Estimating Mortality Rates in Italy Using the Lee-Carter Model: National and Regional Perspectives

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Abstract

This paper implements the Lee-Carter model to estimate and forecast mortality rates in Italy between 2000 and 2023. The analysis is first conducted on national data and then disaggregated into three macro-areas (North, Center, and South). The results reveal both temporal trends and age-specific mortality profiles, highlighting regional heterogeneity. The goodness of fit is assessed using both graphical diagnostics and summary statistics such as MAE, RMSE, and R^2.

1 Introduction

Mortality forecasting is essential for public health, pension planning, and insurance pricing. The Lee-Carter model, first proposed in 1992, provides a parsimonious but powerful method to estimate age-specific mortality patterns over time. In this work, we apply the Lee-Carter methodology to Italian mortality data from 2000 to 2023, using both aggregate and regional-level datasets.

2 Methodology

Let $m_{x,t}$ denote the central mortality rate for age x in year t. The Lee-Carter model specifies:

$$\log m_{x,t} = a_x + b_x \cdot k_t + \varepsilon_{x,t}$$

where:

- a_x captures the average mortality at age x across time,
- b_x measures the sensitivity of log mortality to changes in k_t for age x,
- k_t reflects the overall time trend in mortality,
- $\varepsilon_{x,t}$ is a mean-zero residual.

The model is estimated by applying Singular Value Decomposition (SVD) to the centered log mortality matrix (by subtracting a_x row-wise). The first principal component is retained, and identification constraints are imposed such that $\sum_x b_x = 1$ and $\sum_t k_t = 0$ (implicitly satisfied by scaling).

3 Results

3.1 National-Level Estimation

Figures 1–3 show the national-level parameters:

- a_x increases almost linearly with age, reflecting exponential growth of mortality.
- b_x peaks around young adulthood (20–30 years) and decreases sharply thereafter.
- k_t exhibits a clear decreasing trend, suggesting overall mortality improvement over time.

