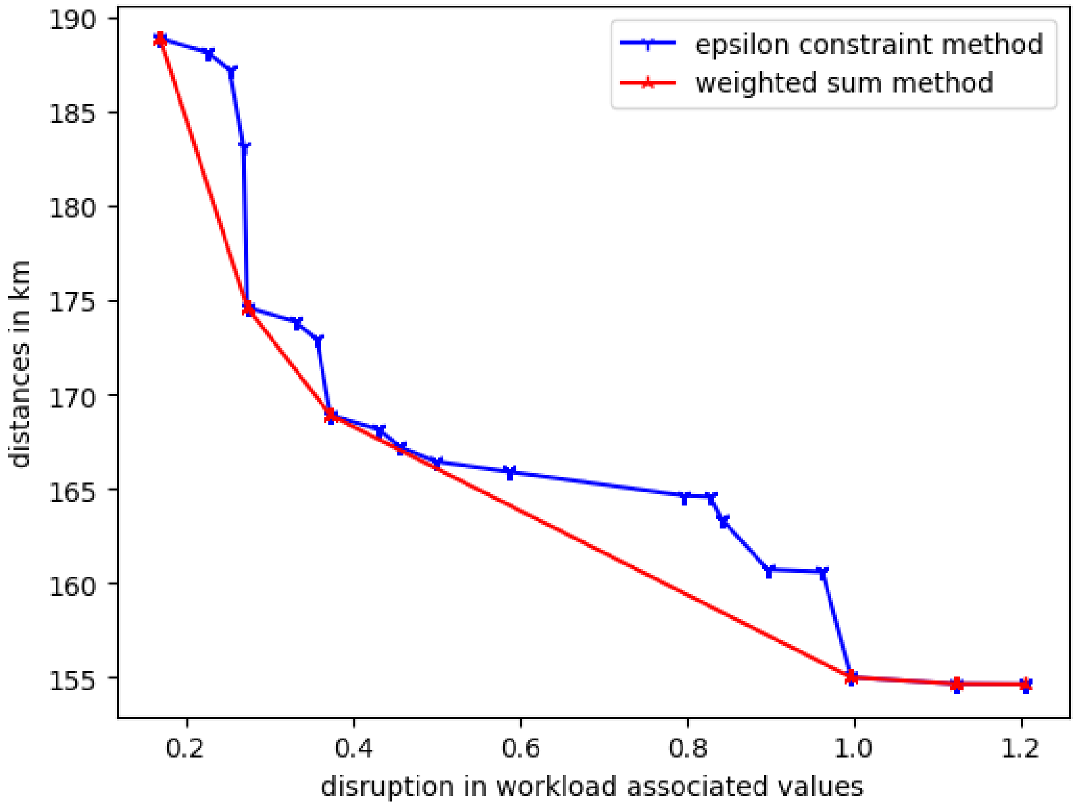

Figure 1: plot of the distances against the disruption using both the weighted sum method and epsilon constraint method.

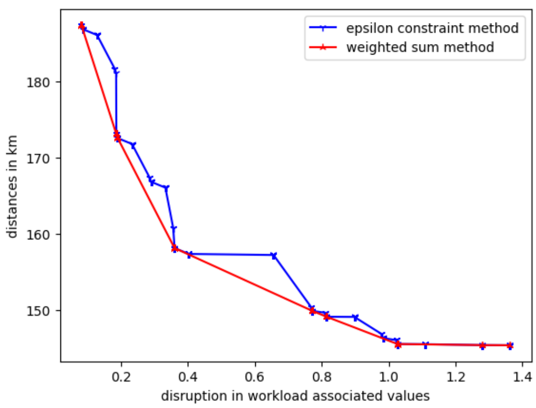

Figure 2: plot of the distances against the disruption for workload balance between 0.7 and 1.3.

