

# 3D gravity gradient inversion by planting density anomalies

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## INTRODUCTION

Large effort directed to 3D gravity inversion.

Traditional methods produce blurred images, e.g., Li and Oldenburg (1998).

Portniaguine and Zhdanov (2002) and Silva Dias et al. (2009) produce sharp images.

Computational bottlenecks:

- Calculating the Jacobian (sensitivity) matrix;
- Solution of large equation system;

Trying to solve bottlenecks, René (1986) grows the solution around "seeds" (but it is 2D).

Camacho et al. (2000) use a systematic search algorithm (doesn't guarantee compactness).

We present a new 3D inversion that uses "seeds", like René (1986), and imposes compactness like Silva Dias et al. (2009).

Doesn't require solving a large equation system.

Only calculates parts of the Jacobian matrix (lazy evaluation).

## INVERSE PROBLEM

$N$  observations of GGT components.

Interpretative model:

- $M$  juxtaposed prisms;

Approximate anomalous density by summed effect of prisms:

$$\mathbf{d} = \mathbf{G}\mathbf{p}$$

Formulate the goal function as:

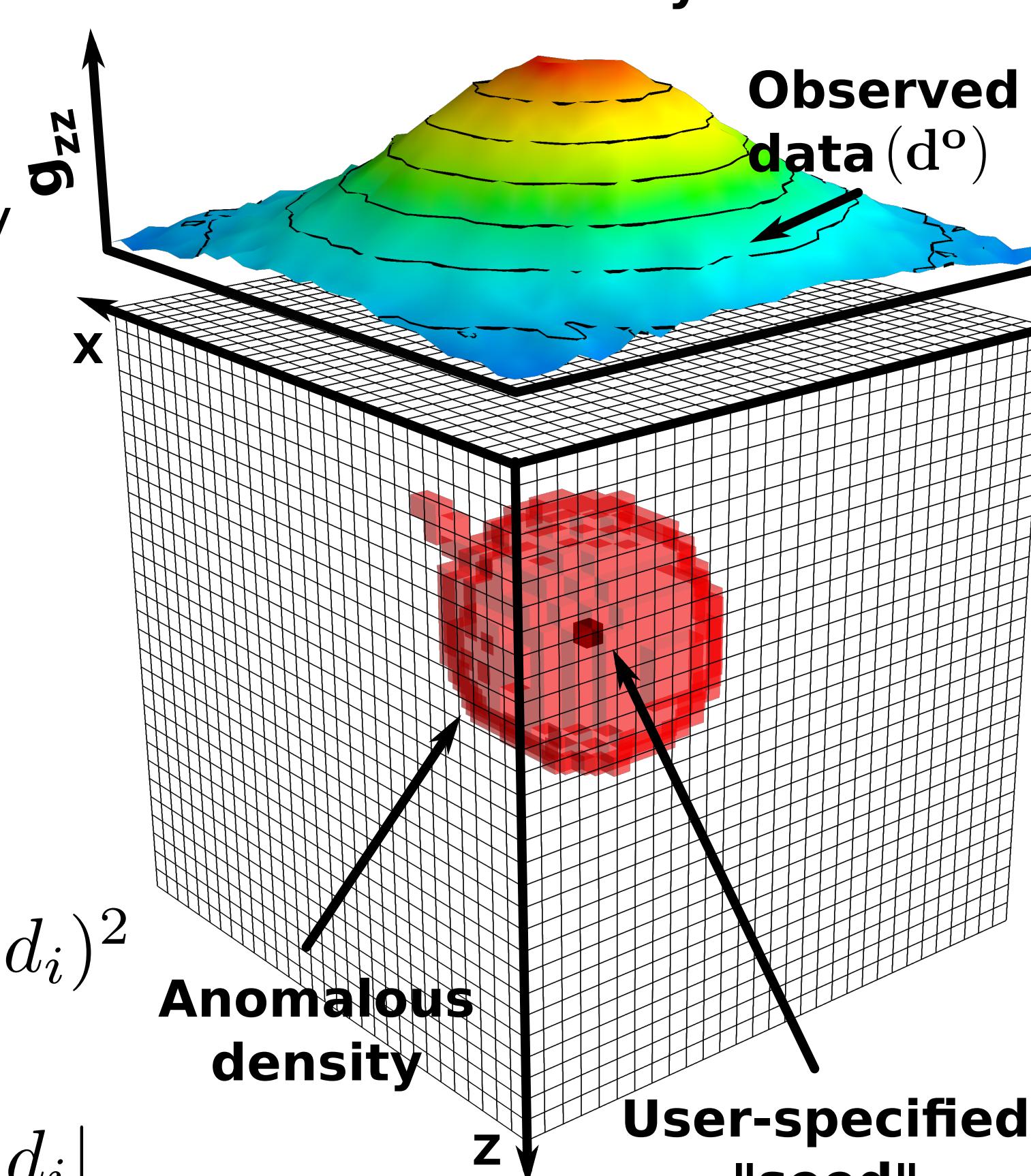
$$\Gamma(\mathbf{p}) = \phi(\mathbf{p}) + \mu\theta(\mathbf{p})$$

Function  $\phi(\mathbf{p})$  is a measure of the data misfit and can be:

- $\ell_2$ -norm:  $\phi(\mathbf{p}) = \sum_{i=1}^N (d_i^o - d_i)^2$
- $\ell_1$ -norm:  $\phi(\mathbf{p}) = \sum_{i=1}^N |d_i^o - d_i|$

Interpretative model, anomalous density and seed

Observed data ( $\mathbf{d}^o$ )



Residuals vectors:  $\mathbf{r} = \mathbf{d}^o - \mathbf{d}$

The  $\ell_1$ -norm can be used for robust inversion.

