PHYS 339 - Measurements Lab in Gen Phys

Project Lab: Electrical Characteristics of Human Muscle Impulses

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Abstract

The electrical activity of the human bicep is measured through the skin via electromyography (EMG). The frequency spectrum of the activity in the contracted muscle is found to follow a broad Gaussian distribution, peaking near 70Hz($\sigma \approx 70Hz$) for two of four subjects and 100Hz ($\sigma \approx 100Hz$) for the other two. There is an average time delay of $(141.5 \pm 117.1)\ ms$ between the drop in electrical activity and absence of force applied while the muscle relaxes. Muscle electrical activity is found to increase linearly with force applied, at a rate of $(6.8 \pm 0.8)\ kg^{-1}$.

1 Introduction

The development of electronic amplifiers has made it possible to study the characteristics of human muscle electrical activity by measuring small voltages present on the surface of the skin above the muscles themselves. These studies have led to many advances in medical science and have dramatically expanded our understanding of the human body. Characterizing neuron-muscle interactions is a key aspect in muscular and neuronal degenerative diseases. In this experiment, an amplifier is used to characterize three aspects of the electrical activity in a human bicep: the frequency spectrum of the signal generated by a contraction, the time delay between the absence of electrical activity and absence of force, and the mathematical relation between the force generated by the muscle and the amplitude of the electrical signal detected.

2 Theory

2.1 Skeletal Muscles

Skeletal muscles are a type of muscle tissue controlled by the voluntary nervous system. Most skeletal muscles are attached to bones, via tendons. They are composed of bundles of muscle fibers, which are in turn composed of smaller elements including sarcomeres, which are the elements responsible for triggering their action. The predominant type of action of skeletal muscles is contraction, which occurs through electrochemical stimulation of the sarcomeres called an action potential, which peaks at approximately 40 mV. Muscles exhibit three principal types of contraction: concentric, when the muscle shortens in length; eccentric, when the muscle lengthens; and isometric when the length of the muscle stays the same. An example of these types is given by the motion of an arm bending at the elbow to lift a weight: concentric contraction as the weight is lifted, eccentric contraction as the weight is lowered, and isometric contraction as the weight is held at one level [1].

2.2 Electromyography

Electromyography is the measurement of electrical signals generated by the contraction of skeletal muscles. The contracting muscle fibers beneath the skin each contribute an action potential, which are summed together as a voltage which is read by the device. This signal (typically on the order of several millivolts) is amplified and then digitally recorded [2].

2.3 Characteristic EMG Signal and noise

The amplitude of typical signal range from 0 to 10 mV (peak-to peak). The electrical signal is mostly random and has a Gaussian distribution with energy distributed from 0 to 300 Hz. The dominant portion is between 50 and 150 Hz. The randomness of the frequencies is due to the fact that the firing rate in the muscle fiber is itself random. [3, 4]

3 Experimental Methods

The experiment consists of measuring electrical activity in the subjects' bicep during two types of motion. In order to reduce the number of variables in the experiment, the movements to be studied are standardized. This standardization is described in detail below for each movement, in Sections 3.2.1 and 3.3.1.

3.1 SpikerShield Box

Electrodes are placed on the subject's bicep, at a position dictated by the ratio 6.6/21 times the length of the upper arm (measured from forearm to inner shoulder)approximately two finger widths apart. These electrodes are connected to an Arduino shield made by Backyard Brains called the Muscle Spiker Shield, whose functional parts consist of a series of amplifiers [5]. The signal is amplified 2.5 times by an instrument amplifier, then approximately 80 times by a set of two operational amplifiers which also act as a bandpass filters with a bandwidth of approximately 900 Hz. This helps reduce noise in the signal.

3.2 Electrical Activity per Force Applied

A system of weights is used to measure the amount of electrical activity generated by the subject's muscles and compare it to the amount of force exerted. The subject is given a handle to which a 1 kg mass is attached. They are asked to hold the mass stationary at a 90^o angle for 10 seconds, an action which involves predominantly the bicep in an isometric contraction. This was repeated three times per trial, with trials for masses of 1, 2, 4, 6 and 8 kilograms. All subjects use their dominant arm.

3.2.1 Standardization of Movement

The standardization of this movement consists, in fact, of minimizing the amount of physical movement. The subject's arm is kept bent at 90^o in order to avoid the electrical activity generated during concentric and eccentric contraction, allowing the EMG to read only the activity created by isometric contraction. An assistant starts the measurement by holding the weight so that the subject is not using their muscle to hold it up. After ten seconds, the assistant lets go of the weights and the subject's bicep exerts the force for the next ten seconds.

3.3 Relation in Time Between Force and Activity

A force-sensitive resistor is attached to the underside of a desk and the seated subject pushes up on it. This motion triggers the bicep to contract, which is recorded by the EMG along with the signal from the force sensor.

3.3.1 Standardization of Movement

In order to avoid extraneous activity or lag, the arm is rested on the subject's knee, in contact with but not pushing on the sensor. Thus, immediately after muscle contraction begins, the sensor is triggered.

4 Results

4.1 Noise Characterization

The subject's arm is at rest for approximately the first two seconds of each measurement in order to quantify the amount of noise inherent to the bicep. A noise baseline is computing by averaging the noise, and the data generated is re-plotted with this noise baseline removed. The filtering code can be found below in Appendix C.

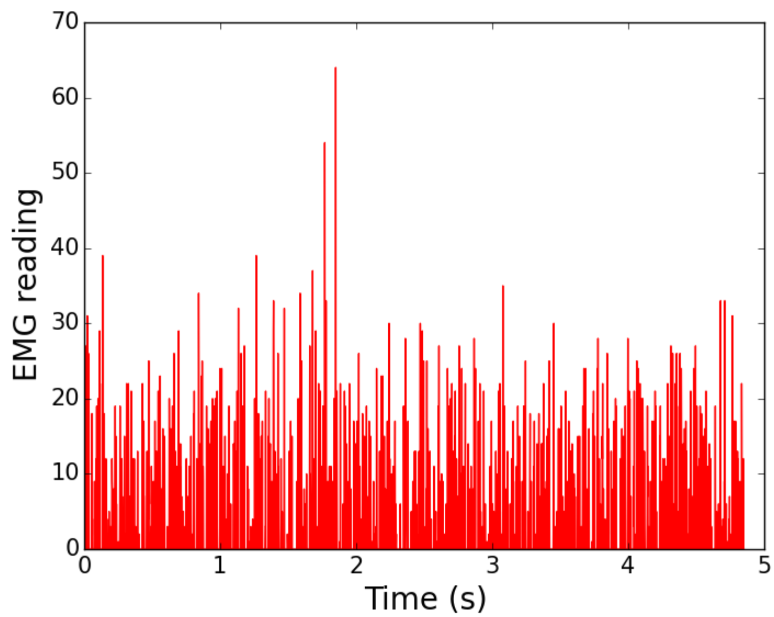

Figure 1: The signal received from a bicep at rest.

