This is the Project Title

Your Name

Master of Science
Computer Science
School of Informatics
University of Edinburgh
2020

Abstract

Formal development of Frank.

Chapter 1

Formalisation of Frank

(data types)	D
(value type variables)	X
(effect type variables)	E
(value types)	$A, B ::= D\,\overline{R}$
	$\mid \{C\} \mid X$
(computation types)	$C ::= \overline{T} \to G$
(argument types)	$T ::= \langle \Delta \rangle A$
(return types)	$G ::= [\Sigma]A$
(type binders)	$Z ::= X \mid [E]$
(type arguments)	$R ::= A \mid [\Sigma]$
(polytypes)	$P ::= \forall \overline{Z}.A$

(interfaces)	I
(term variables)	x, y, z, f
(instance variables)	s, a, b, c
(seeds)	$\sigma ::= \emptyset \mid E$
(abilities)	$\Sigma ::= \sigma \mid \Xi$
(extensions)	$\Xi ::= \iota \mid \Xi, I\,\overline{R}$
(adaptors)	$\Theta ::= \iota \mid \Theta, I(S \to S')$
(adjustments)	$\Delta ::= \Theta \mid \Xi$
(instance patterns)	$S ::= s \mid S\,a$
(kind environments)	$\Phi, \Psi ::= \cdot \mid \Phi, Z$
(type environments)	$\Gamma ::= \cdot \mid \Gamma, x : A \mid \Gamma, f : P$
(instance environments)	$\Omega ::= s : \Sigma \mid \Omega, a : I\,\overline{R}$

Figure 1.1: Types

(constructors)	k
(commands)	c
(uses)	$m ::= x \mid f\,\overline{R} \mid m\,\overline{n} \mid \uparrow(n : A)$
(constructions)	$n ::= \downarrow m \mid k\,\overline{n} \mid c\,\overline{R}\,\overline{n} \mid \{e\}$
	$\mid \ \textbf{let}\ f : P = n\ \textbf{in}\ n' \mid \textbf{letrec}\ \overline{f : P = e}\ \textbf{in}\ n$
	$\mid \ \langle \Theta \rangle\,n$
(computations)	$e ::= \overline{\overline{r} \mapsto n}$
(computation patterns)	$r ::= p \mid \langle c\,\overline{p} \to z \rangle \mid \langle x \rangle$
(value patterns)	$p ::= k\,\overline{p} \mid x$

Figure 1.2: Terms

$$\boxed{\Phi;\Gamma\,[\Sigma]\!\vdash m \Rightarrow A}$$

T-VAR
$$\frac{x:A \in \Gamma}{\Phi;\Gamma\,[\Sigma]\!\vdash x \Rightarrow A}$$

T-POLYVAR
$$\frac{\Phi \vdash \overline{R} \qquad f:\forall \overline{Z}.A \in \Gamma}{\Phi;\Gamma\,[\Sigma]\!\vdash f\,\overline{R} \Rightarrow A[\overline{R}/\overline{Z}]}$$

T-APP
$$\frac{\Sigma' = \Sigma \qquad (\Sigma \vdash \Delta_i \dashv \Sigma'_i)_i \qquad}{\Phi;\Gamma\,[\Sigma]\!\vdash m \Rightarrow \{\overline{\langle \Delta \rangle A} \to [\Sigma']B\} \qquad (\Phi;\Gamma\,[\Sigma_i]\!\vdash n_i:A_i)_i}{\Phi;\Gamma\,[\Sigma]\!\vdash m\,\overline{n} \Rightarrow B}$$

T-ASCRIBE
$$\frac{\Phi;\Gamma\,[\Sigma]\!\vdash n:A}{\Phi;\Gamma\,[\Sigma]\!\vdash \uparrow(n:A) \Rightarrow A}$$

$$\boxed{\Phi;\Gamma\,[\Sigma]\!\vdash n:A}$$

T-SWITCH
$$\frac{\Phi;\Gamma\,[\Sigma]\!\vdash m \Rightarrow A \qquad A = B}{\Phi;\Gamma\,[\Sigma]\!\vdash \downarrow m:B}$$

T-DATA
$$\frac{k\,\overline{A} \in D\,\overline{R} \qquad (\Phi;\Gamma\,[\Sigma]\!\vdash n_j:A_j)_j}{\Phi;\Gamma\,[\Sigma]\!\vdash k\,\overline{n}:D\,\overline{R}}$$

T-COMMAND
$$\frac{\Phi \vdash \overline{R} \qquad c:\forall \overline{Z}.\overline{A} \to B \in \Sigma \qquad (\Phi;\Gamma\,[\Sigma]\!\vdash n_j:A_j[\overline{R}/\overline{Z}])_j}{\Phi;\Gamma\,[\Sigma]\!\vdash c\,\overline{R}\,\overline{n}:B[\overline{R}/\overline{Z}]}$$

