Robotics 1

February 3, 2022

Exercise #1

Figure 1 shows two 3D views, together with top and side views with geometric data, of the Crane-X7 robot (by RT Corporation, Japan), a 7-dof arm with all revolute joints. The base frame RF_0 and the end-effector frame RF_e attached to the gripper are already assigned as in the figure.

i. Define a set of Denavit-Hartenberg (D-H) frames for the robot. The origin of the last D-H frame should coincide with the origin O_e of frame RF_e.

ii. Draw clearly the relevant axes of the D-H frames and fill in the associated table of parameters. Specify therein the signs of the variables q_i, $i = 1, \ldots, 7$, in the shown robot configuration.

iii. Provide the constant rotation matrix ${}^7\boldsymbol{R}_e$.

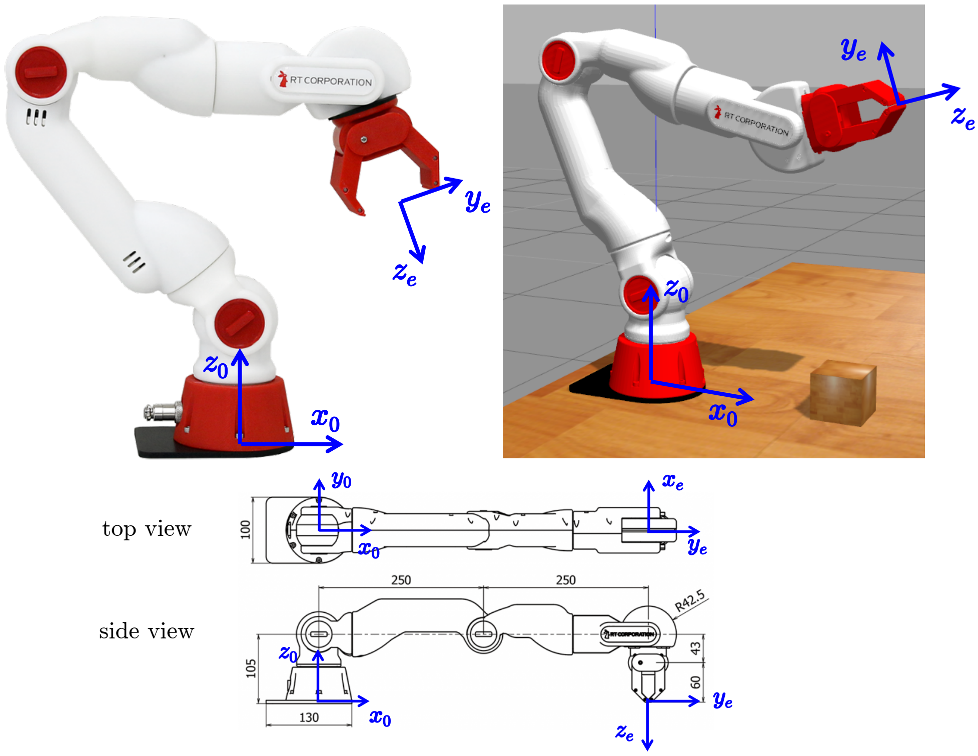

Figure 1: Views of the Crane-X7 robot, with geometric data (in [mm]) and frames RF_0 and RF_e.

Use the Extra Sheet to complete this exercise. Fill in there also the elements of the matrix ${}^7\boldsymbol{R}_e$.

Exercise #2

The absolute initial orientation of the end effector of a 6R robot with a spherical wrist is specified by the YXY sequence of Euler angles $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3) = (45°, -45°, 120°)$. A different orientation is expressed instead by the rotation matrix

$$
{}^0\boldsymbol{R}_f = \begin{pmatrix} 0 & \sin\phi & \cos\phi \\ 0 & \cos\phi & -\sin\phi \\ -1 & 0 & 0 \end{pmatrix}, \qquad \text{with } \phi = \frac{\pi}{3}.
$$

Find an axis-angle representation $(\boldsymbol{r}, \theta)$ of the relative rotation between these two end-effector orientations. Further, if a motion is imposed to the end effector with constant angular velocity $\boldsymbol{\omega} = 1.1 \cdot \boldsymbol{r}$ [rad/s], what will be the time T_ω needed to accomplish this change of orientation?

Exercise #3

Assume that the motion of a 3R planar robot having equal links of unitary length is commanded by the joint acceleration $\ddot{\boldsymbol{q}} \in \mathbb{R}^3$. With reference to Fig. 2, the robot end effector should follow a desired smooth trajectory $\boldsymbol{p}_d(t) = \begin{pmatrix} p_{x,d}(t) & p_{y,d}(t) \end{pmatrix}^T \in \mathbb{R}^2$ in position, while keeping constant its angular speed at some value $\omega_{z,d} \in \mathbb{R}$ (perhaps, after an initial transient).

i. Provide the general form of the command $\ddot{\boldsymbol{q}}$ that executes the full task in nominal conditions.

ii. Study the singularities that may be encountered during the execution of the task.

iii. Compute the numerical value of $\ddot{\boldsymbol{q}}$ when the robot is in the nominal state $\boldsymbol{x}_d = (\boldsymbol{q}_d, \dot{\boldsymbol{q}}_d) \in \mathbb{R}^6$ and for a desired $\ddot{\boldsymbol{p}}_d \in \mathbb{R}^2$, as given by

$$
\boldsymbol{q}_d = \begin{pmatrix} \pi/4 \\ \pi/3 \\ -\pi/2 \end{pmatrix} \text{ [rad]}, \qquad \dot{\boldsymbol{q}}_d = \begin{pmatrix} -0.8 \\ 1 \\ 0.2 \end{pmatrix} \text{ [rad/s]}, \qquad \ddot{\boldsymbol{p}}_d = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ [m/s}^2\text{]}.
$$

What are the values of $\boldsymbol{p}_d$, $\dot{\boldsymbol{p}}_d$, and of $\omega_{z,d}$ in this nominal robot state?

iv. If at some time $t \geq 0$, there is a position and/or a velocity error in the execution of the desired end-effector trajectory $\boldsymbol{p}_d(t)$, how would you modify the commanded acceleration $\ddot{\boldsymbol{q}}(t)$ so as to recover exponentially[1] the error to zero, both in position and velocity? And what if also the angular velocity $\omega_z(t)$ is not the desired one?

