This is the Project Title

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Master of Science
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2020

Abstract

Formal development of Frank.

Chapter 1

Formalisation of Frank

(data types)	D
(value type variables)	X
(effect type variables)	E
(value types)	$A, B ::= D\,\overline{R}$
	$\mid\ \{C\} \mid X$
(computation types)	$C ::= \overline{T \to} G$
(argument types)	$T ::= \langle\Delta\rangle A$
(return types)	$G ::= [\Sigma]A$
(type binders)	$Z ::= X \mid [E]$
(type arguments)	$R ::= A \mid [\Sigma]$
(polytypes)	$P ::= \forall\overline{Z}.A$

(interfaces)	I
(term variables)	x, y, z, f
(instance variables)	s, a, b, c
(seeds)	$\sigma ::= \emptyset \mid E$
(abilities)	$\Sigma ::= \sigma \mid \Xi$
(extensions)	$\Xi ::= \iota \mid \Xi, I\,\overline{R}$
(adaptors)	$\Theta ::= \iota \mid \Theta, I(S \to S')$
(adjustments)	$\Delta ::= \Theta \mid \Xi$
(instance patterns)	$S ::= s \mid S\,a$
(kind environments)	$\Phi, \Psi ::= \cdot \mid \Phi, Z$
(type environments)	$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, f : P$
(instance environments)	$\Omega ::= s : \Sigma \mid \Omega, a : I\,\overline{R}$

Figure 1.1: Types

(constructors)	k
(commands)	c
(uses)	$m ::= x \mid f\,\overline{R} \mid m\,\overline{n} \mid \uparrow(n : A)$
(constructions)	$n ::= \downarrow m \mid k\,\overline{n} \mid c\,\overline{R}\,\overline{n} \mid \{e\}$
	$\mid\ \textbf{let}\ f : P = n\ \textbf{in}\ n' \mid \textbf{letrec}\ \overline{f : P = e}\ \textbf{in}\ n$
	$\mid\ \langle\Theta\rangle\ n$
(computations)	$e ::= \overline{\overline{r} \mapsto n}$
(computation patterns)	$r ::= p \mid \langle c\,\overline{p} \to z\rangle \mid \langle x\rangle$
(value patterns)	$p ::= k\,\overline{p} \mid x$

Figure 1.2: Terms

$$\boxed{\Phi;\Gamma\,[\Sigma]\!\vdash m \Rightarrow A}$$

$$\textsc{T-Var}\ \frac{x : A \in \Gamma}{\Phi;\Gamma\,[\Sigma]\!\vdash x \Rightarrow A} \qquad \textsc{T-PolyVar}\ \frac{\Phi \vdash \overline{R} \qquad f : \forall \overline{Z}.A \in \Gamma}{\Phi;\Gamma\,[\Sigma]\!\vdash f\ \overline{R} \Rightarrow A[\overline{R}/\overline{Z}]}$$

$$\textsc{T-App}\ \frac{\begin{array}{c}\Sigma' = \Sigma \qquad (\Sigma \vdash \Delta_i \dashv \Sigma'_i)_i \\ \Phi;\Gamma\,[\Sigma]\!\vdash m \Rightarrow \{\overline{\langle\Delta\rangle A \to}\ [\Sigma']B\} \qquad (\Phi;\Gamma\,[\Sigma'_i]\!\vdash n_i : A_i)_i\end{array}}{\Phi;\Gamma\,[\Sigma]\!\vdash m\ \overline{n} \Rightarrow B} \qquad \textsc{T-Ascribe}\ \frac{\Phi;\Gamma\,[\Sigma]\!\vdash n : A}{\Phi;\Gamma\,[\Sigma]\!\vdash \uparrow(n : A) \Rightarrow A}$$

