ECE 269

HW 1

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2. **Problem 1: Subspaces of $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$.**
Determine which of the following subsets of $\mathbb{R}^n$, and $\mathbb{R}^{n\times n}$ are subspaces ($n > 2$).

(a) $\{\mathbf{x} \mid x_i \geq 0\}$
(b) $\{\mathbf{x} \mid x_1 = 0\}$
(c) $\{\mathbf{x} \mid x_1 x_2 = 0\}$
(d) $\{\mathbf{x} \mid \mathbf{A}\mathbf{x} = \mathbf{b} \text{ where } \mathbf{b} \neq \mathbf{0}\}$
(e) $\{[x_1, x_2, x_3, x_4] \in \mathbb{R}^4 \mid x_3 = x_1 + x_2,\ x_4 = x_1 - x_2\}$
(f) $\{[x_1, x_2, x_3] \in \mathbb{R}^3 \mid x_1 \leq x_2 \leq x_3\}$
(g) $\{\mathbf{A} \in \mathbb{R}^{3\times 3} \mid [1, 0, 4]^T \in N(\mathbf{A})\}$
(h) All matrices that commute with a given matrix $\mathbf{A} \in \mathbb{R}^{m\times m}$
(i) $\{\mathbf{X} \in \mathbb{R}^{n\times n} \mid \mathbf{X}^2 = \mathbf{X}\}$
(j) $\{\mathbf{X} \in \mathbb{R}^{n\times n} \mid \text{trace}(\mathbf{X}) = 0\}$

(a) not a subspace. negative scalar $\times\, x_i \notin W$

(b) subspace. $x = 0$ is the trivial subspace.

(c) not a subspace. x_1 or $x_2 = 0$ but $2x_1 (x_1 \neq 0) \notin W$

(d). not a subspace, 0 must in W but $x \neq 0$.

(e)
$$\begin{bmatrix} x_1 \\ x_2 \\ x_1+x_2 \\ x_1-x_2 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ y_1+y_2 \\ y_1-y_2 \end{bmatrix} = \begin{bmatrix} x_1+y_1 \\ x_2+y_2 \\ x_1+x_2+y_1+y_2 \\ x_1-x_2+y_1-y_2 \end{bmatrix} \in W \quad \begin{matrix} \\ \\ = x_1' + x_2' \\ = x_1' - x_2' \end{matrix}$$

$$2 \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_1+x_2 \\ x_1-x_2 \end{bmatrix} = \begin{matrix} 2x_1 \\ 2x_2 \\ 2x_1+2x_2 \\ 2x_1-2x_2 \end{matrix} \in W$$

$x_3 = x_1 + x_2 \quad x_4 = x_1 - x_2$

$\boxed{\text{so it is a subspace}}$

(f). $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{matrix} x_1+y_1 \\ x_2+y_2 \\ x_3+y_3 \end{matrix} \in W$

$\alpha \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{matrix} \alpha x_1 \\ \alpha x_2 \\ \alpha x_3 \end{matrix}$ if $\alpha < 0$ $\alpha x_3 < \alpha x_2 < \alpha x_1$

$\boxed{\text{so it is not a subspace.}}$

(g). $A \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} = 0$.

$x_{11} + 3x_{13} = 0 \qquad x_{13} = -\frac{1}{3} x_{11}$

$x_{21} + 3x_{23} = 0 \qquad x_{23} = -\frac{1}{3} x_{21}$

$x_{21} + 3x_{33} = 0 \qquad x_{33} = -\frac{1}{3} x_{31}$.

$A = \begin{bmatrix} x_{11} & x_{12} & -\frac{1}{3}x_{4} \\ x_{21} & x_{22} & -\frac{1}{3}x_{21} \\ x_{31} & x_{32} & -\frac{1}{3}x_{31} \end{bmatrix} \qquad A_1 + A_2 = A_3 \in W$

$\alpha A = \begin{bmatrix} \alpha x_{11} & \alpha x_{12} & \alpha -\frac{1}{3}x_{11} \\ \alpha x_{21} & \alpha x_{22} & \alpha \cdot \frac{1}{3}x_{21} \\ \alpha x_{31} & \alpha x_{32} & \alpha -\frac{1}{3}x_{33} \end{bmatrix} \in W$

So the Subset is a subspace of $\mathbb{R}^{3\times 3}$.

