

# modelagem direta e inversão

de campos gravitacionais em  
coordenadas esféricas



# Leonardo Uieda

Valéria C F Barbosa (orientadora)

# A COORDENADORA DE PÓS-GRADUAÇÃO ADVERTE:

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Efeitos colaterais dessa defesa incluem:

discussão

choro

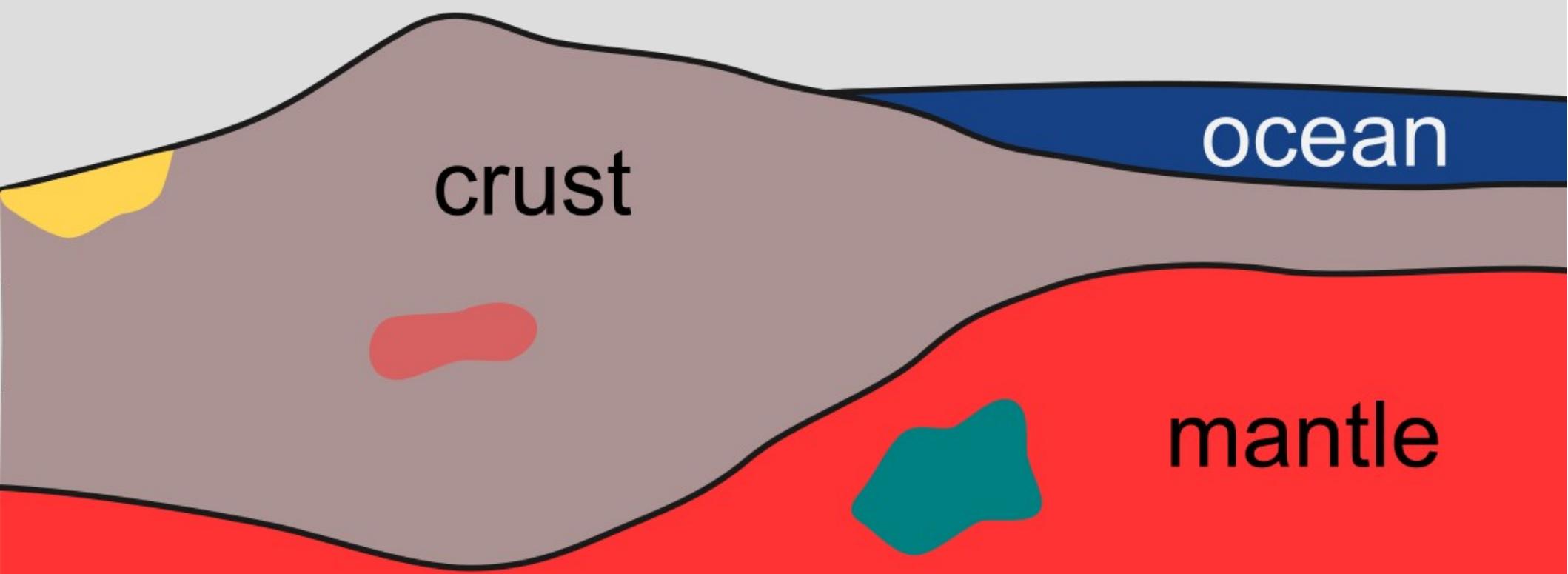
discórdia

aprovação

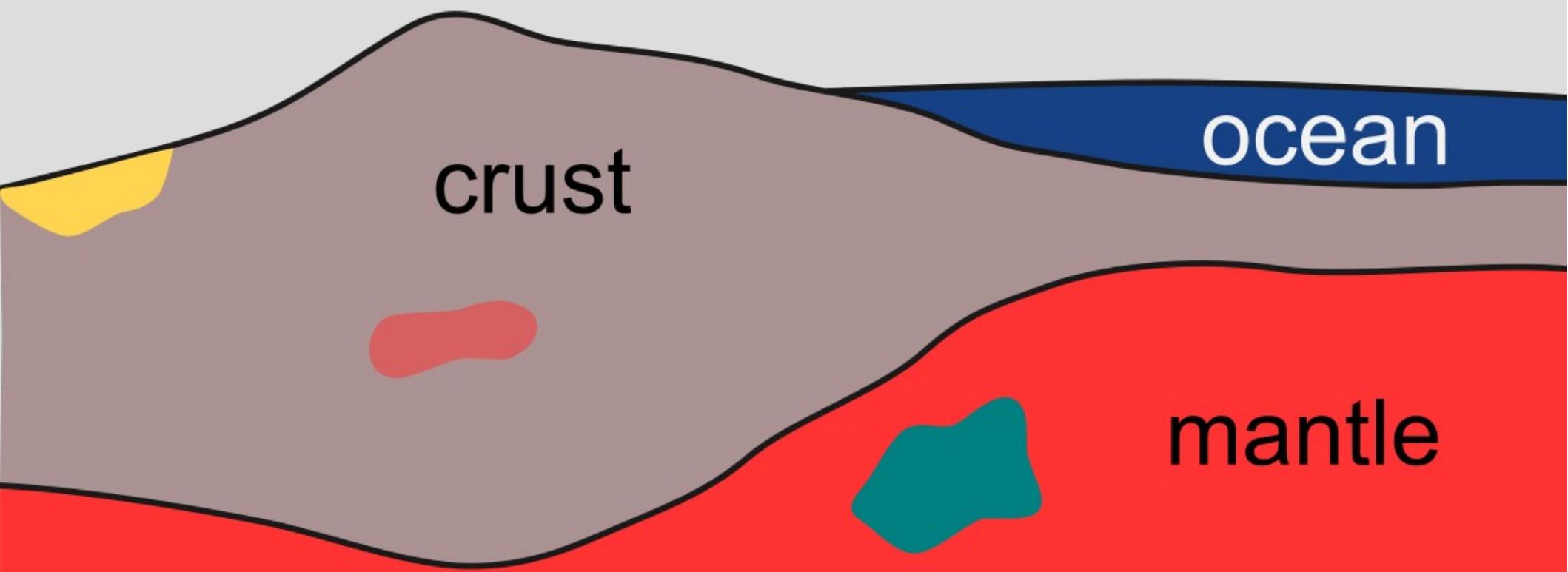
crítica

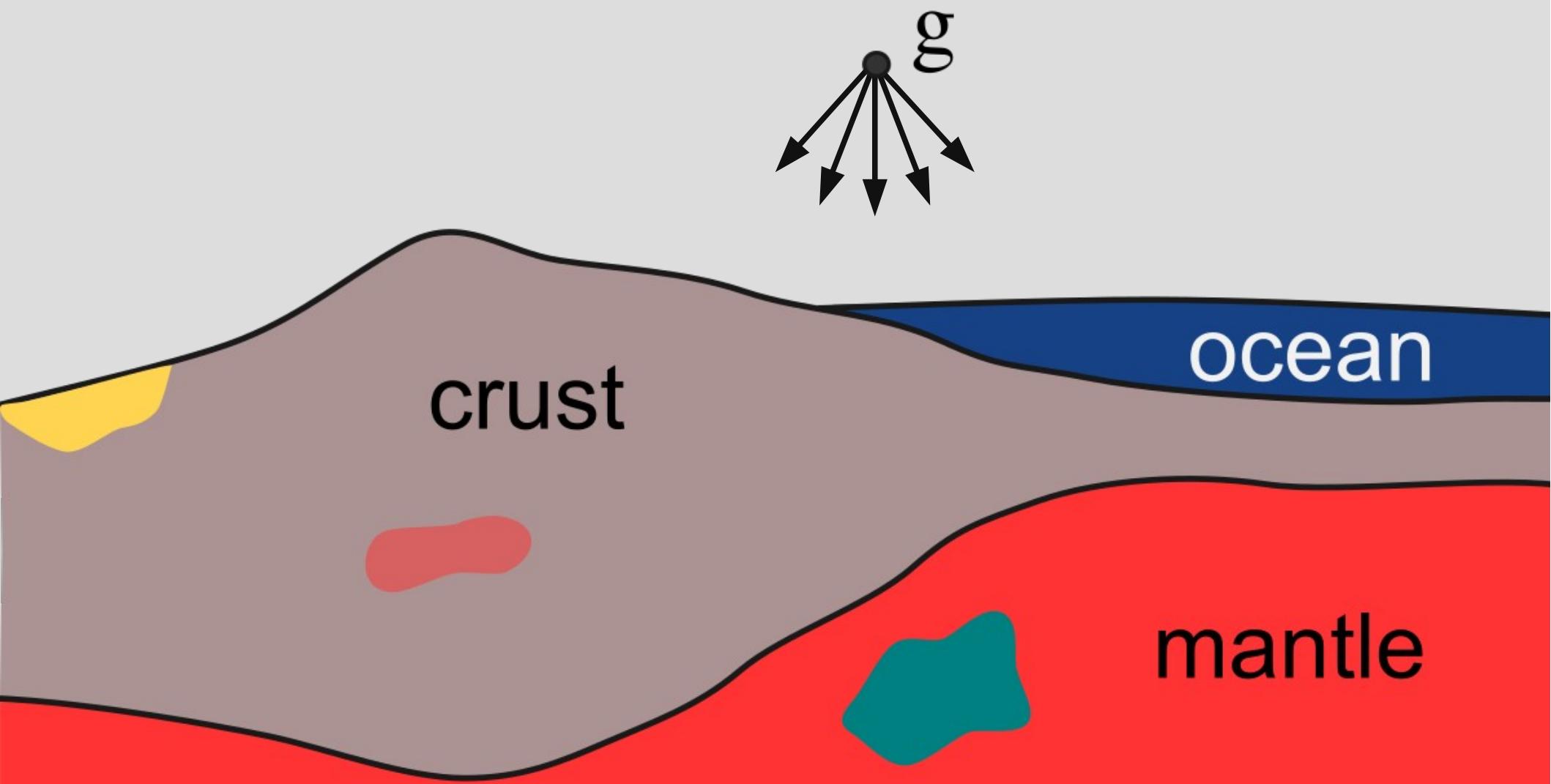
reprovação

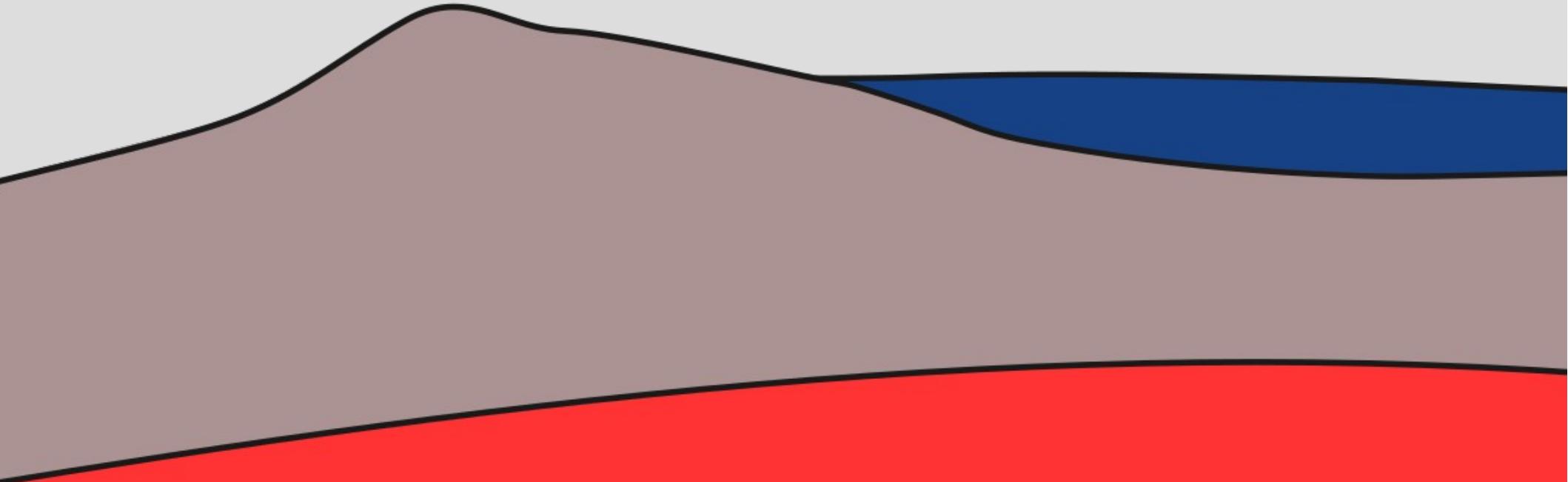
# introdução



.g





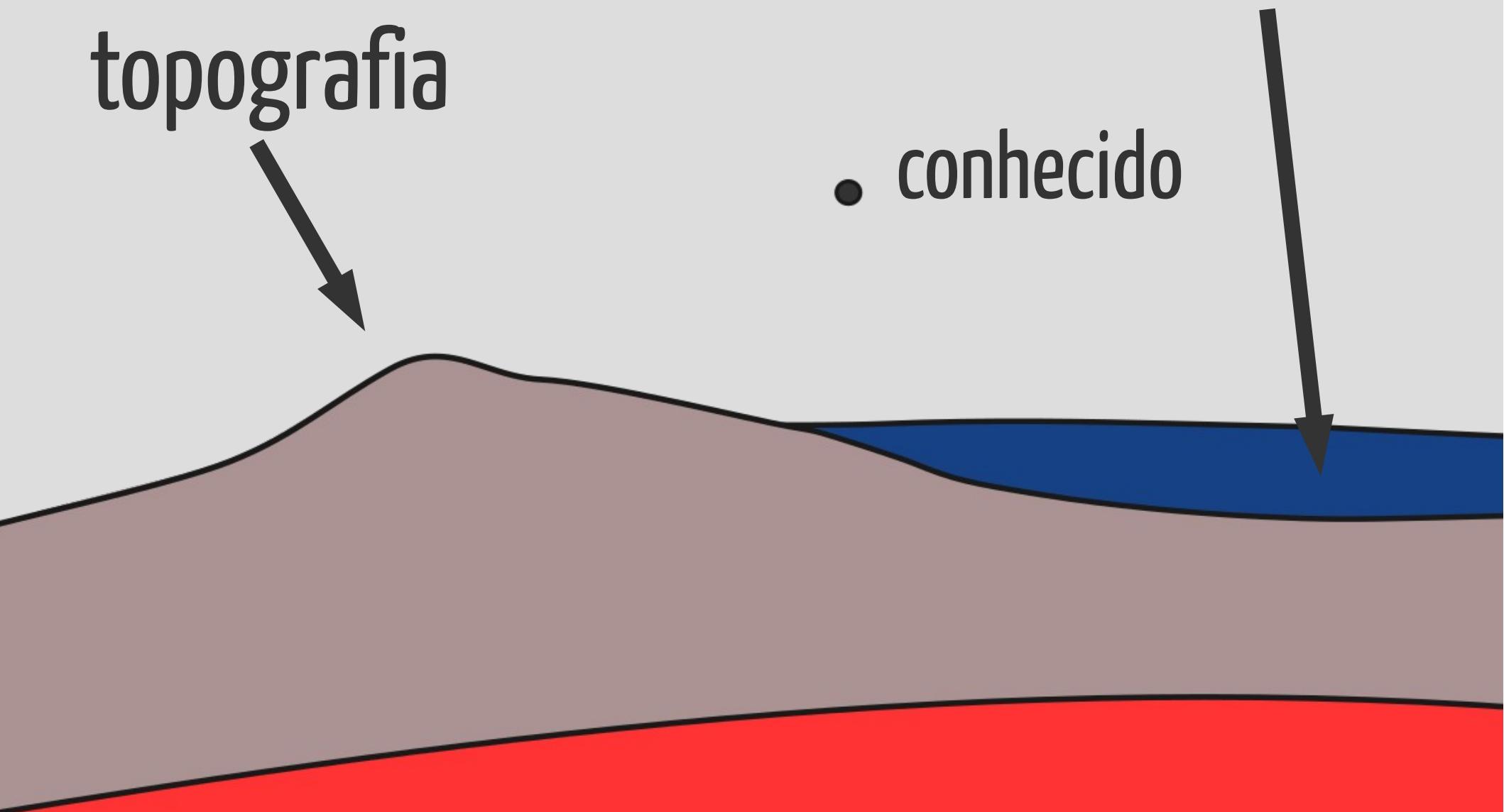


• conhecido

batimetria

topografia

• conhecido



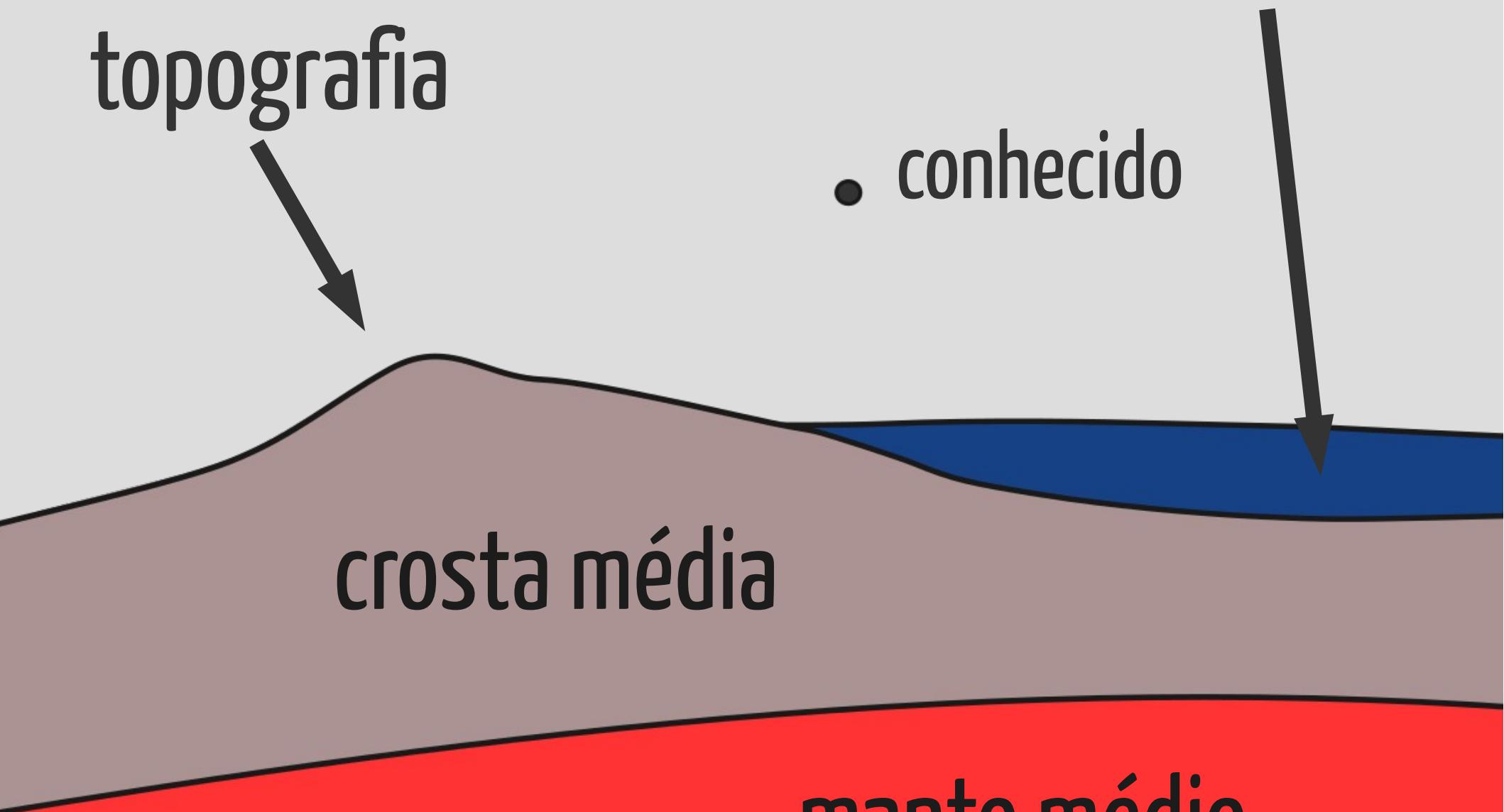
batimetria

topografia

• conhecido

crosta média

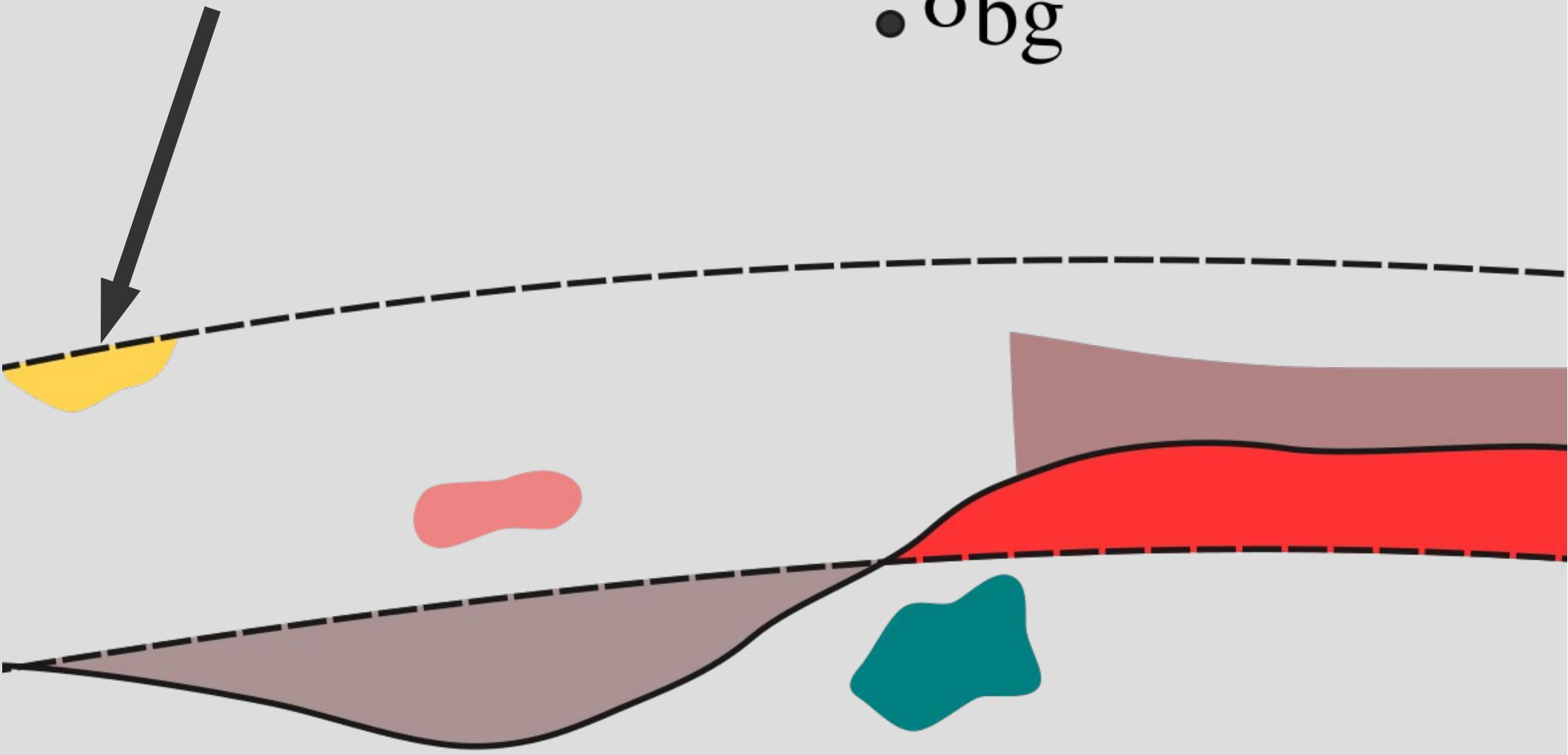
manto médio



•  $\delta_{\text{bg}}$

bacias

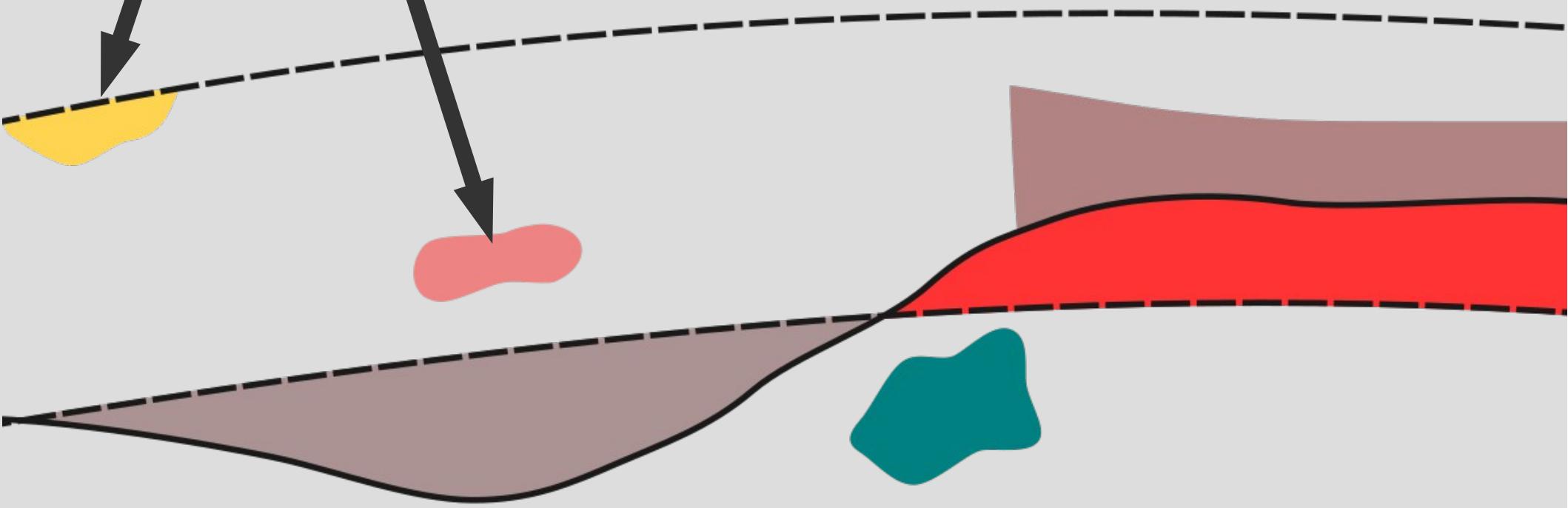
•  $\delta_{bg}$



bacias

intrusões

•  $\delta_{bg}$



bacias

intrusões

•  $\delta_{bg}$

crosta  
oceânica

bacias

intrusões

•  $\delta_{bg}$

crosta  
oceânica

Moho

bacias

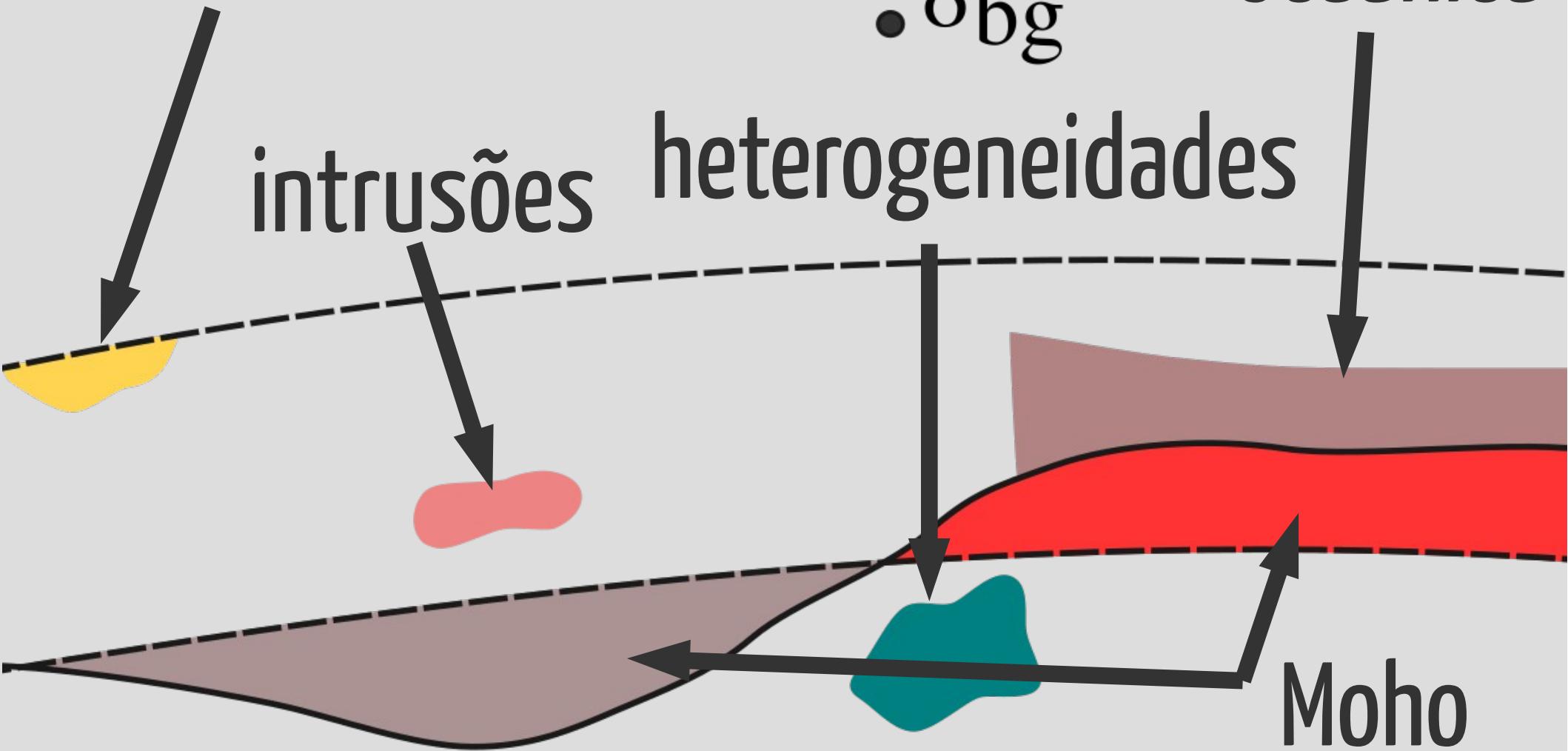
intrusões

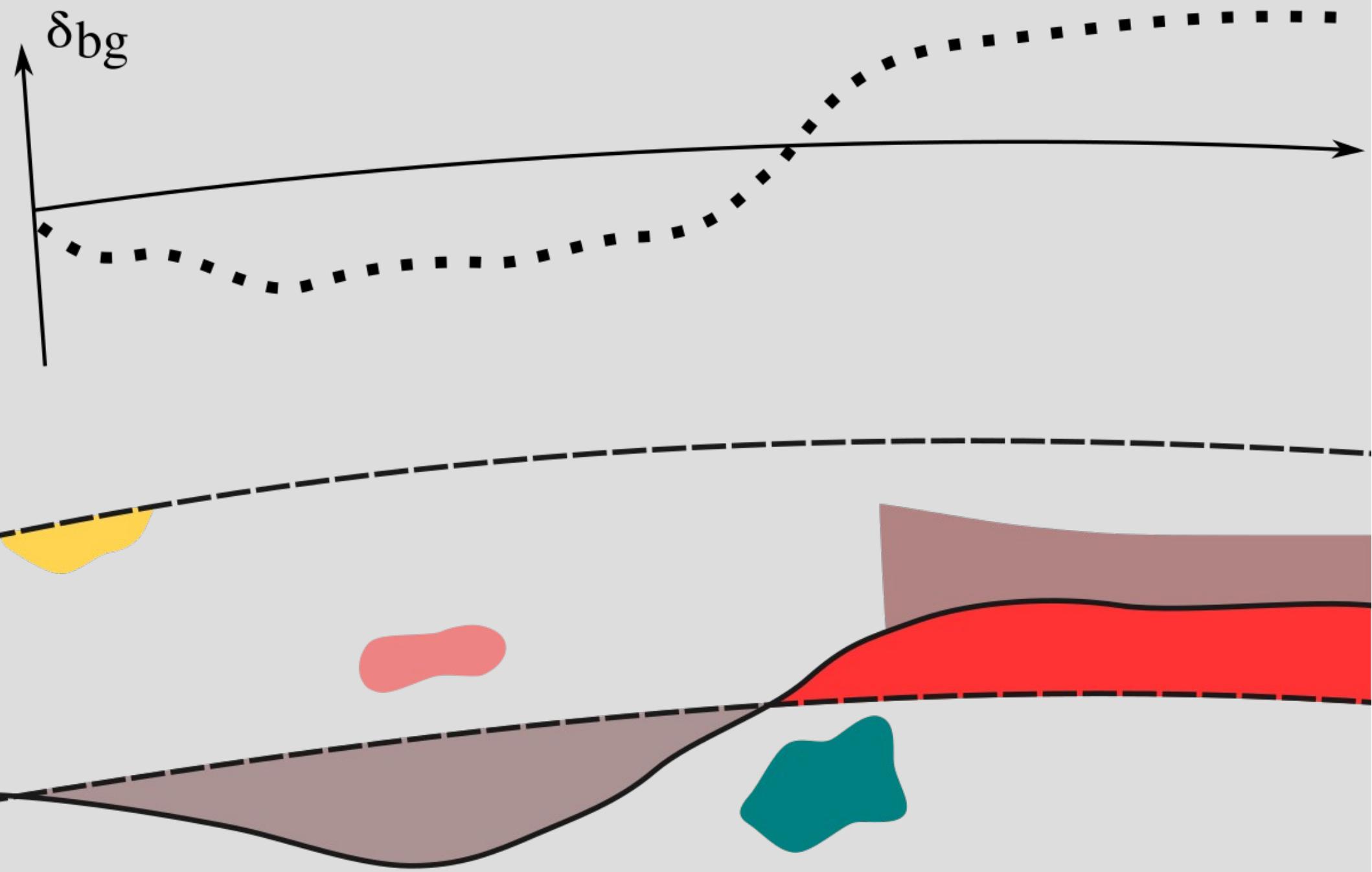
•  $\delta_{bg}$

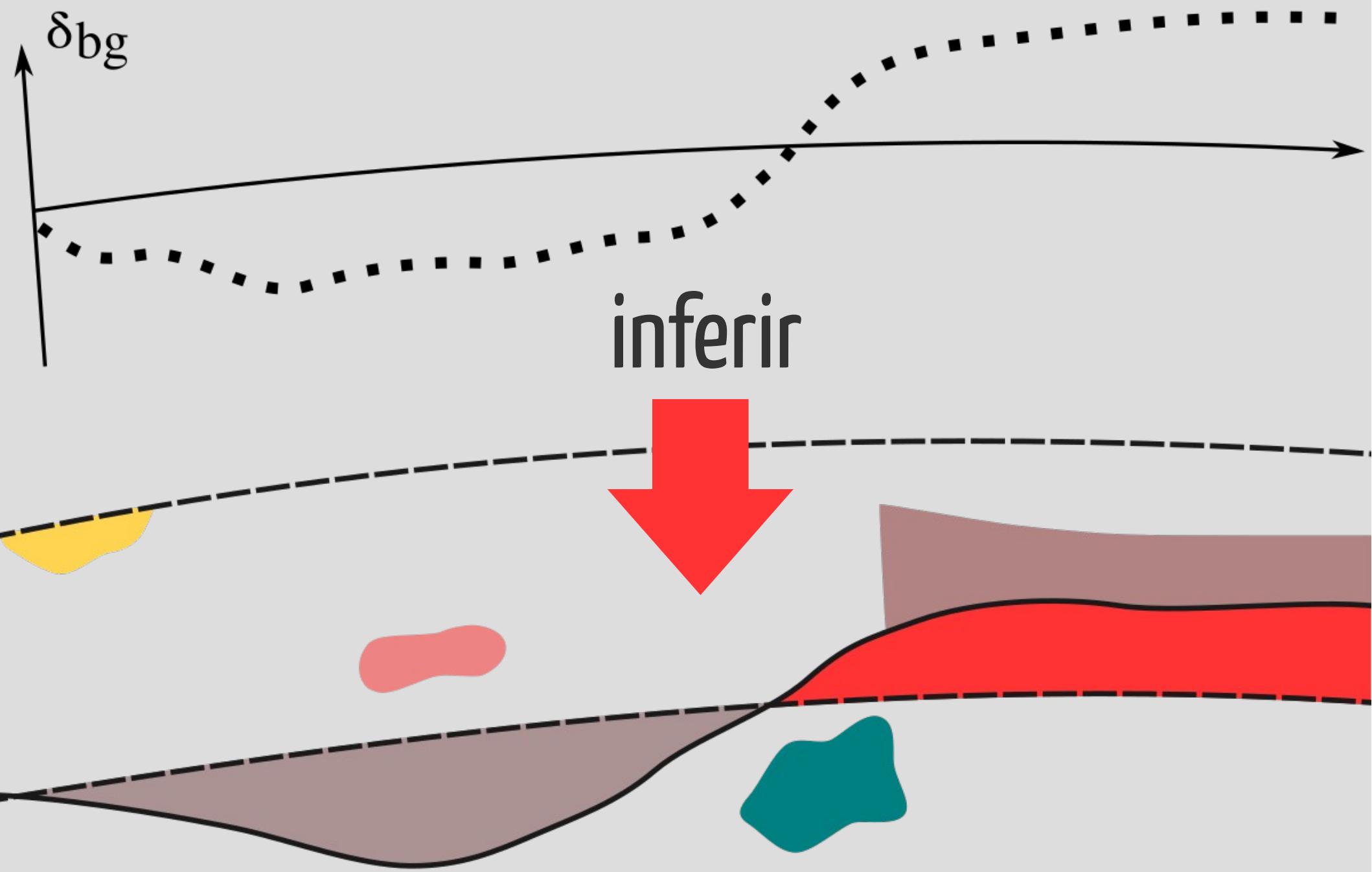
crosta  
oceânica

heterogeneidades

Moho

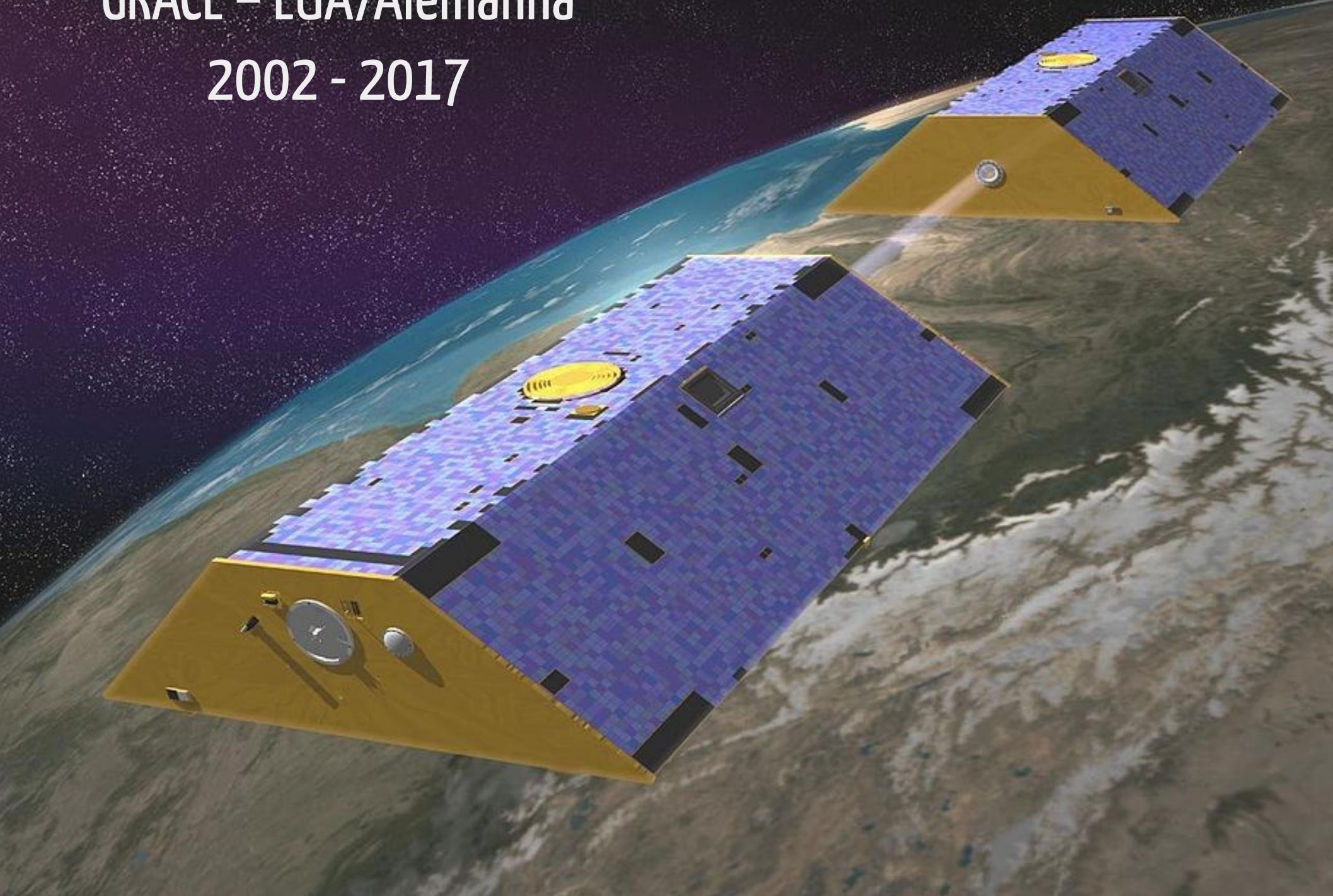


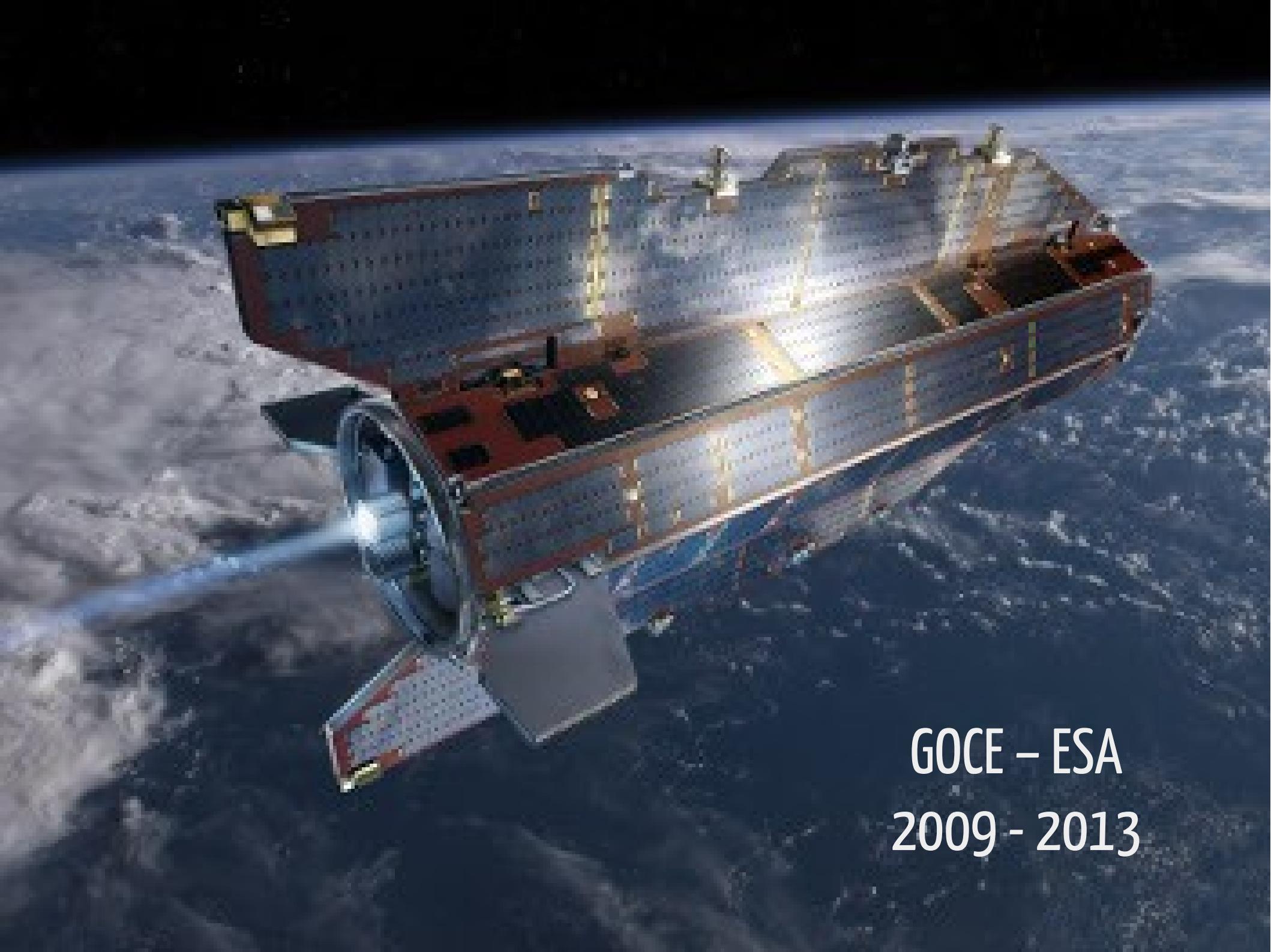




GRACE - EUA/Alemanha

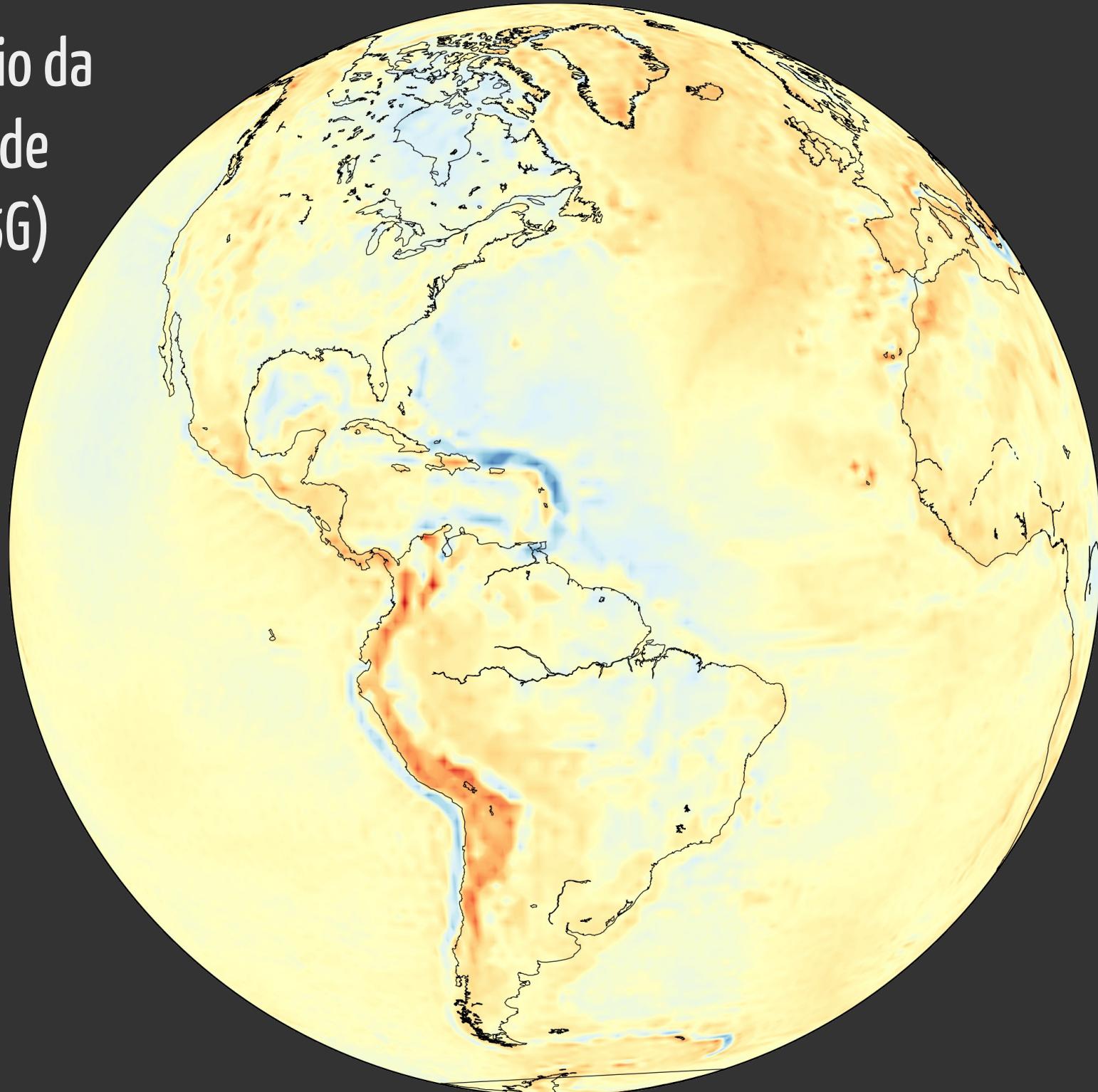
2002 - 2017





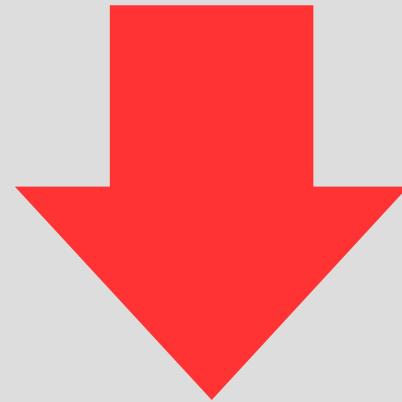
GOCE - ESA  
2009- 2013

# distúrbio da gravidade (GGM05G)



**dados** ✓

**dados** ✓

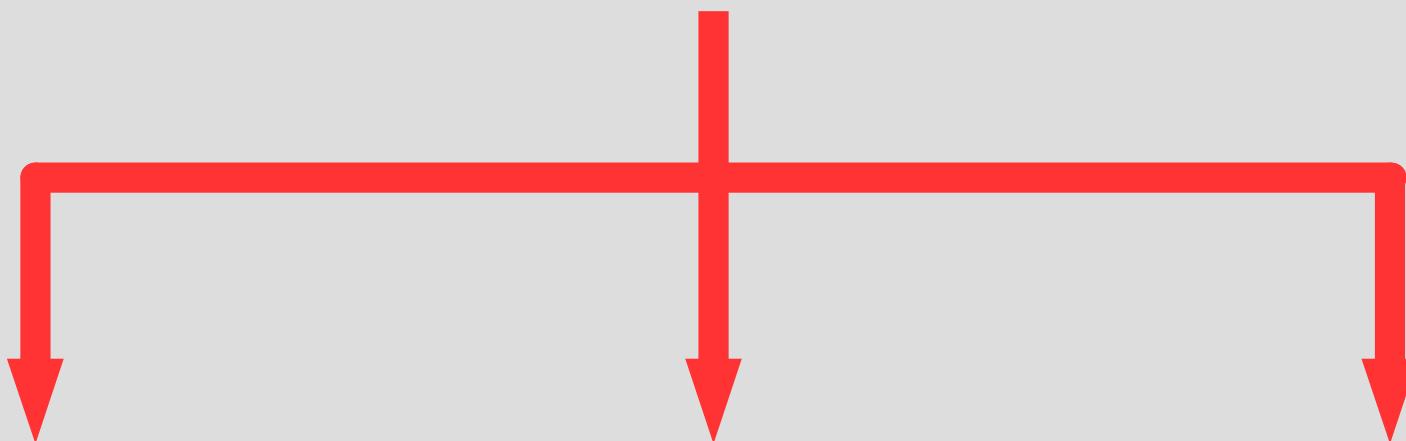


**subsuperfície**

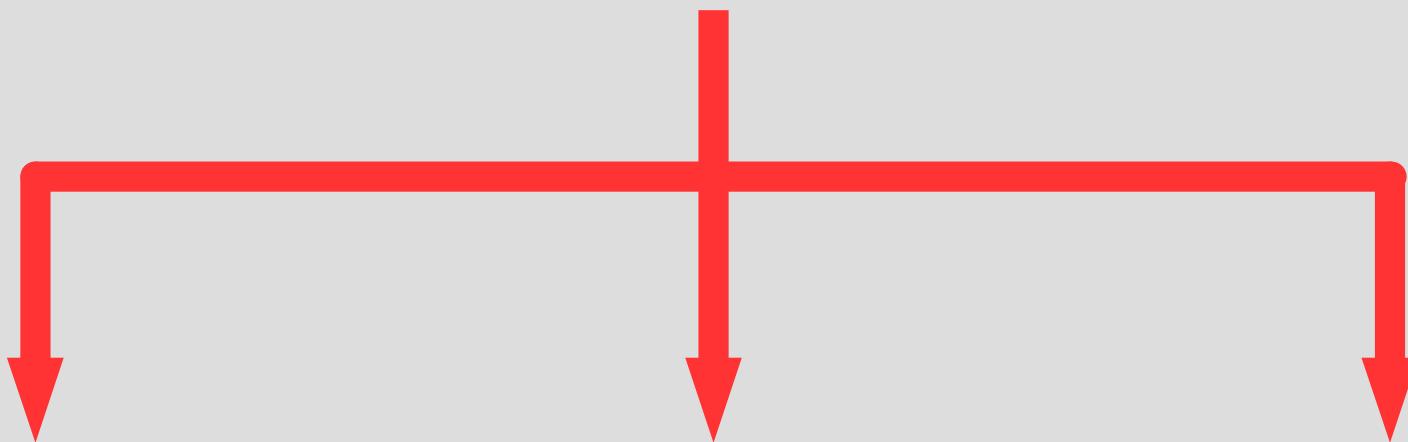
**aproximação esférica**

# problema inverso

# problema inverso

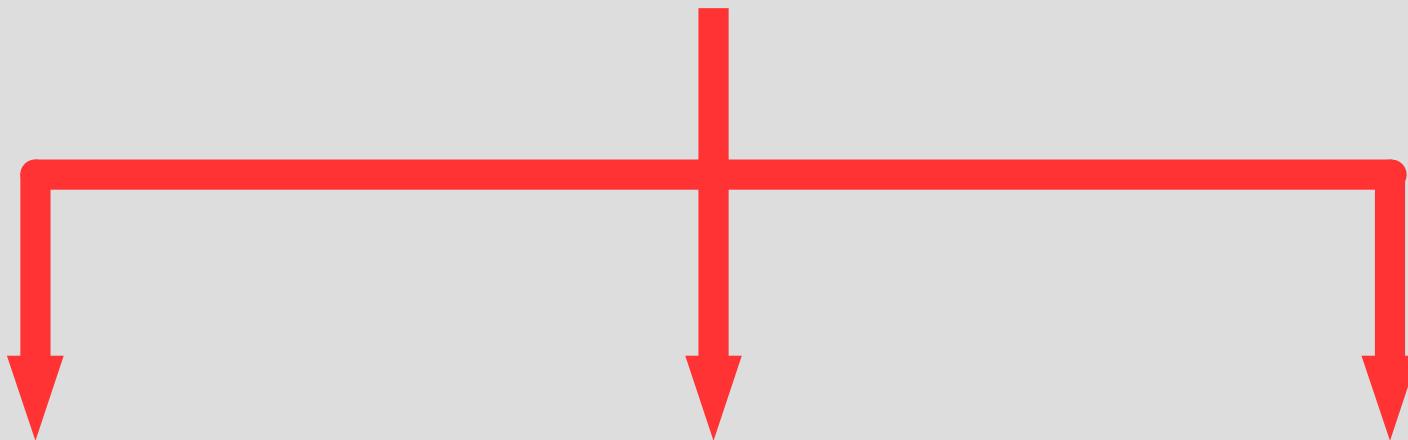


# problema inverso



modelagem  
direta

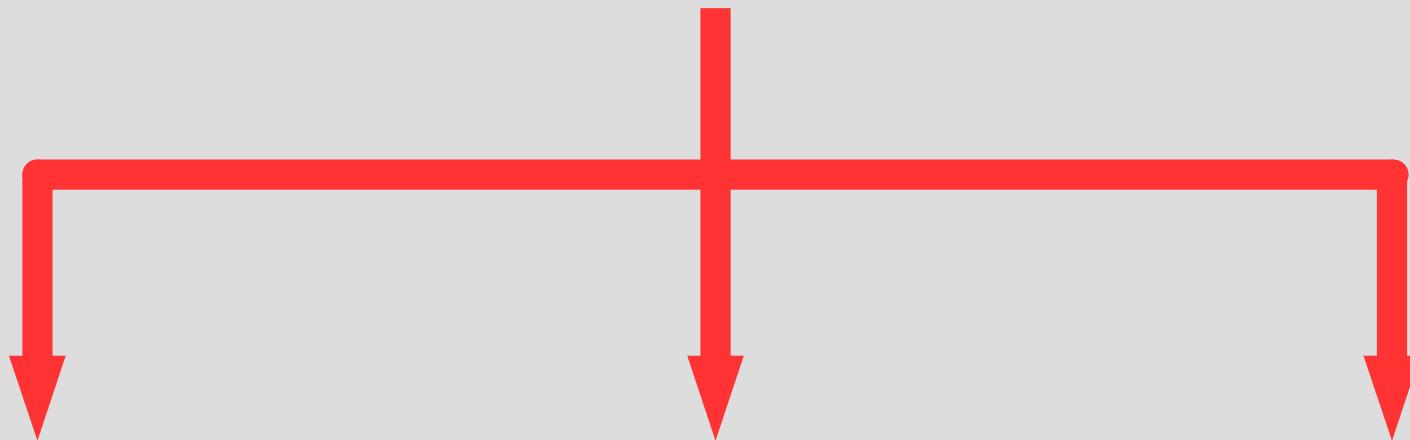
# problema inverso



modelagem  
direta

otimização

# problema inverso



modelagem  
direta

otimização

regularização

# programação

```
cdef inline double kernelxz(double x, double y, double z, double r) nogil:
    return safe_log(y + r)

cdef inline double kernelyy(double x, double y, double z, double r) nogil:
    return -safe_atan2(z*x, y*r)

cdef inline double kernelyz(double x, double y, double z, double r) nogil:
    return safe_log(x + r)

cdef inline double kernelzz(double x, double y, double z, double r) nogil:
    return -safe_atan2(x*y, z*r)

@cython.wraparound(False)
@cython.boundscheck(False)
def tf(numpy.ndarray[DTYPE_T, ndim=1] xp not None,
       numpy.ndarray[DTYPE_T, ndim=1] yp not None,
       numpy.ndarray[DTYPE_T, ndim=1] zp not None,
       double x1, double x2, double y1, double y2, double z1, double z2,
       double mx, double my, double mz, double fx, double fy, double fz,
       numpy.ndarray[DTYPE_T, ndim=1] v1 not None,
       cdef unsigned ti l, iz, i, j, k,
       cdef numpy.ndarray[DTYPE_T, ndim=1] x,
       cdef DTTYPE_T kernelx, v1, v2, v3, v4, v5, v6, bx, vy, vz, vx, fz,
       size = len(xp))
    x = numpy.array([x2, x1], dtype=DTYPE_T)
    y = numpy.array([y2, y1], dtype=DTYPE_T)
