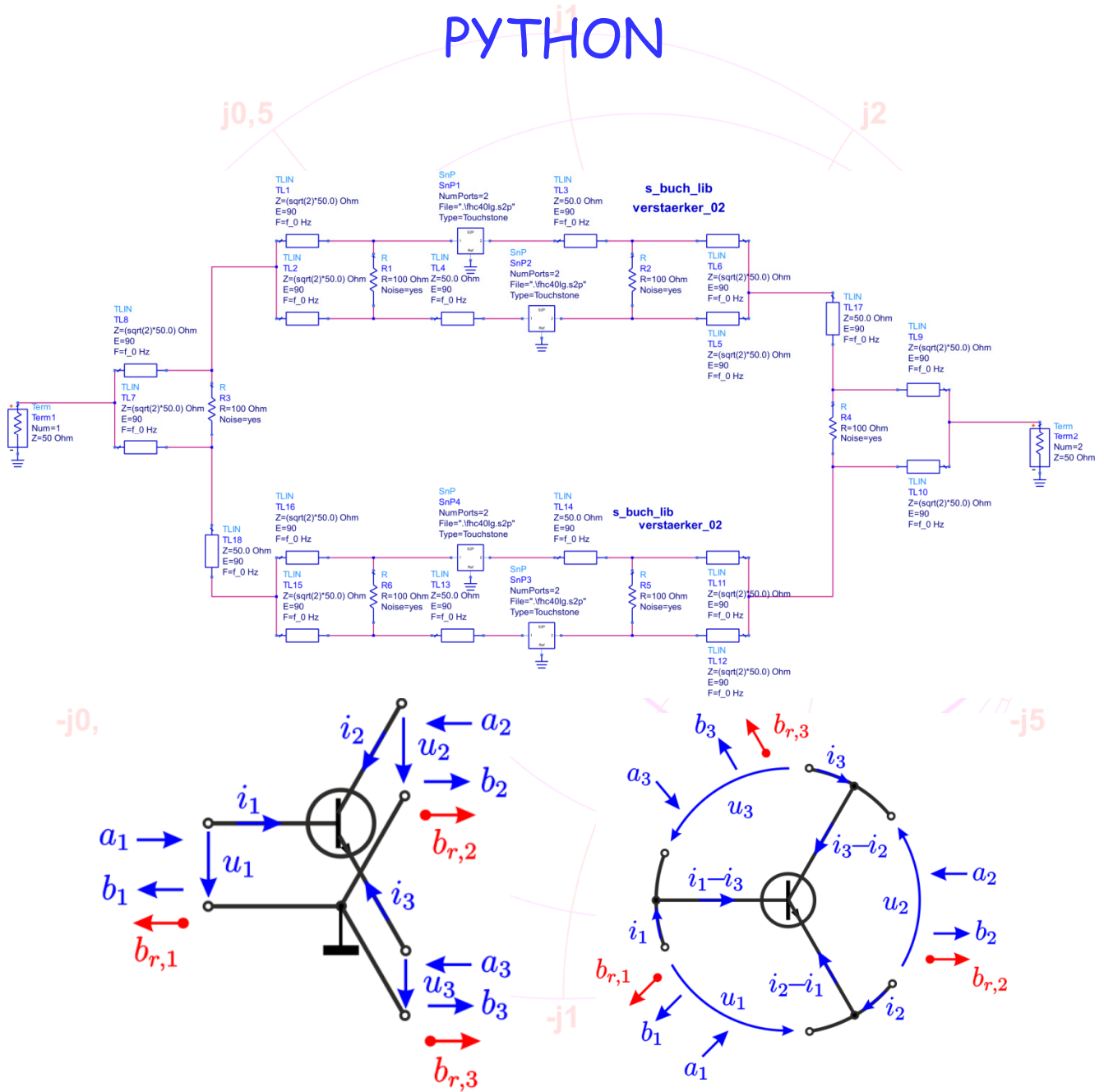
