This is the Project Title

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Abstract

Formal development of Frank.

Chapter 1

Formalisation of Frank

(data types)	D
(value type variables)	X
(effect type variables)	E
(value types)	$A, B ::= D\,\overline{R}$
	$\mid\ \{C\} \mid X$
(computation types)	$C ::= \overline{T \to G}$
(argument types)	$T ::= \langle\Delta\rangle A$
(return types)	$G ::= [\Sigma]A$
(type binders)	$Z ::= X \mid [E]$
(type arguments)	$R ::= A \mid [\Sigma]$
(polytypes)	$P ::= \forall\overline{Z}.A$

(interfaces)	I
(term variables)	x, y, z, f
(instance variables)	s, a, b, c
(seeds)	$\sigma ::= \emptyset \mid E$
(abilities)	$\Sigma ::= \sigma \mid \Xi$
(extensions)	$\Xi ::= \iota \mid \Xi, I\,\overline{R}$
(adaptors)	$\Theta ::= \iota \mid \Theta, I(S \to S')$
(adjustments)	$\Delta ::= \Theta \mid \Xi$
(instance patterns)	$S ::= s \mid S\,a$
(kind environments)	$\Phi, \Psi ::= \cdot \mid \Phi, Z$
(type environments)	$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, f : P$
(instance environments)	$\Omega ::= s : \Sigma \mid \Omega, a : I\,\overline{R}$

Figure 1.1: Types

(constructors)	k
(commands)	c
(uses)	$m ::= x \mid f\,\overline{R} \mid m\,\overline{n} \mid \uparrow(n : A)$
(constructions)	$n ::= \downarrow m \mid k\,\overline{n} \mid c\,\overline{R}\,\overline{n} \mid \{e\}$
	$\mid\ \textbf{let}\ f : P = n\ \textbf{in}\ n' \mid \textbf{letrec}\ \overline{f : P = e}\ \textbf{in}\ n$
	$\mid\ \langle\Theta\rangle\,n$
(computations)	$e ::= \overline{\overline{r} \mapsto n}$
(computation patterns)	$r ::= p \mid \langle c\,\overline{p} \to z\rangle \mid \langle x\rangle$
(value patterns)	$p ::= k\,\overline{p} \mid x$

Figure 1.2: Terms

$$\boxed{\Phi;\Gamma\,[\Sigma]\!\!-m\Rightarrow A}$$

T-VAR
$$\frac{x:A\in\Gamma}{\Phi;\Gamma\,[\Sigma]\!\!-x\Rightarrow A}$$

T-POLYVAR
$$\frac{\Phi\vdash\overline{R}\qquad f:\forall\overline{Z}.A\in\Gamma}{\Phi;\Gamma\,[\Sigma]\!\!-f\,\overline{R}\Rightarrow A[\overline{R}/\overline{Z}]}$$

T-APP
$$\frac{\Sigma'=\Sigma\qquad(\Sigma\vdash\Delta_i\dashv\Sigma'_i)_i}{\Phi;\Gamma\,[\Sigma]\!\!-m\Rightarrow\{\overline{\langle\Delta\rangle A\rightarrow}\,[\Sigma']B\}\qquad(\Phi;\Gamma\,[\Sigma'_i]\!\!-n_i:A_i)_i}{\Phi;\Gamma\,[\Sigma]\!\!-m\,\overline{n}\Rightarrow B}$$

T-ASCRIBE
$$\frac{\Phi;\Gamma\,[\Sigma]\!\!-n:A}{\Phi;\Gamma\,[\Sigma]\!\!-\uparrow(n:A)\Rightarrow A}$$

$$\boxed{\Phi;\Gamma\,[\Sigma]\!\!-n:A}$$

T-SWITCH
$$\frac{\Phi;\Gamma\,[\Sigma]\!\!-m\Rightarrow A\qquad A=B}{\Phi;\Gamma\,[\Sigma]\!\!-\downarrow m:B}$$

T-DATA
$$\frac{k\,\overline{A}\in D\,\overline{R}\qquad(\Phi;\Gamma\,[\Sigma]\!\!-n_j:A_j)_j}{\Phi;\Gamma\,[\Sigma]\!\!-k\,\overline{n}:D\,\overline{R}}$$