T-THUNK
$$\frac{\Phi;\Gamma \vdash e:C}{\Phi;\Gamma\,[\Sigma]\!\vdash \{e\}:\{C\}}$$

T-LET
$$\frac{P = \forall \overline{Z}.A}{\dfrac{\Phi,\overline{Z};\Gamma\,[\emptyset]\!\vdash n:A \qquad \Phi;\Gamma,f:P\,[\Sigma]\!\vdash n':B}{\Phi;\Gamma\,[\Sigma]\!\vdash \mathbf{let}\ f:P = n\ \mathbf{in}\ n':B}}$$

T-LETREC
$$\frac{(P_i = \forall \overline{Z}_i.\{C_i\})_i}{\dfrac{(\Phi,\overline{Z}_i;\Gamma,\overline{f:P} \vdash e_i:C)_i \qquad \Phi;\Gamma,\overline{f:P}\,[\Sigma]\!\vdash n:B}{\Phi;\Gamma\,[\Sigma]\!\vdash \mathbf{letrec}\ \overline{f:P} = e\ \mathbf{in}\ n:B}}$$

T-ADAPT
$$\frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Phi;\Gamma\,[\Sigma']\!\vdash n:A}{\Phi;\Gamma\,[\Sigma]\!\vdash \langle \Theta \rangle\,n:A}$$

$$\boxed{\Phi;\Gamma \vdash e:C}$$

T-COMP
$$\frac{(\Phi \vdash r_{i,j}:T_j \dashv [\Sigma]\,\exists \Psi_{i,j}.\Gamma'_{i,j})_{i,j}}{\dfrac{(\Phi,(\Psi_{i,j})_j;\Gamma,(\Gamma'_{i,j})_j\,[\Sigma]\!\vdash n_i:B)_i \qquad ((r_{i,j})_i\ \text{covers}\ T_j)_j}{\Phi;\Gamma \vdash ((r_{i,j})_j \mapsto n_i)_i:(T_j \to)_j\,[\Sigma]B}}$$

Figure 1.3: Term Typing Rules

$$
\begin{aligned}
\text{(uses)} \quad & m ::= \cdots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \\
\text{(constructions)} \quad & n ::= \cdots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \\
\text{(use values)} \quad & u ::= x \mid f\,\overline{R} \mid {\uparrow}(v:A) \\
\text{(non-use values)} \quad & v ::= k\,\overline{w} \mid \{e\} \\
\text{(construction values)} \quad & w ::= {\downarrow}u \mid v \\
\text{(normal forms)} \quad & t ::= w \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \\
\text{(evaluation frames)} \quad & \mathcal{F} ::= [\,]\,\overline{n} \mid u\,(\overline{t},[\,],\overline{n}) \mid {\uparrow}([\,]:A) \\
& \quad\quad \mid\ {\downarrow}[\,] \mid k\,(\overline{w},[\,],\overline{n}) \mid c\,\overline{R}\,(\overline{w},[\,],\overline{n}) \\
& \quad\quad \mid\ \mathbf{let}\ f:P=[\,]\ \mathbf{in}\ n \mid \langle\Theta\rangle\,[\,] \\
\text{(evaluation contexts)} \quad & \mathcal{E} ::= [\,] \mid \mathcal{F}[\mathcal{E}]
\end{aligned}
$$

Figure 1.4: Runtime Syntax

$$\boxed{\Phi;\Gamma\,[\Sigma] \vdash m \Rightarrow A} \quad \boxed{\Phi;\Gamma\,[\Sigma] \vdash n:A}$$

T-Freeze-Use
$$\frac{\neg(\mathcal{E}\ \mathsf{handles}\ c) \qquad \Phi;\Gamma\,[\Sigma] \vdash \mathcal{E}[c\,\overline{R}\,\overline{w}] \Rightarrow A}{\Phi;\Gamma\,[\Sigma] \vdash \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \Rightarrow A}$$

T-Freeze-Cons
$$\frac{\neg(\mathcal{E}\ \mathsf{handles}\ c) \qquad \Phi;\Gamma\,[\Sigma] \vdash \mathcal{E}[c\,\overline{R}\,\overline{w}]:A}{\Phi;\Gamma\,[\Sigma] \vdash \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil :A}$$

Figure 1.5: Frozen Commands

$$\boxed{m \rightsquigarrow_{\mathrm{u}} m'} \quad \boxed{n \rightsquigarrow_{\mathrm{c}} n'} \quad \boxed{m \longrightarrow_{\mathrm{u}} m'} \quad \boxed{n \longrightarrow_{\mathrm{c}} n'}$$

