Basic Interferometry

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April 7, 2014

Abstract

The idea in this lab is to understand the working of an interferometer. We use two antennas with a baseline separation of $10/meters$, operating with a spectral resolution of about $11/GHz$. With this interferometer, using *Python* codes to remotely control this pair, we point to three kinds of objects: the sun, the moon and a point source. Exploring the least square fitting procedure, we collect data for these three sources and find quantities related to the observation. For three different point sources (Virgo A, Orion Nebula and M17) we calculate the baseline separation. For the Sun and the Moon, we evaluate their radius.

1 Introduction

In astrometry, terferometry is known as the practice of using two or more radio telescopes. It is a powerful method, since based on the geometry of the equipments and the distance between them, we can achieve a good frequency resolution. In this lab, we use our interferometer configuration and perform the data analysis, recovering some of these informations and finding some informations about the geometry of our sources.

In section 2, we introduce some of the quantities related to interferometry that are largelly explored in this experiment, introduce relevant ideas and understand the working of this instrument.

In section 3, via the *PyEphem* module, we develop codes for the rotation matrices, used to manipulate the coordinates of our targets. Also, we outline the basic procedure to track an object and store data during our observations, done in *Python* using functions from the provided *radiolab* module.

In the remaining sections, we analyse the collected data, present the general procedures and later discuss the results.

2 The Interferometer

Our interferometer is a pair of antennas separated by a baseline $B = 10/meters$ working at a about $\nu = 10.7/GHz$. By definition, the spectral resolution of this instrument is given by

$$R = \frac{\lambda}{B}, \tag{1}$$

also known as fringe spacing. Since our antennas are spaced by a vector $\vec{B}$ on the east-west direction, as the source moves in the sky they receive different signal (amount of photons) during the time of observation. It defines the *geometric delay*, τ_g, defined as:

$$\tau_g = \frac{\vec{B} \cdot \hat{s}}{c}, \tag{2}$$

where $\hat{s}$ is a unit vector representing the distance from one of the antennas, so each of them has a τ_g. These quantities are labelled in Fig.1.

What this interferometer does is to multiply the two signals using a mixer (a couple of mixers and amplifiers, in fact), giving a result that is proportional to the flux of the source. This is done by correlating the signals, say E_1 and E_2, that are intrinsically proportional to the time of observation (hence, *tau*):

$$E(t)_1 = cos(2\pi\nu t),\ E(t)_2 = cos(2\pi\nu t + \tau_{tot}), \tag{3}$$

where $\tau_{tot} = \tau_g + \tau_c$ takes into account a constant delay τ_c given by the cable length.

To take into account the unit vector to the source, we write our geometrical delay in terms of the position of the source in the sky, its hour angle h_s:

$$\tau_g(h_s) = \left[\frac{B}{c} cos\delta\right] sinh_s, \tag{4}$$

and δ is the source's declination, which is fixed for sources as Virgo A, Orion and M 17, but varies for the Sun and the Moon on a day.

One thing about correlation: it ends up zeroing signals that are noise, so our result is dependent of the brightness of our sources and not from the sky. So the product of our interferometer is $E_1 \times E_2$:

$$F(t) = cos(2\pi\nu t)cos(2\pi\nu t[t + \tau_{tot}]), \tag{5}$$

what, in term of the hour angle h_s is:

$$F(t) = cos(2\pi\nu[\tau_g(h_s) + \tau_c]). \tag{6}$$

This previous equation is very important. It describes the *fringe pattern*, a quantity that is really important to analyse any data from the interferometer.

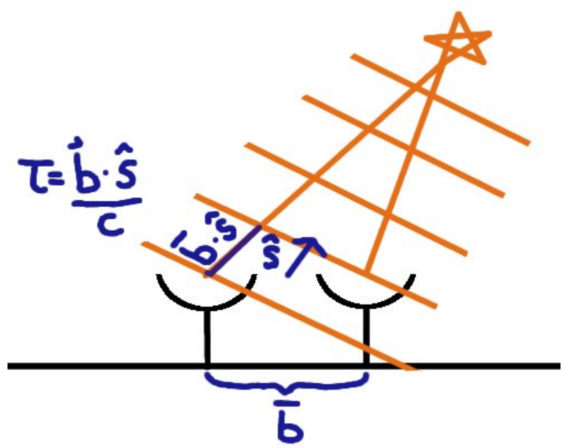

Figure 1: Geometrical description of our interferometer. Credits: Radiolab *casper* website.

3 Using Our Instrument

The interferometer is controlled by a code using the *radiolab* module, which tracks the source in specified intervals and also performs a homing (gets back to rest position) when set to. During data aquisition, our antennas will be following the objects as it rotates with the Celestial Sphere. To describe an object's position in the sky, it all depends on the *Coordinate System* one uses. Our antennas track given values in the *Horizontal Coordinate System* (*Azimuth* and *Altitude*), while the objects positions are initially given in the *Equatorial Coordinate System* (*Right Ascention* and *Declination*). So, first of all, we build mathematical tools that will perform a change of coordinates for our tracking procedure.

