

Weekly Summary Template

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Tuesday, Jan 17

! TIL

Include a *very brief* summary of what you learnt in this class here.
Today, I learnt the following concepts in class:

1. Multicollinearity
2. VIF
3. Stepwise Regression

Provide more concrete details here. You can also use footnotes¹ if you like

```
# package loading
packages <- c("ISLR2","dplyr","tidyr","readr","purrr","glmnet","caret","car")
renv::install(packages)
```

¹You can include some footnotes here

```
Installing ISLR2 [1.3-2] ...
  OK [linked cache in 4.9 milliseconds]
Installing dplyr [1.1.0] ...
  OK [linked cache in 1.5 milliseconds]
Installing purrr [1.0.1] ...
  OK [linked cache in 1.8 milliseconds]
Installing tidyr [1.3.0] ...
  OK [linked cache in 4 milliseconds]
Installing readr [2.1.4] ...
  OK [linked cache in 4.6 milliseconds]
Installing glmnet [4.1-6] ...
  OK [linked cache in 4.1 milliseconds]
Installing caret [6.0-93] ...
  OK [linked cache in 2 milliseconds]
Installing car [3.1-1] ...
  OK [linked cache in 5.8 milliseconds]
```

```
install.packages("ISLR2")
```

```
Installing ISLR2 [1.3-2] ...
  OK [linked cache in 4.3 milliseconds]
```

```
library(dplyr)
```

Attaching package: 'dplyr'

The following objects are masked from 'package:stats':

filter, lag

The following objects are masked from 'package:base':

intersect, setdiff, setequal, union

```
library(tidyr)
library(readr)
library(purrr)
library(glmnet)
```

Loading required package: Matrix

Attaching package: 'Matrix'

The following objects are masked from 'package:tidyr':

expand, pack, unpack

Loaded glmnet 4.1-6

```
library(caret)
```

Loading required package: ggplot2

Loading required package: lattice

Attaching package: 'caret'

The following object is masked from 'package:purrr':

lift

```
library(car)
```

Loading required package: carData

Attaching package: 'car'

The following object is masked from 'package:purrr':

some

The following object is masked from 'package:dplyr':

recode

Multicollinearity

Consider the Boston housing dataset:

```
library(ISLR2)
attach(ISLR2::Boston)
df <- Boston

full_model <- lm(medv~.,df)
summary(full_model)
```