R-HANDLE
$$\frac{k = \min_{i}\{i \mid \exists \overline{\theta}.(r_{i,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \dashv [\Sigma]\, \theta_j)_j\} \qquad (r_{k,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \dashv [\Sigma]\, \theta_j)_j}{\uparrow(\{((r_{i,j})_j \rightarrow n_i)_i\} : \{\overline{\langle \Delta \rangle A} \rightarrow [\Sigma]B\})\, \overline{t} \rightsquigarrow_{\mathrm{u}} \uparrow((\overline{\theta}(n_k) : B)}$$

R-ASCRIBE-USE $\qquad$ R-ASCRIBE-CONS $\qquad$ R-LET

$$\frac{}{\uparrow(\downarrow u : A) \rightsquigarrow_{\mathrm{u}} u} \qquad \frac{}{\downarrow\uparrow(w : A) \rightsquigarrow_{\mathrm{c}} w} \qquad \frac{}{\mathbf{let}\ f : P = w\ \mathbf{in}\ n \rightsquigarrow_{\mathrm{c}} n[\uparrow(w : P)/f]}$$

R-LETREC
$$\qquad\qquad\qquad \frac{e = \overline{\overline{r} \rightarrow n}}{} \qquad\qquad\qquad\qquad\qquad \text{R-ADAPT}$$
$$\frac{}{\mathbf{letrec}\ \overline{f : P = e}\ \mathbf{in}\ n' \rightsquigarrow_{\mathrm{c}} n'[\uparrow(\{\overline{\overline{r} \rightarrow \mathbf{letrec}\ \overline{f : P = e}\ \mathbf{in}\ n}\} : P)/f]} \qquad \frac{}{\langle \Theta \rangle\ w \rightsquigarrow_{\mathrm{c}} w}$$

R-FREEZE-COMM
$$\frac{}{c\ \overline{R}\ \overline{w} \rightsquigarrow_{\mathrm{c}} \lceil c\ \overline{R}\ \overline{w} \rceil}$$

R-FREEZE-FRAME-USE $\qquad\qquad\qquad$ R-FREEZE-FRAME-CONS
$$\frac{\neg(\mathcal{F}[\mathcal{E}]\ \text{handles}\ c)}{\mathcal{F}[\lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil] \rightsquigarrow_{\mathrm{u}} \lceil \mathcal{F}[\mathcal{E}[c\ \overline{R}\ \overline{w}]] \rceil} \qquad \frac{\neg(\mathcal{F}[\mathcal{E}]\ \text{handles}\ c)}{\mathcal{F}[\lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil] \rightsquigarrow_{\mathrm{c}} \lceil \mathcal{F}[\mathcal{E}[c\ \overline{R}\ \overline{w}]] \rceil}$$

R-LIFT-UU $\qquad$ R-LIFT-UC $\qquad$ R-LIFT-CU $\qquad$ R-LIFT-CC
$$\frac{m \rightsquigarrow_{\mathrm{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathrm{u}} \mathcal{E}[m']} \quad \frac{m \rightsquigarrow_{\mathrm{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathrm{c}} \mathcal{E}[m']} \quad \frac{n \rightsquigarrow_{\mathrm{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathrm{u}} \mathcal{E}[n']} \quad \frac{n \rightsquigarrow_{\mathrm{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathrm{c}} \mathcal{E}[n']}$$

Figure 1.6: Operational Semantics

$$\boxed{r : T \leftarrow t \dashv [\Sigma]\ \theta}$$

B-VALUE
$$\Sigma \vdash \Delta \dashv \Sigma'$$
$$\dfrac{p : A \leftarrow w \dashv \theta}{p : \langle \Delta \rangle A \leftarrow w \dashv [\Sigma]\ \theta}$$

B-REQUEST
$$\dfrac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \mathcal{E} \text{ poisedfor } c \qquad \Delta = \Theta \mid \Xi \qquad c : \forall \overline{Z}.\overline{B} \rightarrow B' \in \Xi \qquad (p_i : B_i \leftarrow w_i \dashv \theta_i)_i}{\langle c\ \overline{p} \rightarrow z \rangle : \langle \Delta \rangle A \leftarrow \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \dashv [\Sigma]\ \overline{\theta}[\uparrow(\{x \mapsto \mathcal{E}[x]\} : \{B' \rightarrow [\Sigma']A\})/z]}$$

B-CATCHALL-VALUE
$$\dfrac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x \rangle : \langle \Delta \rangle A \leftarrow w \dashv [\Sigma]\ [\uparrow(\{w\}:\{[\Sigma']A\})/x]}$$