Estimating  $\mathbf{p}$  from  $\mathbf{d}$  is ill-posed and requires constraints:

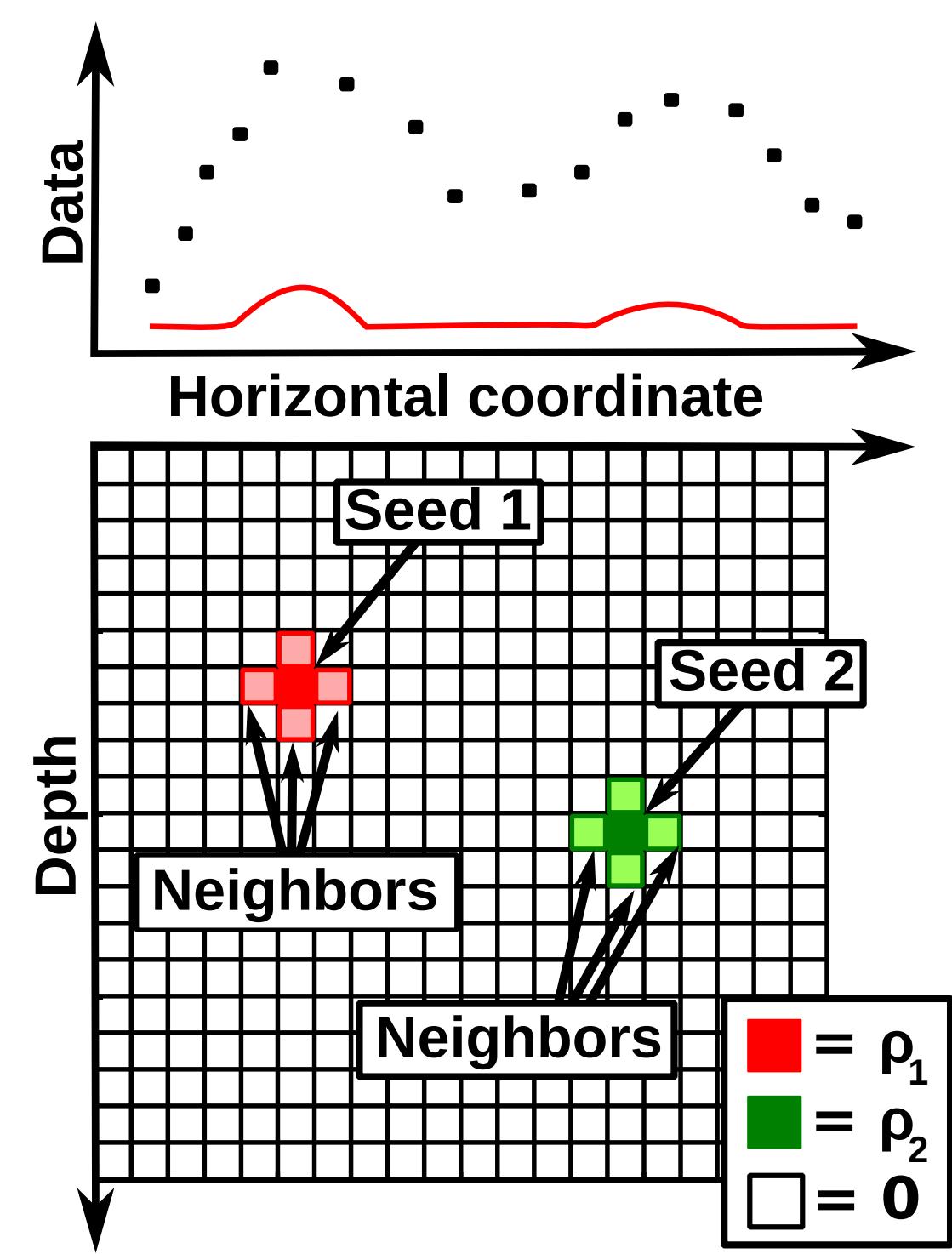
- Mass is concentrated around user-specified prisms (called "seeds");
- Solution is compact;
- Known density contrasts;

Imposed using regularizing function similar to Silva Dias et al. (2009):

$$\theta(\mathbf{p}) = \sum_{i=1}^M \frac{p_i}{p_i + \epsilon} l_i^\beta$$

## PLANTING ALGORITHM

### Starting configuration



- Start with set of seeds with known density contrasts;
- Find the neighbors of each seed;
- Initialize  $\mathbf{p}$  with the density of the seeds;
- All other parameters start with zero;

### Accretion

Temporarily add each neighbor of the seed.  
Add with density contrast of the seed.  
Choose the one that:

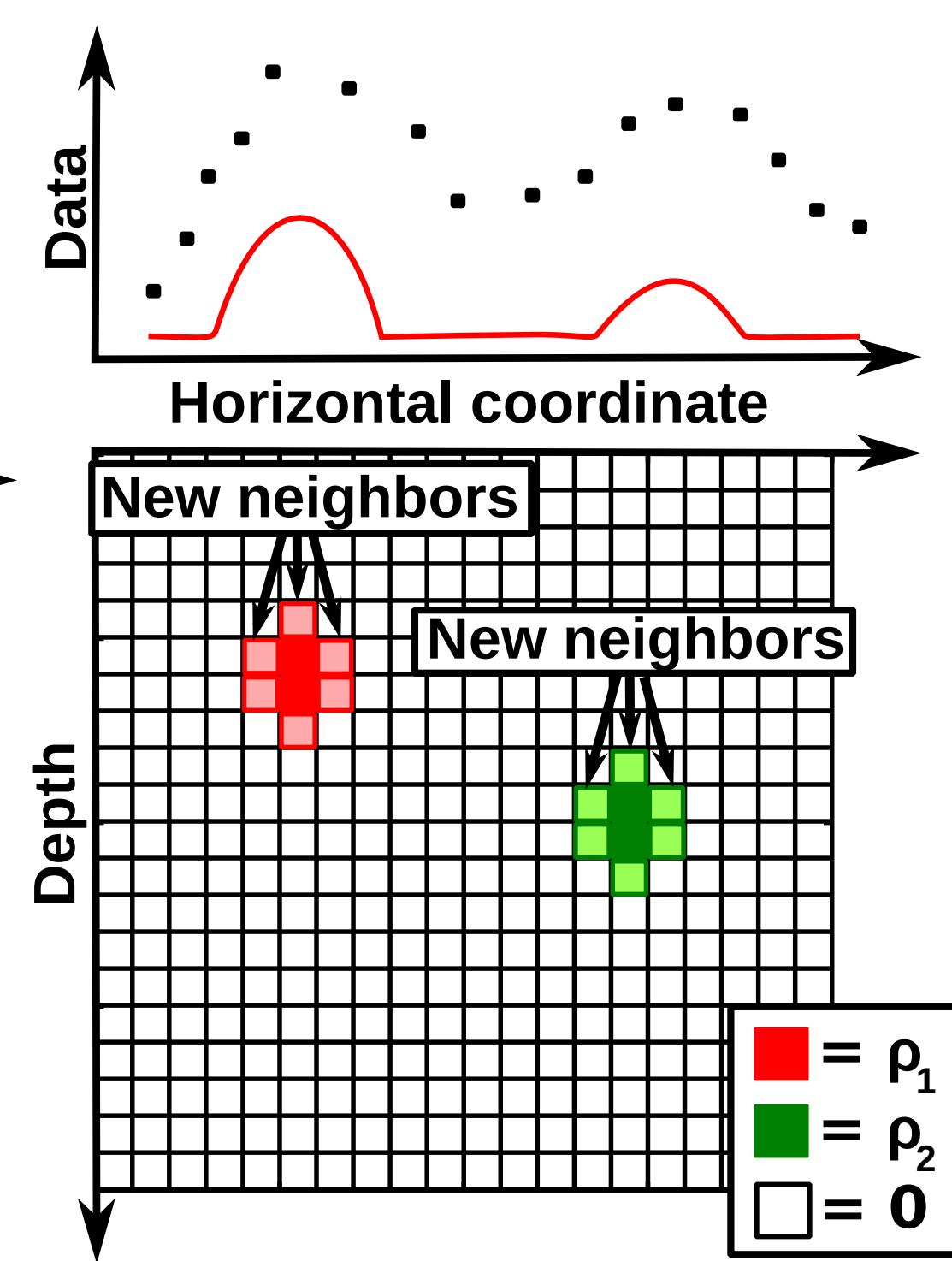
- Reduces the data misfit function;
- Produces the smallest goal function;

If no neighbor meets criteria the seed doesn't grow.  
Update residuals vector.