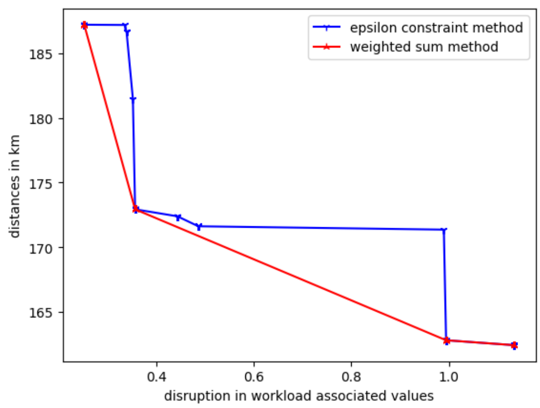

Figure 3: plot of the distances against the disruption for workload balance between 0.9 and 1.1.

6 Advanced models

6.1 Assigning multiple SR to the same brick

We've seen in the previous section that the salesman workload can't be perfectly distributed among themselves. To solve this issue, we can redefine the problem and the assignment matrix variable V_{ij}. By replacing the binary variables of the assignment matrix by continuous variables, we can simulate multiples sales rep assigned to the same brick. Since the rows of the V_{ij} must equal to one, representing that the whole district is assigned to someone, previously the matrix values were binary meaning that only one salesman would be in charge of the whole district. By setting the values of the matrix as continuous, the optimizer can find the best distribution of a district among the sales rep while minimizing the distances and the balance of workload, and assign a fraction of a district to multiple sales rep.
For example, in the specific case were the decision maker want all salesman to have a workload balance of 1 we would have the following assignment of districts (see table1).

	SR1	SR2	SR3	SR4					
0	1.0	0.0	0.0	0.0	11	0.0	1.0	0.0	0.0
1	0.0	0.0	0.0	1.0	12	0.0	1.0	0.0	0.0
2	0.0	0.0	0.0	1.0	13	0.0	0.35	0.0	0.65
3	1.0	0.0	0.0	0.0	14	0.0	0.0	1.0	0.0
4	1.0	0.0	0.0	0.0	15	0.0	0.42	0.58	0.0
5	1.0	0.0	0.0	0.0	16	0.0	0.0	1.0	0.0
6	1.0	0.0	0.0	0.0	17	0.0	1.0	0.0	0.0
7	0.21	0.79	0.0	0.0	18	1.0	0.0	0.0	0.0
8	0.0	0.0	1.0	0.0	19	1.0	0.0	0.0	0.0
9	0.0	1.0	0.0	0.0	20	1.0	0.0	0.0	0.0
10	0.0	0.0	1.0	0.0	21	0.0	0.0	0.0	1.0

Table 7: V_{ij} assignment matrix of sales rep with a workload balance of one.

The workload distribution among the sales representatives are respectively of 1,1,1, and 0.999999, for a minimum total distance of 189.37 km. The distance is larger than in our previous models because multiple sales rep have to travel to the same district. Moreover this distance will increase if we try to reduce the disruption as well.
From figure 5, showing both plot of distances against disruption for either assigning one or multiple sales rep per district, we can see that assigning fractions of districts to different sales has a great influence on disruption.

6.2 Recruiting a new sales representative if workload suddenly increases by 20% in all districts

Our case study also anticipate a sudden increase in workload of each districts, thus necessitating a new sales representative in order to balance the workload between 0.8 and 1.2 for each sales rep. We therefore need to assign him an office and districts to look after. We modelled the case where workload of each districts increased by 20%.
To find the best districts and center brick for the new sale rep, we added a new binary

