Homework3 PA Report

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Part 3. Noiseless case.

In this case, we set up three N values for x, and got 3 different output results.

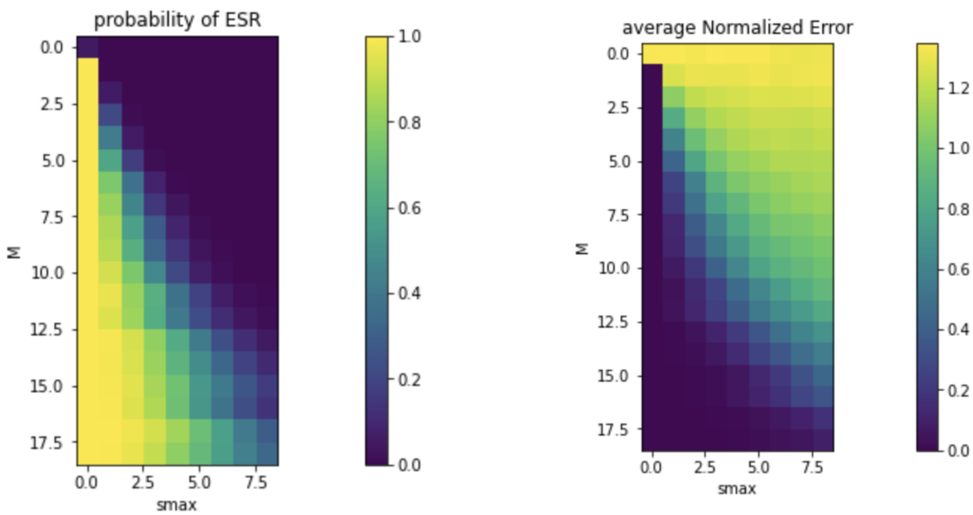


Fig1: N = 20 noiseless phase transition

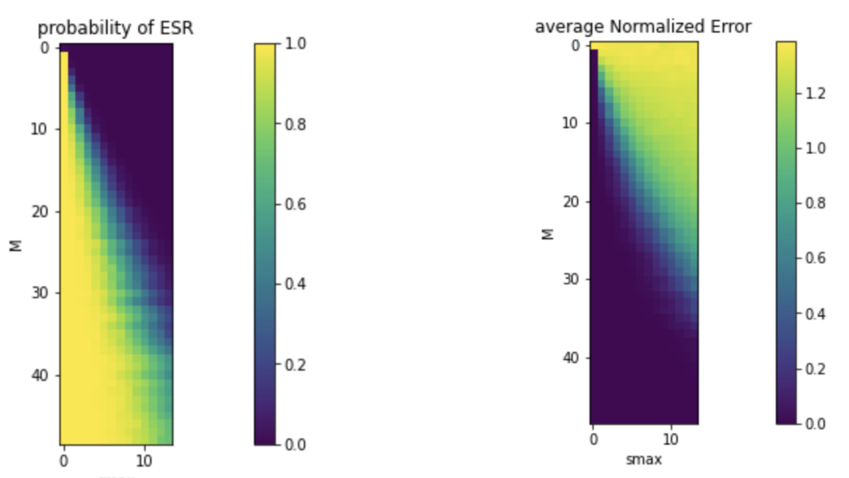


Fig2: N = 50 noiseless phase transition

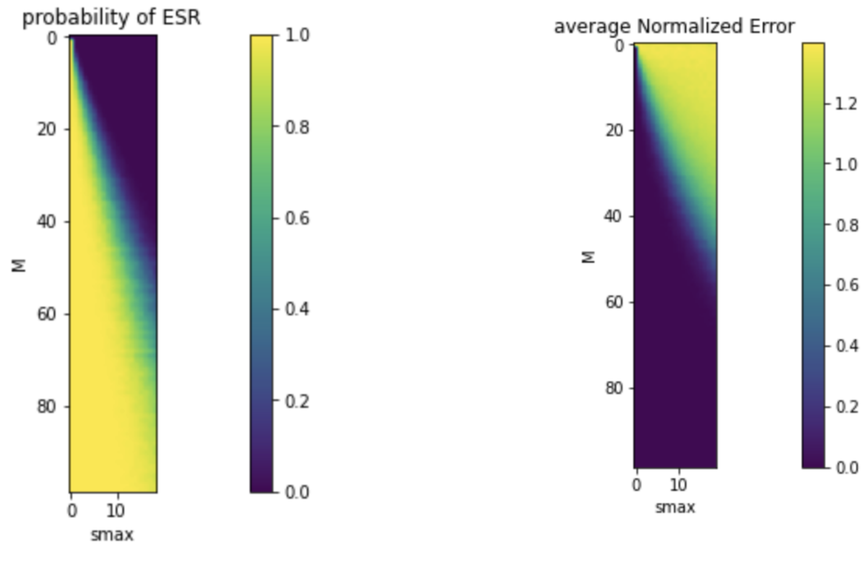


Fig3: N=100 noiseless phase transition

For the Exact Support Recovery, the sharp transition appears when Smax > M, and we can also assume from the plot, when s get larger, the probability of recovery get lower.
For the Normalized Error plot, we could observe that when smax goes larger, the error goes larger, it's almost a reverse plot compared to the ESR plot.
For different N case, when N goes larger, the measurement times M also goes larger, and the result will be better along with the large measurements.

Part 4: Noisy case

(a) In this case, we add a noise to the Ax measurement, and assume we known the sparsity of the signal, so we stop OMP loop at s times, or the residual signal less than 10^-3.
For noise sigma = 0.01:

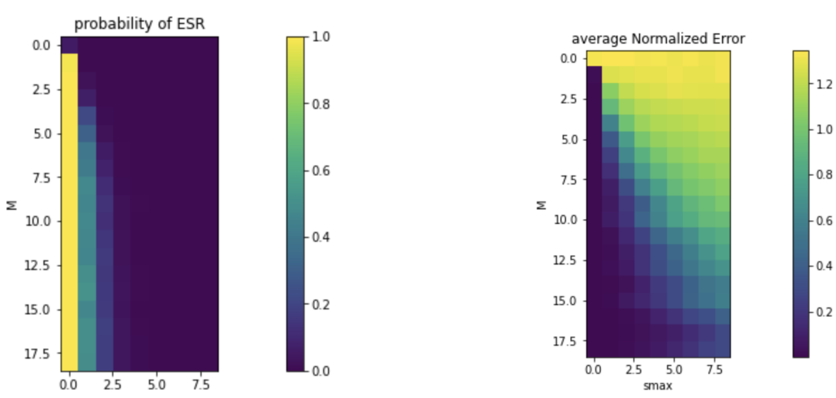


Fig 4: N = 20, sigma = 0.01 noisy phase transition

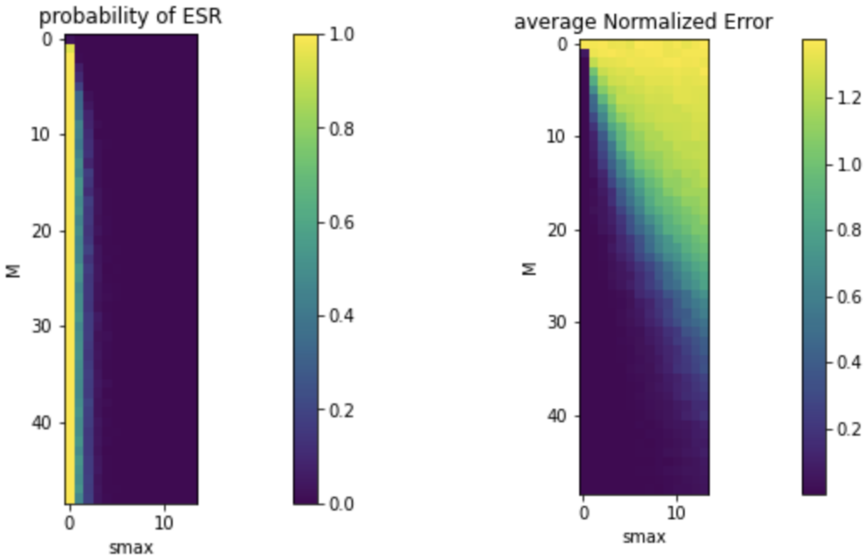


Fig 5: N = 50, sigma = 0.01 noisy phase transition

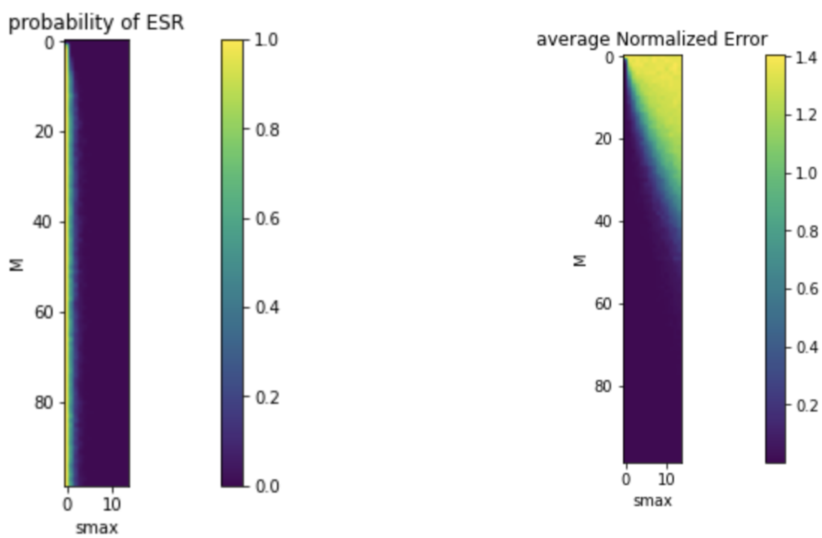
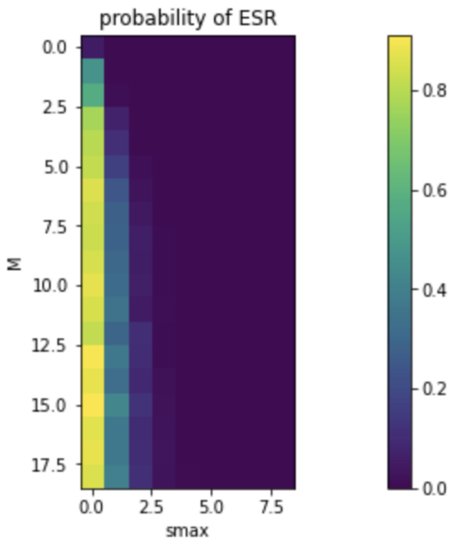