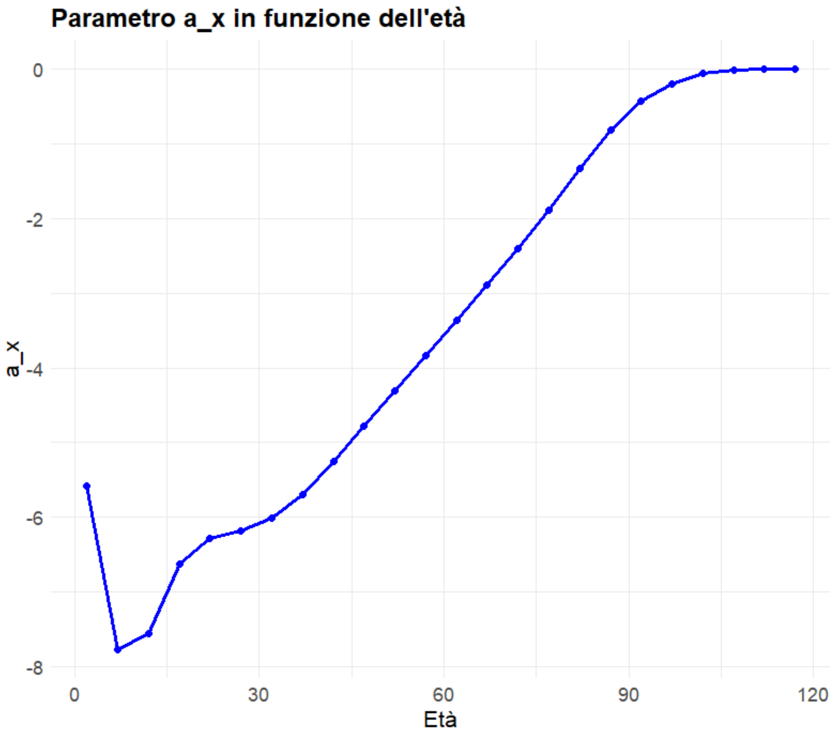

Figure 1: Estimated a_x for Italy

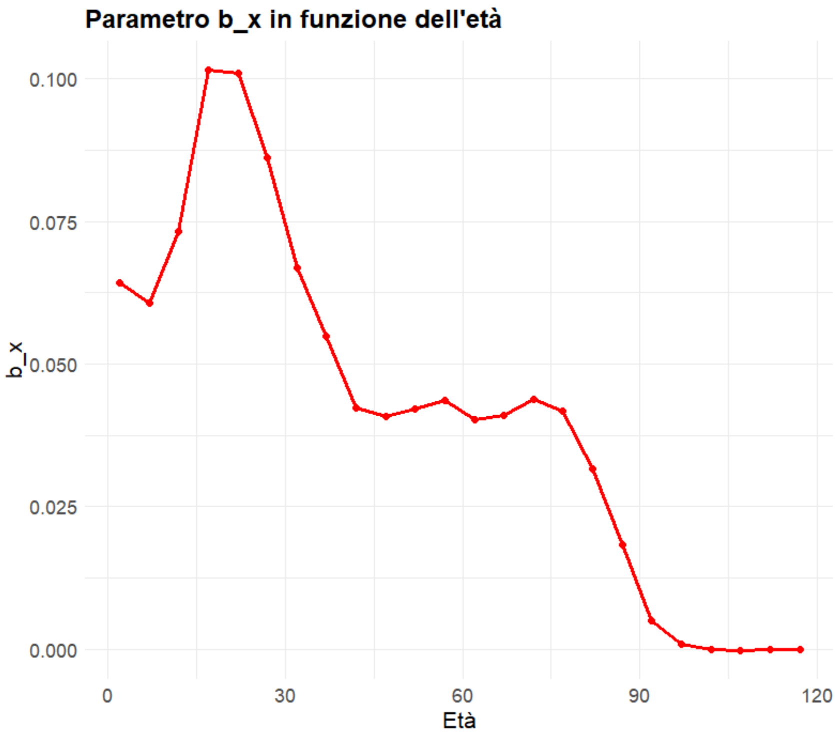

Figure 2: Estimated b_x for Italy

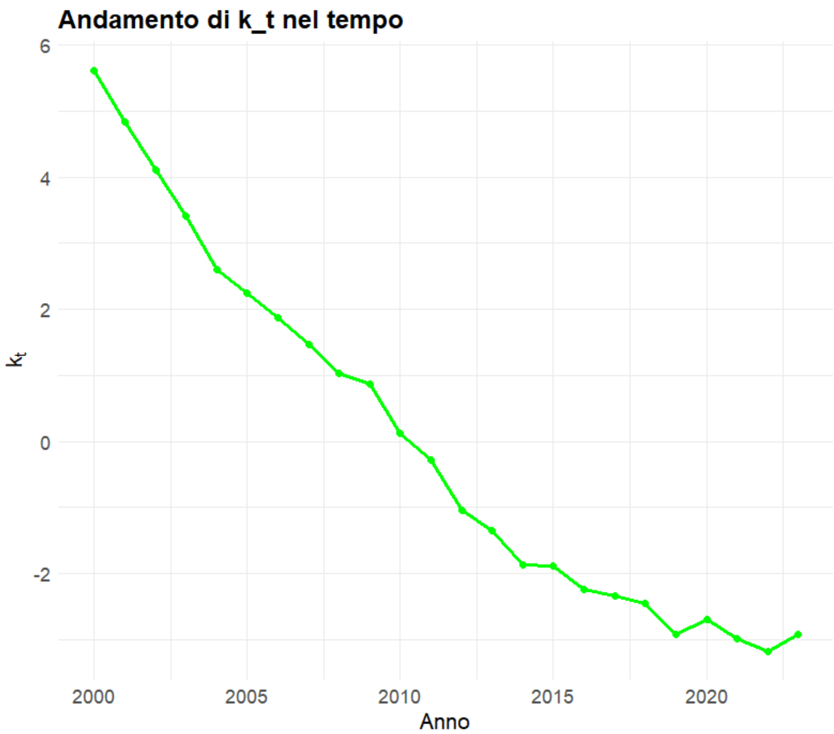

Figure 3: Estimated k_t for Italy (2000–2023)

3.2 Regional Comparison

The same model is estimated separately for the North, Center, and South macro-areas. Figures 4, 5, and 6 show the regional parameters.

