

# **This is the Project Title**

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# Abstract

Formal development of Frank.

# **Chapter 1**

## **Introduction**

(data types)	$D$
(value type variables)	$X$
(effect type variables)	$E$
(value types)	$A, B ::= D \bar{R}$ $\quad \mid \{C\} \mid X$
(computation types)	$C ::= \overline{T \rightarrow G}$
(argument types)	$T ::= \langle \Delta \rangle A$
(return types)	$G ::= [\Sigma] A$
(type binders)	$Z ::= X \mid [E]$
(type arguments)	$R ::= A \mid [\Sigma]$
(polytypes)	$P ::= \forall \bar{Z}. A$
(interfaces)	$I$
(term variables)	$x, y, z, f$
(instance variables)	$s, a, b, c$
(seeds)	$\sigma ::= \emptyset \mid E$
(abilities)	$\Sigma ::= \sigma \mid \Xi$
(extensions)	$\Xi ::= \mathfrak{t} \mid \Xi, I \bar{R}$
(adaptors)	$\Theta ::= \mathfrak{t} \mid \Theta, I(S \rightarrow S')$
(adjustments)	$\Delta ::= \Theta \mid \Xi$
(instance patterns)	$S ::= s \mid S a$
(kind environments)	$\Phi, \Psi ::= \cdot \mid \Phi, Z$
(type environments)	$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, f : P$
(instance environments)	$\Omega ::= s : \Sigma \mid \Omega, a : I \bar{R}$

Figure 1.1: Types

(constructors)	$k$
(commands)	$c$
(uses)	$m ::= x \mid f \bar{R} \mid m \bar{n} \mid \uparrow(n : A)$
(constructions)	$n ::= \downarrow m \mid k \bar{n} \mid c \bar{R} \bar{n} \mid \{e\}$ $\mid \text{let } f : P = n \text{ in } n' \mid \text{letrec } \overline{f : P = e} \text{ in } n$ $\mid \langle \Theta \rangle n$
(computations)	$e ::= \overline{\bar{r} \mapsto n}$
(computation patterns)	$r ::= p \mid \langle c \bar{p} \rightarrow z \rangle \mid \langle x \rangle$
(value patterns)	$p ::= k \bar{p} \mid x$

Figure 1.2: Terms

$$\boxed{\Sigma \vdash \Delta \dashv \Sigma'}$$

$$\begin{array}{c} \text{A-ADJ} \\ \hline \Sigma \vdash \Theta \dashv \Sigma' \quad \Sigma' \vdash \Xi \dashv \Sigma'' \\ \hline \Sigma \vdash \Theta | \Xi \dashv \Sigma'' \end{array}$$

$$\boxed{\Sigma \vdash \Xi \dashv \Sigma'}$$

$$\begin{array}{c} \text{A-EXT-ID} \\ \hline \Sigma \vdash \mathbf{1} \dashv \Sigma \end{array}$$

$$\begin{array}{c} \text{A-EXT-SNOC} \\ \hline \Sigma \vdash \Xi \dashv \Sigma' \\ \hline \Sigma \vdash \Xi, I \bar{R} \dashv \Sigma', I \bar{R} \end{array}$$

$$\boxed{\Sigma \vdash \Theta \dashv \Sigma'}$$

$$\begin{array}{c} \text{A-ADAPT-ID} \\ \hline \Sigma \vdash \mathbf{1} \dashv \Sigma \end{array}$$

$$\begin{array}{c} \text{A-ADAPT-SNOC} \\ \hline \Sigma \vdash \Theta \dashv \Sigma' \quad \Sigma' \vdash I(S \rightarrow S') \dashv \Sigma'' \\ \hline \Sigma \vdash \Theta, I(S \rightarrow S') \dashv \Sigma'' \end{array}$$

$$\boxed{\Sigma \vdash I(S \rightarrow S') \dashv \Sigma'}$$

$$\begin{array}{c} \text{A-ADAPT-COM} \\ \hline \Sigma \vdash S : I \dashv \Sigma'; \Omega \quad \Omega \vdash S' : I \dashv \Xi \quad \Sigma' \vdash \Xi \dashv \Sigma'' \\ \hline \Sigma \vdash I(S \rightarrow S') \dashv \Sigma'' \end{array}$$