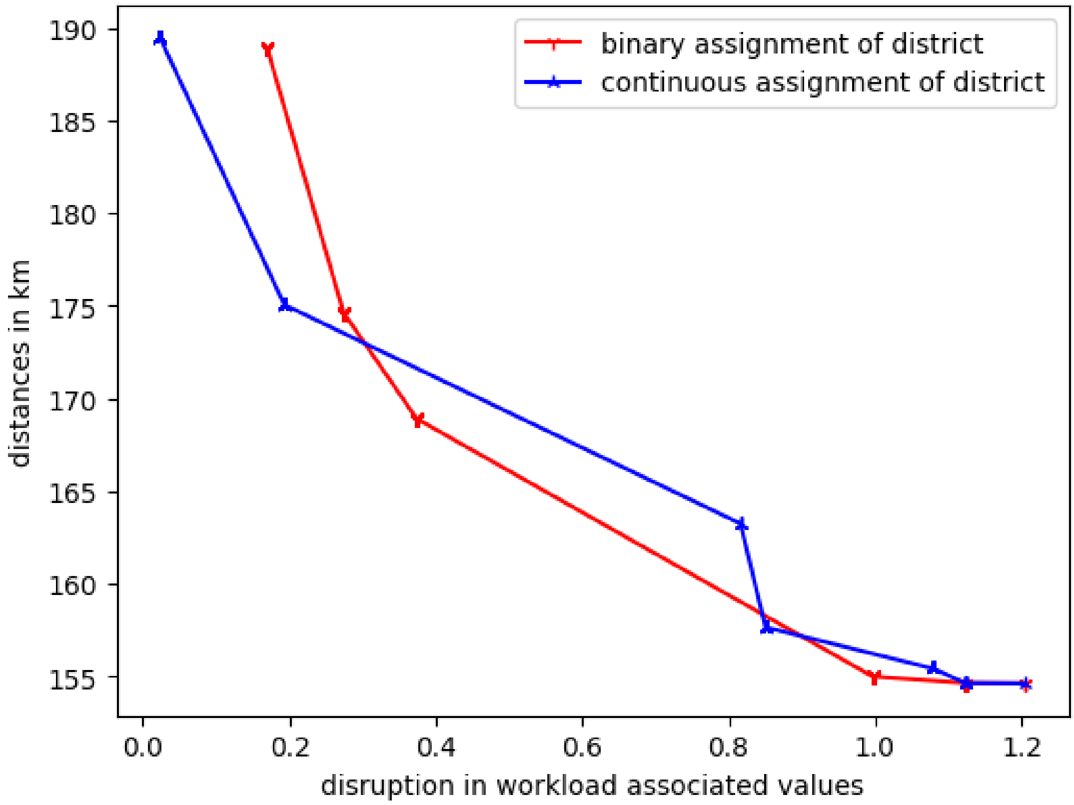

Figure 4: plot of the distances against the disruption using both a binary and continuous assignment matrix. (the disruption was computed using the weighed sum method)

variable "b" to our model. "b" is a 22 values long vector containing one 1 and 0s everywhere else. This new variable represents the center brick of the new sales rep and allows the model to optimize the objective to find the best district for him to set his office, depending on the other districts he is in charge of, given by the V_{ij} matrix.

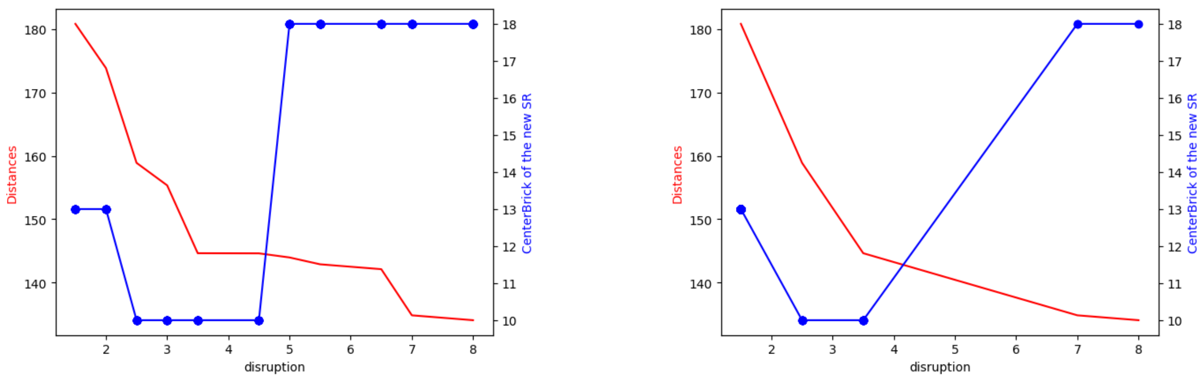

Figure 5: plot of the distances and the district of the center brick of the new sales representatives, against the disruption. (the disruption was computed using the epsilon constraint method on the left, and weighted sum on the right)

