

This is the Project Title

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Abstract

Formal development of Frank.

Chapter 1

Formalisation of Frank

(data types)	D	(interfaces)	I
(value type variables)	X	(term variables)	x, y, z, f
(effect type variables)	E	(instance variables)	s, a, b, c
(value types)	$A, B ::= D \bar{R}$	(seeds)	$\sigma ::= \emptyset \mid E$
	$\mid \{C\} \mid X$	(abilities)	$\Sigma ::= \sigma \mid \Xi$
(computation types)	$C ::= \overline{T \rightarrow G}$	(extensions)	$\Xi ::= \mathfrak{t} \mid \Xi, I \bar{R}$
(argument types)	$T ::= \langle \Delta \rangle A$	(adaptors)	$\Theta ::= \mathfrak{t} \mid \Theta, I(S \rightarrow S')$
(return types)	$G ::= [\Sigma]A$	(adjustments)	$\Delta ::= \Theta \mid \Xi$
(type binders)	$Z ::= X \mid [E]$	(instance patterns)	$S ::= s \mid S a$
(type arguments)	$R ::= A \mid [\Sigma]$	(kind environments)	$\Phi, \Psi ::= \cdot \mid \Phi, Z$
(polytypes)	$P ::= \forall \bar{Z}. A$	(type environments)	$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, f : P$
		(instance environments)	$\Omega ::= s : \Sigma \mid \Omega, a : I \bar{R}$

Figure 1.1: Types

(constructors)	k
(commands)	c
(uses)	$m ::= x \mid f \bar{R} \mid m \bar{n} \mid \uparrow(n : A)$
(constructions)	$n ::= \downarrow m \mid k \bar{n} \mid c \bar{R} \bar{n} \mid \{e\}$ $\mid \text{let } f : P = n \text{ in } n' \mid \text{letrec } \overline{f : P = e} \text{ in } n$ $\mid \langle \Theta \rangle n$
(computations)	$e ::= \overline{\bar{r} \mapsto n}$
(computation patterns)	$r ::= p \mid \langle c \bar{p} \rightarrow z \rangle \mid \langle x \rangle$
(value patterns)	$p ::= k \bar{p} \mid x$

Figure 1.2: Terms

$\Phi; \Gamma [\Sigma] \vdash m \Rightarrow A$	
$\frac{\text{T-VAR} \quad x : A \in \Gamma}{\Phi; \Gamma [\Sigma] \vdash x \Rightarrow A}$	$\frac{\text{T-POLYVAR} \quad \Phi \vdash \bar{R} \quad f : \forall \bar{Z}. A \in \Gamma}{\Phi; \Gamma [\Sigma] \vdash f \bar{R} \Rightarrow A[\bar{R}/\bar{Z}]}$
$\frac{\text{T-APP} \quad \Sigma' = \Sigma \quad (\Sigma \vdash \Delta_i \dashv \Sigma'_i)_i \quad \Phi; \Gamma [\Sigma] \vdash m \Rightarrow \{\langle \Delta \rangle A \rightarrow [\Sigma'] B\} \quad (\Phi; \Gamma [\Sigma'_i] \vdash n_i : A_i)_i}{\Phi; \Gamma [\Sigma] \vdash m \bar{n} \Rightarrow B}$	$\frac{\text{T-ASCRIBE} \quad \Phi; \Gamma [\Sigma] \vdash n : A}{\Phi; \Gamma [\Sigma] \vdash \uparrow(n : A) \Rightarrow A}$
$\Phi; \Gamma [\Sigma] \vdash n : A$	
$\frac{\text{T-SWITCH} \quad \Phi; \Gamma [\Sigma] \vdash m \Rightarrow A \quad A = B}{\Phi; \Gamma [\Sigma] \vdash \downarrow m : B}$	$\frac{\text{T-DATA} \quad k \bar{A} \in D \bar{R} \quad (\Phi; \Gamma [\Sigma] \vdash n_j : A_j)_j}{\Phi; \Gamma [\Sigma] \vdash k \bar{n} : D \bar{R}}$
$\frac{\text{T-COMMAND} \quad \Phi \vdash \bar{R} \quad c : \forall \bar{Z}. \bar{A} \rightarrow B \in \Sigma \quad (\Phi; \Gamma [\Sigma] \vdash n_j : A_j[\bar{R}/\bar{Z}])_j}{\Phi; \Gamma [\Sigma] \vdash c \bar{R} \bar{n} : B[\bar{R}/\bar{Z}]}$	$\frac{\text{T-THUNK} \quad \Phi; \Gamma \vdash e : C}{\Phi; \Gamma [\Sigma] \vdash \{e\} : \{C\}}$
$\frac{\text{T-LET} \quad P = \forall \bar{Z}. A \quad \Phi, \bar{Z}; \Gamma [\emptyset] \vdash n : A \quad \Phi; \Gamma, f : P [\Sigma] \vdash n' : B}{\Phi; \Gamma [\Sigma] \vdash \text{let } f : P = n \text{ in } n' : B}$	
$\frac{\text{T-LETREC} \quad (P_i = \forall \bar{Z}_i. \{C_i\})_i \quad (\Phi, \bar{Z}_i; \Gamma, \bar{f} : \bar{P} \vdash e_i : C)_i \quad \Phi; \Gamma, \bar{f} : \bar{P} [\Sigma] \vdash n : B}{\Phi; \Gamma [\Sigma] \vdash \text{letrec } \bar{f} : \bar{P} = e \text{ in } n : B}$	$\frac{\text{T-ADAPT} \quad \Sigma \vdash \Theta \dashv \Sigma' \quad \Phi; \Gamma [\Sigma'] \vdash n : A}{\Phi; \Gamma [\Sigma] \vdash \langle \Theta \rangle n : A}$
$\Phi; \Gamma \vdash e : C$	
$\frac{\text{T-COMP} \quad (\Phi \vdash r_{i,j} : T_j \dashv [\Sigma] \exists \Psi_{i,j}. \Gamma'_{i,j})_{i,j} \quad (\Phi, (\Psi_{i,j})_j; \Gamma, (\Gamma'_{i,j})_j [\Sigma] \vdash n_i : B)_i \quad ((r_{i,j})_i \text{ covers } T_j)_j}{\Phi; \Gamma \vdash ((r_{i,j})_j \mapsto n_i)_i : (T_j \rightarrow)_j [\Sigma] B}$	