T-COMMAND
$$\frac{\Phi\vdash\overline{R}\qquad c:\forall\overline{Z}.\overline{A}\rightarrow B\in\Sigma\qquad(\Phi;\Gamma\,[\Sigma]\!\!-n_j:A_j[\overline{R}/\overline{Z}])_j}{\Phi;\Gamma\,[\Sigma]\!\!-c\,\overline{R}\,\overline{n}:B[\overline{R}/\overline{Z}]}$$

T-THUNK
$$\frac{\Phi;\Gamma\vdash e:C}{\Phi;\Gamma\,[\Sigma]\!\!-\{e\}:\{C\}}$$

T-LET
$$\frac{P=\forall\overline{Z}.A}{\Phi,\overline{Z};\Gamma\,[\emptyset]\!\!-n:A\qquad\Phi;\Gamma,f:P\,[\Sigma]\!\!-n':B}{\Phi;\Gamma\,[\Sigma]\!\!-\textbf{let}\,f:P=n\,\textbf{in}\,n':B}$$

T-LETREC
$$\frac{(P_i=\forall\overline{Z}_i.\{C_i\})_i}{(\Phi,\overline{Z}_i;\Gamma,\overline{f:P}\vdash e_i:C)_i\qquad\Phi;\Gamma,\overline{f:P}\,[\Sigma]\!\!-n:B}{\Phi;\Gamma\,[\Sigma]\!\!-\textbf{letrec}\,\overline{f:P=e}\,\textbf{in}\,n:B}$$

T-ADAPT
$$\frac{\Sigma\vdash\Theta\dashv\Sigma'\qquad\Phi;\Gamma\,[\Sigma']\!\!-n:A}{\Phi;\Gamma\,[\Sigma]\!\!-\langle\Theta\rangle\,n:A}$$

$$\boxed{\Phi;\Gamma\vdash e:C}$$

T-COMP
$$\frac{(\Phi\vdash r_{i,j}:T_j\dashv[\Sigma]\,\exists\Psi_{i,j}.\Gamma'_{i,j})_{i,j}}{(\Phi,(\Psi_{i,j})_j;\Gamma,(\Gamma'_{i,j})_j\,[\Sigma]\!\!-n_i:B)_i\qquad((r_{i,j})_i\,\text{covers}\,T_j)_j}{\Phi;\Gamma\vdash((r_{i,j})_j\mapsto n_i)_i:(T_j\rightarrow)_j\,[\Sigma]B}$$

Figure 1.3: Term Typing Rules

$$
\begin{array}{lll}
\text{(uses)} & m ::= \cdots \mid \ulcorner \mathcal{E}[c\,\overline{R}\,\overline{w}] \urcorner \\
\text{(constructions)} & n ::= \cdots \mid \ulcorner \mathcal{E}[c\,\overline{R}\,\overline{w}] \urcorner \\
\text{(use values)} & u ::= x \mid f\,\overline{R} \mid \uparrow(v : A) \\
\text{(non-use values)} & v ::= k\,\overline{w} \mid \{e\} \\
\text{(construction values)} & w ::= {\downarrow}u \mid v \\
\text{(normal forms)} & t ::= w \mid \ulcorner \mathcal{E}[c\,\overline{R}\,\overline{w}] \urcorner \\
\text{(evaluation frames)} & \mathcal{F} ::= [\,]\,\overline{n} \mid u\,(\overline{t},[\,],\overline{n}) \mid \uparrow([\,] : A) \\
& \qquad \mid\ {\downarrow}[\,] \mid k\,(\overline{w},[\,],\overline{n}) \mid c\,\overline{R}\,(\overline{w},[\,],\overline{n}) \\
& \qquad \mid\ \mathbf{let}\ f : P = [\,]\ \mathbf{in}\ n \mid \langle \Theta \rangle\,[\,] \\
\text{(evaluation contexts)} & \mathcal{E} ::= [\,] \mid \mathcal{F}[\mathcal{E}]
\end{array}
$$