Call:

```
lm(formula = medv ~ ., data = df)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-15.1304	-2.7673	-0.5814	1.9414	26.2526

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	41.617270	4.936039	8.431	3.79e-16 ***
crim	-0.121389	0.033000	-3.678	0.000261 ***
zn	0.046963	0.013879	3.384	0.000772 ***
indus	0.013468	0.062145	0.217	0.828520
chas	2.839993	0.870007	3.264	0.001173 **
nox	-18.758022	3.851355	-4.870	1.50e-06 ***
rm	3.658119	0.420246	8.705	< 2e-16 ***
age	0.003611	0.013329	0.271	0.786595
dis	-1.490754	0.201623	-7.394	6.17e-13 ***
rad	0.289405	0.066908	4.325	1.84e-05 ***
tax	-0.012682	0.003801	-3.337	0.000912 ***
ptratio	-0.937533	0.132206	-7.091	4.63e-12 ***
lstat	-0.552019	0.050659	-10.897	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

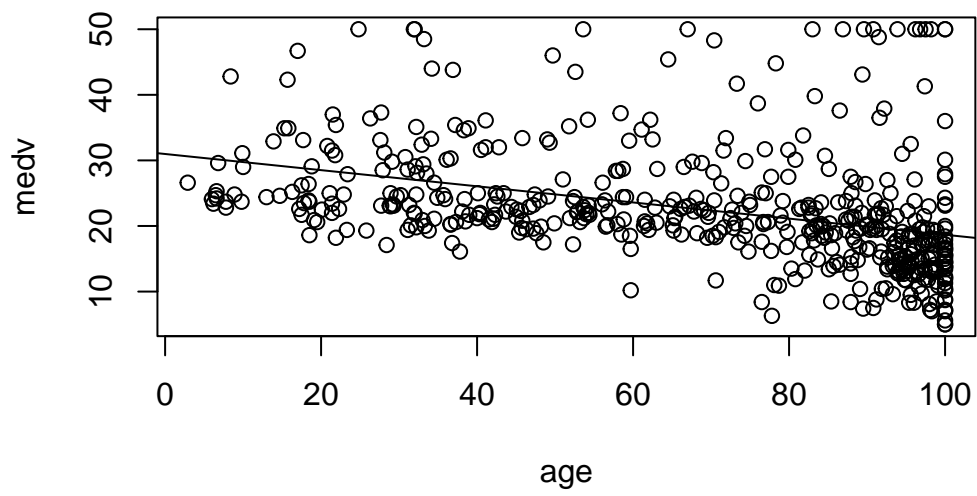
Residual standard error: 4.798 on 493 degrees of freedom

Multiple R-squared: 0.7343, Adjusted R-squared: 0.7278

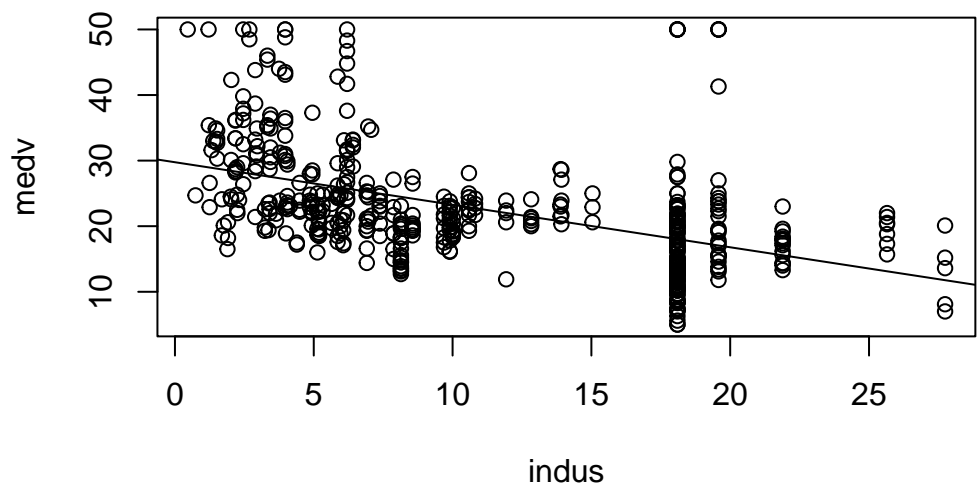
F-statistic: 113.5 on 12 and 493 DF, p-value: < 2.2e-16

Observe insignificant variables:

```
plot(medv~age,df)
abline(lm(medv~age,df))
```



```
plot(medv~indus,df)
abline(lm(medv~indus,df))
```



```
age_model <- lm(medv~age,df)
summary(age_model)
```

Call:

```
lm(formula = medv ~ age, data = df)
```

Residuals:

Min	1Q	Median	3Q	Max
-15.097	-5.138	-1.958	2.397	31.338

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	30.97868	0.99911	31.006	<2e-16 ***
age	-0.12316	0.01348	-9.137	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.527 on 504 degrees of freedom

Multiple R-squared: 0.1421, Adjusted R-squared: 0.1404

F-statistic: 83.48 on 1 and 504 DF, p-value: < 2.2e-16

```
indus_model <- lm(medv~indus,df)
summary(indus_model)
```

Call:

```
lm(formula = medv ~ indus, data = df)
```

Residuals:

Min	1Q	Median	3Q	Max
-13.017	-4.917	-1.457	3.180	32.943

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	29.75490	0.68345	43.54	<2e-16 ***
indus	-0.64849	0.05226	-12.41	<2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.057 on 504 degrees of freedom

Multiple R-squared: 0.234, Adjusted R-squared: 0.2325

F-statistic: 154 on 1 and 504 DF, p-value: < 2.2e-16

Both age and indus are significant in their individual models, so why are they not significant in the full model?