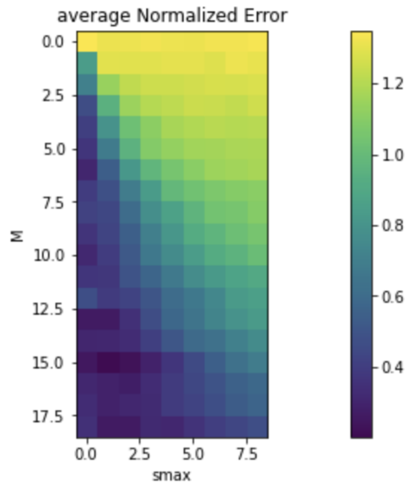


Fig 6: N = 100, sigma = 0.01 noisy phase transition

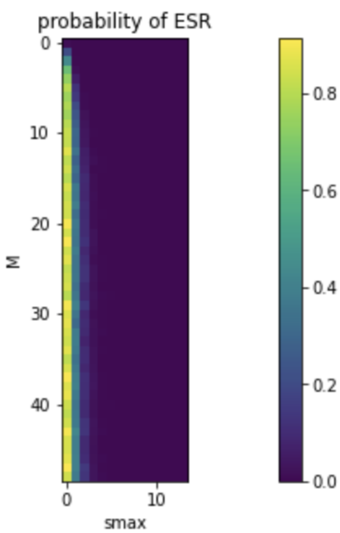
As we can observe, compared to noiseless OMP, the result is worse when smax goes larger, and Error also goes larger.

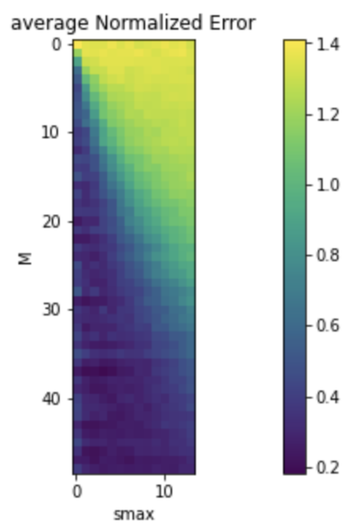
Sigma = 1:



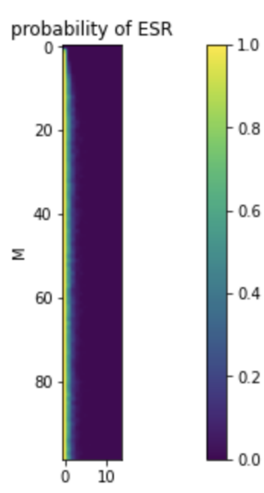


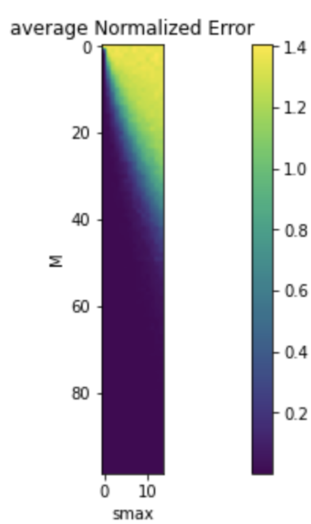
N = 20





N = 50

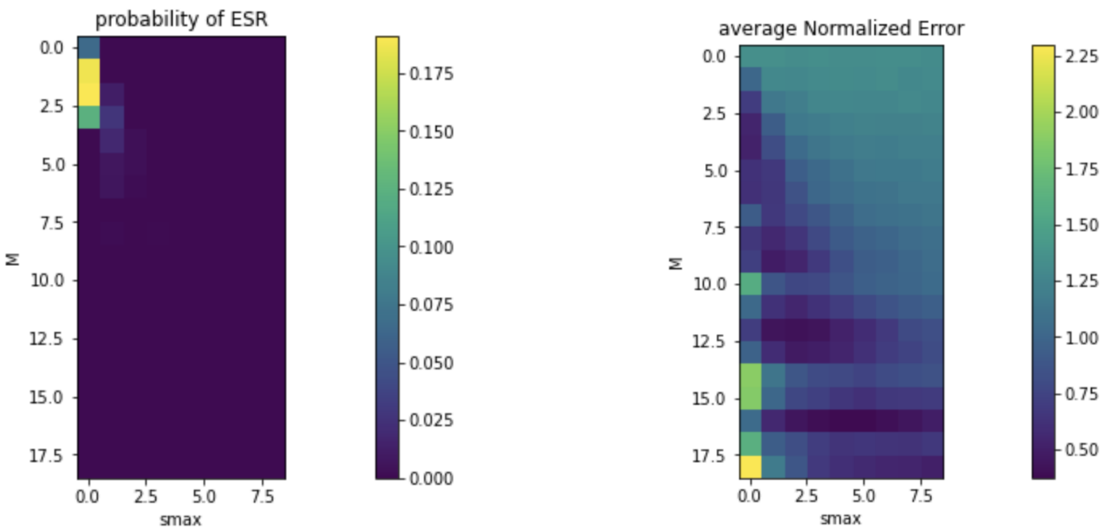




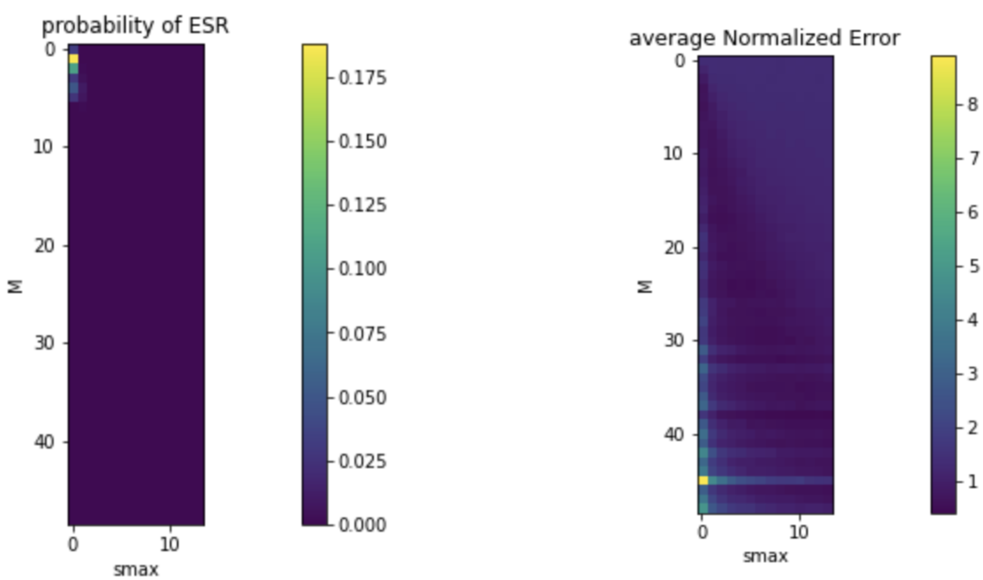
N =100

For sigma = 1 case, the noise is larger, and we can observe that the recovery probability becomes lower, and the error grows larger compared to the sigma = 0.01 case