B-CATCHALL-REQUEST
$$\dfrac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \mathcal{E} \text{ poisedfor } c \qquad \Delta = \Theta \mid \Xi \qquad c : \forall \overline{Z}.\overline{B} \rightarrow B' \in \Xi}{\langle x \rangle : \langle \Delta \rangle A \leftarrow \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \dashv [\Sigma]\ [\uparrow(\{\lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil\}:\{[\Sigma']A\})/x]}$$

$$\boxed{p : A \leftarrow w \dashv \theta}$$

B-VAR
$$\dfrac{}{x : A \leftarrow w \dashv [\uparrow(w : A)/x]}$$

B-DATA
$$\dfrac{k\ \overline{A} \in D\ \overline{R} \qquad (p_i : A_i \leftarrow w_i \dashv \theta_i)_i}{k\ \overline{p} : D\ \overline{R} \leftarrow k\ \overline{w} \dashv \overline{\theta}}$$

Figure 1.7: Pattern Binding

Chapter 2

Arbitrary Thread Interruption

2.1 Relaxing Catches

B-CATCHALL-REQUEST-LOOSE
$$\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x \rangle : \langle \Delta \rangle A \leftarrow \lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil \dashv_\Sigma [\uparrow(\{\lceil \mathcal{E}[c\ \overline{R}\ \overline{w}] \rceil\} : \{[\Sigma']A\})/x]}$$

Figure 2.1: Updated B-CATCHALL-REQUEST

2.2 Interrupting Arbitrary Terms

$$
\begin{aligned}
\text{(uses)} & & m &::= \cdots \mid \lceil \mathcal{E}[c \,\overline{R}\,\overline{w}] \rceil \\
\text{(constructions)} & & n &::= \cdots \mid \lceil \mathcal{E}[c \,\overline{R}\,\overline{w}] \rceil \\
\text{(use values)} & & u &::= x \mid f \,\overline{R} \mid \uparrow(v:A) \\
\text{(non-use values)} & & v &::= k \,\overline{w} \mid \{e\} \\
\text{(construction values)} & & w &::= \downarrow u \mid v \\
\text{(normal forms)} & & t &::= w \mid \lceil \mathcal{E}[c \,\overline{R}\,\overline{w}] \rceil \mid !(m) \\
\text{(evaluation frames)} & & \mathcal{F} &::= [\,] \,\overline{n} \mid u \,(\overline{t},[\,],\overline{n}) \mid \uparrow([\,]:A) \\
& & & \mid \downarrow[\,] \mid k \,(\overline{w},[\,],\overline{n}) \mid c \,\overline{R}\,(\overline{w},[\,],\overline{n}) \\
& & & \mid \mathbf{let}\, f : P = [\,] \,\mathbf{in}\, n \mid \langle \Theta \rangle \,[\,] \\
\text{(evaluation contexts)} & & \mathcal{E} &::= [\,] \mid \mathcal{F}[\mathcal{E}]
\end{aligned}
$$

Figure 2.2: Runtime Syntax, Updated with Suspension of Uses

B-CATCHALL-INTERRUPT
$$
\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x \rangle : \langle \Delta \rangle A \leftarrow !(m) \dashv_{[\Xi]} [\uparrow(\{m\} : \{[\Sigma']A\})/x]}
$$

Figure 2.3: Catching Interrupts rule.

2.3 Freezing

Another way of doing it is to just let any use become frozen, in the same way as commands become frozen once invoked.

TODO: Do we need the 'hoisting' rules for general suspended terms?

2.4 Yielding

Note that R-YIELD-EF relies on the predicate $\mathcal{F}[\mathcal{E}]$ allows c. This states that the Yield interface is in the ability of the term in the evaluation context.

For any frame apart from argument frames, $\mathcal{F}[\mathcal{E}]$ allows $c = $ false. In this case, it is defined as follows;

$$
\uparrow(v : \{\overline{\langle \Delta \rangle A \to} [\Sigma]B\}) \,(\overline{t},[\,],\overline{n}) \text{ allows } c = \begin{cases} \Xi \text{ allows } c & \text{if } |\overline{n}| = 0 \text{ where } \Sigma = \sigma \,|\, \Xi \\ \text{false} & \text{otherwise} \end{cases}
$$

TODO: What happens for 0-ary constructors? I don't know if it's exactly how I think it is.

