1.1) $y(t) = 0$ for all t

$u(t) = 0$

$y_1(t) = 0 \quad (u_1) \qquad u_1 = 0$

$y_2(t) = 0 \quad (u_2) \qquad u_2 = 0$

$y_3(t) = 0 \quad (u_3) \qquad u_3 = 0$

$y_3 = \alpha y_1 + \beta y_2 \qquad u_3 = \alpha u_1 + \beta u_2$

$y_3(t) = \alpha u_1 + \beta u_2$

$= \alpha y_1(t) + \beta y_2(t)$

satisfies add & homogenity, so linear

$y_1(t) = u_1(t)$

$y_2(t) = u_2(t) \qquad y_2(t) = u_2(t-\tau)$

$u_2(t) = u_2(t-\tau) \qquad y_1(t-\tau) = u_1(t-\tau) \qquad y_2(t) = y_1(t-\tau)$ so Time invariant

linear, time invariant

1.2) $y(t) = u^3(t)$

$y_1(t) = u_1^3(t) \qquad u_3 = \alpha u_1 + \beta u_2$

$y_2(t) = u_2^3(t)$

$y_3(t) = u_3^3(t)$

$$y_3 = (\alpha u_1 + \beta u_2)^3$$

$$= \alpha^3 u_1^3 + 3\alpha^2 \beta u_1^2 u_2 + 3\alpha\beta^2 u_1 u_2^2 + \beta^3 u_2^3$$

$$= \alpha^3 y_1 + 3\alpha^2 \beta u_1^2 u_2 + 3\alpha\beta^2 u_1 u_2^2 + \beta^3 y_2$$

non linear as $\neq \alpha y_1 + \beta y_2$

$y_1(t) = u_1^3(t) \qquad u_2(t) = u_1(t-\tau)$

$y_2(t) = u_2^3(t) = u_2^3(t-\tau)$

$y_1(t-\tau) = u_1^3(t-\tau)$

so Time invariant

non linear

1.3) $y(t) = u(3t)$

$y_1(t) = u_1(3t)$ $\quad u_3 = \alpha u_1 + \beta u_2$

$y_2(t) = u_2(3t)$

$y_3(t) = u_3(3t)$

$= \alpha u_1(3t) + \beta u_2(3t)$

$= \alpha y_1(t) + \beta y_2(3t)$ so linear

$y_3 = \alpha y_1 + \beta y_2$

$y_1(t) = u_1(3t) \leftarrow t \to t - \tau$

$y_2(t) = u_2(3t)$

$= u_2(3t - \tau)$ so time variant

$y_1(t-\tau) = u_1(3t - 3\tau)$ linear

1.4) $y(t) = e^{-t} u(t-T)$

$y_1(t) = e^{-t} u_1(t-T)$ $\quad u_3(t-T) = \alpha u_1(t-T) + \beta u_2(t-T)$

$y_2(t) = e^{-t} u_2(t-T)$

$y_3(t) = e^{-t} u_3(t-T)$

$= e^{-t} \alpha u_1(t-T) + e^{-t} \beta u_2(t-T)$

$= \cancel{e^{-t}} \alpha \cdot \dfrac{y_1(t)}{\cancel{e^{-t}}} + \cancel{e^{-t}} \cdot \beta \cdot \dfrac{y_2(t)}{\cancel{e^{-t}}}$

$= \alpha y_1(t) + \beta y_2(t)$ ✓ linear

$y_1(t) = e^{-t} u_1(t-T) \Rightarrow y_1(t-\tau) = \underline{e^{-t-\tau}} u_1(t-T-\tau)$

$y_2(t) = \underline{e^{-t}} u_2(t-T-\tau)$ $\quad e^{-t-\tau} \neq e^{-t}$, so time variant linear

1.5)
$$y(t) = \begin{cases} 0 & t \leq 0 \\ u(t) & t > 0 \end{cases} \qquad u_3 = \alpha u_1 + \beta u_2$$