```
library(corrplot)
```

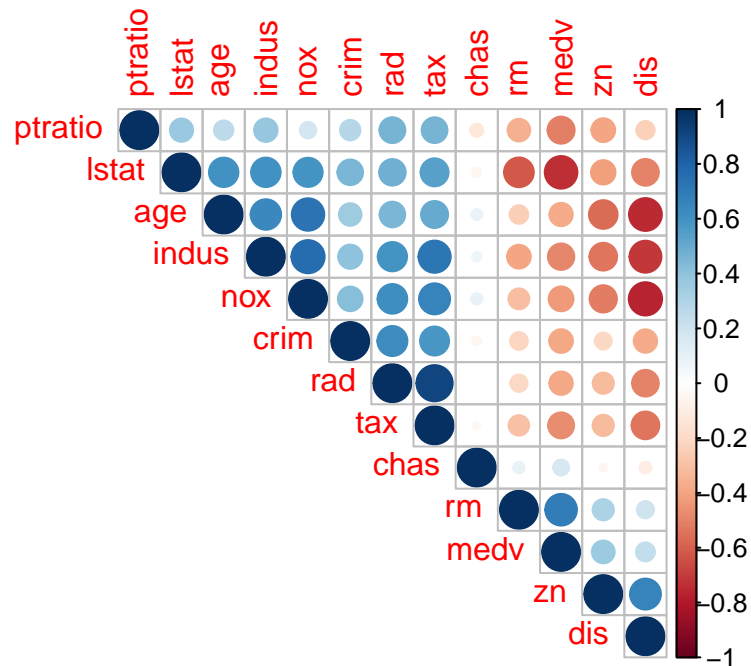
corrplot 0.92 loaded

```
R <- df %>%
  keep(is.numeric)%>%
  cor()
R
```

	crim	zn	indus	chas	nox		
crim	1.00000000	-0.20046922	0.40658341	-0.055891582	0.42097171		
zn	-0.20046922	1.00000000	-0.53382819	-0.042696719	-0.51660371		
indus	0.40658341	-0.53382819	1.00000000	0.062938027	0.76365145		
chas	-0.05589158	-0.04269672	0.06293803	1.00000000	0.09120281		
nox	0.42097171	-0.51660371	0.76365145	0.091202807	1.00000000		
rm	-0.21924670	0.31199059	-0.39167585	0.091251225	-0.30218819		
age	0.35273425	-0.56953734	0.64477851	0.086517774	0.73147010		
dis	-0.37967009	0.66440822	-0.70802699	-0.099175780	-0.76923011		
rad	0.62550515	-0.31194783	0.59512927	-0.007368241	0.61144056		
tax	0.58276431	-0.31456332	0.72076018	-0.035586518	0.66802320		
ptratio	0.28994558	-0.39167855	0.38324756	-0.121515174	0.18893268		
lstat	0.45562148	-0.41299457	0.60379972	-0.053929298	0.59087892		
medv	-0.38830461	0.36044534	-0.48372516	0.175260177	-0.42732077		
	rm	age	dis	rad	tax	ptratio	
crim	-0.21924670	0.35273425	-0.37967009	0.625505145	0.58276431	0.2899456	
zn	0.31199059	-0.56953734	0.66440822	-0.311947826	-0.31456332	-0.3916785	
indus	-0.39167585	0.64477851	-0.70802699	0.595129275	0.72076018	0.3832476	
chas	0.09125123	0.08651777	-0.09917578	-0.007368241	-0.03558652	-0.1215152	
nox	-0.30218819	0.73147010	-0.76923011	0.611440563	0.66802320	0.1889327	
rm	1.00000000	-0.24026493	0.20524621	-0.209846668	-0.29204783	-0.3555015	
age	-0.24026493	1.00000000	-0.74788054	0.456022452	0.50645559	0.2615150	
dis	0.20524621	-0.74788054	1.00000000	-0.494587930	-0.53443158	-0.2324705	
rad	-0.20984667	0.45602245	-0.49458793	1.00000000	0.91022819	0.4647412	
tax	-0.29204783	0.50645559	-0.53443158	0.910228189	1.00000000	0.4608530	
ptratio	-0.35550149	0.26151501	-0.23247054	0.464741179	0.46085304	1.0000000	
lstat	-0.61380827	0.60233853	-0.49699583	0.488676335	0.54399341	0.3740443	
medv	0.69535995	-0.37695457	0.24992873	-0.381626231	-0.46853593	-0.5077867	
	lstat	medv					

crim	0.4556215	-0.3883046
zn	-0.4129946	0.3604453
indus	0.6037997	-0.4837252
chas	-0.0539293	0.1752602
nox	0.5908789	-0.4273208
rm	-0.6138083	0.6953599
age	0.6023385	-0.3769546
dis	-0.4969958	0.2499287
rad	0.4886763	-0.3816262
tax	0.5439934	-0.4685359
ptratio	0.3740443	-0.5077867
lstat	1.0000000	-0.7376627
medv	-0.7376627	1.0000000

