Robotics 1

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Exercise 1

Figure 1 shows two views of TIAGo, a mobile manipulator by PAL Robotics. Disregard the wheeled base and consider only the motion of the 8-dof robotic arm with respect to the on-board reference frame RF_0 as described hereafter. The first prismatic joint of the arm (with axis in blue/dashed) provides elevation to the rest of the structure and is followed by 7 revolute joints (with axes in red/dashed): the axes of joints #1 and #2 are parallel, joints #3 and #4 intersect at the shoulder, joints #4 to #6 intersect at the elbow, while the last three joints (#6 to #8) constitute a spherical wrist with center at W. Videos showing the TIAGo arm mobility can be seen on YouTube[1].

Assign the kinematic frames to the arm links, following the classical Denavit-Hartenberg (DH) convention. Draw clearly the frames and fill in the associated table of parameters using one (or both) of the two extra sheets that have been distributed. Keep the frame RF_0 as first DH frame, and place the origin of the last DH frame at the wrist center point W.

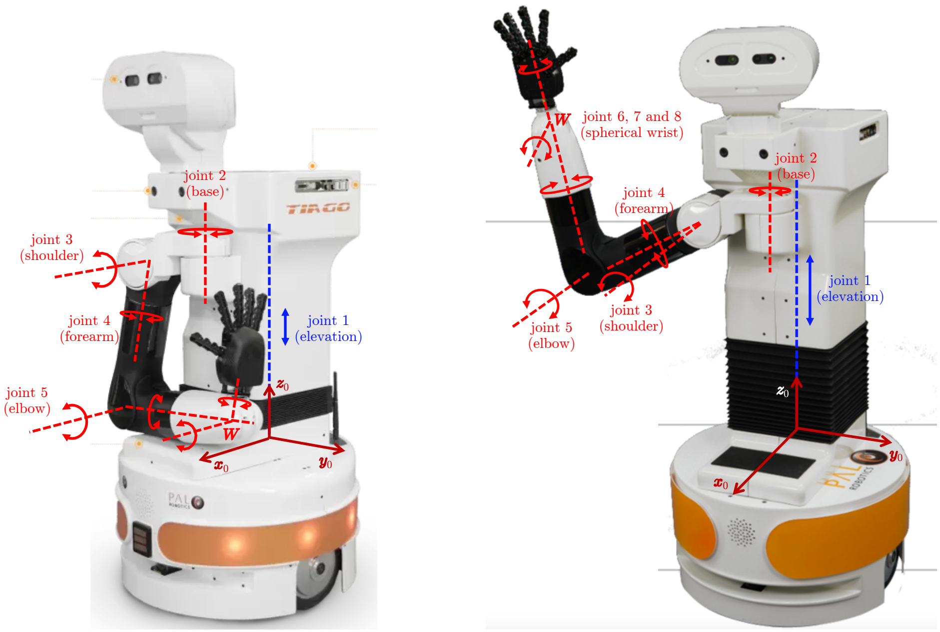

Figure 1: The TIAGo mobile manipulator, with the 8-dof arm shown in two configurations.

Exercise 2

Let the origin of frame RF_0 of the TIAGo robotic arm be at a position ${}^w\boldsymbol{p}_0 = (1.5, -4.5, 0.3)$ [m] with respect to a world frame RF_w placed horizontally on the floor surface. Moreover, let the angle between $\boldsymbol{y}_w$ and $\boldsymbol{y}_0$ axes be $\phi = -45°$. With the TIAGo robotic arm in a generic configuration $\boldsymbol{q}$, use the formula that evaluates the position ${}^w\boldsymbol{p}_W$ of the wrist center W with the minimum number of elementary operations. Provide then the symbolic expression of ${}^w\boldsymbol{p}_W(\boldsymbol{q})$ in explicit form.

[1] TIAGo - Robot Workspace versatility: `https://youtu.be/6BwRqwD066g` (1'08"); TIAGo - Gravity compensation: `https://youtu.be/EjIggPKy0T0` (1'23").