Figure 1.3: Term Typing Rules

(uses)	$m ::= \dots \mid \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil$
(constructions)	$n ::= \dots \mid \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil$
(use values)	$u ::= x \mid f \bar{R} \mid \uparrow(v : A)$
(non-use values)	$v ::= k \bar{w} \mid \{e\}$
(construction values)	$w ::= \downarrow u \mid v$
(normal forms)	$t ::= w \mid \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil$
(evaluation frames)	$\mathcal{F} ::= [] \bar{n} \mid u (\bar{t}, [], \bar{n}) \mid \uparrow([], A)$ $\mid \downarrow[] \mid k (\bar{w}, [], \bar{n}) \mid c \bar{R} (\bar{w}, [], \bar{n})$ $\mid \mathbf{let} f : P = [] \mathbf{in} n \mid \langle \Theta \rangle []$
(evaluation contexts)	$\mathcal{E} ::= [] \mid \mathcal{F}[\mathcal{E}]$

Figure 1.4: Runtime Syntax

$\Phi; \Gamma[\Sigma] \vdash m \Rightarrow A$	$\Phi; \Gamma[\Sigma] \vdash n : A$
$\frac{\text{T-FREEZE-USE} \quad \neg(\mathcal{E} \text{ handles } c) \quad \Phi; \Gamma[\Sigma] \vdash \mathcal{E}[c \bar{R} \bar{w}] \Rightarrow A}{\Phi; \Gamma[\Sigma] \vdash \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil \Rightarrow A}$	
$\frac{\text{T-FREEZE-CONS} \quad \neg(\mathcal{E} \text{ handles } c) \quad \Phi; \Gamma[\Sigma] \vdash \mathcal{E}[c \bar{R} \bar{w}] : A}{\Phi; \Gamma[\Sigma] \vdash \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil : A}$	