(ii) $XA = AX$

$YA = AY$

$(X+Y)A = A(X+Y).$

$(\alpha X)A = A(\alpha X)$

So. the subset is a subspace of $R^{n\times n}$

(ii) $X^2 = X.$ $(X+Y)^2 \neq X+Y$

$Y^2 = Y.$

So the subset is not a subspace,

(iii) $X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$ $x_{11} + x_{22} + x_{33} = 0.$

$y = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix}$ $y_{11} + y_{22} + y_{33} = 0.$

$x+y = \begin{bmatrix} x_{11}+y_{11} & \cdots & \\ \cdots & x_{22}+y_{22} & \cdots \\ & & x_{33}+y_{33} \end{bmatrix}$ $\text{trace}(x+y) = 0$

$x+y \in W$

$$\alpha X = \begin{bmatrix} \alpha x_{11} & \alpha x_{12} & \alpha x_{13} \\ \alpha x_{21} & \alpha x_{22} & \alpha x_{23} \\ \alpha x_{31} & \alpha x_{32} & \alpha x_{33} \end{bmatrix} \quad \text{trace}(\alpha x) = 0.$$

$$\alpha x \in W$$

$\therefore$ $\{X \in R^{n \times n} \mid \text{trace}(X) = 0\}$ is a subspace.

3. **Problem 2: Vector Spaces of Polynomials.**
Consider the set $\mathbb{P}_n(\mathbb{R})$ of all real valued polynomials of degree $\leq n$ with real coefficients:

$$\mathbb{P}_n(\mathbb{R}) = \{f(x) = \sum_{k=0}^{n} c_k x^k, c_0, c_1, \cdots, c_n \in \mathbb{R}\} \tag{1}$$

(a) Show that $\mathbb{P}_n(\mathbb{R})$ is a vector space. What is the dimension of this vector space?

(b) Is the union $\cup_{n=1}^{m} \mathbb{P}_n$ a vector space? Does this contradict or comply with something you learned in class?

(c) Find a basis for $\mathbb{P}_4$ containing $\{x^2+1, x^2-1\}$

(d) Find a basis for $\mathbb{P}_2$ from the set $\{1+x, x+x^2, x+2x^2, 2x+3x^2, 1+2x+x^2\}$

(a). For $a_{0 \sim n}, b_{0 \sim n}, c_{0 \sim n} \in \mathbb{R}$

① $X(x) = \sum_{k=0}^{n} a_k x^k \quad Y(x) = \sum_{k=0}^{n} b_k x^k$ (A1) ✓

$X(x)+Y(x) = \sum_{k=0}^{n} (a_k + b_k) x^k \in P_n(\mathbb{R})$.

② $Z(x) = \sum_{k=0}^{n} c_k x^k$

$(X+Y)+Z = \sum_{k=0}^{n} (a_k+b_k) x^k + \sum_{k=0}^{n} c_k x^k$

$\quad = \sum_{k=0}^{n} (a_k+b_k+c_k) x^k \in P_n(\mathbb{R})$

$\| $

$X+(Y+Z) = \sum_{k=0}^{n} a_k x^k + \sum_{k=0}^{n} (b_k+c_k) x^k$ (A2) ✓

$\quad = \sum_{k=0}^{n} (a_k+b_k+c_k) x^k \in P_n(\mathbb{R}$

③. $X+Y = \sum_{k=0}^{n} a_k x^k + \sum_{k=0}^{n} b_k x^k = \sum_{k=0}^{n} (a_k+b_k)x^k \in P_n(\mathbb{R})$.