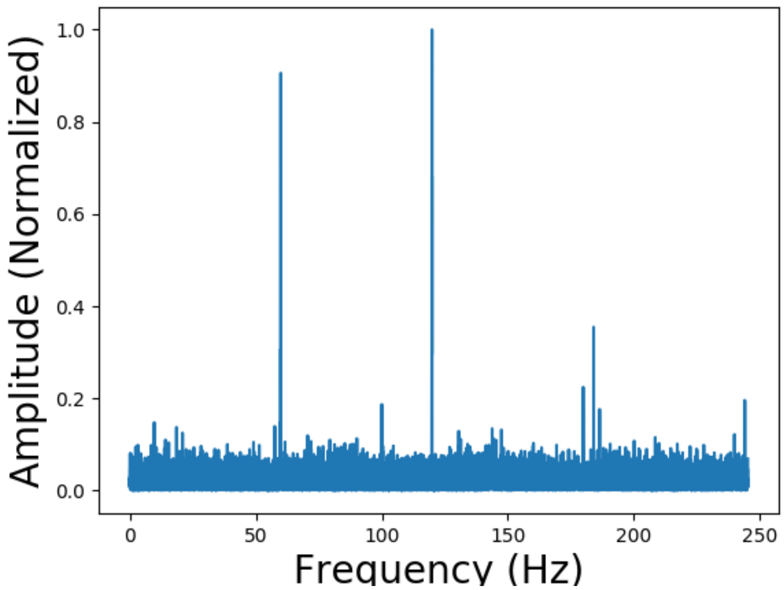

Figure 2: Fourier transform of the signal received when the bicep is at rest with the subject's elbow resting on their knee. The sharp spikes at 60, 120, 180, and 240 Hz are likely harmonics of the 60 Hz signal of nearby power sources. The electrical activity in the bicep when it is at rest is very low and evenly distributed across the frequency spectrum.

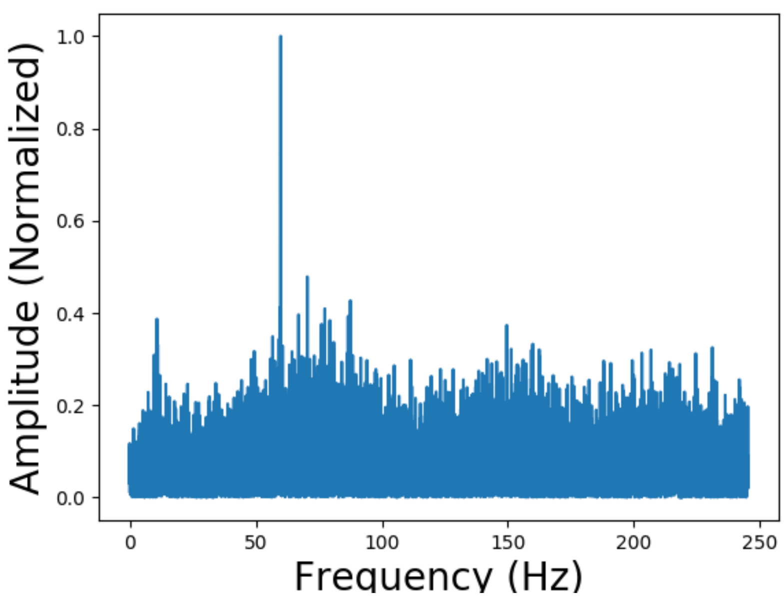

Figure 3: Fourier transform of the bicep with the arm maintained bent at 90°. Here the EMG activity is slightly higher, though still less than the ambient signals picked up by the electrode wires, which as above may be acting as small antennae.

The Fourier analysis of EMG samples when the bicep is at rest confirms that the minimal amount of electrical activity generated by the muscle is random in nature since it has no dominant frequency. We also notice that the EMG electrodes are highly sensitive, and in the absence of clear electrical activity from the muscle, will register signals from nearby electronic devices. We can therefore be confident that EMG readings of a significant amplitude (>100), which are not dominated by frequencies at 60Hz and associated harmonics, are a direct result of muscle activity.

4.2 Spectrum of Activated Muscle

Each subject holds a variable weight (1 kg, 2 kg, 4 kg, 6 kg and 8 kg) over ten second intervals, during which time the EMG signal is recorded. An example can be seen in Figure 4. The goal of having several isolated contractions is to remove effects from muscle fatigue, as extended muscle tension leads to sporadic contractions which are less easily characterizable.