As we can see of figure 6, the center brick of the new sales rep will change depending on the disruption wanted, respectively district 14, 11, and 19,(lets' take into consideration that the solver considers the first district as 0 instead of 1, so the districts shown in figure

6 are similar to the table 8) for low disruption to high disruption. To minimize the total distances without considering the disruption, the new sales rep would have the following assigned districts(lets' take into consideration that the solver considers the first district as 0 instead of 1):

SR#	Center Brick	Bricks assigned
1	2	4,5,6,7,8,9
2	18	13,14,18
3	15	10,11,16,17
4	22	3,15,22
5	19	1,2,12,19,20,21

Table 8: Bricks distribution with a 5th SR

The respective workload for each sales representatives is: 1.04, 0.96, 0.91, 1.08, and 0.8. Finally the total distance is 134.4km.

6.3 Reassigning center brick and districts

Another way to improve the situation is to reassign the offices of the sales representatives to reduce the distances to their districts. This implementation is the most complete one of our study as it tackles perfectly the current issue of this document. To make it possible we reused our work done for adding a new SR and generalized it to every SR. This time around, SR's center bricks are no more predefined but must be set by the model. To do so we represented them with the variable "O" (for office), a 22x4 binary matrix. By multiplying it to the distance matrix, we have a 22x1 matrix of the distances of all the bricks from a center brick. Multiplying this last result by the Vij variable in Gurobi gives us the optimal distance for each SR in only 7 seconds. The optimal solution we found is the following:

SR#	Center Brick	Bricks assigned
1	2	1,2,3,19,20,21,22
2	18	13,14,17,18
3	15	11,11,15,16
4	6	4,5,6,7,8,9,12

Table 9: Bricks distribution of each SR

With the distribution given in table 9 we set the objective function to minimize distances. The optimal solution found is an impressive optimized distance of 103.57 km ! For records, the initial distance we started was 154.62 km. We have truly improved the efficiency.

6.4 Preferences

Finally, to please the sales representatives, our model takes into consideration the preferences for different districts they want to be in charge of. By filling an excel sheet with their preferences for each district, with values from zero to ten, the model can assign them new bricks depending on their preferences. As we can see on figure 7, when their preferences is very importantly taken into account, they have their favorite district , in return for very large distances to travel. To model this situation, the we added weight to the objective function such as the more a SR wants a district, the more likely he is to be assigned to it. To simulate preferences, 3 excel sheet were created with random values from 0 to 10 using the RAND() function.

