This is the Project Title

Your Name

Master of Science

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Abstract

Formal development of Frank.

Chapter 1

Formalisation of Frank

(data types)	D
(value type variables)	X
(effect type variables)	E
(value types)	$A, B ::= D\ \overline{R}$
	$\quad \mid \quad \{C\} \mid X$
(computation types)	$C ::= \overline{T \to}\ G$
(argument types)	$T ::= \langle \Delta \rangle A$
(return types)	$G ::= [\Sigma] A$
(type binders)	$Z ::= X \mid [E]$
(type arguments)	$R ::= A \mid [\Sigma]$
(polytypes)	$P ::= \forall \overline{Z}.A$
(interfaces)	I
(term variables)	x, y, z, f
(instance variables)	s, a, b, c
(seeds)	$\sigma ::= \emptyset \mid E$
(abilities)	$\Sigma ::= \sigma \mid \Xi$
(extensions)	$\Xi ::= \iota \mid \Xi, I\ \overline{R}$
(adaptors)	$\Theta ::= \iota \mid \Theta, I(S \to S')$
(adjustments)	$\Delta ::= \Theta \mid \Xi$
(instance patterns)	$S ::= s \mid S\ a$
(kind environments)	$\Phi, \Psi ::= \cdot \mid \Phi, Z$
(type environments)	$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, f : P$
(instance environments)	$\Omega ::= s : \Sigma \mid \Omega, a : I\ \overline{R}$

Figure 1.1: Types

$$
\begin{array}{lll}
\text{(constructors)} & & k \\
\text{(commands)} & & c \\
\text{(uses)} & m ::= & x \mid f\ \overline{R} \mid m\ \overline{n} \mid \uparrow(n : A) \\
\text{(constructions)} & n ::= & \downarrow m \mid k\ \overline{n} \mid c\ \overline{R}\ \overline{n} \mid \{e\} \\
& & \quad \mid \ \textbf{let}\ f : P = n\ \textbf{in}\ n' \mid \textbf{letrec}\ \overline{f : P = e}\ \textbf{in}\ n \\
& & \quad \mid \ \langle \Theta \rangle\ n \\
\text{(computations)} & e ::= & \overline{\overline{r} \mapsto n} \\
\text{(computation patterns)} & r ::= & p \mid \langle c\ \overline{p} \to z \rangle \mid \langle x \rangle \\
\text{(value patterns)} & p ::= & k\ \overline{p} \mid x
\end{array}
$$

Figure 1.2: Terms

$\boxed{\Phi;\Gamma\,[\Sigma]\vdash m \Rightarrow A}$

T-VAR
$$\frac{x : A \in \Gamma}{\Phi;\Gamma\,[\Sigma]\vdash x \Rightarrow A}$$

T-POLYVAR
$$\frac{\Phi \vdash \overline{R} \qquad f : \forall \overline{Z}.A \in \Gamma}{\Phi;\Gamma\,[\Sigma]\vdash f\ \overline{R} \Rightarrow A[\overline{R}/\overline{Z}]}$$

T-APP
$$\frac{\Sigma' = \Sigma \qquad (\Sigma \vdash \Delta_i \dashv \Sigma'_i)_i \qquad \Phi;\Gamma\,[\Sigma]\vdash m \Rightarrow \{\overline{\langle\Delta\rangle A \to}\ [\Sigma']B\} \qquad (\Phi;\Gamma\,[\Sigma'_i]\vdash n_i : A_i)_i}{\Phi;\Gamma\,[\Sigma]\vdash m\ \overline{n} \Rightarrow B}$$

T-ASCRIBE
$$\frac{\Phi;\Gamma\,[\Sigma]\vdash n : A}{\Phi;\Gamma\,[\Sigma]\vdash \uparrow(n : A) \Rightarrow A}$$

$\boxed{\Phi;\Gamma\,[\Sigma]\vdash n : A}$

T-SWITCH
$$\frac{\Phi;\Gamma\,[\Sigma]\vdash m \Rightarrow A \qquad A = B}{\Phi;\Gamma\,[\Sigma]\vdash \downarrow m : B}$$

T-DATA
$$\frac{k\ \overline{A} \in D\ \overline{R} \qquad (\Phi;\Gamma\,[\Sigma]\vdash n_j : A_j)_j}{\Phi;\Gamma\,[\Sigma]\vdash k\ \overline{n} : D\ \overline{R}}$$