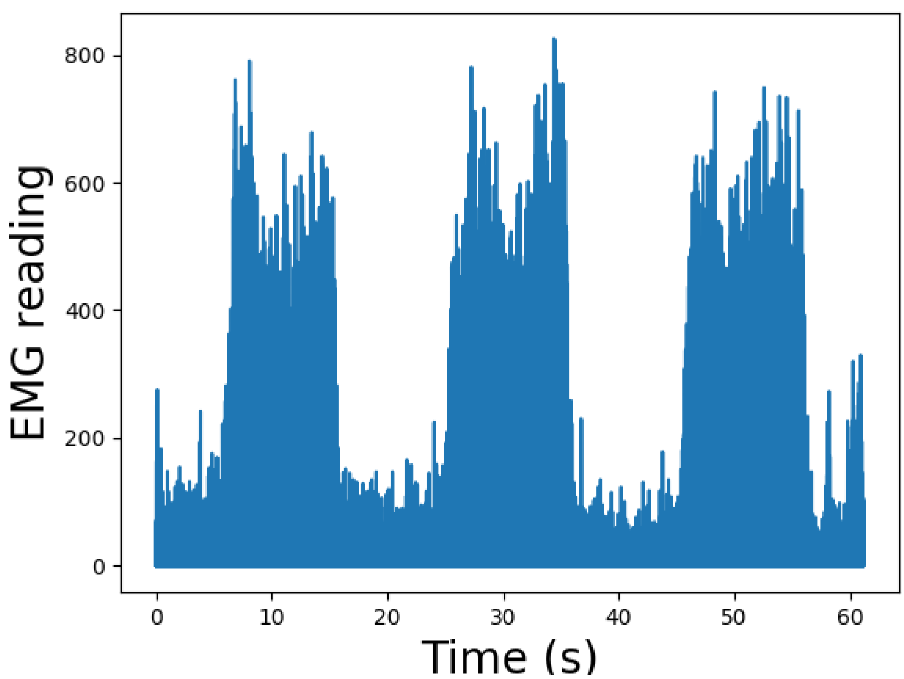

Figure 4: An example of the EMG reading when the subject held an 8 kg weight for 3 intervals of 10 seconds, spaced ten seconds apart. Note the inherent noisiness of the signal, and presence of signal noise (amplitude between 100 and 200) when the muscle is at rest.

To characterize this signal we look the Fourier transform of the sections in which the muscle is activated (see Figure 5). We expect the frequency spectrum to follow a broad Gaussian distribution, since the EMG signal is random in nature [3]. Inherent noisiness of the Fourier spectrum can be mitigated by averaging over intervals of ≈ 5 Hz. This allows us to fit the data to a Gaussian distribution much more closely (see Figure 6).

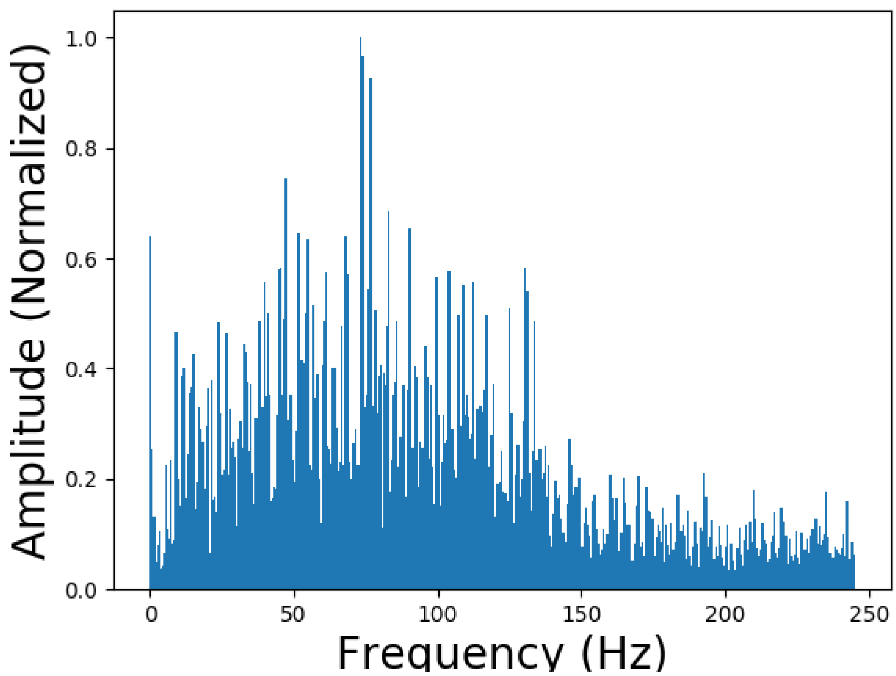

Figure 5: Fourier transform of the EMG signal. Note that inherent noisiness of the signal leads to an equally noisy Fourier spectrum, which is difficult to characterize. Python's curve-fitting function (scipy.optimize.curvefit) could not find optimal fit parameters before reaching a maximal number of function calls.

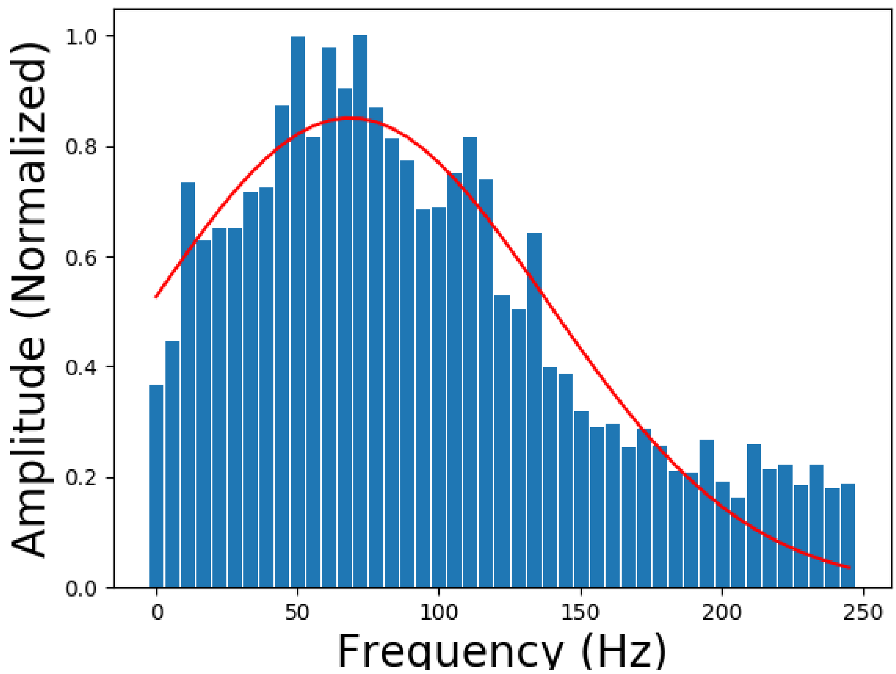

Figure 6: The frequency spectrum of the EMG signal, with values averaged over intervals of 5 Hz to reduce noisiness, with associated Gaussian distribution ($\mu = 68.69$ and $\sigma = 70.07$).

In order to determine whether EMG signal frequency is correlated to to amount of muscle strain, we characterize the mean frequency and standard deviation of the EMG by subject and by weight.