$Y+X = \sum_{k=0}^{n} b_k x^k + \sum_{k=0}^{n} a_k x^k = \sum_{k=0}^{n} (a_k+b_k)x^k \in P_n(\mathbb{R})$

$= x+y$.

(A3) ✓

④. $0 = \sum_{k=0}^{n} c_k x^k \ (c_k = 0)$

(A4) ✓

field 0.

$X+0 = \sum_{k=0}^{n} (a_k+0)\, x^k = \sum_{k=0}^{n} a_k(x^k) = X$

⑤. $-X = \sum_{k=0}^{n} -a_k (X)^k$

(A5) ✓

$X+(-X) = \sum_{k=0}^{n} a_k X^k + \sum_{k=0}^{n} (-a_k) x^k$

$= \sum_{k=0}^{n} (a_k - a_k) x^k = 0$

(M1) ✓

⑥. $\alpha \in \mathbb{R}$

$\alpha x = \alpha \cdot \sum_{k=0}^{n} c_k x^k = \sum_{k=0}^{n} c_k \cdot \alpha x^k \in P_n(\mathbb{R})$.

⑦ . $\alpha \in \mathbb{R}$ $\beta \in \mathbb{R}$ (M₂) ✓

$$(\alpha \cdot \beta) X = (\alpha \cdot \beta) \sum_{k=0}^{n} a_k x^k$$

$$= \alpha \cdot \sum_{k=0}^{n} a_k \cdot \beta \cdot x^k$$

$$\alpha \cdot (\beta X) = \alpha \cdot \sum_{k=0}^{n} a_k \beta \cdot x^k .$$

$$= (\alpha \cdot \beta) x .$$

⑧ . $\alpha(X+Y) = \alpha \cdot \sum_{k=0}^{n} (a_k + b_k) x^k$ (M_3) ✓

$$= \sum_{k=0}^{n} (\alpha a_k + \alpha b_k) x^k$$

$$= \sum_{k=0}^{n} \alpha a_k x^k + \sum_{k=0}^{n} \alpha b_k x^k$$

$$\alpha X + \alpha Y = \alpha \sum_{k=0}^{n} a_k x^k + \alpha \sum_{k=0}^{n} b_k x^k$$

$$= \sum_{k=0}^{n} \alpha a_k x^k + \sum_{k=0}^{n} \alpha b_k x^k = \alpha(X+Y)$$

⑨ . $(\alpha+\beta) X = (\alpha+\beta) \sum_{k=0} a_k x^k$ (M_4) ✓

$$= \alpha \sum_{k=0}^{n} a_k x^k + \beta \sum_{k=0}^{n} a_k x^k$$

$$= \alpha X + \beta X .$$

(a). $1 \in \mathbb{R}$ $\quad 1 \cdot X = 1 \cdot \sum_{k=0}^{n} a_k x^k$ (M5)✓

$$= \sum_{k=0}^{n} 1 \cdot a_k x^k$$

$$= \sum_{k=0}^{n} a_k x^k = X$$

A1~5 M1~5 Are axioms in Book.

One of the basis of $P_n(\mathbb{R})$ =

$\{1, x, x^2, \cdots, x^n\}$ which has

$n+1$ vectors in the basis.

So $\dim(P_n(\mathbb{R})) = \boxed{n+1}$

(b). $\bigcup_{n=1}^{m} P_n = P_1 \cup P_2 \cup P_3 \cdots \cup P_m$

where $P_1 \sim P_{m-1} \subseteq P_m$

So related to the course, union of two subspaces of W is a subspace iff one of the subspaces is contained in the other.

Therefore, $\bigcup_{n=1}^{m} P_n$ is a subspace, $P_n(\mathbb{R})$ is a vector space

so $\bigcup_{n=1}^{m} P_n$ is also a vector space.