$$\text{R-INTERRUPT}$$

$$\frac{}{m \rightsquigarrow_u !(m)}$$

Figure 2.4: Use interruption rule

$$\boxed{m \rightsquigarrow_u m'} \quad \boxed{n \rightsquigarrow_c n'} \quad \boxed{m \longrightarrow_u m'} \quad \boxed{n \longrightarrow_c n'}$$

$$\text{R-FREEZE-USE} \qquad\qquad \text{R-FREEZE-COMM}$$

$$\frac{}{m \rightsquigarrow_u \lceil m \rceil} \qquad\qquad \frac{}{c \; \overline{R} \; \overline{w} \rightsquigarrow_c \lceil c \; \overline{R} \; \overline{w} \rceil}$$

$$\text{R-FREEZE-FRAME-USE} \qquad\qquad \text{R-FREEZE-FRAME-CONS}$$

$$\frac{}{\mathcal{F}[\lceil m \rceil] \rightsquigarrow_u \lceil \mathcal{F}[m] \rceil} \qquad\qquad \frac{}{\mathcal{F}[\lceil m \rceil] \rightsquigarrow_c \lceil \mathcal{F}[m] \rceil}$$

$$\text{R-FREEZE-FRAME-USE} \qquad\qquad \text{R-FREEZE-FRAME-CONS}$$

$$\frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[\lceil \mathcal{E}[c \; \overline{R} \; \overline{w}] \rceil] \rightsquigarrow_u \lceil \mathcal{F}[\mathcal{E}[c \; \overline{R} \; \overline{w}]] \rceil} \qquad \frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[\lceil \mathcal{E}[c \; \overline{R} \; \overline{w}] \rceil] \rightsquigarrow_c \lceil \mathcal{F}[\mathcal{E}[c \; \overline{R} \; \overline{w}]] \rceil}$$

Figure 2.5: Updated Freezing

For an extension Ξ, the allows predicate is defined as

$$\imath \text{ allows } c = \text{false}$$

$$(\Xi, I \; \overline{R}) \text{ allows } c = \begin{cases} \text{true} & \text{if } c \in I \\ \Xi \text{ allows } c & \text{otherwise} \end{cases}$$

Informally, $\mathcal{F}[\mathcal{E}]$ allows c is true when $\mathcal{F}[\mathcal{E}]$ is a handler with the command c as a member of an interface in its ability, *and* all of the arguments have been evaluated (this is what the $|\overline{n}| = 0$ constraint expresses).

2.5 Counting

In practise we count up through the amount of R-HANDLE rules we apply and only insert the yield when this count exceeds a threshold value t_y.

So we supplement the operational semantics with a *counter c_y*, so that our transitions are e.g. $m; c_y \rightsquigarrow_u m'; c_y'$. We adopt the convention that, when the counter is not

$$(\text{uses}) \qquad m ::= \dots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \mid \lceil m \rceil$$

$$(\text{constructions}) \qquad n ::= \dots \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \mid \lceil m \rceil$$

$$(\text{use values}) \qquad u ::= x \mid f\,\overline{R} \mid \uparrow(v:A)$$

$$(\text{non-use values}) \qquad v ::= k\,\overline{w} \mid \{e\}$$

$$(\text{construction values}) \quad w ::= \downarrow u \mid v$$

$$(\text{normal forms}) \qquad t ::= w \mid \lceil \mathcal{E}[c\,\overline{R}\,\overline{w}] \rceil \mid \lceil m \rceil$$

$$(\text{evaluation frames}) \quad \mathcal{F} ::= [\,]\,\overline{n} \mid u\,(\overline{t},[\,],\overline{n}) \mid \uparrow([\,]:A)$$
$$\mid \quad \downarrow[\,] \mid k\,(\overline{w},[\,],\overline{n}) \mid c\,\overline{R}\,(\overline{w},[\,],\overline{n})$$
$$\mid \quad \textbf{let } f:P = [\,] \textbf{ in } n \mid \langle \Theta \rangle\,[\,]$$

$$(\text{evaluation contexts}) \quad \mathcal{E} ::= [\,] \mid \mathcal{F}[\mathcal{E}]$$

Figure 2.6: Runtime Syntax, Updated with Freezing of Uses

B-CATCHALL-INTERRUPT
$$\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\langle x \rangle : \langle \Delta \rangle A \leftarrow \lceil m \rceil \dashv [\Sigma]\;[\uparrow(\{m\}:\{[\Sigma']A\})/x]}$$

Figure 2.7: Catching Frozen Terms rule.

mentioned in a transition[1], the counter stays the same. Hence e.g. $m \rightsquigarrow_u m'$ desugars to $m; c_y \rightsquigarrow_u m'; c_y$.