$$\boxed{\Phi;\Gamma\,[\Sigma]\!\vdash n : A}$$

$$\textsc{T-Switch}\ \frac{\Phi;\Gamma\,[\Sigma]\!\vdash m \Rightarrow A \qquad A = B}{\Phi;\Gamma\,[\Sigma]\!\vdash \downarrow m : B} \qquad \textsc{T-Data}\ \frac{k\ \overline{A} \in D\ \overline{R} \qquad (\Phi;\Gamma\,[\Sigma]\!\vdash n_j : A_j)_j}{\Phi;\Gamma\,[\Sigma]\!\vdash k\ \overline{n} : D\ \overline{R}}$$

$$\textsc{T-Command}\ \frac{\Phi \vdash \overline{R} \qquad c : \forall \overline{Z}.\overline{A \to}\ B \in \Sigma \qquad (\Phi;\Gamma\,[\Sigma]\!\vdash n_j : A_j[\overline{R}/\overline{Z}])_j}{\Phi;\Gamma\,[\Sigma]\!\vdash c\ \overline{R}\ \overline{n} : B[\overline{R}/\overline{Z}]} \qquad \textsc{T-Thunk}\ \frac{\Phi;\Gamma \vdash e : C}{\Phi;\Gamma\,[\Sigma]\!\vdash \{e\} : \{C\}}$$

$$\textsc{T-Let}\ \frac{\begin{array}{c}P = \forall \overline{Z}.A \\ \Phi,\overline{Z};\Gamma\,[\emptyset]\!\vdash n : A \qquad \Phi;\Gamma, f : P\,[\Sigma]\!\vdash n' : B\end{array}}{\Phi;\Gamma\,[\Sigma]\!\vdash \mathbf{let}\ f : P = n\ \mathbf{in}\ n' : B}$$

$$\textsc{T-LetRec}\ \frac{\begin{array}{c}(P_i = \forall \overline{Z}_i.\{C_i\})_i \\ (\Phi,\overline{Z}_i;\Gamma,\overline{f : P} \vdash e_i : C)_i \qquad \Phi;\Gamma,\overline{f : P}\,[\Sigma]\!\vdash n : B\end{array}}{\Phi;\Gamma\,[\Sigma]\!\vdash \mathbf{letrec}\ \overline{f : P = e}\ \mathbf{in}\ n : B} \qquad \textsc{T-Adapt}\ \frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Phi;\Gamma\,[\Sigma']\!\vdash n : A}{\Phi;\Gamma\,[\Sigma]\!\vdash \langle\Theta\rangle\ n : A}$$

$$\boxed{\Phi;\Gamma \vdash e : C}$$

$$\textsc{T-Comp}\ \frac{\begin{array}{c}(\Phi \vdash r_{i,j} : T_j \dashv\![\Sigma]\ \exists \Psi_{i,j}.\Gamma'_{i,j})_{i,j} \\ (\Phi,(\Psi_{i,j})_j;\Gamma,(\Gamma'_{i,j})_j\,[\Sigma]\!\vdash n_i : B)_i \qquad ((r_{i,j})_i \text{ covers } T_j)_j\end{array}}{\Phi;\Gamma \vdash ((r_{i,j})_j \mapsto n_i)_i : (T_j \to)_j\ [\Sigma]B}$$

Figure 1.3: Term Typing Rules

$$
\begin{array}{lrl}
\text{(uses)} & m & ::= \cdots \mid \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \\
\text{(constructions)} & n & ::= \cdots \mid \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \\
\text{(use values)} & u & ::= x \mid f\ \overline{R} \mid \uparrow(v : A) \\
\text{(non-use values)} & v & ::= k\ \overline{w} \mid \{e\} \\
\text{(construction values)} & w & ::= \downarrow u \mid v \\
\text{(normal forms)} & t & ::= w \mid \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \\
\text{(evaluation frames)} & \mathcal{F} & ::= [\,]\ \overline{n} \mid u\ (\bar{t}, [\,], \overline{n}) \mid \uparrow([\,] : A) \\
 & & \mid\ \ \downarrow[\,] \mid k\ (\overline{w}, [\,], \overline{n}) \mid c\ \overline{R}\ (\overline{w}, [\,], \overline{n}) \\
 & & \mid\ \ \textbf{let}\ f : P = [\,]\ \textbf{in}\ n \mid \langle \Theta \rangle\ [\,] \\
\text{(evaluation contexts)} & \mathcal{E} & ::= [\,] \mid \mathcal{F}[\mathcal{E}]
\end{array}
$$