(c) . $P_4 = f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4$

$a(x^2+1) + b(x^2-1) + c = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4$

$ax^2 + a + bx^2 - b + c = \sim$

$(a+b)x^2 + (a-b) + c = \sim$

$a+b = c_2 \quad a-b = c_0 . \quad c = c_1 x + c_3 x^3 + c_4 x^4$

basis = $\boxed{\{x^2+1, x^2-1, x, x^3, x^4\}}$.

(d) $P_2 = c_0 + c_1 x + c_2 x^2$

$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

$a\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + c\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

$a = 0 \qquad b + c = 0 \qquad b = c = 0.$

$b + 2c = 0.$

$\therefore \boxed{\{1+x,\ x+x^2,\ x+2x^2\} \text{ is a basis for } P_2.}$

14) Problem 3.

ii) $\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n = 0$. $\alpha_1 \sim \alpha_n = 0$

$$\alpha_1 \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \alpha_2 \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \cdots + \alpha_n \begin{bmatrix} x_n \\ y_n \end{bmatrix} = 0.$$

$\alpha_1 x_1 + \alpha_2 x_2 + \cdots + \alpha_n x_n = 0.$

$\alpha_1 y_1 + \alpha_2 y_2 + \cdots + \alpha_n y_n = 0.$

$\because$ $x_1 \cdots x_n$ is linear independent

$\therefore$ $\alpha_1 = \alpha_2 = \alpha_3 = \cdots = \alpha_n = 0$. $= \alpha_1 \sim \alpha_n = 0$.

$\boxed{\text{So } z_1 \sim z_n \text{ are linearly independent}}$

(iii) If $x_1 \sim x_n$ is linear dependent,

We can have $\alpha_1 \sim \alpha_n \in F$ not all equal to 0.

However, if $y_1 \sim y_n$ are linear independent.

$z_1 \sim z_n$ can be linear independent

So, $\boxed{\text{it is not.}}$

(b). $\{x, y, z\}$ is a basis, which means $\{x, y, z\}$ are linear independent, and spanning the vector space V.

For $\{x+y, y+z, z+x\}$.

$$a(x+y) + b(y+z) + c(z+x) = 0. \quad ①$$

$$ax + ay + by + bz + cz + cx = 0. \quad ②$$

$$(a+c)x + (a+b)y + (b+c)z = 0. \quad ③$$

$\because$ x, y, z are linear independent,

$\therefore$ $a+c=0 \quad a+b=0 \quad b+c=0.$

$\therefore$ $a=b=c=0.$

$\therefore$ $x+y, y+z, z+x$ are also linearly independent

Also from equation ① $\to$ ③ we could see that $\{x+y, y+z, z+x\}$ could also produce all linear combination as $\{x, y, z\}$, s.t $\{x+y, y+z, z+x\}$ is a spanning set for V, so. $\{x+y, y+z, z+x\}$ is also a basis.

(5) Probem 4.

(a)
$$\begin{bmatrix} \alpha_{11} \sim \alpha_{1n} \\ \alpha_{21} \sim \alpha_{2n} \\ \alpha_{n1} \sim \alpha_{nn} \end{bmatrix} \times \begin{bmatrix} \beta_{11} \sim \beta_{1n} \\ \beta_{21} \sim \beta_{2n} \\ \beta_{n1} \sim \beta_{nn} \end{bmatrix} = 0.$$

$$\underset{A}{\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}} \times \underset{B}{\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}} = \begin{bmatrix} -1\times1+1\times1 & -1\times0+1\times0 \\ 0\times1+0\times1 & 0\times0+0\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

0

So $A \neq 0 \neq B \neq 0$.

$\boxed{\text{False}}$

(b)
$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} a\cdot a+b\cdot c & a\cdot b+b\cdot d \\ c\cdot a+d\cdot c & c\cdot b+d\cdot d \end{bmatrix}.$$

So if $a=b=d=0$ or $a=c=d=0$

result $=0$.

and $\begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix} \neq 0$.

$\boxed{\text{False}}$

(C) $A = a_{ij} \ \forall 1 \le i,j \le k \quad A^T = a_{ji}$

$B = A * A^T = b_{ij} = \sum_{n=1}^{k} a_{ik} a_{jk}$

Because $B = 0$.