3.1 Rotation Matrices

Although astronomical objects pass by in the sky as the Earth rotates, if you look at a catalog we have some of these objects specified at fixed points in the sky. As said previously, it all depends on the Coordinate System you are looking at, since each one of them have different characteristics (most important, the origin).

As said before, our antennas will track Azimuth (az) and Altitude (alt) values of the object, but only after specified their Right Ascentions (ra) and Declinations (dec), which we get from catalogs. But, for the

Sun and the moon, since these values will vary within a day, we enter new values in the Equatorial System each time we evaluate their value in the Horizontal System. Reason: the origin of the Equatorial System is the point where the Sun cross the Equator at the Spring's Equinox, known as *Vernal Point*. Since the Sun change its position relative to this point during the year (as goes through its path in the sky, the Eclipt), it will have relevant variations in a day. This effect is even greater for the Moon, due to its proximity to the Earth. For distant sources as stars and other galaxies, the variation of this point would be accounted due to the Earth's spin variation, the Precession, that ends up moving the origin of this system since the Equator changes in relation to the Eclipt. But the Precession is negligible in a day, so only the Sun and Moon will have different ra's and dec's at each measurement of the interferometer.

To perform the change of coordinates, we will make a two-step transformation described by the following equation:

$$\mathbf{R}_{(\alpha,\delta)\to(az,alt)} = \mathbf{R}^2_{(ha,\delta)\to(az,alt)} \cdot \mathbf{R}^1_{(\alpha,\delta)\to(ha,\delta)}, \tag{7}$$

where we are first applying R^1 that does not change the Coordinate System, but simply rerwites the Azimuth of the object into Hour Angle via the simple relation $ha = lst - \alpha$. lst is our *Local Sidereal Time*, calculated via the *PyEphem* and the *Time* modules in *Python*. Our R^1 matrix is then defined as

$$\mathbf{R^1} = \begin{bmatrix} cos(lst) & sin(lst) & 0 \\ sin(lst) & -cos(lst) & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{8}$$

At second, we apply R^2, and since (ha, δ) are Earth-based coordinates, this matrix depends only on our terrestrial latitude ϕ:

$$\mathbf{R^2} = \begin{bmatrix} -sin(\phi) & 0 & cos(\phi) \\ 0 & -1 & 0 \\ cos(\phi) & 0 & sin(\phi) \end{bmatrix}, \tag{9}$$

ϕ being 37.87 *degrees*.

So R is comprised of these two matrices, that we use to solve the equation $\vec{r'}_{(az,alt)} = \mathbf{R} \times \vec{r}_{(\alpha,\delta)}$. To build our $\vec{r}$ vector, we use our values of (α, δ):

$$\vec{r} = \begin{bmatrix} cos(\delta) \times cos(\alpha) \\ cos(\delta) \times sin(\alpha) \\ sin(\delta) \end{bmatrix}, \tag{10}$$

in order to work with the angles in rectangular coordinates and apply the rotation.

The output of Eq.(7) applied to Eq.(10), $\vec{r'}_{(az,alt)} = \mathbf{R} \times \vec{r}_{(\alpha,\delta)}$, will then be a 3×1 matrix, $\vec{r'}_{3\times1}$, and our (az, alt) will be evaluated as follows:

$$\begin{aligned} az &= \arctan\left(\frac{\vec{r'}_{2,1}}{\vec{r'}_{1,1}}\right), \\ alt &= \arcsin\left(\vec{r'}_{3,1}\right). \end{aligned} \tag{11}$$

The codes that built the rotation matrices and performed the Coordinate System transformation, that were used through the data aquisition for the analyzed objects, can be found at my *Github* account here.

3.2 The Controlling Codes

The values of Eq.(11) are the input of our codes to control the antennas. For so, together with the support of the *Time* module, we imported the *radiolab* module and developed a code that performed the data aquisition for this lab. Using the function *pntTo*, which takes the arguments (alt,az), we pointed the antennas to a desired position in the sky. Since the Earth rotates at a speed of 15 degrees per hour, we call this function every 30 *seconds* to keep track of our objects. Besides that, we perform a homing every 1 *hour* because, after around 100 pointing commands, the interferometer starts to loose precision on doing this task.

This all is done via a *thread* in our codes. Another *thread* is set to run to record the data via the function *recordDVM*. This function stores the data in *.npz* files with the following information: the Right Ascention

(ra) of the object in decimal hours, the Declination (dec) in decimal degrees, the voltage measurement of our sources, in units of $[V]^2$ (see Eq.(5)), and the Local Sidereal Time (lst) (in decimal hours) and the Julian Date (jd) (in decimal days) at the time of the measurement.

The developed codes that controlled the telescope for the Moon, Sun and point sources, storing the data as it performed the pointing at desired intervals, can be found in my *Github* account here.