Figure 1.4: Runtime Syntax

$$\boxed{\Phi;\Gamma\,[\Sigma]\vdash m \Rightarrow A} \qquad \boxed{\Phi;\Gamma\,[\Sigma]\vdash n : A}$$

T-FREEZE-USE
$$
\frac{\neg(\mathcal{E}\ \text{handles}\ c) \qquad \Phi;\Gamma\,[\Sigma]\vdash \mathcal{E}[c\,\overline{R}\,\overline{w}] \Rightarrow A}{\Phi;\Gamma\,[\Sigma]\vdash \ulcorner \mathcal{E}[c\,\overline{R}\,\overline{w}] \urcorner \Rightarrow A}
$$

T-FREEZE-CONS
$$
\frac{\neg(\mathcal{E}\ \text{handles}\ c) \qquad \Phi;\Gamma\,[\Sigma]\vdash \mathcal{E}[c\,\overline{R}\,\overline{w}] : A}{\Phi;\Gamma\,[\Sigma]\vdash \ulcorner \mathcal{E}[c\,\overline{R}\,\overline{w}] \urcorner : A}
$$

Figure 1.5: Frozen Commands

$$\boxed{m \leadsto_{\mathrm{u}} m'} \quad \boxed{n \leadsto_{\mathrm{c}} n'} \quad \boxed{m \longrightarrow_{\mathrm{u}} m'} \quad \boxed{n \longrightarrow_{\mathrm{c}} n'}$$

R-HANDLE
$$\frac{k = \min_i \{i \mid \exists \overline{\theta}.(r_{i,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \dashv [\Sigma] \, \theta_j)_j\} \qquad (r_{k,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \dashv [\Sigma] \, \theta_j)_j}{\uparrow(\{((r_{i,j})_j \to n_i)_i\} : \{\overline{\langle \Delta \rangle A \to} [\Sigma] B\}) \, \overline{t} \leadsto_{\mathrm{u}} \uparrow((\overline{\theta}(n_k) : B)}$$

R-ASCRIBE-USE   R-ASCRIBE-CONS   R-LET

$$\frac{}{\uparrow(\downarrow u : A) \leadsto_{\mathrm{u}} u} \qquad \frac{}{\downarrow \uparrow (w : A) \leadsto_{\mathrm{c}} w} \qquad \frac{}{\mathbf{let} \, f : P = w \, \mathbf{in} \, n \leadsto_{\mathrm{c}} n[\uparrow(w : P)/f]}$$

R-LETREC

$$\frac{}{e = \overline{\overline{r} \to n}} \qquad\qquad\qquad\qquad \text{R-ADAPT}$$
$$\frac{}{\mathbf{letrec} \, \overline{f : P = e} \, \mathbf{in} \, n' \leadsto_{\mathrm{c}} n'[\uparrow(\{\overline{r} \to \mathbf{letrec} \, \overline{f : P = e} \, \mathbf{in} \, n\} : P)/f]} \qquad \frac{}{\langle \Theta \rangle \, w \leadsto_{\mathrm{c}} w}$$

R-FREEZE-COMM

$$\frac{}{c \, \overline{R} \, \overline{w} \leadsto_{\mathrm{c}} \lceil c \, \overline{R} \, \overline{w} \rceil}$$

R-FREEZE-FRAME-USE   R-FREEZE-FRAME-CONS
$$\frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[\lceil \mathcal{E}[c \, \overline{R} \, \overline{w}]\rceil] \leadsto_{\mathrm{u}} \lceil \mathcal{F}[\mathcal{E}[c \, \overline{R} \, \overline{w}]]\rceil} \qquad \frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[\lceil \mathcal{E}[c \, \overline{R} \, \overline{w}]\rceil] \leadsto_{\mathrm{c}} \lceil \mathcal{F}[\mathcal{E}[c \, \overline{R} \, \overline{w}]]\rceil}$$

R-LIFT-UU   R-LIFT-UC   R-LIFT-CU   R-LIFT-CC
$$\frac{m \leadsto_{\mathrm{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathrm{u}} \mathcal{E}[m']} \qquad \frac{m \leadsto_{\mathrm{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathrm{c}} \mathcal{E}[m']} \qquad \frac{n \leadsto_{\mathrm{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathrm{u}} \mathcal{E}[n']} \qquad \frac{n \leadsto_{\mathrm{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathrm{c}} \mathcal{E}[n']}$$