So to get our counting semantics we just need to supplement Figure **?** with the updated rule in

---

[1] That is, the transition is of the form $m \rightsquigarrow_u m'$.

$$\boxed{m \rightsquigarrow_{\mathrm{u}} m'}$$

R-YIELD

$$\Sigma = \sigma \,|\, \Xi \qquad \mathsf{Yield} \in \Xi$$

$$\uparrow(\{n\} : \{[\Sigma]A\}) \rightsquigarrow_{\mathrm{u}} \uparrow(\quad \mathbf{let}\ f_n : \{\mathsf{Unit} \to [\Sigma]A\} = \{\_ \mapsto n\}\ \mathbf{in}$$
$$\mathbf{let}\ y : \{[\Sigma]\mathsf{Unit}\} = \{\mathsf{yield!}\}\ \mathbf{in}\ \{f_n\ (\downarrow(y!))\} : \{[\Sigma]A\})$$

R-YIELD-EF

$$\mathcal{F}[\mathcal{E}]\ \text{allows}\ \mathsf{Yield}$$

$$\mathcal{F}[\mathcal{E}[m]] \rightsquigarrow_{\mathrm{u}} \mathcal{F}[\mathcal{E}[\mathsf{yield!}; m]]$$

Figure 2.8: Inserting Yields

> **TODO:** In R-Yield-EF we have to let terms inside eval CTXs be also uses. Does this not mean that we can then put any term in either one? Is that a problem?
>
> **TODO:** Not sure if we want to yield /inside/ the ctx, or just before it?

$$\boxed{m \rightsquigarrow_{\mathrm{u}} m'} \quad \boxed{n \rightsquigarrow_{\mathrm{c}} n'} \quad \boxed{m \longrightarrow_{\mathrm{u}} m'} \quad \boxed{n \longrightarrow_{\mathrm{c}} n'}$$

R-HANDLE

$$k = \min_{i} \{i \mid \exists \overline{\theta}.(r_{i,j} : \langle \Delta_j \rangle A_j \leftarrow t_j -\!\![\Sigma]\ \theta_j)_j\} \qquad (r_{k,j} : \langle \Delta_j \rangle A_j \leftarrow t_j -\!\![\Sigma]\ \theta_j)_j$$

$$\uparrow(\{((r_{i,j})_j \to n_i)_i\} : \{\overline{\langle \Delta \rangle A \to} [\Sigma]B\})\ \bar{t}; c_y \rightsquigarrow_{\mathrm{u}} \uparrow((\overline{\theta}(n_k) : B)); c_y + 1$$

Figure 2.9: R-Handle with counting.

$$\boxed{m \rightsquigarrow_{\mathrm{u}} m'}$$

R-YIELD

$$c_y \geq t_y$$

$$m; c_y \rightsquigarrow_{\mathrm{u}} \mathbf{let}\ \mathsf{yield} : [\mathsf{Yield}]\mathsf{Unit} = y\,\mathbf{in}\ m; 0$$

Figure 2.10: Inserting Yields, when over counter

Chapter 3

Use-Only Reductions

Here we talk about how to change the operational semantics so that only uses may reduce.

$$\boxed{m \rightsquigarrow_{\mathrm{u}} m'} \quad \boxed{n \rightsquigarrow_{\mathrm{c}} n'} \quad \boxed{m \longrightarrow_{\mathrm{u}} m'} \quad \boxed{n \longrightarrow_{\mathrm{c}} n'}$$

R-HANDLE
$$\frac{k = \min_{i} \{i \mid \exists \overline{\theta}.(r_{i,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \text{-}[\Sigma]\, \theta_j)_j\} \qquad (r_{k,j} : \langle \Delta_j \rangle A_j \leftarrow t_j \text{-}[\Sigma]\, \theta_j)_j}{\uparrow(\{((r_{i,j})_j \rightarrow n_i)_i\} : \{\overline{\langle \Delta \rangle A \rightarrow [\Sigma]B}\})\, \overline{t} \rightsquigarrow_{\mathrm{u}} \uparrow((\overline{\theta}(n_k) : B)}$$

R-ASCRIBE-USE
$$\frac{}{\uparrow(\downarrow u : A) \rightsquigarrow_{\mathrm{u}} u}$$

R-ASCRIBE-CONS
$$\frac{}{\downarrow\uparrow(w : A) \rightsquigarrow_{\mathrm{c}} w}$$

R-LET
$$\frac{}{\uparrow(\textbf{let } f : P = w \textbf{ in } n : A) \rightsquigarrow_{\mathrm{u}} \uparrow(n[\uparrow(w : P)/f] : A)}$$

R-LETREC
$$\frac{\overline{e = \overline{\overline{r} \rightarrow n}}}{\uparrow(\textbf{letrec } \overline{f : P = e} \textbf{ in } n' : A) \rightsquigarrow_{\mathrm{u}} \uparrow(n'[\overline{\uparrow(\{\overline{r} \rightarrow \textbf{letrec } \overline{f : P = e} \textbf{ in } n\} : P)/f}] : A)}$$