4 Point Sources

We analyze measurements for three point sources: Orion Nebula, Virgo A and M17. As outlined before, the output of a measurement is the product of the intensities seen by both antennas, as in Eq.(6). Using trigonometry identities, we can rewrite Eq.(6) into another meaningful equation:

$$F(h_s) = Acos\left[2\pi\left(\frac{B}{\lambda}cos(\delta)\right)sin(h_s)\right] - Bsin\left[2\pi\left(\frac{B}{\lambda}cos(\delta)\right)sin(h_s)\right], \tag{12}$$

where A and B are constants that replace the values $cos(2\pi\nu\tau_c)$ and $sin(2\pi\nu\tau_c)$, respectively. Also, the ν from the previous equation was written as λ via the relation $c = \lambda\nu$. This is still the fringe pattern, but now presented only with values that can be directly measured from our interferometer configuration and source's position in the sky. Our objective here is to perform a least square in our data and determine the A and B values. For so, we write into the arguments of the cos and sin the follwing value:

$$C = 2\pi\left(\frac{B}{\lambda}cos(\delta)\right), \tag{13}$$

also known as the *fringe frequency* f_f for our point sources.

To proceed with the least square fitting, we outline:

· Our values for $F(h_s)$ are stored in our data as $volts$.

· We can get the corresponding values for h_s from our source data by subtracting its lst values at each measurement by the ra of this source.

· Relevant physical values are:

Information Table		
Targets	ra	dec
Orion	$5^h34^m17.3s$	$-05°23'28''$
Virgo A	$12^h30^m49.423s$	$+12°23'28.04''$
M17	18^h20^m26s	$-16°10.6'$
$B = 10\ meters$		
$\lambda = 0.028\ meters$		

Table 1: Relevant quantities to perform the least square fit.

4.1 Managing Data

Our measurements are the products of measured intesities from the two antennas. Besides astronomical effects, such as small brightness from the sources, problems related to the instrument require a previous manipulation of our data before any analysis. First of all, not all intensities not corresponding to the source are zeroed out, so our data comes with a lot of noise. The electric equipment ends up adding some information as well, such as the DC offset.

Some problems are due to the tracking/homing procedure. Our tracking might not be optimal, so our measurements are not precisely what we theoretically expect. Also, the homing give spikes of minima though the data, so we start our analysis by getting rid of them.

Fig.(4) shows the data before and after the manipulation. The top plot is the data for Orion with a fitting curve in red. This function is evaluated using a window and finding the average of values inside this window and storing the result in the center. By subtracting the measured volts by this curve we end up with a smoother data, much more reliable to work with. Also, the spikes from the top plot were eliminated since they are not actual measurements, but a result of the homing process.

This was done for the three point sources, and also for the Sun and the moon. Now we may proceed to the least square using this smoothed data.

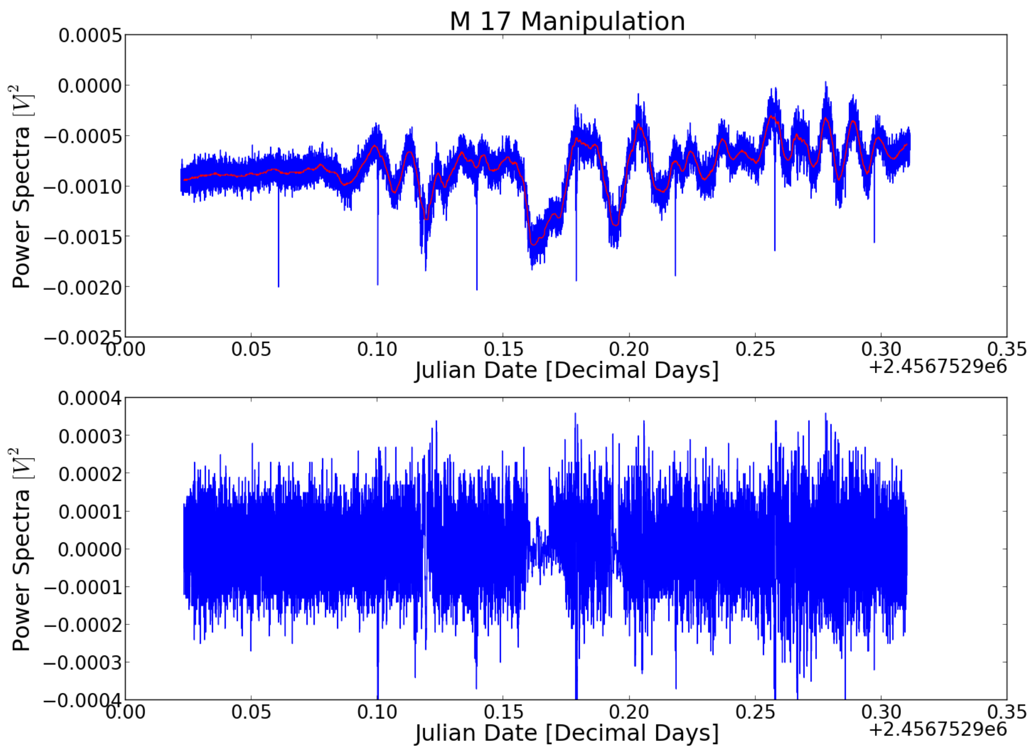

Figure 2: Data manipulation for M17. The red curve on the top plot is done by finding averages of a window across the data. The bottom plot is the measured data minus the red curve, giving a smoothed function.