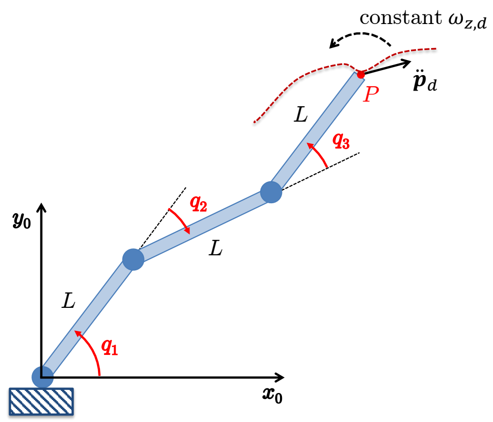

Figure 2: A 3R planar robot executing the desired Cartesian task.

[1]This rate is a property that automatically follows from the linearity of an asymptotically stable error dynamics.

Exercise #4

Consider the situation in Fig. 3, with all data defined therein in symbolic form. The PR robot starts at rest with its end effector placed in $P_{start} = (S, L)$ and should move the end effector to $P_{goal} = (S + \Delta, L)$ in a given time T and stop there, without colliding with the obstacle $\mathcal{O}_{obs}$ located at $(S + (\Delta/2), L/2)$. Design a joint trajectory $\boldsymbol{q}_d(t) \in \mathbb{R}^2$, $t \in [0, T]$, that realizes the task with continuous acceleration $\ddot{\boldsymbol{q}}_d(t)$ and no instant of zero velocity in the open interval $(0, T)$. The solution should be parametric with respect to $L > 0$ (length of the second link of the robot), $S > 0$ (x-coordinate of P_{start}), $\Delta > L/2$ (distance of the two Cartesian points in the x-direction), and T (motion time). Provide then a numerical example, sketching the plot of $\boldsymbol{q}_d(t)$.

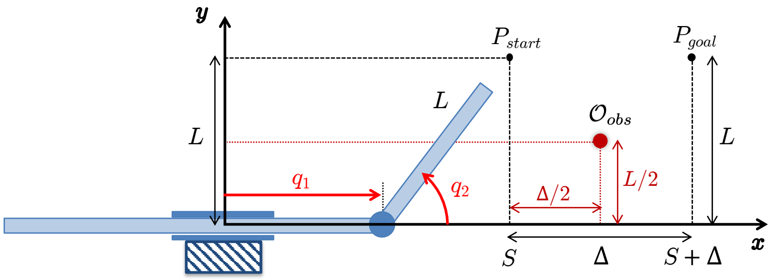

Figure 3: A PR robot should move its end effector from P_{start} to P_{goal}, avoiding the obstacle $\mathcal{O}_{obs}$.

Exercise #5

A transmission/reduction system that displaces rotary motion from the motor axis to the joint axis of a link of length L is sketched in Fig. 4. The system involves two toothed gears and two pulleys, connected by a belt at a distance D. The radius of each of the two gear wheels and of the two pulleys is denoted as r_i, $i = 1, \ldots, 4$. At $t = 0$, the link is in the position shown in the figure. If the motor spins on its axis $\boldsymbol{z}_m$ with a constant angular speed $\dot{\theta}_m > 0$, how much time T_θ will it take for the link to rotate by 90°? Will the link rotate clockwise (CW) or counterclockwise (CCW) w.r.t. its joint axis $\boldsymbol{z}_j$? Evaluate then T_θ using the following data:

$$\dot{\theta}_m = 10 \ \text{[rad/s]}, \qquad r_1 = 20,\ r_2 = 60,\ r_3 = 8,\ r_4 = 32 \ \text{[mm]}, \qquad D = 0.15,\ L = 0.3 \ \text{[m]}.$$

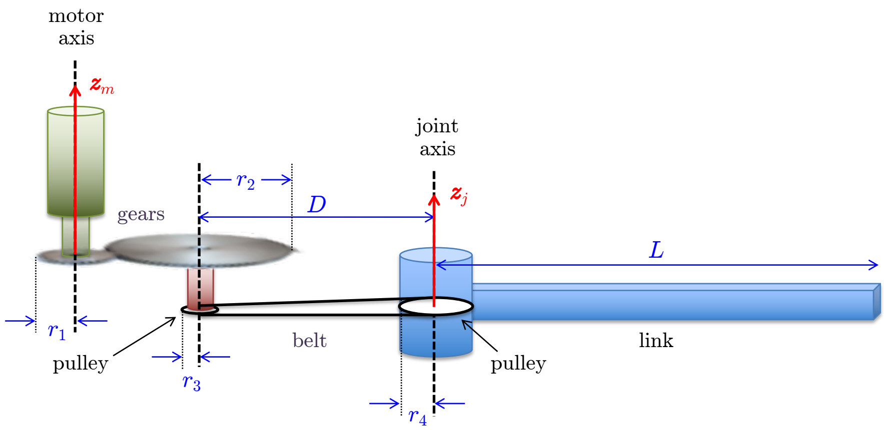

Figure 4: A transmission/reduction system for a motor/link pair.

[210 minutes (3.5 hours); open books]