Figure 1.5: Frozen Commands

$$\begin{array}{c}
\boxed{m \rightsquigarrow_{\mathbf{u}} m'} \quad \boxed{n \rightsquigarrow_{\mathbf{c}} n'} \quad \boxed{m \longrightarrow_{\mathbf{u}} m'} \quad \boxed{n \longrightarrow_{\mathbf{c}} n'} \\
\\
\text{R-HANDLE} \\
\frac{k = \min_i \{i \mid \exists \bar{\theta}. (r_{i,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \neg[\Sigma] \theta_j)_j\} \quad (r_{k,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \neg[\Sigma] \theta_j)_j}{\uparrow(\{((r_{i,j})_j \rightarrow n_i)_i\} : \{\langle \Delta \rangle A \rightarrow [\Sigma] B\}) \bar{t} \rightsquigarrow_{\mathbf{u}} \uparrow((\bar{\theta}(n_k) : B))} \\
\\
\begin{array}{ccc}
\text{R-ASCRIBE-USE} & \text{R-ASCRIBE-CONS} & \text{R-LET} \\
\frac{}{\uparrow(\downarrow u : A) \rightsquigarrow_{\mathbf{u}} u} & \frac{}{\downarrow \uparrow(w : A) \rightsquigarrow_{\mathbf{c}} w} & \frac{}{\mathbf{let} \ f : P = w \ \mathbf{in} \ n \rightsquigarrow_{\mathbf{c}} n[\uparrow(w : P)/f]} \\
\\
\text{R-LETREC} & \frac{}{e = \bar{r} \rightarrow n} & \text{R-ADAPT} \\
\frac{}{\mathbf{letrec} \ f : P = e \ \mathbf{in} \ n' \rightsquigarrow_{\mathbf{c}} n'[\uparrow(\{\bar{r} \rightarrow \mathbf{letrec} \ f : P = e \ \mathbf{in} \ n\} : P)/f]} & & \frac{}{\langle \Theta \rangle w \rightsquigarrow_{\mathbf{c}} w} \\
\\
\text{R-FREEZE-COMM} \\
\frac{}{c \ \bar{R} \ \bar{w} \rightsquigarrow_{\mathbf{c}} [c \ \bar{R} \ \bar{w}]} \\
\\
\begin{array}{cc}
\text{R-FREEZE-FRAME-USE} & \text{R-FREEZE-FRAME-CONS} \\
\frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[[\mathcal{E}[c \ \bar{R} \ \bar{w}]]] \rightsquigarrow_{\mathbf{u}} [\mathcal{F}[\mathcal{E}[c \ \bar{R} \ \bar{w}]]]} & \frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[[\mathcal{E}[c \ \bar{R} \ \bar{w}]]] \rightsquigarrow_{\mathbf{c}} [\mathcal{F}[\mathcal{E}[c \ \bar{R} \ \bar{w}]]]} \\
\\
\begin{array}{cccc}
\text{R-LIFT-UU} & \text{R-LIFT-UC} & \text{R-LIFT-CU} & \text{R-LIFT-CC} \\
\frac{m \rightsquigarrow_{\mathbf{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathbf{u}} \mathcal{E}[m']} & \frac{m \rightsquigarrow_{\mathbf{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathbf{c}} \mathcal{E}[m']} & \frac{n \rightsquigarrow_{\mathbf{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathbf{u}} \mathcal{E}[n']} & \frac{n \rightsquigarrow_{\mathbf{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathbf{c}} \mathcal{E}[n']}
\end{array}
\end{array}
\end{array}$$

Figure 1.6: Operational Semantics

$$\boxed{r : T \leftarrow t \neg[\Sigma] \theta}$$

$$\begin{array}{c} \text{B-VALUE} \\ \Sigma \vdash \Delta \dashv \Sigma' \\ p : A \leftarrow w \dashv \theta \\ \hline p : \langle \Delta \rangle A \leftarrow w \neg[\Sigma] \theta \end{array}$$

B-REQUEST

$$\begin{array}{c} \Sigma \vdash \Delta \dashv \Sigma' \quad \mathcal{E} \text{ poised for } c \\ \Delta = \Theta \mid \Xi \quad c : \forall \bar{Z}. \bar{B} \rightarrow B' \in \Xi \quad (p_i : B_i \leftarrow w_i \dashv \theta_i)_i \\ \hline \langle c \bar{p} \rightarrow z \rangle : \langle \Delta \rangle A \leftarrow [\mathcal{E}[c \bar{R} \bar{w}]] \neg[\Sigma] \bar{\theta}[\uparrow(\{x \mapsto \mathcal{E}[x]\} : \{B' \rightarrow [\Sigma']A\})/z] \end{array}$$

