Robotics I

July 8, 2022

Exercise 1

Assume that the time-varying orientation of a robot end-effector is expressed by the triplet of angles $\boldsymbol{\phi}(t) = (\alpha(t), \beta(t), \gamma(t))$ defined around the sequence of fixed axes XZY (an RPY-type representation). Determine the relationship between $\dot{\boldsymbol{\phi}} = (\dot{\alpha}, \dot{\beta}, \dot{\gamma})$ and the angular velocity $\boldsymbol{\omega} \in \mathbb{R}^3$ of the end-effector and find the representation singularities of this mapping. In one such singularity, determine all vectors $\boldsymbol{\omega}$ that cannot be represented by a choice of $\dot{\boldsymbol{\phi}} = (\dot{\alpha}, \dot{\beta}, \dot{\gamma})$ and, conversely, find all non-trivial values for $\dot{\boldsymbol{\phi}} = (\dot{\alpha}, \dot{\beta}, \dot{\gamma})$ that are associated to $\boldsymbol{\omega} = \mathbf{0}$.

Exercise 2

Figure 1 shows a spatial 3R robot with its Denavit-Hartenberg (D-H) frames and the definition of joint variables and kinematic parameters. The direct kinematics for the end-effector position (point O_3) of this robot is

$$\boldsymbol{p} = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} L\cos q_1 + N\cos(q_1+q_2)\cos q_3 \\ L\sin q_1 + N\sin(q_1+q_2)\cos q_3 \\ M + N\sin q_3 \end{pmatrix} = \boldsymbol{f}(\boldsymbol{q}). \tag{1}$$

Provide the closed-form expression of all inverse kinematics solutions $\boldsymbol{q} = \boldsymbol{f}^{-1}(\boldsymbol{p}_d)$ that realize a desired end-effector position $\boldsymbol{p}_d \in WS_1$. Apply your formulas with the following numerical data

$$L = M = N = 0.5 \text{ [m]}, \qquad \boldsymbol{p}_d = \begin{pmatrix} 0.3 & -0.3 & 0.7 \end{pmatrix}^T \text{ [m]},$$

and check at the end the correctness of the obtained numerical solutions by using the direct kinematics (1).

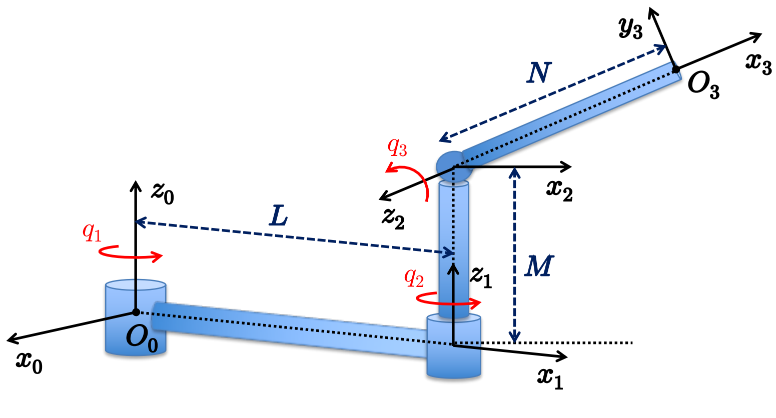

Figure 1: A spatial 3R robot with its D-H frames.

Exercise 3

Using the same numerical data of Exercise 2, solve the inverse kinematics problem with an iterative scheme based on Newton method. Start with the initial guess $\boldsymbol{q}^{\{0\}} = (-\pi/4, \pi/4, \pi/4)$ [rad] and list the values $\boldsymbol{q}^{\{k\}}$, $k = 1, 2, \ldots$, of the first few iterations, until the error $\boldsymbol{e}^{\{k\}} = \boldsymbol{p}_d - \boldsymbol{f}(\boldsymbol{q}^{\{k\}})$ is such that $\left\|\boldsymbol{e}^{\{k\}}\right\| \le \epsilon = 10^{-3}$ [m] (i.e., small enough, meaning that convergence has been achieved). How would you search for another inverse kinematics solution using this iterative method?

Exercise 4

Consider a trajectory planning problem for the 3R robot in Fig. 1. The robot should move from the start configuration $\boldsymbol{q}_s = (-\pi/4, \pi/4, \pi/4)$ [rad] to the goal configuration $\boldsymbol{q}_g = (0, 0, \pi/4)$ [rad] in a time $T = 2$ s, with continuity up to the acceleration over the whole interval $t \in [0, T]$. The initial joint velocity is chosen so that the end-effector velocity starts with $\dot{\boldsymbol{p}}(0) = (1, -1, 0)$ [m/s], while the final velocity should be zero. Provide the values of the coefficients of the *doubly normalized* joint trajectories satisfying all the given conditions. Sketch the plots of joint position, velocity and acceleration.

[240 minutes, open books]