$\therefore b_{ii} = 0$.

$\because b_{ii} = \sum_{n=1}^{i} \sum_{n=1}^{k} a_{ik}^2 = 0$.

$\therefore a_{ij}^2 = 0$

$\therefore$ every $a_{ij} = 0$ for $a \in R$

$\therefore A = 0$.

True

Program Assignment.

1. H is a column Full Rank Matrix, which means row

$n-k$ in H is linearly independent. And columns are linearly dependent.

$x \in N(H)$

$Hx = 0$.

$Rank(H) = n-k$ $\quad$ $Rank(H) + \dim(N(H)) = n$.

So $\dim(N(H)) = n-k-n$

$= k$.

$\because x \in F_2^n$

$\therefore x$ has 2^k possibilities.

So there are 2^k codewords in C.

H is not unique because, u $\quad$ y can general

H $n-k \times k$ colums. from C.

2 - $R(H) = \{ y \in F_2^{n-k}, y = Hx, x \in F_2^{?} \}$

$= \text{span}\{ h_1, h_2, \cdots, h_{n-k} \}$.

$\because$ h_1 ~ h_{n-k} is linearly independent

$\therefore$ cardinality of $R(H) = \dim(H)$

$= n-k$.

PA
3.

```
function output = checkCodeword(H,x)
    Out = H*x;
    Out = mod(Out,2);
    disp(Out)
    output = true;
    for i = 1:length(Out)
        a = mod(Out(i),2);
        if a ~= 0
            output = false;

        end
    end
end
```

Output1^T = [0 0 1 1]
Output2^T = [0 0 0 0]
Output3^T = [0 1 0 0]

4.

```
function [S,E] = buildTable(H)
    [~,col] = size(H);
    E1 = zeros(col,col+1);
    for i = 1:col
        E1(i,i)=1;
    end
    E = E1;
    S = mod(H*E,2);
end
```

E:

15x16 double

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
16																
17																

S:

4x16 double

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1	0
2	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1	0
3	0	0	1	0	0	1	0	1	0	1	1	0	1	1	1	0
4	0	0	0	1	0	0	1	0	1	1	0	1	1	1	1	0

5. For special H in eq(2).

H is in $F_2^{4\times 15}$, So E is in $F_2^{15\times 16}$.

Which S is in $F_2^{4\times 16}$. where fit $2^4=16$

so S contains every s in F_2^4,

which mean, any s in F_2^4 is a syndrome

of a e where e's l_1 norm atmost 1.

6.
(a)

```
function [e,x] = channelDecode(r)
    H1 = [1 0 0 0 1 1 1 0 0 0 1 1 1 0 1;
          0 1 0 0 1 0 0 1 1 0 1 1 0 1 1;
          0 0 1 0 0 1 0 1 0 1 1 0 1 1 1;
          0 0 0 1 0 0 1 0 1 1 0 1 1 1 1];

    [S,E] = buildTable(H1);
    s = mod(H1*r,2);
    S = transpose(S);
    s = transpose(s);
    %disp(s);
    locVector = ismember(S,s,'rows');
    %disp(locVector);
    loc = find(locVector);
    e = E(:,loc);
    x = abs(r-e);
end
```

(b)

```
[e,x] = channelDecode(x3);
%q6(b)
x4 = transpose([1 1 1 1 1 0 0 0 0 1 0 0 0 0 0]);
count_x4 = 0;
for i = 1:15
    r = x4;
    r(i,1) = abs(r(i,1)-1);
    [e,x] = channelDecode(r);
    if x == x4
        count_x4 = count_x4+1;
    end
end
x2 = [1 0 0 1 1 0 0 1 0 1 0 0 0 0 0];
x2 = transpose(x2);
count_x2 = 0;
for i = 1:15
    r = x2;
    r(i,1) = abs(r(i,1)-1);
    [e,x] = channelDecode(r);
    if x == x2
        count_x2 = count_x2+1;
    end
end
```

6(b). 15 times decoded successful.