T-COMMAND
$$\frac{\Phi \vdash \overline{R} \qquad c : \forall \overline{Z}.\overline{A \to}\ B \in \Sigma \qquad (\Phi;\Gamma\,[\Sigma]\vdash n_j : A_j[\overline{R}/\overline{Z}])_j}{\Phi;\Gamma\,[\Sigma]\vdash c\ \overline{R}\ \overline{n} : B[\overline{R}/\overline{Z}]}$$

T-THUNK
$$\frac{\Phi;\Gamma \vdash e : C}{\Phi;\Gamma\,[\Sigma]\vdash \{e\} : \{C\}}$$

T-LET
$$\frac{P = \forall \overline{Z}.A \qquad \Phi,\overline{Z};\Gamma\,[\emptyset]\vdash n : A \qquad \Phi;\Gamma, f : P\,[\Sigma]\vdash n' : B}{\Phi;\Gamma\,[\Sigma]\vdash \textbf{let}\ f : P = n\ \textbf{in}\ n' : B}$$

T-LETREC
$$\frac{(P_i = \forall \overline{Z}_i.\{C_i\})_i \qquad (\Phi,\overline{Z}_i;\Gamma,\overline{f : P} \vdash e_i : C)_i \qquad \Phi;\Gamma,\overline{f : P}\,[\Sigma]\vdash n : B}{\Phi;\Gamma\,[\Sigma]\vdash \textbf{letrec}\ \overline{f : P = e}\ \textbf{in}\ n : B}$$

T-ADAPT
$$\frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Phi;\Gamma\,[\Sigma']\vdash n : A}{\Phi;\Gamma\,[\Sigma]\vdash \langle\Theta\rangle\ n : A}$$

$\boxed{\Phi;\Gamma \vdash e : C}$

T-COMP
$$\frac{(\Phi \vdash r_{i,j} : T_j \dashv [\Sigma]\ \exists \Psi_{i,j}.\Gamma'_{i,j})_{i,j} \qquad (\Phi,(\Psi_{i,j})_j;\Gamma,(\Gamma'_{i,j})_j\,[\Sigma]\vdash n_i : B)_i \qquad ((r_{i,j})_i \text{ covers } T_j)_j}{\Phi;\Gamma \vdash ((r_{i,j})_j \mapsto n_i)_i : (T_j \to)_j\ [\Sigma]B}$$

Figure 1.3: Term Typing Rules

$$
\begin{array}{lll}
\text{(uses)} & m & ::= \cdots \mid \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \\
\text{(constructions)} & n & ::= \cdots \mid \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \\
\text{(use values)} & u & ::= x \mid f\ \overline{R} \mid \uparrow(v : A) \\
\text{(non-use values)} & v & ::= k\ \overline{w} \mid \{e\} \\
\text{(construction values)} & w & ::= \downarrow u \mid v \\
\text{(normal forms)} & t & ::= w \mid \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \\
\text{(evaluation frames)} & \mathcal{F} & ::= [\,]\ \overline{n} \mid u\ (\overline{t}, [\,], \overline{n}) \mid \uparrow([\,] : A) \\
 & & \quad \mid \downarrow[\,] \mid k\ (\overline{w}, [\,], \overline{n}) \mid c\ \overline{R}\ (\overline{w}, [\,], \overline{n}) \\
 & & \quad \mid \textbf{let}\ f : P = [\,]\ \textbf{in}\ n \mid \langle \Theta \rangle\ [\,] \\
\text{(evaluation contexts)} & \mathcal{E} & ::= [\,] \mid \mathcal{F}[\mathcal{E}]
\end{array}
$$

Figure 1.4: Runtime Syntax

$\boxed{\Phi; \Gamma\ [\Sigma] \vdash m \Rightarrow A}$ $\boxed{\Phi; \Gamma\ [\Sigma] \vdash n : A}$

$$
\text{T-FREEZE-USE}\quad \frac{\neg(\mathcal{E}\ \mathsf{handles}\ c) \qquad \Phi; \Gamma\ [\Sigma] \vdash \mathcal{E}[c\ \overline{R}\ \overline{w}] \Rightarrow A}{\Phi; \Gamma\ [\Sigma] \vdash \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \Rightarrow A}
$$

$$
\text{T-FREEZE-CONS}\quad \frac{\neg(\mathcal{E}\ \mathsf{handles}\ c) \qquad \Phi; \Gamma\ [\Sigma] \vdash \mathcal{E}[c\ \overline{R}\ \overline{w}] : A}{\Phi; \Gamma\ [\Sigma] \vdash \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil : A}
$$