    z = numpy.array([z2, z1], dtype=DTYPE_T)
    for l in range(size):
        # Evaluate the integration limits
        for k in range(2):
            dz = z[k] - zp[l]
            for j in range(2):
                dy = y[j] - yp[l]
                for i in range(2):
                    dx = x[i] - xp[l]
                    r = sqrt(dx**2 + dy**2 + dz**2)
                    v1 = kernelxz(dx, dy, dz, r)
                    v2 = kernelxy(dx, dy, dz, r)
                    v3 = kernelxz(dx, dy, dz, r)
                    v4 = kernelyy(dx, dy, dz, r)
                    v5 = kernelyz(dx, dy, dz, r)
                    v6 = kernelzz(dx, dy, dz, r)
                    bx = (v1*mx + v2*my + v3*mz)
```

# program

```
9
10 * :class:`~fatiando.inversion.misfit.Mi
11
12 See the documentation for :mod:`fatiand
13 ``Misfit``.
14
15 ----
16
17 """
18 from __future__ import division
19 import copy
20 from abc import abstractmethod
21 import numpy as np
22 import scipy.sparse
23
24 from ..utils import safe_dot
25 from .base import (OptimizerMixin, Oper
26                      CachedMethodPermanen
27
28
29 class Misfit(OptimizerMixin, OperatorMi
30     r"""
31     A l2-norm data-misfit function.
32
33     This is a kind of objective function
34     that compares observed data :math:`\bar{d}^o` and
35     parameters :math:`\bar{d} = \bar{f}(p)` via
36
37     The l2-norm data-misfit is defined
38
39     .. math::
40
41         \phi(\bar{p}) = \bar{r}^T \bar{r}
42
43     where :math:`\bar{r} = \bar{d}^o - \bar{f}` and
44     :math:`N` is the number of data.
45
46     When subclassing this class, you must
47
48     * ``predicted(self, p)``: calculate
49       :math:`\bar{d}` for a given param
50
51     If you want to use any gradient-bas
```

1

2

3

programa A

programa B

Novo método

 Tesseroids

modelagem direta  
aprox. esférica

programa B

Novo método



modelagem direta  
aprox. esférica



fatiando a terra  
modelagem direta  
otimização  
regularização, etc

# Novo método

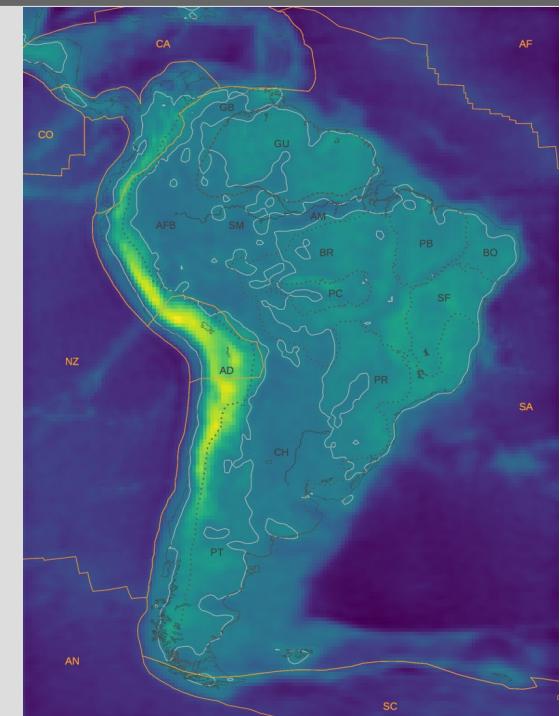
# Tesseroids

modelagem direta  
aprox. esférica



modelagem direta  
otimização  
regularização, etc

método: inversão não-linear rápida  
aplicação: Moho da América do Sul



Introdução

Tesseroids

Fatiando a Terra

Inversão Moho

Conclusão

Introdução

# Tesseroids

Fatiando a Terra

Inversão Moho

Conclusão



modelagem direta em coordenadas esféricas

# GEOPHYSICS

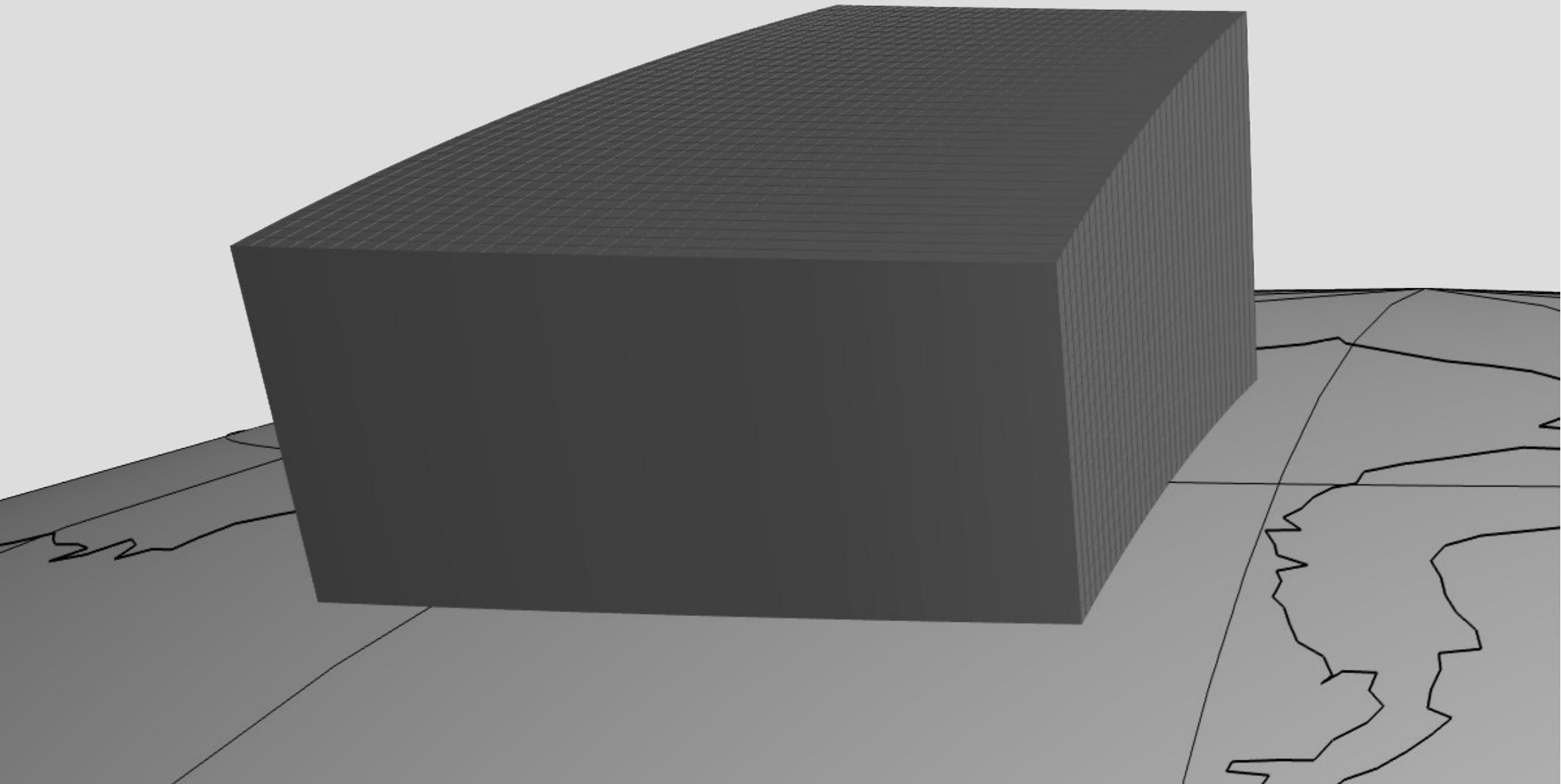
Geophysical Software and Algorithms

Submetido: Mar 2015

R1: Nov 2015

Aceito: Abr 2016

**tesseroid = prisma esférico**



$$g_z = G \rho \iiint\limits_{\lambda_1 \varphi_1 r_1}^{\lambda_2 \varphi_2 r_2} K_z dr d\varphi d\lambda$$

cte.

$$g_z = G \rho \iiint_{\lambda_1 \varphi_1 r_1}^{\lambda_2 \varphi_2 r_2} K_z dr d\varphi d\lambda$$

densidade

cte.

$$g_z = G \rho \iiint_{\lambda_1 \varphi_1 r_1}^{\lambda_2 \varphi_2 r_2} K_z dr d\varphi d\lambda$$

densidade

cte.

$$g_z = G \rho \iiint_{\lambda_1 \varphi_1 r_1}^{\lambda_2 \varphi_2 r_2} K_z dr d\varphi d\lambda$$

kernel

(Grombein et al., 2013)

