Optimal forward calculation of the Marussi tensor due to a geologic structure at GOCE height

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1 - Overview

New GOCE data requires modeling to take into account the sphericity of the Earth;

Solution: use tesseroids (Figure 1) in the discretization (Asgharzadeh *et al.*, 2007; Heck & Seitz, 2007);

No closed formulas for tesseroids so solve with numerical integration: Gauss-Legendre Cubature (GLC);

GLC is slow so there is need to optimize the computations;

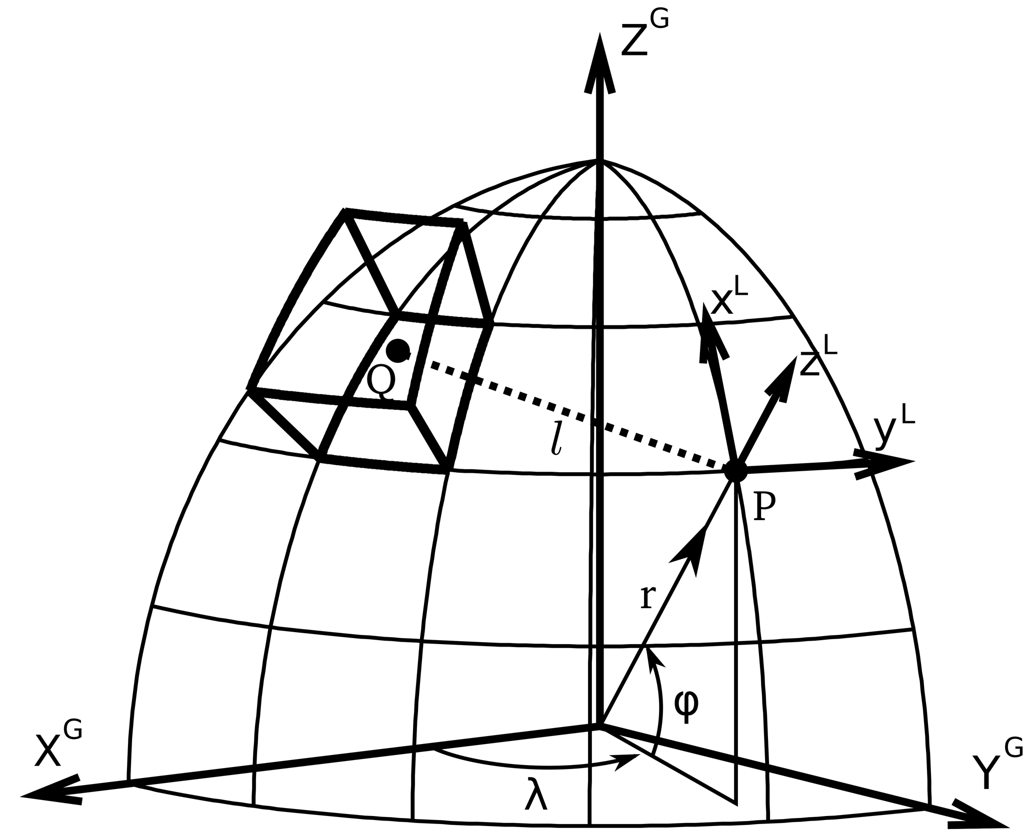

Figure 1: View of a tesseroid in the geocentric coordinate system. Point Q is an integration point. The observation point P is shown with its local coordinate system.

2 - Numerical Integration

GLC approximates the integration by a weighted sum over integration limits:

$$\int_{x_1}^{x_2} f(x)dx \approx \frac{x_2 - x_1}{2} \sum_{i=1}^{N} w_i f(x_i)$$

Points x_i are called GLC nodes;

Accuracy of depends on:

- Number of nodes;
- Distance from tesseroid to observation point;

Ku (1977) showed that for g_z the distance should be larger than the size of the geometric element;

Need to find the smallest distance acceptable for tesseroid and tensor components

3 - Optimization Scheme

Instead of using high order for all tesseroids:

- Keep number of GLC nodes fixed;
- Divide the tesseroid into smaller ones ONLY when computation point is too close;

Need to know how much is "too close";

Compare effect of tesseroid with prism of same mass;