Figure 1.5: Frozen Commands

$\boxed{m \rightsquigarrow_{\mathrm{u}} m'}$ $\boxed{n \rightsquigarrow_{\mathrm{c}} n'}$ $\boxed{m \longrightarrow_{\mathrm{u}} m'}$ $\boxed{n \longrightarrow_{\mathrm{c}} n'}$

R-HANDLE

$$\frac{k = \min_i \{i \mid \exists \overline{\theta}.(r_{i,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \dashv [\Sigma]\ \theta_j)_j\} \qquad (r_{k,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \dashv [\Sigma]\ \theta_j)_j}{\uparrow(\{((r_{i,j})_j \to n_i)_i\} : \{\overline{\langle \Delta \rangle A \to}\ [\Sigma] B\})\ \overline{t} \rightsquigarrow_{\mathrm{u}} \uparrow((\overline{\theta}(n_k) : B)}$$

R-ASCRIBE-USE

$$\frac{}{\uparrow(\downarrow u : A) \rightsquigarrow_{\mathrm{u}} u}$$

R-ASCRIBE-CONS

$$\frac{}{\downarrow\uparrow(w : A) \rightsquigarrow_{\mathrm{c}} w}$$

R-LET

$$\frac{}{\mathbf{let}\ f : P = w\ \mathbf{in}\ n \rightsquigarrow_{\mathrm{c}} n[\uparrow(w : P)/f]}$$

R-LETREC

$$\frac{\overline{e = \overline{\overline{r} \to n}}}{\mathbf{letrec}\ \overline{f : P = e}\ \mathbf{in}\ n' \rightsquigarrow_{\mathrm{c}} n'[\uparrow(\{\overline{\overline{r} \to \mathbf{letrec}\ \overline{f : P = e}\ \mathbf{in}\ n}\} : P)/f]}$$

R-ADAPT

$$\frac{}{\langle \Theta \rangle\ w \rightsquigarrow_{\mathrm{c}} w}$$

R-FREEZE-COMM

$$\frac{}{c\ \overline{R}\ \overline{w} \rightsquigarrow_{\mathrm{c}} \lceil c\ \overline{R}\ \overline{w} \rceil}$$

R-FREEZE-FRAME-USE

$$\frac{\neg(\mathcal{F}\,[\mathcal{E}]\ \mathsf{handles}\ c)}{\mathcal{F}\,[\lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil] \rightsquigarrow_{\mathrm{u}} \lceil \mathcal{F}\,[\mathcal{E}[c\ \overline{R}\ \overline{w}]] \rceil}$$

R-FREEZE-FRAME-CONS

$$\frac{\neg(\mathcal{F}\,[\mathcal{E}]\ \mathsf{handles}\ c)}{\mathcal{F}\,[\lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil] \rightsquigarrow_{\mathrm{c}} \lceil \mathcal{F}\,[\mathcal{E}[c\ \overline{R}\ \overline{w}]] \rceil}$$

R-LIFT-UU

$$\frac{m \rightsquigarrow_{\mathrm{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathrm{u}} \mathcal{E}[m']}$$

R-LIFT-UC

$$\frac{m \rightsquigarrow_{\mathrm{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathrm{c}} \mathcal{E}[m']}$$

R-LIFT-CU

$$\frac{n \rightsquigarrow_{\mathrm{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathrm{u}} \mathcal{E}[n']}$$

R-LIFT-CC

$$\frac{n \rightsquigarrow_{\mathrm{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathrm{c}} \mathcal{E}[n']}$$

Figure 1.6: Operational Semantics

$\boxed{r : T \leftarrow t \dashv[\Sigma]\ \theta}$

$$\textsc{B-Value}\ \frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad p : A \leftarrow w \dashv \theta}{p : \langle \Delta \rangle A \leftarrow w \dashv[\Sigma]\ \theta}$$

$$\textsc{B-Request}\ \frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \mathcal{E}\ \mathsf{poisedfor}\ c \qquad \Delta = \Theta \mid \Xi \qquad c : \forall \overline{Z}.\overline{B \to} B' \in \Xi \qquad (p_i : B_i \leftarrow w_i \dashv \theta_i)_i}{\langle c\ \overline{p} \to z \rangle : \langle \Delta \rangle A \leftarrow \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \dashv[\Sigma]\ \overline{\theta}[\uparrow(\{x \mapsto \mathcal{E}[x]\} : \{B' \to [\Sigma']A\})/z]}$$