Figure 1.4: Runtime Syntax

$\boxed{\Phi; \Gamma\ [\Sigma]\!\!-\ m \Rightarrow A}$ $\boxed{\Phi; \Gamma\ [\Sigma]\!\!-\ n : A}$

$$
\text{T-FREEZE-USE}\ \frac{\neg(\mathcal{E}\ \text{handles}\ c) \qquad \Phi; \Gamma\ [\Sigma]\!\!-\ \mathcal{E}[c\ \overline{R}\ \overline{w}] \Rightarrow A}{\Phi; \Gamma\ [\Sigma]\!\!-\ \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \Rightarrow A}
$$

$$
\text{T-FREEZE-CONS}\ \frac{\neg(\mathcal{E}\ \text{handles}\ c) \qquad \Phi; \Gamma\ [\Sigma]\!\!-\ \mathcal{E}[c\ \overline{R}\ \overline{w}] : A}{\Phi; \Gamma\ [\Sigma]\!\!-\ \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil : A}
$$

Figure 1.5: Frozen Commands

$\boxed{m \rightsquigarrow_{\mathrm{u}} m'}$ $\boxed{n \rightsquigarrow_{\mathrm{c}} n'}$ $\boxed{m \longrightarrow_{\mathrm{u}} m'}$ $\boxed{n \longrightarrow_{\mathrm{c}} n'}$

$$\textsc{R-Handle}\ \frac{k = \min_i \{i \mid \exists\overline{\theta}.(r_{i,j} : \langle\Delta_j\rangle A_j \leftarrow t_j \dashv[\Sigma]\ \theta_j)_j\} \qquad (r_{k,j} : \langle\Delta_j\rangle A_j \leftarrow t_j \dashv[\Sigma]\ \theta_j)_j}{\uparrow(\{((r_{i,j})_j \to n_i)_i\} : \{\overline{\langle\Delta\rangle A \to}\ [\Sigma]B\})\ \overline{t} \rightsquigarrow_{\mathrm{u}} \uparrow((\overline{\theta}(n_k) : B)}$$

$$\textsc{R-Ascribe-Use}\ \frac{}{\uparrow(\downarrow u : A) \rightsquigarrow_{\mathrm{u}} u} \qquad \textsc{R-Ascribe-Cons}\ \frac{}{\downarrow\uparrow(w : A) \rightsquigarrow_{\mathrm{c}} w} \qquad \textsc{R-Let}\ \frac{}{\mathbf{let}\ f : P = w\ \mathbf{in}\ n \rightsquigarrow_{\mathrm{c}} n[\uparrow(w : P)/f]}$$

$$\textsc{R-LetRec}\ \frac{\overline{e = \overline{\overline{r} \to n}}}{\mathbf{letrec}\ \overline{f : P = e}\ \mathbf{in}\ n' \rightsquigarrow_{\mathrm{c}} n'\overline{[\uparrow(\{\overline{\overline{r} \to \mathbf{letrec}\ \overline{f : P = e}\ \mathbf{in}\ n}\} : P)/f]}} \qquad \textsc{R-Adapt}\ \frac{}{\langle\Theta\rangle\ w \rightsquigarrow_{\mathrm{c}} w}$$

$$\textsc{R-Freeze-Comm}\ \frac{}{c\ \overline{R}\ \overline{w} \rightsquigarrow_{\mathrm{c}} \lceil c\ \overline{R}\ \overline{w} \rceil}$$