$y_1(t) = u_1(t) \quad t > 0$

$y_2(t) = u_2(t) \quad t > 0$

$$Y_3(t) = u_3(t) \quad t \geq 0$$
$$= \alpha u_1 + \beta u_2 \quad t \geq 0 \qquad \text{so linear}$$
$$= \alpha Y_1 + \beta Y_2$$

for $t \leq 0$, $Y_3 = 0$, $Y_2 = 0$, $Y_1 = 0$

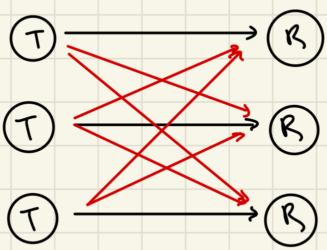
$$Y_1(t) = u_1(t) \quad t \geq 0 \implies t - \tau$$
$$Y_2(t) = u_2(t) \quad t \geq 0$$

$$Y_1(t-\tau) = u_1(t-\tau)$$
$$Y_1(t) = u_2(t-\tau)$$

Time invariant

linear

2.1)



noise + interference: $q_i = \sigma^2 + \sum_{j \neq i} G_{ij} P_j$

$$S_i(k) = s_i(k) / q_i(k) = \alpha\gamma$$

$$s_i = G_{ii} P_i$$

$$P_i(k+1) = P_i(k)\left(\alpha\gamma / S_i(k)\right)$$

$$P_i(k+1) = P_i(k) \cdot \alpha\gamma \cdot \frac{q_i(k)}{s_i(k)} = P_i(k) \cdot \alpha\gamma \cdot \frac{\sigma^2 + \sum_{j \neq i} G_{ij} P_j(k)}{G_{ii} P_i(k)}$$

$$= P_i\cancel{(k)} \cdot \alpha\gamma \cdot \frac{\sigma^2}{G_{ii} P_i \cancel{(k)}}$$

$$+ P_i\cancel{(k)} \cdot \alpha\gamma \cdot \frac{\sum_{j \neq i} G_{ij} P_j(k)}{G_{ii} P_i \cancel{(k)}}$$

$$P_i(k+1) = \frac{\alpha\gamma}{G_{ii}} \sigma^2 + \frac{\alpha\gamma}{G_{ii}} \cdot \sum_{j \neq i} G_{ij} P_j(k)$$

$$= \begin{bmatrix} \frac{dr}{G_{11}} \\ \frac{dr}{G_{22}} \\ \frac{dr}{G_{33}} \end{bmatrix} \sigma^2 + \begin{bmatrix} \frac{dr}{G_{11}} \\ \frac{dr}{G_{22}} \\ \frac{dr}{G_{33}} \end{bmatrix} \cdot \begin{bmatrix} 0 & G_{12} & G_{13} \\ G_{21} & 0 & G_{23} \\ G_{31} & G_{32} & 0 \end{bmatrix} P_j(K)$$

$$= \underbrace{dr \begin{bmatrix} 0 & \frac{G_{12}}{G_{11}} & \frac{G_{13}}{G_{11}} \\ \frac{G_{21}}{G_{22}} & 0 & \frac{G_{23}}{G_{22}} \\ \frac{G_{31}}{G_{33}} & \frac{G_{32}}{G_{33}} & 0 \end{bmatrix}}_{A} P_j(K) + dr \underbrace{\begin{bmatrix} \frac{1}{G_{11}} \\ \frac{1}{G_{22}} \\ \frac{1}{G_{33}} \end{bmatrix}}_{B} \sigma^2$$

