Robotics 1

February 4, 2021

There are 8 questions. Provide answers with short texts, completed with drawings and derivations needed for the solutions. Students with confirmed midterm grade should do only the second set of 4 questions.

Question #1 [students without midterm]

The orientation of a rigid body is defined by the axis-angle pair $\boldsymbol{r} = \begin{pmatrix} 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}^T$ and $\theta = \pi/6$ [rad]. Determine the angles (α, β, γ) of the Roll-Pitch-Yaw sequence XYZ of fixed axes that provide the same orientation. Check the correctness of the obtained result. Find the singular cases for this RPY representation and provide an example of an axis-angle pair $(\boldsymbol{r}_s, \theta_s)$ that would fall in this class.

Question #2 [students without midterm]

Figure 1 shows a top view of a planar two-jaw articulated gripper. This robotic gripper has a revolute joint at its base, followed by two independent revolute joints for each jaw (the first joints in the two jaws share the same axis). This 5-dof robotic system has a tree structure for which the usual Denavit-Hartenberg frame assignment can also be applied (to each branch). Define the joint coordinates accordingly, together with the two DH tables. Provide then the symbolic expression of some task variables that are relevant for gripping operations, defined as follows:

- position of the midpoint P_c between the tips of the two jaws;
- distance d between the two tips;
- relative angle α_{rel} of the left jaw w.r.t. the right jaw;
- orientation angle β w.r.t. the $\boldsymbol{x}_0$ axis of the jaw pair (from the right jaw tip to the left one).

When the gripper links have all the same length $L = 0.05$ [m], compute the numerical value of such task variables in the configuration $\boldsymbol{q} = (q_1, q_{r2}, q_{r3}, q_{l2}, q_{l3}) = (-\pi/2, -\pi/2, 3\pi/4, \pi/2, -3\pi/4)$. Subscripts r and l stand respectively for DH variables pertaining to the right or left jaw only.

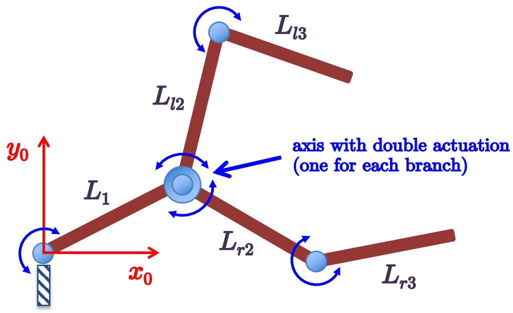

Figure 1: A planar 5-dof two-jaw gripper.

Question #3 [students without midterm]

A planar 2R robot has incremental encoders at the joints measuring the configuration $\boldsymbol{\theta} = (\theta_1, \theta_2)$ used in the computation of its direct kinematics. Because of a bad mounting of the encoders, the two measures are affected by (very) small angular errors δ_1 and δ_2. When using these readings, which of the following statements is correct in terms of Cartesian accuracy of the end-effector position? *A)* there is always an error; *B)* there are configurations at which there may be no error; *C)* the error is always negligible (e.g., below the sensor resolution). Provide a detailed explanation of your answer!

Question #4 [students without midterm]

The prismatic joints of the planar PPR robot in Fig. 2 have bounded ranges, $q_{i,min} \leq q_i \leq q_{i,max}$, for $i = 1, 2$, while the revolute joint q_3 has an unlimited motion range. Draw accurately the primary workspace WS_1 and the secondary workspace WS_2 of this robot, under the following assumption for the third link length: $L < \min \{(q_{1,max} - q_{1,min})/2, (q_{2,max} - q_{2,min})/2\}$.

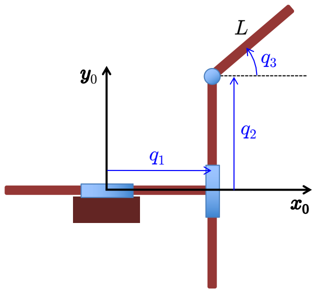

Figure 2: A planar PPR robot.

Question #5 [all students]

The direct kinematics and the initial configuration of a planar RP robot are given by

$$\boldsymbol{p} = \boldsymbol{f}(\boldsymbol{q}) = \begin{pmatrix} q_2 \cos q_1 \\ q_2 \sin q_1 \end{pmatrix}, \qquad \boldsymbol{q}^{\{0\}} = \begin{pmatrix} \pi/4 \\ \varepsilon \end{pmatrix},$$

where $0 < \epsilon \ll 1$ is a very small number. Given the following desired end-effector positions,

$$\boldsymbol{p}_{d,\mathrm{I}} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad \text{and} \quad \boldsymbol{p}_{d,\mathrm{II}} = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

compute the first iteration (i.e., $\boldsymbol{q}^{\{1\}}$) of a Newton method and of a Gradient method for solving the two inverse kinematics problems. Discuss what happens in each of the four cases when $\varepsilon \to 0$.