```
corrplot(R, type="upper", order="hclust")
```



Age and indus are clearly related to multiple other variables.

Variance Inflation Factors (VIF)

```
library(car)
vif_model <- lm(medv~.,df)
vif(vif_model)
```

	crim	zn	indus	chas	nox	rm	age	dis
	1.767486	2.298459	3.987181	1.071168	4.369093	1.912532	3.088232	3.954037
	rad	tax	ptratio	lstat				
	7.445301	9.002158	1.797060	2.870777				

VIF>2 means that there is high variance inflation and correlation with other covariates.

Stepwise Regression

```
null_model<-lm(medv~1,df)
full_model<-lm(medv~.,df)
```

Forward selection for stepwise regression:

```
forward_model<-step(null_model,direction="forward",scope=formula(full_model))
```

Start: AIC=2246.51

medv ~ 1

	Df	Sum of Sq	RSS	AIC
+ lstat	1	23243.9	19472	1851.0
+ rm	1	20654.4	22062	1914.2
+ ptratio	1	11014.3	31702	2097.6
+ indus	1	9995.2	32721	2113.6
+ tax	1	9377.3	33339	2123.1
+ nox	1	7800.1	34916	2146.5
+ crim	1	6440.8	36276	2165.8
+ rad	1	6221.1	36495	2168.9
+ age	1	6069.8	36647	2171.0
+ zn	1	5549.7	37167	2178.1
+ dis	1	2668.2	40048	2215.9
+ chas	1	1312.1	41404	2232.7
<none>			42716	2246.5

Step: AIC=1851.01

medv ~ lstat

	Df	Sum of Sq	RSS	AIC
+ rm	1	4033.1	15439	1735.6
+ ptratio	1	2670.1	16802	1778.4
+ chas	1	786.3	18686	1832.2
+ dis	1	772.4	18700	1832.5
+ age	1	304.3	19168	1845.0
+ tax	1	274.4	19198	1845.8
+ zn	1	160.3	19312	1848.8
+ crim	1	146.9	19325	1849.2
+ indus	1	98.7	19374	1850.4
<none>			19472	1851.0
+ rad	1	25.1	19447	1852.4
+ nox	1	4.8	19468	1852.9

Step: AIC=1735.58

medv ~ lstat + rm

	Df	Sum of Sq	RSS	AIC
+ ptratio	1	1711.32	13728	1678.1
+ chas	1	548.53	14891	1719.3
+ tax	1	425.16	15014	1723.5
+ dis	1	351.15	15088	1725.9
+ crim	1	311.42	15128	1727.3
+ rad	1	180.45	15259	1731.6
+ indus	1	61.09	15378	1735.6
<none>			15439	1735.6
+ zn	1	56.56	15383	1735.7
+ age	1	20.18	15419	1736.9
+ nox	1	14.90	15424	1737.1

Step: AIC=1678.13

medv ~ lstat + rm + ptratio

	Df	Sum of Sq	RSS	AIC
+ dis	1	499.08	13229	1661.4
+ chas	1	377.96	13350	1666.0
+ crim	1	122.52	13606	1675.6
+ age	1	66.24	13662	1677.7
<none>			13728	1678.1

+ tax	1	44.36	13684	1678.5
+ nox	1	24.81	13703	1679.2
+ zn	1	14.96	13713	1679.6
+ rad	1	6.07	13722	1679.9
+ indus	1	0.83	13727	1680.1

Step: AIC=1661.39

medv ~ lstat + rm + ptratio + dis

	Df	Sum of Sq	RSS	AIC
+ nox	1	759.56	12469	1633.5
+ chas	1	267.43	12962	1653.1
+ indus	1	242.65	12986	1654.0
+ tax	1	240.34	12989	1654.1
+ crim	1	233.54	12995	1654.4
+ zn	1	144.81	13084	1657.8
+ age	1	61.36	13168	1661.0
<none>			13229	1661.4
+ rad	1	22.40	13206	1662.5

Step: AIC=1633.47

medv ~ lstat + rm + ptratio + dis + nox

	Df	Sum of Sq	RSS	AIC