$$\textsc{R-Freeze-Frame-Use}\ \frac{\neg(\mathcal{F}[\mathcal{E}]\ \text{handles}\ c)}{\mathcal{F}[\lceil\mathcal{E}[c\ \overline{R}\ \overline{w}]\rceil] \rightsquigarrow_{\mathrm{u}} \lceil\mathcal{F}[\mathcal{E}[c\ \overline{R}\ \overline{w}]]\rceil} \qquad \textsc{R-Freeze-Frame-Cons}\ \frac{\neg(\mathcal{F}[\mathcal{E}]\ \text{handles}\ c)}{\mathcal{F}[\lceil\mathcal{E}[c\ \overline{R}\ \overline{w}]\rceil] \rightsquigarrow_{\mathrm{c}} \lceil\mathcal{F}[\mathcal{E}[c\ \overline{R}\ \overline{w}]]\rceil}$$

$$\textsc{R-Lift-UU}\ \frac{m \rightsquigarrow_{\mathrm{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathrm{u}} \mathcal{E}[m']} \qquad \textsc{R-Lift-UC}\ \frac{m \rightsquigarrow_{\mathrm{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathrm{c}} \mathcal{E}[m']} \qquad \textsc{R-Lift-CU}\ \frac{n \rightsquigarrow_{\mathrm{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathrm{u}} \mathcal{E}[n']} \qquad \textsc{R-Lift-CC}\ \frac{n \rightsquigarrow_{\mathrm{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathrm{c}} \mathcal{E}[n']}$$

Figure 1.6: Operational Semantics

$\boxed{r : T \leftarrow t \dashv_{[\Sigma]} \theta}$

$$\textsc{B-Value}\quad \frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad p : A \leftarrow w \dashv \theta}{p : \langle\Delta\rangle A \leftarrow w \dashv_{[\Sigma]} \theta}$$

$$\textsc{B-Request}\quad \frac{\begin{array}{c}\Sigma \vdash \Delta \dashv \Sigma' \qquad \mathcal{E}\ \text{poisedfor}\ c \\ \Delta = \Theta \mid \Xi \qquad c : \forall \overline{Z}.\overline{B \to}\ B' \in \Xi \qquad (p_i : B_i \leftarrow w_i \dashv \theta_i)_i\end{array}}{\langle c\ \overline{p} \to z\rangle : \langle\Delta\rangle A \leftarrow \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \dashv_{[\Sigma]} \overline{\theta}[\uparrow(\{x \mapsto \mathcal{E}[x]\} : \{B' \to [\Sigma']A\})/z]}$$

$$\textsc{B-CatchAll-Value}\quad \frac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x\rangle : \langle\Delta\rangle A \leftarrow w \dashv_{[\Sigma]} [\uparrow(\{w\}:\{[\Sigma']A\})/x]}$$

$$\textsc{B-CatchAll-Request}\quad \frac{\begin{array}{c}\Sigma \vdash \Delta \dashv \Sigma' \qquad \mathcal{E}\ \text{poisedfor}\ c \\ \Delta = \Theta \mid \Xi \qquad c : \forall \overline{Z}.\overline{B \to}\ B' \in \Xi\end{array}}{\langle x\rangle : \langle\Delta\rangle A \leftarrow \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \dashv_{[\Sigma]} [\uparrow(\{\lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil\}:\{[\Sigma']A\})/x]}$$

$\boxed{p : A \leftarrow w \dashv \theta}$

$$\textsc{B-Var}\quad \frac{}{x : A \leftarrow w \dashv [\uparrow(w : A)/x]} \qquad\qquad \textsc{B-Data}\quad \frac{k\ \overline{A} \in D\ \overline{R} \qquad (p_i : A_i \leftarrow w_i \dashv \theta_i)_i}{k\ \overline{p} : D\ \overline{R} \leftarrow k\ \overline{w} \dashv \overline{\theta}}$$