Use small tesseroid (< 100m) to ignore sphericity of the Earth;

Find the size ratio (d/L) that gives difference bellow a given limit;

Use size ratio to automatically divide tesseroids when needed (Figure 2);

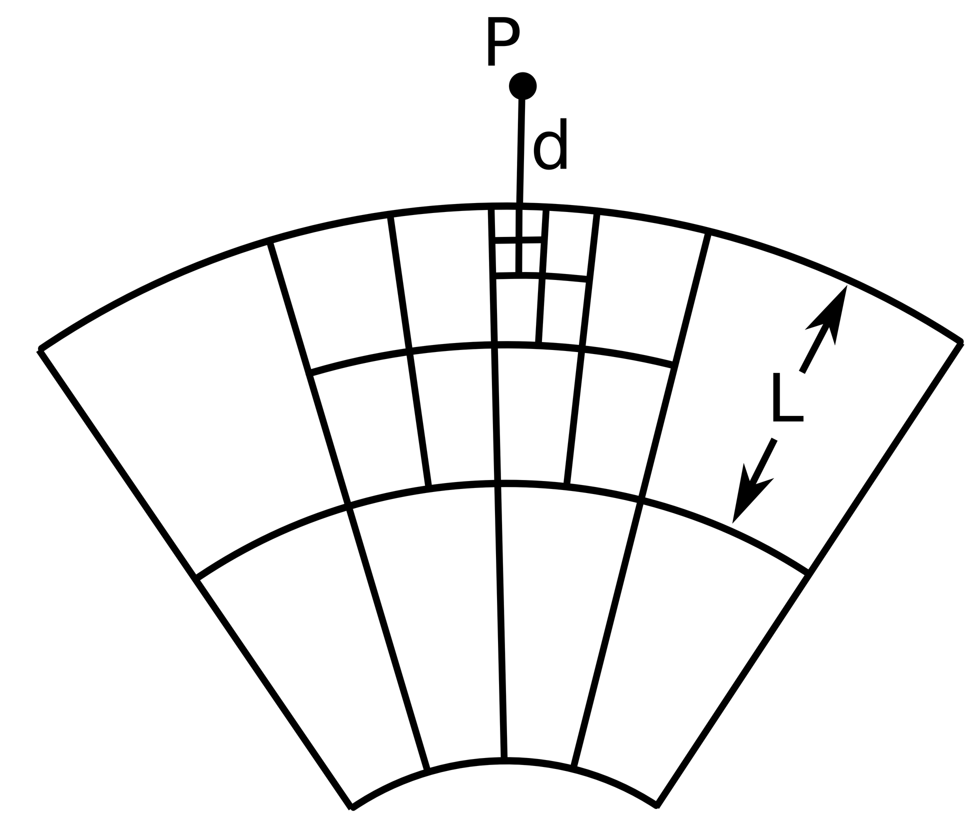

Figure 2: Sketch of the optimization algorithm. Adaptative resizing of tesseroids when the distance-to-size ratio is bellow a certain threshold.

4 - Results

Used 2 GLC nodes in each direction;

Figures 3 and 4 show difference for a tesseroid of size 0.001° X 0.001° X 100m;

Set maximum approximation error to 0.01 mGal for g_z and 0.001 Eötvös for tensor components;

Found a size ratio of 1 for g_z and 4-6 for tensor components;

For tesseroids of 10m and 50m (not shown) the ratios where the same;

Implemented ratio and optimization scheme in *Tesseroids* software; (http://code.google.com/p/tesseroids)

Speed increase of 3-5 times over using fixed size and 6 GLC nodes;