Subject N^o	$\mu_{Frequency}$	$\sigma_{Frequency}$
1	64.15	11.67
2	105.01	2.18
3	102.91	7.81
4	68.82	2.53

Table 1: Average EMG signal frequency per subject, over 4 kg, 6 kg, and 8 kg trials, with associated standard deviations. Averages for trials at 1kg and 2kg are not included because of noise from power sources dominating the signal in some subjects (see appendix A.2, figure 13

Weight	$\mu_{Frequency}$	$\sigma_{Frequency}$
4kg	87.75	23.97
6kg	82.45	26.82
8kg	85.51	15.69

Table 2: Average EMG signal frequency per weight, over all subjects, with associated standard deviations. Some trials at 1 and 2 kg could not be included due to overwhelming presence of background noise.

While the mean signal frequency stayed constant as the weight changed, the associated standard deviation is much larger, whereas the mean signal frequency per person had a much smaller standard deviation, indicating that the EMG signal frequency is more closely correlated to the subject than the weight.

4.3 Time Delay

As described in section 2.1, electrical activity in the motor neuron triggers a release of Ca^{2+}, which in turn triggers the muscle contraction. While electrical activity may have ceased in the muscle, the muscle may experience a slight delay before relaxing, as Ca^{2+} ions are cleared from the channels. [4] In order to characterize this time delay, we will use a force sensor to measure bicep muscle activation, while recording the corresponding EMG signal (see figure 7).

In an attempt to remove contributions to the force sensor's reading from muscles other than the bicep, the pressure sensor is placed on the underside of the table surface such that the subjects, with their arm at a right angle, uses only bicep activation to apply force upwards onto the sensor.

The pressure sensor is sensitive up to about 10N. In comparison, all four subjects can reasonably hold 10kg for a short period of time, indicating that their biceps output force on the order of 100N. Since the bicep output force in a range much larger than the accuracy range of the force sensor, this sensor saturates very quickly and effectively acts as an on/off sensor.

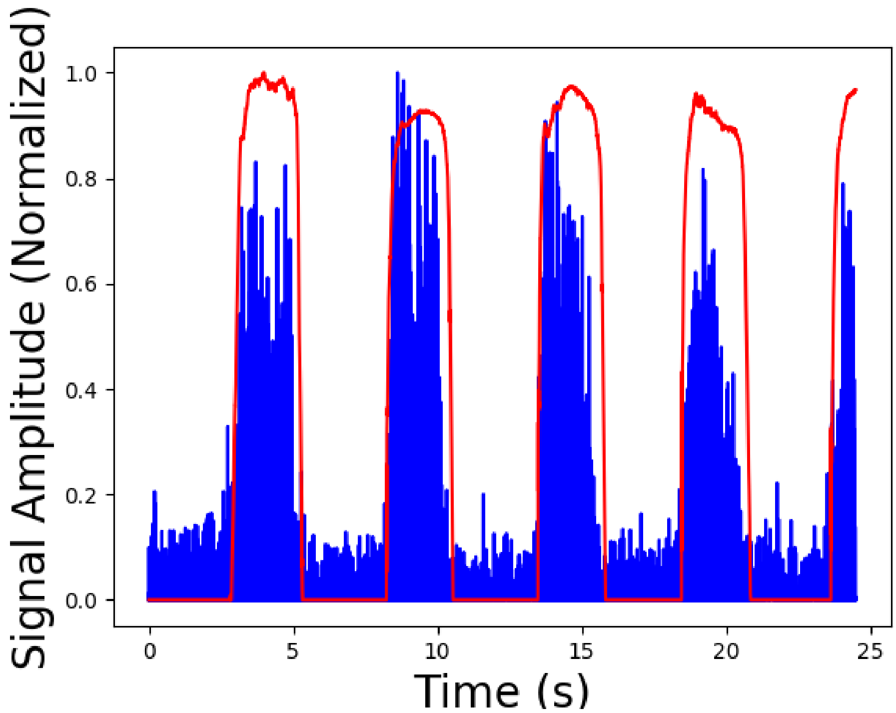

Figure 7: EMG signal (blue) with force sensor signal (red). Notice that the EMG signal drops slightly before the force signal, on the leading edge of each spike.

To characterize this time delay, we measure the time difference between the leading edge of the EMG and force signals for each spike. Noticing that the EMG signal noise is ≈ 15% of its max value, we define the threshold value at which we take the time difference to be 20% of the maximal value of that spike. Since the max value of the force sensor is 10N, we are measuring at the point where the force output is $2N$, and where the EMG activity spike fades into noise. This is, to a good approximation, the point where electrical activity and force have both ceased. Each subject performed 3 trials with between 3 and 5 spikes each. The mean time delays are shown in table 3.The significance of these results will be discussed later.

Subject N°	μ_{DELAY} (s)	σ_{DELAY} (s)
1	0.0780	0.0640
2	0.1733	0.0824
3	0.0730	0.0540
4	0.2434	0.1425
Avg	0.1415	0.1171

Table 3: Mean time delay between absence of EMG activity and muscle force output

4.4 Activity Versus Force

Figure 17 in Appendix A.3 shows the signal obtained while holding a weight of 8 kg 3 times 10 seconds, within a minute. A low-pass filter function is applied to the raw signal in order to reduce the amount of signal clutter (as shown on Figure 18). A cut-off frequency of the sampling frequency divided by 2000 is chosen, as much of the undesirable noise on the unfiltered signal comes from the Arduino's sampling frequency, and because the ratio between the lifting period (3 lifts per minutes) and the sampling period is around 2000. As visible in the figures, this reduces much of the noise and gives a quantitative method to get a clearer value for the signal.

By averaging the values on the upper plateau, the ratio of the signal measured to weight held is calculated. The four figures in Appendix A.1 show the signal versus the mass lifted for the four subjects individually. Figure 8 shows the mean of these data with a linear fit. The slope of the linear fit is $(6.8 \pm 0.8)\ kg^{-1}$ and the intercept is 7.5. Since the signal's amplitude is affected by the filter, the quantitative value of each individual point is meaningless, but the linear relation between them is meaningful (as discussed in Section 5).