Figure 1.7: Pattern Binding

Chapter 2

Arbitrary Thread Interruption

2.1 Relaxing Catches

$$\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x \rangle : \langle \Delta \rangle A \leftarrow \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \dashv_{[\Sigma]} [\uparrow(\{\lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil\}:\{[\Sigma']A\})/x]}\ \textsc{B-CatchAll-Request-Loose}$$

Figure 2.1: Updated B-CatchAll-Request

2.2 Interrupting Arbitrary Terms

$$
\begin{array}{lrl}
\text{(uses)} & m ::= & \cdots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \\
\text{(constructions)} & n ::= & \cdots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \\
\text{(use values)} & u ::= & x \mid f\,\overline{R} \mid \uparrow(v : A) \\
\text{(non-use values)} & v ::= & k\,\overline{w} \mid \{e\} \\
\text{(construction values)} & w ::= & \downarrow u \mid v \\
\text{(normal forms)} & t ::= & w \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \mid !(m) \\
\text{(evaluation frames)} & \mathcal{F} ::= & [\,]\,\overline{n} \mid u\,(\overline{t},[\,],\overline{n}) \mid \uparrow([\,] : A) \\
 & \mid & \downarrow[\,] \mid k\,(\overline{w},[\,],\overline{n}) \mid c\,\overline{R}\,(\overline{w},[\,],\overline{n}) \\
 & \mid & \textbf{let}\ f : P = [\,]\ \textbf{in}\ n \mid \langle\Theta\rangle\ [\,] \\
\text{(evaluation contexts)} & \mathcal{E} ::= & [\,] \mid \mathcal{F}[\mathcal{E}]
\end{array}
$$

Figure 2.2: Runtime Syntax, Updated with Suspension of Uses

$$
\textsc{B-CatchAll-Interrupt}\quad
\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x \rangle : \langle \Delta \rangle A \leftarrow !(m) \dashv_{\Sigma} [\uparrow(\{m\}:\{[\Sigma']A\})/x]}
$$

Figure 2.3: Catching Interrupts rule.

2.3 Freezing

Another way of doing it is to just let any use become frozen, in the same way as commands become frozen once invoked.

TODO: Do we need the 'hoisting' rules for general suspended terms?

2.4 Yielding

2.5 Counting

In practise we count up through the amount of R-HANDLE rules we apply and only insert the yield when this count exceeds a threshold value t_y.

So we supplement the operational semantics with a *counter* c_y, so that our transitions are e.g. $m; c_y \leadsto_{\mathrm{u}} m'; c_y{}'$. We adopt the convention that, when the counter is not mentioned in a transition[1], the counter stays the same. Hence e.g. $m \leadsto_{\mathrm{u}} m'$ desugars

[1] That is, the transition is of the form $m \leadsto_{\mathrm{u}} m'$.

R-INTERRUPT

$$\frac{}{m \rightsquigarrow_{\mathrm{u}} !(m)}$$

Figure 2.4: Use interruption rule

$\boxed{m \rightsquigarrow_{\mathrm{u}} m'}$ $\boxed{n \rightsquigarrow_{\mathrm{c}} n'}$ $\boxed{m \longrightarrow_{\mathrm{u}} m'}$ $\boxed{n \longrightarrow_{\mathrm{c}} n'}$

R-FREEZE-USE

$$\frac{}{m \rightsquigarrow_{\mathrm{u}} \lceil m \rceil}$$

R-FREEZE-COMM

$$\frac{}{c\,\overline{R}\,\overline{w} \rightsquigarrow_{\mathrm{c}} \lceil c\,\overline{R}\,\overline{w} \rceil}$$

R-FREEZE-FRAME-USE

$$\frac{}{\mathcal{F}[\lceil m \rceil] \rightsquigarrow_{\mathrm{u}} \lceil \mathcal{F}[m] \rceil}$$

R-FREEZE-FRAME-CONS

$$\frac{}{\mathcal{F}[\lceil m \rceil] \rightsquigarrow_{\mathrm{c}} \lceil \mathcal{F}[m] \rceil}$$

R-FREEZE-FRAME-USE

$$\frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[\lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil] \rightsquigarrow_{\mathrm{u}} \lceil \mathcal{F}[\mathcal{E}[c\,\overline{R}\,\overline{w}]] \rceil}$$

R-FREEZE-FRAME-CONS

$$\frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[\lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil] \rightsquigarrow_{\mathrm{c}} \lceil \mathcal{F}[\mathcal{E}[c\,\overline{R}\,\overline{w}]] \rceil}$$

Figure 2.5: Updated Freezing

to $m; c_y \rightsquigarrow_{\mathrm{u}} m'; c_y$.