B-CATCHALL-VALUE

$$\begin{array}{c} \Sigma \vdash \Delta \dashv \Sigma' \\ \hline \langle x \rangle : \langle \Delta \rangle A \leftarrow w \neg[\Sigma] [\uparrow(\{w\} : \{[\Sigma']A\})/x] \end{array}$$

B-CATCHALL-REQUEST

$$\begin{array}{c} \Sigma \vdash \Delta \dashv \Sigma' \quad \mathcal{E} \text{ poised for } c \\ \Delta = \Theta \mid \Xi \quad c : \forall \bar{Z}. \bar{B} \rightarrow B' \in \Xi \\ \hline \langle x \rangle : \langle \Delta \rangle A \leftarrow [\mathcal{E}[c \bar{R} \bar{w}]] \neg[\Sigma] [\uparrow(\{[\mathcal{E}[c \bar{R} \bar{w}]]\} : \{[\Sigma']A\})/x] \end{array}$$

$$\boxed{p : A \leftarrow w \dashv \theta}$$

B-VAR

$$\frac{}{x : A \leftarrow w \dashv [\uparrow(w : A)/x]}$$

B-DATA

$$\frac{k \bar{A} \in D \bar{R} \quad (p_i : A_i \leftarrow w_i \dashv \theta_i)_i}{k \bar{p} : D \bar{R} \leftarrow k \bar{w} \dashv \bar{\theta}}$$

Figure 1.7: Pattern Binding

Chapter 2

Arbitrary Thread Interruption

2.1 Relaxing Catches

$$\begin{array}{c}
 \text{B-CATCHALL-REQUEST-LOOSE} \\
 \Sigma \vdash \Delta \dashv \Sigma' \\
 \hline
 \langle x \rangle : \langle \Delta \rangle A \leftarrow [\mathcal{E}[c \ \bar{R} \ \bar{w}]] \text{-}[\Sigma] \ [\uparrow(\{[\mathcal{E}[c \ \bar{R} \ \bar{w}]]\}:\{[\Sigma']A\})/x]
 \end{array}$$

Figure 2.1: Updated B-CATCHALL-REQUEST

2.2 Interrupting Arbitrary Terms

(uses)	$m ::= \dots \mid \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil$
(constructions)	$n ::= \dots \mid \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil$
(use values)	$u ::= x \mid f \bar{R} \mid \uparrow(v : A)$
(non-use values)	$v ::= k \bar{w} \mid \{e\}$
(construction values)	$w ::= \downarrow u \mid v$
(normal forms)	$t ::= w \mid \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil \mid !(m)$
(evaluation frames)	$\mathcal{F} ::= [] \bar{n} \mid u(\bar{t}, [], \bar{n}) \mid \uparrow([], : A)$ $\mid \downarrow[] \mid k(\bar{w}, [], \bar{n}) \mid c \bar{R}(\bar{w}, [], \bar{n})$ $\mid \text{let } f : P = [] \text{ in } n \mid \langle \Theta \rangle []$
(evaluation contexts)	$\mathcal{E} ::= [] \mid \mathcal{F}[\mathcal{E}]$

Figure 2.2: Runtime Syntax, Updated with Suspension of Uses

$$\begin{array}{c}
\text{B-CATCHALL-INTERRUPT} \\
\hline
\Sigma \vdash \Delta \dashv \Sigma' \\
\hline
\langle x \rangle : \langle \Delta \rangle A \leftarrow !(m) \dashv [\Sigma] [\uparrow(\{m\} : \{\Sigma' A\}) / x]
\end{array}$$

Figure 2.3: Catching Interrupts rule.

2.3 Freezing

Another way of doing it is to just let any use become frozen, in the same way as commands become frozen once invoked.

TODO: Do we need the 'hoisting' rules for general suspended terms?

2.4 Yielding

2.5 Counting

In practise we count up through the amount of R-HANDLE rules we apply and only insert the yield when this count exceeds a threshold value t_y .

So we supplement the operational semantics with a *counter* c_y , so that our transitions are e.g. $m; c_y \rightsquigarrow_u m'; c_y'$. We adopt the convention that, when the counter is not mentioned in a transition¹, the counter stays the same. Hence e.g. $m \rightsquigarrow_u m'$ desugars

¹That is, the transition is of the form $m \rightsquigarrow_u m'$.

