This is the Project Title

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Abstract

Formal development of Frank.

Chapter 1

Introduction

(data types)	D
(value type variables)	X
(effect type variables)	E
(value types)	$A, B ::= D\,\overline{R}$
	$\mid\ \ \{C\} \mid X$
(computation types)	$C ::= \overline{T} \to G$
(argument types)	$T ::= \langle\Delta\rangle A$
(return types)	$G ::= [\Sigma]A$
(type binders)	$Z ::= X \mid [E]$
(type arguments)	$R ::= A \mid [\Sigma]$
(polytypes)	$P ::= \forall\overline{Z}.A$
(interfaces)	I
(term variables)	x, y, z, f
(instance variables)	s, a, b, c
(seeds)	$\sigma ::= \emptyset \mid E$
(abilities)	$\Sigma ::= \sigma \mid \Xi$
(extensions)	$\Xi ::= \iota \mid \Xi, I\,\overline{R}$
(adaptors)	$\Theta ::= \iota \mid \Theta, I(S \to S')$
(adjustments)	$\Delta ::= \Theta \mid \Xi$
(instance patterns)	$S ::= s \mid S\,a$
(kind environments)	$\Phi, \Psi ::= \cdot \mid \Phi, Z$
(type environments)	$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, f : P$
(instance environments)	$\Omega ::= s : \Sigma \mid \Omega, a : I\,\overline{R}$

Figure 1.1: Types

$$\begin{array}{ll}
\text{(constructors)} & k \\[4pt]
\text{(commands)} & c \\[4pt]
\text{(uses)} & m ::= x \mid f\,\overline{R} \mid m\,\overline{n} \mid \uparrow(n:A) \\[4pt]
\text{(constructions)} & n ::= \downarrow m \mid k\,\overline{n} \mid c\,\overline{R}\,\overline{n} \mid \{e\} \\[4pt]
& \quad\mid\ \textbf{let } f:P=n \textbf{ in } n' \mid \textbf{letrec } \overline{f:P=e} \textbf{ in } n \\[4pt]
& \quad\mid\ \langle\Theta\rangle\,n \\[4pt]
\text{(computations)} & e ::= \overline{\overline{r} \mapsto n} \\[4pt]
\text{(computation patterns)} & r ::= p \mid \langle c\,\overline{p} \to z\rangle \mid \langle x\rangle \\[4pt]
\text{(value patterns)} & p ::= k\,\overline{p} \mid x
\end{array}$$

Figure 1.2: Terms

$$\boxed{\Phi;\Gamma\,[\Sigma]\!\!\vdash m \Rightarrow A}$$

T-VAR
$$\frac{x:A \in \Gamma}{\Phi;\Gamma\,[\Sigma]\!\!\vdash x \Rightarrow A}$$

T-POLYVAR
$$\frac{\Phi \vdash \overline{R} \qquad f:\forall \overline{Z}.A \in \Gamma}{\Phi;\Gamma\,[\Sigma]\!\!\vdash f\,\overline{R} \Rightarrow A[\overline{R}/\overline{Z}]}$$

T-APP
$$\frac{\Sigma' = \Sigma \qquad (\Sigma \vdash \Delta_i \dashv \Sigma'_i)_i}{\Phi;\Gamma\,[\Sigma]\!\!\vdash m \Rightarrow \{\overline{\langle\Delta\rangle A \to}\,[\Sigma']B\} \qquad (\Phi;\Gamma\,[\Sigma'_i]\!\!\vdash n_i:A_i)_i}{\Phi;\Gamma\,[\Sigma]\!\!\vdash m\,\overline{n} \Rightarrow B}$$

T-ASCRIBE
$$\frac{\Phi;\Gamma\,[\Sigma]\!\!\vdash n:A}{\Phi;\Gamma\,[\Sigma]\!\!\vdash \uparrow(n:A) \Rightarrow A}$$

$$\boxed{\Phi;\Gamma\,[\Sigma]\!\!\vdash n:A}$$

T-SWITCH
$$\frac{\Phi;\Gamma\,[\Sigma]\!\!\vdash m \Rightarrow A \qquad A = B}{\Phi;\Gamma\,[\Sigma]\!\!\vdash \downarrow m:B}$$

T-DATA
$$\frac{k\,\overline{A} \in D\,\overline{R} \qquad (\Phi;\Gamma\,[\Sigma]\!\!\vdash n_j:A_j)_j}{\Phi;\Gamma\,[\Sigma]\!\!\vdash k\,\overline{n}:D\,\overline{R}}$$