R-ADAPT
$$\frac{}{\langle \Theta \rangle\, w \rightsquigarrow_{\mathrm{c}} w}$$

R-FREEZE-COMM
$$\frac{}{c\, \overline{R}\, \overline{w} \rightsquigarrow_{\mathrm{c}} \lceil c\, \overline{R}\, \overline{w} \rceil}$$

R-FREEZE-FRAME-USE
$$\frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[\lceil \mathcal{E}[c\, \overline{R}\, \overline{w}] \rceil] \rightsquigarrow_{\mathrm{u}} \lceil \mathcal{F}[\mathcal{E}[c\, \overline{R}\, \overline{w}]] \rceil}$$

R-FREEZE-FRAME-CONS
$$\frac{\neg(\mathcal{F}[\mathcal{E}] \text{ handles } c)}{\mathcal{F}[\lceil \mathcal{E}[c\, \overline{R}\, \overline{w}] \rceil] \rightsquigarrow_{\mathrm{c}} \lceil \mathcal{F}[\mathcal{E}[c\, \overline{R}\, \overline{w}]] \rceil}$$

R-LIFT-UU
$$\frac{m \rightsquigarrow_{\mathrm{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathrm{u}} \mathcal{E}[m']}$$

R-LIFT-UC
$$\frac{m \rightsquigarrow_{\mathrm{u}} m'}{\mathcal{E}[m] \longrightarrow_{\mathrm{c}} \mathcal{E}[m']}$$

R-LIFT-CU
$$\frac{n \rightsquigarrow_{\mathrm{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathrm{u}} \mathcal{E}[n']}$$

R-LIFT-CC
$$\frac{n \rightsquigarrow_{\mathrm{c}} n'}{\mathcal{E}[n] \longrightarrow_{\mathrm{c}} \mathcal{E}[n']}$$

Figure 3.1: Operational Semantics

Appendix A

Remaining Formalisms

$$\boxed{\Sigma \vdash \Delta \dashv \Sigma'}$$

A-ADJ
$$\frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Sigma' \vdash \Xi \dashv \Sigma''}{\Sigma \vdash \Theta \mid \Xi \dashv \Sigma''}$$

$$\boxed{\Sigma \vdash \Xi \dashv \Sigma'}$$

A-EXT-ID
$$\frac{}{\Sigma \vdash \iota \dashv \Sigma}$$

A-EXT-SNOC
$$\frac{\Sigma \vdash \Xi \dashv \Sigma'}{\Sigma \vdash \Xi, I\,\overline{R} \dashv \Sigma', I\,\overline{R}}$$

$$\boxed{\Sigma \vdash \Theta \dashv \Sigma'}$$

A-ADAPT-ID
$$\frac{}{\Sigma \vdash \iota \dashv \Sigma}$$

A-ADAPT-SNOC
$$\frac{\Sigma \vdash \Theta \dashv \Sigma' \qquad \Sigma' \vdash I(S \to S') \dashv \Sigma''}{\Sigma \vdash \Theta, I(S \to S') \dashv \Sigma''}$$

$$\boxed{\Sigma \vdash I(S \to S') \dashv \Sigma'}$$

A-ADAPT-COM
$$\frac{\Sigma \vdash S : I \dashv \Sigma'; \Omega \qquad \Omega \vdash S' : I \dashv \Xi \qquad \Sigma' \vdash \Xi \dashv \Sigma''}{\Sigma \vdash I(S \to S') \dashv \Sigma''}$$

$$\boxed{\Sigma \vdash S : I \dashv \Sigma'; \Omega}$$

I-PAT-ID
$$\frac{}{\Sigma \vdash s : I \dashv \Sigma; s : \Sigma}$$

I-PAT-BIND
$$\frac{\Sigma \vdash S : I \dashv \Sigma'; \Omega}{\Sigma, I\,\overline{R} \vdash S\,a : I \dashv \Sigma'; \Omega, a : I\,\overline{R}}$$

I-PAT-SKIP
$$\frac{\Sigma \vdash S\,a : I \dashv \Sigma'; \Omega \qquad I \neq I'}{\Sigma, I'\,\overline{R} \vdash S\,a : I \dashv \Sigma', I'\,\overline{R}; \Omega}$$

$$\boxed{\Omega \vdash S : I \dashv \Xi}$$

I-INST-ID
$$\frac{s \in \mathsf{dom}(\Omega)}{\Omega \vdash s : I \dashv \iota}$$

I-INST-LKP
$$\frac{a \in \mathsf{dom}(\Omega) \qquad \Omega \vdash S : I \dashv \Xi \qquad \Omega(a) = I\,\overline{R}}{\Omega \vdash S\,a : I \dashv \Xi, I\,\overline{R}}$$