4.2 Least Square for Point Sources

Given the values of Table 1 and the information from our data, our least square is done to solve for the coefficients A and B. But they are constants that tell about the amplitude of our measurements, and our goal here is to find quantities that we can compare to previously measured values (B, for example). So we proceed as follows:

1. We make a 'guess' for C (Eq.(13)) based on our information from Table 1, and create an array around this value.

2. for each element inside this array, we will evaluate an A and B in order to have a fit and calculate the residuals.

3. By plotting the square of the residuals against our array of $C's$, we can find the one that minimizes the curve and assume this is our best C representation.

With this value that minimizes the squared residuals, C_{min}, we get back to Eq.(13) and evaluate the baseline separation B. This is the procedure done for each source, and the resulting plots are presented next.

On Fig.(3) one least square fit for Orion is presented (in red). The bottom plot is a zoom that shows how is the behavior of our least square fit compared to the data. Although this result seems not to be actually fitting the measurements, since the data is noisy and hard to describe, not necessarily this fit will not be able to statistically give informations about our data. In fact we move on with this fit and calculate the residuals.

Fig.(4) is a graphical repesentation of the method for evaluation of C_{min}. Plotted on the $y\ axis$ we have the square of the residuals for the performed least squares of Orion, done for each value of C (plotted on the $x\ axis$). When the y value reaches a minimum, it means that the function evaluated has the closest value to our data. With the C corresponding to this minimum, we refer back to Eq.(13) and, assuming known $cos(\delta)$ and known λ, we solve for the corresponding baseline B.

The least square plot for the other point sources are at my *Github* account page, on the folder called *lab_interf*. On Table 2 the evaluated C_{min} and baselines B are presented.

Final Evaluated Quantities		
Target	C_{min}	Baseline (B)
Orion	352.5695	9.9156
Virgo A	320.8382	9.1973
M17	341.0037	9.9417

Table 2: Evaluated values for each point source.

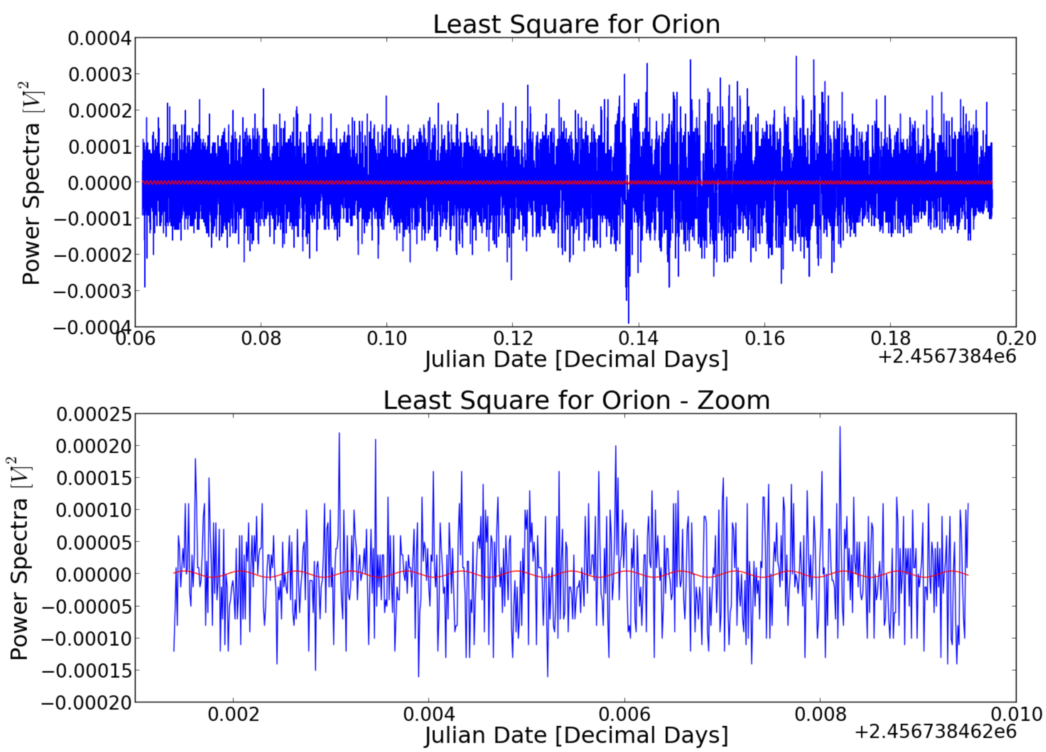

Figure 3: Least Square fit for Orion. The bottom plot is a zoom of the top one, to evidentiate the fitting pattern.