T-COMMAND
$$\frac{\Phi \vdash \overline{R} \qquad c:\forall \overline{Z}.\overline{A} \to B \in \Sigma \qquad (\Phi;\Gamma\,[\Sigma]\!\!\vdash n_j:A_j[\overline{R}/\overline{Z}])_j}{\Phi;\Gamma\,[\Sigma]\!\!\vdash c\,\overline{R}\,\overline{n}:B[\overline{R}/\overline{Z}]}$$

T-THUNK
$$\frac{\Phi;\Gamma \vdash e:C}{\Phi;\Gamma\,[\Sigma]\!\!\vdash \{e\}:\{C\}}$$

T-LET
$$\frac{P = \forall \overline{Z}.A \\ \Phi,\overline{Z};\Gamma\,[\emptyset]\!\!\vdash n:A \qquad \Phi;\Gamma,f:P\,[\Sigma]\!\!\vdash n':B}{\Phi;\Gamma\,[\Sigma]\!\!\vdash \mathbf{let}\ f:P = n\ \mathbf{in}\ n':B}$$

T-LETREC
$$\frac{(P_i = \forall \overline{Z}_i.\{C_i\})_i \\ (\Phi,\overline{Z}_i;\Gamma,\overline{f:P} \vdash e_i:C)_i \qquad \Phi;\Gamma,\overline{f:P}\,[\Sigma]\!\!\vdash n:B}{\Phi;\Gamma\,[\Sigma]\!\!\vdash \mathbf{letrec}\ \overline{f:P = e}\ \mathbf{in}\ n:B}$$

T-ADAPT
$$\frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Phi;\Gamma\,[\Sigma']\!\!\vdash n:A}{\Phi;\Gamma\,[\Sigma]\!\!\vdash \langle\Theta\rangle\,n:A}$$

$$\boxed{\Phi;\Gamma \vdash e:C}$$

T-COMP
$$\frac{(\Phi \vdash r_{i,j}:T_j \dashv[\Sigma]\ \exists\Psi_{i,j}.\Gamma'_{i,j})_{i,j} \\ (\Phi,(\Psi_{i,j})_j;\Gamma,(\Gamma'_{i,j})_j\,[\Sigma]\!\!\vdash n_i:B)_i \qquad ((r_{i,j})_i\ \text{covers}\ T_j)_j}{\Phi;\Gamma \vdash ((r_{i,j})_j \mapsto n_i)_i : (T_j \to)_j\,[\Sigma]B}$$

Figure 1.3: Term Typing Rules

$$
\begin{array}{lll}
\text{(uses)} & m ::= \cdots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \\[4pt]
\text{(constructions)} & n ::= \cdots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \\[4pt]
\text{(use values)} & u ::= x \mid f\,\overline{R} \mid {\uparrow}(v:A) \\[4pt]
\text{(non-use values)} & v ::= k\,\overline{w} \mid \{e\} \\[4pt]
\text{(construction values)} & w ::= {\downarrow}u \mid v \\[4pt]
\text{(normal forms)} & t ::= w \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \\[4pt]
\text{(evaluation frames)} & \mathcal{F} ::= [\,]\,\overline{n} \mid u\,(\overline{t},[\,],\overline{n}) \mid {\uparrow}([\,]:A) \\[4pt]
& \qquad \mid\ {\downarrow}[\,] \mid k\,(\overline{w},[\,],\overline{n}) \mid c\,\overline{R}\,(\overline{w},[\,],\overline{n}) \\[4pt]
& \qquad \mid\ \mathbf{let}\ f:P = [\,]\ \mathbf{in}\ n \mid \langle \Theta \rangle\,[\,] \\[4pt]
\text{(evaluation contexts)} & \mathcal{E} ::= [\,] \mid \mathcal{F}[\mathcal{E}]
\end{array}
$$

Figure 1.4: Runtime Syntax

$$
\boxed{\Phi;\Gamma\,[\Sigma]\!\!\vdash m \Rightarrow A} \qquad \boxed{\Phi;\Gamma\,[\Sigma]\!\!\vdash n : A}
$$

T-FREEZE-USE
$$
\frac{\neg(\mathcal{E}\ \mathsf{handles}\ c) \qquad \Phi;\Gamma\,[\Sigma]\!\!\vdash \mathcal{E}[c\,\overline{R}\,\overline{w}] \Rightarrow A}{\Phi;\Gamma\,[\Sigma]\!\!\vdash \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \Rightarrow A}
$$