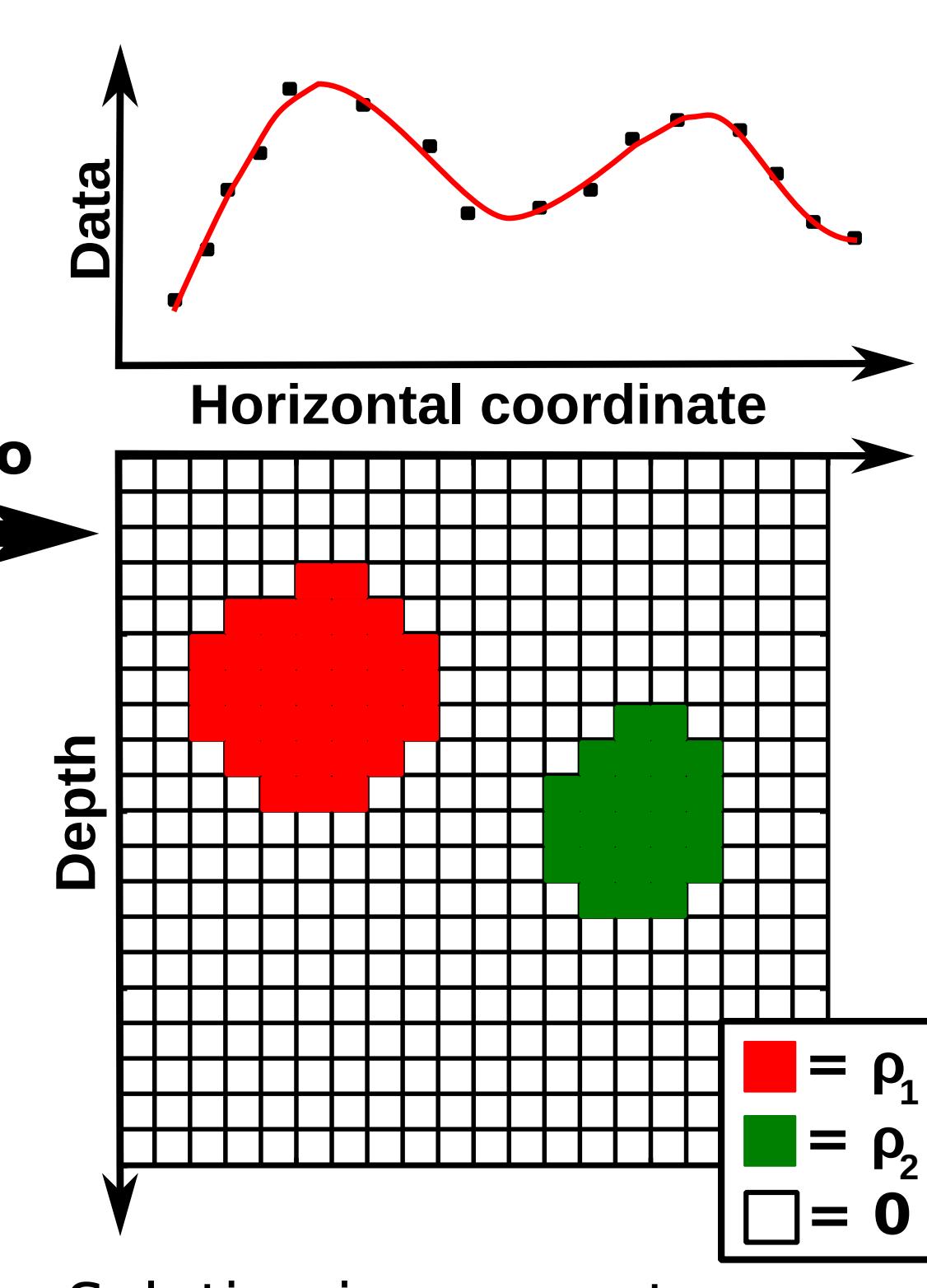
LOOP OVER SEEDS

### Growth

#### End of growth iteration



### Final result



- Solution is compact;
- Seeds produce bodies of different sizes;
- Estimate fits observed data (according to  $\ell_1$ - or  $\ell_2$ -norms);
- No equation system to solve;

## LAZY EVALUATION

The search is limited to neighboring prisms of the current solution.

Each column of  $\mathbf{G}$  refers to one parameter (prism in the interpretative model).

So don't need to know all columns of  $\mathbf{G}$ .

Only calculate the ones corresponding to neighboring prisms.

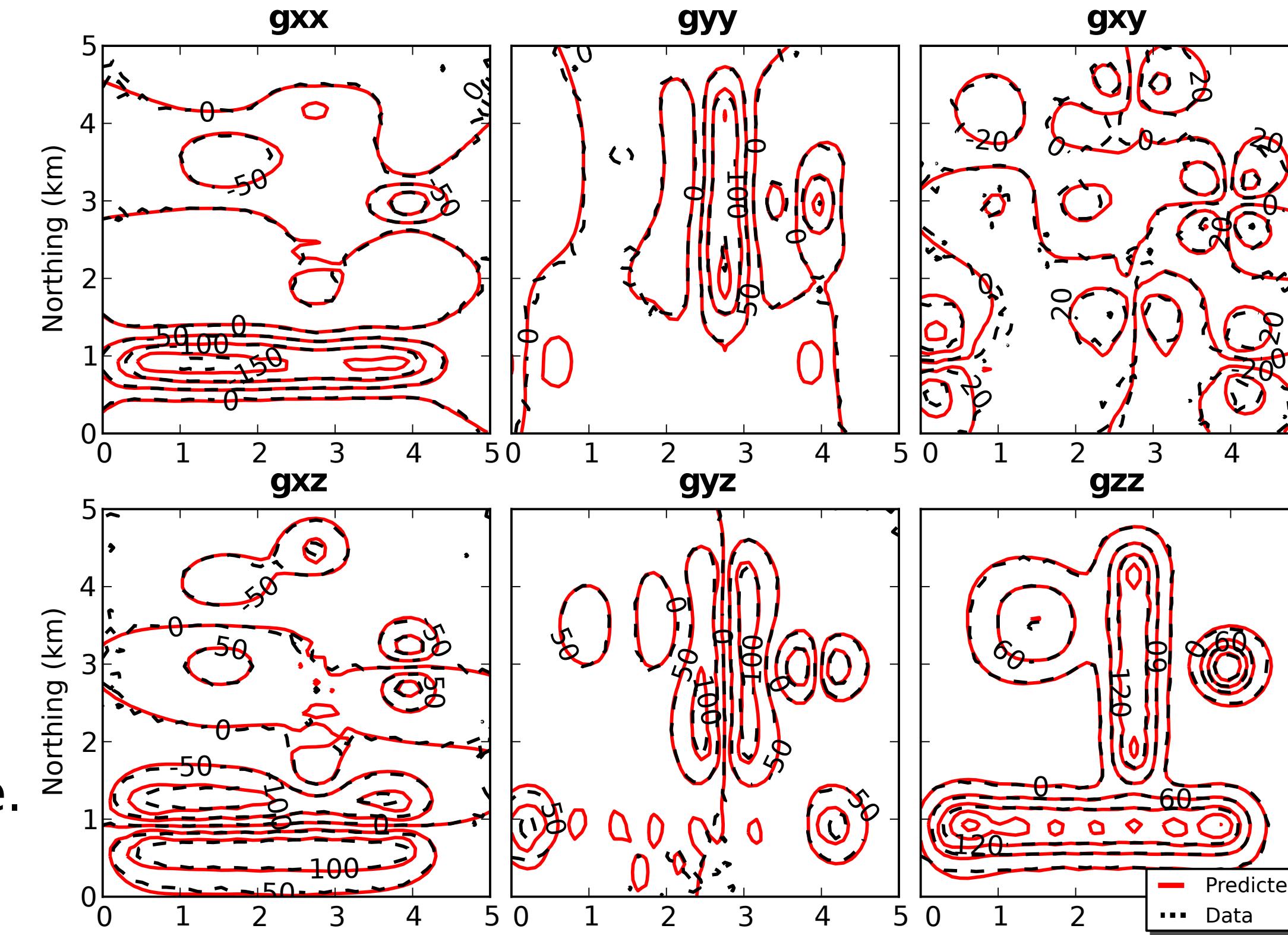
Can delete the column once a prism is added to the estimate.