+ chas	1	328.27	12141	1622.0
+ zn	1	151.71	12318	1629.3
+ crim	1	141.43	12328	1629.7
+ rad	1	53.48	12416	1633.3
<none>			12469	1633.5
+ indus	1	17.10	12452	1634.8
+ tax	1	10.50	12459	1635.0
+ age	1	0.25	12469	1635.5

Step: AIC=1621.97

medv ~ lstat + rm + ptratio + dis + nox + chas

	Df	Sum of Sq	RSS	AIC
+ zn	1	164.406	11977	1617.1
+ crim	1	116.330	12025	1619.1
+ rad	1	58.556	12082	1621.5
<none>			12141	1622.0
+ indus	1	26.274	12115	1622.9
+ tax	1	4.187	12137	1623.8

+ age 1 2.331 12139 1623.9

Step: AIC=1617.07

medv ~ lstat + rm + ptratio + dis + nox + chas + zn

	Df	Sum of Sq	RSS	AIC
+ crim	1	170.902	11806	1611.8
<none>			11977	1617.1
+ tax	1	31.773	11945	1617.7
+ rad	1	28.311	11948	1617.9
+ indus	1	27.377	11949	1617.9
+ age	1	0.071	11977	1619.1

Step: AIC=1611.8

medv ~ lstat + rm + ptratio + dis + nox + chas + zn + crim

	Df	Sum of Sq	RSS	AIC
+ rad	1	155.006	11651	1607.1
<none>			11806	1611.8
+ indus	1	24.957	11781	1612.7
+ tax	1	1.418	11804	1613.7
+ age	1	0.178	11806	1613.8

Step: AIC=1607.11

medv ~ lstat + rm + ptratio + dis + nox + chas + zn + crim +
rad

	Df	Sum of Sq	RSS	AIC
+ tax	1	298.573	11352	1596.0
<none>			11651	1607.1
+ indus	1	44.346	11606	1607.2
+ age	1	0.581	11650	1609.1

Step: AIC=1595.98

medv ~ lstat + rm + ptratio + dis + nox + chas + zn + crim +
rad + tax

	Df	Sum of Sq	RSS	AIC
<none>			11352	1596.0
+ age	1	1.6865	11350	1597.9
+ indus	1	1.0784	11351	1597.9

```
summary(forward_model)
```

Call:

```
lm(formula = medv ~ lstat + rm + ptratio + dis + nox + chas +  
    zn + crim + rad + tax, data = df)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-15.1814	-2.7625	-0.6243	1.8448	26.3920

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	41.451747	4.903283	8.454	3.18e-16	***
lstat	-0.546509	0.047442	-11.519	< 2e-16	***
rm	3.672957	0.409127	8.978	< 2e-16	***
ptratio	-0.930961	0.130423	-7.138	3.39e-12	***
dis	-1.515951	0.187675	-8.078	5.08e-15	***
nox	-18.262427	3.565247	-5.122	4.33e-07	***
chas	2.871873	0.862591	3.329	0.000935	***
zn	0.046191	0.013673	3.378	0.000787	***
crim	-0.121665	0.032919	-3.696	0.000244	***
rad	0.283932	0.063945	4.440	1.11e-05	***
tax	-0.012292	0.003407	-3.608	0.000340	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.789 on 495 degrees of freedom

Multiple R-squared: 0.7342, Adjusted R-squared: 0.7289

F-statistic: 136.8 on 10 and 495 DF, p-value: < 2.2e-16

Since including either age or indus would increase the AIC value at the final step, the forward stepwise regression ends and outputs the model that does not include age and indus, since it is the lowest AIC model that this method could find.