Figure A.1: Action of an Adjustment on an Ability and Auxiliary Judgements

$$X ::= A \mid C \mid T \mid G \mid Z \mid R \mid P \mid \sigma \mid \Sigma \mid \Xi \mid \Theta \mid \Delta \mid \Gamma \mid \exists \Psi.\Gamma \mid \Omega$$

$$\boxed{\Phi \vdash X}$$

$$
\frac{}{\Phi, X \vdash X} \quad \text{WF-VAL}
\qquad
\frac{}{\Phi, [E] \vdash E} \quad \text{WF-EFF}
\qquad
\frac{\Phi, \overline{Z} \vdash A}{\Phi \vdash \forall \overline{Z}.A} \quad \text{WF-POLY}
$$

$$
\frac{(\Phi \vdash R)_i}{\Phi \vdash D\,\overline{R}} \quad \text{WF-DATA}
\qquad
\frac{\Phi \vdash C}{\Phi \vdash \{C\}} \quad \text{WF-THUNK}
\qquad
\frac{(\Phi \vdash T)_i \quad \Phi \vdash G}{\Phi \vdash \overline{T \to G}} \quad \text{WF-COMP}
\qquad
\frac{\Phi \vdash \Delta \quad \Phi \vdash A}{\Phi \vdash \langle \Delta \rangle A} \quad \text{WF-ARG}
$$

$$
\frac{\Phi \vdash \Sigma \quad \Phi \vdash A}{\Phi \vdash [\Sigma]A} \quad \text{WF-RET}
\qquad
\frac{\Phi \vdash \Sigma}{\Phi \vdash [\Sigma]} \quad \text{WF-ABILITY}
\qquad
\frac{}{\Phi \vdash \emptyset} \quad \text{WF-PURE}
\qquad
\frac{}{\Phi \vdash \iota} \quad \text{WF-ID}
\qquad
\frac{\Phi \vdash \Xi \quad (\Phi \vdash R)_i}{\Phi \vdash \Xi, I\,\overline{R}} \quad \text{WF-EXT}
$$

$$
\frac{\Phi \vdash \Theta}{\Phi \vdash \Theta, I\,(S \to S')} \quad \text{WF-ADAPT}
$$

$$
\frac{}{\Phi \vdash \cdot} \quad \text{WF-EMPTY}
\qquad
\frac{\Phi \vdash \Gamma \quad \Phi \vdash A}{\Phi \vdash \Gamma, x : A} \quad \text{WF-MONO}
\qquad
\frac{\Phi \vdash \Gamma \quad \Phi \vdash P}{\Phi \vdash \Gamma, f : P} \quad \text{WF-POLY}
$$

$$
\frac{\Phi, \Psi \vdash \Gamma}{\Phi \vdash \exists \Psi.\Gamma} \quad \text{WF-EXISTENTIAL}
\qquad
\frac{\Phi \vdash \Omega \quad (\Phi \vdash R)_i}{\Phi \vdash \Omega, x : I\,\overline{R}} \quad \text{WF-INTERFACE}
$$

Figure A.2: Well-Formedness Rules

$$\boxed{\Phi \vdash p : A \dashv \Gamma}$$

P-VAR
$$\frac{}{\Phi \vdash x : A \dashv x : A}$$

P-DATA
$$\frac{k\,\overline{A} \in D\,\overline{R} \qquad (\Phi \vdash p_i : A_i \dashv \Gamma)_i}{\Phi \vdash k\,\overline{p} : D\,\overline{R} \dashv \overline{\Gamma}}$$

$$\boxed{\Phi \vdash r : T \dashv_{[\Sigma]} \exists \Psi.\Gamma}$$

P-VALUE
$$\frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \Phi \vdash p : A \dashv \Gamma}{\Phi \vdash p : \langle\Delta\rangle A \dashv_{[\Sigma]} \Gamma}$$

P-CATCHALL
$$\frac{\Sigma \vdash \Delta \dashv \Sigma'}{\Phi \vdash \langle x \rangle : \langle\Delta\rangle A \dashv_{[\Sigma]} x : \{[\Sigma']A\}}$$

P-COMMAND
$$\frac{\Sigma \vdash \Delta \dashv \Sigma' \qquad \Delta = \Theta \mid \Xi \qquad c : \forall \overline{Z}.\overline{A} \to B \in \Xi \qquad (\Phi, \overline{Z} \vdash p_i : A_i \dashv \Gamma_i)_i}{\Phi \vdash \langle c\,\overline{p} \to z \rangle : \langle\Delta\rangle B' \dashv_{[\Sigma]} \exists \overline{Z}.\overline{\Gamma}, z : \{\langle \iota \mid \iota \rangle B \to [\Sigma']B'\}}$$

Figure A.3: Pattern Matching Typing Rules