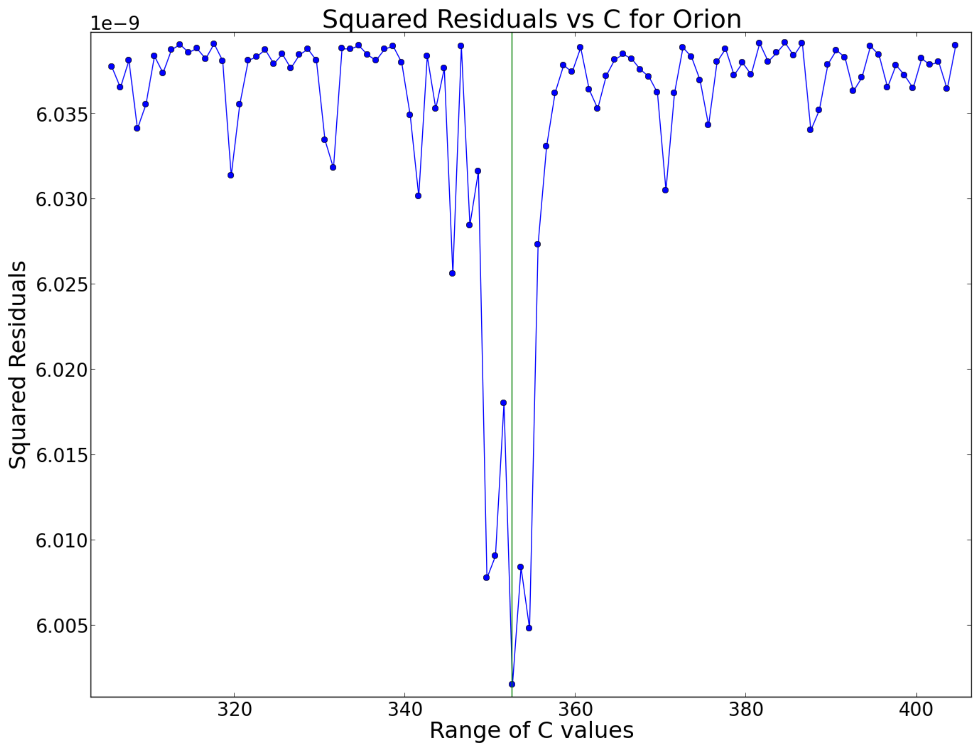

Figure 4: Plot of the squared residuals for each value of C. The green line spot the value where the squaerd residuals reach a minima, corresponding to a good value of C (that best describes the data).

5 Sun and Moon

For the Sun and the Moon we get some particularities that changes our process of taking and analysing the data. First of all, as discussed before, their Right Ascention and Declination change within a day, so their tracking function is different from the one used to the point sources, since keep updating their ra and dec from time to time.

About the data analysis, these objects are in fact resolved in the sky, which means that their angular size are appreciable hence are not perceived as a dot by our interferometer beam. What it does to the data is that our fringe pattern will have an amplitude variation. As the Sun and the Moon move through the sky, the contribution of both antennas will eventually get cancelled. An equation that describes this effect is

$$R(h_s) = F(h_s) \times \int I(\Delta h) cos(2\pi f_f \Delta h) d\Delta h, \tag{14}$$

where we see from Eq.(12) that the first factor is the point source fringe, and the second is a modulator. I is the intensity of the source, $\Delta h = h - h_s$ is the hour angle h relative to the h_s of the source center and f_f is the fringe frequency, a very important quantity to be discussed on the followinf sections. This second factor had to be taken into account for extended sources, while for objects as Virgo, M 17 and Orion, $\Delta h = 0$ and we had $R(h_s) = F(h_s)$ for them.

5.1 Least Square for Sun and Moon

Here we investigate the modulation of our data. Zeros of this modulation should occur when integral number of fringe periods occur inside the source width, where there will be an equal contribution of negative and positive values of the fringe, leading to a net integral equals to zero.

The next plot shows the Sun and Moon data, presenting the expected oscilation, specially for the Sun (top plot).