Now do the opposite, start with the full model and procedurally remove variables:

```
backward_model<-step(full_model,direction="backward",scope=formula(full_model))
```

Start: AIC=1599.85

```
medv ~ crim + zn + indus + chas + nox + rm + age + dis + rad +  
    tax + ptratio + lstat
```

	Df	Sum of Sq	RSS	AIC
- indus	1	1.08	11350	1597.9
- age	1	1.69	11351	1597.9
<none>			11349	1599.8
- chas	1	245.31	11595	1608.7
- tax	1	256.28	11606	1609.2
- zn	1	263.59	11613	1609.5
- crim	1	311.49	11661	1611.6
- rad	1	430.71	11780	1616.7
- nox	1	546.10	11896	1621.6
- ptratio	1	1157.70	12507	1647.0
- dis	1	1258.52	12608	1651.1
- rm	1	1744.36	13094	1670.2
- lstat	1	2733.54	14083	1707.0

Step: AIC=1597.9

medv ~ crim + zn + chas + nox + rm + age + dis + rad + tax +
ptratio + lstat

	Df	Sum of Sq	RSS	AIC
- age	1	1.69	11352	1596.0
<none>			11350	1597.9
- chas	1	251.21	11602	1607.0
- zn	1	262.99	11614	1607.5
- tax	1	299.68	11650	1609.1
- crim	1	313.07	11664	1609.7
- rad	1	453.61	11804	1615.7
- nox	1	574.23	11925	1620.9
- ptratio	1	1168.01	12518	1645.5
- dis	1	1333.19	12684	1652.1
- rm	1	1750.50	13101	1668.5
- lstat	1	2743.21	14094	1705.4

Step: AIC=1595.98

medv ~ crim + zn + chas + nox + rm + dis + rad + tax + ptratio +
lstat

	Df	Sum of Sq	RSS	AIC
<none>			11352	1596.0
- chas	1	254.21	11606	1605.2
- zn	1	261.75	11614	1605.5
- tax	1	298.57	11651	1607.1

```

- crim      1      313.27 11666 1607.8
- rad       1      452.16 11804 1613.7
- nox       1      601.74 11954 1620.1
- ptratio   1     1168.51 12521 1643.5
- dis       1     1496.35 12848 1656.6
- rm        1     1848.38 13201 1670.3
- lstat     1     3043.23 14395 1714.2

```

```
summary(backward_model)
```

Call:

```
lm(formula = medv ~ crim + zn + chas + nox + rm + dis + rad +
    tax + ptratio + lstat, data = df)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-15.1814	-2.7625	-0.6243	1.8448	26.3920

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	41.451747	4.903283	8.454	3.18e-16 ***
crim	-0.121665	0.032919	-3.696	0.000244 ***
zn	0.046191	0.013673	3.378	0.000787 ***
chas	2.871873	0.862591	3.329	0.000935 ***
nox	-18.262427	3.565247	-5.122	4.33e-07 ***
rm	3.672957	0.409127	8.978	< 2e-16 ***
dis	-1.515951	0.187675	-8.078	5.08e-15 ***
rad	0.283932	0.063945	4.440	1.11e-05 ***
tax	-0.012292	0.003407	-3.608	0.000340 ***
ptratio	-0.930961	0.130423	-7.138	3.39e-12 ***
lstat	-0.546509	0.047442	-11.519	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.789 on 495 degrees of freedom

Multiple R-squared: 0.7342, Adjusted R-squared: 0.7289

F-statistic: 136.8 on 10 and 495 DF, p-value: < 2.2e-16

Again, for the final step, removing any more variables would hurt the AIC, so it stops with the exact same model as the forward method produces. While they did the same thing here,

they do not always produce the same model. In general, look at both methods and compare the differences.