So to get our counting semantics we just need to supplement Figure **?** with the updated rule in

$$
\begin{array}{lrl}
\text{(uses)} & m ::= & \ldots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \mid \boxed{\lceil m \rceil} \\
\text{(constructions)} & n ::= & \ldots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \mid \boxed{\lceil m \rceil} \\
\text{(use values)} & u ::= & x \mid f\,\overline{R} \mid \uparrow(v : A) \\
\text{(non-use values)} & v ::= & k\,\overline{w} \mid \{e\} \\
\text{(construction values)} & w ::= & \downarrow u \mid v \\
\text{(normal forms)} & t ::= & w \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \mid \boxed{\lceil m \rceil} \\
\text{(evaluation frames)} & \mathcal{F} ::= & [\,]\,\overline{n} \mid u\,(\bar{t},[\,],\overline{n}) \mid \uparrow([\,] : A) \\
& \mid & \downarrow[\,] \mid k\,(\overline{w},[\,],\overline{n}) \mid c\,\overline{R}\,(\overline{w},[\,],\overline{n}) \\
& \mid & \mathbf{let}\ f : P = [\,]\ \mathbf{in}\ n \mid \langle \Theta \rangle\,[\,] \\
\text{(evaluation contexts)} & \mathcal{E} ::= & [\,] \mid \mathcal{F}[\mathcal{E}]
\end{array}
$$

Figure 2.6: Runtime Syntax, Updated with Freezing of Uses

B-CATCHALL-INTERRUPT

$$
\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x \rangle : \langle \Delta \rangle A \leftarrow \lceil m \rceil \dashv_{[\Sigma]} [\uparrow(\{m\}:\{[\Sigma']A\})/x]}
$$

Figure 2.7: Catching Frozen Terms rule.

$\boxed{m \leadsto_{\mathrm{u}} m'}$

R-YIELD

$$
\frac{\Delta = \Theta \mid \Xi \qquad \mathsf{Yield} \in \Xi}{\uparrow(n : [\Delta]A) \leadsto_{\mathrm{u}} \uparrow(\mathbf{let}\ \mathsf{yield!} : [\Delta]\mathsf{Unit} = \_\ \mathbf{in}\ m : [\Delta]A)}
$$

Figure 2.8: Inserting Yields

$\boxed{m \leadsto_{\mathrm{u}} m'}$ $\boxed{n \leadsto_{\mathrm{c}} n'}$ $\boxed{m \longrightarrow_{\mathrm{u}} m'}$ $\boxed{n \longrightarrow_{\mathrm{c}} n'}$

R-HANDLE

$$
\frac{k = \min_i \{ i \mid \exists \overline{\theta}.(r_{i,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \dashv_{[\Sigma]} \theta_j)_j \} \qquad (r_{k,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \dashv_{[\Sigma]} \theta_j)_j}{\uparrow(\{((r_{i,j})_j \to n_i)_i\} : \{\overline{\langle \Delta \rangle A \to}\ [\Sigma]B\})\ \bar{t}; c_y \leadsto_{\mathrm{u}} \uparrow((\overline{\theta}(n_k) : B)); c_y + 1}
$$

Figure 2.9: R-Handle with counting.

