# README for constrmag.F

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#### Introduction 1

We added a new constrained non-collinear magnetism method to VASP. It can be implemented easily by recompiling VASP with our modified constrmag.F. Please cite the following if you are using this method.

Pui-Wai Ma and S. L. Dudarev, Constrained density functional for noncollinear magnetism, Physical Review B 91, 054420 (2015)

#### 2 Theory

We define the magnetic moment of an atom as

$$\mathbf{M}_{I} = \int_{\Omega_{I}} \mathbf{m}(\mathbf{r}) d^{3}r, \tag{1}$$

where  $\mathbf{m}(\mathbf{r})$  is a spatially varying magnetization density, and  $\Omega_I$  is a sphere centred at atom I.

Instead of using  $\mathbf{M}_I$  directly, we use an alternative definition of the local magnetic moment, namely

$$\mathbf{M}_{I}^{F} = \int_{\Omega_{I}} \mathbf{m}(\mathbf{r}) F_{I}(|\mathbf{r} - \mathbf{r}_{I}|) d^{3}r$$
 (2)

where  $F_I(|\mathbf{r} - \mathbf{r}_I|)$  is a scalar function decreasing monotonically to zero towards the boundary of the atomic sphere. Such definition of  $\mathbf{M}_{I}^{F}$  was adopted in VASP for other constrained method.

The constrained total energy functional has the form:

$$E = E_0 + E_p \tag{3}$$

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$$= E_0 + \sum_I \lambda_I \left( \left| \mathbf{M}_I^F \right| - \mathbf{e}_I \cdot \mathbf{M}_I^F \right)$$
(3)
$$(4)$$

where  $E_0$  is the DFT energy of the material,  $E_p$  is the penalty energy term,  $\mathbf{e}_I$  is a unit vector in the desired direction of the local magnetic moment, and  $\lambda_I$  is a Lagrange multiplier associated with site I. The dimensionality of  $\lambda_I$  is the same as of external magnetic field.

The penalty energy term in (4) introduces an effective extra potential inside each sphere  $\Omega_I$  centred at atom I, given by

$$V_I(\mathbf{r}) = -\mathbf{b}_p(\mathbf{r}) \cdot \boldsymbol{\sigma} \tag{5}$$

where  $\sigma$  is the vector of Pauli matrices, and

$$\mathbf{b}_{p}(\mathbf{r}) = -\frac{\delta E_{p}}{\delta \mathbf{m}(\mathbf{r})} \tag{6}$$

$$= -\lambda_I \left( \frac{\mathbf{M}_I^F}{|\mathbf{M}_I^F|} - \mathbf{e}_I \right) F_I(|\mathbf{r} - \mathbf{r}_I|) \tag{7}$$

is an additional penalty "field" in the Kohn-Sham equations. Equations 4 and 7 show that both  $E_p$  and  $V_I(\mathbf{r})$  terms vanish only if vector  $\mathbf{M}_I^F$  points in the same direction as  $\mathbf{e}_I$ .

From equation (6) we see that functions  $F_I(|\mathbf{r} - \mathbf{r}_I|)$  eliminate discontinuities of the effective potential at the boundaries of atomic spheres  $\Omega_I$ . We do not need to separate the core and interstitial regions. The part played by the penalty term in this respect is similar to the action of local spatially-varying external magnetic field.

For more details, please refer to our Phys. Rev. B paper.

## 3 Controls

Our constrained method can be activated by using:

### I\_CONSTRAINED\_M=4

Other details of setup are the same as using I\_CONSTRAINED\_M=1.

Additionally, if VASP is compiled with "-Dconstrmagtanh", it replaces the function

$$F_I(|\mathbf{r} - \mathbf{r}_I|) = \sin(x)/x,\tag{8}$$

where  $x = \pi(|\mathbf{r} - \mathbf{r}_I|)/R_I$  by

$$F_I(|\mathbf{r} - \mathbf{r}_I|) = -\tanh((x/\pi - 1)/c) \tag{9}$$

where we set c = 0.01. It is useful for I\_CONSTRAINED\_M=2.