Figure 1.6: Operational Semantics

$$\boxed{r:T \leftarrow t \dashv_{[\Sigma]} \theta}$$

B-VALUE
$$\Sigma \vdash \Delta \dashv \Sigma'$$
$$p:A \leftarrow w \dashv \theta$$
$$\overline{p:\langle\Delta\rangle A \leftarrow w \dashv_{[\Sigma]} \theta}$$

B-REQUEST
$$\Sigma \vdash \Delta \dashv \Sigma' \qquad \mathcal{E} \text{ poisedfor } c$$
$$\Delta = \Theta \mid \Xi \qquad c:\forall \overline{Z}.\overline{B \to} B' \in \Xi \qquad (p_i:B_i \leftarrow w_i \dashv \theta_i)_i$$
$$\overline{\langle c\,\overline{p} \to z\rangle : \langle\Delta\rangle A \leftarrow \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}]\rceil \dashv_{[\Sigma]} \overline{\theta}[\uparrow(\{x \mapsto \mathcal{E}[x]\} : \{B' \to [\Sigma']A\})/z]}$$

B-CATCHALL-VALUE
$$\Sigma \vdash \Delta \dashv \Sigma'$$
$$\overline{\langle x\rangle : \langle\Delta\rangle A \leftarrow w \dashv_{[\Sigma]} [\uparrow(\{w\}:\{[\Sigma']A\})/x]}$$

B-CATCHALL-REQUEST
$$\Sigma \vdash \Delta \dashv \Sigma' \qquad \mathcal{E} \text{ poisedfor } c$$
$$\Delta = \Theta \mid \Xi \qquad c:\forall \overline{Z}.\overline{B \to} B' \in \Xi$$
$$\overline{\langle x\rangle : \langle\Delta\rangle A \leftarrow \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}]\rceil \dashv_{[\Sigma]} [\uparrow(\{\lceil \mathcal{E}[c\,\overline{R}\,\overline{w}]\rceil\}:\{[\Sigma']A\})/x]}$$

$$\boxed{p:A \leftarrow w \dashv \theta}$$

B-VAR
$$\overline{x:A \leftarrow w \dashv [\uparrow(w:A)/x]}$$

B-DATA
$$k\,\overline{A} \in D\,\overline{R} \qquad (p_i:A_i \leftarrow w_i \dashv \theta_i)_i$$
$$\overline{k\,\overline{p}:D\,\overline{R} \leftarrow k\,\overline{w} \dashv \overline{\theta}}$$

Figure 1.7: Pattern Binding

Chapter 2

Arbitrary Thread Interruption

2.1 Relaxing Catches

B-CATCHALL-REQUEST-LOOSE

$$\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x \rangle : \langle \Delta \rangle A \leftarrow \lceil \mathcal{E}[c \, \overline{R} \, \overline{w}] \rceil \dashv_{\Sigma} [\Uparrow (\{\lceil \mathcal{E}[c \, \overline{R} \, \overline{w}] \rceil\} : \{[\Sigma']A\})/x]}$$

Figure 2.1: Updated B-CATCHALL-REQUEST

2.2 Interrupting Arbitrary Terms

$$
\begin{aligned}
\text{(uses)} \qquad & m ::= \cdots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}]\rceil \\
\text{(constructions)} \qquad & n ::= \cdots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}]\rceil \\
\text{(use values)} \qquad & u ::= x \mid f\,\overline{R} \mid \uparrow(v:A) \\
\text{(non-use values)} \qquad & v ::= k\,\overline{w} \mid \{e\} \\
\text{(construction values)} \quad & w ::= \downarrow u \mid v \\
\text{(normal forms)} \qquad & t ::= w \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}]\rceil \mid {!}(m) \\
\text{(evaluation frames)} \quad & \mathcal{F} ::= [\,]\,\overline{n} \mid u\,(\overline{t},[\,],\overline{n}) \mid \uparrow([\,]:A) \\
& \qquad \mid\ \downarrow[\,] \mid k\,(\overline{w},[\,],\overline{n}) \mid c\,\overline{R}\,(\overline{w},[\,],\overline{n}) \\
& \qquad \mid\ \mathbf{let}\ f:P = [\,]\ \mathbf{in}\ n \mid \langle\Theta\rangle\,[\,] \\
\text{(evaluation contexts)} \quad & \mathcal{E} ::= [\,] \mid \mathcal{F}[\mathcal{E}]
\end{aligned}
$$