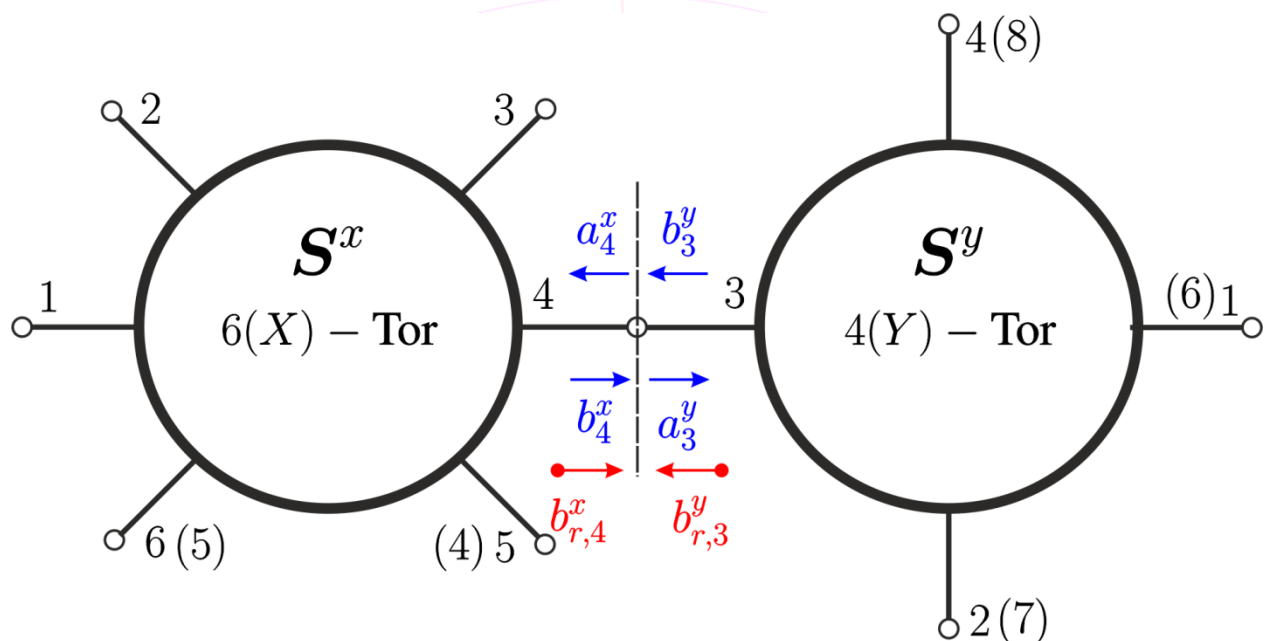