# Quadratura

## Gauss-Legendre

$$\int\limits_a^bf\left( x\right) dx$$

$$\int\limits_a^bf\left( x\right) dx\approx\frac{b-a}{2}\sum\limits_{i=1}^NW_if\left( x_i\right)$$

ordem

$$\int_a^b f(x) dx \approx \frac{b-a}{2} \sum_{i=1}^N w_i f(x_i)$$

$$\int_a^b f(x) dx \approx \frac{b-a}{2} \sum_{i=1}^N w_i f(x_i)$$

ordem

pesos

$$\int_a^b f(x) dx \approx \frac{b-a}{2} \sum_{i=1}^N w_i f(x_i)$$

Diagram illustrating the components of the quadrature formula:

- ordem** (Order) points to the term  $\frac{b-a}{2}$ .
- pesos** (Weights) points to the term  $w_i$ .
- raízes de  $P_N(x)$**  (Roots of  $P_N(x)$ ) points to the term  $f(x_i)$ .

$$\iiint\limits_{\Omega} K_z\!\left(r\,,\varphi\,,\lambda\right) d\,\Omega$$

$$\iiint_{\Omega} K_z(r,\varphi,\lambda) d\Omega$$



$$A \sum_{i=1}^{N_r} \sum_{j=1}^{N_\varphi} \sum_{k=1}^{N_\lambda} W_i W_j W_k K_z(r_i, \varphi_j, \lambda_k)$$

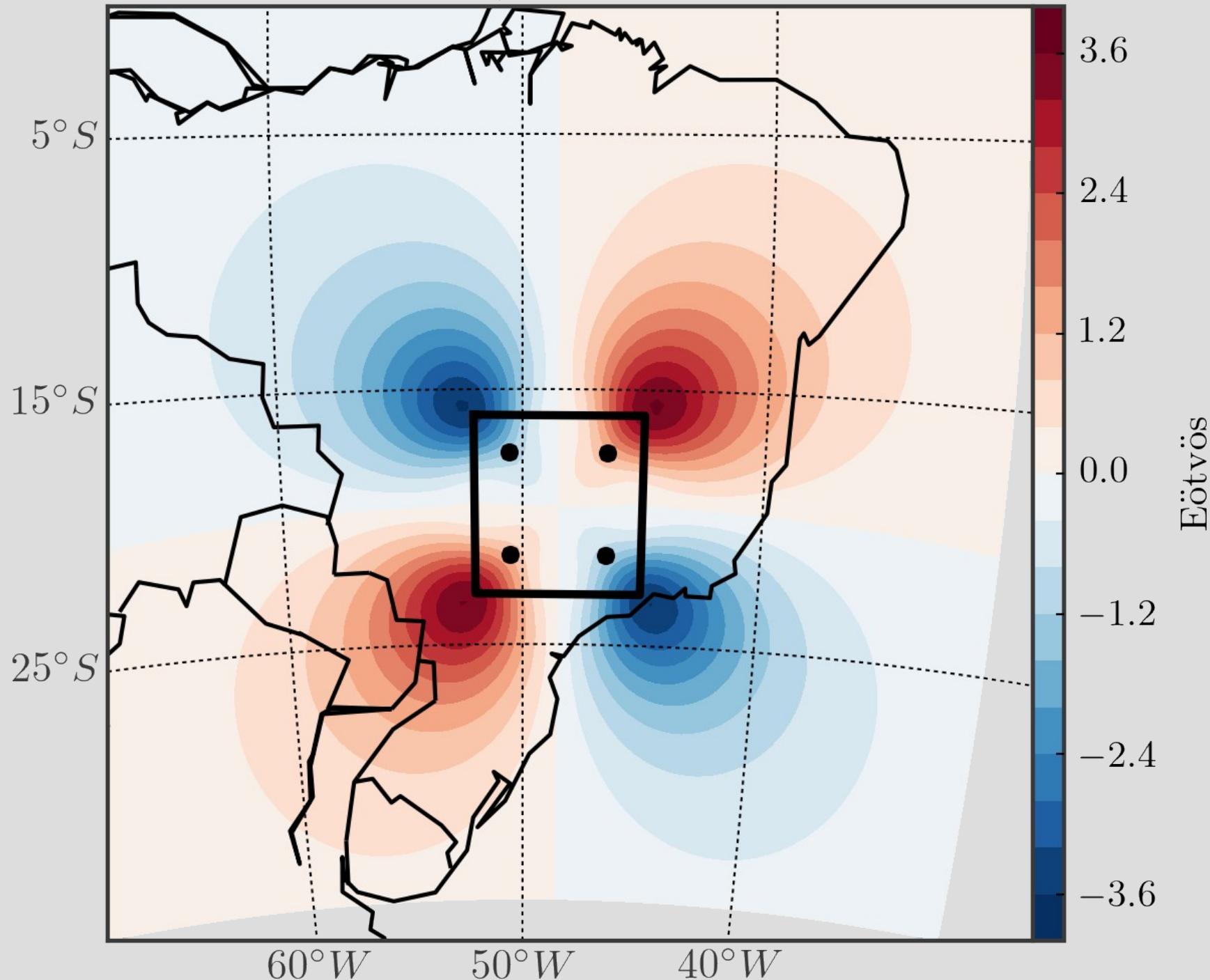
$$\iiint_{\Omega} K_z(r, \varphi, \lambda) d\Omega$$



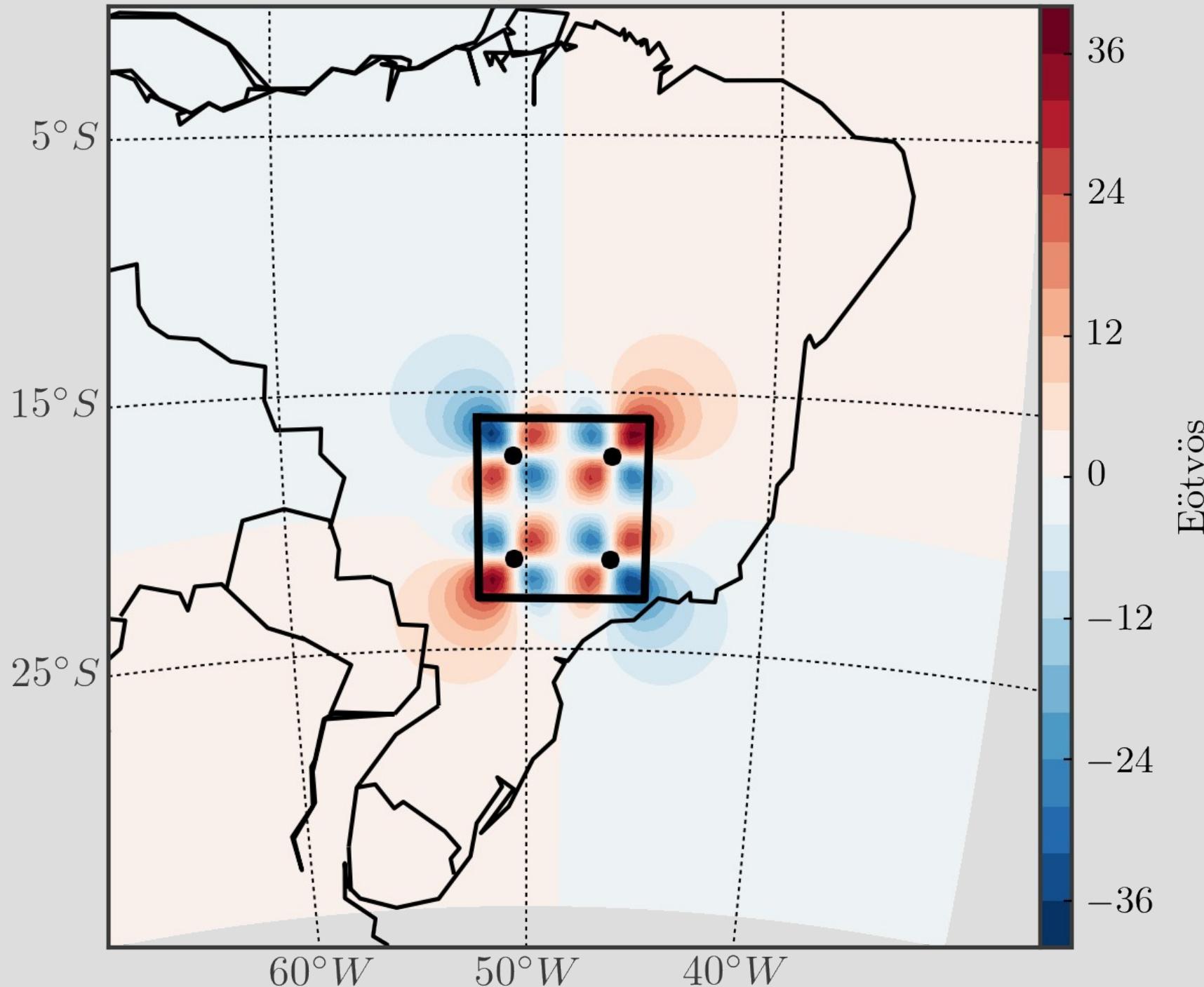
massa pontual

$$A \sum_{i=1}^{N_r} \sum_{j=1}^{N_\varphi} \sum_{k=1}^{N_\lambda} W_i W_j W_k K_z(r_i, \varphi_j, \lambda_k)$$

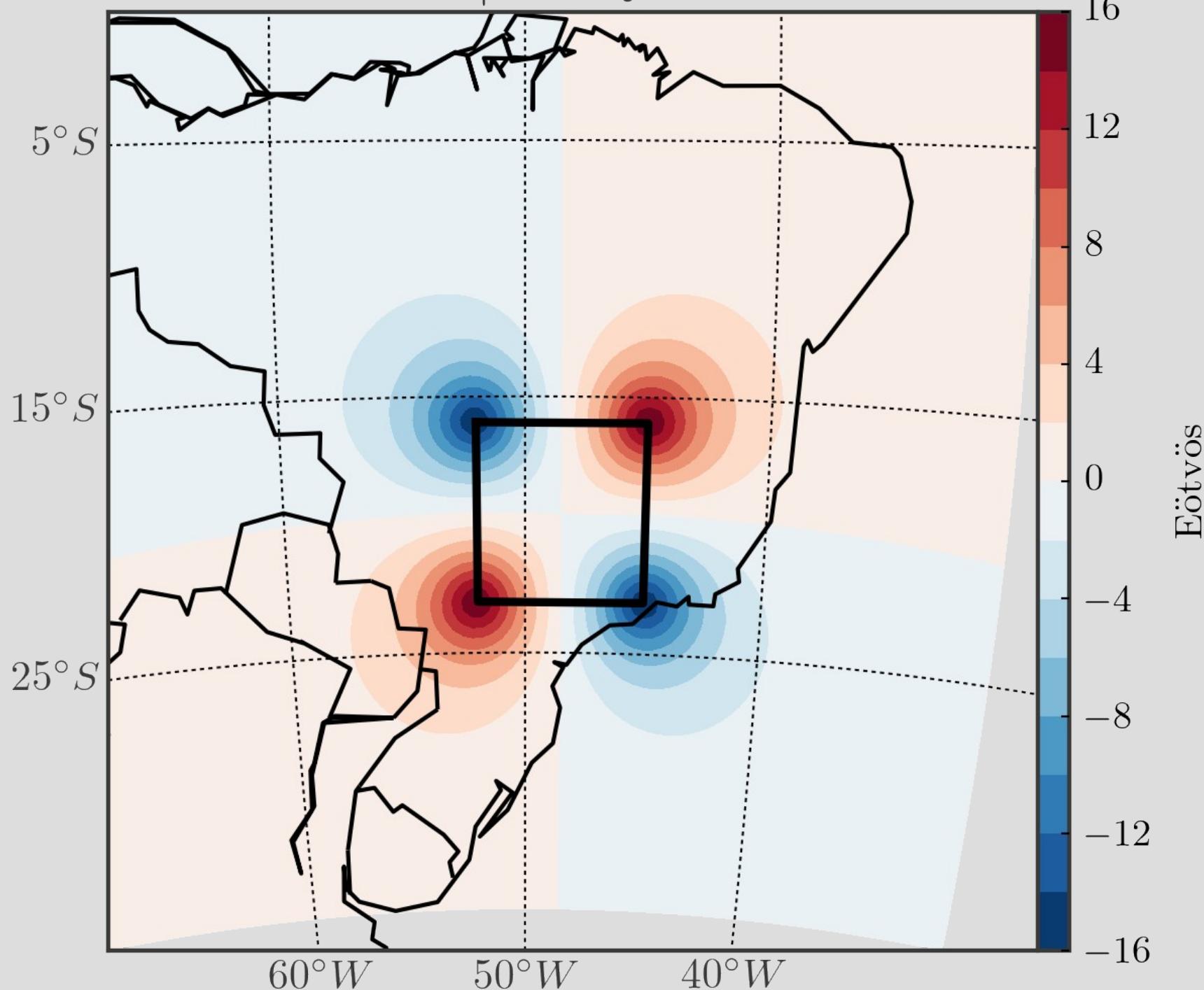
# 400 km | GLQ order 2



# 150 km | GLQ order 2



150 km | GLQ order 30

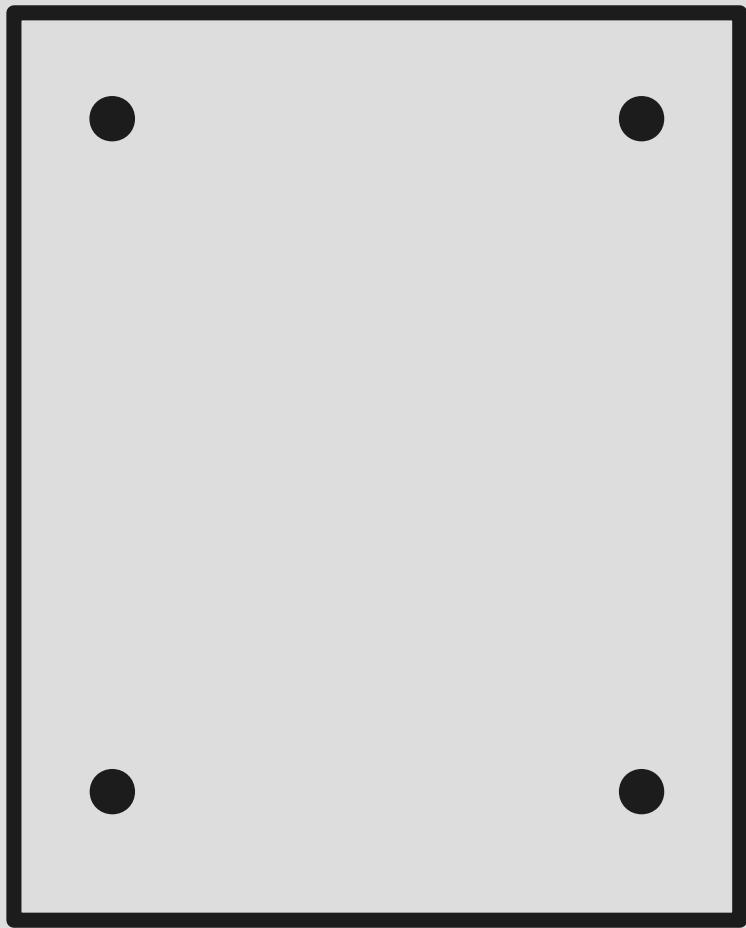


# Ku (1977)

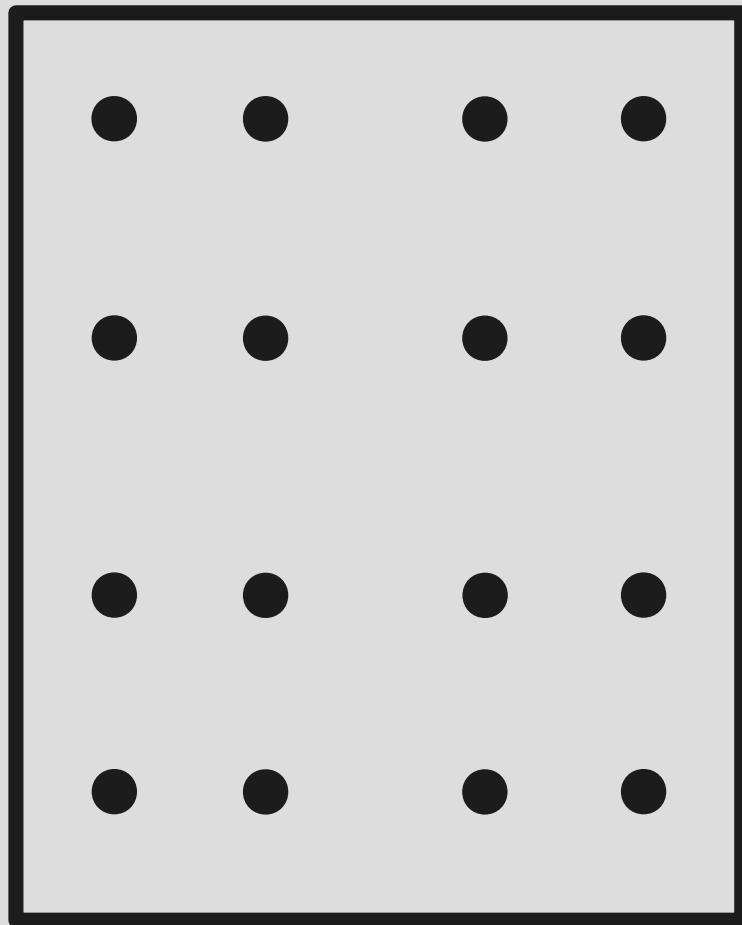
dist. até  
observação



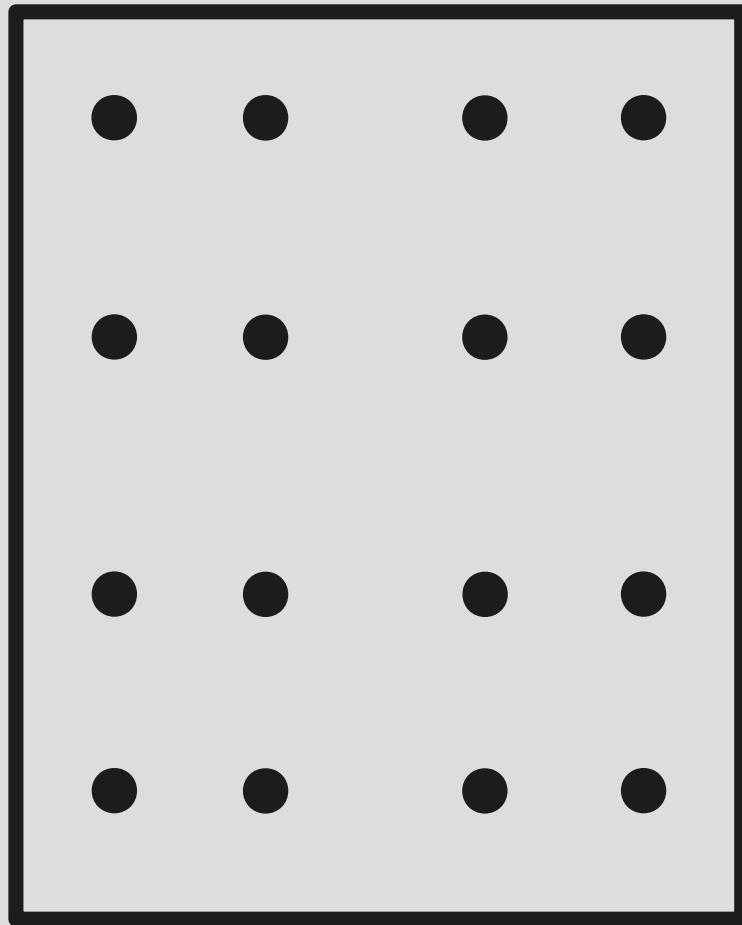
dist. entre  
massas



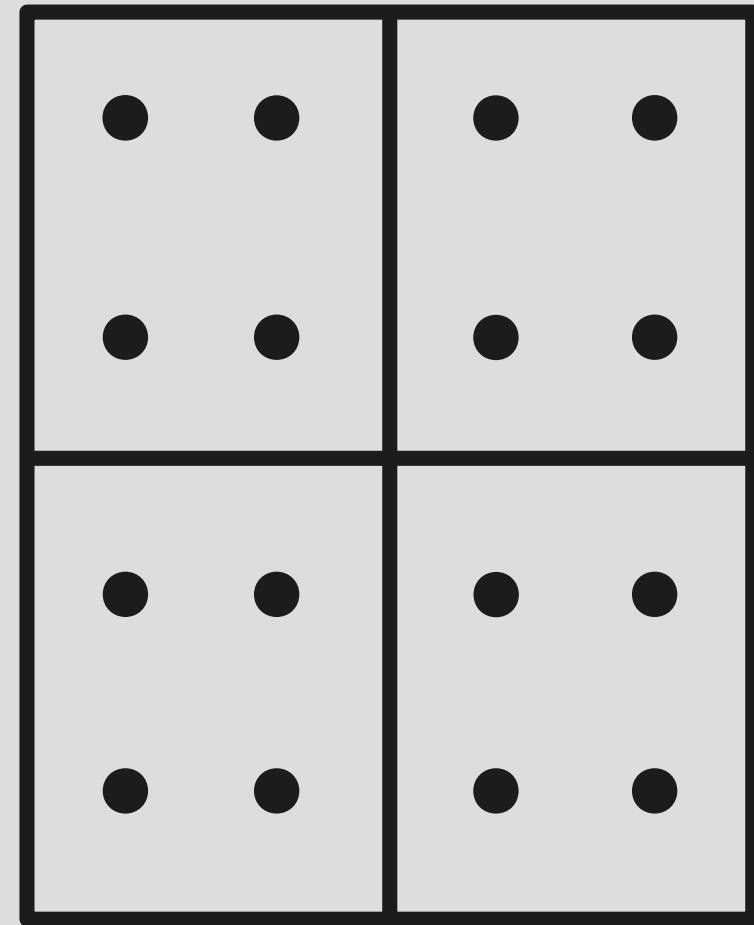
+ massas



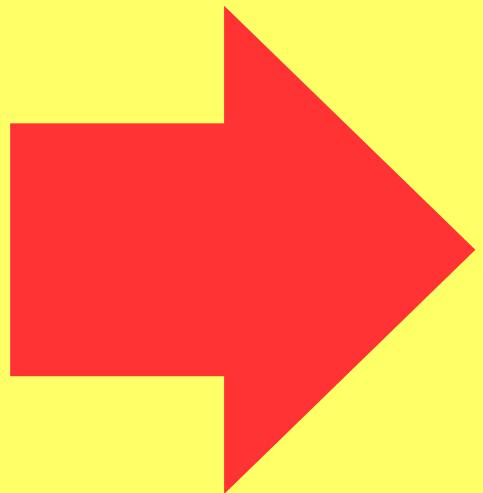
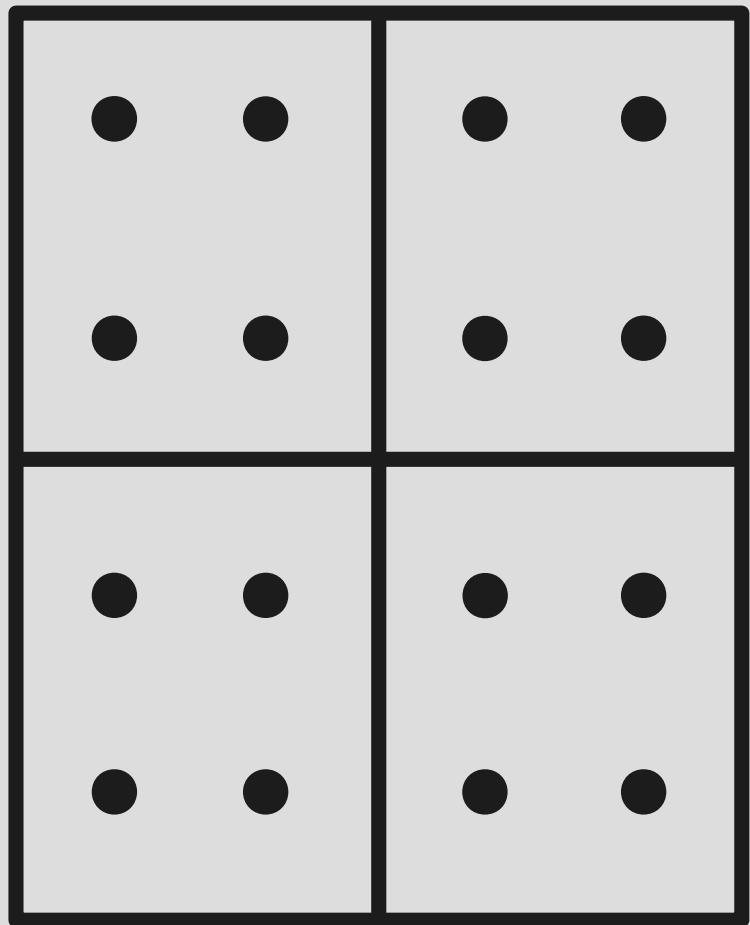
+ massas

