# Robotics I

**June 10, 2022**

## Exercise 1

Consider the spatial 3R robot in Fig. 1.

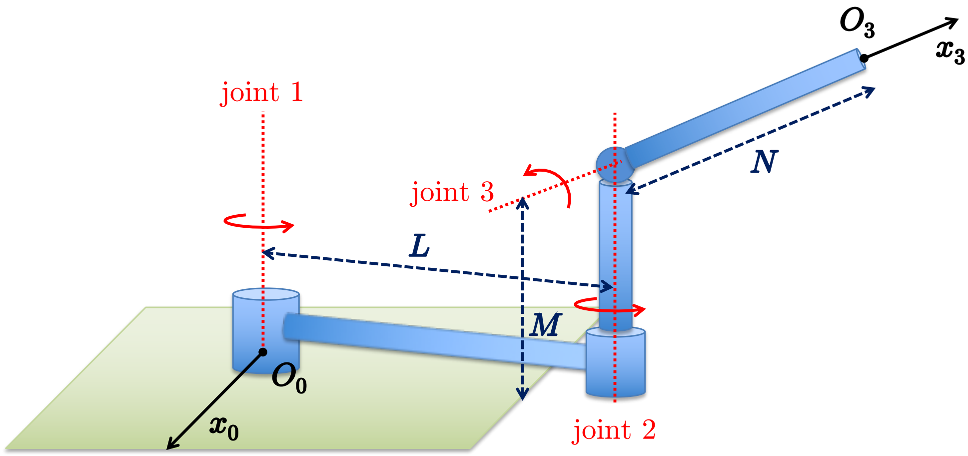


Figure 1: A spatial 3R robot.

- Assign a set of frames to this robot according to the Denavit-Hartenberg (D-H) convention and provide the associated table of parameters. Keep the origins $O_0$ and $O_3$ and the axes $x_0$ and $x_3$ as shown in the figure, respectively in frame $RF_0$ and frame $RF_3$. Indicate also the signs taken by the joint variables $q_i$, $i = 1, 2, 3$, in the robot configuration shown in Fig. 1.
- Compute the direct kinematics for the position $\boldsymbol{p} = \boldsymbol{p}_3$ of the end effector, i.e., the point $O_3$.
- Draw accurately the primary workspace of this robot.
- Provide the $3 \times 3$ Jacobian matrix $\boldsymbol{J}(\boldsymbol{q})$ of the robot in

$$\boldsymbol{v} = \dot{\boldsymbol{p}} = \boldsymbol{J}(\boldsymbol{q})\dot{\boldsymbol{q}},$$

  and determine all the kinematic singularities, each with the associated rank of $\boldsymbol{J}(\boldsymbol{q})$.
- In a singularity $\boldsymbol{q}_s$ where rank $\boldsymbol{J}(\boldsymbol{q}_s) = 1$, find an admissible end-effector velocity $\boldsymbol{v}_s \in \mathbb{R}^3$ and a joint velocity $\dot{\boldsymbol{q}}_s \in \mathbb{R}^3$ such that $\boldsymbol{J}(\boldsymbol{q}_s)\dot{\boldsymbol{q}}_s = \boldsymbol{v}_s \neq \boldsymbol{0}$. Is such $\dot{\boldsymbol{q}}_s$ unique for a given admissible end-effector velocity $\boldsymbol{v}_s$?

## Exercise 2

A planar RP robot is commanded at the acceleration level. Its end-effector position is given by

$$\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{q}) = \begin{pmatrix} q_2 \cos q_1 \\ q_2 \sin q_1 \end{pmatrix}. \tag{1}$$

If the robot is in a generic nonsingular configuration $\boldsymbol{q}$ and with non-zero velocities for both joints, determine the explicit expression of a command $\ddot{\boldsymbol{q}}$ such that the end-effector acceleration is instantaneously $\ddot{\boldsymbol{p}} = \boldsymbol{0}$. Is this command unique?

## Exercise 3

Consider again the same RP robot of Exercise #2. Suppose that the generalized forces $\boldsymbol{\tau} \in \mathbb{R}^2$ that the robot actuators can provide at the two joints are bounded componentwise as

$$|\tau_1| \leq \tau_{max,1} = 10\,[\text{Nm}], \qquad |\tau_2| \leq \tau_{max,2} = 5\,[\text{N}].$$

In the configuration $\boldsymbol{q} = (\pi/3, 1.5)$ [rad,m], find the set of feasible Cartesian forces $\boldsymbol{F} = (F_x, F_y) \in \mathbb{R}^2$ (expressed in [N]) which can applied to the end effector and that the robot can sustain while in static equilibrium.

## Exercise 4

The end-effector of a 2R planar robot with unitary link lengths has to track a linear path with constant speed $v_d = 0.5$ [m/s] between $\boldsymbol{P}_1 = (1, 0.5)$ and $\boldsymbol{P}_2 = (1, 1.5)$ [m]. However, at the initial time $t = 0$, the end effector is positioned in $\boldsymbol{P}_0 = (0.5, 0.5)$ [m]. The robot is commanded by a joint velocity $\dot{\boldsymbol{q}}$ that is limited componentwise as

$$|\dot{q}_1| \leq V_{max,1} = 3\,[\text{rad/s}], \qquad |\dot{q}_2| \leq V_{max,2} = 2\,[\text{rad/s}].$$

Design a kinematic control law that is able to achieve the fastest exponential convergence to zero of the trajectory tracking error, uniformly in all Cartesian directions, while being still feasible in terms of robot commands at time $t = 0$ for the given task. Provide some discussion on where/how fast the return to the original trajectory will be achieved.

**[180 minutes, open books]**