# S- and noise parameters calculations of electrical networks with PYTHON

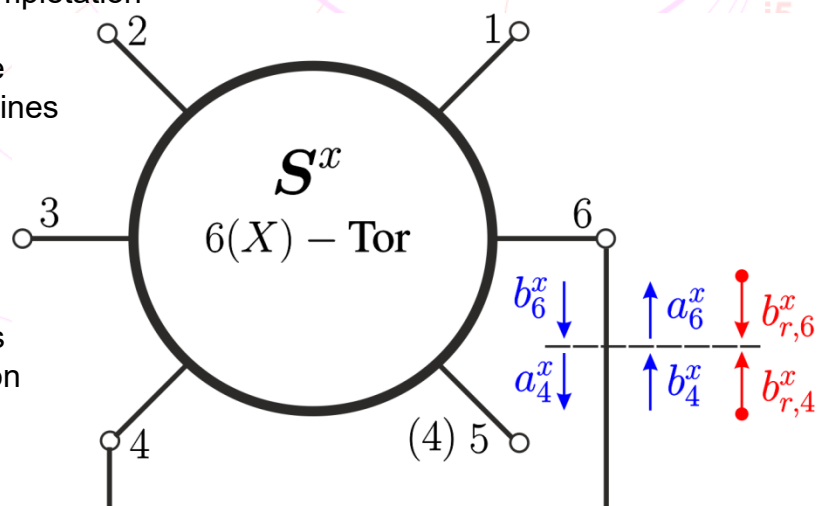


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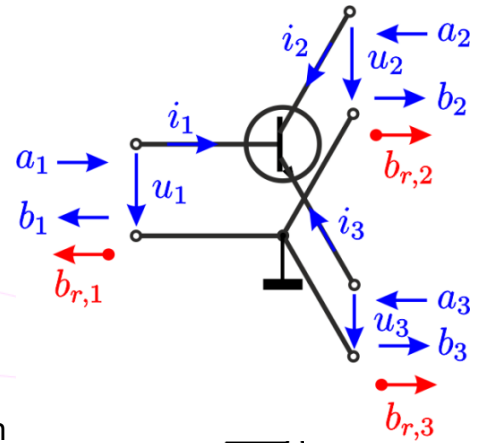
In part 1 is shown how you can compute S- and noise parameters of simple electrical networks by combinations of little MATLAB subroutines. Except the series  $z$  and parallel  $y$  connections at a two port all others are cascade circuits. This is not the general case. Often it is required to connect two n-ports with different numbers of ports at one or more ports. One way for solution this task is to connect the two n-ports at one port pair only, one port from n-port X and one port from n-port Y as seen in the figure below [1],[2]



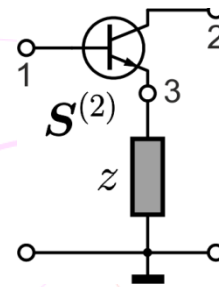
That makes the subroutines [Nnpc\\_xy.m](#), [Snpc\\_xy.m](#), N for signal and noise, S for signal only [2]. Important is the order of port numbers before and after connection. This is suggested in the figure by numbers in brackets. At the connection port numbers there are no restrictions. For the completion of circuit the opposite connection at two ports at one n-port is necessary. The subroutines [Nnpc\\_x.m](#), [Snpc\\_x.m](#) makes that for signal and noise or signal only. Inputs for the port numbers must be in increasing numbering. See the order of port numbers before and after the connection again. .



For a general port connection with transistors it is required a three port representation, here in **common earth** configuration [2]. In the most tables (\*.s2p, \*.cti) the two port S- and noise parameters are given for common emitter/source.



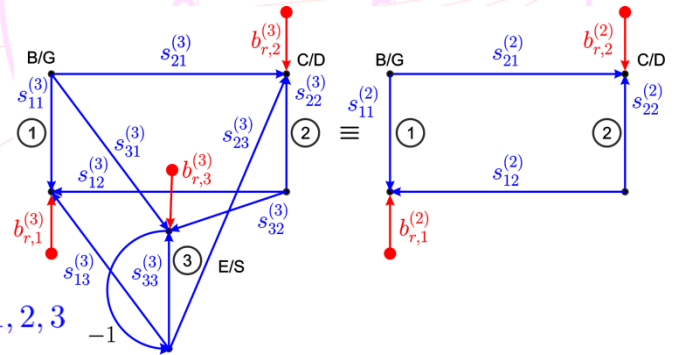
In the figure of the right side there is a transistor with series feedback  $z$  at emitter/source. By choosing  $z = 1$  it is the measuring circuit for the upper left block in the three port transistor matrix. With the equations in the subroutine **sssZ.m** ( $S^{(2)}, 1$ ) in part 1 the four elements of the upper block can be computed [2]. At a common earth three port with no sources like here must be



$$\sum_{i=1}^3 s_{j,i} = 1; j = 1, 2, 3 \quad \sum_{i=1}^3 s_{i,j} = 1; j = 1, 2, 3$$

which means sums of rows and columns are one. Therefore with the four block elements all others are defined.