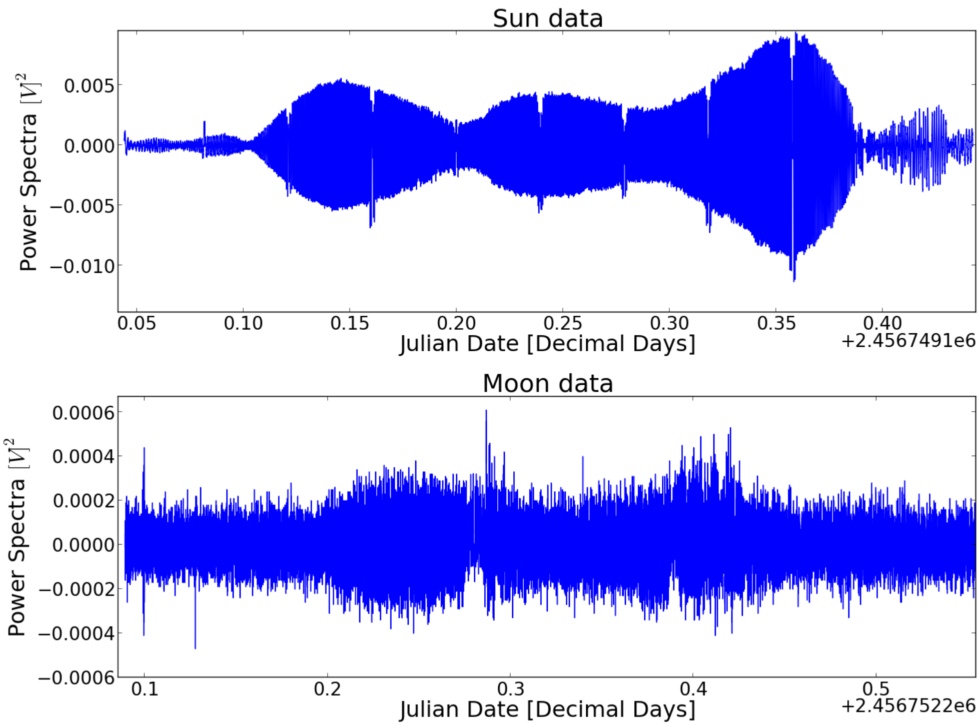

Figure 5: Plot of Sun (top) and Moon (bottom). Their shape are modulated by the factor presented on Eq.(14), and compared to the point source data it indeed shows a variation of its amplitude.

We now apply a least square to the Eq.(14) to determine the fringe modulator function. Initially we are suppose to find an optimal value for ϕ in the follwing equation:

$$F(h_s) = cos(2\pi \frac{B}{\lambda} cos(\delta) sin(ha) + \phi), \tag{15}$$

where ϕ is the phase offset of the fringe source, and $F(h_S)$ describes the amplitude of the data. This equation is achieved via trigonometry functions from the Eq.(14).

We now choose a window of values from our data that contains an apparent drop of amplitude being exposed (see Fig.(6) and Fig.(7)). We solve a least square by fitting a parabola to the data from this window, and, similarly to what was done to calculate C_{min}, we get an optimal value for ϕ from a range of guesses, by storing the sum of the squared residuals for different least squares with different $\phi's$, and later with a plot we spot the ϕ_{min} that minimizes the residuals.

Our parabola for the least square is like the following:

$$X = \begin{pmatrix} F(h_s) & F(h_s) \times ha & F(h_s) \times ha^2 \\ F(h_s) & F(h_s) \times ha & F(h_s) \times ha^2 \\ F(h_s) & F(h_s) \times ha & F(h_s) \times ha^2 \end{pmatrix}, \tag{16}$$

based on the hour angle measurements of our object (here, either the Sun or the Moon). After finding the ϕ_{min}, we proceed to a last least square fit and get a shape of what would best describe the parabola of the window. This optimized shape has the information of one of the zeros across our data, described by the fringe frequency,

$$f_f = \left(\frac{B}{\lambda} cos(\delta) \right) cos(h_s). \tag{17}$$

The index of the minimum value of this parabola fit, presented on the bottom of Fig.6 for the Sun and of Fig.7 for the Moon, will give the values we use to calculate the previous quantity f_f for each of them, since the declination and the hour angle ($h_s = lst - \alpha$) corresponding to this index indeed represent a point where our data reaches a minimum.