$$\boxed{\Sigma \vdash S : I \dashv \Sigma'; \Omega}$$

$$\text{I-PAT-ID}$$

$$\Sigma \vdash s : I \dashv \Sigma; s : \Sigma$$

$$\text{I-PAT-BIND}$$

$$\Sigma \vdash S : I \dashv \Sigma'; \Omega$$

$$\Sigma, I \bar{R} \vdash S a : I \dashv \Sigma'; \Omega, a : I \bar{R}$$

$$\text{I-PAT-SKIP}$$

$$\Sigma \vdash S a : I \dashv \Sigma'; \Omega \quad I \neq I'$$

$$\Sigma, I' \bar{R} \vdash S a : I \dashv \Sigma', I' \bar{R}; \Omega$$

$$\boxed{\Omega \vdash S : I \dashv \Xi}$$

$$\begin{array}{c} \text{I-INST-ID} \\ \hline s \in \text{dom}(\Omega) \\ \hline \Omega \vdash s : I \dashv \mathbf{1} \end{array}$$

$$\begin{array}{c} \text{I-INST-LKP} \\ \hline a \in \text{dom}(\Omega) \quad \Omega \vdash S : I \dashv \Xi \quad \Omega(a) = I \bar{R} \\ \hline \Omega \vdash S a : I \dashv \Xi, I \bar{R} \end{array}$$

Figure 1.3: Action of an Adjustment on an Ability and Auxiliary Judgements

$$\begin{array}{c}
\mathcal{X} ::= A \mid C \mid T \mid G \mid Z \mid R \mid P \mid \sigma \mid \Sigma \mid \Xi \mid \Theta \mid \Delta \mid \Gamma \mid \exists \Psi. \Gamma \mid \Omega \\
\boxed{\Phi \vdash \mathcal{X}} \\
\\
\begin{array}{ccc}
\text{WF-VAL} & \text{WF-EFF} & \text{WF-POLY} \\
\frac{}{\Phi, X \vdash X} & \frac{}{\Phi, [E] \vdash E} & \frac{\Phi, \bar{Z} \vdash A}{\Phi \vdash \forall \bar{Z}. A} \\
\\
\begin{array}{ccc}
\text{WF-DATA} & \text{WF-THUNK} & \text{WF-COMP} \\
\frac{(\Phi \vdash R)_i}{\Phi \vdash D \bar{R}} & \frac{\Phi \vdash C}{\Phi \vdash \{C\}} & \frac{(\Phi \vdash T)_i \quad \Phi \vdash G}{\Phi \vdash \bar{T} \rightarrow G} \\
\\
\text{WF-RET} & \text{WF-ABILITY} & \text{WF-PURE} & \text{WF-ID} & \text{WF-EXT} \\
\frac{\Phi \vdash \Sigma \quad \Phi \vdash A}{\Phi \vdash [\Sigma] A} & \frac{\Phi \vdash \Sigma}{\Phi \vdash [\Sigma]} & \frac{}{\Phi \vdash \emptyset} & \frac{}{\Phi \vdash \mathbf{1}} & \frac{\Phi \vdash \Xi \quad (\Phi \vdash R)_i}{\Phi \vdash \Xi, I \bar{R}}
\end{array} \\
\\
\begin{array}{c}
\text{WF-ADAPT} \\
\frac{\Phi \vdash \Theta}{\Phi \vdash \Theta, I (S \rightarrow S')}
\end{array} \\
\\
\begin{array}{ccc}
\text{WF-EMPTY} & \text{WF-MONO} & \text{WF-POLY} \\
\frac{}{\Phi \vdash \cdot} & \frac{\Phi \vdash \Gamma \quad \Phi \vdash A}{\Phi \vdash \Gamma, x : A} & \frac{\Phi \vdash \Gamma \quad \Phi \vdash P}{\Phi \vdash \Gamma, f : P} \\
\\
\begin{array}{ccc}
\text{WF-EXISTENTIAL} & & \text{WF-INTERFACE} \\
\frac{\Phi, \Psi \vdash \Gamma}{\Phi \vdash \exists \Psi. \Gamma} & & \frac{\Phi \vdash \Omega \quad (\Phi \vdash R)_i}{\Phi \vdash \Omega, x : I \bar{R}}
\end{array}
\end{array}
\end{array}$$