The right figure shows how the noise waves of the three port suppose at ports 1 and 2 if we make a short at port 3 (emitter/source). Analog to the equations above is at the noise wave matrix [2]



$$\sum_{i=1}^3 cs_{j,i} = 0; j = 1, 2, 3 \quad \sum_{i=1}^3 cs_{i,j} = 0; j = 1, 2, 3$$

With this information must be

$$\begin{pmatrix} b_{r,1}^{(3)} \\ b_{r,2}^{(3)} \\ b_{r,3}^{(3)} \end{pmatrix} = 0.5 \begin{pmatrix} 2 - s_{13}^{(3)} & -s_{13}^{(3)} \\ -s_{23}^{(3)} & 2 - s_{23}^{(3)} \\ -(1 + s_{33}^{(3)}) & -(1 + s_{33}^{(3)}) \end{pmatrix} \begin{pmatrix} b_{r,1}^{(2)} \\ b_{r,2}^{(2)} \end{pmatrix} = M_1 \begin{pmatrix} b_{r,1}^{(2)} \\ b_{r,2}^{(2)} \end{pmatrix}$$

and

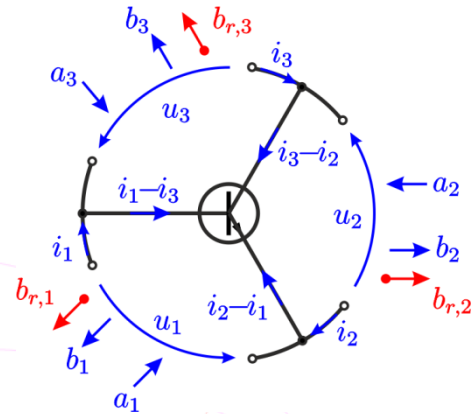
$$C_s^3 = M_1 C_s^2 M_1^*$$

$$C_s^2 = (M_1^* M_1)^{-1} M_1^* C_s^3 M_1 (M_1^* M_1)^{-1}$$

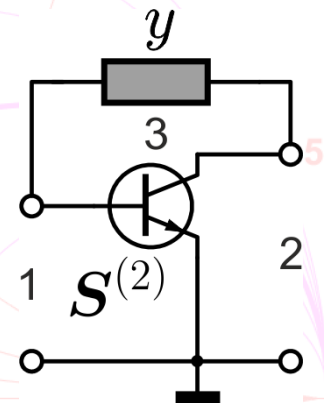
An other three port description of transistor is an **earth free** one. The sum of voltages around the transistor must be zero. So the equations of row and column sums are [2]

$$\sum_{i=1}^3 s_{j,i} = -1; j = 1, 2, 3$$

$$\sum_{i=1}^3 s_{i,j} = -1; j = 1, 2, 3$$



With  $y = 1$  in the parallel feedback circuit from part 1 there is the measuring circuit for the three port parameters in the upper right block of matrix. The formulas are given in the subroutine `sspy.m` ( $S^{(2)}, 1$ ) of part 1 [3]. But it is important to note that the polarity of port two at free earth version is opposite to the common earth one. So a minus sign must be used for the transmission parameters. The other S parameters of three port are the complement to row and column sum -1.



With an open at port three, the flow graphs from figure above and the same conditions as before

$$\sum_{i=1}^3 cs_{j,i} = 0; j = 1, 2, 3 \quad \sum_{i=1}^3 cs_{i,j} = 0; j = 1, 2, 3$$

the relation between the three and two port  $C^s$  parameters of earth free version is

$$\begin{pmatrix} b_{r,1}^{(3)} \\ b_{r,2}^{(3)} \\ b_{r,3}^{(3)} \end{pmatrix} = 0.5 \begin{pmatrix} 2 + s_{13}^{(3)} & -s_{13}^{(3)} \\ s_{23}^{(3)} & -(2 + s_{23}^{(3)}) \\ s_{33}^{(3)} - 1 & 1 - s_{33}^{(3)} \end{pmatrix} \begin{pmatrix} b_{r,1}^{(2)} \\ b_{r,2}^{(2)} \end{pmatrix} = M_2 \begin{pmatrix} b_{r,1}^{(2)} \\ b_{r,2}^{(2)} \end{pmatrix}$$