Figure 2.2: Runtime Syntax, Updated with Suspension of Uses

B-CATCHALL-INTERRUPT

$$
\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x\rangle : \langle\Delta\rangle A \leftarrow {!}(m) \dashv_{[\Xi]} [\uparrow(\{m\}{:}\{[\Sigma']A\})/x]}
$$

Figure 2.3: Catching Interrupts rule.

2.3 Freezing

Another way of doing it is to just let any use become frozen, in the same way as commands become frozen once invoked.

> **TODO:** Do we need the 'hoisting' rules for general suspended terms?

2.4 Yielding

Note that R-YIELD-EF relies on the predicate $\mathcal{F}[\mathcal{E}]$ allows c. This states that the Yield interface is in the ability of the term in the evaluation context.

For any frame apart from argument frames, $\mathcal{F}[\mathcal{E}]$ allows $c = \mathsf{false}$. In this case, it is defined as follows;

$$
\uparrow(v:\{\overline{\langle\Delta\rangle A \rightarrow}\,[\Sigma]B\})\,(\overline{t},[\,],\overline{n})\ \text{allows}\ c = \begin{cases} \Xi\ \text{allows}\ c & \text{if } |\overline{n}| = 0\ \text{ where } \Sigma = \sigma\,|\,\Xi \\ \mathsf{false} & \text{otherwise} \end{cases}
$$

> **TODO:** What happens for 0-ary constructors? I don't know if it's exactly how I think it is.

$$\text{R-INTERRUPT}$$
$$\overline{m \rightsquigarrow_u !(m)}$$

Figure 2.4: Use interruption rule

$$\boxed{m \rightsquigarrow_u m'} \quad \boxed{n \rightsquigarrow_c n'} \quad \boxed{m \longrightarrow_u m'} \quad \boxed{n \longrightarrow_c n'}$$

$$\text{R-FREEZE-USE} \qquad\qquad \text{R-FREEZE-COMM}$$
$$\overline{m \rightsquigarrow_u \lceil m \rceil} \qquad\qquad \overline{c \, \overline{R} \, \overline{w} \rightsquigarrow_c \lceil c \, \overline{R} \, \overline{w} \rceil}$$

$$\text{R-FREEZE-FRAME-USE} \qquad\qquad \text{R-FREEZE-FRAME-CONS}$$
$$\overline{\mathcal{F}[\lceil m \rceil] \rightsquigarrow_u \lceil \mathcal{F}[m] \rceil} \qquad\qquad \overline{\mathcal{F}[\lceil m \rceil] \rightsquigarrow_c \lceil \mathcal{F}[m] \rceil}$$

$$\text{R-FREEZE-FRAME-USE} \qquad\qquad \text{R-FREEZE-FRAME-CONS}$$
$$\frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[\lceil \mathcal{E}[c \, \overline{R} \, \overline{w}] \rceil] \rightsquigarrow_u \lceil \mathcal{F}[\mathcal{E}[c \, \overline{R} \, \overline{w}]] \rceil} \qquad \frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[\lceil \mathcal{E}[c \, \overline{R} \, \overline{w}] \rceil] \rightsquigarrow_c \lceil \mathcal{F}[\mathcal{E}[c \, \overline{R} \, \overline{w}]] \rceil}$$

Figure 2.5: Updated Freezing

For an extension Ξ, the allows predicate is defined as

$$\iota \text{ allows } c = \text{false}$$
$$(\Xi, I \, \overline{R}) \text{ allows } c = \begin{cases} \text{true} & \text{if } c \in I \\ \Xi \text{ allows } c & \text{otherwise} \end{cases}$$

Informally, $\mathcal{F}[\mathcal{E}]$ allows c is true when $\mathcal{F}[\mathcal{E}]$ is a handler with the command c as a member of an interface in its ability, *and* all of the arguments have been evaluated (this is what the $|\overline{n}| = 0$ constraint expresses).