Figure 1.4: Well-Formedness Rules

$\Phi; \Gamma [\Sigma] \vdash m \Rightarrow A$	
$\frac{\text{T-VAR} \quad x : A \in \Gamma}{\Phi; \Gamma [\Sigma] \vdash x \Rightarrow A}$	$\frac{\text{T-POLYVAR} \quad \Phi \vdash \bar{R} \quad f : \forall \bar{Z}. A \in \Gamma}{\Phi; \Gamma [\Sigma] \vdash f \bar{R} \Rightarrow A[\bar{R}/\bar{Z}]}$
$\frac{\text{T-APP} \quad \Sigma' = \Sigma \quad (\Sigma \vdash \Delta_i \dashv \Sigma'_i)_i \quad \Phi; \Gamma [\Sigma] \vdash m \Rightarrow \{\langle \Delta \rangle A \rightarrow [\Sigma'] B\} \quad (\Phi; \Gamma [\Sigma'_i] \vdash n_i : A_i)_i}{\Phi; \Gamma [\Sigma] \vdash m \bar{n} \Rightarrow B}$	$\frac{\text{T-ASCRIBE} \quad \Phi; \Gamma [\Sigma] \vdash n : A}{\Phi; \Gamma [\Sigma] \vdash \uparrow(n : A) \Rightarrow A}$
$\Phi; \Gamma [\Sigma] \vdash n : A$	
$\frac{\text{T-SWITCH} \quad \Phi; \Gamma [\Sigma] \vdash m \Rightarrow A \quad A = B}{\Phi; \Gamma [\Sigma] \vdash \downarrow m : B}$	$\frac{\text{T-DATA} \quad k \bar{A} \in D \bar{R} \quad (\Phi; \Gamma [\Sigma] \vdash n_j : A_j)_j}{\Phi; \Gamma [\Sigma] \vdash k \bar{n} : D \bar{R}}$
$\frac{\text{T-COMMAND} \quad \Phi \vdash \bar{R} \quad c : \forall \bar{Z}. \bar{A} \rightarrow B \in \Sigma \quad (\Phi; \Gamma [\Sigma] \vdash n_j : A_j[\bar{R}/\bar{Z}])_j}{\Phi; \Gamma [\Sigma] \vdash c \bar{R} \bar{n} : B[\bar{R}/\bar{Z}]}$	$\frac{\text{T-THUNK} \quad \Phi; \Gamma \vdash e : C}{\Phi; \Gamma [\Sigma] \vdash \{e\} : \{C\}}$
$\frac{\text{T-LET} \quad P = \forall \bar{Z}. A \quad \Phi, \bar{Z}; \Gamma [\emptyset] \vdash n : A \quad \Phi; \Gamma, f : P [\Sigma] \vdash n' : B}{\Phi; \Gamma [\Sigma] \vdash \text{let } f : P = n \text{ in } n' : B}$	
$\frac{\text{T-LETREC} \quad (P_i = \forall \bar{Z}_i. \{C_i\})_i \quad (\Phi, \bar{Z}_i; \Gamma, \bar{f} : \bar{P} \vdash e_i : C)_i \quad \Phi; \Gamma, \bar{f} : \bar{P} [\Sigma] \vdash n : B}{\Phi; \Gamma [\Sigma] \vdash \text{letrec } \bar{f} : \bar{P} = e \text{ in } n : B}$	$\frac{\text{T-ADAPT} \quad \Sigma \vdash \Theta \dashv \Sigma' \quad \Phi; \Gamma [\Sigma'] \vdash n : A}{\Phi; \Gamma [\Sigma] \vdash \langle \Theta \rangle n : A}$
$\Phi; \Gamma \vdash e : C$	
$\frac{\text{T-COMP} \quad (\Phi \vdash r_{i,j} : T_j \dashv [\Sigma] \exists \Psi_{i,j}. \Gamma'_{i,j})_{i,j} \quad (\Phi, (\Psi_{i,j})_j; \Gamma, (\Gamma'_{i,j})_j [\Sigma] \vdash n_i : B)_i \quad ((r_{i,j})_i \text{ covers } T_j)_j}{\Phi; \Gamma \vdash ((r_{i,j})_j \mapsto n_i)_i : (T_j \rightarrow)_j [\Sigma] B}$	

Figure 1.5: Term Typing Rules



$$\boxed{\Phi \vdash p : A \dashv \Gamma}$$

$$\begin{array}{c}
\text{P-VAR} \\
\hline
\Phi \vdash x : A \dashv x : A
\end{array}
\qquad
\begin{array}{c}
\text{P-DATA} \\
\hline
k \bar{A} \in D \bar{R} \quad (\Phi \vdash p_i : A_i \dashv \Gamma)_i \\
\hline
\Phi \vdash k \bar{p} : D \bar{R} \dashv \bar{\Gamma}
\end{array}$$

$$\boxed{\Phi \vdash r : T \dashv [\Sigma] \exists \Psi. \Gamma}$$

$$\begin{array}{c}
\text{P-VALUE} \\
\hline
\Sigma \vdash \Delta \dashv \Sigma' \quad \Phi \vdash p : A \dashv \Gamma \\
\hline
\Phi \vdash p : \langle \Delta \rangle A \dashv [\Sigma] \Gamma
\end{array}
\qquad
\begin{array}{c}
\text{P-CATCHALL} \\
\hline
\Sigma \vdash \Delta \dashv \Sigma' \\
\hline
\Phi \vdash \langle x \rangle : \langle \Delta \rangle A \dashv [\Sigma] x : \{[\Sigma'] A\}
\end{array}$$