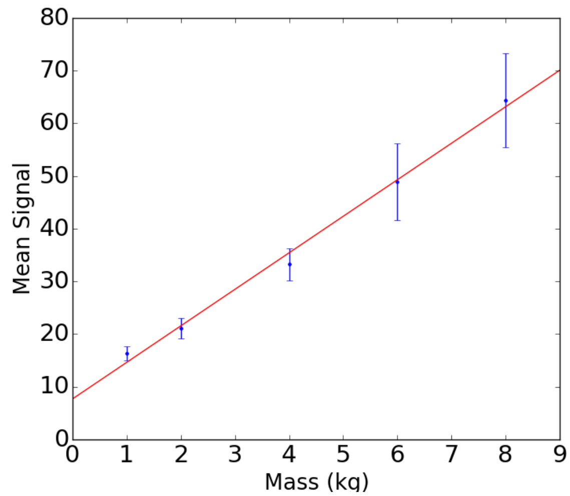

Figure 8: Average of the filtered signals. The slope of the line is $(6.8 \pm 0.8)\ kg^{-1}$ and the intercept is 7.5.

5 Discussion

5.1 EMG signal frequency spectrum

EMG signal frequency is random in nature, and is found to follow broad Gaussian distribution, as expected [6]. We notice some some distributions to be slightly bimodal (Appendix A.2, figure 14), indicating the presence of signal from type I (slow twitch) and type II (fast twitch) muscle, which peak between 75 and 125 Hz and between 125 and 250 Hz respectively [7]. We also notice some distributions, although Gaussian, are narrower than others (figure 15 compared to figure 16), this is likely due to the placement of the EMG pads closer to the innervation zone of the muscle (where the motor neuron enters the muscle), near the top of the bicep [6].

We notice that the mean frequency varies more by weight held than it does by person, indicating that EMG signal frequency is not necessarily correlated to muscle force output, although EMG signal amplitude clearly is (this is discussed in a following section). A larger sample size of subjects would allow for a more complete analysis of the distribution of EMG signal frequency by person. One would expect a Gaussian distribution centered around 80Hz, but characterization by hours of exercise per week could lead to interesting insight on learned/developed neuro-muscular interaction.

5.2 Time Delay

There exists a noticeable time delay between the drop in electrical activity of the muscle and the drop of force output, on the order of 0.1 to 0.2 seconds. This is lower than the range found by Ferris-Hood and colleagues (1996), who found relaxation time of the knee-extensor to be delayed between 239 and 300 ms after the drop of EMG activity [8]. The value range found in this experiment is, however, larger on average than the value found by Robert and Gabaldon (2008) for muscle relaxation time in turkeys, which varied linearly with walking/running speed, on the order of 80 to 50ms [9]. There is no noticeable correlation between EMG signal frequency distribution and muscle relaxation delay.

The process of muscle relaxation is complex and dependent on many factors unaccounted for in this experiment, such as nutrition, dehydration, and fatigue.

Due to the more rapid onset of muscle force following EMG activity we were not able to characterize the time delay in this case. Cavanaugh and Komi (1979) characterize this time delay, for isometric contractions, at 59.3 ms [10], an order of magnitude smaller than the force offset delay found in this experiment. In this case, the muscle reacts so quickly that the EMG spike is hidden by the noise, but filtering this noise alters the shape of EMG curve and would lead to inaccurate results. To characterize this delay more easily, finewire or needle electrodes inserted directly into the muscle could be used to reduce signal noise. A pressure sensor with a higher saturation point would allow for the force onset to be fitted to a time dependent function (as opposed to its current step function) which could be compared with a smoothed signal, and an average time difference over the entire muscle activation process would be obtained.

Finally a second set of electrodes could be used to characterize the the EMG activity of the antagonist muscle (the triceps, in the case of this experiment). The time delay between the offset of the bicep and the onset of the triceps and vice-versa is likely correlated with the activation time of either muscle alone.

5.3 Signal Linearity

The linearity of the signal with force applied can be intuitively understood using a simple model. Electrical signal is sent to a muscle fiber for its activation. If we consider this signal to be the same for each fiber, the signal strength is proportional to the number of fibers involved. By Newton's law, $F = ma$, the force applied is proportional to the mass lifted and this is naturally also the case for the muscle fibers involved in lifting the weight. The non-zero intercept is due to the body's inherent electrical noise, as well as interference from the environment.

5.4 Multiple Muscle Involvement

During the lifting test, the weight is hung on the hand. Therefore, the energy spent by the body does not come only from the bicep, but also from all the muscles involved keeping the hand and forearm steady. Mechanically, the force applied to keep the arm at 90^o is still on the bicep (the forearm muscle only keeps the hand straight), but the human body is highly complex and isolating one aspect of it is very difficult, especially to establish dependent relations and across multiple subjects. A more effective way of involving only the bicep could have been to suspend the weight directly off the forearm, thereby eliminating the need to stabilize the hand.

5.5 Multiple Sets of Electrodes

Having multiple points of measurement for the same trial may have allowed for better noise reduction, by permitting cross-referencing signals between them. A second set of electrodes (connected to a second amplifier, interfaced through the same Arduino) could be placed near the first set, in order to capture the same muscle in action. This is a technique commonly used in medical EMGs, for example the network of electrodes placed all over the chest during cardiac activity measurements [11]. The additional electrodes could also be used to measure other muscles suspected of being secondarily involved in the action, or to confirm the measurement of the time delays.

Unfortunately, limits on time were prohibitive to the construction and application of a second EMG device.

6 Conclusion

Analysis of the electric signal and force output of the bicep was successful in determining the power spectrum, time delay and relation between force output and EMG activity of a human bicep. The power spectrum of the activated bicep was found to follow a broad Gaussian distribution with the dominant frequencies in the 50-150 Hz region. A time delay of the order of $100ms$ was found between the end of electric activity of the bicep and its relaxation (absence of force on sensor). A linear relation of $(6.8 \pm 0.8)\ kg^{-1}$ was found between the force exerted by the bicep and corresponding EMG activity.

Obstacles faced in this investigation included the presence of noise in the recorded EMG signals, low saturation points of force sensors and limited sample size. An additional set of electrodes could help remove noise from the signal in order to find a more precise EMG signal frequency range by probing the same muscle in a different place, or could be used to probe the antagonist muscle to isolate the time delay in force offset due the muscle's relaxation from the time delay due the antagonist muscle's contraction onset. A pressure sensor with a higher saturation would characterize force onset and offset as functions of time, which would allow for a more accurate time delay comparison with EMG signal onset and offset. A larger sample size would allow for more significant mean signal frequency and amplitude measurements, and would help characterize these by factors other than weight lifted, such as age, exercise level, and BMI.