$$\textsc{B-CatchAll-Value}\ \frac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x \rangle : \langle \Delta \rangle A \leftarrow w \dashv[\Sigma]\ [\uparrow(\{w\} : \{[\Sigma']A\})/x]}$$

$$\textsc{B-CatchAll-Request}\ \frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \mathcal{E}\ \mathsf{poisedfor}\ c \qquad \Delta = \Theta \mid \Xi \qquad c : \forall \overline{Z}.\overline{B \to} B' \in \Xi}{\langle x \rangle : \langle \Delta \rangle A \leftarrow \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \dashv[\Sigma]\ [\uparrow(\{\lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil\} : \{[\Sigma']A\})/x]}$$

$\boxed{p : A \leftarrow w \dashv \theta}$

$$\textsc{B-Var}\ \frac{}{x : A \leftarrow w \dashv [\uparrow(w : A)/x]} \qquad \textsc{B-Data}\ \frac{k\ \overline{A} \in D\ \overline{R} \qquad (p_i : A_i \leftarrow w_i \dashv \theta_i)_i}{k\ \overline{p} : D\ \overline{R} \leftarrow k\ \overline{w} \dashv \overline{\theta}}$$

Figure 1.7: Pattern Binding

Chapter 2

Arbitrary Thread Interruption

2.1 Motivation

One important part of our asynchronous effect handling system is the ability to interrupt arbitrary computations. This is essential for pre-emptive concurrency, which relies on being able to suspend computations for resumption later.

For instance, consider the two programs below;

```
controller : {[Stop, Go, Console] Unit}
controller! = go!; stop!; print ``stop 1'';
              go!; stop!; print ``stop 2'';
              go!; stop!; print ``stop 3''

runner : {[Console] Unit}
runner! = print ``1 ''; print ``2 ''; print ``3 '';
```

We ideally want a multihandler that can run these two programs in parallel, such that the result will be `1 stop 1 2 stop 2 3 stop 3`; that is to say, the `stop` and `go` operations from

2.2 Interruption with Yields

One way we can get this behaviour is using the `Yield` interface. This offers a single operation, `yield : Unit`. With this, we can write a multihandler `suspend`;

```
runner : {[Console, Yield] Unit}
runner! = print "1 "; yield!; print "2 "; yield!; print "3 ";
   yield!
```

```
suspend : {<Yield> Unit -> <Stop, Go> Unit -> Maybe {[Console,
    Yield] Unit} -> [Console] Unit}
suspend <yield -> k> <stop -> l> _ = suspend unit (l unit) (
   just {k unit})
suspend <k>          <go -> l> (just res) = suspend (res!) (l
   unit) nothing
suspend <yield -> k> <m> maybe = suspend (k unit) m! maybe
suspend unit _ _ = unit
suspend _ unit _ = unit
```

TODO: Maybe make more concise?

TODO: change to not be letter l cos it looks like a 1

Running `suspend runner! controller! nothing` then prints out `1 stop 1 2 stop 2 3 stop 3` as desired. So far so good; this works as planned. However, observe that we had to change the code of the original runner program to `yield` every time it prints. We would rather not have this requirement; the threads should be suspendable without knowing in advance they will be suspended, and thus without needing to explicitly `yield`. Furthermore, see that the `yield` operation adds no more information; it is just used as a placeholder operation; any operation would work. As such, we keep searching for a better solution.

2.3 Relaxing Catches

The key to this lies in the catchall pattern, $\langle x \rangle$, and the pattern binding rules of Figure 1.7.

Recall that the catchall pattern $\langle x \rangle$ matches either a value — e.g. `unit` — or a command that is handled in the surrounding ability. For instance, the pattern `<k>` in the code above matches either `unit` or `<yield -> k>`. The variable `k` is then bound to whatever this match is, leaving the (potentially) invoked effect unhandled. This is expressed in the B-CATCHALL-REQUEST rule in Figure 1.7.

Important to note is that only effects that are handled in that position are able to be caught. `runner` also makes use of the `print` effect, but these are not able to be caught by the catchall command. Formally, this is due to the fourth requirement of B-CATCHALL-REQUEST; that the command c that is invoked is a member of Ξ.