If the x is in the code book, the count is still 15

6(c). 0 times successfully recover

because the E is only for L1-norm at most 1

and when we have 2 bit wrong, we couldnot

find a correct syndrome for an orignall signal.

Same result will happen for error bit=3.

(c)

```
%p6(c)
count_2x2 = 0;
count = 0;
for i = 1:15
    for j = 1:15
        if i == j
            continue;
        end
        count = count+1;
        r = x2;
        r(i,1) = abs(r(i,1)-1);
        r(j,1) = abs(r(j,1)-1);
        [e,x] = channelDecode(r);
        if x == x2
            count_2x2 = count_2x2+1;
        end
    end
end
```

All code:

```
H1 = [1 0 0 0 1 1 1 0 0 0 1 1 1 0 1;
      0 1 0 0 1 0 0 1 1 0 1 1 0 1 1;
      0 0 1 0 0 1 0 1 0 1 1 0 1 1 1;
      0 0 0 1 0 0 1 0 1 1 0 1 1 1 1];
x1 = [1 1 1 1 1 1 1 0 0 1 0 0 0 0 0];
x2 = [1 0 0 1 1 0 0 1 0 1 0 0 0 0 0];
x3 = [1 0 0 0 1 0 0 0 0 0 0 0 0 0 0];
x1 = transpose(x1);
x2 = transpose(x2);
x3 = transpose(x3);
o1 = checkCodeword(H1,x1);
o2 = checkCodeword(H1,x2);
o3 = checkCodeword(H1,x3);
[S,E] = buildTable(H1);
[e,x] = channelDecode(x3);
%q6(b)
x4 = transpose([1 1 1 1 1 0 0 0 0 1 0 0 0 0 0]);
count_x4 = 0;
for i = 1:15
   r = x4;
   r(i,1) = abs(r(i,1)-1);
   [e,x] = channelDecode(r);
   if x == x4
       count_x4 = count_x4+1;
   end
end
x2 = [1 0 0 1 1 0 0 1 0 1 0 0 0 0 0];
x2 = transpose(x2);
count_x2 = 0;
for i = 1:15
   r = x2;
   r(i,1) = abs(r(i,1)-1);
   [e,x] = channelDecode(r);
   if x == x2
       count_x2 = count_x2+1;
   end
end
%p6(c)
count_2x2 = 0;
count = 0;
for i = 1:15
   for j = 1:15
       if i == j
           continue;
       end
       count = count+1;
       r = x2;
```

```
        r(i,1) = abs(r(i,1)-1);
        r(j,1) = abs(r(j,1)-1);
        [e,x] = channelDecode(r);
        if x == x2
            count_2x2 = count_2x2+1;
        end
    end
end
function output = checkCodeword(H,x)
    Out = H*x;
    Out = mod(Out,2);
    disp(Out)
    output = true;
    for i = 1:length(Out)
        a = mod(Out(i),2);
        if a ~= 0
            output = false;

        end
    end
end
function [S,E] = buildTable(H)
    [~,col] = size(H);
    E1 = zeros(col,col+1);
    for i = 1:col
        E1(i,i)=1;
    end
    E = E1;
    S = mod(H*E,2);
end
function [e,x] = channelDecode(r)
    H1 = [1 0 0 0 1 1 1 0 0 0 1 1 1 0 1;
          0 1 0 0 1 0 0 1 1 0 1 1 0 1 1;
          0 0 1 0 0 1 0 1 0 1 1 0 1 1 1;
          0 0 0 1 0 0 1 0 1 1 0 1 1 1 1];
    [S,E] = buildTable(H1);
    s = mod(H1*r,2);
    S = transpose(S);
    s = transpose(s);
    %disp(s);
    locVector = ismember(S,s,'rows');
    %disp(locVector);
    loc = find(locVector);
    e = E(:,loc);
    x = abs(r-e);
end
```