Question #6 [all students]

The 3R robotic device in Fig. 3 has joint axes that intersect two by two. The second joint axis is inclined by an angle $\delta \approx 20°$. This structure is mainly intended for pointing the final axis $\boldsymbol{n}$ at a moving target in 3D. Provide the explicit expression of the square angular part $\boldsymbol{J}_A(\boldsymbol{q})$ of the geometric Jacobian of this robot. Find the singularities, if any, of the mapping $\boldsymbol{\omega} = \boldsymbol{J}_A(\boldsymbol{q})\dot{\boldsymbol{q}}$. Compute the relation between $\dot{\boldsymbol{q}} \in \mathbb{R}^3$ and the time derivative $\dot{\boldsymbol{n}}$ of the pointing axis.

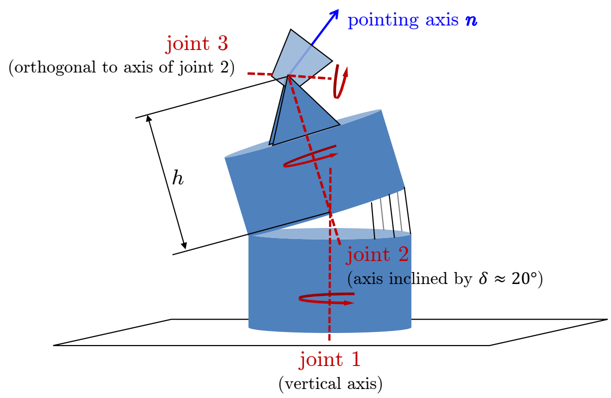

Figure 3: A 3-dof robotic pointing device.

Question #7 [all students]

The desired initial and final orientation of the end effector of a certain robot are specified at time $t = 0$ and $t = T$, respectively by

$$\boldsymbol{R}(0) = \boldsymbol{R}_{in} = \begin{pmatrix} 0.5 & 0 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & 0 & -0.5 \\ 0 & 1 & 0 \end{pmatrix}, \qquad \boldsymbol{R}(T) = \boldsymbol{R}_{fin} = \begin{pmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ -0.5 & -0.5 & -\sqrt{2}/2 \\ 0.5 & 0.5 & -\sqrt{2}/2 \end{pmatrix}.$$

The end effector should start with zero angular velocity and acceleration ($\boldsymbol{\omega}_{in} = \dot{\boldsymbol{\omega}}_{in} = \boldsymbol{0}$, at $t = 0$) and reach the final orientation with angular velocity and acceleration given by

$$\boldsymbol{\omega}(T) = \boldsymbol{\omega}_{fin} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \text{ [rad/s]}, \qquad \dot{\boldsymbol{\omega}}(T) = \dot{\boldsymbol{\omega}}_{fin} = \boldsymbol{0}.$$

Plan a smooth and coordinated trajectory for the end-effector orientation that satisfies all the given boundary conditions for a generic motion time $T > 0$. Setting next $T = 1$, compute at the mid-time instant $t = T/2$ the numerical values of the resulting orientation $\boldsymbol{R}(T/2)$ and angular velocity $\boldsymbol{\omega}(T/2)$ of the robot end effector.

Question #8 [all students]

The planar 3R robot with unitary link lengths shown in Fig. 4 is initially in the configuration $\boldsymbol{q}_{in} = (-\pi/9,\ 11\pi/18,\ -\pi/4)$. Commanded by a joint velocity $\dot{\boldsymbol{q}}(t)$ that uses feedback from the current $\boldsymbol{q}(t)$, the robot should perform a self-motion so as to reach asymptotically the final value $q_{3,fin} = -\pi/2$ for the third joint, while keeping the position of its end-effector always at the same initial point P_{in}. Verify first that such task is feasible. Design then a control scheme that completes the task in a robust way, i.e., by rejecting also possible transient errors and without encountering any singular situation in which the control law is ill conditioned. *Hint: Use an approach based on joint space decomposition.*

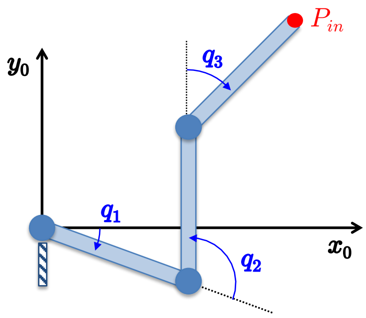

Figure 4: A 3R robot that should perform a self-motion task with constant end-effector position.

[240 minutes (4 hours) for the full exam; open books]
[120 minutes (2 hours) for students with midterm; open books]