3) $\ddot{y} + (1+y)\dot{y} - 2y + 0.5y^3 = 0$

$x_1 = y \;,\; x_2 = \dot{y} \;,\; \dot{x}_1 = x_2 \;,\; \dot{x}_2 = -(1+x_1)x_2 + 2x_1 - 0.5x_1^3$

for $x_2 = 0$,

$$-(1+x_1)0 + 2x_1 - 0.5x_1^3 = 0$$

$$2x_1 - 0.5x_1^3 = 0$$

$$x_1(2 - 0.5x_1^2) = 0 \qquad \begin{cases} x_1 = 0 \\ x_1 = 2 \\ x_1 = -2 \end{cases}$$

$$2 - 0.5x_1^2 = 0$$

$$x_1 = \sqrt{\frac{2}{0.5}}$$

$$= 2, -2$$

J for $\dot{x} = f(x)$, $J = \left(\frac{\partial f_1}{\partial x_1} \;\; \frac{\partial f_1}{\partial x_2} \;\; \frac{\partial f_2}{\partial x_1} \;\; \frac{\partial f_2}{\partial x_2} \right)$

$f_1 = x_2 \;,\; f_2 = -x_2 - x_1x_2 + 2x_1 - 0.5x_1^3$

$\frac{\partial f_1}{\partial x_1} = 0 \;,\; \frac{\partial f_1}{\partial x_2} = 1 \;,\; \frac{\partial f_2}{\partial x_1} = -x_2 + 2 - 1.5x_1^2 \;,\; \frac{\partial f_2}{\partial x_2} = -1 - x_1$

$$J = \begin{bmatrix} 0 & 1 \\ -x_2 + 2 - 1.5x_1^2 & -1 - x_1 \end{bmatrix}$$

For $x_1 = 0, x_2 = 0$, $J = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$

$x_1 = 2, x_2 = 0$, $J = \begin{bmatrix} 0 & 1 \\ -4 & -3 \end{bmatrix}$

$x_1 = -2, x_2 = 0$, $J = \begin{bmatrix} 0 & 1 \\ -4 & 1 \end{bmatrix}$

4.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ -g\left(\frac{D}{x_1(t)+D}\right)^2 + \frac{\ln(u)}{m} \end{bmatrix}$$

$$x_2(t) = 0$$

$$-g\left(\frac{D}{x_1(t)+D}\right)^2 + \frac{\ln(u)}{m} = 0$$

$$\left(\frac{D}{x_1(t)+D}\right)^2 = \frac{\ln(u)}{mg}$$

$$\frac{D}{x_1(t)+D} = \sqrt{\frac{\ln(u)}{mg}}$$

$$x_1(t) = \frac{D}{\sqrt{\frac{\ln(u)}{mg}}} - D$$

$$\chi = \begin{bmatrix} \frac{D}{\sqrt{\frac{\ln(u)}{mg}}} - D \\ 0 \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 \\ -\frac{2gD^2}{(x_1+D)^3} & 0 \end{bmatrix}$$

$$\frac{\partial f_2}{\partial x_1} = -g\left(\frac{D^2}{(x_1(t)+D)^2}\right)$$

$$= -g \cdot D^2 \cdot \frac{\frac{\partial}{\partial x}(x+D)^2}{((x+D)^2)^2}$$

$$= -gD^2 \cdot \frac{\frac{\partial}{\partial x}(x+D)^2 \cdot \frac{\partial}{\partial x}(x+D)}{(x+D)^4}$$

$$= -gD^2 \cdot \frac{2\cancel{(x+D)} \cdot 1}{(x+D)^{\cancel{4}\,3}}$$

$$= -\frac{2gD^2}{(x+D)^3}$$

$$= \begin{bmatrix} 0 & 1 \\ \frac{-2gD^2}{\left(\frac{D}{\sqrt{\frac{\ln(u)}{mg}}}\right)^3} & 0 \end{bmatrix}$$

$$\frac{-2g\cancel{D^2}}{\frac{D^{\cancel{3}}}{\left(\frac{\ln(u)}{mg}\right)^{\frac{3}{2}}}} \Rightarrow -\frac{2g\left(\frac{\ln(u)}{mg}\right)^{\frac{3}{2}}}{D}$$

$$\delta\dot{x} = A\delta x = \begin{bmatrix} 0 & 1 \\ -\frac{2g}{D}\left(\frac{\ln(u)}{mg}\right)^{\frac{3}{2}} & 0 \end{bmatrix}\begin{bmatrix} x_1 & \dot{x}_1 \\ x_2 & \dot{x}_2 \end{bmatrix}$$