Further investigations to consider include linearity of EMG activity with muscle size or maximum force output; does the bicep have a smaller EMG signal amplitude than the quadricep? Are both muscles activated by similar signal frequencies? Lastly one could test for whether muscle force onset and offset is most closely correlated to muscle length (since contractions are caused by muscle fibers "pulling" against each other along their length) or cross-sectional area (since nerves are distributed throughout the muscle), by testing muscles of different cross-sectional areas but similar lengths, such as the calf and quadriceps muscles.

References

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A Miscellaneous Data

A.1 Signal Versus Weight for Each Person

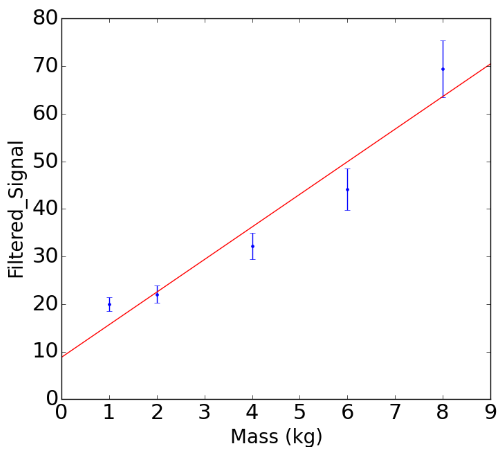

Figure 9: Average of the signal for Subject 1

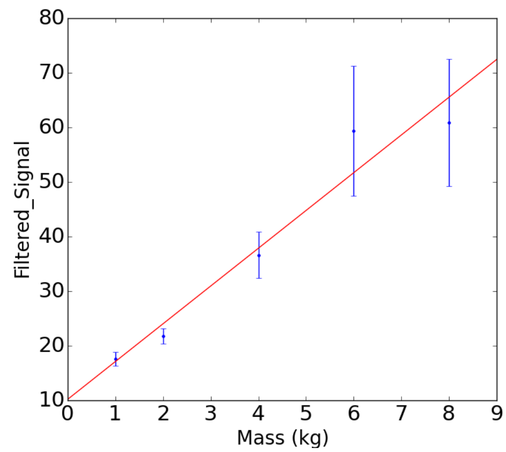

Figure 10: Average of the signal for Subject 2.

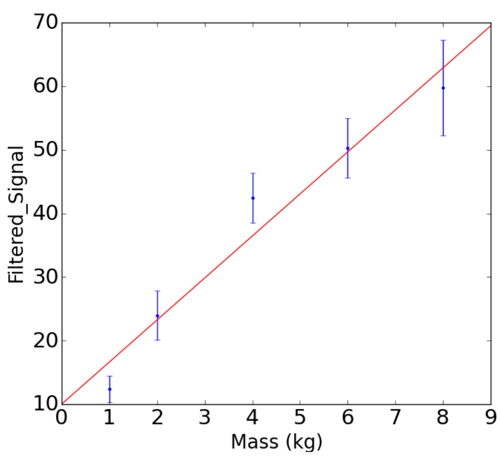

Figure 11: Average of the signal for Subject 3.

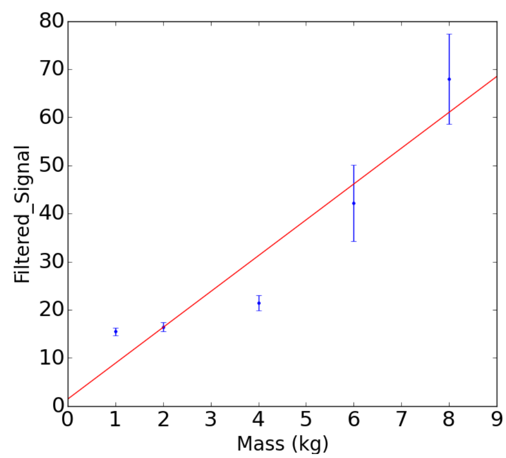

Figure 12: Average of the signal for Subject 4.

A.2 Representative frequency spectra of EMG signals

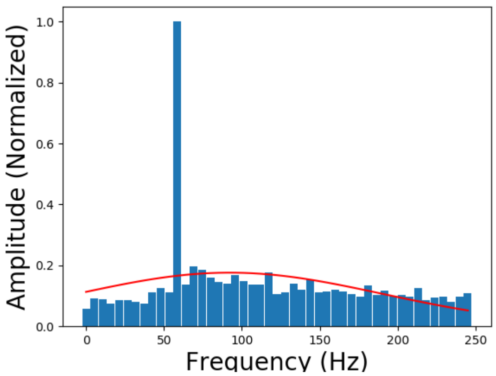

Figure 13: Frequency spectrum of Subject 2's EMG signal at 2kg. Dominating component at 60Hz is likely result of noise from a nearby power source, possibly amplified by the apparatus, or even the subject themself.

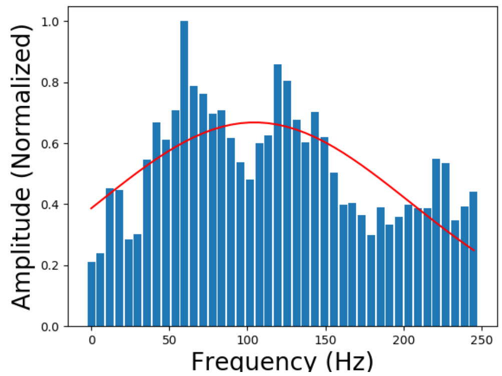

Figure 14: Frequency spectrum of Subject 3's EMG signal at 6kg. Note the bimodal distribution. This may due to different kinds of muscle fiber slow twitch firing near 75-125 Hz, and fast twitch firing near 125-250 Hz, or it may be due to periodic muscle spasms as the muscle becomes fatigued after maintaining a near constant contraction.