- k_t is decreasing across all areas, though slightly faster in the North.
- b_x shows greater volatility in the South and more regularity in the North.
- a_x is remarkably stable across regions.

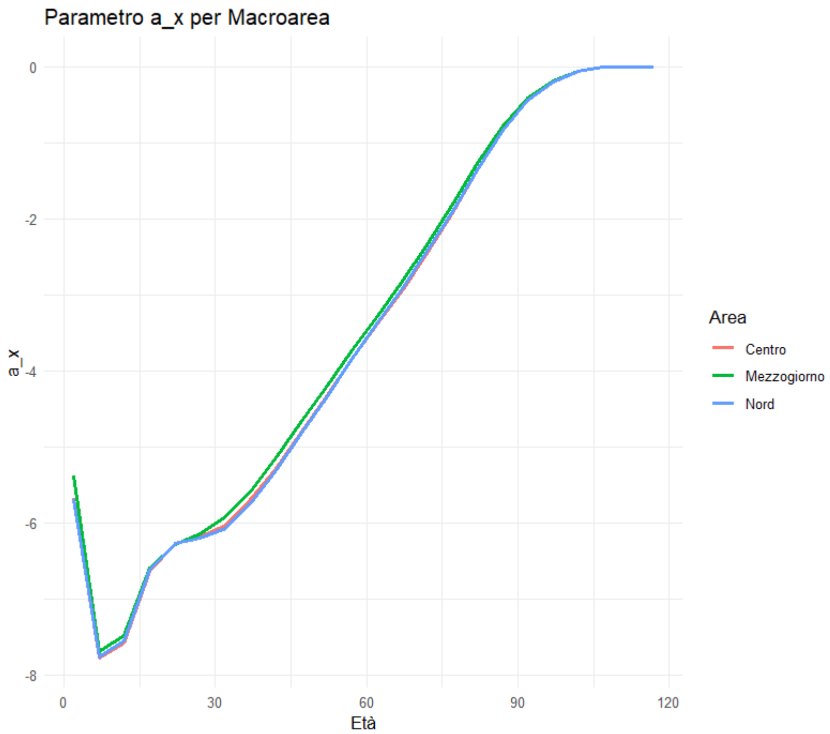

Figure 4: k_t by macroarea (North, Center, South)

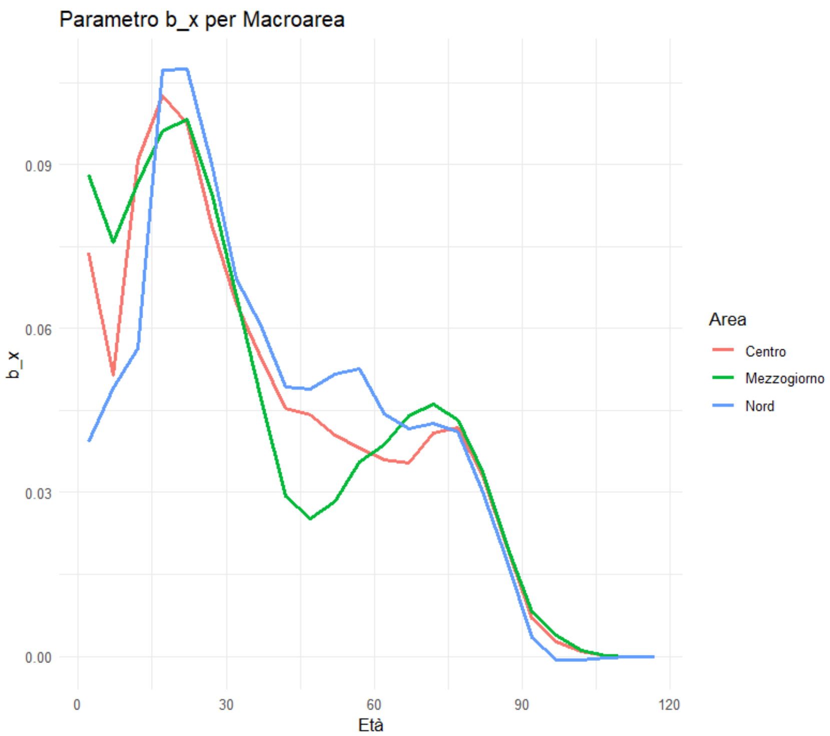

Figure 5: b_x by macroarea (North, Center, South)