and again

$$C_s^3 = M_2 C_s^2 M_2^*$$

$$C_s^2 = (M_2^* M_2)^{-1} M_2^* C_s^3 M_2 (M_2^* M_2)^{-1}$$

The program collection has the main directory [s\\_matrix](#) and the subroutine/function directories [./lib\\_func](#), [./transistors](#), [./plots](#). The file [./lib\\_func/contens\\_lib\\_func.py](#) describes the subroutines/functions. In front of each subroutine/function there is short comment for the meaning of input parameters. In the subdirectory/function [lib\\_N\(S\)circuits](#) are files with the circuit topology and the data of transistors are in the subdirectory [transistors](#). The files in the main directory will be described in the recommended order of use to get a feeling for the programs.

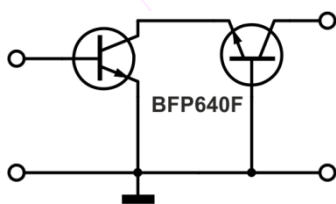
[bfp640f\\_test\\_00\\_3p.py](#) ..... test program to show the transform from two port to three port parameters, common earth and earth free

[bfp640f\\_test\\_01.py](#) ..... test program to show how you must use the programs to connect an inductor in the emitter line. The inductor will be represented by the reflection coefficient [gamma\\_z](#) as a one port. If you set the display parameter [dsp=1](#) in [Nnpc\\_xy](#) you get a short information about ports order and connection.

[bfp640f\\_test\\_02ce.py](#) ..... test program to show the inductor connection and a parallel feedback resistor connection with the common earth three port model and two three port splitters.

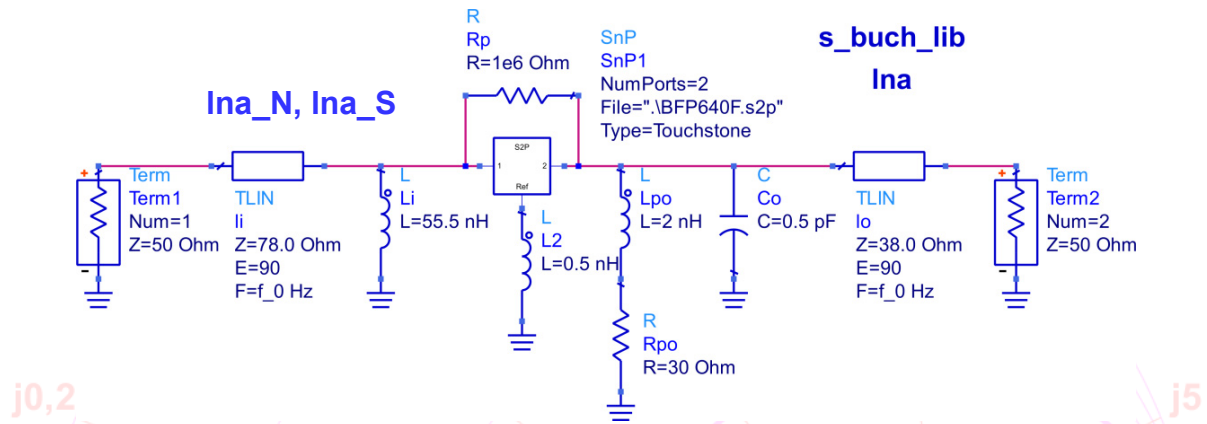
[bfp640f\\_test\\_02ef.py](#) ..... program with the same task as before but now with the free earth model and the connection of a resistor like a inductor as an reflection coefficient [gamma\\_y](#) at port three of transistor with inductor. Because the different port polarity at the models common earth and free earth the sign of transmission parameters must be changed .