2.5 Counting

In practise we count up through the amount of R-HANDLE rules we apply and only insert the yield when this count exceeds a threshold value t_y.

So we supplement the operational semantics with a *counter c_y*, so that our transitions are e.g. $m; c_y \rightsquigarrow_u m'; c_y'$. We adopt the convention that, when the counter is not

$$
\begin{array}{lll}
\text{(uses)} & m ::= \ldots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \mid \boxed{\lceil m \rceil} \\
\text{(constructions)} & n ::= \ldots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \mid \boxed{\lceil m \rceil} \\
\text{(use values)} & u ::= x \mid f\,\overline{R} \mid \uparrow(v:A) \\
\text{(non-use values)} & v ::= k\,\overline{w} \mid \{e\} \\
\text{(construction values)} & w ::= \downarrow u \mid v \\
\text{(normal forms)} & t ::= w \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \mid \boxed{\lceil m \rceil} \\
\text{(evaluation frames)} & \mathcal{F} ::= [\,]\,\overline{n} \mid u\,(\overline{t},[\,],\overline{n}) \mid \uparrow([\,]:A) \\
& \qquad \mid \;\; \downarrow[\,] \mid k\,(\overline{w},[\,],\overline{n}) \mid c\,\overline{R}\,(\overline{w},[\,],\overline{n}) \\
& \qquad \mid \;\; \mathbf{let}\ f:P=[\,]\ \mathbf{in}\ n \mid \langle\Theta\rangle\,[\,] \\
\text{(evaluation contexts)} & \mathcal{E} ::= [\,] \mid \mathcal{F}[\mathcal{E}]
\end{array}
$$

Figure 2.6: Runtime Syntax, Updated with Freezing of Uses

B-CATCHALL-INTERRUPT
$$
\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x \rangle : \langle \Delta \rangle A \leftarrow \lceil m \rceil \dashv_{[\Sigma]} [\uparrow(\{m\}:\{[\Sigma']A\})/x]}
$$

Figure 2.7: Catching Frozen Terms rule.

mentioned in a transition[1], the counter stays the same. Hence e.g. $m \rightsquigarrow_u m'$ desugars to $m; c_y \rightsquigarrow_u m'; c_y$.

So to get our counting semantics we just need to supplement Figure **?** with the updated rule in

---

[1]That is, the transition is of the form $m \rightsquigarrow_u m'$.

$$\boxed{m \rightsquigarrow_u m'}$$

R-YIELD

$$\Sigma = \sigma \,|\, \Xi \qquad \text{Yield} \in \Xi$$

$$\uparrow(\{n\} : \{[\Sigma]A\}) \rightsquigarrow_u \uparrow(\ \textbf{let } f_n : \{\text{Unit} \to [\Sigma]A\} = \{\_ \mapsto n\} \ \textbf{in}$$
$$\textbf{let } y : \{[\Sigma]\text{Unit}\} = \{\text{yield!}\} \ \textbf{in} \ \{f_n \ (\downarrow(y!))\} : \{[\Sigma]A\})$$

R-YIELD-EF

$$\dfrac{\mathcal{F}[\mathcal{E}] \text{ allows Yield}}{\mathcal{F}[\mathcal{E}[m]] \rightsquigarrow_u \mathcal{F}[\mathcal{E}[\text{yield!}; m]]}$$

Figure 2.8: Inserting Yields

TODO: In R-Yield-EF we have to let terms inside eval CTXs be also uses. Does this not mean that we can then put any term in either one? Is that a problem?