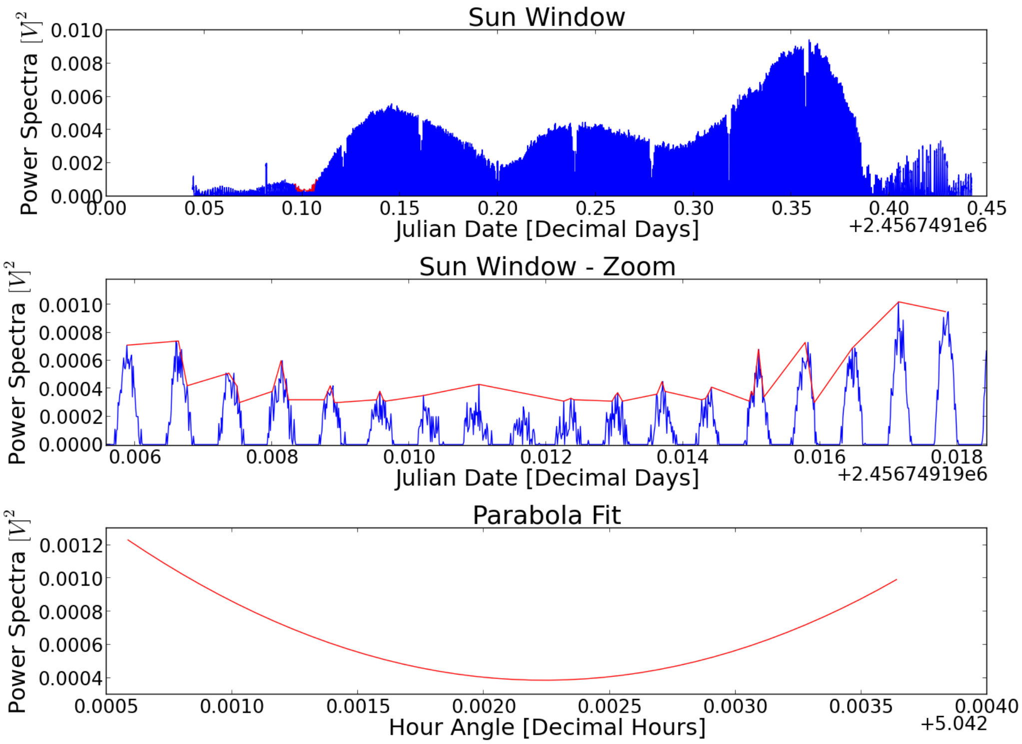

Figure 6: Figure showing the selected window position (top), the envelope fitting (middle), and the final least square fitting (bottom) for the Sun.

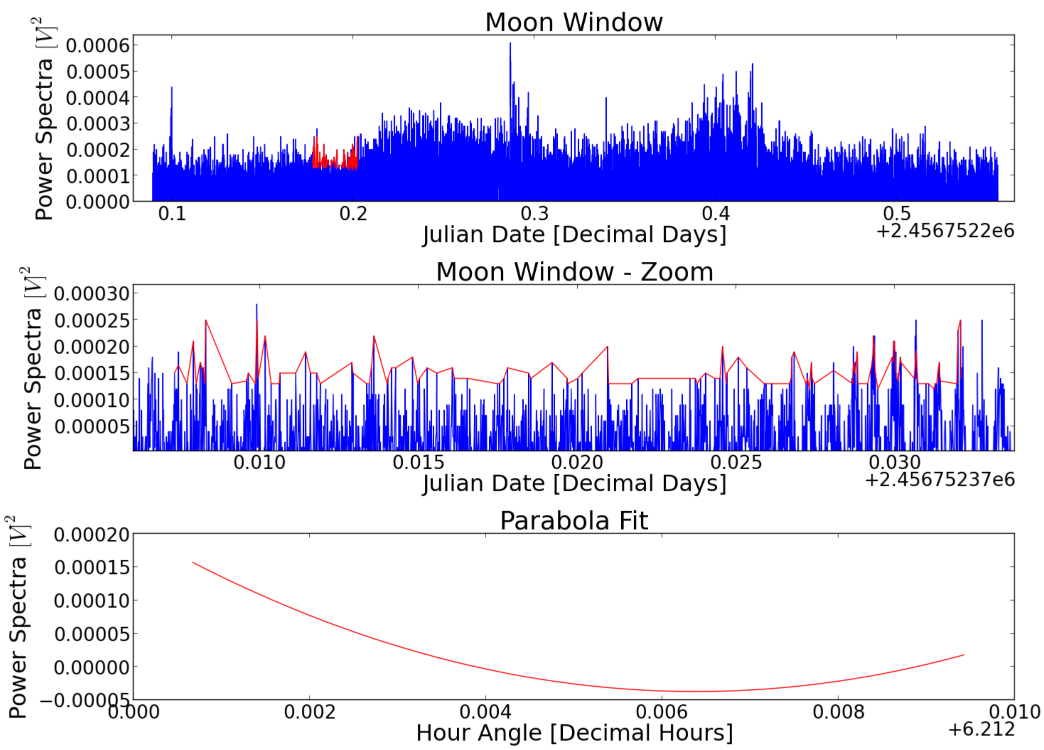

Figure 7: Figure showing the selected window position (top), the envelope fitting (middle), and the final least square fitting (bottom) for the Moon.

5.2 The Modulating Function and Results

The fringe modulator of Eq.(14) can be evaluated by a numerical solution of an integral that only depends on our fringe frequency f_f and on the Radius of the source R. This integral solution is an approximation of a Bessel Function, which is the result of a Fourier Transform of a circle. This is expected since *the modulating function is a Fourier Transform of the source intensity distribution on the sky.* The integral is defined as (see handout for further details):

$$MF_{theory} \approx \frac{R}{N} \sum_{n=-N}^{n=+N} \left[1 - \left(\frac{n}{N}\right)\right]^{1/2} cos\left(\frac{2\pi f_f R n}{N}\right). \tag{18}$$

Since this integral only depends on the product Rf_f, the zero occurances will give important information about the source structure, since they occur, as commented previously, when the whole disk of the Sun (or of the Moon) are providing a minimum on their data.

