

# Useful Equations

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$$X = \frac{\cos \delta \sin(\alpha - \alpha_0)}{\cos \delta_0 \cos \delta \cos(\alpha - \alpha_0) + \sin \delta \sin \delta_0}, \quad (1)$$

$$Y = -\frac{\sin \delta_0 \cos \delta \cos(\alpha - \alpha_0) - \cos \delta_0 \sin \delta}{\cos \delta_0 \cos \delta \cos(\alpha - \alpha_0) + \sin \delta \sin \delta_0}, \quad (2)$$

$$x = f\left(\frac{X}{p}\right) + x_0, \quad (3)$$

$$x = \frac{f}{p}(X \cos \theta - Y \sin \theta) + x_0, \quad (4)$$

$$y = \frac{f}{p}(X \cos \theta - Y \sin \theta) + y_0. \quad (5)$$

$$az = \arctan\left(\frac{\vec{r}_{2,1}^j}{\vec{r}_{1,1}^j}\right), \quad (6)$$

$$alt = \arcsin(\vec{r}_{3,1}^j).$$

$$\mathbf{x} = \mathbf{T}\mathbf{X}, \quad (7)$$

$$\mathbf{T} = \begin{pmatrix} (f/p)a_{11} & (f/p)a_{12} & x_0 \\ (f/p)a_{21} & (f/p)a_{22} & y_0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (8)$$

$$\mathbf{a} = \mathbf{B}\mathbf{c}, \quad (9)$$

$$\mathbf{a} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix}, \mathbf{B} = \begin{pmatrix} (f/p)X_1 & (f/p)Y_1 & 1 \\ (f/p)X_2 & (f/p)Y_2 & 1 \\ \vdots & \vdots & \vdots \\ (f/p)X_N & (f/p)Y_N & 1 \end{pmatrix}, \text{ and } \mathbf{c} = \begin{pmatrix} a_{11} \\ a_{12} \\ x_0 \end{pmatrix} \quad (10)$$

$$\mathbf{X} = \mathbf{T}^{-1}\mathbf{x}. \quad (11)$$

$$\mathbf{r} = \mathbf{R} + \rho\mathbf{s}, \quad (12)$$

$$\rho = k^2 \left( \frac{1}{R^3} - \frac{1}{r^3} \right) \frac{\dot{\mathbf{s}} \cdot (\mathbf{R} \times \mathbf{s})}{\dot{\mathbf{s}} \cdot (\dot{\mathbf{s}} \times \mathbf{s})}, \quad (13)$$

$$r^2 = \rho^2 + R^2 + 2\rho\mathbf{R} \cdot \mathbf{s}, \quad (14)$$

$$s = \begin{pmatrix} \cos \alpha \cos \delta \\ \sin \alpha \cos \delta \\ \sin \delta \end{pmatrix}, \quad (15)$$

$$\begin{aligned} \dot{\mathbf{s}}_2 &= \frac{\tau_3(\mathbf{s}_2 - \mathbf{s}_1)}{\tau_1(\tau_1 + \tau_3)} + \frac{\tau_1(\mathbf{s}_3 - \mathbf{s}_2)}{\tau_3(\tau_1 + \tau_3)}, \\ \ddot{\mathbf{s}}_2 &= \frac{2(\mathbf{s}_3 - \mathbf{s}_2)}{\tau_3(\tau_1 + \tau_3)} - \frac{2(\mathbf{s}_2 - \mathbf{s}_1)}{\tau_1(\tau_1 + \tau_3)}. \end{aligned} \quad (16)$$

$$\begin{aligned} \mathbf{T} &= \begin{pmatrix} 0.9685 & 0.0235 & 507.6056 \\ -0.0023 & 1.0123 & 517.7182 \\ 0 & 0 & 1 \end{pmatrix} \text{ for day 17,} \\ \text{and } \mathbf{T} &= \begin{pmatrix} 0.9928 & 0.0394 & 504.6182 \\ 0.0193 & 0.9959 & 491.0023 \\ 0 & 0 & 1 \end{pmatrix} \text{ for day 21.} \end{aligned} \quad (17)$$