Robotics 1

October 21, 2022

Exercise 1a

For the spatial RPR robot of Fig. 1, complete the assignment of Denavit-Hartenberg (DH) frames and fill in the associated table of parameters. The origin of the last frame should be placed at the point P. Moreover, the frame assignment should be such that all constant DH parameters are non-negative and the value of the joint variables q_i, $i = 1, 2, 3$, are strictly positive in the shown configuration. Compute then the direct kinematics $\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{q})$ for the position of the point P.

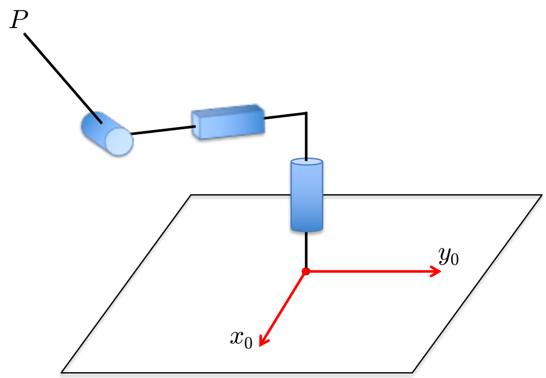

Figure 1: A spatial RPR robot.

Exercise 1b

Provide the Jacobian $\boldsymbol{J}(\boldsymbol{q})$ of this robot relating the joint velocity $\dot{\boldsymbol{q}} \in \mathbb{R}^3$ to the velocity $\boldsymbol{v} = \dot{\boldsymbol{p}} \in \mathbb{R}^3$ of P and determine all its singularities. For each singularity, determine the rank of $\boldsymbol{J}$, a basis for the null space motion, and the Cartesian direction(s) where instantaneous mobility of P is lost.

Exercise 1c

Determine a joint velocity control law that will eventually bring the robot end-effector to a generic desired position $\boldsymbol{p}_d \in \mathbb{R}^3$ in the reachable workspace, starting from any initial position $\boldsymbol{p}(0)$ and moving the end-effector always along a straight line without the need of planning a trajectory.

Exercise 2

A planar 2R robot having link lengths $L_1 = 2$ [m] and $L_2 = 1$ [m] is commanded by joint accelerations $\ddot{\boldsymbol{q}}$ with a bang-bang profile, under the joint velocity limits $|\dot{q}_1| \le V_{max,1} = 2$ [rad/s] and $|\dot{q}_2| \le V_{max,2} = 1.5$ [rad/s]. The robot should move its end-effector between the two points

$$P_{in} = \begin{pmatrix} 2 + 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \text{ [m]} \qquad \rightarrow \qquad P_{fin} = \begin{pmatrix} 3/\sqrt{2} \\ -1/\sqrt{2} \end{pmatrix} \text{ [m]},$$

i) with zero initial and final velocity, *ii)* in minimum time, *iii)* in a coordinated way, with both joints starting and ending their motion at the same instant, and *iv)* without crossing any singular configuration. Provide the minimum time T and the maximum absolute values $A_i > 0$, $i = 1, 2$, of the joint accelerations. Draw the time-optimal profiles of $\ddot{q}_1(t)$ and $\ddot{q}_2(t)$, for $t \in [0, T]$.

[180 minutes, open books]