T-FREEZE-CONS
$$
\frac{\neg(\mathcal{E}\ \mathsf{handles}\ c) \qquad \Phi;\Gamma\,[\Sigma]\!\!\vdash \mathcal{E}[c\,\overline{R}\,\overline{w}] : A}{\Phi;\Gamma\,[\Sigma]\!\!\vdash \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil : A}
$$

Figure 1.5: Frozen Commands

$$\boxed{m \rightsquigarrow_{\mathrm{u}} m'} \quad \boxed{n \rightsquigarrow_{\mathrm{c}} n'} \quad \boxed{m \longrightarrow_{\mathrm{u}} m'} \quad \boxed{n \longrightarrow_{\mathrm{c}} n'}$$

R-HANDLE
$$\frac{k = \min_i \{i \mid \exists \overline{\theta}.(r_{i,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \dashv [\Sigma] \, \theta_j)_j\} \qquad (r_{k,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \dashv [\Sigma] \, \theta_j)_j}{\uparrow(\{((r_{i,j})_j \rightarrow n_i)_i\} : \{\overline{\langle \Delta \rangle A \rightarrow} [\Sigma] B\}) \, \overline{t} \rightsquigarrow_{\mathrm{u}} \uparrow((\overline{\theta}(n_k) : B)}$$

R-ASCRIBE-USE

$$\frac{}{\uparrow(\downarrow u : A) \rightsquigarrow_{\mathrm{u}} u}$$

R-ASCRIBE-CONS

$$\frac{}{\downarrow\uparrow(w : A) \rightsquigarrow_{\mathrm{c}} w}$$

R-LET

$$\frac{}{\mathbf{let} \, f : P = w \, \mathbf{in} \, n \rightsquigarrow_{\mathrm{c}} n[\uparrow(w : P)/f]}$$

R-LETREC

R-ADAPT

$$\frac{e = \overline{\overline{r} \rightarrow n}}{\mathbf{letrec} \, \overline{f : P = e} \, \mathbf{in} \, n' \rightsquigarrow_{\mathrm{c}} n'[\uparrow(\{\overline{r} \rightarrow \mathbf{letrec} \, \overline{f : P = e} \, \mathbf{in} \, n\} : P)/f]} \qquad \frac{}{\langle \Theta \rangle \, w \rightsquigarrow_{\mathrm{c}} w}$$

R-FREEZE-COMM

$$\frac{}{c \, \overline{R} \, \overline{w} \rightsquigarrow_{\mathrm{c}} \lceil c \, \overline{R} \, \overline{w} \rceil}$$

R-FREEZE-FRAME-USE
$$\frac{\neg(\mathcal{F}[\mathcal{E}] \, \mathrm{handles} \, c)}{\mathcal{F}[[\mathcal{E}[c \, \overline{R} \, \overline{w}]]] \rightsquigarrow_{\mathrm{u}} \lceil \mathcal{F}[\mathcal{E}[c \, \overline{R} \, \overline{w}]] \rceil}$$

R-FREEZE-FRAME-CONS
$$\frac{\neg(\mathcal{F}[\mathcal{E}] \, \mathrm{handles} \, c)}{\mathcal{F}[[\mathcal{E}[c \, \overline{R} \, \overline{w}]]] \rightsquigarrow_{\mathrm{c}} \lceil \mathcal{F}[\mathcal{E}[c \, \overline{R} \, \overline{w}]] \rceil}$$

R-LIFT-UU
$$\frac{m \rightsquigarrow_{\mathrm{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathrm{u}} \mathcal{E}[m']}$$

R-LIFT-UC
$$\frac{m \rightsquigarrow_{\mathrm{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathrm{c}} \mathcal{E}[m']}$$

R-LIFT-CU
$$\frac{n \rightsquigarrow_{\mathrm{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathrm{u}} \mathcal{E}[n']}$$

R-LIFT-CC
$$\frac{n \rightsquigarrow_{\mathrm{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathrm{c}} \mathcal{E}[n']}$$

Figure 1.6: Operational Semantics

$$\boxed{r : T \leftarrow t \dashv_{[\Sigma]} \theta}$$

B-VALUE
$$\dfrac{\begin{array}{c} \Sigma \vdash \Delta \dashv \Sigma' \\ p : A \leftarrow w \dashv \theta \end{array}}{p : \langle \Delta \rangle A \leftarrow w \dashv_{[\Sigma]} \theta}$$