Called *lazy evaluation* in computer science.

Reduce computation time and memory usage.

No need for data compression algorithms, e.g., Portniaguine and Zhdanov (2002).

## SYNTHETIC EXAMPLE USING THE L2-NORM



Dashed black contours = synthetic gradiometry data produced by (a).

Calculated at 150 m height.

Added 2 Eötvös pseudorandom Gaussian noise.

Used set of 18 seeds shown in (b).

Data set = 15,000 measurements.

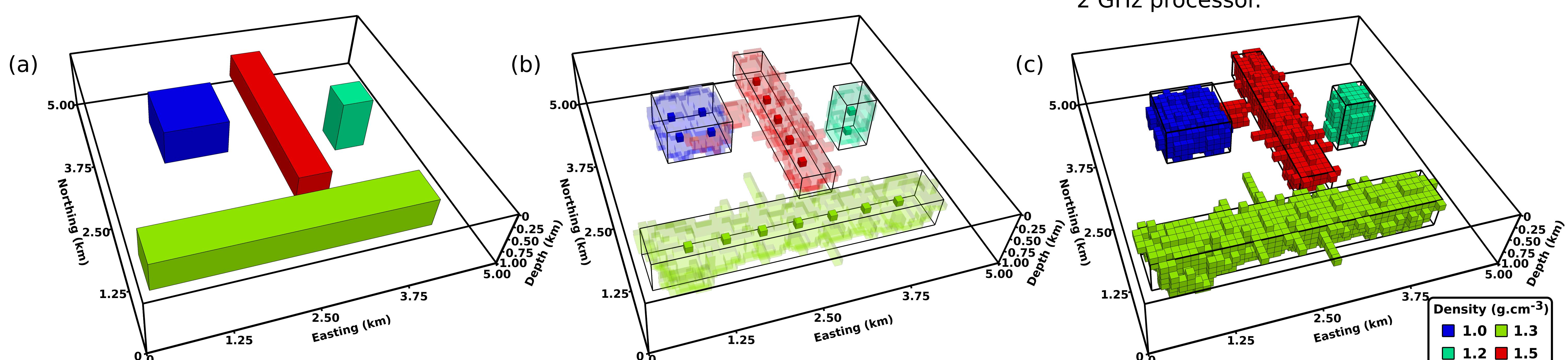
Interpretative model = 25,000 prisms.

Estimate in (c) is close to the model in (a).