$\boxed{m \rightsquigarrow_{\mathrm{u}} m'}$

$$\textsc{R-Yield}\quad \frac{c_y \geq t_y}{m; c_y \rightsquigarrow_{\mathrm{u}} \textbf{let}\ \mathsf{yield} : [\mathsf{Yield}]\mathsf{Unit} = \_\ \textbf{in}\ m; 0}$$

Figure 2.10: Inserting Yields, when over counter

Appendix A

Remaining Formalisms

$\boxed{\Sigma \vdash \Delta \dashv \Sigma'}$

$$\textsc{A-Adj}\ \frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Sigma' \vdash \Xi \dashv \Sigma''}{\Sigma \vdash \Theta \mid \Xi \dashv \Sigma''}$$

$\boxed{\Sigma \vdash \Xi \dashv \Sigma'}$

$$\textsc{A-Ext-Id}\ \frac{}{\Sigma \vdash \iota \dashv \Sigma} \qquad \textsc{A-Ext-Snoc}\ \frac{\Sigma \vdash \Xi \dashv \Sigma'}{\Sigma \vdash \Xi, I\,\overline{R} \dashv \Sigma', I\,\overline{R}}$$

$\boxed{\Sigma \vdash \Theta \dashv \Sigma'}$

$$\textsc{A-Adapt-Id}\ \frac{}{\Sigma \vdash \iota \dashv \Sigma} \qquad \textsc{A-Adapt-Snoc}\ \frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Sigma' \vdash I(S \to S') \dashv \Sigma''}{\Sigma \vdash \Theta, I(S \to S') \dashv \Sigma''}$$

$\boxed{\Sigma \vdash I(S \to S') \dashv \Sigma'}$

$$\textsc{A-Adapt-Com}\ \frac{\Sigma \vdash S : I \dashv \Sigma';\Omega \qquad \Omega \vdash S' : I \dashv \Xi \qquad \Sigma' \vdash \Xi \dashv \Sigma''}{\Sigma \vdash I(S \to S') \dashv \Sigma''}$$

$\boxed{\Sigma \vdash S : I \dashv \Sigma';\Omega}$

$$\textsc{I-Pat-Id}\ \frac{}{\Sigma \vdash s : I \dashv \Sigma; s : \Sigma} \qquad \textsc{I-Pat-Bind}\ \frac{\Sigma \vdash S : I \dashv \Sigma';\Omega}{\Sigma, I\,\overline{R} \vdash S\,a : I \dashv \Sigma';\Omega, a : I\,\overline{R}} \qquad \textsc{I-Pat-Skip}\ \frac{\Sigma \vdash S\,a : I \dashv \Sigma';\Omega \qquad I \neq I'}{\Sigma, I'\,\overline{R} \vdash S\,a : I \dashv \Sigma', I'\,\overline{R};\Omega}$$

$\boxed{\Omega \vdash S : I \dashv \Xi}$

$$\textsc{I-Inst-Id}\ \frac{s \in \mathsf{dom}(\Omega)}{\Omega \vdash s : I \dashv \iota} \qquad \textsc{I-Inst-Lkp}\ \frac{a \in \mathsf{dom}(\Omega) \qquad \Omega \vdash S : I \dashv \Xi \qquad \Omega(a) = I\,\overline{R}}{\Omega \vdash S\,a : I \dashv \Xi, I\,\overline{R}}$$

Figure A.1: Action of an Adjustment on an Ability and Auxiliary Judgements

$$\mathcal{X} ::= A \mid C \mid T \mid G \mid Z \mid R \mid P \mid \sigma \mid \Sigma \mid \Xi \mid \Theta \mid \Delta \mid \Gamma \mid \exists\Psi.\Gamma \mid \Omega$$