$$\begin{array}{c}
\text{P-COMMAND} \\
\hline
\Sigma \vdash \Delta \dashv \Sigma' \quad \Delta = \Theta \mid \Xi \quad c : \forall \bar{Z}. \bar{A} \rightarrow \bar{B} \in \Xi \quad (\Phi, \bar{Z} \vdash p_i : A_i \dashv \Gamma_i)_i \\
\hline
\Phi \vdash \langle c \bar{p} \rightarrow z \rangle : \langle \Delta \rangle B' \dashv [\Sigma] \exists \bar{Z}. \bar{\Gamma}, z : \{ \langle \iota \mid \iota \rangle B \rightarrow [\Sigma'] B' \}
\end{array}$$

Figure 1.6: Pattern Matching Typing Rules

$$\begin{array}{ll}
(\text{uses}) & m ::= \dots \mid \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil \\
(\text{constructions}) & n ::= \dots \mid \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil \\
(\text{use values}) & u ::= x \mid f \bar{R} \mid \uparrow(v : A) \\
(\text{non-use values}) & v ::= k \bar{w} \mid \{e\} \\
(\text{construction values}) & w ::= \downarrow u \mid v \\
(\text{normal forms}) & t ::= w \mid \lceil \mathcal{E}[c \bar{R} \bar{w}] \rceil \\
(\text{evaluation frames}) & \mathcal{F} ::= [] \bar{n} \mid u(\bar{t}, [], \bar{n}) \mid \uparrow([], : A) \\
& \mid \downarrow[] \mid k(\bar{w}, [], \bar{n}) \mid c \bar{R}(\bar{w}, [], \bar{n}) \\
& \mid \mathbf{let} f : P = [] \mathbf{in} n \mid \langle \Theta \rangle [] \\
(\text{evaluation contexts}) & \mathcal{E} ::= [] \mid \mathcal{F}[\mathcal{E}]
\end{array}$$

Figure 1.7: Runtime Syntax

$$\boxed{\Phi; \Gamma [\Sigma] \vdash m \Rightarrow A} \quad \boxed{\Phi; \Gamma [\Sigma] \vdash n : A}$$

$$\begin{array}{c}
\text{T-FREEZE-USE} \\
\frac{\neg(\mathcal{E} \text{ handles } c) \quad \Phi; \Gamma [\Sigma] \vdash \mathcal{E}[c \bar{R} \bar{w}] \Rightarrow A}{\Phi; \Gamma [\Sigma] \vdash [\mathcal{E}[c \bar{R} \bar{w}]] \Rightarrow A}
\end{array}$$

$$\begin{array}{c}
\text{T-FREEZE-CONS} \\
\frac{\neg(\mathcal{E} \text{ handles } c) \quad \Phi; \Gamma [\Sigma] \vdash \mathcal{E}[c \bar{R} \bar{w}] : A}{\Phi; \Gamma [\Sigma] \vdash [\mathcal{E}[c \bar{R} \bar{w}]] : A}
\end{array}$$

Figure 1.8: Frozen Commands

$$\boxed{m \rightsquigarrow_u m'} \quad \boxed{n \rightsquigarrow_c n'} \quad \boxed{m \longrightarrow_u m'} \quad \boxed{n \longrightarrow_c n'}$$

$$\begin{array}{c}
\text{R-HANDLE} \\
\frac{k = \min_i \{i \mid \exists \bar{\theta}. (r_{i,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \neg[\Sigma] \theta_j)_j\} \quad (r_{k,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \neg[\Sigma] \theta_j)_j}{\uparrow(\{((r_{i,j})_j \rightarrow n_i)_i\} : \{\langle \Delta \rangle A \rightarrow [\Sigma] B\}) \bar{t} \rightsquigarrow_u \uparrow((\bar{\theta}(n_k) : B))}
\end{array}$$

$$\begin{array}{ccc}
\text{R-ASCRIBE-USE} & \text{R-ASCRIBE-CONS} & \text{R-LET} \\
\frac{}{\uparrow(\downarrow u : A) \rightsquigarrow_u u} & \frac{}{\downarrow \uparrow(w : A) \rightsquigarrow_c w} & \frac{}{\mathbf{let} f : P = w \mathbf{in} n \rightsquigarrow_c n[\uparrow(w : P)/f]}
\end{array}$$