```
backward_model<-step(full_model,direction="both",scope=formula(full_model))
```

Start: AIC=1599.85

```
medv ~ crim + zn + indus + chas + nox + rm + age + dis + rad +  
      tax + ptratio + lstat
```

	Df	Sum of Sq	RSS	AIC
- indus	1	1.08	11350	1597.9
- age	1	1.69	11351	1597.9
<none>			11349	1599.8
- chas	1	245.31	11595	1608.7
- tax	1	256.28	11606	1609.2
- zn	1	263.59	11613	1609.5
- crim	1	311.49	11661	1611.6
- rad	1	430.71	11780	1616.7
- nox	1	546.10	11896	1621.6
- ptratio	1	1157.70	12507	1647.0
- dis	1	1258.52	12608	1651.1
- rm	1	1744.36	13094	1670.2
- lstat	1	2733.54	14083	1707.0

Step: AIC=1597.9

```
medv ~ crim + zn + chas + nox + rm + age + dis + rad + tax +  
      ptratio + lstat
```

	Df	Sum of Sq	RSS	AIC
- age	1	1.69	11352	1596.0
<none>			11350	1597.9
+ indus	1	1.08	11349	1599.8
- chas	1	251.21	11602	1607.0
- zn	1	262.99	11614	1607.5
- tax	1	299.68	11650	1609.1
- crim	1	313.07	11664	1609.7
- rad	1	453.61	11804	1615.7
- nox	1	574.23	11925	1620.9
- ptratio	1	1168.01	12518	1645.5
- dis	1	1333.19	12684	1652.1
- rm	1	1750.50	13101	1668.5
- lstat	1	2743.21	14094	1705.4

Step: AIC=1595.98

medv ~ crim + zn + chas + nox + rm + dis + rad + tax + ptratio +
lstat

	Df	Sum of Sq	RSS	AIC
<none>			11352	1596.0
+ age	1	1.69	11350	1597.9
+ indus	1	1.08	11351	1597.9
- chas	1	254.21	11606	1605.2
- zn	1	261.75	11614	1605.5
- tax	1	298.57	11651	1607.1
- crim	1	313.27	11666	1607.8
- rad	1	452.16	11804	1613.7
- nox	1	601.74	11954	1620.1
- ptratio	1	1168.51	12521	1643.5
- dis	1	1496.35	12848	1656.6
- rm	1	1848.38	13201	1670.3
- lstat	1	3043.23	14395	1714.2

```
summary(backward_model)
```

Call:

```
lm(formula = medv ~ crim + zn + chas + nox + rm + dis + rad +  
tax + ptratio + lstat, data = df)
```

Residuals:

	Min	1Q	Median	3Q	Max
	-15.1814	-2.7625	-0.6243	1.8448	26.3920

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	41.451747	4.903283	8.454	3.18e-16	***
crim	-0.121665	0.032919	-3.696	0.000244	***
zn	0.046191	0.013673	3.378	0.000787	***
chas	2.871873	0.862591	3.329	0.000935	***
nox	-18.262427	3.565247	-5.122	4.33e-07	***
rm	3.672957	0.409127	8.978	< 2e-16	***
dis	-1.515951	0.187675	-8.078	5.08e-15	***
rad	0.283932	0.063945	4.440	1.11e-05	***
tax	-0.012292	0.003407	-3.608	0.000340	***

```
ptratio      -0.930961    0.130423   -7.138 3.39e-12 ***
lstat        -0.546509    0.047442  -11.519 < 2e-16 ***
---
```

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.789 on 495 degrees of freedom

Multiple R-squared: 0.7342, Adjusted R-squared: 0.7289

F-statistic: 136.8 on 10 and 495 DF, p-value: < 2.2e-16

The “both” method considers both adding and dropping variables for a blend between the methods. Once again, it produces a model without age and indus. Next class we will look at the *LASSO* method for model selection, which is a faster method.

Thursday, Jan 19

! TIL

Include a *very brief* summary of what you learnt in this class here.

There was no class on Thursday.