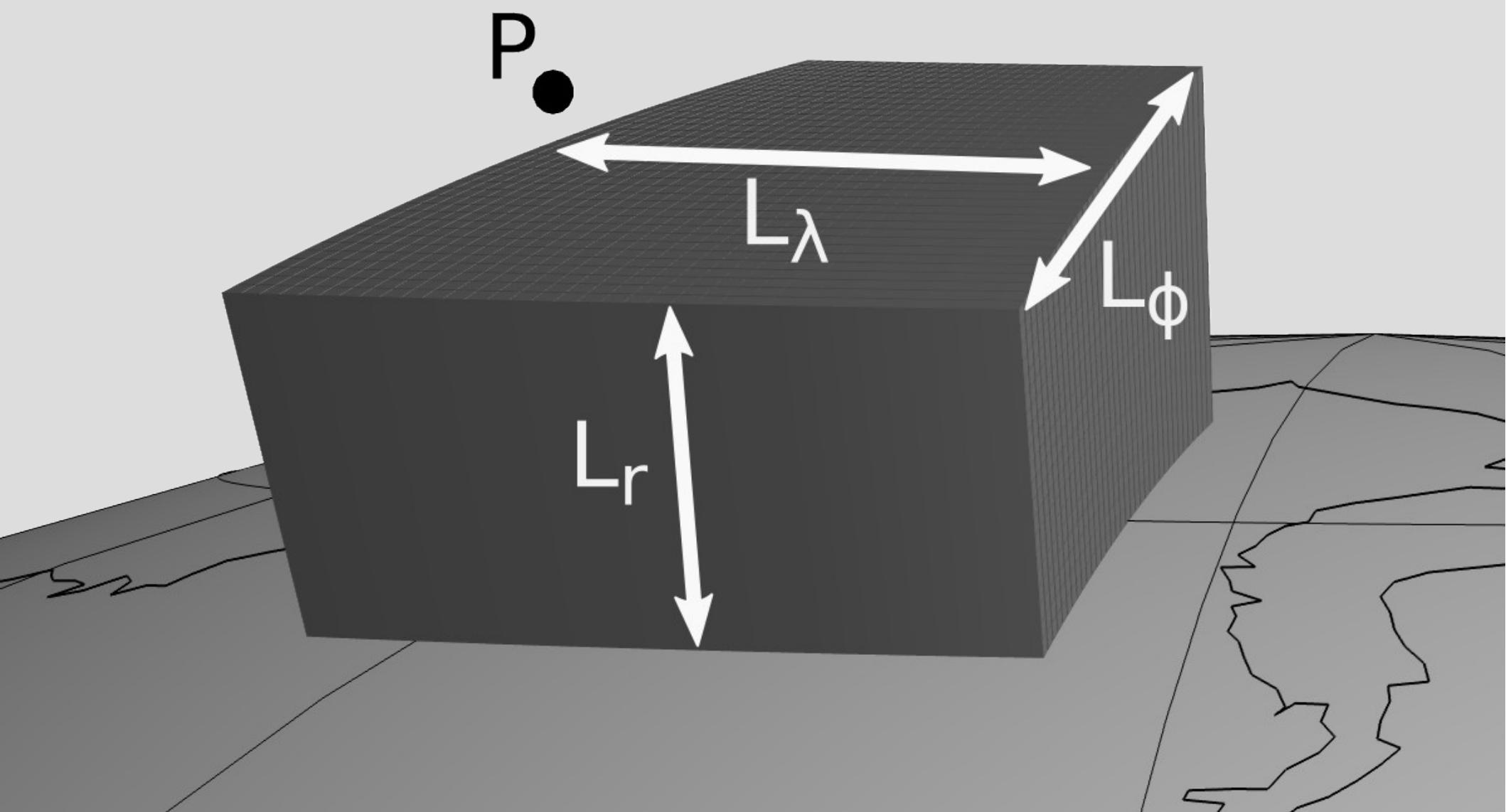


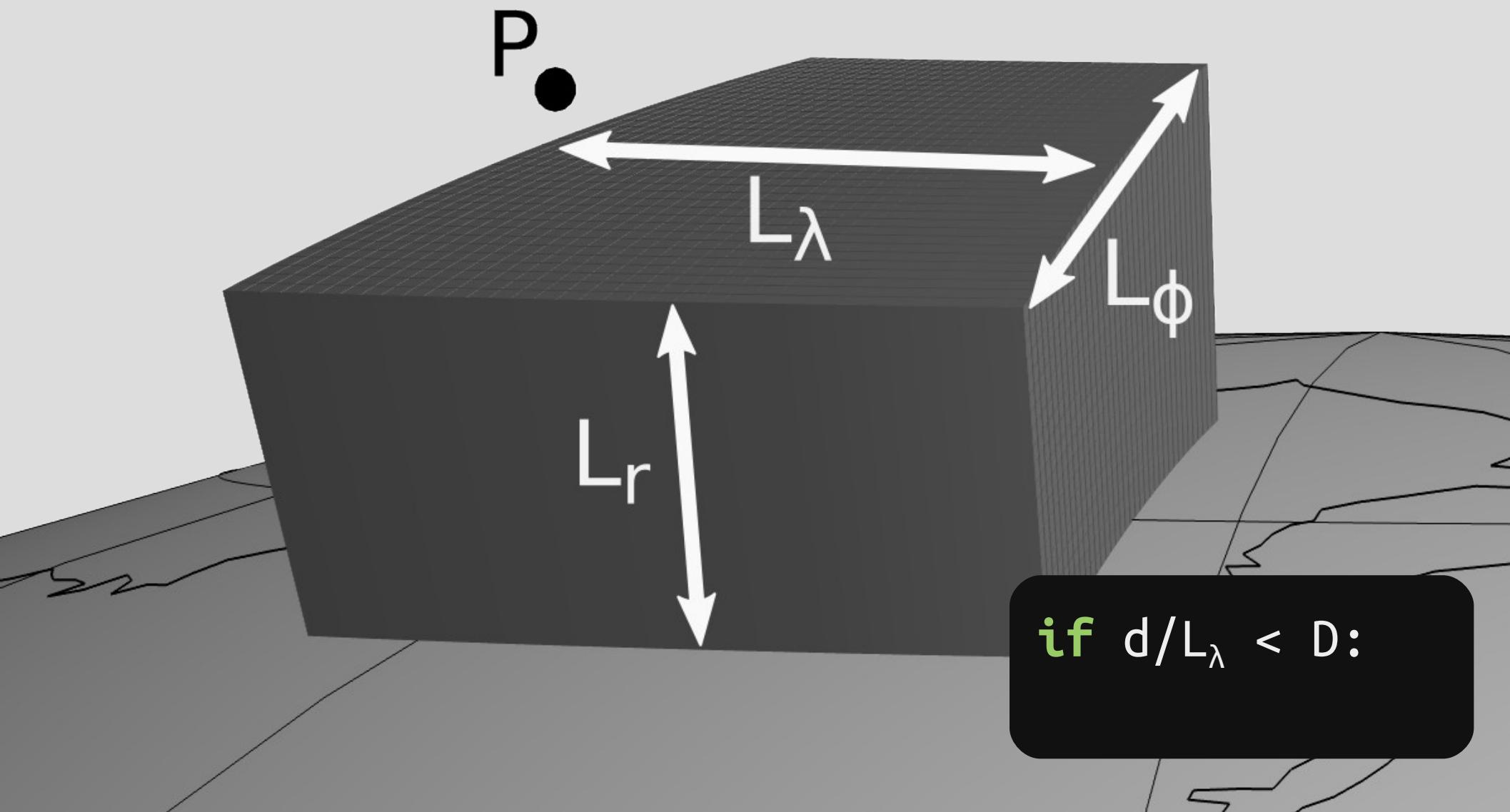
+ tesseroides



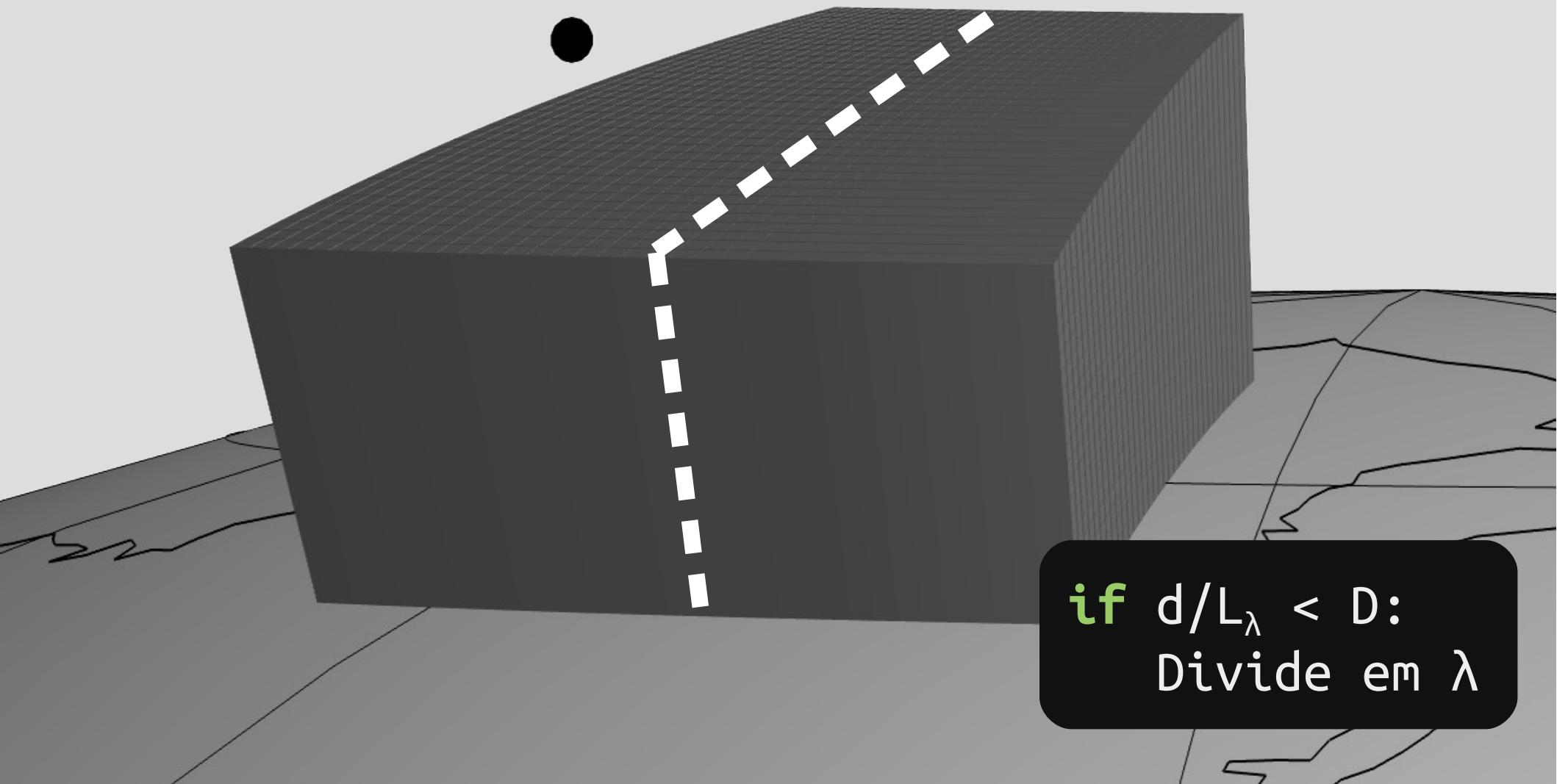
+ tesseroides



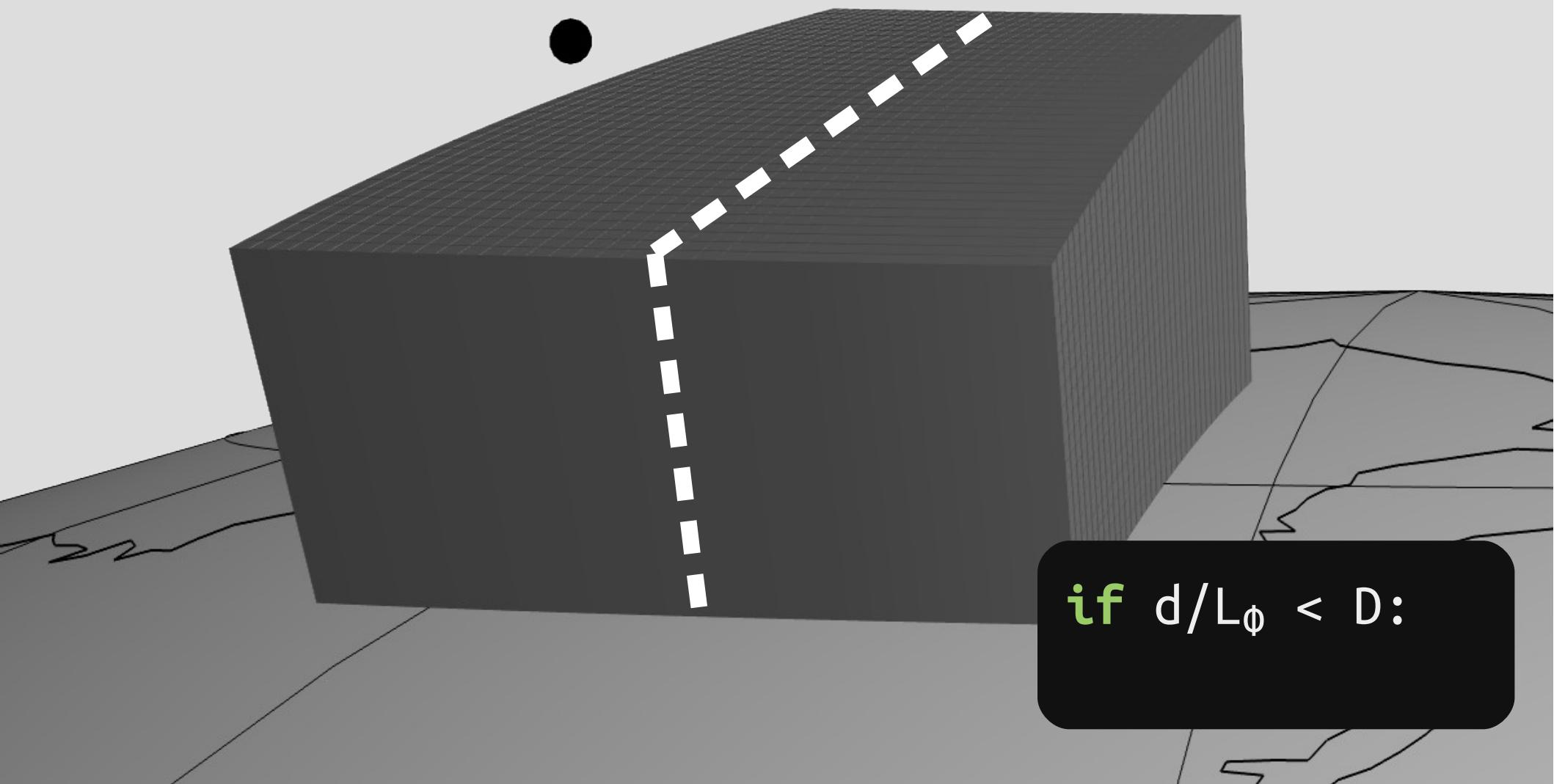




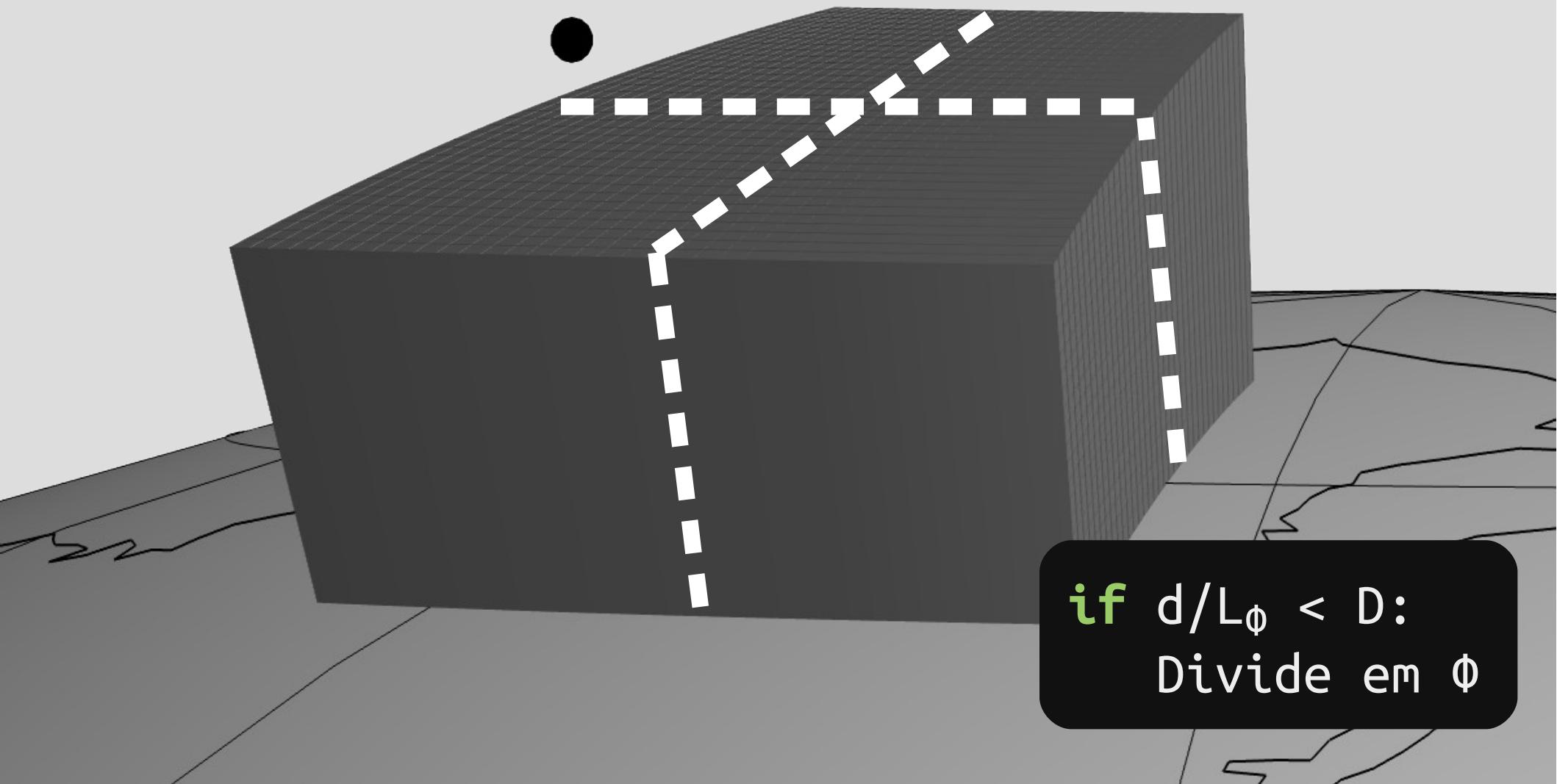
if  $d/L_\lambda < D:$



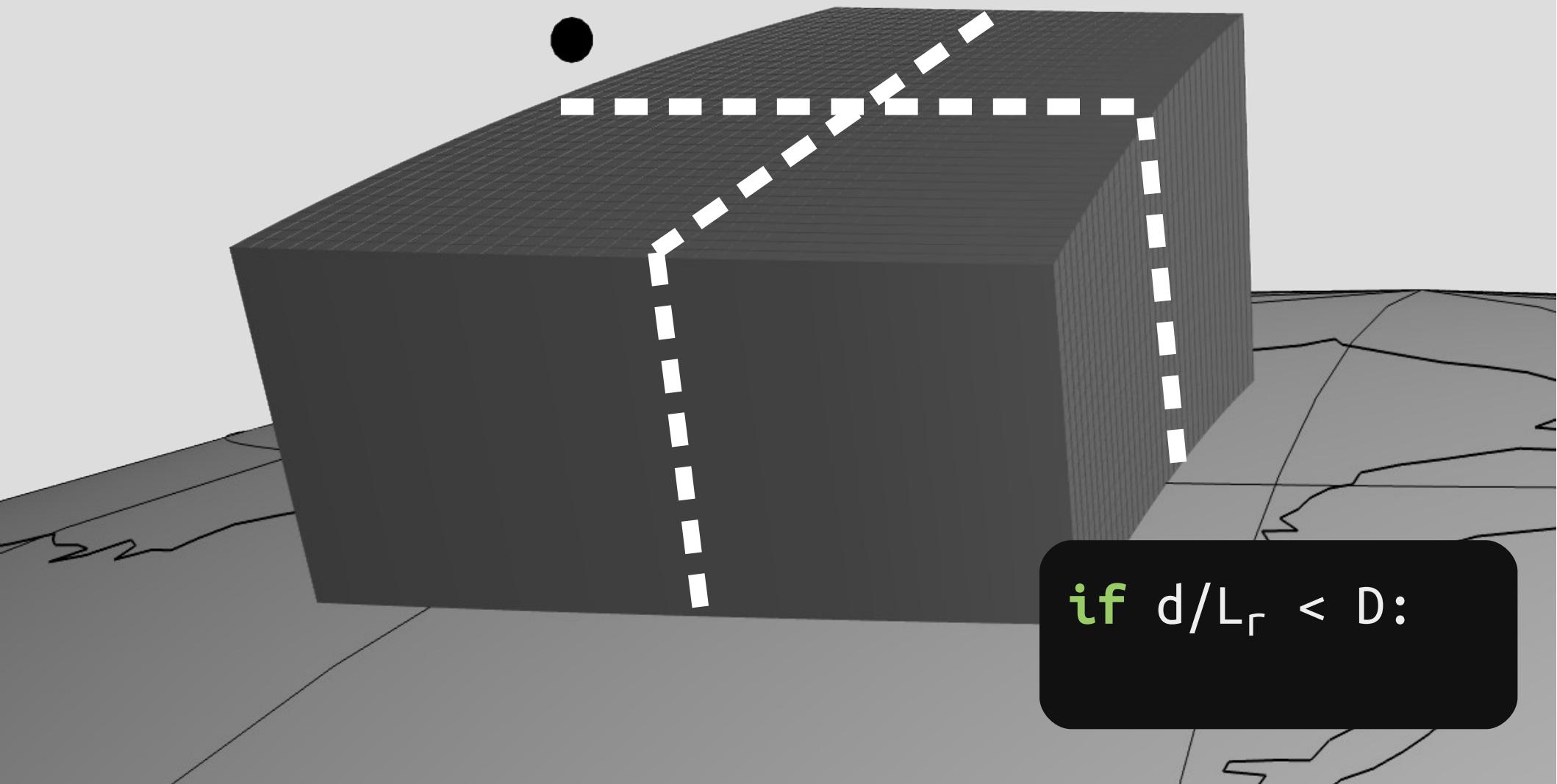
if  $d/L_\lambda < D$ :  
Divide em  $\lambda$



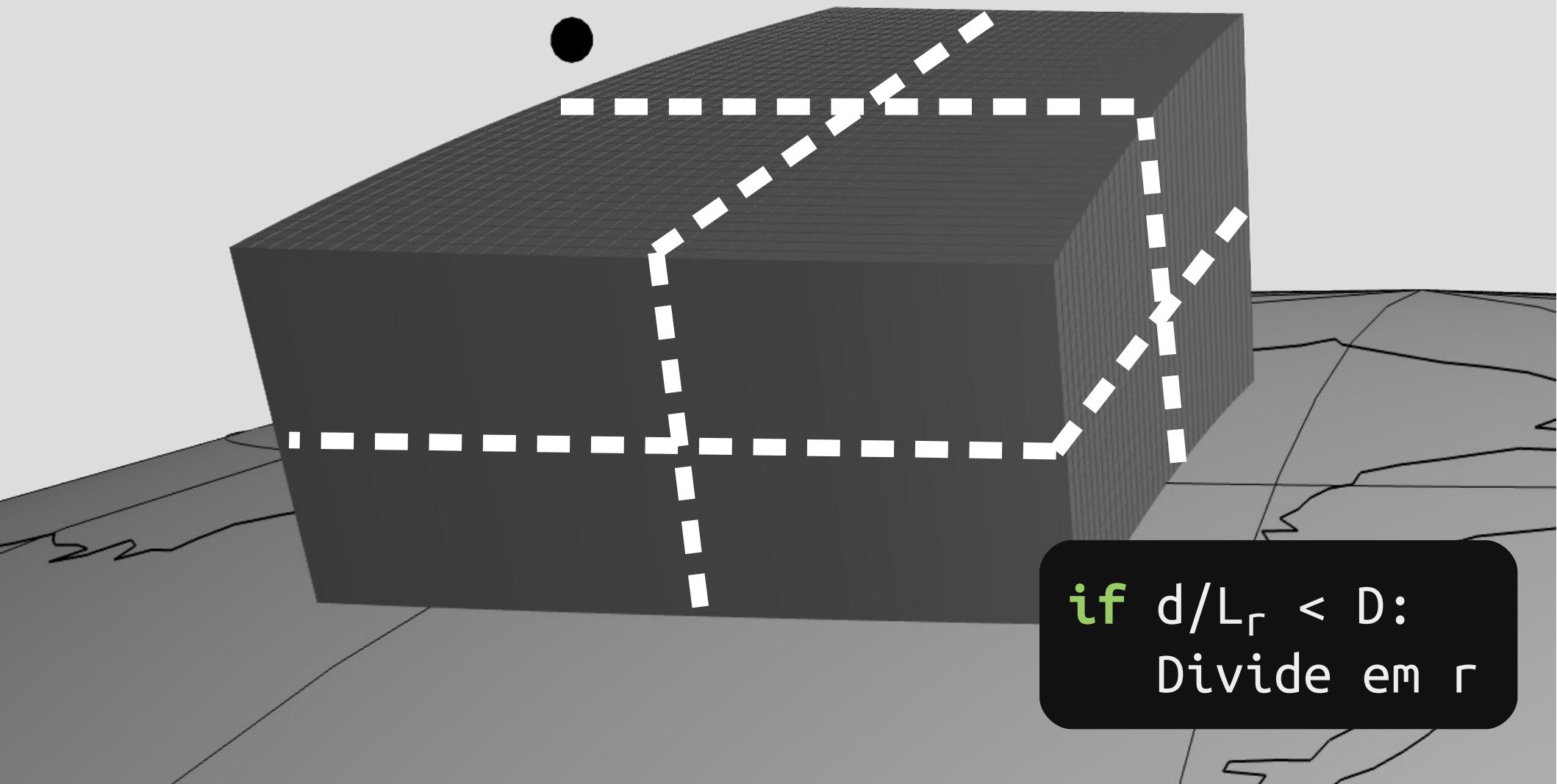
if  $d/L_\phi < D$ :



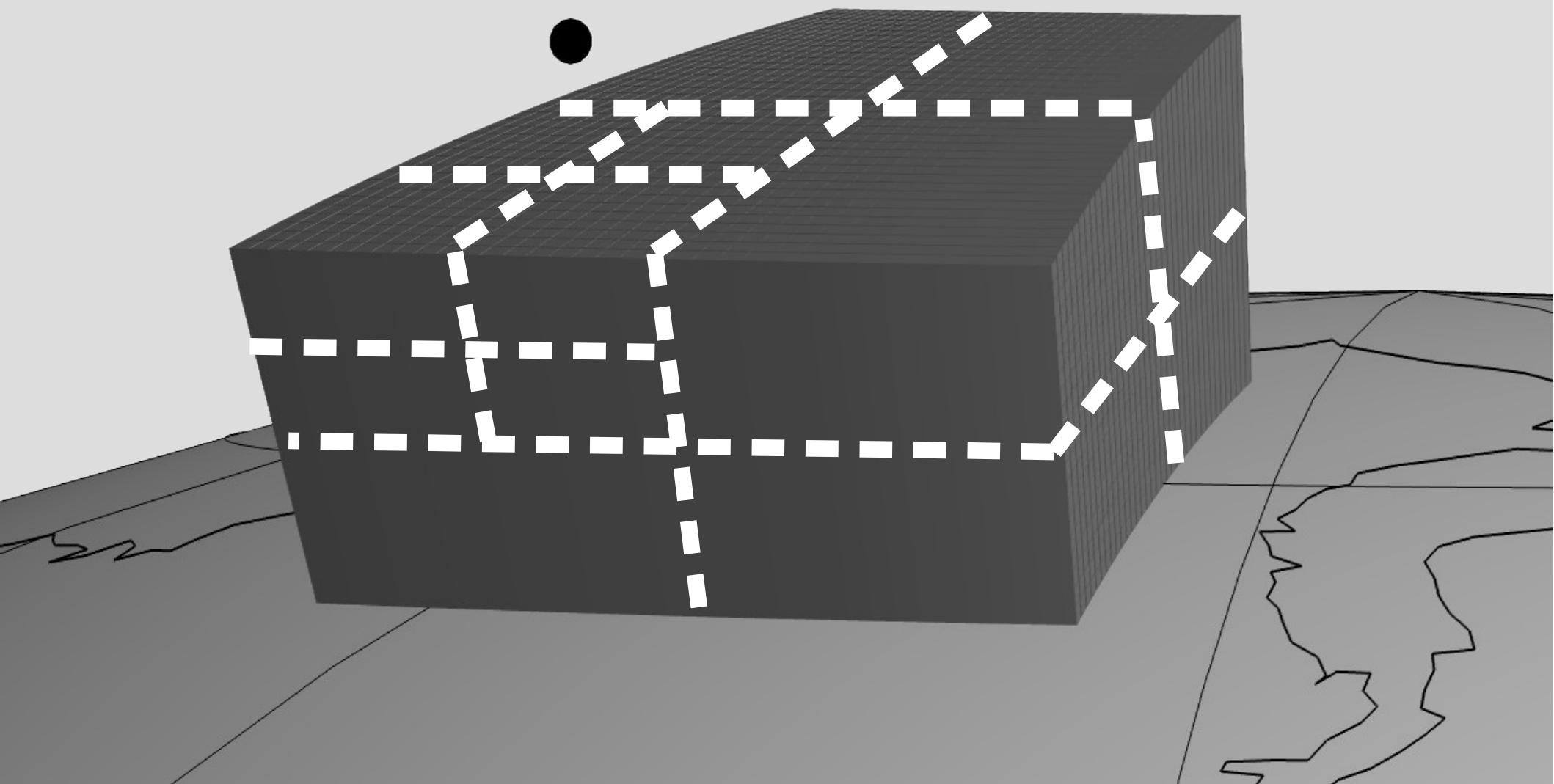
**if**  $d/L_\phi < D$ :  
Divide em  $\Phi$

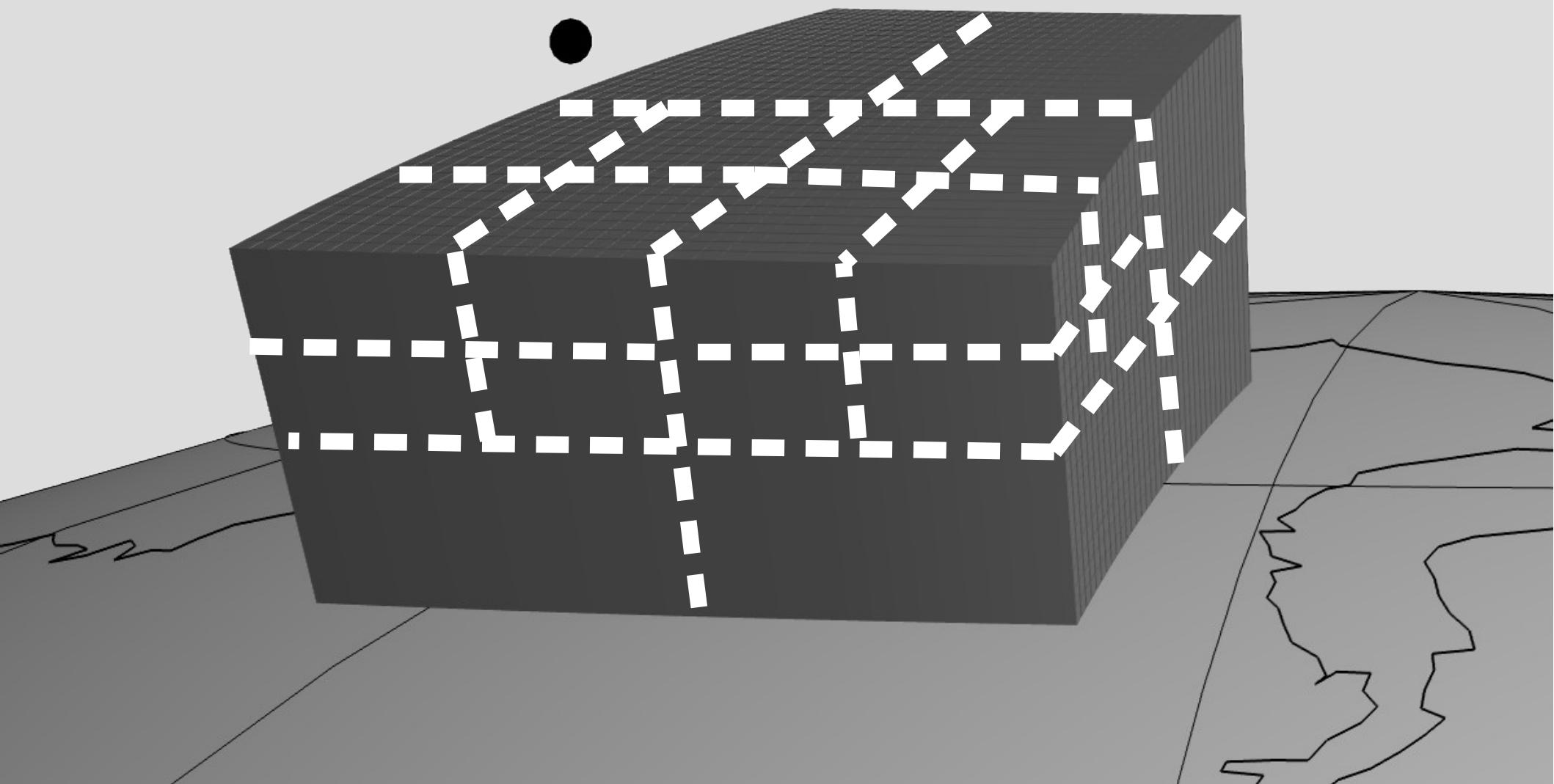


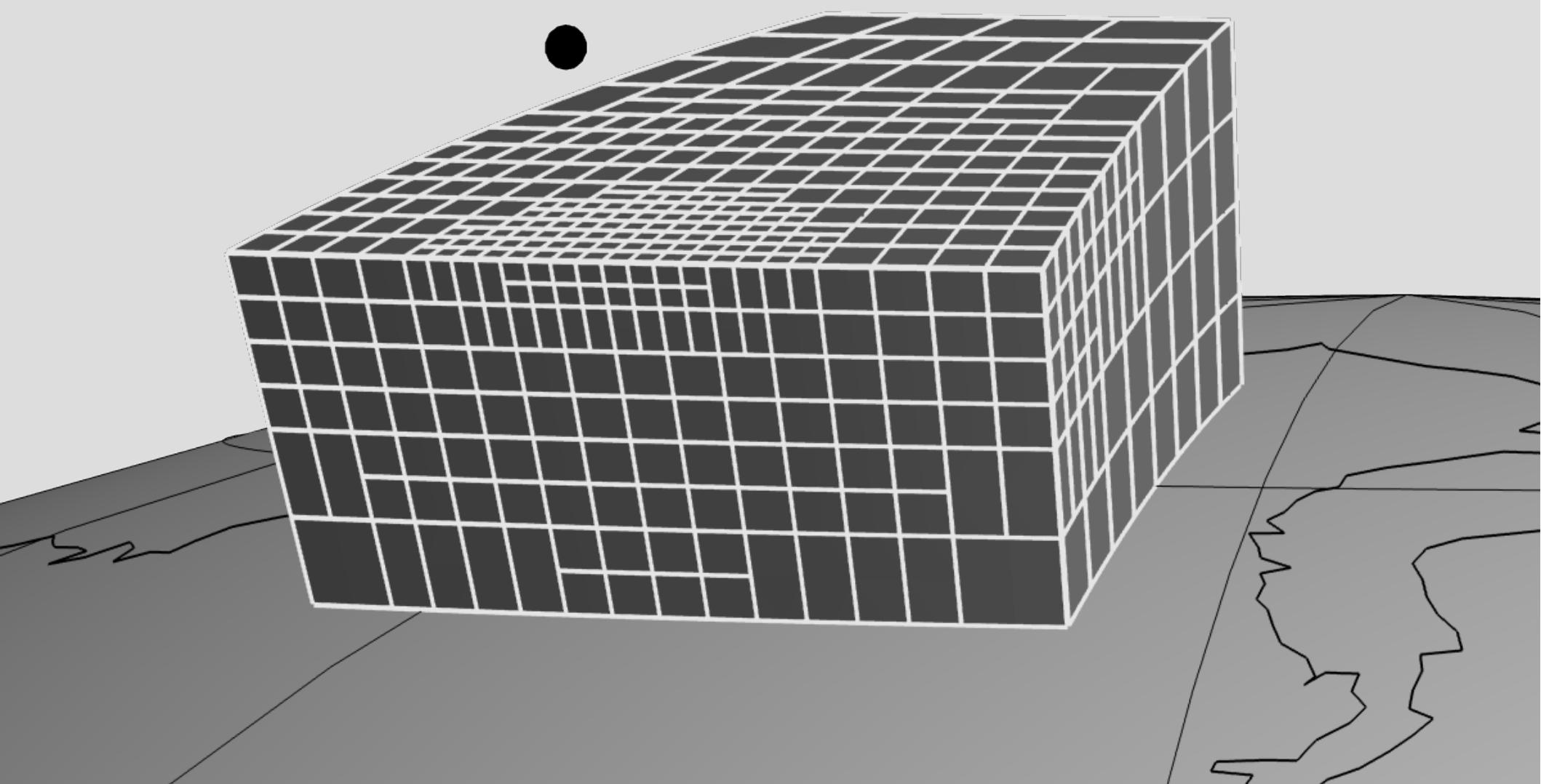
**if**  $d/L_r < D:$



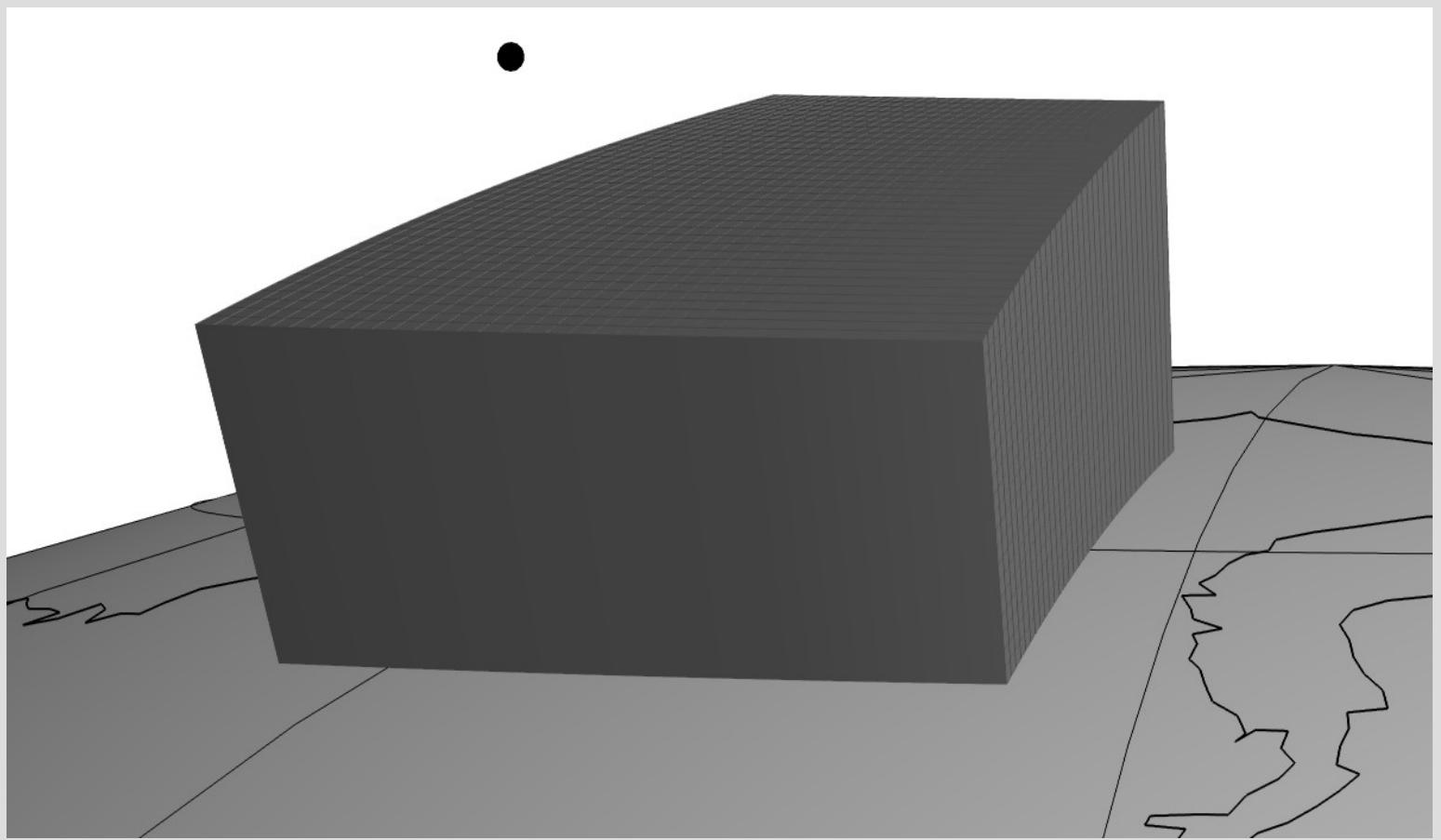
**if**  $d/L_r < D$ :  
Divide em r



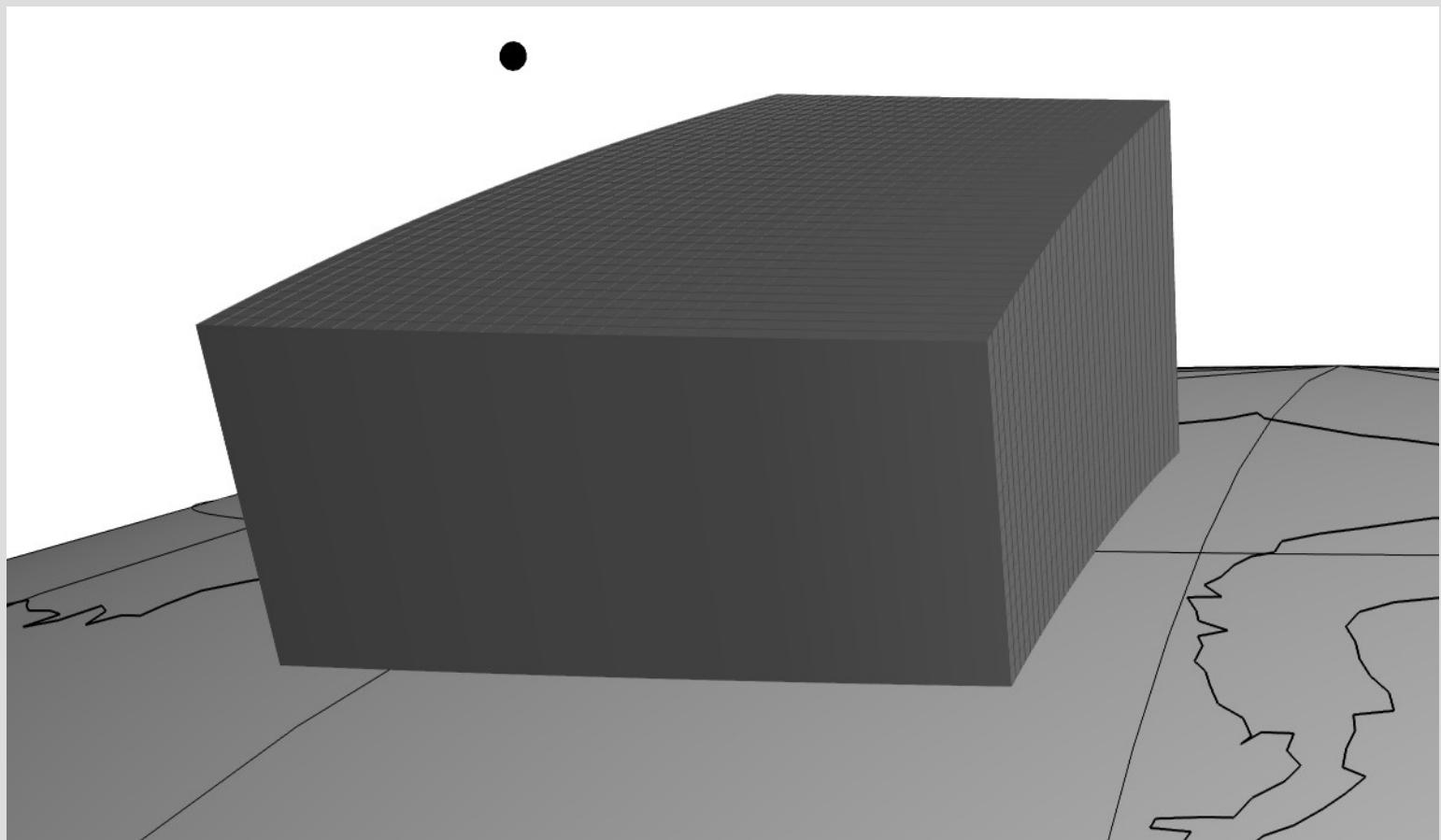




algoritmo

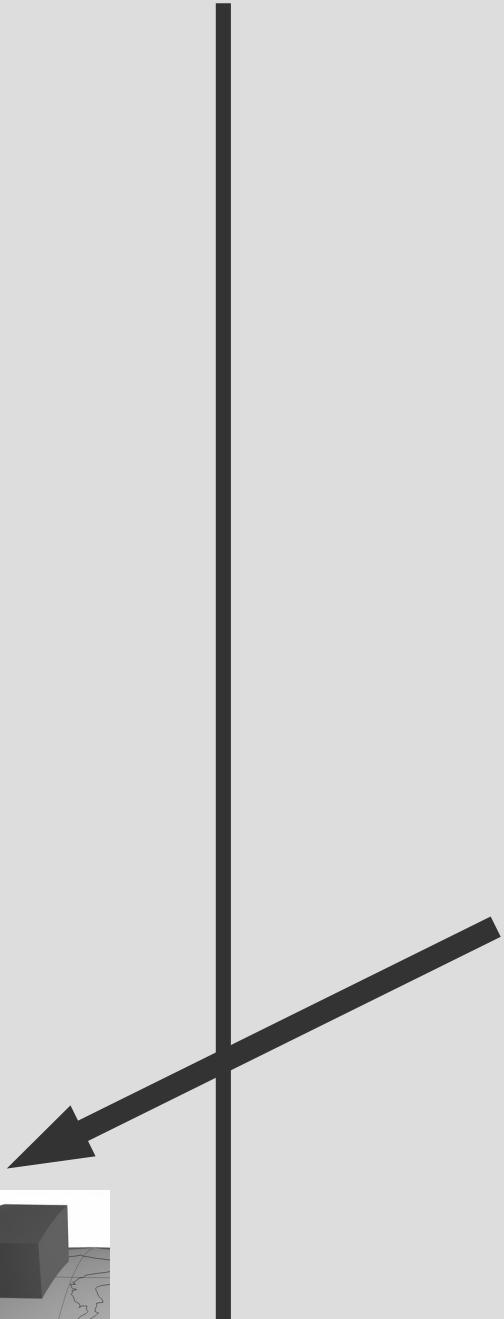
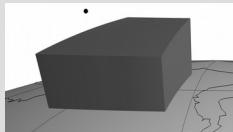