The next plot shows the approximation of a Bessel function for the Sun. At each zero crossing we can evaluate R, since it will correspond to a zero at Eq.(18), hence the argument of the *cos* must be one, and since all values inside this function are constant, the variation is given by our product Rf_f that can be solved for R since we know f_f from Eq.(15). One can compare the zero positions on this graph with the minimum positions along the Sun's data on the top of Fig.'s (5) or (6), and try to guess which one corresponds to which.

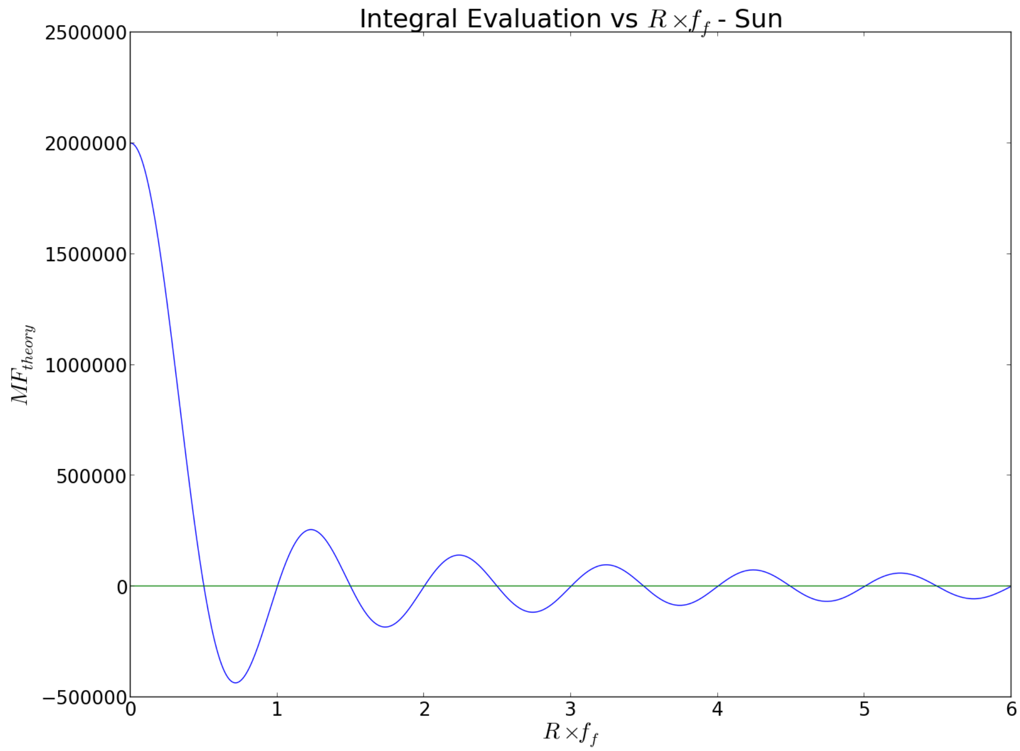

Figure 8: Sun's modulating function. Each zero crossing is a minima on the Sun's data, corresponding to a number tha makes possible the evaluation of the Radius of the sun for a known f_f.

The next Table presents the evaluated svalues for the Radius of the Sun and the Moon:

Final Evaluated Quantities			
Target	f_f	R in Radians	R in arcmin
Sun	115.786669	0.004318	14.844190
Moon	356.302406	0.004209	14.469476

Table 3: Results for the Sun and Moon.

6 Conclusion

In this lab we learned the working of an interferometer by dealing with its physical properties, and via data analysis we recovered informations about the instrument and source's geometry. And, in order to perform data aquisition, we went through Coordinate Systems, so that we were able to use the *radiolab* module and control the interferometer antennas. These procedures were correctly done, based on the reliability of our data and the decent values for the evaluated quantities, evidenciating the success of the developed codes.

For the point sources, the values for Virgo A are much different from the expected, $B = 10$ *meters*, reaching 91.97% of the expected value, while for Orion we achieved 99.15% and for M 17 99.41% as estimatives of the nominal value for the baseline. It can be justified by the fact that, the greater the brighness of the source, more information about it we collect, hence more reliable are our measurements. For the three of them, Virgo A is the one with the smallest value of *Flux Density* (~ 34 Jy), while Orion, our second best estimative, has (~ 340 Jy), and M17, our best result, has (~ 500 Jy).

Also, these results indicate that, although the fitting from the least square procedure was not ideal (see Fig.(3)), it indeed was able to provide statistical informations that allowed us to find reasonable values.

The extended sources had values around the minimum expected. For the Sun, the found value is around 93.95% of the minimum angular size of the Sun in the sky. For the Moon, it is 98.76% from its minimum size, what is a good estimative. For the Sun, the chosen window was the first zero of our data, and for the Moom it was the second one.