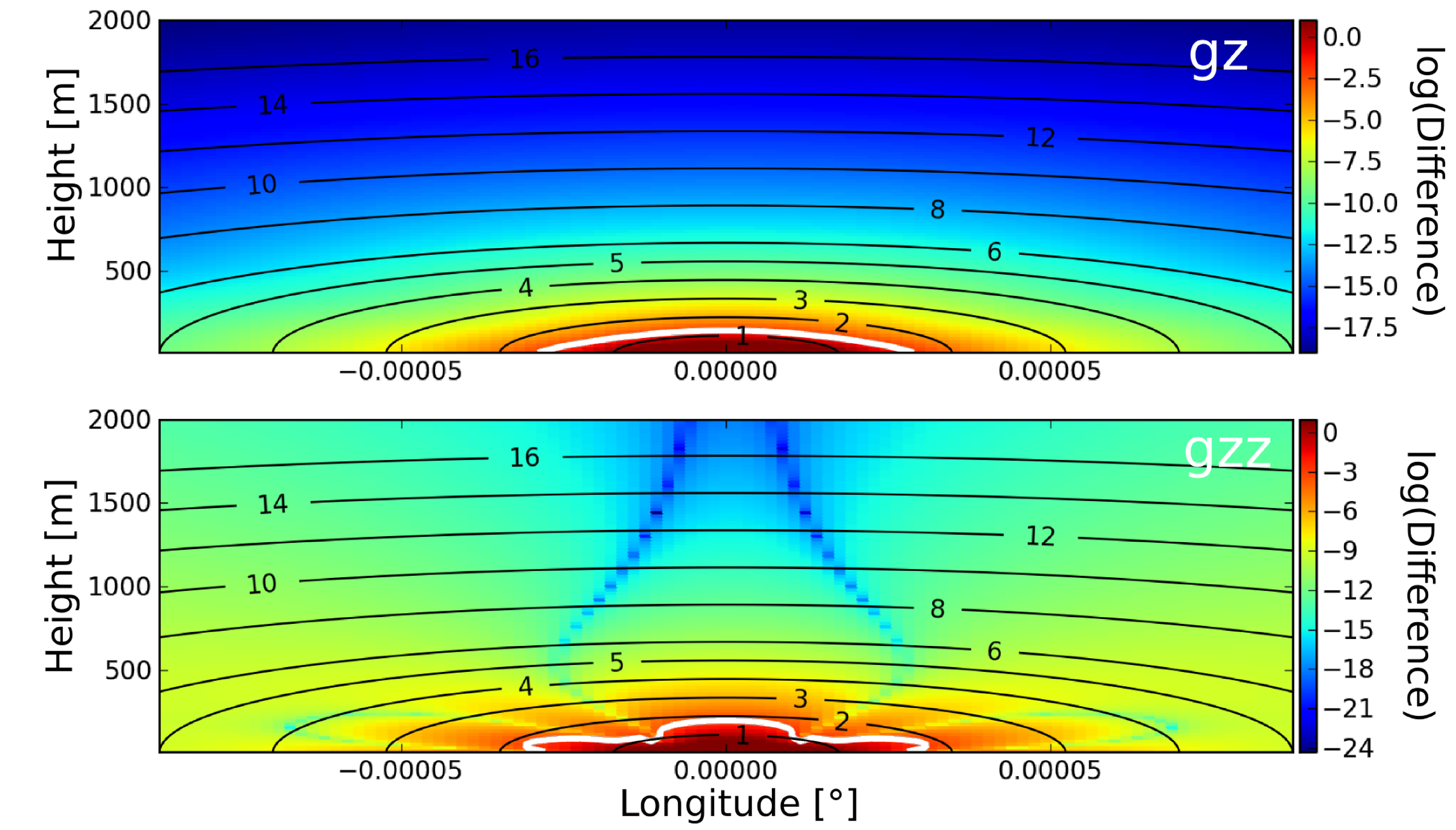

Figure 3: Difference between prism and tesseroid along a profile. Black contours are the relative distance (d/L) to the top of the tesseroid. White contours are differences of 0.01 mGal and 0.001 Eötvös.