# pilha



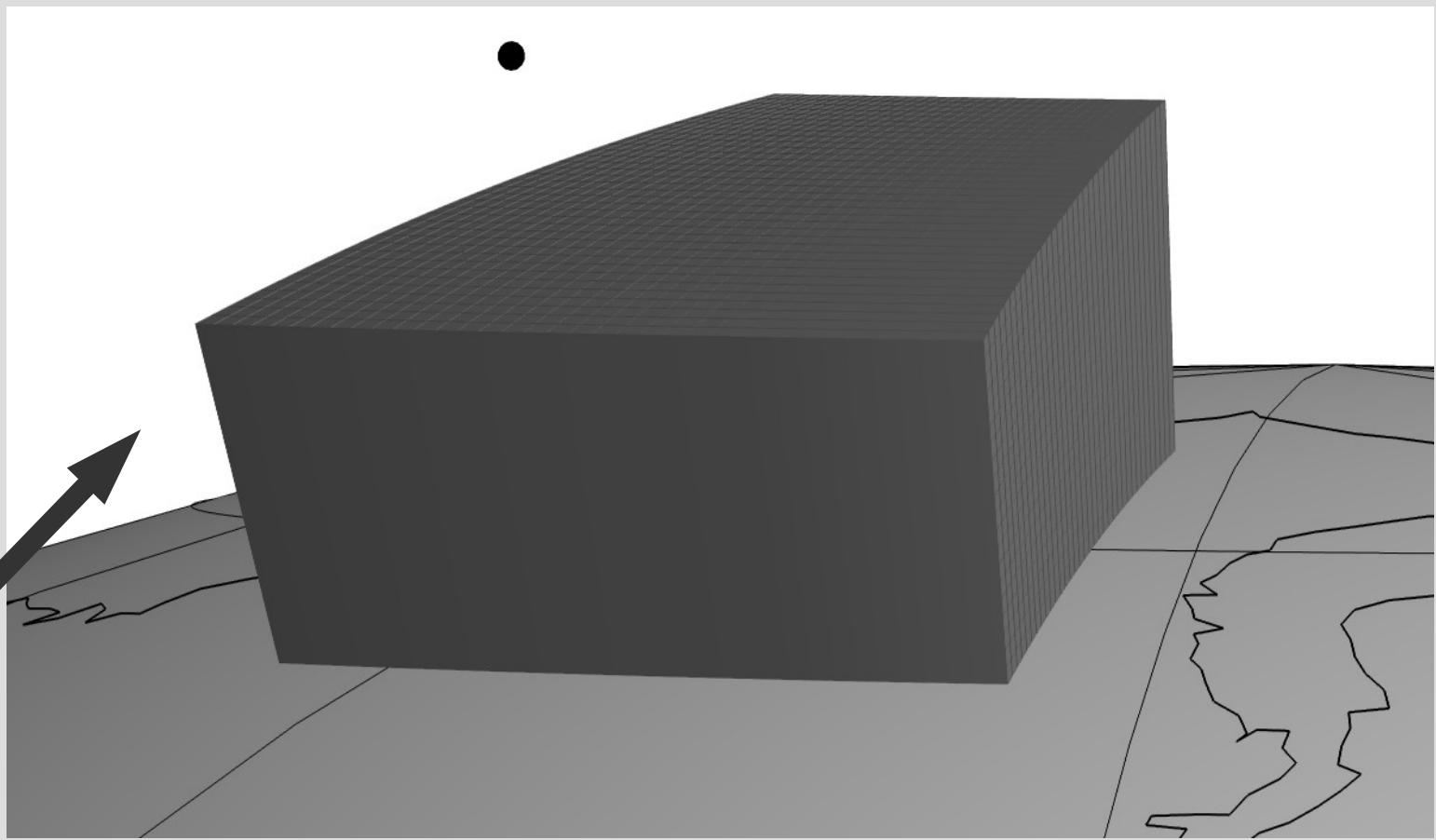
# pilha

adicionar no  
topo da pilha



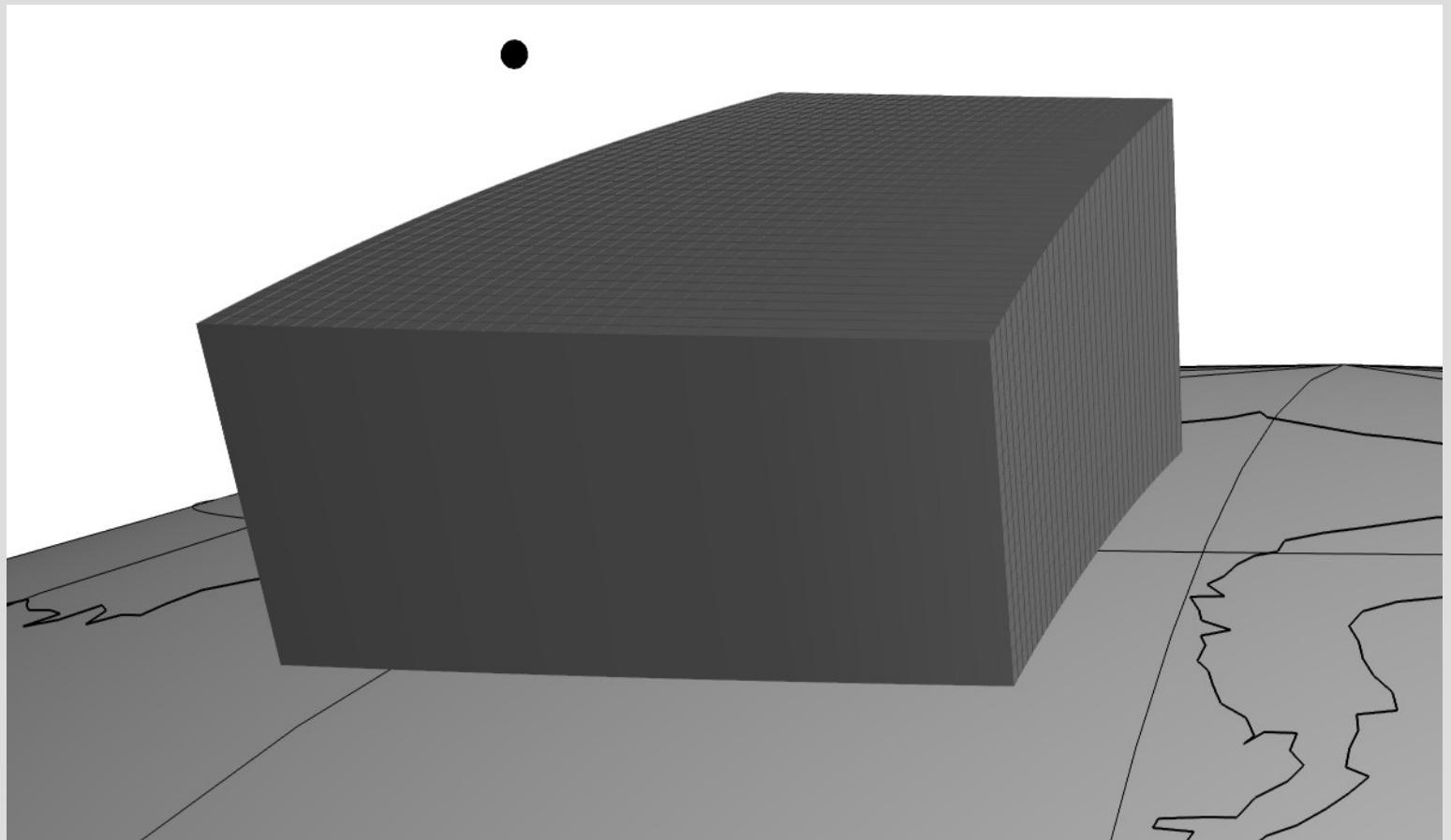
# pilha

“pop”



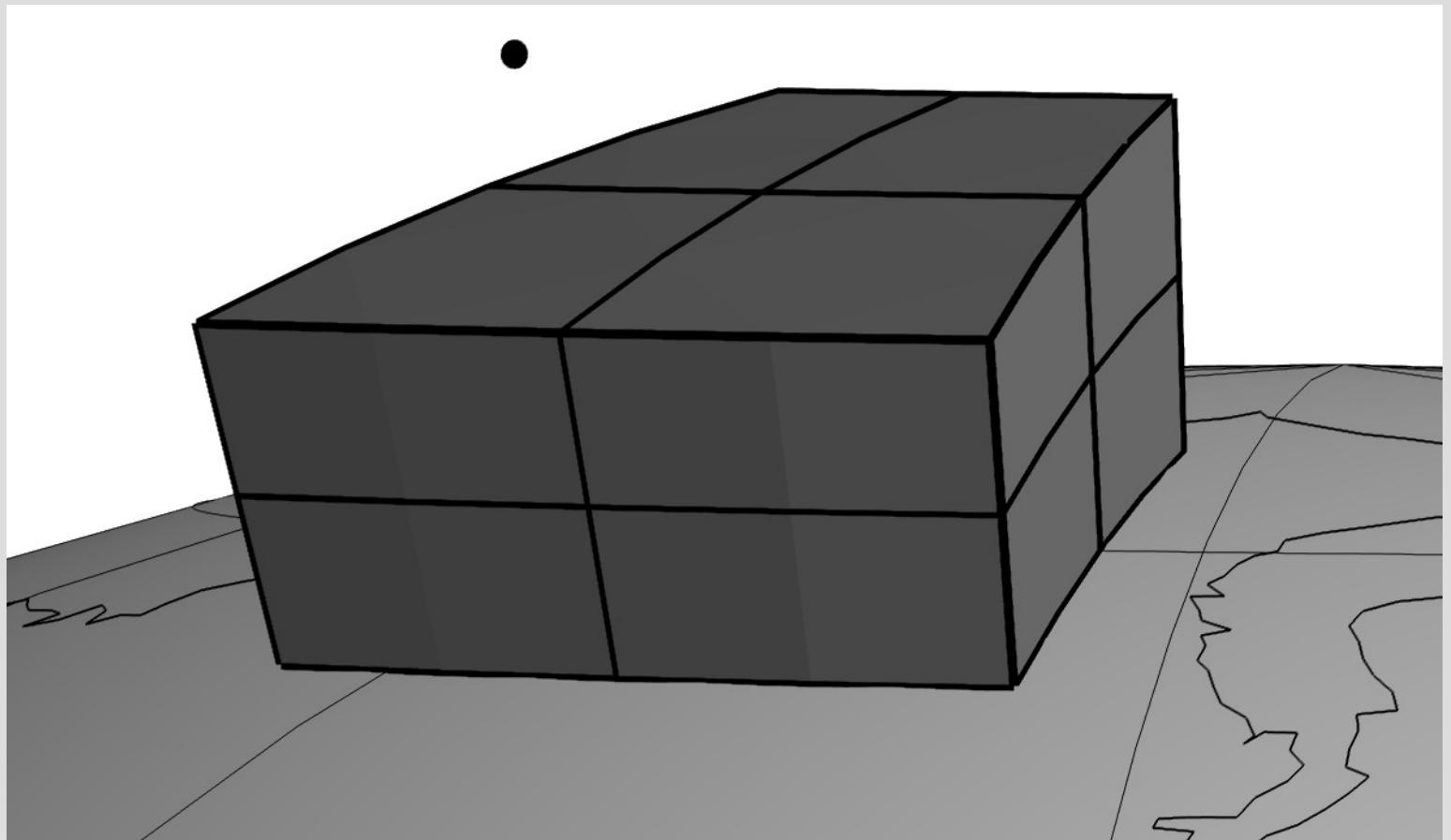
# pilha

if  $d/L_x < D$ :  
Divide em x

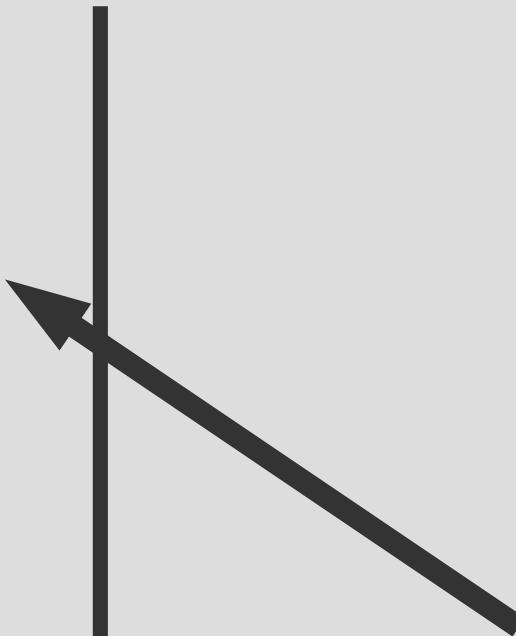
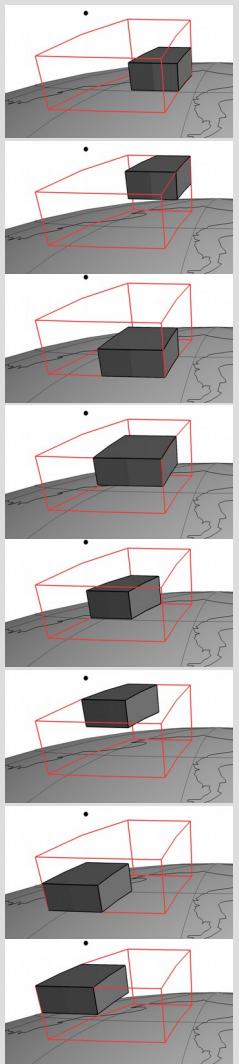


# pilha

if  $d/L_x < D$ :  
Divide em x

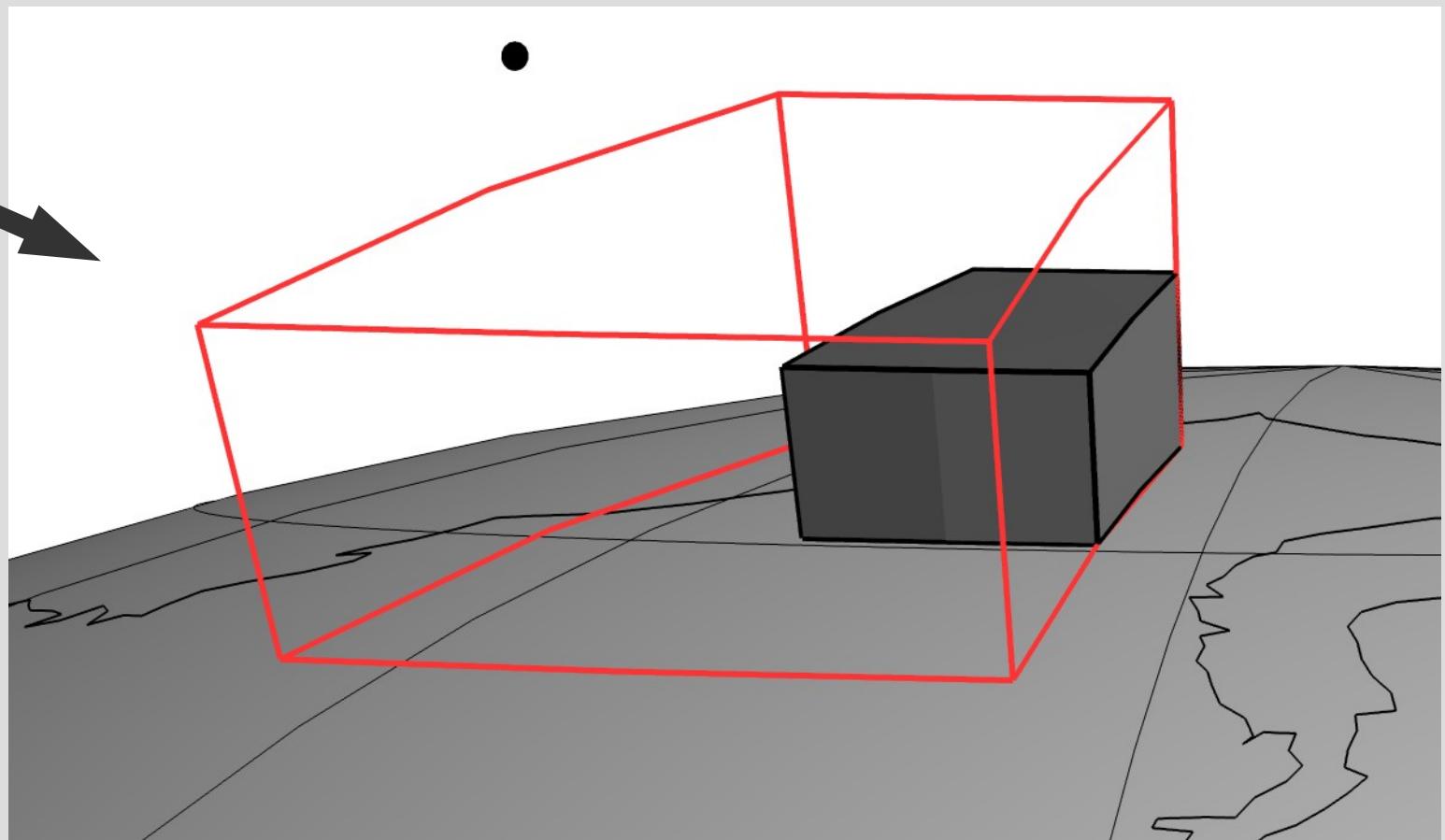
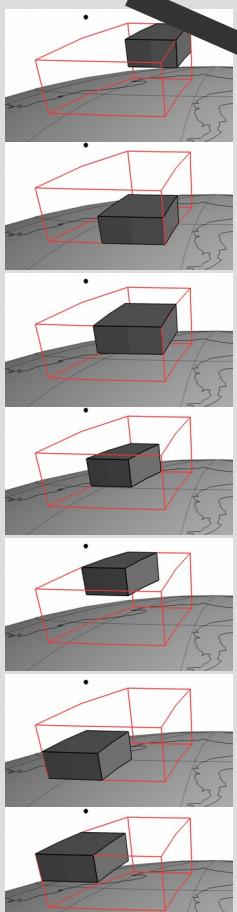


# pilha



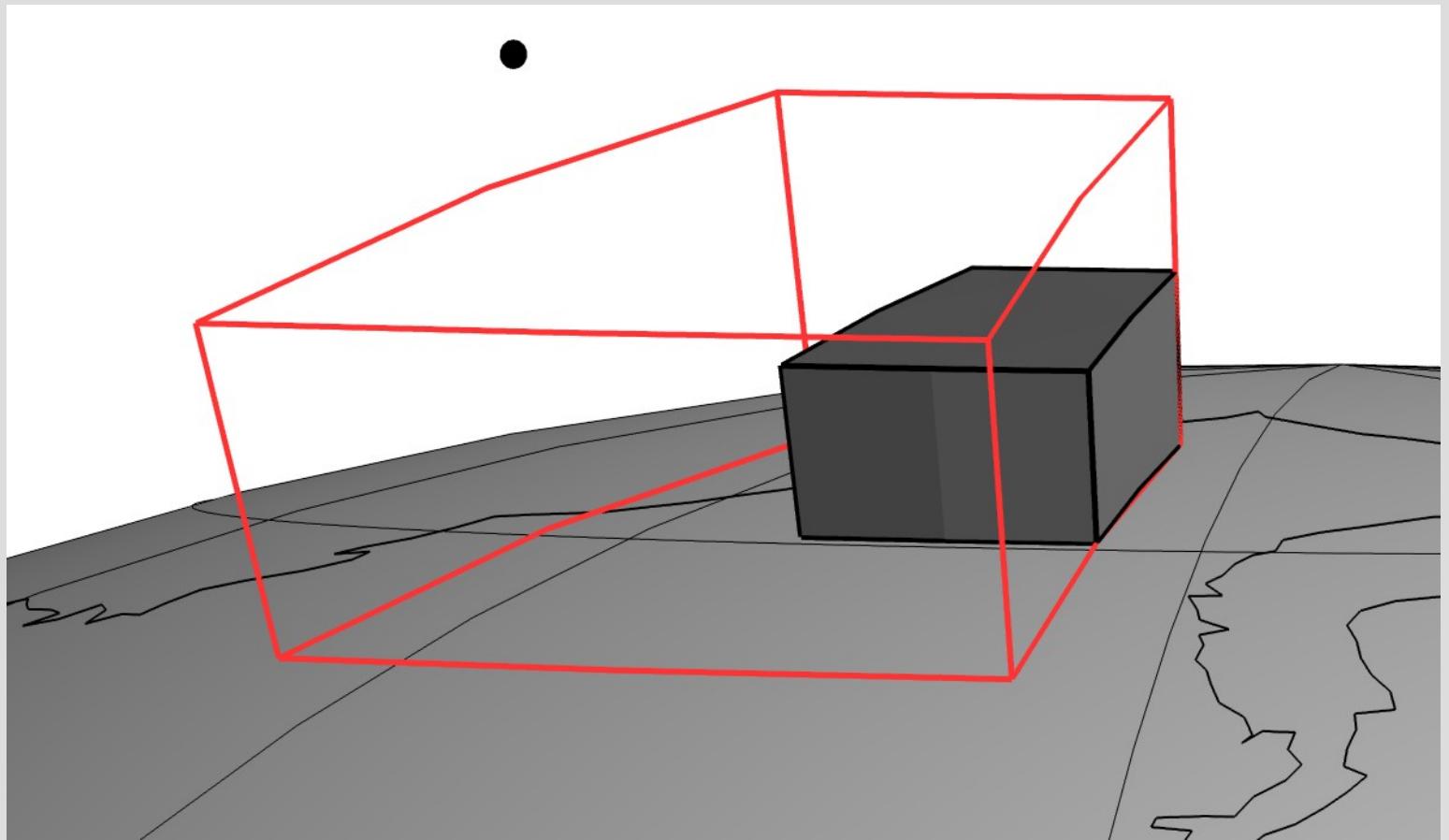
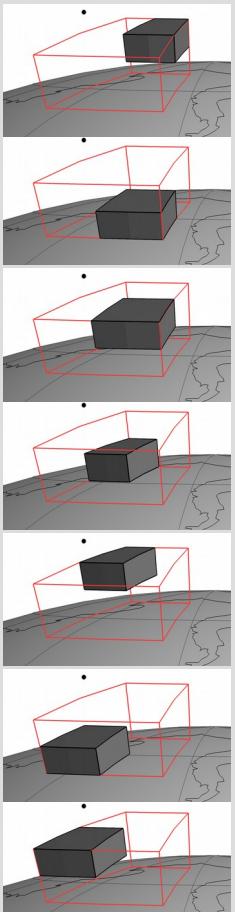
# pilha

“pop”



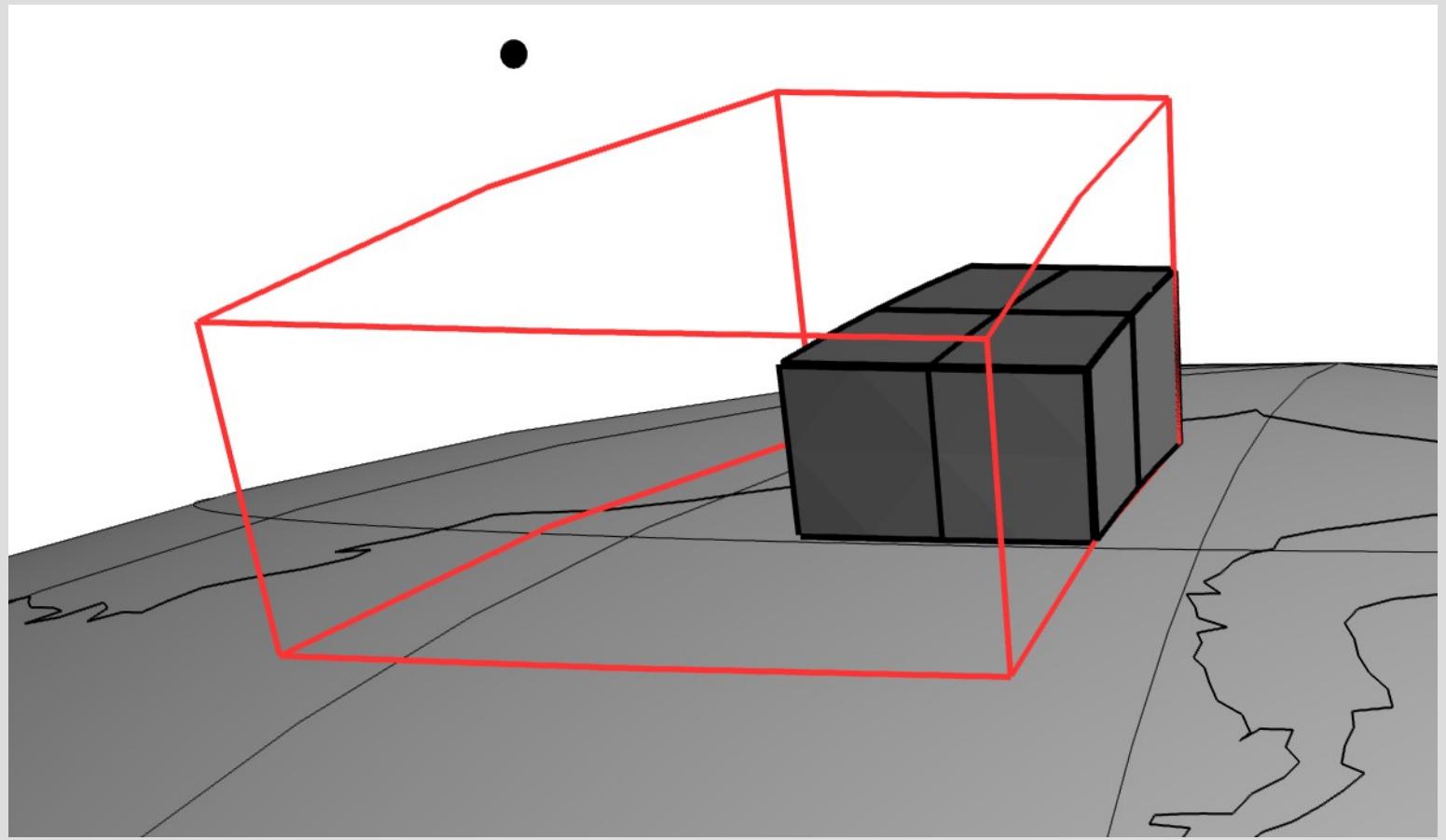
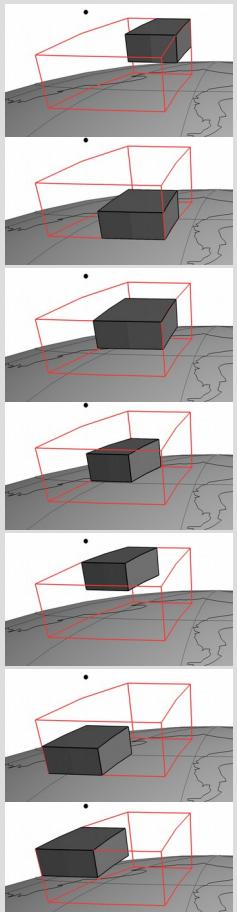
# pilha

if  $d/L_x < D$ :  
Divide em x

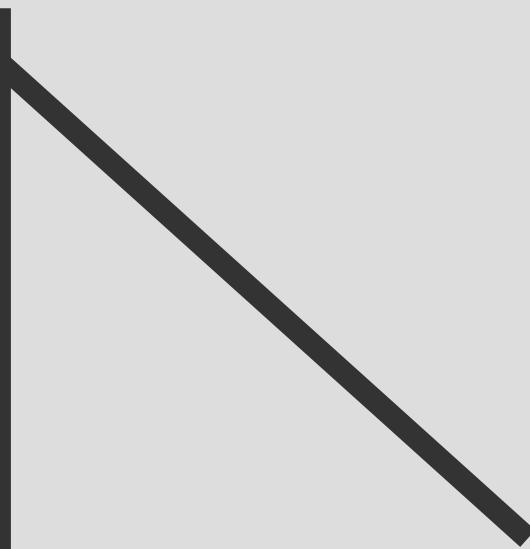
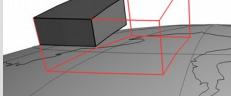
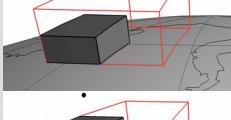
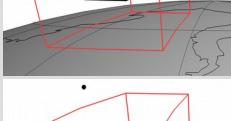
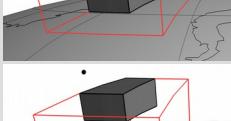
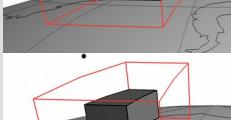
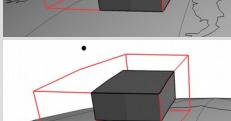
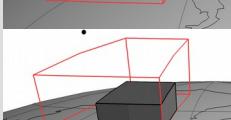
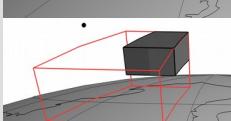
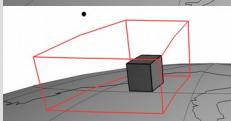
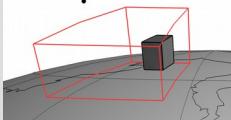
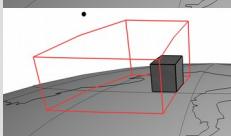
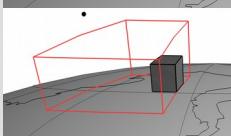