R-INTERRUPT

$$\frac{}{m \rightsquigarrow_u!(m)}$$

Figure 2.4: Use interruption rule

$m \rightsquigarrow_u m'$	$n \rightsquigarrow_c n'$	$m \longrightarrow_u m'$	$n \longrightarrow_c n'$
R-FREEZE-USE		R-FREEZE-COMM	
$\frac{}{m \rightsquigarrow_u [m]}$		$\frac{}{c \bar{R} \bar{w} \rightsquigarrow_c [c \bar{R} \bar{w}]}$	
R-FREEZE-FRAME-USE		R-FREEZE-FRAME-CONS	
$\frac{}{\mathcal{F}[[m]] \rightsquigarrow_u [\mathcal{F}[m]]}$		$\frac{}{\mathcal{F}[[m]] \rightsquigarrow_c [\mathcal{F}[m]]}$	
R-FREEZE-FRAME-USE		R-FREEZE-FRAME-CONS	
$\frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[[\mathcal{E}[c \bar{R} \bar{w}]]] \rightsquigarrow_u [\mathcal{F}[\mathcal{E}[c \bar{R} \bar{w}]]]}$		$\frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[[\mathcal{E}[c \bar{R} \bar{w}]]] \rightsquigarrow_c [\mathcal{F}[\mathcal{E}[c \bar{R} \bar{w}]]]}$	

Figure 2.5: Updated Freezing

to $m; c_y \rightsquigarrow_u m'; c_y$.

So to get our counting semantics we just need to supplement Figure ? with the updated rule in

(uses)	$m ::= \dots \mid \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil \mid \boxed{m}$
(constructions)	$n ::= \dots \mid \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil \mid \boxed{m}$
(use values)	$u ::= x \mid f \bar{R} \mid \uparrow(v : A)$
(non-use values)	$v ::= k \bar{w} \mid \{e\}$
(construction values)	$w ::= \downarrow u \mid v$
(normal forms)	$t ::= w \mid \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil \mid \boxed{m}$
(evaluation frames)	$\mathcal{F} ::= [] \bar{n} \mid u (\bar{t}, [], \bar{n}) \mid \uparrow([], : A)$ $\mid \downarrow[] \mid k (\bar{w}, [], \bar{n}) \mid c \bar{R} (\bar{w}, [], \bar{n})$ $\mid \text{let } f : P = [] \text{ in } n \mid \langle \Theta \rangle []$
(evaluation contexts)	$\mathcal{E} ::= [] \mid \mathcal{F}[\mathcal{E}]$

Figure 2.6: Runtime Syntax, Updated with Freezing of Uses

$$\begin{array}{c}
 \text{B-CATCHALL-INTERRUPT} \\
 \Sigma \vdash \Delta \dashv \Sigma' \\
 \hline
 \langle x \rangle : \langle \Delta \rangle A \leftarrow \boxed{m} \dashv [\Sigma] [\uparrow(\{m\} : \{\Sigma' \} A)] / x
 \end{array}$$

Figure 2.7: Catching Frozen Terms rule.

$$\boxed{m \rightsquigarrow_u m'}$$

R-YIELD

$$m \rightsquigarrow_u \text{let yield : [Yield]Unit} = \text{in } m$$

Figure 2.8: Inserting Yields

TODO: Only if yield is in the ability of m?

TODO: How do we access m 's type?

$$\boxed{m \rightsquigarrow_{\mathbf{u}} m'} \quad \boxed{n \rightsquigarrow_{\mathbf{c}} n'} \quad \boxed{m \longrightarrow_{\mathbf{u}} m'} \quad \boxed{n \longrightarrow_{\mathbf{c}} n'}$$

R-HANDLE

$$\frac{k = \min_i \{i \mid \exists \bar{\theta}. (r_{i,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \text{--} [\Sigma] \theta_j)_j\} \quad (r_{k,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \text{--} [\Sigma] \theta_j)_j}{\uparrow(\{((r_{i,j})_j \rightarrow n_i)_i : \{\overline{\langle \Delta \rangle A} \rightarrow [\Sigma] B\}) \bar{t}; c_y \rightsquigarrow_{\mathbf{u}} \uparrow((\bar{\theta}(n_k) : B)); c_y + 1)}$$

Figure 2.9: R-Handle with counting.