$$\begin{array}{ccc}
\text{R-LETREC} & & \text{R-ADAPT} \\
\frac{\overline{e = \bar{r} \rightarrow n}}{\mathbf{letrec} \bar{f} : P = \bar{e} \mathbf{in} n' \rightsquigarrow_c n'[\uparrow(\{\bar{r} \rightarrow \mathbf{letrec} \bar{f} : P = \bar{e} \mathbf{in} n\} : P)/f]} & & \frac{}{\langle \Theta \rangle w \rightsquigarrow_c w}
\end{array}$$

$$\begin{array}{c}
\text{R-FREEZE-COMM} \\
\frac{}{c \bar{R} \bar{w} \rightsquigarrow_c [c \bar{R} \bar{w}]}
\end{array}$$

$$\begin{array}{cc}
\text{R-FREEZE-FRAME-USE} & \text{R-FREEZE-FRAME-CONS} \\
\frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[[\mathcal{E}[c \bar{R} \bar{w}]]] \rightsquigarrow_u [\mathcal{F}[\mathcal{E}[c \bar{R} \bar{w}]]]} & \frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[[\mathcal{E}[c \bar{R} \bar{w}]]] \rightsquigarrow_c [\mathcal{F}[\mathcal{E}[c \bar{R} \bar{w}]]]}
\end{array}$$

$$\begin{array}{cccc}
\text{R-LIFT-UU} & \text{R-LIFT-UC} & \text{R-LIFT-CU} & \text{R-LIFT-CC} \\
\frac{m \rightsquigarrow_u m'}{\mathcal{E}[m] \longrightarrow_u \mathcal{E}[m']} & \frac{m \rightsquigarrow_u m'}{\mathcal{E}[m] \longrightarrow_c \mathcal{E}[m']} & \frac{n \rightsquigarrow_c n'}{\mathcal{E}[n] \longrightarrow_u \mathcal{E}[n']} & \frac{n \rightsquigarrow_c n'}{\mathcal{E}[n] \longrightarrow_c \mathcal{E}[n']}
\end{array}$$

Figure 1.9: Operational Semantics

$$\boxed{r : T \leftarrow t \neg[\Sigma] \theta}$$

$$\begin{array}{c} \text{B-VALUE} \\ \Sigma \vdash \Delta \dashv \Sigma' \\ p : A \leftarrow w \dashv \theta \\ \hline p : \langle \Delta \rangle A \leftarrow w \neg[\Sigma] \theta \end{array}$$

B-REQUEST

$$\begin{array}{c} \Sigma \vdash \Delta \dashv \Sigma' \quad \mathcal{E} \text{ poised for } c \\ \Delta = \Theta \mid \Xi \quad c : \forall \bar{Z}. \bar{B} \rightarrow B' \in \Xi \quad (p_i : B_i \leftarrow w_i \dashv \theta_i)_i \\ \hline \langle c \bar{p} \rightarrow z \rangle : \langle \Delta \rangle A \leftarrow [\mathcal{E}[c \bar{R} \bar{w}]] \neg[\Sigma] \bar{\theta}[\uparrow(\{x \mapsto \mathcal{E}[x]\} : \{B' \rightarrow [\Sigma']A\})/z] \end{array}$$

B-CATCHALL-VALUE

$$\begin{array}{c} \Sigma \vdash \Delta \dashv \Sigma' \\ \hline \langle x \rangle : \langle \Delta \rangle A \leftarrow w \neg[\Sigma] [\uparrow(\{w\} : \{[\Sigma']A\})/x] \end{array}$$

B-CATCHALL-REQUEST

$$\begin{array}{c} \Sigma \vdash \Delta \dashv \Sigma' \quad \mathcal{E} \text{ poised for } c \\ \Delta = \Theta \mid \Xi \quad c : \forall \bar{Z}. \bar{B} \rightarrow B' \in \Xi \\ \hline \langle x \rangle : \langle \Delta \rangle A \leftarrow [\mathcal{E}[c \bar{R} \bar{w}]] \neg[\Sigma] [\uparrow(\{[\mathcal{E}[c \bar{R} \bar{w}]]\} : \{[\Sigma']A\})/x] \end{array}$$

$$\boxed{p : A \leftarrow w \dashv \theta}$$

B-VAR

$$\frac{}{x : A \leftarrow w \dashv [\uparrow(w : A)/x]}$$

B-DATA

$$\frac{k \bar{A} \in D \bar{R} \quad (p_i : A_i \leftarrow w_i \dashv \theta_i)_i}{k \bar{p} : D \bar{R} \leftarrow k \bar{w} \dashv \bar{\theta}}$$

Figure 1.10: Pattern Binding

# **Appendix A**

## **First appendix**