# pilha

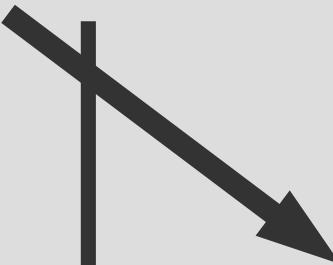
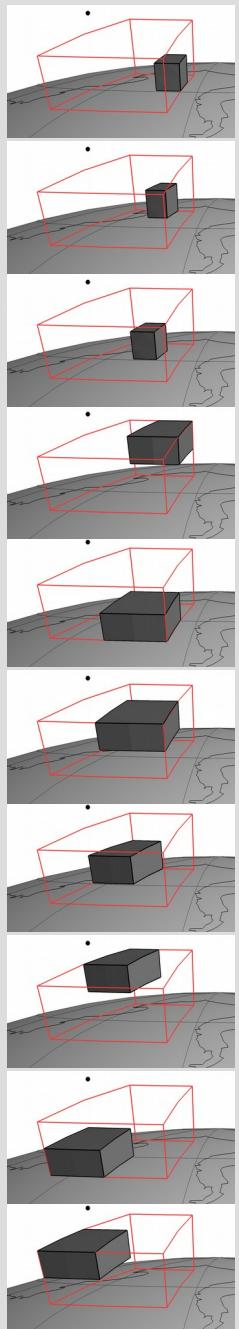
if  $d/L_x < D$ :  
Divide em x



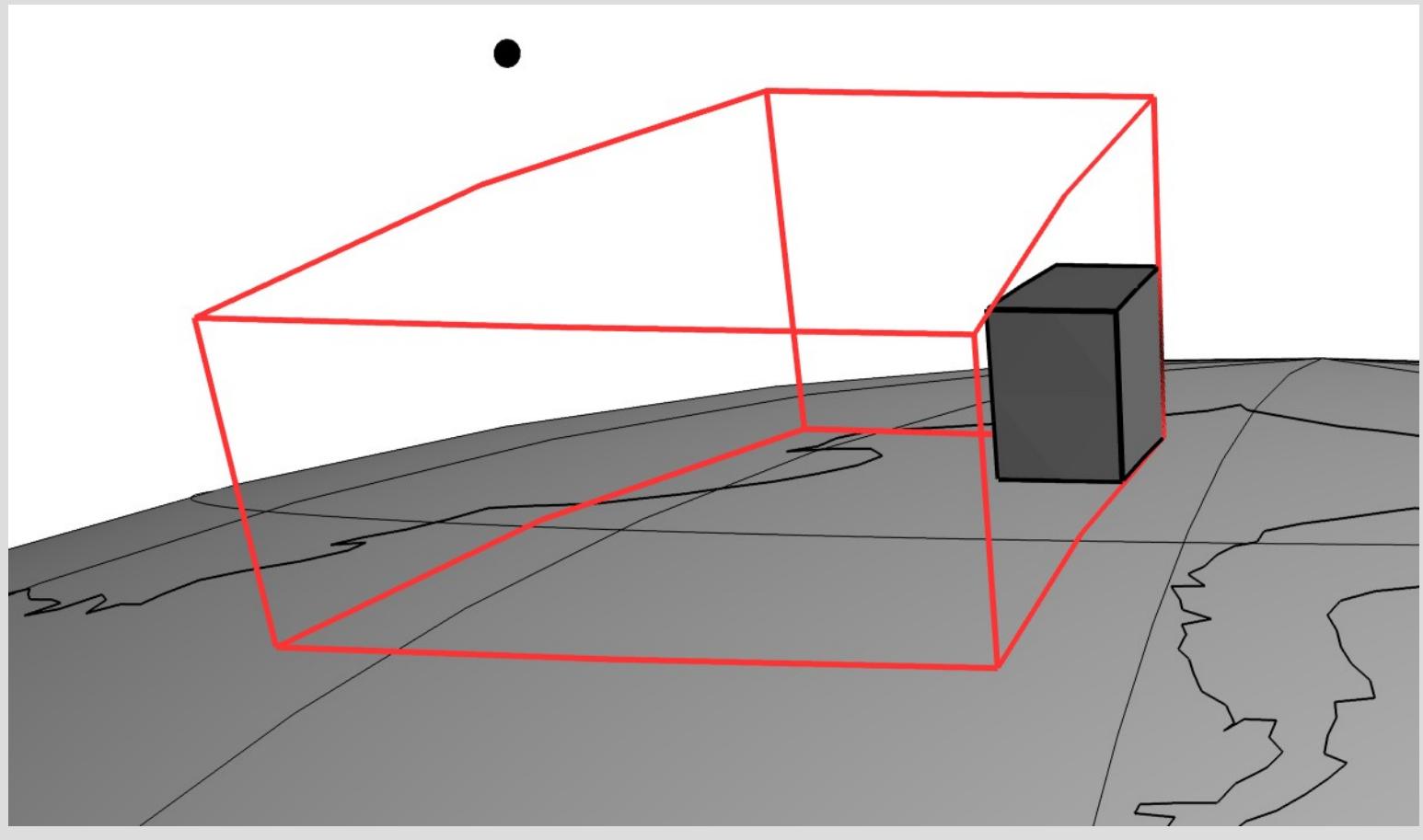
pilha



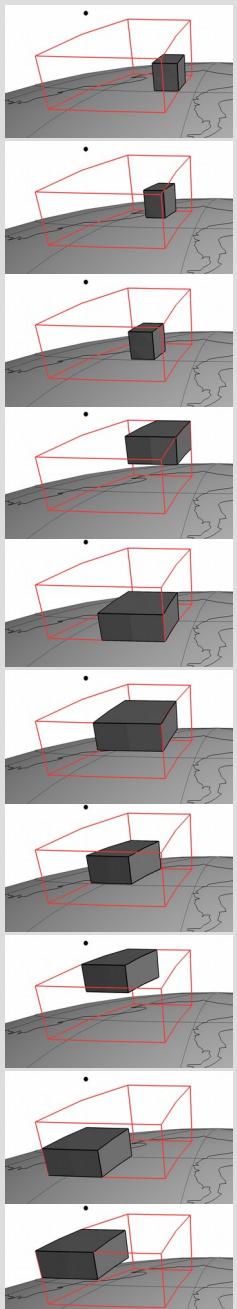
# pilha



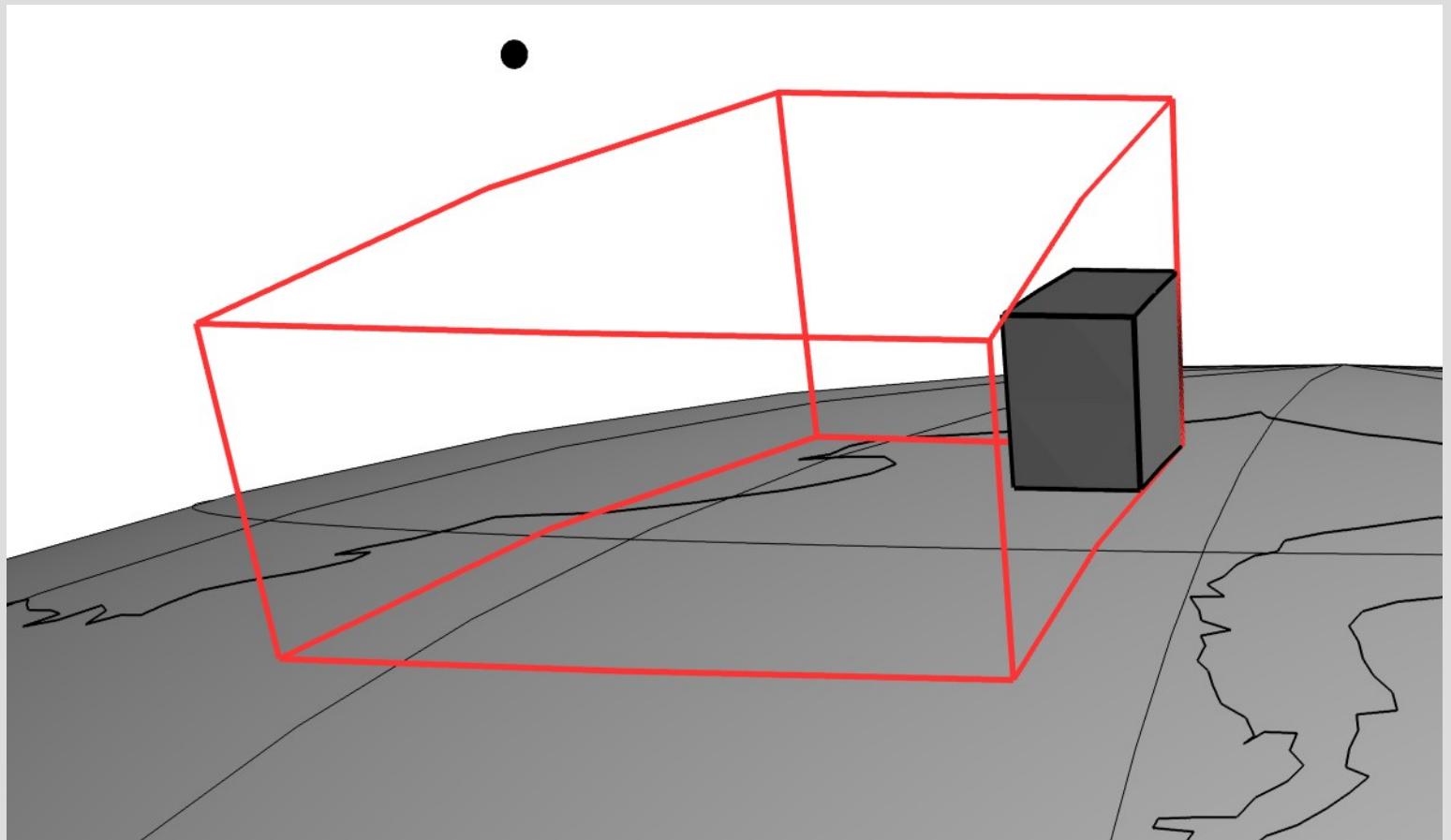
“pop”



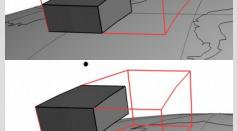
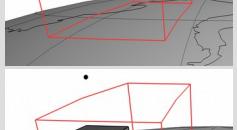
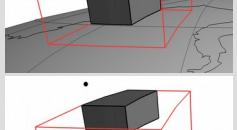
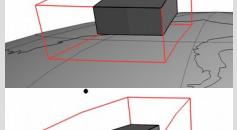
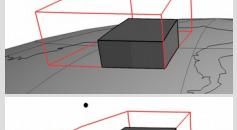
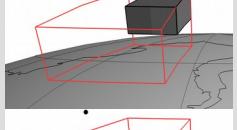
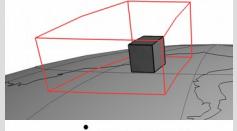
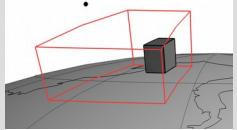
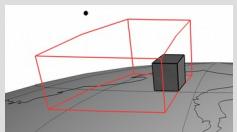
# pilha



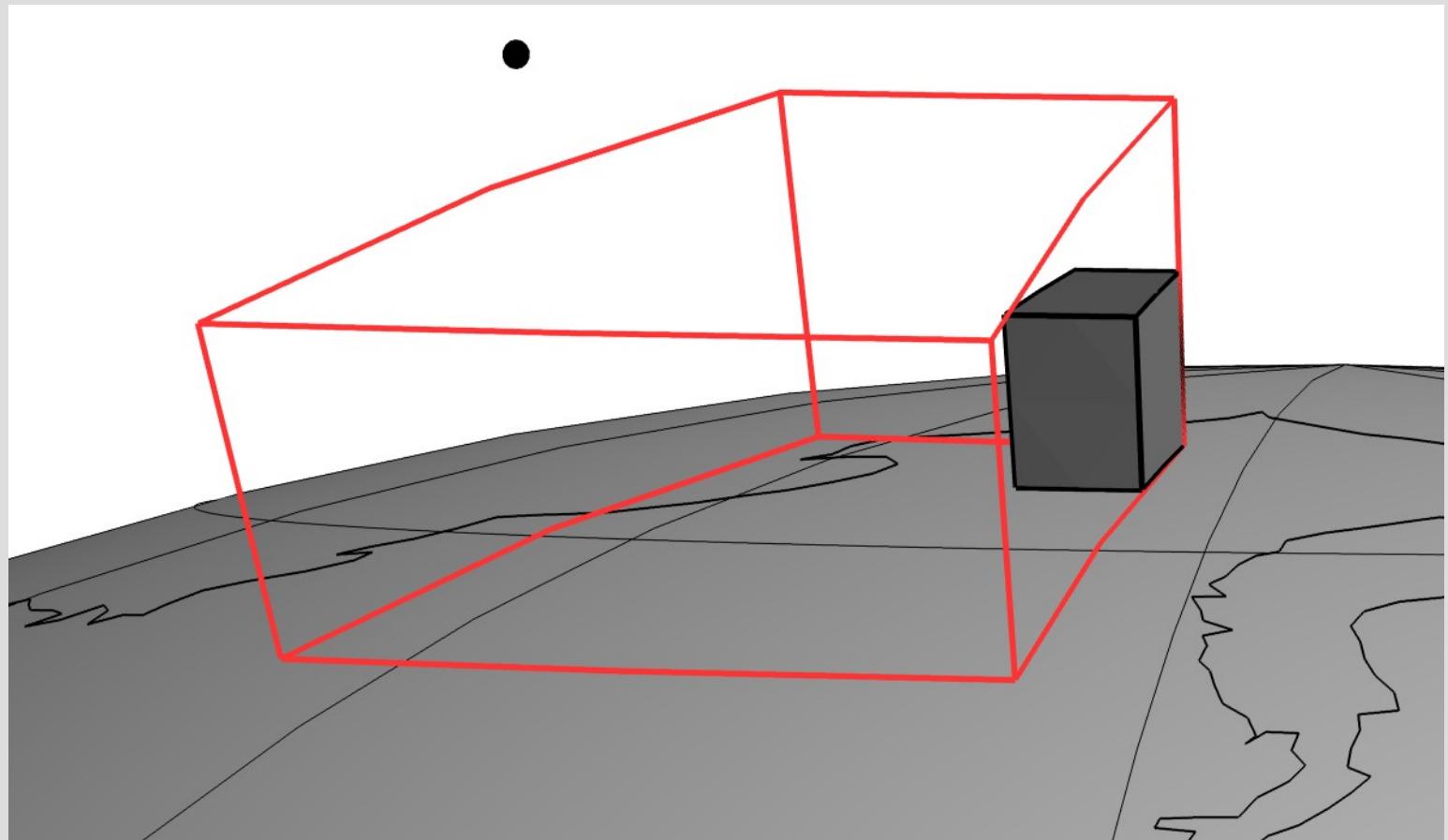
if  $d/L_x < D$ :  
Divide em x



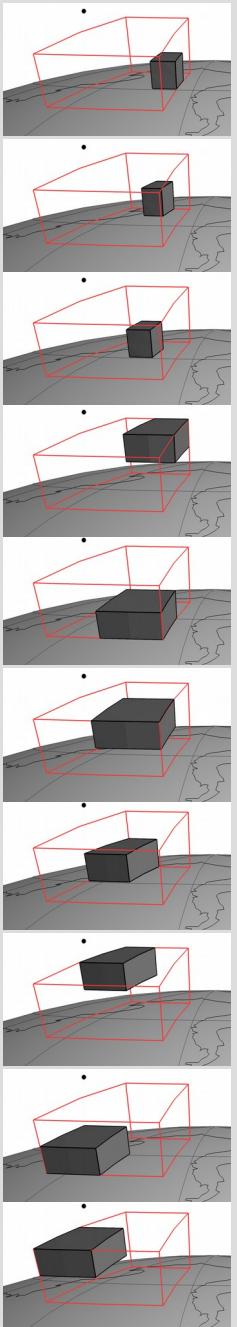
# pilha



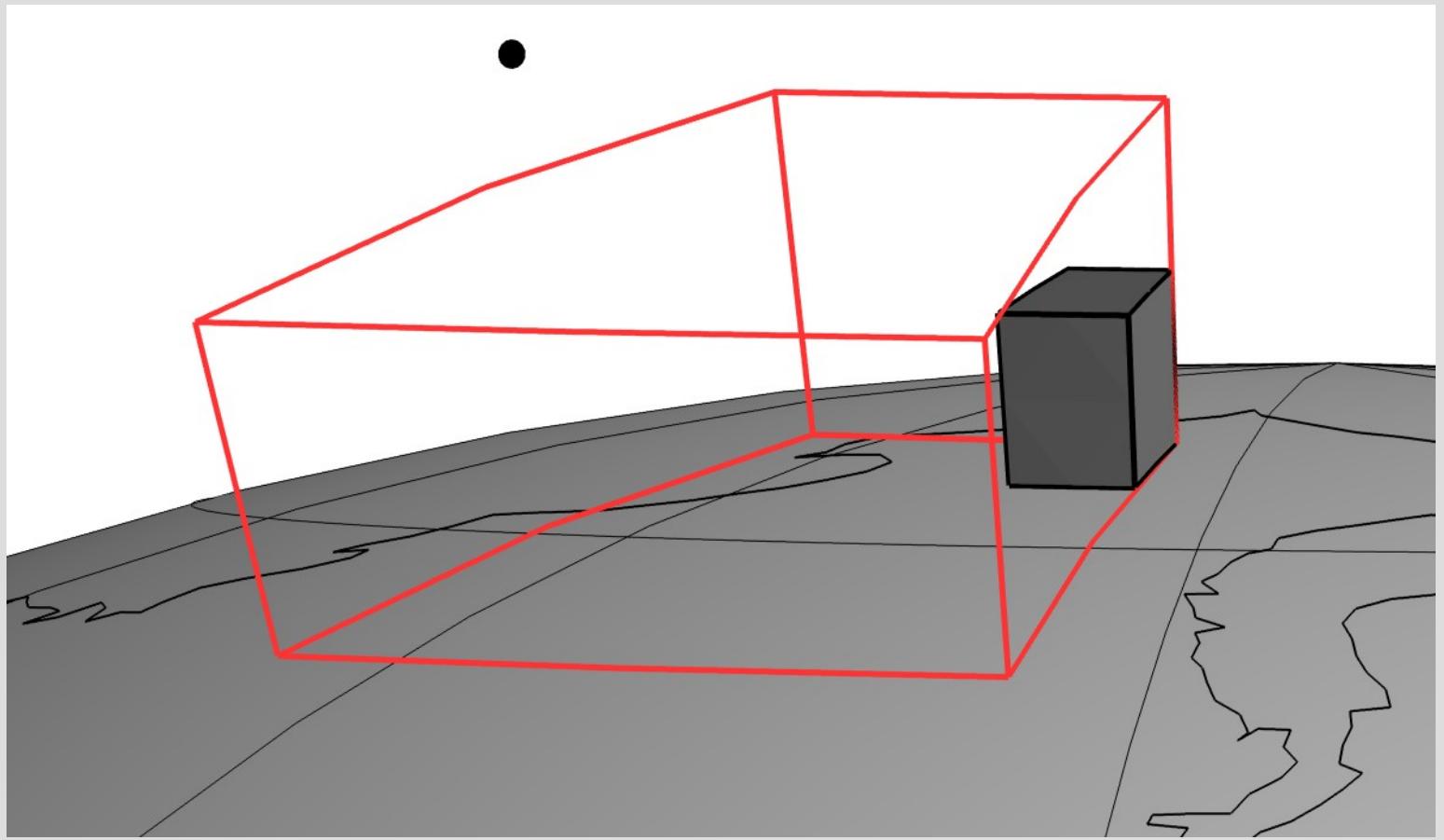
~~if  $d/x \leq D$ :  
Divide em x~~



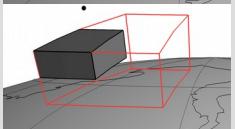
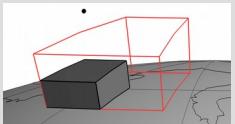
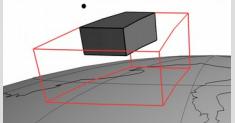
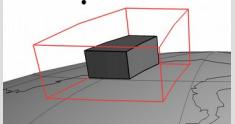
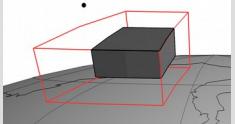
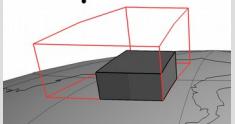
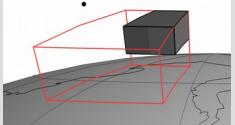
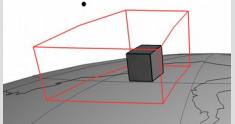
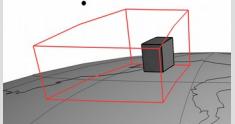
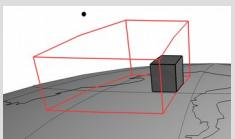
# pilha



$g_{\text{total}} += g_{\text{tesseroide}}$

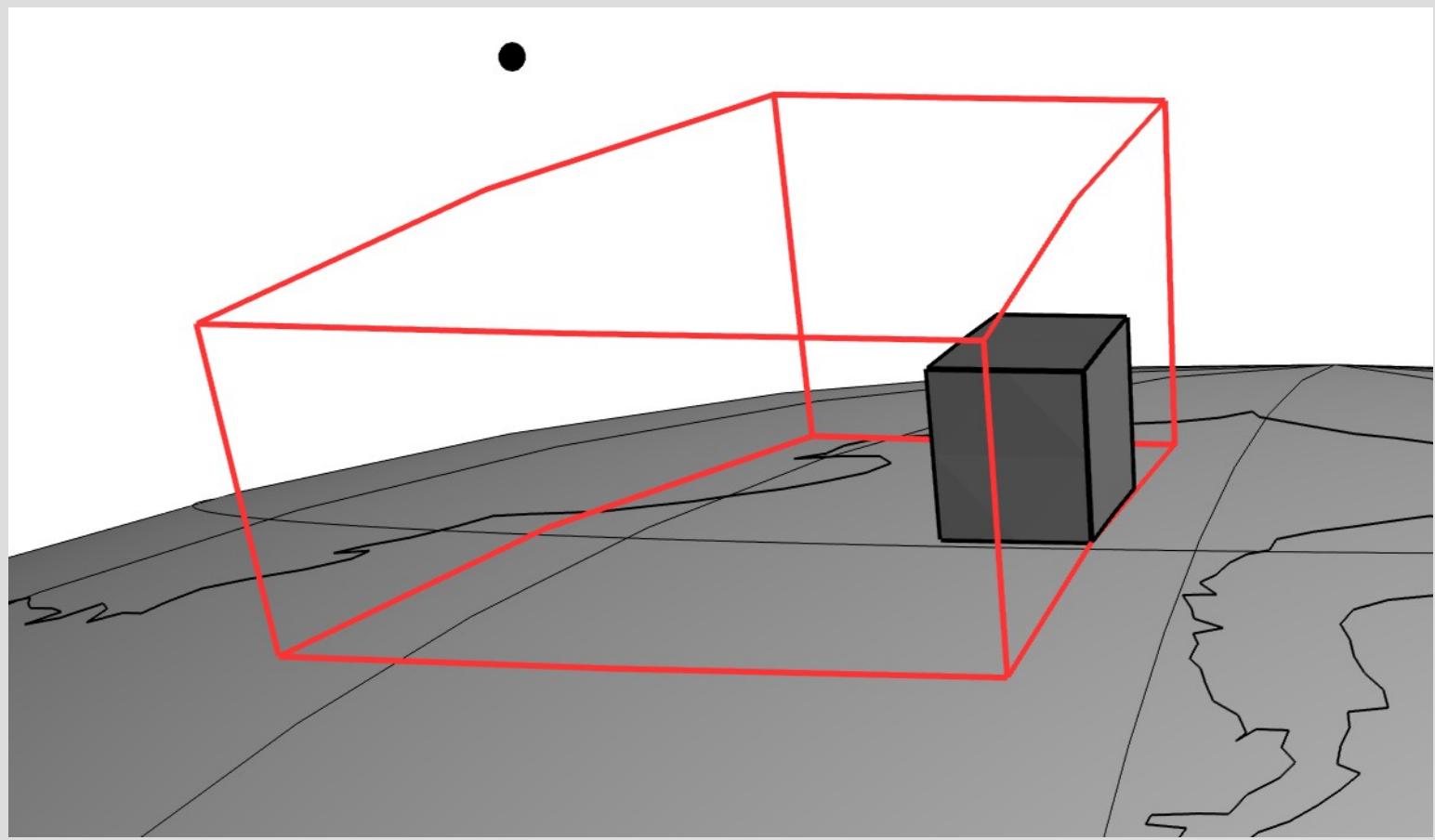
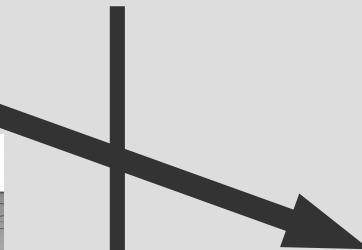
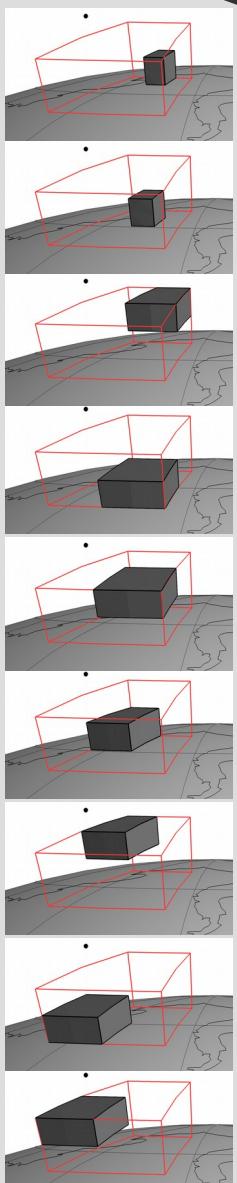


# pilha



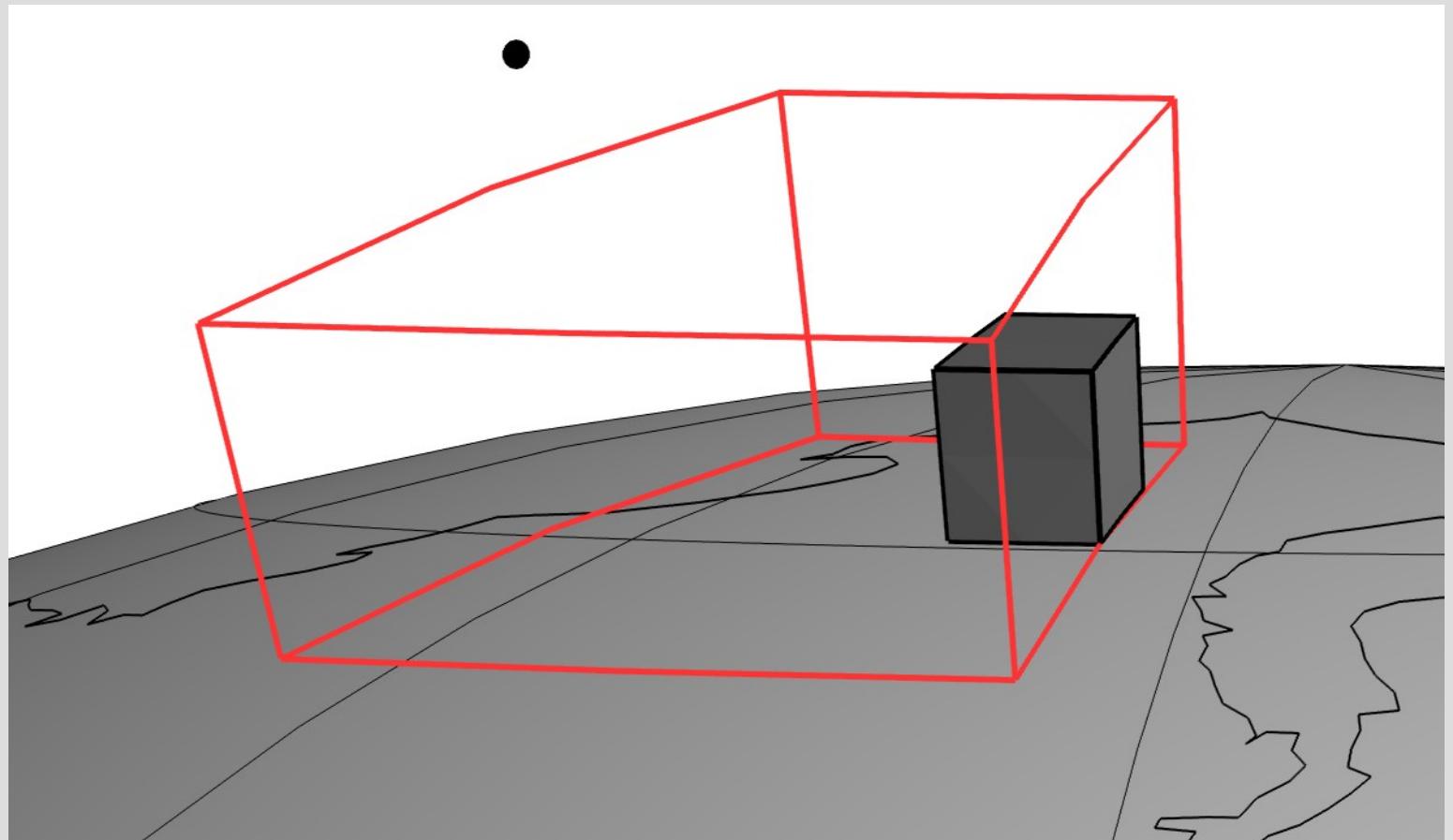
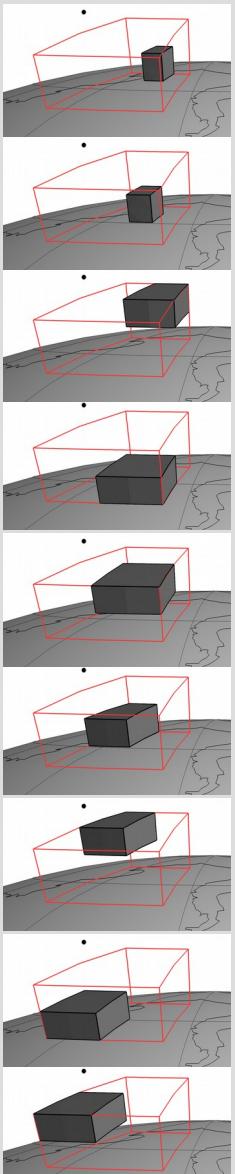
# pilha

“pop”