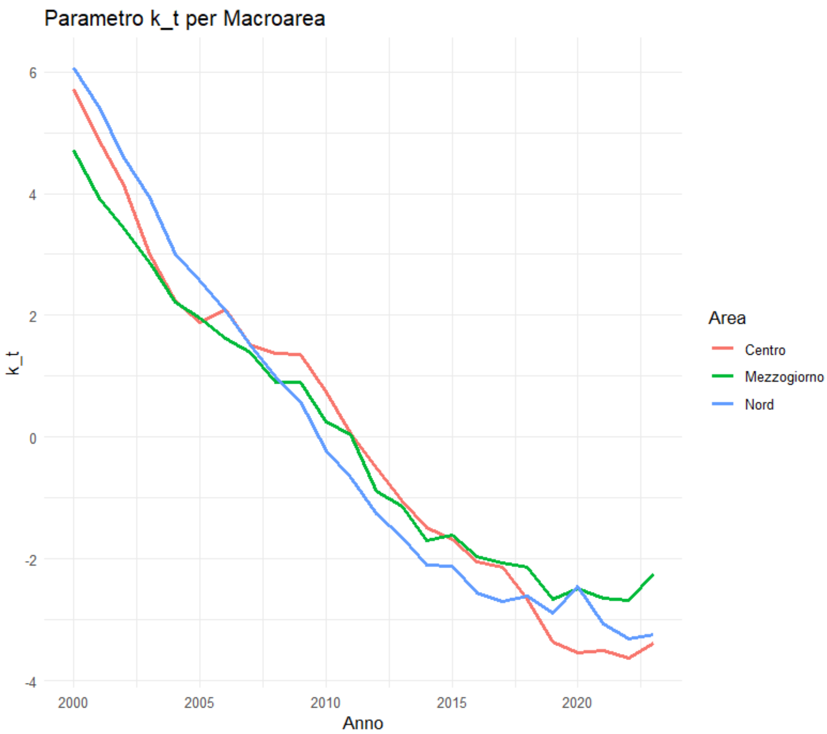

Figure 6: a_x by macroarea (North, Center, South)

3.3 Summary Statistics

Table 1: Mean and Standard Deviation of Estimated Parameters by Region

Parameter	North	Center	South
Mean a_x	-3.50	-3.49	-3.42
SD a_x	2.69	2.69	2.66
Mean b_x	0.0417	0.0417	0.0417
SD b_x	0.0321	0.0314	0.0327
Mean k_t	6.4×10^{-16}	-2.3×10^{-15}	1.4×10^{-16}
SD k_t	2.99	2.84	2.33

4 Policy Implications and Interpretation

The downward trend of k_t reflects a general improvement in life expectancy, particularly evident in the North. This trend suggests the need for long-term adjustments in:

- **Pension systems**: As people live longer, the retirement age and contribution structures may need revision to maintain actuarial balance.
- **Healthcare resource allocation**: The sharp decline in mortality at younger ages and rising longevity requires stronger focus on geriatric care, prevention, and chronic disease management.
- **Regional equity**: Differences in the volatility and speed of k_t decline highlight persistent gaps between macroareas, particularly between the North and South. Policymakers may consider targeted health interventions and investment in Southern regions.

Moreover, the Lee-Carter model, despite its strength, may underestimate sudden shocks (e.g., COVID-19) and regional disparities due to its linear structure. Therefore, integrating nonlinear components or regime-switching models could offer more resilience for policy scenario planning.

5 Conclusion

The Lee-Carter model provides an effective and interpretable method for mortality forecasting. At the national level, mortality rates in Italy have shown a continuous decline, particularly since the early 2000s. Across regions, while mortality improvement is consistent, the speed and variability of such improvements differ slightly—especially for the k_t component, where the North shows a more stable and linear decline compared to the South.

Despite some deviations from normality in residuals, the model fits the data well, with low error metrics (MAE, RMSE) and high R^2. However, the model does not account for sudden shocks or nonlinear trends, suggesting future extensions might integrate regime-switching or covariate-augmented models.