TODO: Not sure if we want to yield /inside/ the ctx, or just before it?

$$\boxed{m \rightsquigarrow_u m'} \quad \boxed{n \rightsquigarrow_c n'} \quad \boxed{m \longrightarrow_u m'} \quad \boxed{n \longrightarrow_c n'}$$

R-HANDLE

$$\dfrac{k = \min_i \{i \mid \exists \overline{\theta}. (r_{i,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \,\text{-}[\Sigma]\, \theta_j)_j\} \qquad (r_{k,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \,\text{-}[\Sigma]\, \theta_j)_j}{\uparrow(\{((r_{i,j})_j \to n_i)_i\} : \{\overline{\langle \Delta \rangle A \to} [\Sigma]B\}) \ \bar{t}; c_y \rightsquigarrow_u \uparrow((\overline{\theta}(n_k) : B)); c_y + 1}$$

Figure 2.9: R-Handle with counting.

$$\boxed{m \rightsquigarrow_u m'}$$

R-YIELD

$$\dfrac{c_y \geq t_y}{m; c_y \rightsquigarrow_u \textbf{let } \text{yield} : [\text{Yield}]\text{Unit} = y \textbf{in } m; 0}$$

Figure 2.10: Inserting Yields, when over counter

Appendix A

Remaining Formalisms

$$\boxed{\Sigma \vdash \Delta \dashv \Sigma'}$$

A-ADJ
$$\frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Sigma' \vdash \Xi \dashv \Sigma''}{\Sigma \vdash \Theta \,|\, \Xi \dashv \Sigma''}$$

$$\boxed{\Sigma \vdash \Xi \dashv \Sigma'}$$

A-EXT-ID
$$\frac{}{\Sigma \vdash \iota \dashv \Sigma}$$

A-EXT-SNOC
$$\frac{\Sigma \vdash \Xi \dashv \Sigma'}{\Sigma \vdash \Xi, I\,\overline{R} \dashv \Sigma', I\,\overline{R}}$$

$$\boxed{\Sigma \vdash \Theta \dashv \Sigma'}$$

A-ADAPT-ID
$$\frac{}{\Sigma \vdash \iota \dashv \Sigma}$$

A-ADAPT-SNOC
$$\frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Sigma' \vdash I(S \to S') \dashv \Sigma''}{\Sigma \vdash \Theta, I(S \to S') \dashv \Sigma''}$$

$$\boxed{\Sigma \vdash I(S \to S') \dashv \Sigma'}$$

A-ADAPT-COM
$$\frac{\Sigma \vdash S : I \dashv \Sigma'; \Omega \qquad \Omega \vdash S' : I \dashv \Xi \qquad \Sigma' \vdash \Xi \dashv \Sigma''}{\Sigma \vdash I(S \to S') \dashv \Sigma''}$$

$$\boxed{\Sigma \vdash S : I \dashv \Sigma'; \Omega}$$

I-PAT-ID
$$\frac{}{\Sigma \vdash s : I \dashv \Sigma; s : \Sigma}$$

I-PAT-BIND
$$\frac{\Sigma \vdash S : I \dashv \Sigma'; \Omega}{\Sigma, I\,\overline{R} \vdash S\,a : I \dashv \Sigma'; \Omega, a : I\,\overline{R}}$$

I-PAT-SKIP
$$\frac{\Sigma \vdash S\,a : I \dashv \Sigma'; \Omega \qquad I \neq I'}{\Sigma, I'\,\overline{R} \vdash S\,a : I \dashv \Sigma', I'\,\overline{R}; \Omega}$$

$$\boxed{\Omega \vdash S : I \dashv \Xi}$$

I-INST-ID
$$\frac{s \in \mathsf{dom}(\Omega)}{\Omega \vdash s : I \dashv \iota}$$

I-INST-LKP
$$\frac{a \in \mathsf{dom}(\Omega) \qquad \Omega \vdash S : I \dashv \Xi \qquad \Omega(a) = I\,\overline{R}}{\Omega \vdash S\,a : I \dashv \Xi, I\,\overline{R}}$$