[cascode\\_frequency.....\\_N.py](#) .. cascode circuit with the circuit program [./lib\\_func/lib\\_5](#)  
[cascode\\_frequency.....\\_S.py](#) [Ncircuit\(def cascode\\_circuit\\_N\)](#) for signal and noise. To get a common base a short must be made at the port one of the transistor three port. The number of collector is two, from emitter three, before short. After short the numbers will be changed in one and two. But the emitter must be one and the collector two. This modification can be handled with [Numpy](#) command [flip](#). After this, common emitter and common base will be connected. For first use un-comment the lines for single frequency analysis and compare these results, the computing time and the structures of circuits programs with them. The programs with great letter S are for signal analysis only.

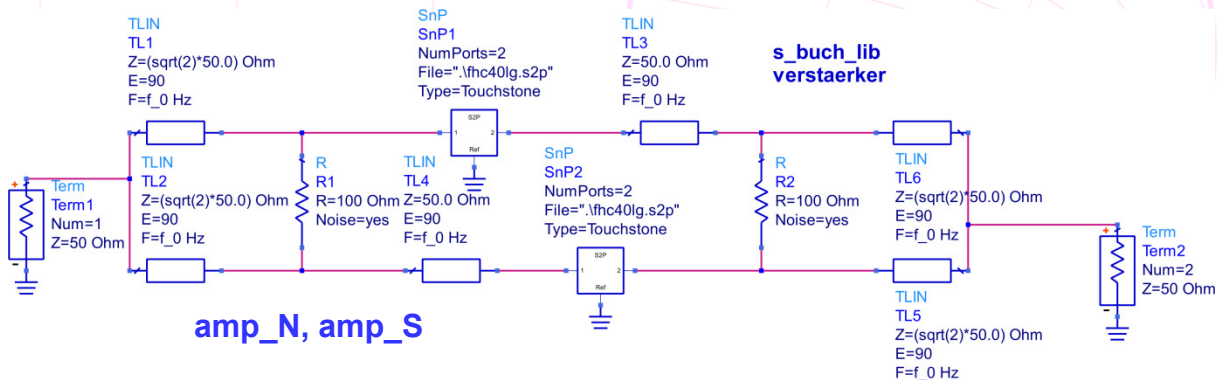


[cascode\\_N, cascode\\_S](#)

[Ina\\_freq\\_N\\_analysis.py](#) ..... programs for analysis of a LNA, circuit structure below. The amplifier structure program is in the `..circuit..` programs.  
[Ina\\_freq\\_S\\_analysis.py](#)

