Robotics 1

January 11, 2022

Exercise #1

Consider the planar RPR robot with L-shaped forearm in Fig. 1, shown with the base reference frame RF_0 and the end-effector frame RF_e attached to the gripper.

i. Assign a set of Denavit-Hartenberg (D-H) frames to the robot. The origin of the last D-H frame should be at the point P.

ii. Fill in the associated table of parameters.

iii. Draw the robot in the configuration $\boldsymbol{q} = \mathbf{0}$.

iv. Give the expression of the position $\boldsymbol{p}$ of point P and of the orientation ${}^0\boldsymbol{R}_3$ of the D-H frame RF_3 when the robot is in the configuration $\boldsymbol{q} = \mathbf{0}$.

v. Determine the constant homogeneous matrix ${}^3\boldsymbol{T}_e$.

vi. Give the symbolic expression of all triples $(\alpha_1, \alpha_2, \alpha_3)$ of XYX Euler angles that realize the rotation matrix ${}^3\boldsymbol{R}_e$. Provide the numerical values of these Euler angles when $L = M = 1$.

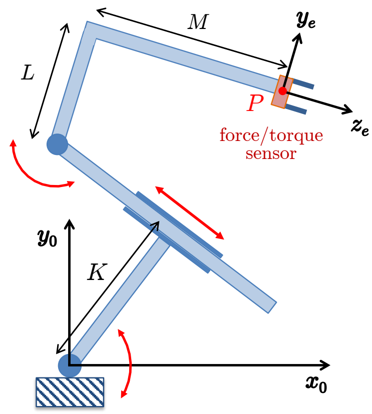

Figure 1: A RPR robot with L-shaped forearm. A force/torque sensor is mounted at the gripper.

Exercise #2

Let a task vector associated to the RPR robot of Fig. 1 be defined as

$$\boldsymbol{r} = \begin{pmatrix} p_x \\ p_y \\ \phi \end{pmatrix} = \boldsymbol{t}(\boldsymbol{q}) \in \mathbb{R}^3,$$

with the Cartesian coordinates (p_x, p_y) of the point P in the plane and the orientation angle ϕ of the D-H axis $\boldsymbol{x}_3$ w.r.t. the base axis $\boldsymbol{x}_0$.

i. Determine the closed-form expression of the inverse kinematics for a given $\boldsymbol{r}_d = \begin{pmatrix} p_{xd} & p_{yd} & \phi_d \end{pmatrix}^T$.

ii. Provide the numerical values of all inverse solutions for the following data: $K = L = M = 1$ [m]; $p_{xd} = 2$, $p_{yd} = 1$ [m]; $\phi = -\pi/6$ [rad].

Exercise #3

i. Compute the 3×3 task Jacobian $\boldsymbol{J}_t(\boldsymbol{q})$ associated to the task vector function $\boldsymbol{r} = \boldsymbol{t}(\boldsymbol{q})$ defined in Exercise #2.

ii. Find all the singularities of the matrix $\boldsymbol{J}_t(\boldsymbol{q})$.

iii. In a singular configuration $\boldsymbol{q}_s$, determine a basis for the null space $\mathcal{N}\{\boldsymbol{J}_t(\boldsymbol{q}_s)\}$ and a basis for the range space $\mathcal{R}\{\boldsymbol{J}_t(\boldsymbol{q}_s)\}$. Both bases should be *globally* defined, namely they should have a constant dimension for all possible $\boldsymbol{q}$ such that $\boldsymbol{J}_t(\boldsymbol{q})$ is singular.

iv. Set now $K = L = M = 1$ [m]. Find a task velocity $\dot{\boldsymbol{r}}_f \in \mathcal{R}\{\boldsymbol{J}_t(\boldsymbol{q}_s)\}$ and an associated joint velocity $\dot{\boldsymbol{q}}_f \in \mathbb{R}^3$ realizing it, i.e., such that $\boldsymbol{J}_t(\boldsymbol{q}_s)\,\dot{\boldsymbol{q}}_f = \dot{\boldsymbol{r}}_f$. Is this $\dot{\boldsymbol{q}}_f$ unique?

Exercise #4

Make again reference to the RPR robot shown in Fig. 1. The robot has a force/torque sensor mounted at the gripper which measures in the reference frame RF_e the two linear components ${}^e f_y$ and ${}^e f_z$ of the force ${}^e\boldsymbol{f} \in \mathbb{R}^3$ and the angular component ${}^e m_x$ of the torque ${}^e\boldsymbol{m} \in \mathbb{R}^3$. The other force/torque components are zero. Define the gripper *wrench* as ${}^e\boldsymbol{F} = \left({}^e\boldsymbol{f}^T \; {}^e\boldsymbol{m}^T \right)^T \in \mathbb{R}^6$, when expressed in frame RF_e. Assume again $K = L = M = 1$ [m] and that the robot is in the configuration $\bar{\boldsymbol{q}} = (\pi/2, -1, 0)$ [rad,m,rad], with the gripper in contact with an external environment.

i. If the sensor measures

$$ {}^e f_y = -1 \text{ N}, \qquad {}^e f_z = -2 \text{ N}, \qquad {}^e m_x = 2 \text{ Nm}, $$

what is the value of the gripper wrench $\boldsymbol{F} = \left(\boldsymbol{f}^T \; \boldsymbol{m}^T \right)^T \in \mathbb{R}^6$, as expressed in the absolute frame RF_0?

ii. Compute the vector $\boldsymbol{\tau} \in \mathbb{R}^3$ of forces/torques at the three joints that balances in static conditions the gripper wrench measured by the sensor.