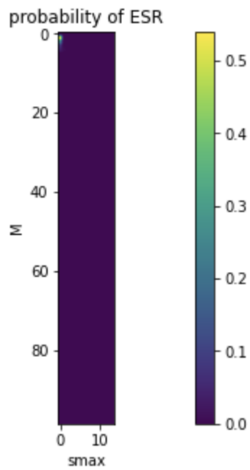
(b) For the case we only know ||n||_2, I also choose sigma = 0.01, and sigma =1 cases.

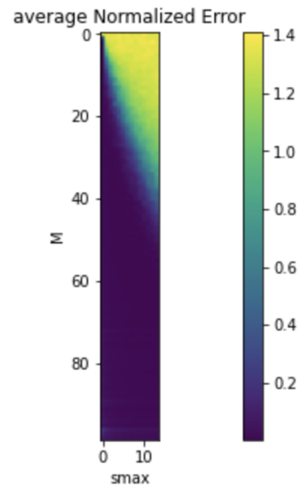


N = 20 sigma = 0.01



N = 50 sigma = 0.01





N = 100

As we can see from the plots, it is hard for OMP to recover the exact x. I think the reason is the ||n||_2 is still too large to become an error. In my code, my ||n||_2 always greater than 1, which means the loop will stop when the residual error is less than a big number. Therefore, it does not even converge to a right signal s. And in this case, the normalized error metric, does not work, because we only focus on the error not the exact sparsity which x has recovered, so the metric is not convincing for this case.

Problem 5:

a) I could not guess the message.

b)

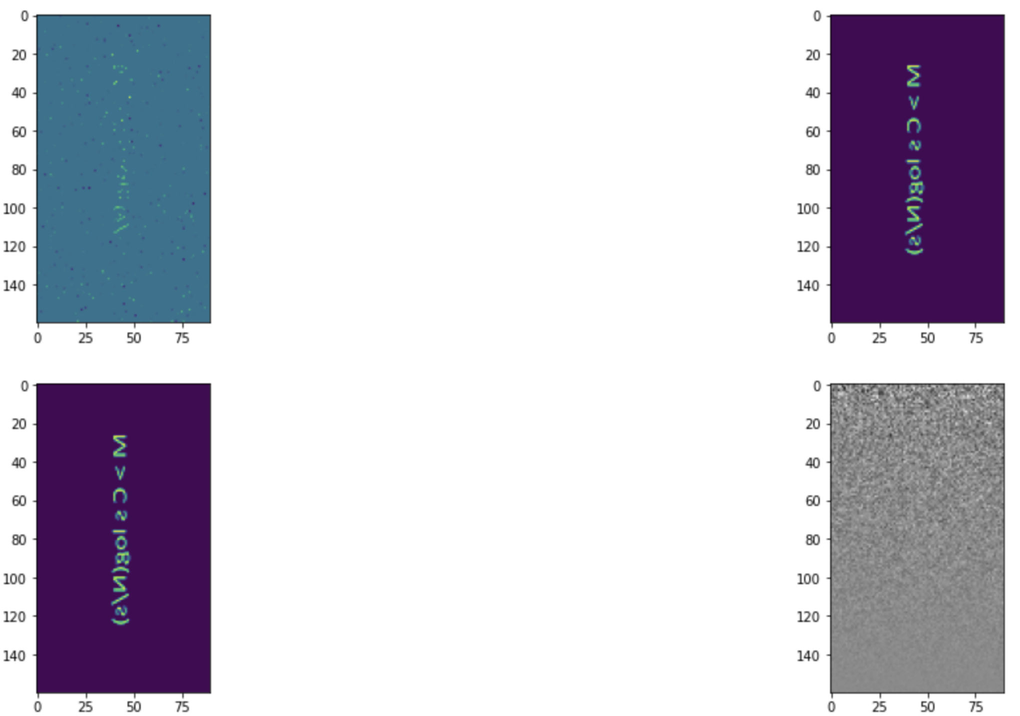


Fig 7: left-right, up-down: A1, A2, A3, least square result.

c) The image recovered from A3 has the best result, because A3 has dimension of 2880x144400, which has more measurement columns than A1, and A2, so the result is also better.

d) M>C^2*log(N/2), maybe means, measurements need to be larger than log(N/2)