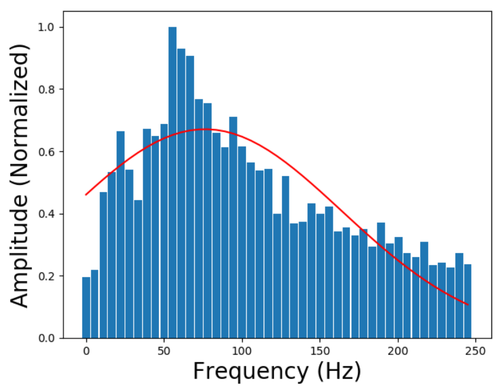

Figure 15: Frequency spectrum of Subject 1's EMG signal at 8kg. The narrower peak is due to electrode placement closer to the muscle's insertion point (on a bicep this is the part nearest to the elbow).

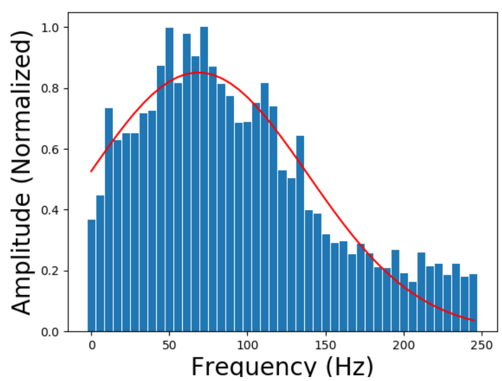

Figure 16: Frequency spectrum of Subject 4's EMG signal at 8kg. The wider distribution is indicative of electrode placement closer to the center of the muscle.

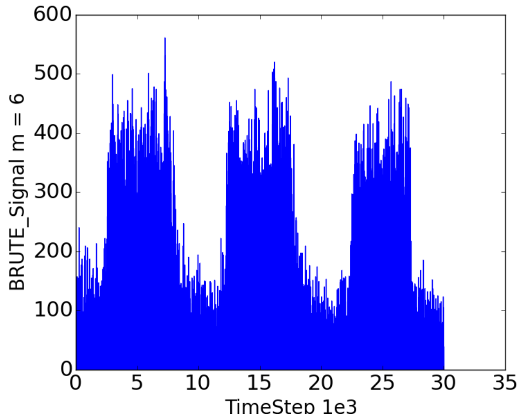

Figure 17: Lifting test result for Michael C. lifting 6 kg 3 times within a minute.

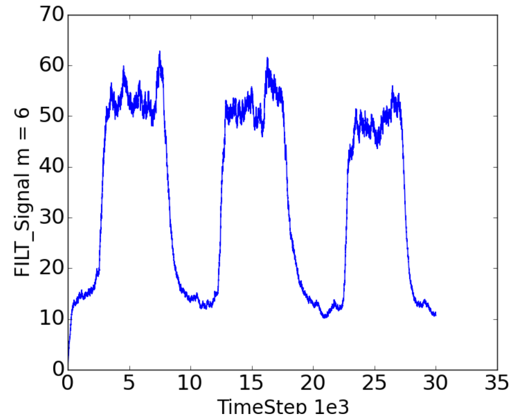

Figure 18: Same signal low-pass filtered with a cut-off frequency of 2000 times less than the sampling frequency.

A.3 Sample of Lifting Test

B Raw data

Weight (kg)	$\mu_{Frequency}$ (Hz)	$\sigma_{Frequency}$ (Hz)	Covariance (μ)	Covariance (σ)
Subject 1				
1	59.23	110.59	18.74	19.80
2	64.06	112.49	16.12	17.98
4	63.04	116.19	14.46	16.19
6	53.07	125.37	20.13	21.02
8	76.34	88.00	6.98	8.07
Subject 2				
1	-	-	-	-
2	-	-	-	-
4	106.79	99.81	10.28	15.18
6	105.80	90.81	9.63	13.17
8	102.61	101.25	12.12	17.74
Michael C.				
1	103.70	129.70	16.36	29.67
2	-	-	-	-
4	109.75	101.54	6.82	10.33
6	104.59	99.92	7.51	10.99
8	94.40	94.01	7.50	9.97
Michael T.				
1	67.34	88.16	8.51	9.24
2	-	-	-	-
4	71.41	79.18	6.23	6.91
6	66.35	65.37	4.29	4.73
8	68.69	70.07	3.43	3.79

Table 4: Table of average EMG signal frequencies, and associated standard deviations, as found by a Gaussian fit to the Fourier transform of EMG signals, by subject and by weight held.

C Arduino and Python Code

The following is the data acquisition code with Python, inspired largely from the Mark's code.

Fourier transform code, used to find power spectrum of EMG signal:

```
def fourier(data, Ts, fs):
    t = np.arange(0,1,Ts)
    n = len(data)
    k = np.arange(n)
    T = n/fs
    frq = k/T
    frq = frq[1:int(n/2)]
    Y = np.fft.fft(data)
    Y = Y[1:int(n/2)]
    Y = (np.real(Y)**2)
    return frq, Y
```

Frequency spectrum averaging function, averages over 50 step intervals (5 Hz):

```
nsteps = 50
afrq= np.arange(0, len(frq), nsteps)
aY = np.zeros(len(afrq))
for i in range(len(afrq)):
    a = i*nsteps
    b = ((i+1)*nsteps )-1
    aY[i] = np.mean(Y[a:b])


afrq = np.linspace(0, np.amax(frq), len(aY))
```

Gaussian curve fitting function to fit smoothed EMG signal:

```
def gaus(x,a,x0,sigma):
    return a*np.exp(-(x-x0)**2/(2*sigma**2))
popt,pcov = curve_fit(gaus,frq,filtY)
fit = gaus(frq,*popt)

# ARDUINO DATA COLLECTION


delay = 5
a = Arduino()
steps = 5 *600*10^6
a.getResp();
a.send("START")


time= numpy.zeros(steps)
signal = numpy.zeros(steps)
force = numpy.zeros(steps)
ind = 0


while True:
    resp = a.getResp()


    if 22== len(resp) and resp[12] == ':':
        words = string.split(resp,":")
        time[ind] = words[0]
        signal[ind]=words[1]
        force[ind]= words[2]
    else:
        print("Unexpected response: %s"%(resp))
        print("Length: %d"%(len(resp)))


    if ind == steps-1:
```

```
        break
    else:ind = ind +1

a.send("STOP")

def Low_Pass_Filter(fs,fc,signal):
    L = numpy.size(signal)
    result = numpy.zeros(L)
    alpha = 2*numpy.pi*fc/fs
    beta = 1-alpha
    signal = numpy.copy(signal) # arrays are passed by reference
    #signal[:,1] *= signal[:,1]  # square the errorbars
    result[0] = alpha*signal[0]
    for i in range(1,L):
        result[i] = alpha*signal[i]+beta*result[i-1]
    #signal
    #result[:,1] = numpy.sqrt(result[:,1])
    return result

# find sample frequency from time step array
def freq(time):
    diffs= numpy.zeros(len(time)-1)
    for i in range(0, len(time)-1):
        diffs[i]= time[i+1]-time[i]

    return (10**6)/(numpy.mean(diffs))


signal_filtered = Low_Pass_Filter(freq(time), freq(time)/200, signal)
force_filtered = Low_Pass_Filter(freq(time), freq(time)/200, force)
```

```
# define threshold value to remove noise from signal by averaging over # first second of
def thresh(time, signal_filtered):
    steps = numpy.where(time<=10**6)
    avg = numpy.mean(signal_filtered[steps])
    var = numpy.var(signal_filtered[steps])
    return avg+(numpy.sqrt(var))
```