$\boxed{\Phi \vdash \mathcal{X}}$

$$\frac{}{\Phi, X \vdash X}\ \textsc{WF-Val} \qquad \frac{}{\Phi, [E] \vdash E}\ \textsc{WF-Eff} \qquad \frac{\Phi, \overline{Z} \vdash A}{\Phi \vdash \forall \overline{Z}.A}\ \textsc{WF-Poly}$$

$$\frac{(\Phi \vdash R)_i}{\Phi \vdash D\,\overline{R}}\ \textsc{WF-Data} \qquad \frac{\Phi \vdash C}{\Phi \vdash \{C\}}\ \textsc{WF-Thunk} \qquad \frac{(\Phi \vdash T)_i \quad \Phi \vdash G}{\Phi \vdash \overline{T \to} \ G}\ \textsc{WF-Comp} \qquad \frac{\Phi \vdash \Delta \quad \Phi \vdash A}{\Phi \vdash \langle \Delta \rangle A}\ \textsc{WF-Arg}$$

$$\frac{\Phi \vdash \Sigma \quad \Phi \vdash A}{\Phi \vdash [\Sigma]A}\ \textsc{WF-Ret} \qquad \frac{\Phi \vdash \Sigma}{\Phi \vdash [\Sigma]}\ \textsc{WF-Ability} \qquad \frac{}{\Phi \vdash \emptyset}\ \textsc{WF-Pure} \qquad \frac{}{\Phi \vdash \iota}\ \textsc{WF-Id} \qquad \frac{\Phi \vdash \Xi \quad (\Phi \vdash R)_i}{\Phi \vdash \Xi, I\,\overline{R}}\ \textsc{WF-Ext}$$

$$\frac{\Phi \vdash \Theta}{\Phi \vdash \Theta, I\ (S \to S')}\ \textsc{WF-Adapt}$$

$$\frac{}{\Phi \vdash \cdot}\ \textsc{WF-Empty} \qquad \frac{\Phi \vdash \Gamma \quad \Phi \vdash A}{\Phi \vdash \Gamma, x : A}\ \textsc{WF-Mono} \qquad \frac{\Phi \vdash \Gamma \quad \Phi \vdash P}{\Phi \vdash \Gamma, f : P}\ \textsc{WF-Poly}$$

$$\frac{\Phi, \Psi \vdash \Gamma}{\Phi \vdash \exists\Psi.\Gamma}\ \textsc{WF-Existential} \qquad \frac{\Phi \vdash \Omega \quad (\Phi \vdash R)_i}{\Phi \vdash \Omega, x : I\,\overline{R}}\ \textsc{WF-Interface}$$

Figure A.2: Well-Formedness Rules

$$\boxed{\Phi \vdash p : A \dashv \Gamma}$$

$$\frac{}{\Phi \vdash x : A \dashv x : A}\ \textsc{P-Var} \qquad \frac{k\,\overline{A} \in D\,\overline{R} \qquad (\Phi \vdash p_i : A_i \dashv \Gamma)_i}{\Phi \vdash k\,\overline{p} : D\,\overline{R} \dashv \overline{\Gamma}}\ \textsc{P-Data}$$

$$\boxed{\Phi \vdash r : T \dashv_{[\Sigma]} \exists \Psi.\Gamma}$$

$$\frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \Phi \vdash p : A \dashv \Gamma}{\Phi \vdash p : \langle \Delta \rangle A \dashv_{[\Sigma]} \Gamma}\ \textsc{P-Value} \qquad \frac{\Sigma \vdash \Delta \dashv \Sigma'}{\Phi \vdash \langle x \rangle : \langle \Delta \rangle A \dashv_{[\Sigma]} x : \{[\Sigma']A\}}\ \textsc{P-CatchAll}$$

$$\frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \Delta = \Theta \mid \Xi \qquad c : \forall \overline{Z}.\overline{A \to} B \in \Xi \qquad (\Phi, \overline{Z} \vdash p_i : A_i \dashv \Gamma_i)_i}{\Phi \vdash \langle c\,\overline{p} \to z \rangle : \langle \Delta \rangle B' \dashv_{[\Sigma]} \exists \overline{Z}.\overline{\Gamma}, z : \{\langle \iota \mid \iota \rangle B \to [\Sigma']B'\}}\ \textsc{P-Command}$$

Figure A.3: Pattern Matching Typing Rules