As such, we propose to remove this constraint from B-CATCHALL-REQUEST. The

$$\textsc{B-CatchAll-Request-Loose}\quad \frac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x \rangle : \langle \Delta \rangle A \leftarrow \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \dashv [\Sigma]\ [\uparrow(\{\lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil\} : \{[\Sigma']A\})/x]}$$

Figure 2.1: Updated B-CatchAll-Request

resulting rule can then be seen in Figure 2.1. This lets us update the previous `suspend` code to the following, which yields the same results as last time;

```
runner : {[Console] Unit}
runner! = print "1 "; print "2 "; print "3 "

suspend : {Unit -> <Stop, Go> Unit -> Maybe {[Console] Unit}
   -> [Console] Unit}
suspend <k> <stop -> l> _           = suspend unit   (l unit) (
   just {k unit})
suspend <k> <go -> l>   (just res) = suspend (res!) (l unit)
   nothing
suspend unit _ _ = unit
suspend _ unit _ = unit
```

TODO: Verify that this particular example really works.

Why does this work?

TODO: explain

Appendix A

Remaining Formalisms

$\boxed{\Sigma \vdash \Delta \dashv \Sigma'}$

$$\textsc{A-Adj}\ \frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Sigma' \vdash \Xi \dashv \Sigma''}{\Sigma \vdash \Theta \mid \Xi \dashv \Sigma''}$$

$\boxed{\Sigma \vdash \Xi \dashv \Sigma'}$

$$\textsc{A-Ext-Id}\ \frac{}{\Sigma \vdash \iota \dashv \Sigma} \qquad \textsc{A-Ext-Snoc}\ \frac{\Sigma \vdash \Xi \dashv \Sigma'}{\Sigma \vdash \Xi, I\ \overline{R} \dashv \Sigma', I\ \overline{R}}$$

$\boxed{\Sigma \vdash \Theta \dashv \Sigma'}$

$$\textsc{A-Adapt-Id}\ \frac{}{\Sigma \vdash \iota \dashv \Sigma} \qquad \textsc{A-Adapt-Snoc}\ \frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Sigma' \vdash I(S \to S') \dashv \Sigma''}{\Sigma \vdash \Theta, I(S \to S') \dashv \Sigma''}$$

$\boxed{\Sigma \vdash I(S \to S') \dashv \Sigma'}$

$$\textsc{A-Adapt-Com}\ \frac{\Sigma \vdash S : I \dashv \Sigma'; \Omega \qquad \Omega \vdash S' : I \dashv \Xi \qquad \Sigma' \vdash \Xi \dashv \Sigma''}{\Sigma \vdash I(S \to S') \dashv \Sigma''}$$

$\boxed{\Sigma \vdash S : I \dashv \Sigma'; \Omega}$

$$\textsc{I-Pat-Id}\ \frac{}{\Sigma \vdash s : I \dashv \Sigma; s : \Sigma} \qquad \textsc{I-Pat-Bind}\ \frac{\Sigma \vdash S : I \dashv \Sigma'; \Omega}{\Sigma, I\ \overline{R} \vdash S\ a : I \dashv \Sigma'; \Omega, a : I\ \overline{R}} \qquad \textsc{I-Pat-Skip}\ \frac{\Sigma \vdash S\ a : I \dashv \Sigma'; \Omega \qquad I \neq I'}{\Sigma, I'\ \overline{R} \vdash S\ a : I \dashv \Sigma', I'\ \overline{R}; \Omega}$$

$\boxed{\Omega \vdash S : I \dashv \Xi}$

$$\textsc{I-Inst-Id}\ \frac{s \in \mathsf{dom}(\Omega)}{\Omega \vdash s : I \dashv \iota} \qquad \textsc{I-Inst-Lkp}\ \frac{a \in \mathsf{dom}(\Omega) \qquad \Omega \vdash S : I \dashv \Xi \qquad \Omega(a) = I\ \overline{R}}{\Omega \vdash S\ a : I \dashv \Xi, I\ \overline{R}}$$

Figure A.1: Action of an Adjustment on an Ability and Auxiliary Judgements

$$\mathcal{X} ::= A \mid C \mid T \mid G \mid Z \mid R \mid P \mid \sigma \mid \Sigma \mid \Xi \mid \Theta \mid \Delta \mid \Gamma \mid \exists \Psi.\Gamma \mid \Omega$$