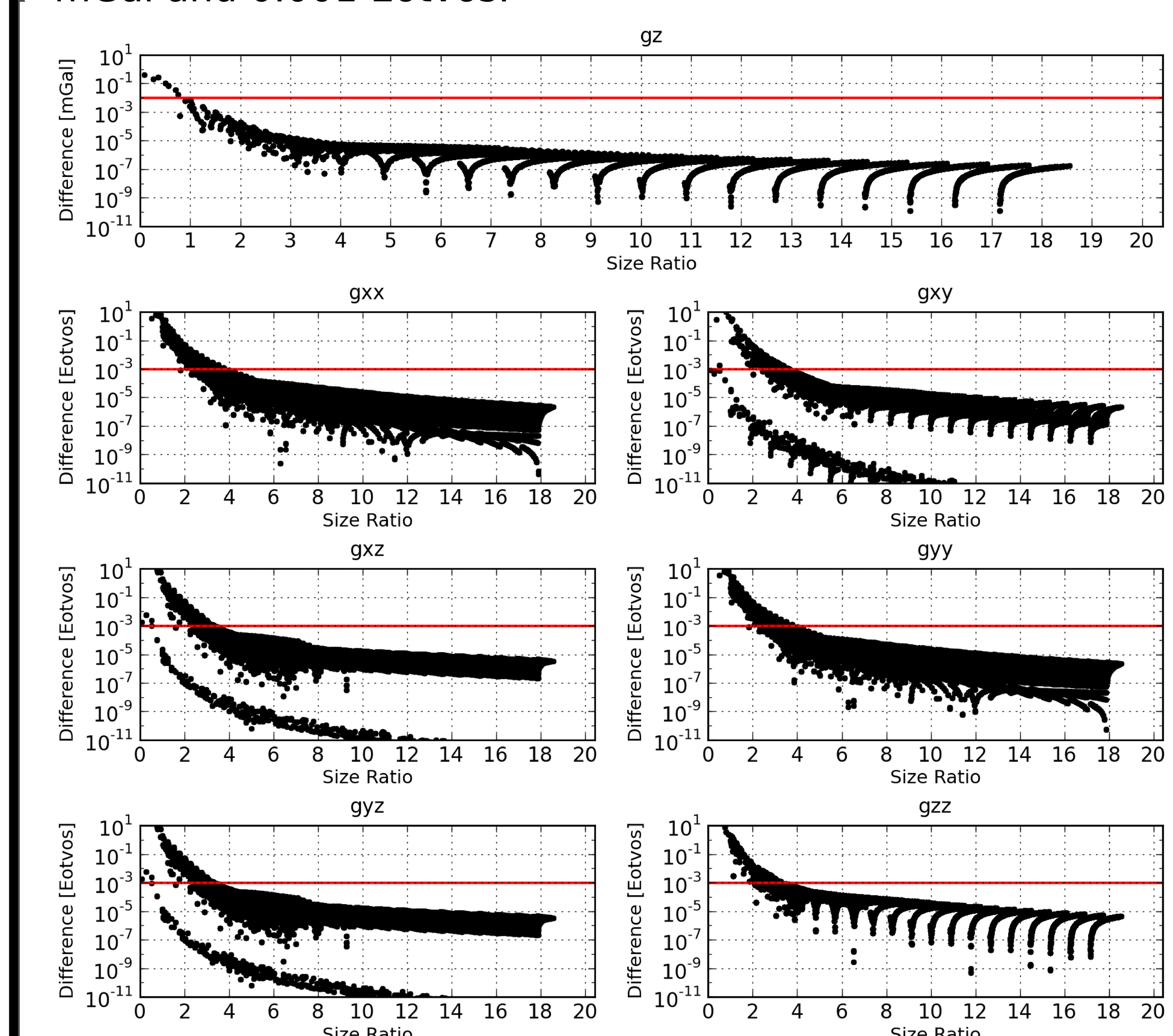

Figure 4: Difference between prism and tesseroid per relative distance (d/L) to the top of the tesseroid. Red lines represent the minimum desired error. For tensor components the distance should be 4-6 times the size of the tesseroid.

5 - Conclusions

Automatic resizing of tesseroids provides lower computation times;

Size ratio is different for g_z and tensor;

Need to assess other sources of error:

- Size limit for sphericity to affect;
- Distance limit for sphericity to affect;

6 - References

Asgharzadeh, M.F., Von Frese, R.R.B., Kim, H.R.; Leftwich, T.E., Kim, J.W., 2007. Spherical prism gravity effects by Gauss-Legendre quadrature integration. Geoph. J. Int., v. 169, p. 1-11.

Heck, B., Seitz, K., 2007. A comparison of the tesseroid, prism and point-mass approaches for mass reductions in gravity field modelling. Journal of Geodesy, v. 81, p. 121 - 136.

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