$$\boxed{m \rightsquigarrow_{\mathbf{u}} m'}$$

R-YIELD

$$\frac{c_y \geq t_y}{m; c_y \rightsquigarrow_{\mathbf{u}} \mathbf{let} \text{ yield} : [\text{Yield}] \text{Unit} = \mathbf{in} \ m; 0}$$

Figure 2.10: Inserting Yields, when over counter

Appendix A

Remaining Formalisms

$$\boxed{\Sigma \vdash \Delta \dashv \Sigma'}$$

$$\begin{array}{c} \text{A-ADJ} \\ \Sigma \vdash \Theta \dashv \Sigma' \quad \Sigma' \vdash \Xi \dashv \Sigma'' \\ \hline \Sigma \vdash \Theta | \Xi \dashv \Sigma'' \end{array}$$

$$\boxed{\Sigma \vdash \Xi \dashv \Sigma'}$$

$$\begin{array}{c} \text{A-EXT-ID} \\ \hline \Sigma \vdash \mathbf{1} \dashv \Sigma \end{array}$$

$$\begin{array}{c} \text{A-EXT-SNOC} \\ \Sigma \vdash \Xi \dashv \Sigma' \\ \hline \Sigma \vdash \Xi, I \bar{R} \dashv \Sigma', I \bar{R} \end{array}$$

$$\boxed{\Sigma \vdash \Theta \dashv \Sigma'}$$

$$\begin{array}{c} \text{A-ADAPT-ID} \\ \hline \Sigma \vdash \mathbf{1} \dashv \Sigma \end{array}$$

$$\begin{array}{c} \text{A-ADAPT-SNOC} \\ \Sigma \vdash \Theta \dashv \Sigma' \quad \Sigma' \vdash I(S \rightarrow S') \dashv \Sigma'' \\ \hline \Sigma \vdash \Theta, I(S \rightarrow S') \dashv \Sigma'' \end{array}$$

$$\boxed{\Sigma \vdash I(S \rightarrow S') \dashv \Sigma'}$$

$$\begin{array}{c} \text{A-ADAPT-COM} \\ \Sigma \vdash S : I \dashv \Sigma'; \Omega \quad \Omega \vdash S' : I \dashv \Xi \quad \Sigma' \vdash \Xi \dashv \Sigma'' \\ \hline \Sigma \vdash I(S \rightarrow S') \dashv \Sigma'' \end{array}$$

$$\boxed{\Sigma \vdash S : I \dashv \Sigma'; \Omega}$$

$$\text{I-PAT-ID}$$

$$\Sigma \vdash s : I \dashv \Sigma; s : \Sigma$$

$$\text{I-PAT-BIND}$$

$$\Sigma \vdash S : I \dashv \Sigma'; \Omega$$

$$\Sigma, I \bar{R} \vdash S a : I \dashv \Sigma'; \Omega, a : I \bar{R}$$

$$\text{I-PAT-SKIP}$$

$$\Sigma \vdash S a : I \dashv \Sigma'; \Omega \quad I \neq I'$$

$$\Sigma, I' \bar{R} \vdash S a : I \dashv \Sigma', I' \bar{R}; \Omega$$

$$\boxed{\Omega \vdash S : I \dashv \Xi}$$

$$\begin{array}{c} \text{I-INST-ID} \\ s \in \text{dom}(\Omega) \\ \hline \Omega \vdash s : I \dashv \mathbf{1} \end{array}$$

$$\begin{array}{c} \text{I-INST-LKP} \\ a \in \text{dom}(\Omega) \quad \Omega \vdash S : I \dashv \Xi \quad \Omega(a) = I \bar{R} \\ \hline \Omega \vdash S a : I \dashv \Xi, I \bar{R} \end{array}$$

Figure A.1: Action of an Adjustment on an Ability and Auxiliary Judgements

$$\mathcal{X} ::= A \mid C \mid T \mid G \mid Z \mid R \mid P \mid \sigma \mid \Sigma \mid \Xi \mid \Theta \mid \Delta \mid \Gamma \mid \exists \Psi. \Gamma \mid \Omega$$