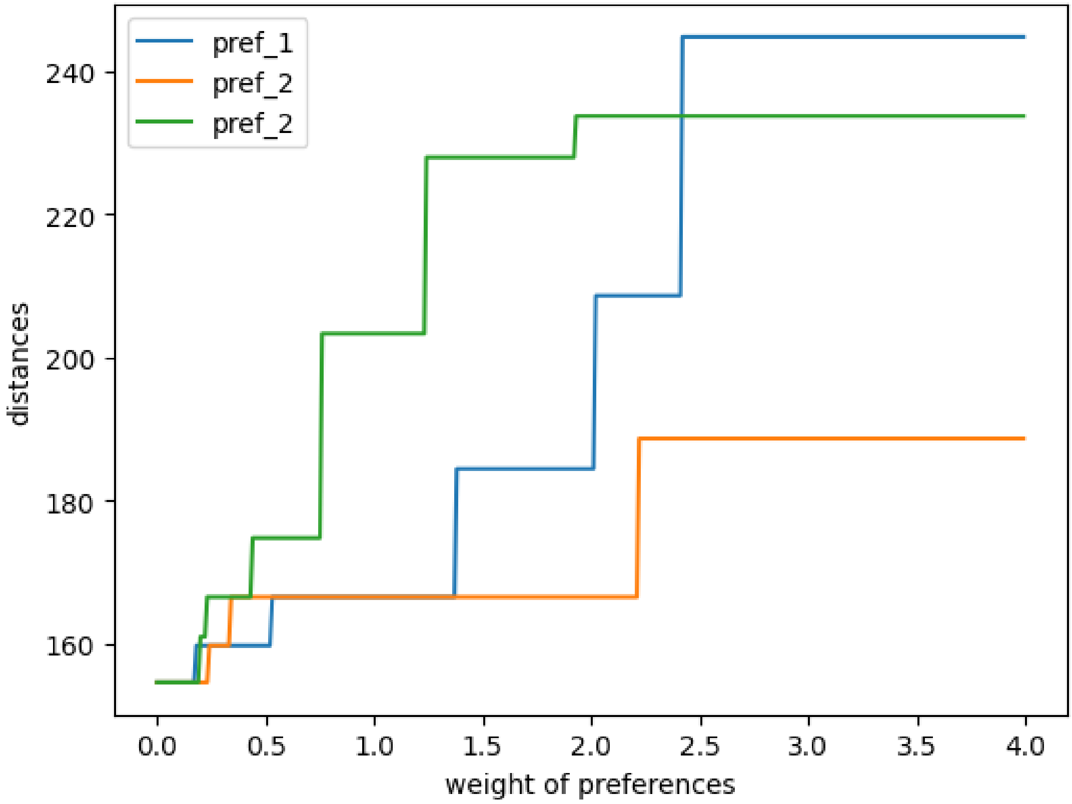

Figure 6: plot of the distances against the weight of preferences for three different preferences table.

7 Finals Recommendations

To conclude, we can affirm that our model covers gives the decision maker a very good insight about the relations between distances travelled y the sales representatives, the disruption caused by new assignments of districts, and workload balance between the sales rep. He can now chose from a vast range of possibilities to re assign the regions to different SR according to many criteria. For example if the distance is more important than the disruption, re assigning the center bricks of each sales representatives can reduce by 50 kilometers the total distance travelled. Or, if the workload balance is very important, multiples sales rep can be assigned to the same region to balance the workload very fairly. The model also prepares many situations in advance, such as anticipating the increase in workload and hiring a new SR. Finally the model is very flexible and can be adjusted according to the specific actual situation.

8 Limits of our case study

Even though our models takes into consideration many variables, it is almost impossible to simulates real life problem. Many details are too random or complicated to predict. For example, in this case study, the main variable to be minimized is the distance between SR's offices and their attributed districts. But the traffic on the road or the type if transports can sometimes have a bigger influence than the total distance. Or similarly the workload balance can be irrelevant if a sale representative doesn't perform as efficiently as the other. For example if one of the SR has less experience than the others, such as an intern, the workload balance can be too much for him even though it is the amount same for the others.

9 Appendix python code

The different functions in the python code are the following:

-Initialisation of the variables
-minimization of the distances
-minimization of disruption
-minimize both distance and disruption using weighted sum method + plot of distances against disruption with increasing values of weight
-minimize both distance and disruption using epsilon constraint method + plot of distances against disruption with increasing values of epsilon
-plot of both epsilon constraint and weighted sum methods for distances against disruption
-assignment of multiples SR to the same brick
-plot of distance against disruption for binary or continuous assignment matrix.
-assign a new SR if demand increases by 20% modelled using weighted sum +plot of disruption and center brick in function of disruption
-assign a new SR if demand increases by 20% modelled using epsilon constraint +plot of disruption and center brick in function of disruption
-reassign every SR to a new district
-assign in function of preferences + plot of the distance against the importance of the preference.