B-REQUEST
$$\dfrac{\begin{array}{c} \Sigma \vdash \Delta \dashv \Sigma' \qquad \mathcal{E} \text{ poisedfor } c \\ \Delta = \Theta \,|\, \Xi \qquad c : \forall \overline{Z}.\overline{B} \to B' \in \Xi \qquad (p_i : B_i \leftarrow w_i \dashv \theta_i)_i \end{array}}{\langle c \, \overline{p} \to z \rangle : \langle \Delta \rangle A \leftarrow \ulcorner \mathcal{E}[c \, \overline{R} \, \overline{w}] \urcorner \dashv_{[\Sigma]} \overline{\theta}[\uparrow(\{x \mapsto \mathcal{E}[x]\} : \{B' \to [\Sigma']A\})/z]}$$

B-CATCHALL-VALUE
$$\dfrac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x \rangle : \langle \Delta \rangle A \leftarrow w \dashv_{[\Sigma]} [\uparrow(\{w\}:\{[\Sigma']A\})/x]}$$

B-CATCHALL-REQUEST
$$\dfrac{\begin{array}{c} \Sigma \vdash \Delta \dashv \Sigma' \qquad \mathcal{E} \text{ poisedfor } c \\ \Delta = \Theta \,|\, \Xi \qquad c : \forall \overline{Z}.\overline{B} \to B' \in \Xi \end{array}}{\langle x \rangle : \langle \Delta \rangle A \leftarrow \ulcorner \mathcal{E}[c \, \overline{R} \, \overline{w}] \urcorner \dashv_{[\Sigma]} [\uparrow(\{\ulcorner \mathcal{E}[c \, \overline{R} \, \overline{w}] \urcorner\}:\{[\Sigma']A\})/x]}$$

$$\boxed{p : A \leftarrow w \dashv \theta}$$

B-VAR
$$\dfrac{}{x : A \leftarrow w \dashv [\uparrow(w : A)/x]}$$

B-DATA
$$\dfrac{k \, \overline{A} \in D \, \overline{R} \qquad (p_i : A_i \leftarrow w_i \dashv \theta_i)_i}{k \, \overline{p} : D \, \overline{R} \leftarrow k \, \overline{w} \dashv \overline{\theta}}$$

Figure 1.7: Pattern Binding

Appendix A

Remaining Formalisms

$$\boxed{\Sigma \vdash \Delta \dashv \Sigma'}$$

A-ADJ
$$\frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Sigma' \vdash \Xi \dashv \Sigma''}{\Sigma \vdash \Theta \mid \Xi \dashv \Sigma''}$$

$$\boxed{\Sigma \vdash \Xi \dashv \Sigma'}$$

A-EXT-ID
$$\frac{}{\Sigma \vdash \iota \dashv \Sigma}$$

A-EXT-SNOC
$$\frac{\Sigma \vdash \Xi \dashv \Sigma'}{\Sigma \vdash \Xi, I\,\overline{R} \dashv \Sigma', I\,\overline{R}}$$

$$\boxed{\Sigma \vdash \Theta \dashv \Sigma'}$$

A-ADAPT-ID
$$\frac{}{\Sigma \vdash \iota \dashv \Sigma}$$

A-ADAPT-SNOC
$$\frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Sigma' \vdash I(S \to S') \dashv \Sigma''}{\Sigma \vdash \Theta, I(S \to S') \dashv \Sigma''}$$

$$\boxed{\Sigma \vdash I(S \to S') \dashv \Sigma'}$$

A-ADAPT-COM
$$\frac{\Sigma \vdash S : I \dashv \Sigma'; \Omega \qquad \Omega \vdash S' : I \dashv \Xi \qquad \Sigma' \vdash \Xi \dashv \Sigma''}{\Sigma \vdash I(S \to S') \dashv \Sigma''}$$

$$\boxed{\Sigma \vdash S : I \dashv \Sigma'; \Omega}$$

I-PAT-ID
$$\frac{}{\Sigma \vdash s : I \dashv \Sigma; s : \Sigma}$$

I-PAT-BIND
$$\frac{\Sigma \vdash S : I \dashv \Sigma'; \Omega}{\Sigma, I\,\overline{R} \vdash S\,a : I \dashv \Sigma'; \Omega, a : I\,\overline{R}}$$

I-PAT-SKIP
$$\frac{\Sigma \vdash S\,a : I \dashv \Sigma'; \Omega \qquad I \neq I'}{\Sigma, I'\,\overline{R} \vdash S\,a : I \dashv \Sigma', I'\,\overline{R}; \Omega}$$

$$\boxed{\Omega \vdash S : I \dashv \Xi}$$

I-INST-ID
$$\frac{s \in \mathsf{dom}(\Omega)}{\Omega \vdash s : I \dashv \iota}$$