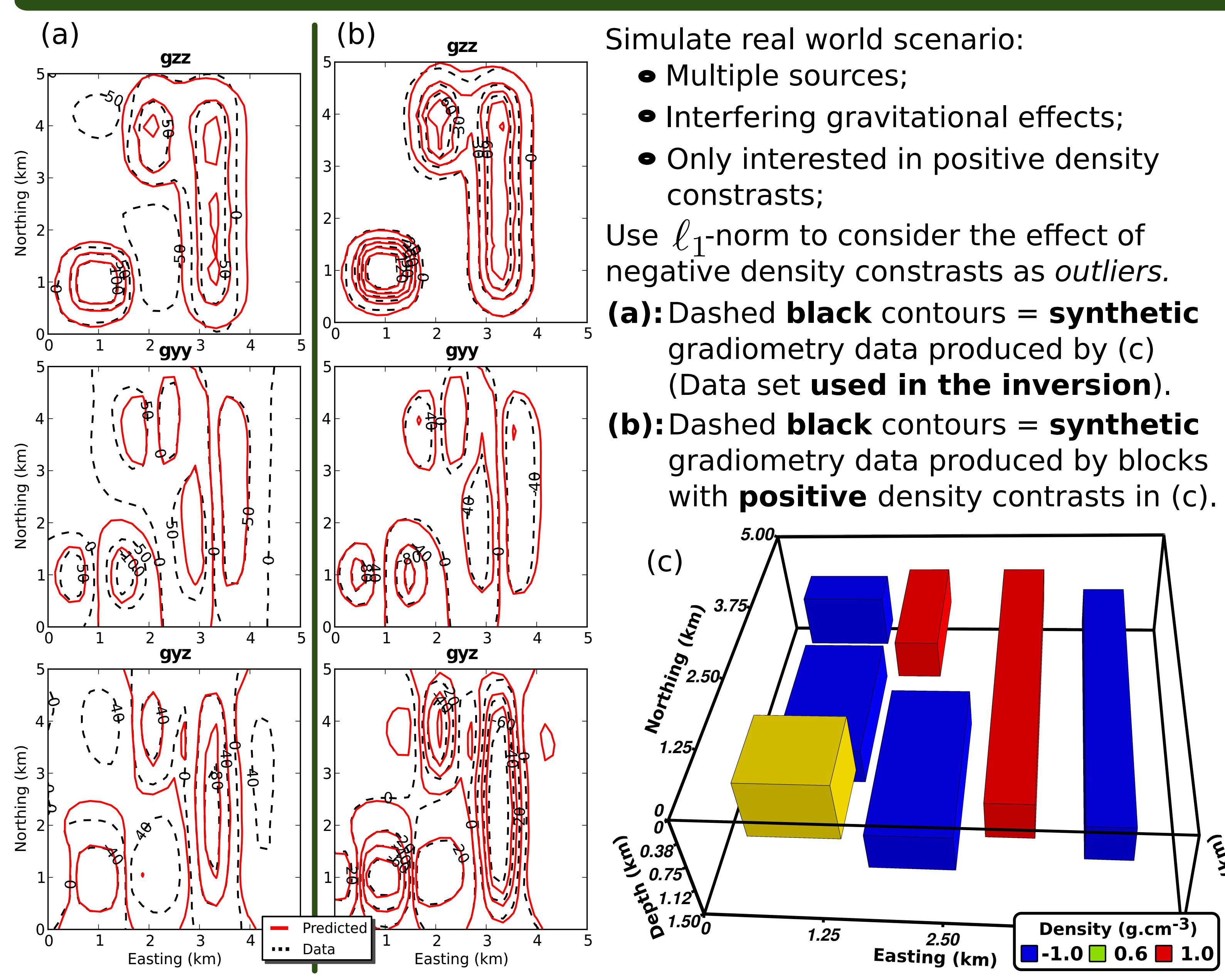
Red contours = data predicted by (c).

Very close fit to synthetic data with noise.

Inversion time ~ 7 min. on Intel Core 2 Duo 2 GHz processor.



## SYNTHETIC EXAMPLE USING THE L1-NORM



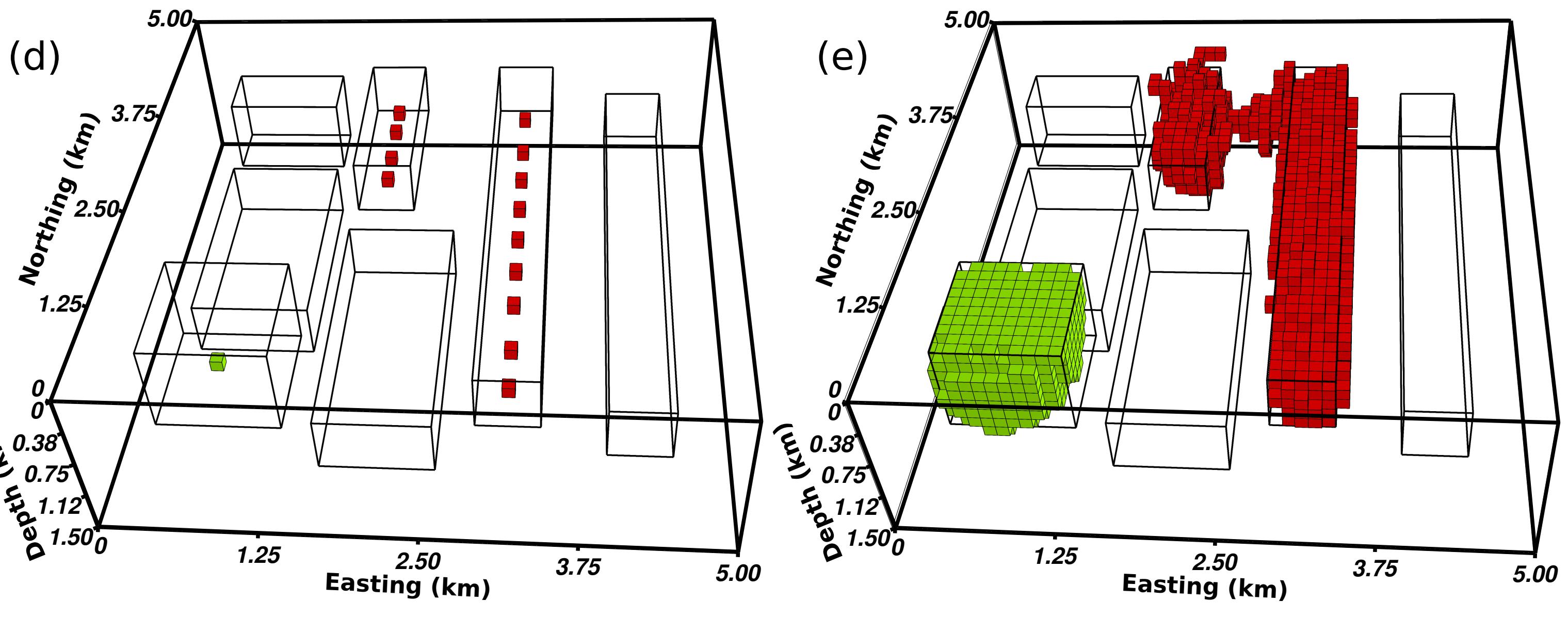
Calculated at 150 m height.  
Added 0.5 Eötvös pseudorandom Gaussian noise.  
Used set of 14 seeds shown in (d).  
Data set = 1,875 measurements.  
Interpretative model = 37,500 prisms.