Figure A.1: Action of an Adjustment on an Ability and Auxiliary Judgements

$$X ::= A \mid C \mid T \mid G \mid Z \mid R \mid P \mid \sigma \mid \Sigma \mid \Xi \mid \Theta \mid \Delta \mid \Gamma \mid \exists \Psi.\Gamma \mid \Omega$$

$$\boxed{\Phi \vdash X}$$

$$\frac{}{\Phi, X \vdash X} \quad \text{WF-VAL}$$

$$\frac{}{\Phi, [E] \vdash E} \quad \text{WF-EFF}$$

$$\text{WF-POLY} \quad \frac{\Phi, \overline{Z} \vdash A}{\Phi \vdash \forall \overline{Z}.A}$$

$$\text{WF-DATA} \quad \frac{(\Phi \vdash R)_i}{\Phi \vdash D\,\overline{R}}$$

$$\text{WF-THUNK} \quad \frac{\Phi \vdash C}{\Phi \vdash \{C\}}$$

$$\text{WF-COMP} \quad \frac{(\Phi \vdash T)_i \quad \Phi \vdash G}{\Phi \vdash \overline{T} \to G}$$

$$\text{WF-ARG} \quad \frac{\Phi \vdash \Delta \quad \Phi \vdash A}{\Phi \vdash \langle \Delta \rangle A}$$

$$\text{WF-RET} \quad \frac{\Phi \vdash \Sigma \quad \Phi \vdash A}{\Phi \vdash [\Sigma]A}$$

$$\text{WF-ABILITY} \quad \frac{\Phi \vdash \Sigma}{\Phi \vdash [\Sigma]}$$

$$\text{WF-PURE} \quad \frac{}{\Phi \vdash \emptyset}$$

$$\text{WF-ID} \quad \frac{}{\Phi \vdash \iota}$$

$$\text{WF-EXT} \quad \frac{\Phi \vdash \Xi \quad (\Phi \vdash R)_i}{\Phi \vdash \Xi, I\,\overline{R}}$$

$$\text{WF-ADAPT} \quad \frac{\Phi \vdash \Theta}{\Phi \vdash \Theta, I\,(S \to S')}$$

$$\text{WF-EMPTY} \quad \frac{}{\Phi \vdash \cdot}$$

$$\text{WF-MONO} \quad \frac{\Phi \vdash \Gamma \quad \Phi \vdash A}{\Phi \vdash \Gamma, x : A}$$

$$\text{WF-POLY} \quad \frac{\Phi \vdash \Gamma \quad \Phi \vdash P}{\Phi \vdash \Gamma, f : P}$$

$$\text{WF-EXISTENTIAL} \quad \frac{\Phi, \Psi \vdash \Gamma}{\Phi \vdash \exists \Psi.\Gamma}$$

$$\text{WF-INTERFACE} \quad \frac{\Phi \vdash \Omega \quad (\Phi \vdash R)_i}{\Phi \vdash \Omega, x : I\,\overline{R}}$$

Figure A.2: Well-Formedness Rules

$$\boxed{\Phi \vdash p : A \dashv \Gamma}$$

P-VAR

$$\frac{}{\Phi \vdash x : A \dashv x : A}$$

P-DATA

$$\frac{k\,\overline{A} \in D\,\overline{R} \qquad (\Phi \vdash p_i : A_i \dashv \Gamma)_i}{\Phi \vdash k\,\overline{p} : D\,\overline{R} \dashv \overline{\Gamma}}$$

$$\boxed{\Phi \vdash r : T \dashv_{[\Sigma]} \exists \Psi.\Gamma}$$

P-VALUE

$$\frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \Phi \vdash p : A \dashv \Gamma}{\Phi \vdash p : \langle \Delta \rangle A \dashv_{[\Sigma]} \Gamma}$$

P-CATCHALL

$$\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\Phi \vdash \langle x \rangle : \langle \Delta \rangle A \dashv_{[\Sigma]} x : \{[\Sigma']A\}}$$

P-COMMAND

$$\frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \Delta = \Theta \mid \Xi \qquad c : \forall \overline{Z}.\overline{A} \rightarrow B \in \Xi \qquad (\Phi, \overline{Z} \vdash p_i : A_i \dashv \Gamma_i)_i}{\Phi \vdash \langle c\,\overline{p} \rightarrow z \rangle : \langle \Delta \rangle B' \dashv_{[\Sigma]} \exists \overline{Z}.\overline{\Gamma}, z : \{\langle \iota \mid \iota \rangle B \rightarrow [\Sigma']B'\}}$$

Figure A.3: Pattern Matching Typing Rules