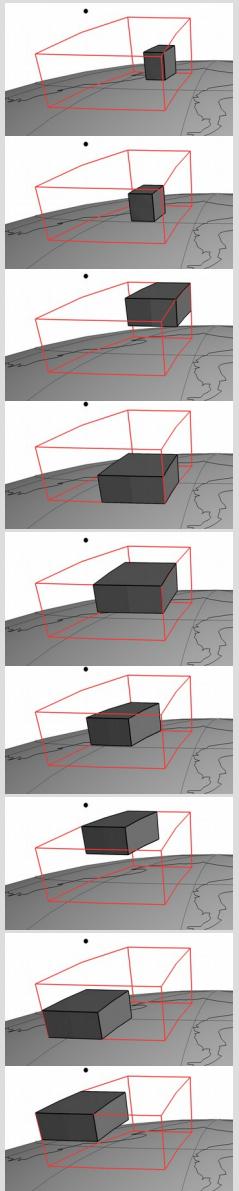


# pilha

if  $d/L_x < D$ :  
Divide em x



# pilha



pilha

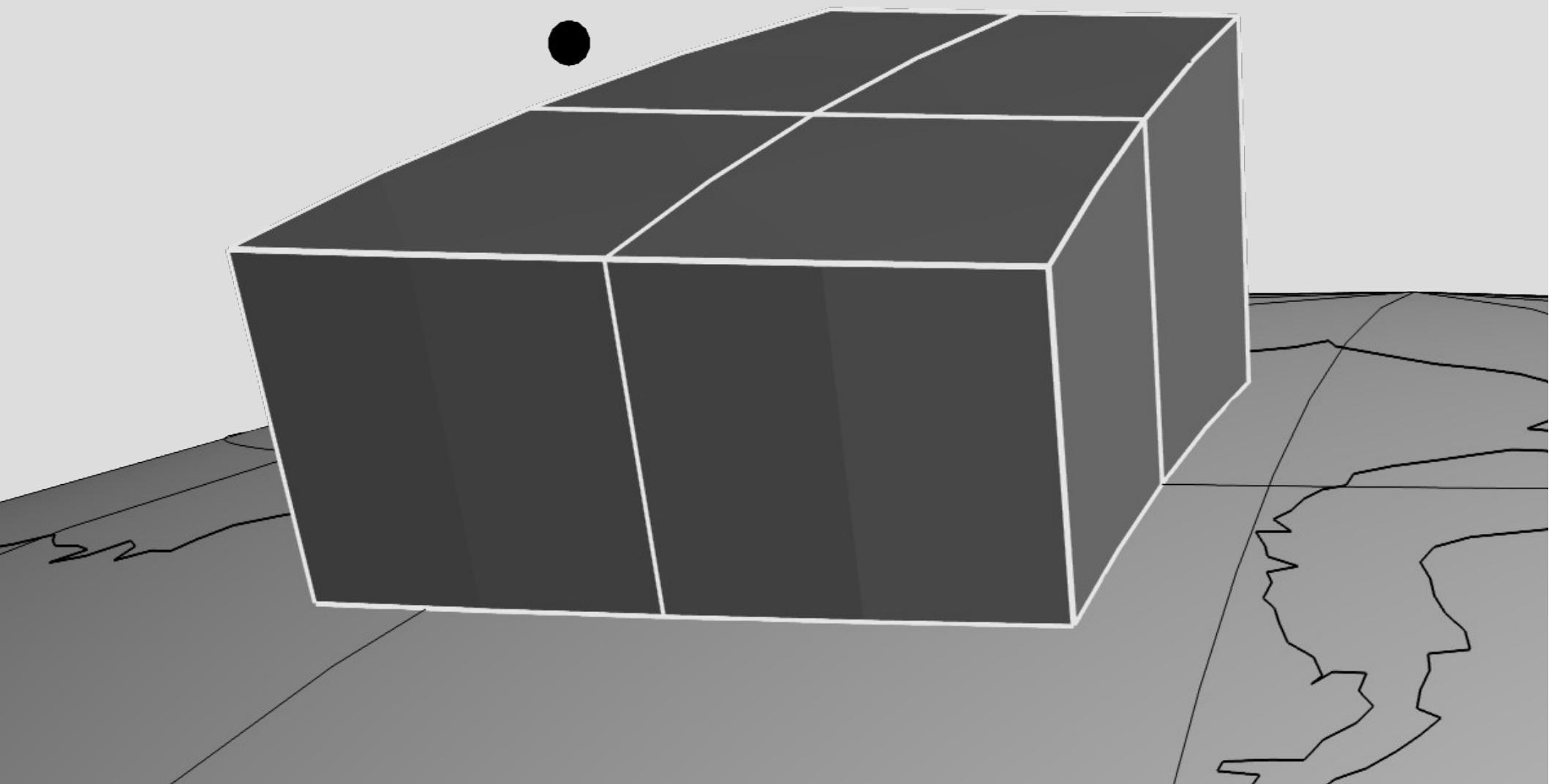
vazia

fim  
g total

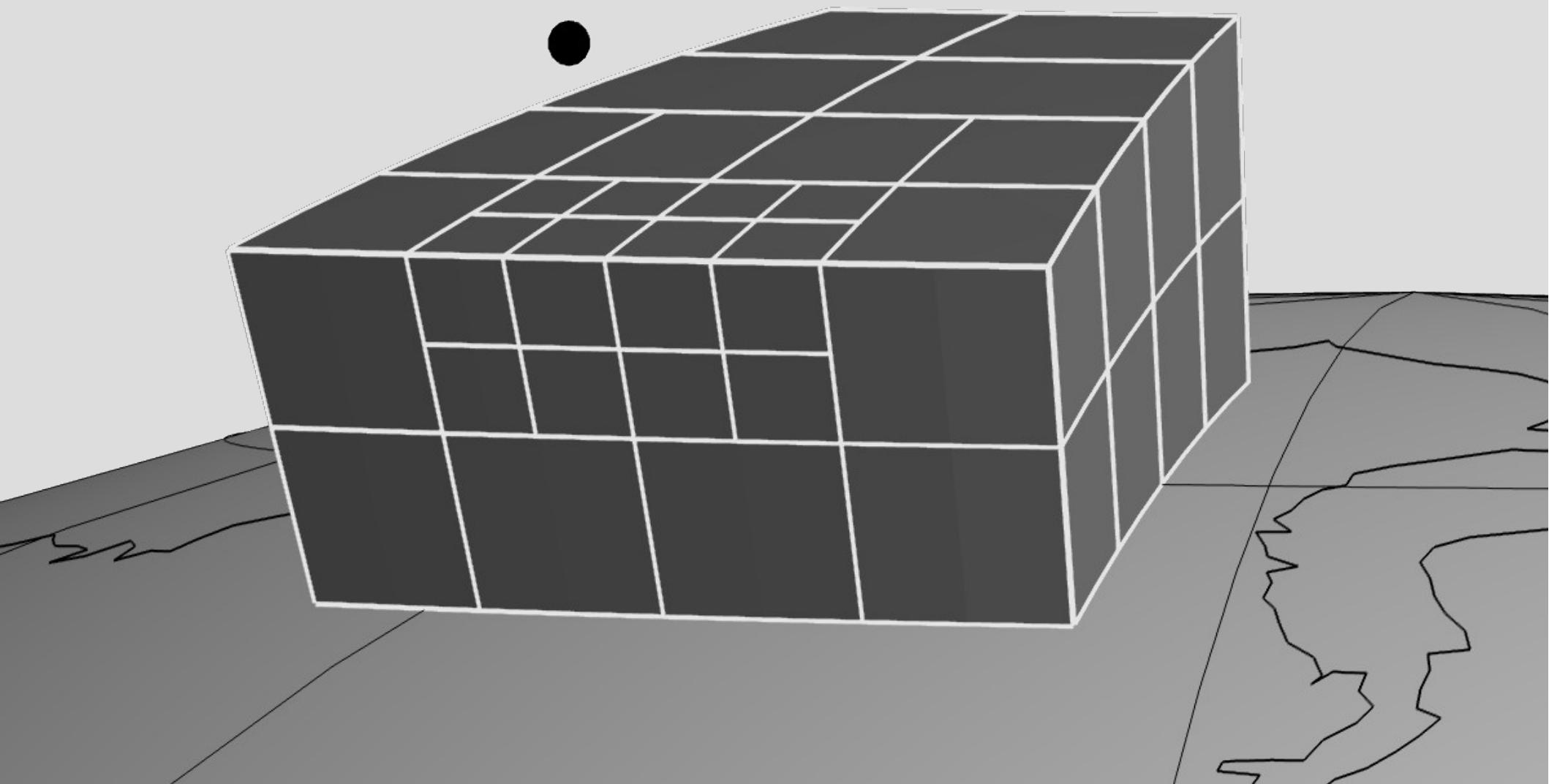
D

d/L<sub>x</sub> < D

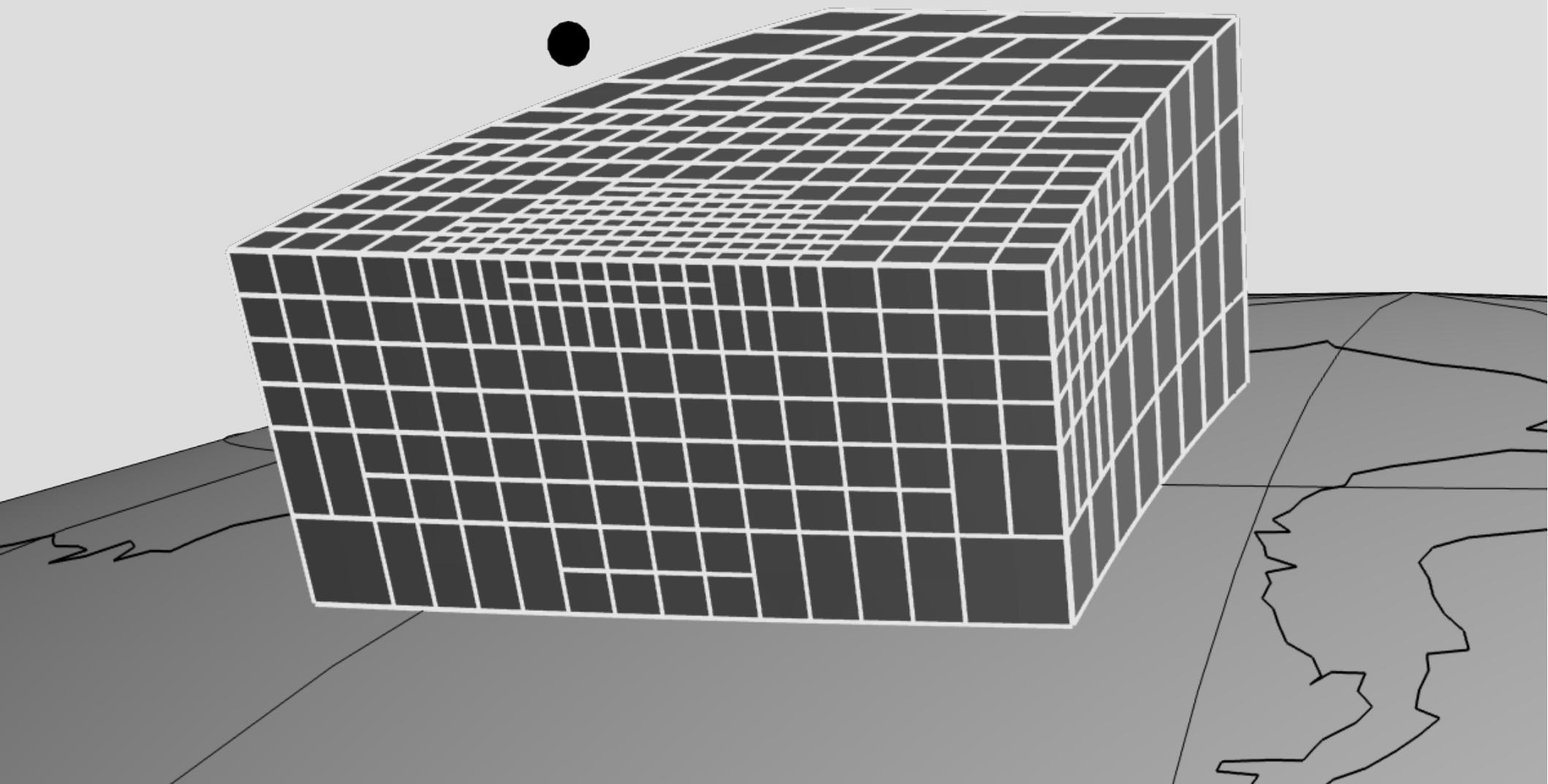
D=1 4 tesseroides



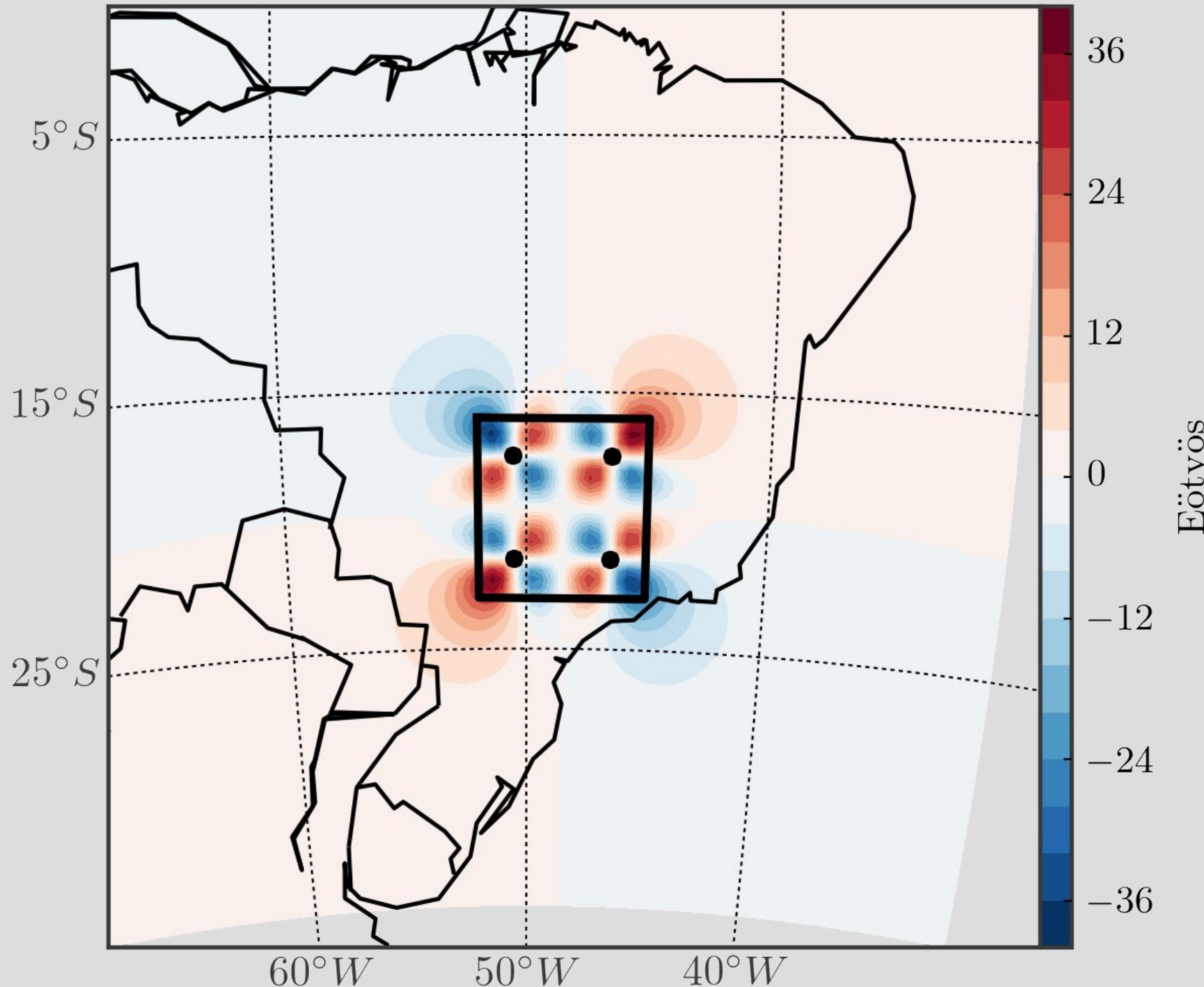
D=2 38 tesseroides



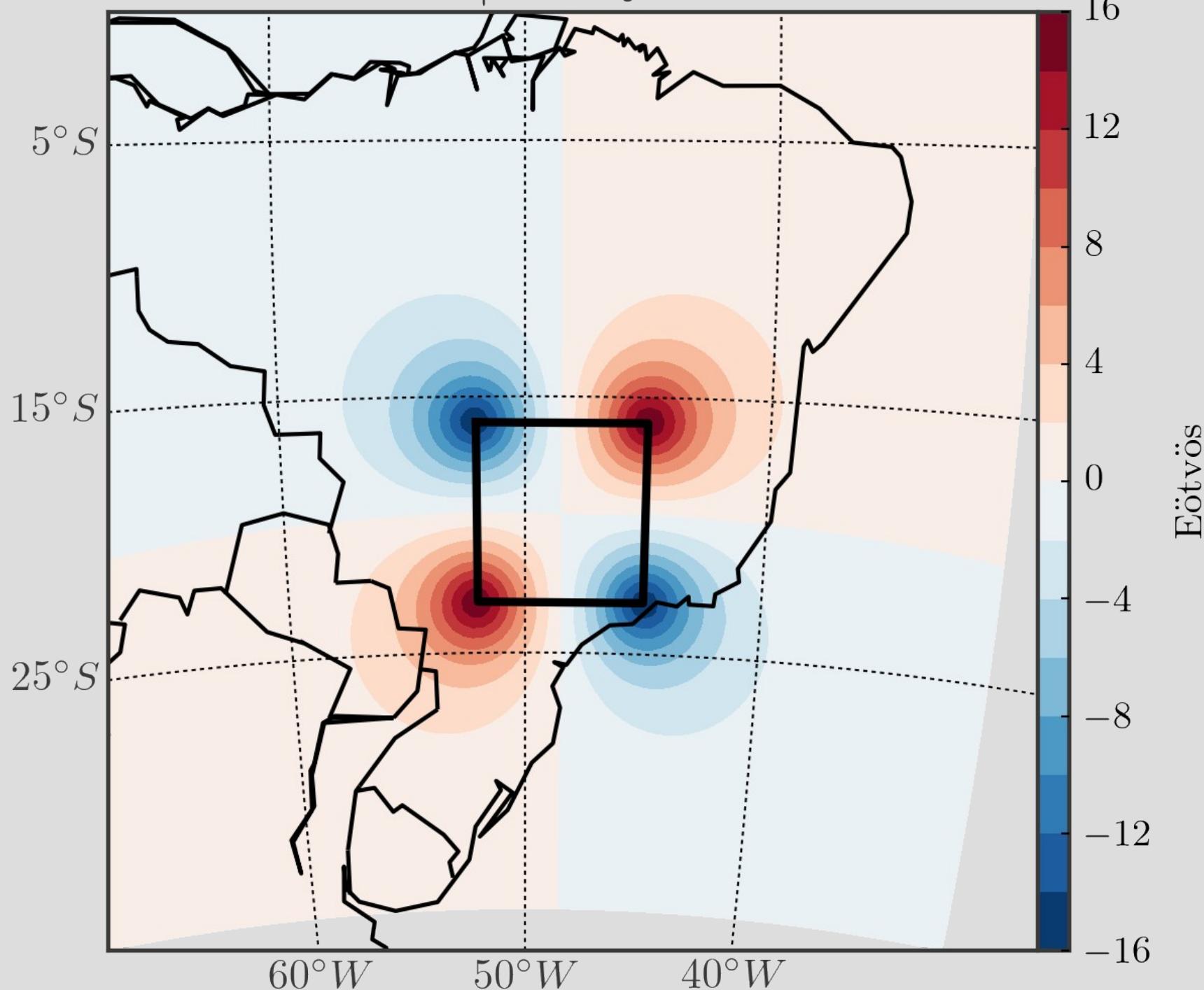
D=6 936 tesseroides



# 150 km | GLQ order 2



150 km | GLQ order 30

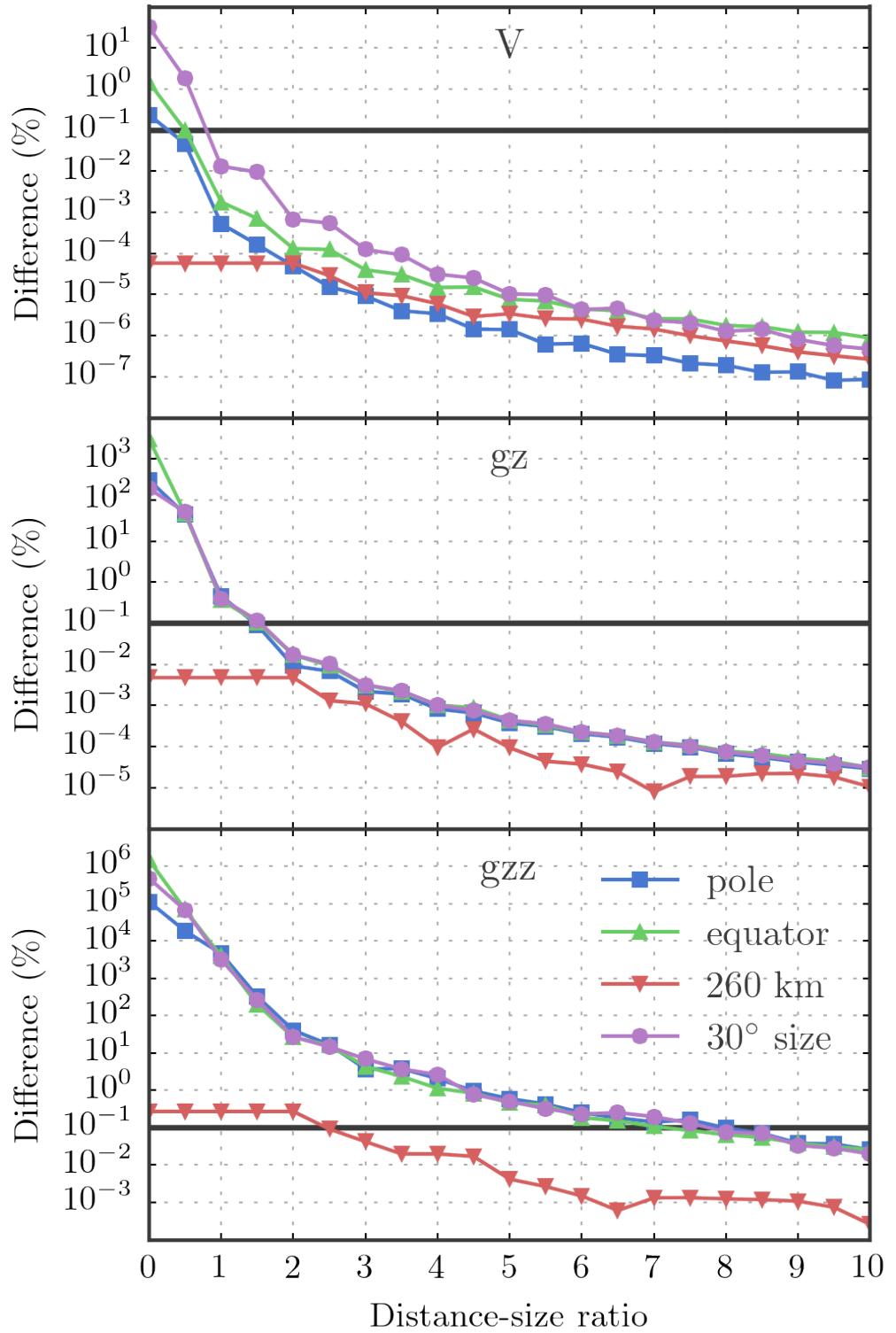


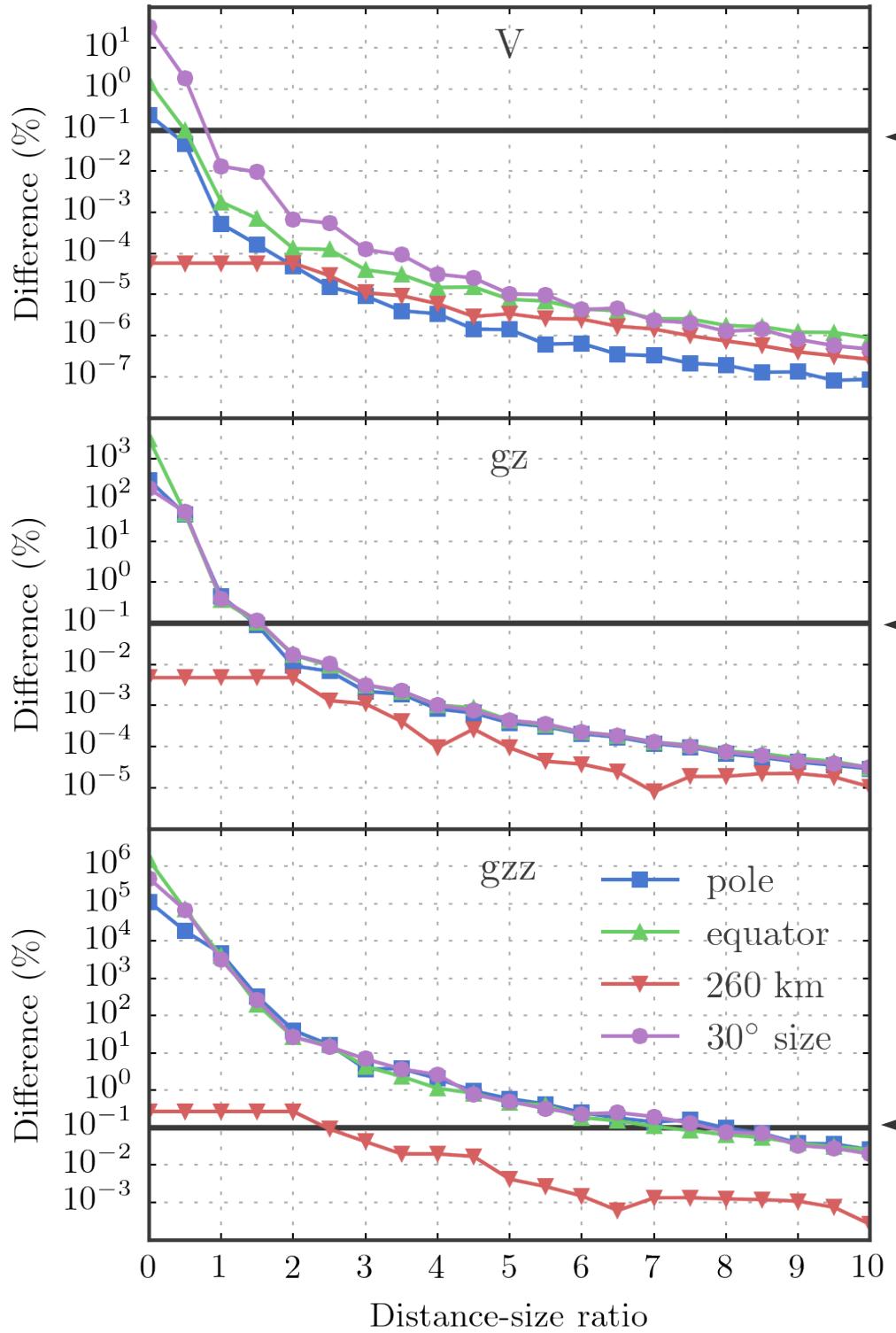
Melhor D?



# Casca x Quadratura

|            |                  |  |
|------------|------------------|--|
| 1. pólo    | $h=2\text{km}$   | $1^\circ \times 1^\circ \times 1\text{km}$   |
| 2. equador | $h=2\text{km}$   | $1^\circ \times 1^\circ \times 1\text{km}$   |
| 3. pólo    | $h=260\text{km}$ | $1^\circ \times 1^\circ \times 1\text{km}$   |
| 4. pólo    | $h=2\text{km}$   | $30^\circ \times 30^\circ \times 1\text{km}$ |

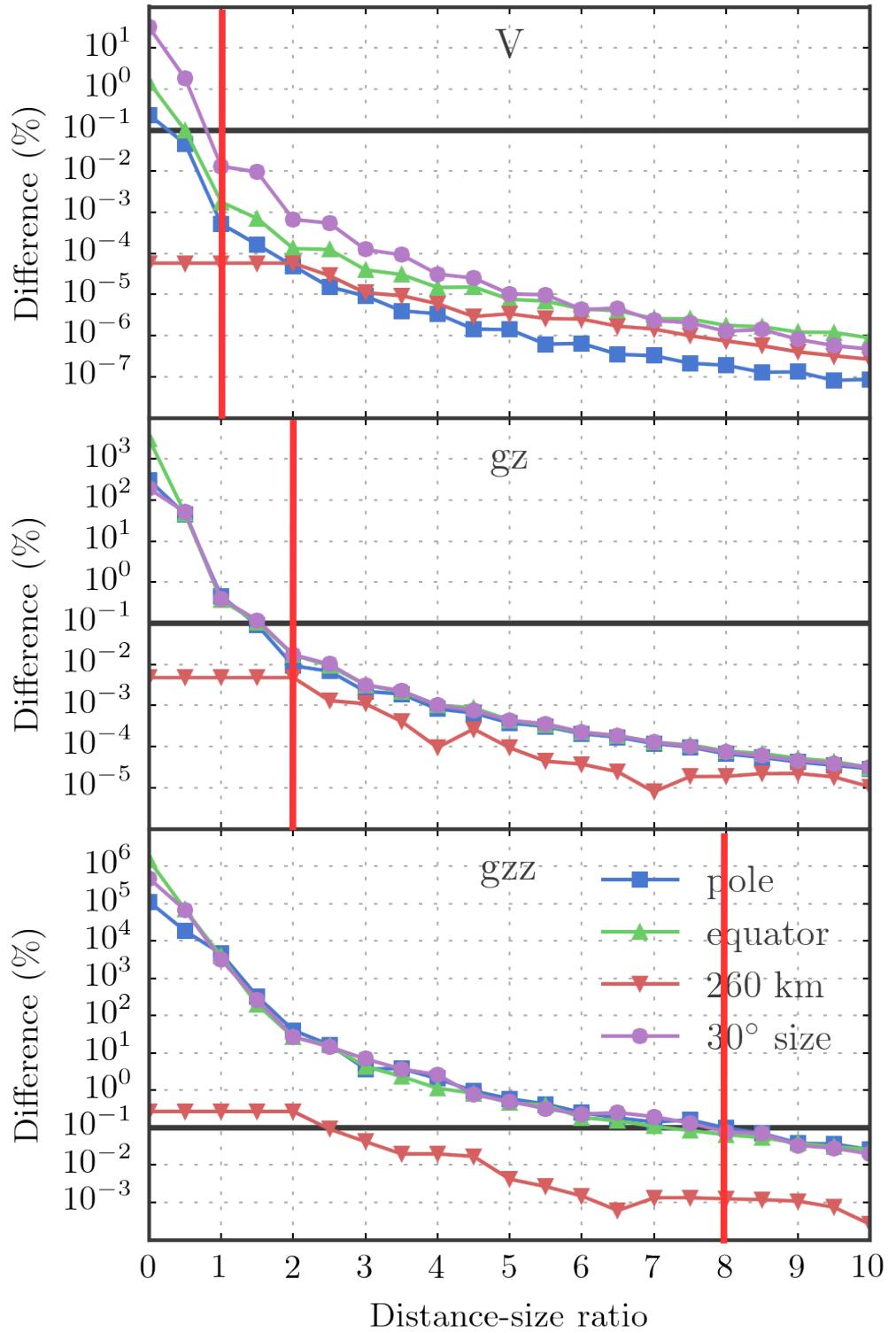




← 0.1% erro

← 0.1% erro

← 0.1% erro



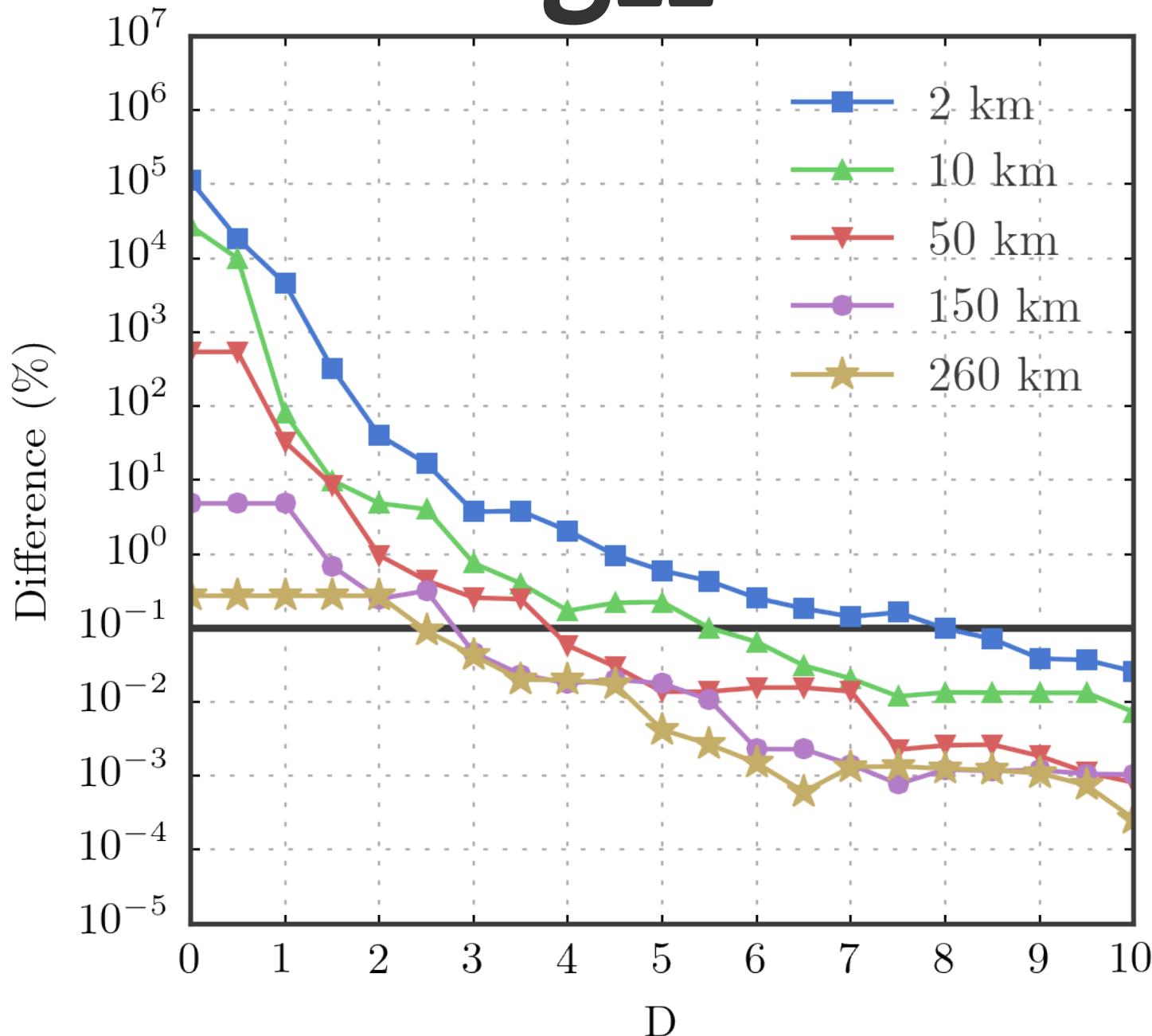
1

1.5

D

8

# $g_{zz}$



software



# Tesseroids

29 programas

linguagem C

open-source: BSD license

# linha de comando

```
leo@gauss:~$ tess
tess2prism    tessgxx      tessgyy      tesslayers
tessdefaults  tessgxy      tessgyz      tessmass
tessgrd       tessgxz      tessgz       tessmodgen
tessgx        tessgy       tessgzz      tesspot
leo@gauss:~$ tessgz -h
Usage: tessgz MODELFILE [OPTIONS]
```

Calculate the gz component due to a tesseroid model on specified observation points.

Values are calculated in the local coordinate system of the observation point: x-> North y-> East z-> Up (away from the center of the Earth).

In order to maintain mainstream convention, component gz is calculated with z-> Down.

All units either SI or degrees!



latest

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THE  
DOCS**

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## Tesseroids: forward modeling in spherical coordinates

# Tesseroids

A collection of command-line programs for modeling the gravitational potential, acceleration, and gradient tensor. *Tesseroids* supports models and computation grids in Cartesian and spherical coordinates.

Developed by [Leonardo Uieda](#) in cooperation with [Carla Braitenberg](#).

**Official site:** <http://tesseroids.leouieda.com>

**License:** BSD 3-clause

**Source code:** <https://github.com/leouieda/tesseroids>

**Latest release:** v1.2.0 (doi:[10.5281/zenodo.16033](https://doi.org/10.5281/zenodo.16033))

 **ⓘ Note**

*Tesseroids* is research software. Please consider [citing](#) it in your publications if you use it for your research.

 **ⓘ Warning**

[tesseroids.leouieda.com](http://tesseroids.leouieda.com)



latest

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## Tesseroids: forward modeling in spherical coordinates

# 34 citações (google scholar)

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**Warning**

[tesseroids.leouieda.com](http://tesseroids.leouieda.com)

# conclusão

# Discretização adaptativa: algoritmo melhor definido      pilha x recursivo

Discretização adaptativa:  
algoritmo melhor definido      pilha x recursivo

Quantificação do erro:  
D ótimo      Ds diferentes: V, gz, gzz

Discretização adaptativa:  
algoritmo melhor definido      pilha x recursivo

Quantificação do erro:

D ótimo      Ds diferentes: V, gz, gzz

Implementação:

open-source      usuários      desenvolvedores

 Tesseroids

modelagem direta  
aprox. esférica

programa B

Novo método

Introdução

Tesseroids

Fatiando a Terra

Inversão Moho

Conclusão



**fatiando a terra**  
modelagem direta e inversa para geofísica



# SciPy 2013

“Modeling the Earth with Fatiando a Terra”



# SciPy 2013

“Modeling the Earth with Fatiando a Terra”

v0.1



# SciPy 2013

“Modeling the Earth with Fatiando a Terra”

v0.1

YouTube 





# fatiando a terra

An open-source Python library for modeling and inversion in geophysics.