$\Phi \vdash \mathcal{X}$

$\frac{}{\Phi, X \vdash X}$ <p>WF-VAL</p>	$\frac{}{\Phi, [E] \vdash E}$ <p>WF-EFF</p>	$\frac{\Phi, \bar{Z} \vdash A}{\Phi \vdash \forall \bar{Z}. A}$ <p>WF-POLY</p>
$\frac{(\Phi \vdash R)_i}{\Phi \vdash D \bar{R}}$ <p>WF-DATA</p>	$\frac{\Phi \vdash C}{\Phi \vdash \{C\}}$ <p>WF-THUNK</p>	$\frac{(\Phi \vdash T)_i \quad \Phi \vdash G}{\Phi \vdash \bar{T} \rightarrow G}$ <p>WF-COMP</p>
$\frac{\Phi \vdash \Delta \quad \Phi \vdash A}{\Phi \vdash \langle \Delta \rangle A}$ <p>WF-ARG</p>		
$\frac{\Phi \vdash \Sigma \quad \Phi \vdash A}{\Phi \vdash [\Sigma] A}$ <p>WF-RET</p>	$\frac{\Phi \vdash \Sigma}{\Phi \vdash [\Sigma]}$ <p>WF-ABILITY</p>	$\frac{}{\Phi \vdash \emptyset}$ <p>WF-PURE</p>
$\frac{}{\Phi \vdash \mathbf{1}}$ <p>WF-ID</p>		$\frac{\Phi \vdash \Xi \quad (\Phi \vdash R)_i}{\Phi \vdash \Xi, I \bar{R}}$ <p>WF-EXT</p>
$\frac{\Phi \vdash \Theta}{\Phi \vdash \Theta, I (S \rightarrow S')}$ <p>WF-ADAPT</p>		
$\frac{}{\Phi \vdash \cdot}$ <p>WF-EMPTY</p>	$\frac{\Phi \vdash \Gamma \quad \Phi \vdash A}{\Phi \vdash \Gamma, x : A}$ <p>WF-MONO</p>	$\frac{\Phi \vdash \Gamma \quad \Phi \vdash P}{\Phi \vdash \Gamma, f : P}$ <p>WF-POLY</p>
$\frac{\Phi, \Psi \vdash \Gamma}{\Phi \vdash \exists \Psi. \Gamma}$ <p>WF-EXISTENTIAL</p>		$\frac{\Phi \vdash \Omega \quad (\Phi \vdash R)_i}{\Phi \vdash \Omega, x : I \bar{R}}$ <p>WF-INTERFACE</p>

Figure A.2: Well-Formedness Rules

$$\boxed{\Phi \vdash p : A \dashv \Gamma}$$

$$\begin{array}{c}
\text{P-VAR} \\
\hline
\Phi \vdash x : A \dashv x : A
\end{array}
\qquad
\begin{array}{c}
\text{P-DATA} \\
\frac{k \bar{A} \in D \bar{R} \quad (\Phi \vdash p_i : A_i \dashv \Gamma)_i}{\Phi \vdash k \bar{p} : D \bar{R} \dashv \bar{\Gamma}}
\end{array}$$

$$\boxed{\Phi \vdash r : T \dashv [\Sigma] \exists \Psi. \Gamma}$$

$$\begin{array}{c}
\text{P-VALUE} \\
\frac{\Sigma \vdash \Delta \dashv \Sigma' \quad \Phi \vdash p : A \dashv \Gamma}{\Phi \vdash p : \langle \Delta \rangle A \dashv [\Sigma] \Gamma}
\end{array}
\qquad
\begin{array}{c}
\text{P-CATCHALL} \\
\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\Phi \vdash \langle x \rangle : \langle \Delta \rangle A \dashv [\Sigma] x : \{[\Sigma'] A\}}
\end{array}$$

$$\begin{array}{c}
\text{P-COMMAND} \\
\frac{\Sigma \vdash \Delta \dashv \Sigma' \quad \Delta = \Theta \mid \Xi \quad c : \forall \bar{Z}. \bar{A} \rightarrow \bar{B} \in \Xi \quad (\Phi, \bar{Z} \vdash p_i : A_i \dashv \Gamma_i)_i}{\Phi \vdash \langle c \bar{p} \rightarrow z \rangle : \langle \Delta \rangle B' \dashv [\Sigma] \exists \bar{Z}. \bar{\Gamma}, z : \{\langle \mathbf{1} \mid \mathbf{1} \rangle B \rightarrow [\Sigma'] B'\}}
\end{array}$$

Figure A.3: Pattern Matching Typing Rules