5)

$$\ddot{r} = r\dot{\theta}^2 - \frac{k}{r^2} + u_1 \quad , \quad \ddot{\theta} = -2\frac{\dot{\theta}}{r}\dot{r} + \frac{u_2}{r}$$

@ reference orbit: $u_1 = u_2 = 0$, $r(t) = p$, $\theta(t) = wt$, $\dot{r} = 0$ $\ddot{r} = 0$, $\dot{\theta} = \omega$ $\ddot{\theta} = 0$

$$\ddot{r} = r\dot{\theta}^2 - \frac{k}{r^2} \quad , \quad \ddot{\theta} = -2\frac{\dot{\theta}}{r}\dot{r}$$

$$0 = pw^2 - \frac{k}{p^2} \quad , \quad 0 = -2\frac{\omega}{p}\cdot 0$$

$$pw^2 = \frac{k}{p^2} \Rightarrow k = p^3\omega^2$$

5.2)

$$r = p + \delta r \quad , \quad \theta = wt + \delta\theta \qquad u_1 = \delta u_1$$

$$\dot{r} = \delta\dot{r} \quad , \quad \dot{\theta} = \omega + \delta\dot{\theta} \qquad u_2 = \delta u_2$$

$$\ddot{r} = \delta\ddot{r} \quad , \quad \ddot{\theta} = \delta\ddot{\theta}$$

$$\delta\ddot{r} = (p+\delta r)(w+\delta\dot{\theta})^2 - \frac{k}{(p+\delta r)^2} + \delta u_1$$

$$= (p+\delta r)(w^2 + 2w\delta\dot{\theta} + \cancel{\delta\dot{\theta}^2}) - \frac{p^3w^2}{(p+\delta r)^2} + \delta u_1$$

$$= pw^2 + 2pw\delta\dot{\theta} + \cancel{p\delta\dot{\theta}^2} + w^2\delta r + \cancel{2w\delta r}\delta\dot{\theta} + \cancel{\delta r\delta\dot{\theta}^2} - \underbrace{p^3w^2p^{-2}\left(1+\frac{\delta r}{p}\right)^{-2}}_{pw^2\left(1-2\frac{\delta r}{p}\right)} + \delta u_1$$

$$= \cancel{pw^2} + 2pw\delta\dot{\theta} + w^2\delta r - \cancel{pw^2} + 2w^2\delta r + \delta u_1$$

$$\delta\ddot{r} = 3w^2\delta r + 2pw\delta\dot{\theta} + \delta u_1$$

$$\frac{\partial \ddot{\theta}}{\partial r} = \frac{2\dot{\theta}\dot{r}}{r^2} - \frac{u_2}{r^2} \qquad \frac{\partial \ddot{\theta}}{\partial \theta} = 0 \qquad \frac{\partial \ddot{\theta}}{\partial u_2} = \frac{1}{r}$$

$$\frac{\partial \ddot{\theta}}{\partial \dot{r}} = \frac{-2\dot{\theta}}{r} \qquad \frac{\partial \ddot{\theta}}{\partial \dot{\theta}} = -\frac{2\dot{r}}{r}$$

$$\delta\ddot{\theta} = -\frac{2w}{p}\delta\dot{r} + \frac{u_2}{p}$$

$$\begin{bmatrix} \delta r \\ \delta \dot{r} \\ \delta \theta \\ \delta \dot{\theta} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\delta\ddot{r} = 3w^2 x_1 + 2pw x_4 + u_1$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3w^2 & 0 & 0 & 2pw \\ 0 & 0 & 1 & 0 \\ 0 & -\frac{2w}{p} & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & \frac{1}{p} \end{bmatrix}$$