Estimate produced by the inversion shown in (e).  
Able to recover blocks with positive density contrasts despite interfering gravitational effects.

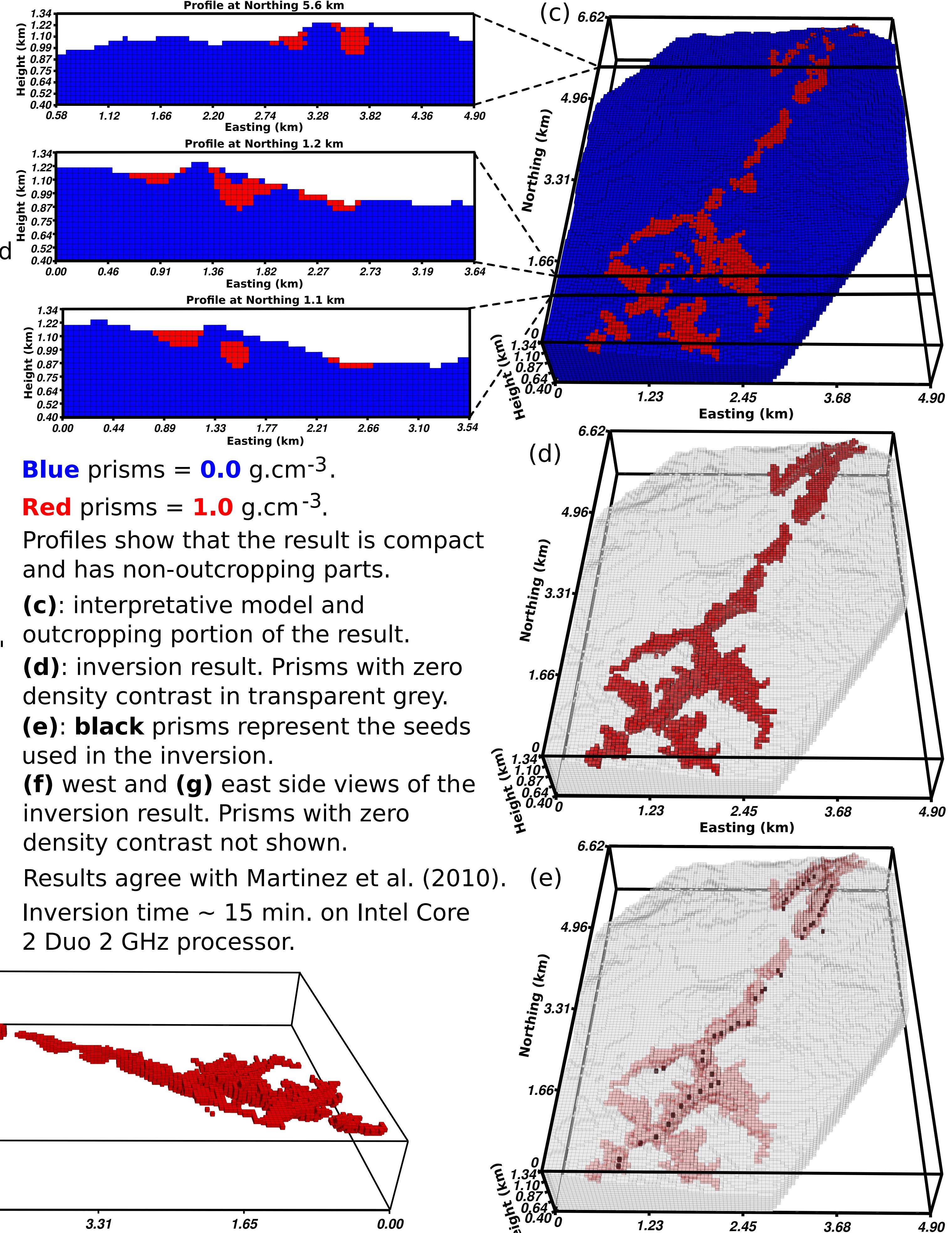
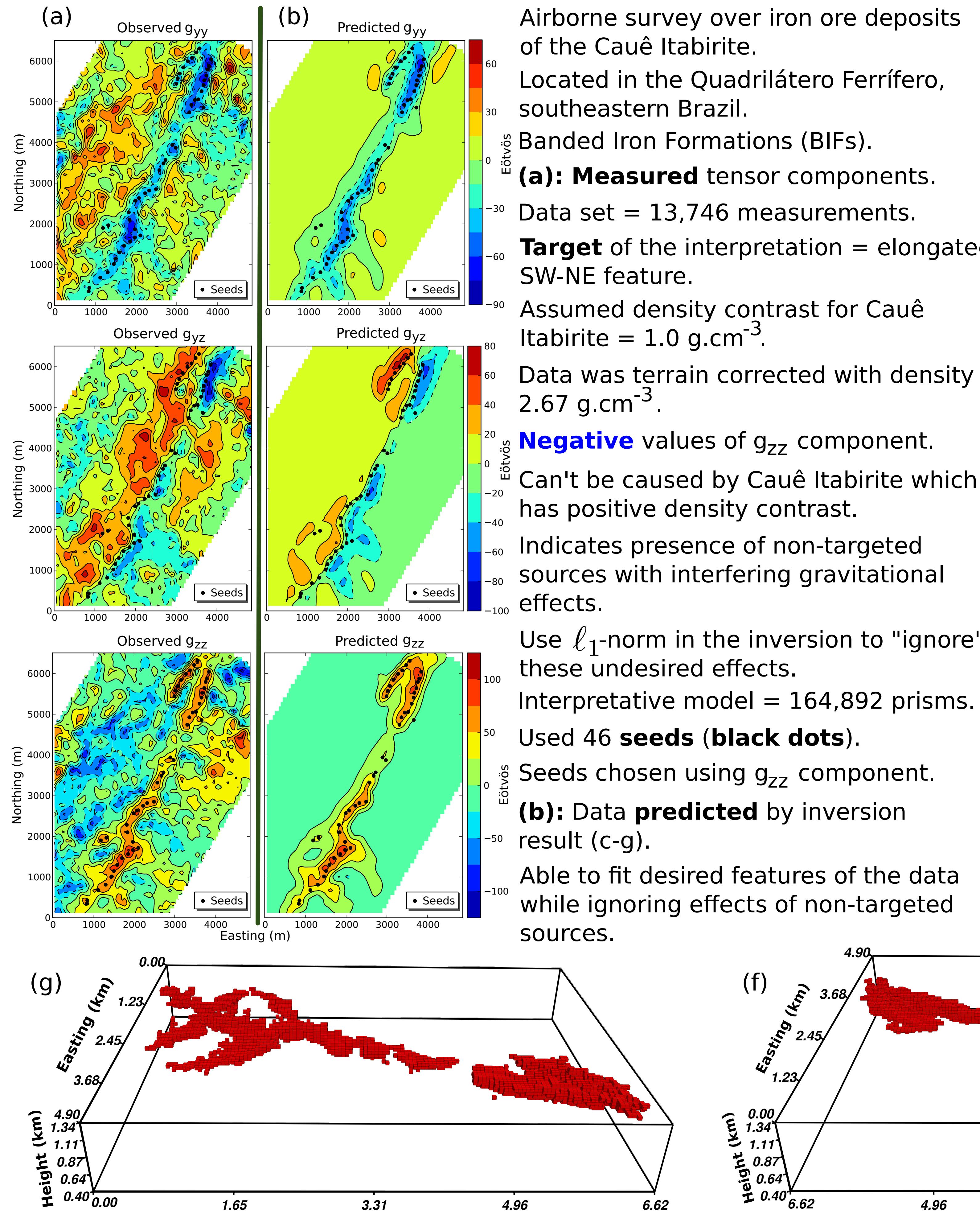
**Red** contours = data **predicted** by (e).  
Inversion performed on **FULL DATA SET** (a) is able to fit the effect of only positive density constraints (b) despite the presence of interfering gravitational effects.

$\ell_1$ -norm allows large residuals where effect of negative density contrasts is dominant.

Inversion time  $\sim$  1.7 min. on Intel Core 2 Duo 2 GHz processor.



## APPLICATION TO REAL DATA



## CONCLUSIONS

Presented a new method for 3D inversion of gravity gradient data based on a systematic search algorithm.

Prior information is incorporated by means of user-specified "seeds".

Enforces compactness of the solution using a regularizing function.

Can perform a robust inversion through  $\ell_1$ -norm of the residuals.

Fast inversion and able to handle large data sets and fine interpretative models:

- Search is limited to neighboring prisms of the current solution;
- Implementation of *lazy evalutation* of the Jacobian matrix  $\mathbf{G}$ ;

Synthetic tests show that:

- Can recover multiple sources with different density contrasts;
- Is able to handle the presence of non-targeted sources with interfering gravitational effects, a common case in real world scenarios;

Results of application to real data from the Quadrilátero Ferrífero agree with previous interpretations.

## ACKNOWLEDGEMENTS

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