$\boxed{\Phi \vdash \mathcal{X}}$

$$\textsc{WF-Val}\ \frac{}{\Phi, X \vdash X} \qquad \textsc{WF-Eff}\ \frac{}{\Phi, [E] \vdash E} \qquad \textsc{WF-Poly}\ \frac{\Phi, \overline{Z} \vdash A}{\Phi \vdash \forall \overline{Z}.A}$$

$$\textsc{WF-Data}\ \frac{(\Phi \vdash R)_i}{\Phi \vdash D\ \overline{R}} \qquad \textsc{WF-Thunk}\ \frac{\Phi \vdash C}{\Phi \vdash \{C\}} \qquad \textsc{WF-Comp}\ \frac{(\Phi \vdash T)_i \qquad \Phi \vdash G}{\Phi \vdash \overline{T \to} G} \qquad \textsc{WF-Arg}\ \frac{\Phi \vdash \Delta \qquad \Phi \vdash A}{\Phi \vdash \langle \Delta \rangle A}$$

$$\textsc{WF-Ret}\ \frac{\Phi \vdash \Sigma \qquad \Phi \vdash A}{\Phi \vdash [\Sigma] A} \qquad \textsc{WF-Ability}\ \frac{\Phi \vdash \Sigma}{\Phi \vdash [\Sigma]} \qquad \textsc{WF-Pure}\ \frac{}{\Phi \vdash \emptyset} \qquad \textsc{WF-Id}\ \frac{}{\Phi \vdash \iota} \qquad \textsc{WF-Ext}\ \frac{\Phi \vdash \Xi \qquad (\Phi \vdash R)_i}{\Phi \vdash \Xi, I\ \overline{R}}$$

$$\textsc{WF-Adapt}\ \frac{\Phi \vdash \Theta}{\Phi \vdash \Theta, I\ (S \to S')}$$

$$\textsc{WF-Empty}\ \frac{}{\Phi \vdash \cdot} \qquad \textsc{WF-Mono}\ \frac{\Phi \vdash \Gamma \qquad \Phi \vdash A}{\Phi \vdash \Gamma, x : A} \qquad \textsc{WF-Poly}\ \frac{\Phi \vdash \Gamma \qquad \Phi \vdash P}{\Phi \vdash \Gamma, f : P}$$

$$\textsc{WF-Existential}\ \frac{\Phi, \Psi \vdash \Gamma}{\Phi \vdash \exists \Psi.\Gamma} \qquad \textsc{WF-Interface}\ \frac{\Phi \vdash \Omega \qquad (\Phi \vdash R)_i}{\Phi \vdash \Omega, x : I\ \overline{R}}$$

Figure A.2: Well-Formedness Rules

$\boxed{\Phi \vdash p : A \dashv \Gamma}$

$$\textsc{P-Var}\ \frac{}{\Phi \vdash x : A \dashv x : A} \qquad \textsc{P-Data}\ \frac{k\ \overline{A} \in D\ \overline{R} \qquad (\Phi \vdash p_i : A_i \dashv \Gamma)_i}{\Phi \vdash k\ \overline{p} : D\ \overline{R} \dashv \overline{\Gamma}}$$

$\boxed{\Phi \vdash r : T \dashv[\Sigma]\ \exists \Psi.\Gamma}$

$$\textsc{P-Value}\ \frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \Phi \vdash p : A \dashv \Gamma}{\Phi \vdash p : \langle \Delta \rangle A \dashv[\Sigma]\ \Gamma} \qquad \textsc{P-CatchAll}\ \frac{\Sigma \vdash \Delta \dashv \Sigma'}{\Phi \vdash \langle x \rangle : \langle \Delta \rangle A \dashv[\Sigma]\ x : \{[\Sigma']A\}}$$

$$\textsc{P-Command}\ \frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \Delta = \Theta \mid \Xi \qquad c : \forall \overline{Z}.\overline{A \to B} \in \Xi \qquad (\Phi, \overline{Z} \vdash p_i : A_i \dashv \Gamma_i)_i}{\Phi \vdash \langle c\ \overline{p} \to z \rangle : \langle \Delta \rangle B' \dashv[\Sigma]\ \exists \overline{Z}.\overline{\Gamma}, z : \{\langle \iota \mid \iota \rangle B \to [\Sigma']B'\}}$$

Figure A.3: Pattern Matching Typing Rules