Exercise 3

For which interval of values of the angle $\theta_2 \in (-\pi, \pi]$ does the transcendental equation

$$\sin\theta_1 + 2\cos(\theta_1 + \theta_2) = 2 \tag{1}$$

have real solutions for the angle θ_1?

Exercise 4

For a 4-dof robot, consider the task vector

$$\boldsymbol{r} = \boldsymbol{f}(\boldsymbol{q}) = \begin{pmatrix} q_2 \cos q_1 + q_4 \cos(q_1 + q_3) \\ q_2 \sin q_1 + q_4 \sin(q_1 + q_3) \\ q_1 + q_3 \end{pmatrix}. \tag{2}$$

Determine all singular configurations for the corresponding analytic robot Jacobian $\boldsymbol{J}(\boldsymbol{q})$. Moreover, find *if possible*:

- a joint velocity $\dot{\boldsymbol{q}}_0 \neq \boldsymbol{0}$ such that $\dot{\boldsymbol{r}} = \boldsymbol{0}$ when the robot is in a regular configuration;
- all joint velocities $\dot{\boldsymbol{q}}$ such that $\dot{\boldsymbol{r}} = \boldsymbol{0}$ when the robot is in a singular configuration;
- the direction(s) along which no task velocity can be realized when the robot is in the chosen singular configuration;
- a generalized task force $\boldsymbol{f}_0 \neq \boldsymbol{0}$ that is statically balanced by the joint torque $\boldsymbol{\tau} = \boldsymbol{0}$ when the robot is in a regular configuration;
- all generalized task forces $\boldsymbol{f}$ that can be statically balanced by zero joint torque when the robot is in the chosen singular configuration.

Exercise 5

The end-effector of a robot manipulator should follow an helical path $\boldsymbol{p} = \boldsymbol{p}(s)$, parametrized by the scalar $s \geq 0$. The helix is right-handed, with radius $r = 0.4$ m and pitch $2\pi h$, with $h = 0.3$ m, starting from the position $\boldsymbol{p}_0 = (0, 0, 2r)$ at $s = 0$. Its axis passes through the point $C = (0, 0, r)$ and is parallel to the $\boldsymbol{y}$-axis. In the time interval $t \in [0, T]$, the robot end-effector should trace two complete turns of the helix, starting and ending its (rest-to-rest) motion with zero velocity, i.e., with $\dot{\boldsymbol{p}}(0) = \dot{\boldsymbol{p}}(T) = \boldsymbol{0}$.

Plan a timing law $s = s(t)$ that minimizes the motion time T under the following bounds on the norm of the velocity and on the (absolute) tangential and normal accelerations,

$$\|\dot{\boldsymbol{p}}\| \leq V, \qquad \left|\ddot{\boldsymbol{p}}^T \boldsymbol{t}\right| \leq A, \qquad \left|\ddot{\boldsymbol{p}}^T \boldsymbol{n}\right| \leq A, \tag{3}$$

where $\boldsymbol{t} = \boldsymbol{t}(s)$ and $\boldsymbol{n} = \boldsymbol{n}(s)$ are the unit axes of the Frenet frame tangent and normal to the path. Determine the minimum time T^* when $V = 2$ m/s and $A = 4.5$ m/s^2. Sketch the profiles of $s(t)$, $\dot{s}(t)$ and $\ddot{s}(t)$ in the obtained time-optimal solution.

Consider next a spatial elbow-type 3R robot manipulator with its base on the plane $z = 0$, height $L_1 = 0.8$ m between base and shoulder, and length of the second and third link $L_2 = L_3 = 1.5$ m. Determine a good placement (x_b, y_b) of the robot base, such that the complete helical path belongs to the primary workspace of the robot and kinematic singularities are not encountered while the end-effector is tracing the path.

[270 minutes, open books]