The following is the code to process the data.

```
import matplotlib.pyplot as plt
import numpy as np
import matplotlib as mpl



#####For Maclean
data1 = np.load('./maclean_1kg.npy')
data2 = np.load('./maclean_2kg.npy')
data4 = np.load('./maclean_4kg.npy')
data6 = np.load('./maclean_6kg.npy')
data8 = np.load('./maclean_8kg.npy')



m = [1,2,4,6,8]
data = [data1,data2,data4,data6,data8]
time = np.linspace(0, (len(data1)), len(data1))
cutOffFactor = 2000



# *** Time interval Maclean
#1kg
```

```
tstamps1 = [[1623,6350],[11290,16229],[20675,26532]]
#2kg
tstamps2 = [[2328,7762],[12913,17923],[22933,28084]]
#4kg
tstamps4 = [[3387,7479],[13195,17782],[22510,27590]]
#6kg
tstamps6 = [[3034,7621],[13054,17641],[22510,27520]]
#8kg
tstamps8 = [[3034,7762],[12983,17429],[23074,27802]]



tstampsAll = [tstamps1,tstamps2, tstamps4, tstamps6, tstamps8]



#Compute the average for each step for each weight
avgs = np.zeros(len(tstampsAll))
err = np.zeros(len(tstampsAll))
avgsFILT = np.zeros(len(tstampsAll))
errFILT = np.zeros(len(tstampsAll))
j = 0
for b in tstampsAll:
    i = 0 #tell which weight we are
    avgs1 = 0 #temporary variable
    err1 = 0 #temporary variable
    avgs1FILT = 0 #temporary variable
    err1FILT = 0 #temporary variable
    dataFILT = Low_Pass_Filter(freq(time), freq(time)/cutOffFactor, data[j])

    for a in b:#Here we compute the average/error for the intervals for one weight
        i = i+1 #increment
```

```
        y = data[j][a[0]:a[1]]
        nonzeros = np.argwhere(y>0)
        y = y[nonzeros] #cancel the zeros
        #Compute the average and error
        avgs1 = avgs1 + np.mean(y)
        err1 = err1 + np.std(y)

        #Filtered stuff
        yFILT = dataFILT[a[0]:a[1]]
        nonzeros = np.argwhere(yFILT>0)
        yFILT = yFILT[nonzeros] #cancel the zeros
        #Compute the average and error
        avgs1FILT = avgs1FILT + np.mean(yFILT)
        err1FILT = err1FILT + np.std(yFILT)


    avgs[j] = avgs1/i
    err[j] = err1/i
    avgsFILT[j] = avgs1FILT/i
    errFILT[j] = err1FILT/i
    j = j+1

m = [1,2,4,6,8] #mass in kg
p = np.poly1d(np.polyfit(m, avgs, 1))
x = np.linspace(0,9, 100)
y = p(x)
plt.figure()
plt.errorbar(m, avgs, err, fmt='.')
plt.plot(x,y, '-r')
plt.xlabel("Mass (kg)",fontsize=20)
```

```
plt.ylabel("Brute_Signal AVGS", fontsize=20)
plt.show()

##Filtered
m = [1,2,4,6,8] #mass in kg
p = np.poly1d(np.polyfit(m, avgsFILT, 1))
x = np.linspace(0,9, 100)
y = p(x)
plt.figure()
plt.errorbar(m, avgsFILT, errFILT, fmt='.')
plt.plot(x,y, '-r')
plt.xlabel("Mass (kg)",fontsize=20)
plt.ylabel("Filtered_Signal", fontsize=20)
plt.show()



print("AverageFiltered:",avgsFILT)
print("ErrorFiltered:",errFILT)



#Stack all togeter
avgsMicC = [ 12.39803743,  23.99014542,  42.46239865,  50.31237426,  59.75426601]
avgsLeo = [ 20.05943047,  22.06596444,  32.42348681,  43.78277691,  68.86407594]
avgsMicT = [ 15.44427065,  16.44685339,  21.45307482,  42.18625762,  67.98348934]
avgsMac = [ 17.5508906 ,  21.70893173,  36.62011453,  59.37731109,  60.85886261]

errMicC = [ 2.06093549,  3.84131496,  3.88078339,  4.64514206,  7.506241  ]
errLeo = [ 1.32411266,  1.71750389,  2.46304424,  4.72210142,  6.92007086]
errMicT = [ 0.81738681,  0.96826119,  1.59310487,  7.96857961,  9.37591907]
errMac = [  1.24096077,   1.37762804,   4.29268622,  11.89902117,  11.69145359]
```

```
avgArray = [avgsMicC, avgsLeo, avgsMicT, avgsMac]
errArray = [errMicC, errLeo, errMicT, errMac]

mean = np.zeros(len(avgsMicC))
error = np.zeros(len(avgsMicC))
for i in [0,1,2,3]:
    mean = mean + avgArray[i]
    error = error + errArray[i]
mean = mean/4
error = error/4

m = [1,2,4,6,8] #mass in kg
p = np.poly1d(np.polyfit(m, mean, 1))
x = np.linspace(0,9, 100)
y = p(x)
plt.figure()
plt.errorbar(m, mean, error, fmt='.')
plt.plot(x,y, '-r')
plt.xlabel("Mass (kg)",fontsize=20)
plt.ylabel("Mean Signal", fontsize=20)
plt.show()
```