Hint: It is convenient here to work with the complete geometric *Jacobian of the robot.*

Exercise #5

Consider the elliptic path shown in Fig. 2, with major (horizontal) semi-axis of length $a > 0$ and minor (vertical) semi-axis of length $b < a$.

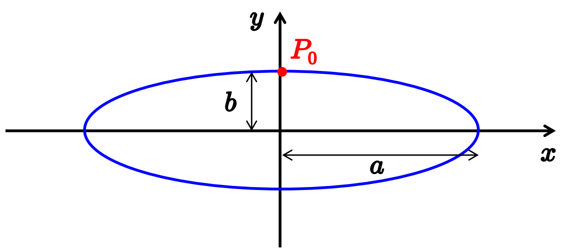

Figure 2: An elliptic path to be parametrized by $\boldsymbol{p}_d(s)$.

i. Choose a smooth parametrization $\boldsymbol{p}_d(s) \in \mathbb{R}^2$, with $s \in [0, 1]$, of the full elliptic path starting at $P_0 = (0, b)$.

ii. Provide a timing law $s = s(t)$ that traces the path counterclockwise with a constant speed $v > 0$ on the path. What will be the motion time T for completing the full ellipse?

iii. The following bounds on the norms of the velocity and of the acceleration should be satisfied along the resulting trajectory $\boldsymbol{p}_d(t) \in \mathbb{R}^2$, for all $t \in [0, T]$:

$$\|\dot{\boldsymbol{p}}_d(t)\| \leq V_{max}, \qquad \|\ddot{\boldsymbol{p}}_d(t)\| \leq A_{max}, \qquad \text{with } V_{max} > 0 \text{ and } A_{max} > 0.$$

Accordingly, what will be the maximum feasible speed v_f for this motion?

iv. Provide the numerical values of the maximum feasible speed v_f and of the resulting motion time T_f for the following data: $a = 1$, $b = 0.3$ [m]; $V_{max} = 3$ [m/s]; $A_{max} = 6$ [m/s^2].

Exercise #6

A planar 2R robot has its base placed at the center of the ellipse of Fig. 2, as shown in Fig. 3. The robot has the first link of length a and the second link of length $b < a$, the *same* values of the semi-axes of the ellipse. The position $\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{q})$ of its end effector (point P) should follow the trajectory $\boldsymbol{p}_d(t)$ defined in a parametric way in Exercise #5, with a path speed $v = 0.4$ [s^{-1}].

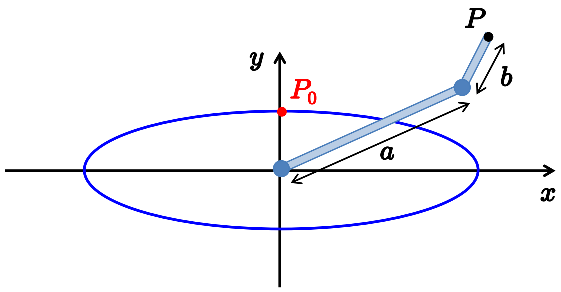

Figure 3: The placement of the 2R robot with respect to the ellipse of Fig. 2.

i. What are the conditions on $a > b > 0$ in order for the robot to be able to reach all points of the desired trajectory $\boldsymbol{p}_d(t)$ while avoiding any robot singularity? Choose numerical values for a and for $b < a$ that satisfy these conditions and keep these values for the rest of this exercise.

ii. Choose an initial robot configuration $\boldsymbol{q}_n(0)$ so as to match the desired trajectory $\boldsymbol{p}_d(t)$ at time $t = 0$, i.e., with initial Cartesian error $\boldsymbol{e}(0) = \boldsymbol{p}_d(0) - \boldsymbol{f}(\boldsymbol{q}_n(0)) = \boldsymbol{0}$.

iii. What nominal joint velocity command $\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}}_n(t)$ should be given for $t \in [0, T]$ in order to execute perfectly the entire trajectory $\boldsymbol{p}_d(t)$ with matched initial conditions?

iv. Choose another initial configuration $\boldsymbol{q}(0)$ such that $\boldsymbol{e}(0) \neq \boldsymbol{0}$, but with the y-component of the error $e_y(0) = 0$. Design a joint velocity control law $\dot{\boldsymbol{q}} = \dot{\boldsymbol{q}}_c(\boldsymbol{q}, t)$, with a feedback term depending on the current configuration $\boldsymbol{q}$, that will let $e_x(t)$ converge to zero with exponential decaying rate $r = 5$ and keep $e_y(t) = 0$ for all $t \geq 0$.

v. With the available data, compute the numerical values of the initial nominal joint velocity command $\dot{\boldsymbol{q}}_n(0) \in \mathbb{R}^2$ and of the initial joint velocity control law $\dot{\boldsymbol{q}}_c(\boldsymbol{q}(0), 0) \in \mathbb{R}^2$.

Exercise #7

The joint of the final flange of a 6R robot has a range of 700°. The driving motor is connected to the joint through a transmission with reduction ratio $n_r = 30$ and mounts a multi-turn absolute encoder. If we want to count the motor revolutions needed to cover the entire joint range and obtain an angular resolution of the final flange of less than 0.02°, how many bits should the multi-turn absolute encoder have at least?

[270 minutes (4.5 hours); open books]