I-INST-LKP
$$\frac{a \in \mathsf{dom}(\Omega) \qquad \Omega \vdash S : I \dashv \Xi \qquad \Omega(a) = I\,\overline{R}}{\Omega \vdash S\,a : I \dashv \Xi, I\,\overline{R}}$$

Figure A.1: Action of an Adjustment on an Ability and Auxiliary Judgements

$$X ::= A \mid C \mid T \mid G \mid Z \mid R \mid P \mid \sigma \mid \Sigma \mid \Xi \mid \Theta \mid \Delta \mid \Gamma \mid \exists \Psi.\Gamma \mid \Omega$$

$$\boxed{\Phi \vdash X}$$

WF-VAL
$$\frac{}{\Phi, X \vdash X}$$

WF-EFF
$$\frac{}{\Phi, [E] \vdash E}$$

WF-POLY
$$\frac{\Phi, \overline{Z} \vdash A}{\Phi \vdash \forall \overline{Z}.A}$$

WF-DATA
$$\frac{(\Phi \vdash R)_i}{\Phi \vdash D \, \overline{R}}$$

WF-THUNK
$$\frac{\Phi \vdash C}{\Phi \vdash \{C\}}$$

WF-COMP
$$\frac{(\Phi \vdash T)_i \qquad \Phi \vdash G}{\Phi \vdash \overline{T \to G}}$$

WF-ARG
$$\frac{\Phi \vdash \Delta \qquad \Phi \vdash A}{\Phi \vdash \langle \Delta \rangle A}$$

WF-RET
$$\frac{\Phi \vdash \Sigma \qquad \Phi \vdash A}{\Phi \vdash [\Sigma]A}$$

WF-ABILITY
$$\frac{\Phi \vdash \Sigma}{\Phi \vdash [\Sigma]}$$

WF-PURE
$$\frac{}{\Phi \vdash \emptyset}$$

WF-ID
$$\frac{}{\Phi \vdash \iota}$$

WF-EXT
$$\frac{\Phi \vdash \Xi \qquad (\Phi \vdash R)_i}{\Phi \vdash \Xi, I \, \overline{R}}$$

WF-ADAPT
$$\frac{\Phi \vdash \Theta}{\Phi \vdash \Theta, I \, (S \to S')}$$

WF-EMPTY
$$\frac{}{\Phi \vdash \cdot}$$

WF-MONO
$$\frac{\Phi \vdash \Gamma \qquad \Phi \vdash A}{\Phi \vdash \Gamma, x : A}$$

WF-POLY
$$\frac{\Phi \vdash \Gamma \qquad \Phi \vdash P}{\Phi \vdash \Gamma, f : P}$$

WF-EXISTENTIAL
$$\frac{\Phi, \Psi \vdash \Gamma}{\Phi \vdash \exists \Psi.\Gamma}$$

WF-INTERFACE
$$\frac{\Phi \vdash \Omega \qquad (\Phi \vdash R)_i}{\Phi \vdash \Omega, x : I \, \overline{R}}$$

Figure A.2: Well-Formedness Rules

$$\boxed{\Phi \vdash p : A \dashv \Gamma}$$

P-Var

$$\frac{}{\Phi \vdash x : A \dashv x : A}$$

P-Data

$$\frac{k\,\overline{A} \in D\,\overline{R} \qquad (\Phi \vdash p_i : A_i \dashv \Gamma)_i}{\Phi \vdash k\,\overline{p} : D\,\overline{R} \dashv \overline{\Gamma}}$$

$$\boxed{\Phi \vdash r : T \dashv [\Sigma]\, \exists \Psi.\Gamma}$$

P-Value

$$\frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \Phi \vdash p : A \dashv \Gamma}{\Phi \vdash p : \langle \Delta \rangle A \dashv [\Sigma]\, \Gamma}$$

P-Catchall

$$\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\Phi \vdash \langle x \rangle : \langle \Delta \rangle A \dashv [\Sigma]\, x : \{[\Sigma']A\}}$$

P-Command

$$\frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \Delta = \Theta \mid \Xi \qquad c : \forall \overline{Z}.\overline{A} \to B \in \Xi \qquad (\Phi, \overline{Z} \vdash p_i : A_i \dashv \Gamma_i)_i}{\Phi \vdash \langle c\,\overline{p} \to z \rangle : \langle \Delta \rangle B' \dashv [\Sigma]\, \exists \overline{Z}.\overline{\Gamma}, z : \{\langle \iota \mid \iota \rangle B \to [\Sigma']B'\}}$$

Figure A.3: Pattern Matching Typing Rules