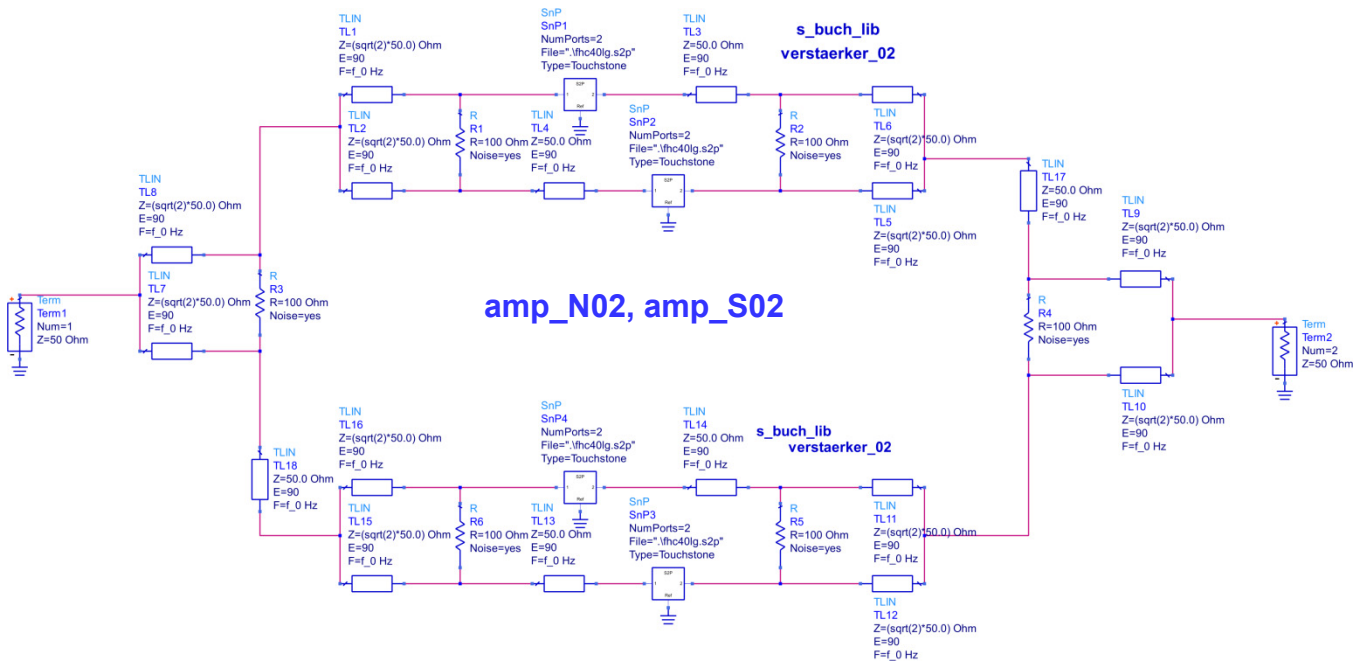


[amp\\_freq\\_N\\_analysis.py](#) ..... programs for analysis of a balanced amplifier  
[amp\\_freq\\_S\\_analysis.py](#)



[amp\\_freq\\_N02\\_analysis.py](#) ..... programs for analysis of a double balanced amplifier next page to demonstrate the multiple data for signal and noise matching, stability, gain and noise figure of the amplifier in comparison to the single transistor.  
[amp\\_freq\\_S02\\_analysis.py](#)





The presented circuits and amplifiers are examples to show, what is possible with the algorithm. We use scattering and noise parameters respectively waves only for signal and noise in contrast to currents and voltages at the most commercial circuit programs.

advantages:

- waves are normalized, there are no poles or very large values, so we have a good numerical stability,
- scattering parameters from data sheets are the direct input values,
- the results are the scattering and noise parameters again and we have a direct information about the significant values as gain, matching, stability, noise and others,
- the computing time is short, so we can use this method of analysis for optimization tasks, the CPU time will be reduced further if the routines are written in C, C++.

disadvantages:

- because we use matrices this method is for linear circuits only!
- for input the circuit topology it takes perhaps more time and more thought or an IT specialist find a way for an easier handling of input.

hint: many programs in the subdirectory `./lib_func` can be used as stand-alone one or as subroutines/supplements for your own programs, the programs were tested with [Spyder 5.5.1](#), [Python 3.11.7](#) in [Anaconda 2.6.1](#) environment

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## References:

- [1] Martius, Siegfried: *Netzwerk-CAD mit Streumatrix und Rauschwellen-Korrelationsmatrix*  
Nachrichtentechnik, Elektronik, Berlin, 38 (1988) 1, 17 - 23, 3. Umschlagseite
- [2] Martius, Siegfried: *Wellenbeschreibung elektrischer Netzwerke mit der Streumatrix*  
Wiesbaden: Springer Fachmedien, 1st ed. 2023  
<https://doi.org/10.1007/978-3-658-38875-1>
- [3] Martius, Siegfried: *Scattering matrix (S) and noise parameters calculations*  
<https://www.mathworks.com/matlabcentral/fileexchange/165956-scattering-matrix-s-and-noise-parameters-calculations>, MATLAB Central File Exchange  
June 2024