Our goal is provide a comprehensive and extensible framework for geophysical data analysis and the development of new methodologies.

**Research:** Fatiando allows you to write Python scripts to perform your data analysis and generate figures in a reproducible way.

**Development:** Designed for extensibility, Fatiando offers tools for users to build upon the existing infrastructure and develop new inversion methods. We take care of the boilerplate.

**Teaching:** Fatiando can be combined with the [Jupyter notebook](#) to make rich, interactive documents. Great for teaching fundamental concepts of geophysics.

## Overview

### Gravity and magnetics

Modeling, inversion, and processing for potential field methods.

*3D forward modeling with prisms, polygonal prisms, spheres, and tesseroids. Handles the potential, acceleration, gradient tensor, magnetic induction, total field magnetic anomaly.*

### Seismology and Seismics

Simple modeling functions for

*Toy problems for: Cartesian site estimation. Experimental finite*

**fatiando.org**



# fatiando a terra

An open-source Python library for modeling and inversion in geophysics

Our goal is provide

development of new

# ~950 downloads/mês

# 7 citações

## Overview

### Gravity and magnetics

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*3D forward modeling with prisms, polygonal prisms, spheres, and tesseroids. Handles the potential, acceleration, gradient tensor, magnetic induction, total field magnetic anomaly.*

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Simple modeling functions for

*Toy problems for: Cartesian sti  
estimation. Experimental finite*

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**Editor's summary**

Dan Bassett *et al.* use residual gravity anomalies to characterize structures in the region of the Tohoku megathrust earthquake of March 2011. Their results point to a ...

**NATURE | LETTER****日本語要約**

# Upper-plate controls on co-seismic slip in the 2011 magnitude 9.0 Tohoku-oki earthquake

**Dan Bassett, David T. Sandwell, Yuri Fialko & Anthony B. Watts****Affiliations | Contributions | Corresponding author***Nature* **531**, 92–96 (03 March 2016) | doi:10.1038/nature16945

Received 26 June 2015 | Accepted 11 December 2015 | Published online 02 March 2016



Full text



PDF



Citation



Reprints



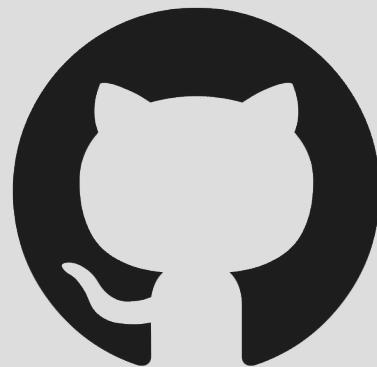
Rights &amp; permissions



Article metrics

The March 2011 Tohoku-oki earthquake was only the second giant (moment magnitude  $M_w \geq 9.0$ ) earthquake to occur in the last 50 years and is the most recent to be recorded using modern geophysical techniques. Available data place high-resolution constraints on the kinematics of earthquake rupture<sup>1</sup>, which have challenged prior knowledge about how much a fault can slip in a

# código github.com



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fatiando / fatiando

Code Issues 87 Pull requests 16 Wiki Pulse Graphs Settings

Python toolkit for modelling and inversion in geophysics <http://www.fatiando.org> — Edit

2,021 commits 16 branches 5 releases 10 contributors

Branch: master New pull request New file Upload files Find file HTTPS https://github.com/f Download ZIP

leouleda Use OSX in TravisCI (#268) ... Latest commit e1b46e2 5 days ago

github Create PULL\_REQUEST\_TEMPLATE.md 10 days ago

benchmarks Merge branch 'master' into tesseroid-kernels a year ago

d-tools Use OSX in TravisCI (#268) 5 days ago

cookbook Final PEP8 adjustments. 3 months ago

doc Put logo on top of name in front page 11 days ago

fatiando Fix broken test in fatiando.inversion doctests 11 days ago

test Final adjustment in reflectivity function to support 1D and 2D calcul... 3 months ago

.coveragerc Remove interactive from coverage report a year ago

.gitattributes Added versioneer support 2 years ago

.gitignore Ignore packaging files 12 days ago

.travis.yml Use OSX in TravisCI (#268) 5 days ago

CITATION.rst Update citation information to cite the Sopy proceedings 4 months ago

LICENSE.txt Updated year in license text 2 years ago

MANIFEST.in Rename readme extension to rst a year ago

Makefile Makefile rule to build a conda env 6 months ago

README.rst Change links to crate.io to pypl 12 days ago

requirements.txt Replace the Imaging package with pillow 4 months ago

setup.py Removed OMP prange from gravmag forward modeling a year ago

versioneer.py Added versioneer support 2 years ago

README.rst

The fatiando logo, featuring a stylized globe composed of blue and green leaf-like shapes.

## fatiando a terra

[Website](#) | [Docs](#) | [Mailing list](#)

An open-source Python library for modelling and inversion in geophysics.

pypi v0.4 download 93.8m monthly build passing coverage 20% health 88% doi 10.5281/zenodo.49087 gitter JOIN CHAT

# Implement tilt filter #261



mtb-za wants to merge 7 commits into `fatiando:master` from `mtb-za:tilt_filter`

Conversation 25

Commits 7

Files changed 3



mtb-za commented 25 days ago

Fatiando a Terra member



This will create a filter to determine the tilt of a potential field, as developed by Miller and Singh (1994).

I could use some suggestions on how to create a sensible test for this.

This is the first of a few potential field filters that I want to implement.

## Checklist:

- Make tests for new code
- Create/update docstrings
- Include relevant equations and citations in docstrings
- Code follows PEP8 style conventions
- Code and docs have been spellchecked
- Include new dependencies in docs, requirements.txt, README, and .travis.yml
- Documentation builds properly
- All tests pass

# Implement tilt filter #261

 Open mtb-za wants to merge 7 commits into `fatiando:master` from `mtb-za:tilt_filter`

 Conversation 25  Commits 7  Files changed 3



mtb-za commented 25 days ago

Fatiando a Terra member

+ 

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- Documentation builds properly
- All tests pass

# Pad arrays #239

 Open drandykass wants to merge 22 commits into `fatiando:master` from `drandykass:pad_arrays`

 Conversation 67  Commits 22  Files changed 5



drandykass commented on Dec 2, 2015

Fatiando a Terra member

+ 

This is an incomplete pull request for the addition of array padding and unpadding routines to gridded.

Some talking points:

- The padding function expects the array to be of the same dimension as intended--that is, no 'flattened' arrays.
- The unpadding function currently is set up to return unpadded coordinate vectors. However, I think that may be completely unnecessary.
- Still working on cleaning up the docs, the actual code, and need to create the test functions.

Update 03-12-2015 (@leouieda): Added the checklist below.

## Checklist:

- Make tests for new code
- Create/update docstrings
- Include relevant equations and citations in docstrings

## Implement tilt filter #261



mtb-za wants to merge 7 commits into `fatiando:master` from `mtb-za:tilt_filter`

Conversation 25

Commits 7

Files changed 3



mtb-za commented 25 days ago

Fatiando a Terra member



This will create a filter to determine the tilt of a potential field, as developed by Miller and Singh (1994).

I could use some suggestions on how to create a sensible test for this.

This is the first of a few potential field filters that I want to implement.

### Checklist:

- Make tests for new code
- Create/update docstrings
- Include relevant equations and citations in docstrings
- Code follows PEP8 style conventions
- Code and docs have been spellchecked
- Include new dependencies in docs, requirements.txt, README, and .travis.yml
- Documentation builds properly
- All tests pass

## Pad arrays #239



drandykass wants to merge 22 commits into `fatiando:master` from `drandykass:pad_arrays`

Conversation 67

Commits 22

Files changed 5



drandykass commented on Dec 2, 2015

Fatiando a Terra member



This is an incomplete pull request for the addition of array padding and unpadding routines to gridded.

Some talking points:

- The padding function expects the array to be of the same dimension as intended--that is, no 'flattened' arrays.
- The unpadding function currently is set up to return unpadded coordinate vectors. However, I think that may be completely unnecessary.
- Still working on cleaning up the docs, the actual code, and need to create the test functions.

Update 03-12-2015 (@leouieda): Added the checklist below.

### Checklist:

- Make tests for new code
- Create/update docstrings
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## Radial Average of Power Spectrum #230



santis19 wants to merge 2 commits into `fatiando:master` from `santis19:fft-radial-average`

Conversation 7

Commits 2

Files changed 2



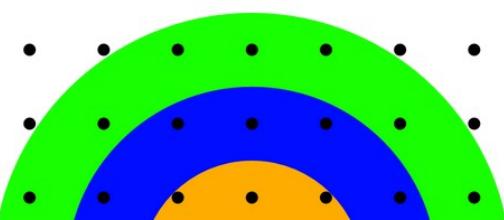
santis19 commented on Aug 11, 2015



I've made a first try to implement the Radial Average of a Power Spectrum (or any real fft function) proposed in #224.

It needs a Power Spectrum in a square grid with odd number of points per axis in order to make sure that there is a grid point in the zero wave number.

The radial average is made by concentric rings using a grid of indexes in order to avoid troubles with frequencies. The rings have an inner radius equal to  $i - 1/2$  and outer radius  $i + 1/2$ , where  $i$  is an integer index ranging from 0 to half the grid's number of points per axis. The first ring ( $i=0$ ) is a circle containing only the zero frequency point. The next is a ring surrounding this circle, and so on.



## Implement tilt filter #261

**Open** mtb-za wants to merge 7 commits into `fatiando:master` from `mtb-za:tilt_filter`

Conversation 25 Commits 7 Files changed 3



mtb-za commented 25 days ago

Fatiando a Terra member

+

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I could use some suggestions on how to create a sensible test for this.

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### Checklist:

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## Pad arrays #239

**Open** drandykass wants to merge 22 commits into `fatiando:master` from `drandykass:pad_arrays`

Conversation 67 Commits 22 Files changed 5



drandykass commented on Dec 2, 2015

Fatiando a Terra member

+

This is an incomplete pull request for the addition of array padding and unpadding routines to gridded.

Some talking points:

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Update 03-12-2015 (@leouieda): Added the checklist below.

### Checklist:

- Make tests for new code
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## Radial Average of Power Spectrum #230

**Open** santis19 wants to merge 2 commits into `fatiando:master` from `santis19:fft-radial-average`

Conversation 7 Commits 2 Files changed 2



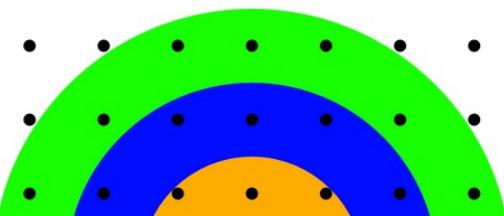
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## Convolutional model for seismic data (using a depth mode) #190

**Merged** leouieda merged 65 commits into `fatiando:master` from `victortxa:convolutional_model` on Feb 2

Conversation 77 Commits 65 Files changed 9



victortxa commented on Mar 30, 2015

Fatiando a Terra member

+

Generate a convolutional model from a depth geological model ( $V_p$ ; density optional). It's useful to put direct in geology instead of a model in time, avoiding lose the main information. It can be helpful in situations when it's boring to convert from depth to time.

### Checklist:

- Make tests for new code
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24 commits

16 branches

5 releases

10 contributors

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New pull request

New file

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<https://github.com/fatiando/fatiando>

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leouieda Restructured the repos. Made fatiando python package with directmods,...

Latest commit 928515b on Apr 30, 2010

... math, utils and geoinv. Includes direct models for prism gravity, sclimate signal from heat well log, and simple cartesian tomography. Geoinv has the inversion program for the single perturbation climate signal, and simple tomography (including using an image as a model).

--HG--

extra : convert\_revision : svn%3A2c9857fa-f4c4-11dd-ada4-5153b8187bf2/trunk%4037

|            |  |             |
|------------|--|-------------|
| c          | Restructured the repos. Made fatiando python package with directmods,... | 6 years ago |
| fatiando   | Restructured the repos. Made fatiando python package with directmods,... | 6 years ago |
| old        | Moved the old src of the project to trunk/old. Don't think it'll be u... | 6 years ago |
| SConstruct | Restructured the repos. Made fatiando python package with directmods,... | 6 years ago |
| test.py    | Restructured the repos. Made fatiando python package with directmods,... | 6 years ago |





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24 commits

## Made fatiando python package

10 contributors

Tree: 928515... ▾

New pull request



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|  |  |             |
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fatiando

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fatiando

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6 years ago

test.py

Restructured the repos. Made fatiando python package with directmods,...

6 years ago

Apr 30, 2010

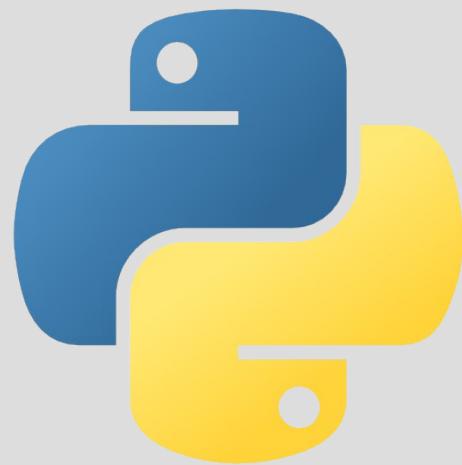
juntar código

disciplinas, teses

dissertações, etc

# biblioteca

funções, classes, etc



TM

python

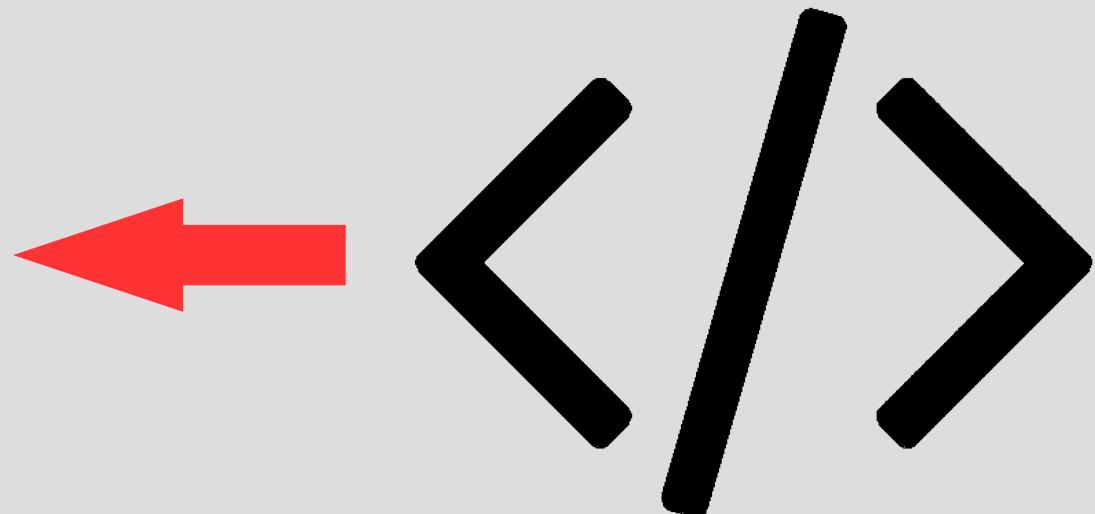
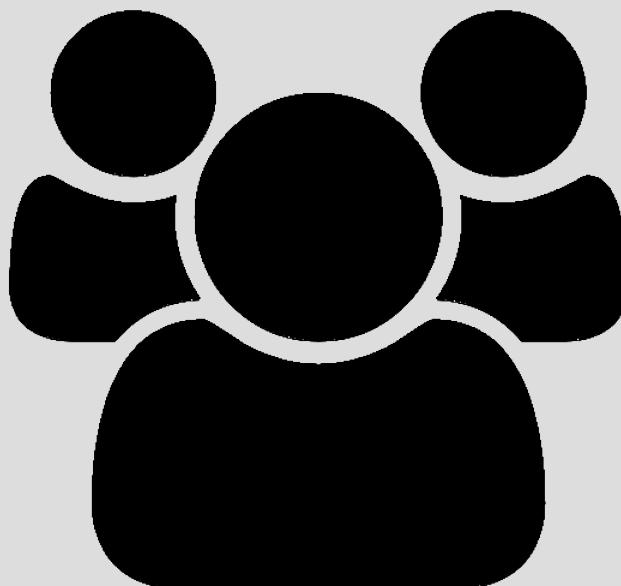
fácil de aprender

rápido de implementar

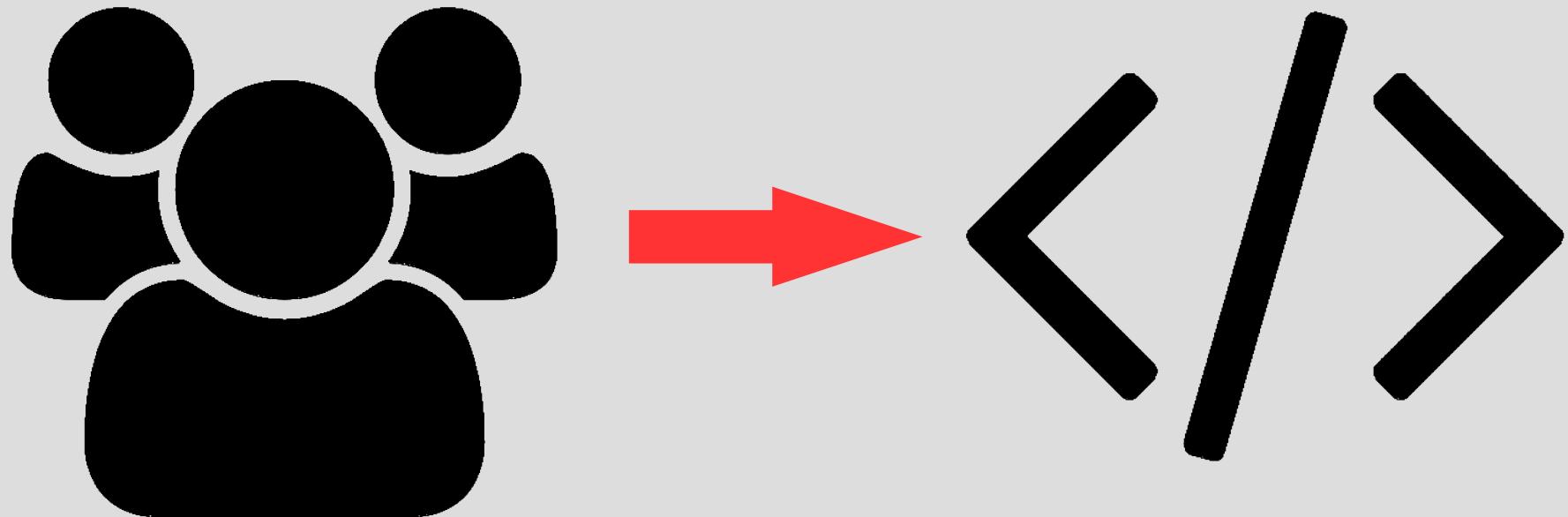
open-source

“batteries included”

1 código  
++ usuários



1 código  
++ programadores



a biblioteca

fatiando.griddeder  
mesher  
utils  
vis  
gravmag  
seismic  
inversion

## Módulos

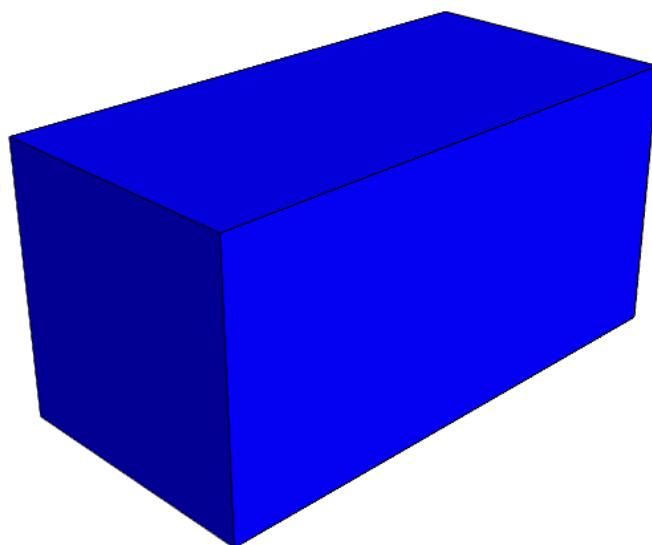
fatiando.

griddeder  
mesher  
utils  
vis  
**gravmag**  
seismic  
inversion

Módulos

```
In: from fatiando.meshes import Prism
modelo = [
    Prism(0, 1000, 0, 2000, 1500, 2500,
          props={'magnetization': [4,-1,3]})]
from fatiando.vis import myv
myv.figure()
myv.prisms(modelo)
myv.show()
```

Out:



In: `from fatiando import gridder`

  
`area = (-5000, 5000, -5000, 5000)`  
`shape = (50, 50)`  
`x, y, z = gridder.regular(area, shape, z=0)`  
  
`print(x)`  
`print(x.size)`

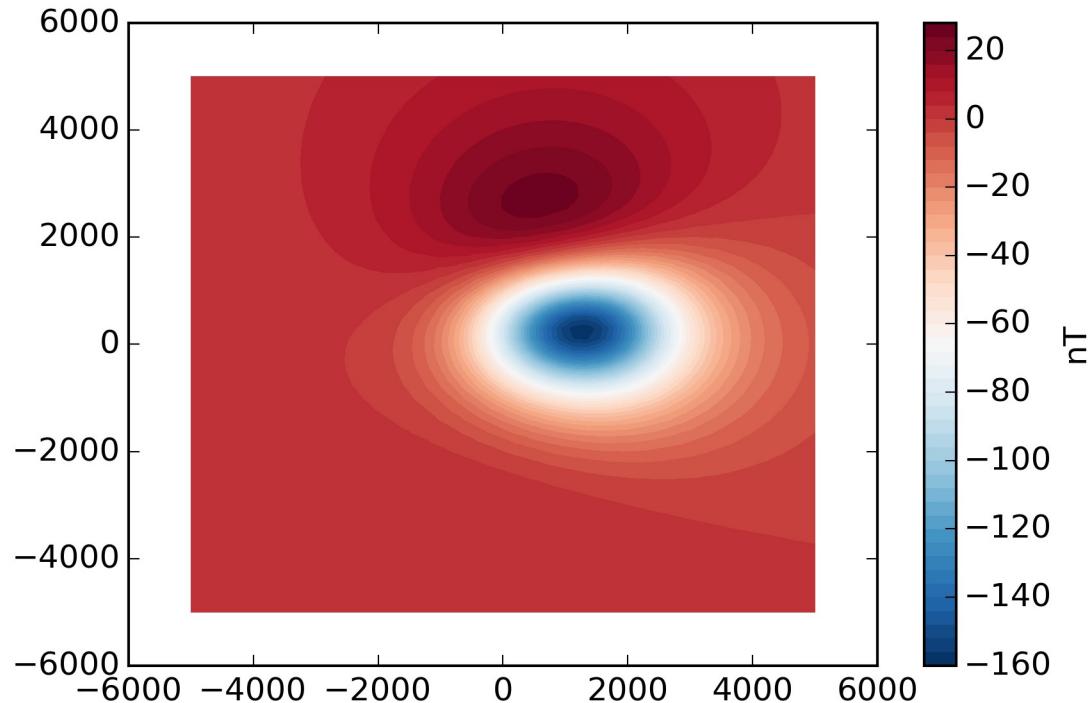
Out: `[-5000. -5000. -5000. ... 5000. 5000. 5000.]`  
`2500`

In:

```
from fatiando.gravmag import prism
mag = prism.tf(x, y, z, modelo,
                 inc=-60, dec=20)
```

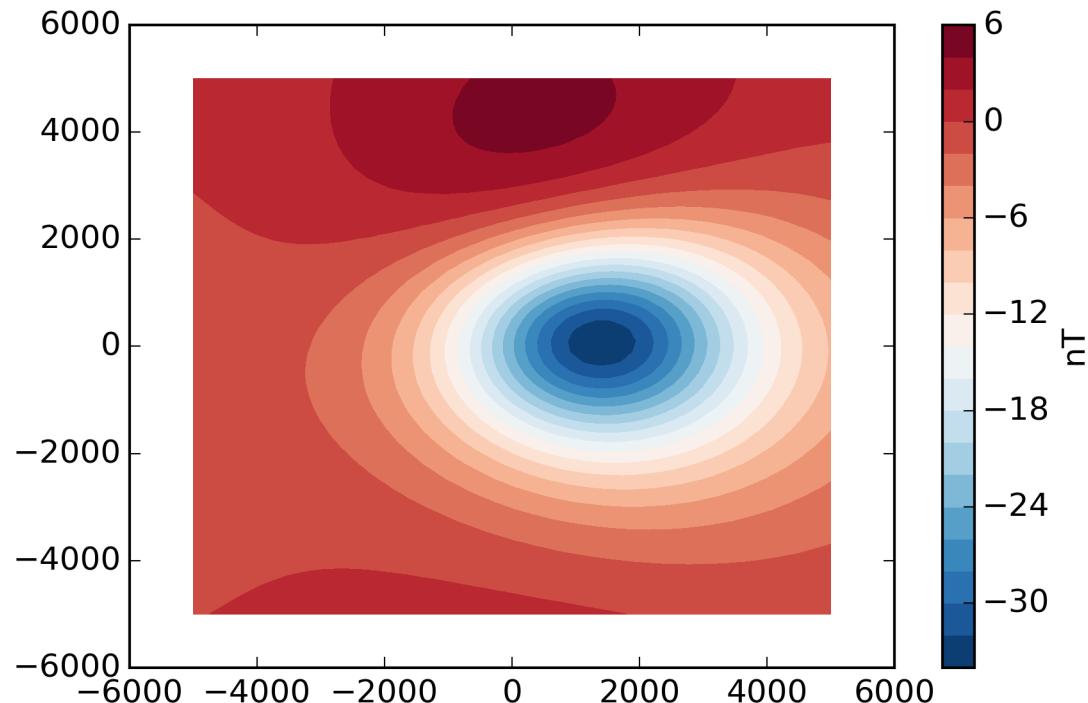
```
import matplotlib.pyplot as plt
plt.tricontourf(y, x, mag, 50, cmap="RdBu_r")
plt.colorbar().set_label('nT')
```

Out:



```
In: from fatiando.gravmag import transform  
up = transform.upcontinue(x, y, mag, shape,  
                           1500)  
  
plt.tricontourf(y, x, up, 20, cmap="RdBu_r")  
plt.colorbar().set_label('nT')
```

Out:



In:

```
from fatiando.meshes import Tesseroid
mod = [Tesseroid(-60, -50, -25, -20, 2000, 0,
                  props={'density': 2670})]
myv.figure()
myv.tesseroids(mod, scale=(1, 1, 500))
myv.earth()
myv.continents()
```

Out:



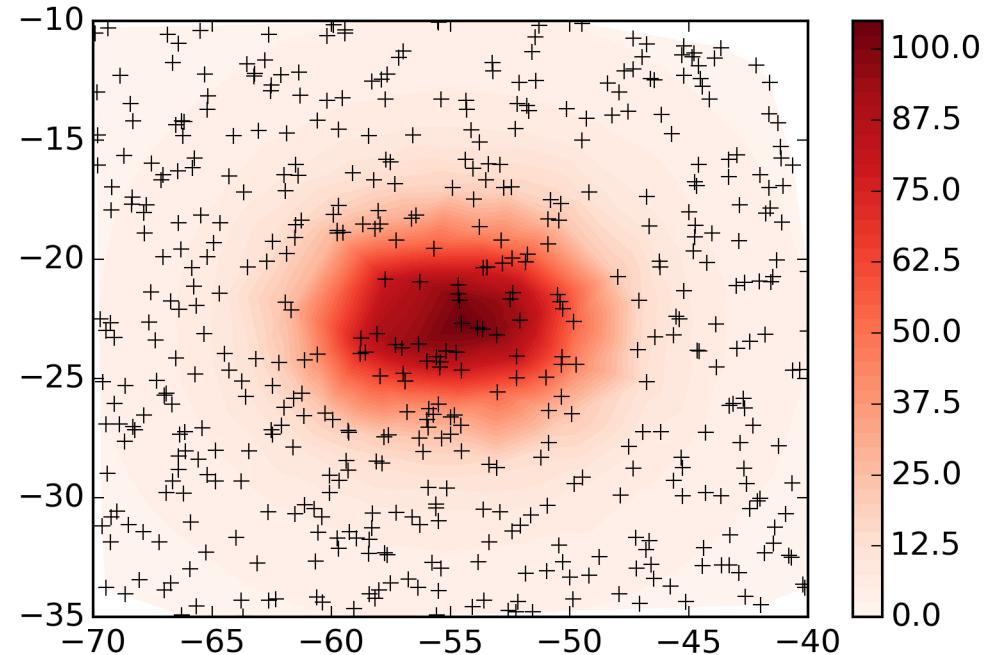
In:

```
lon, lat, h = gridded.scatter(  
    (-70, -40, -35, -10), 500, z=250e3)
```

```
from fatiando.gravmag import tesseral  
g = tesseral.gz(lon, lat, h, mod)
```

```
plt.tricontourf(lon, lat, g, 50, cmap="Reds")  
plt.colorbar()
```

Out:



fatiando.

griddeder  
mesher  
utils  
vis  
**gravmag**  
seismic  
inversion

# Módulos

fatiando.griddeder

mesher

utils

vis

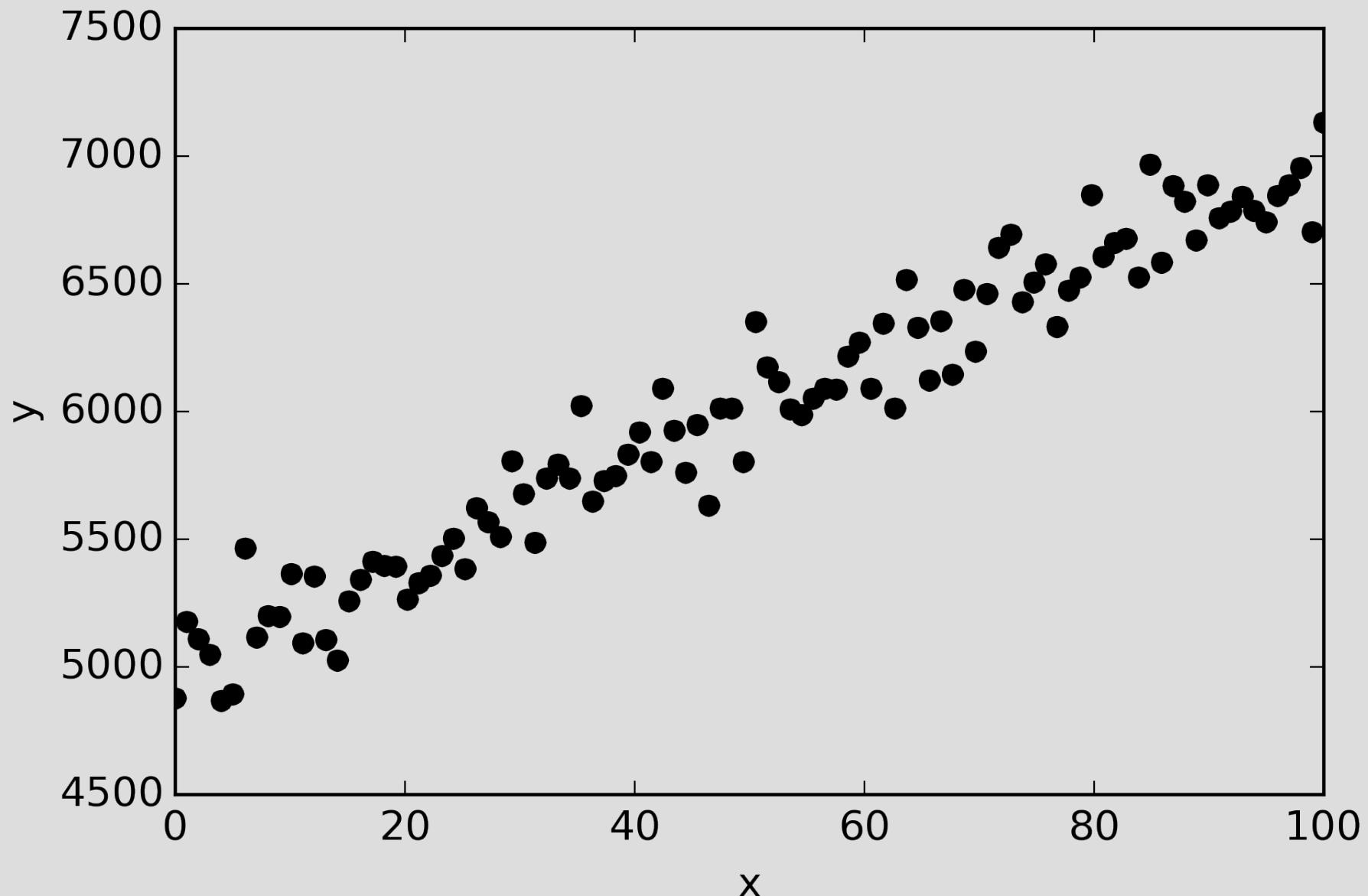
gravmag

seismic

inversion

Módulos

fatiando.inversion



$$y_i = a x_i + b$$

$$y_1 = a x_1 + b$$

$$y_2 = a x_2 + b$$

•  
•  
•

$$y_N = a x_N + b$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\bar{d} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$\bar{d}$

$=$

$\bar{\bar{A}}$

$\begin{bmatrix} a \\ b \end{bmatrix}$

$$\bar{d} \equiv \bar{\bar{A}} \bar{p}$$

$$\bar{d} = \bar{\mathbf{A}} \bar{p}$$

*dados preditos*      *matriz Jacobiana*      *vetor de parâmetros*

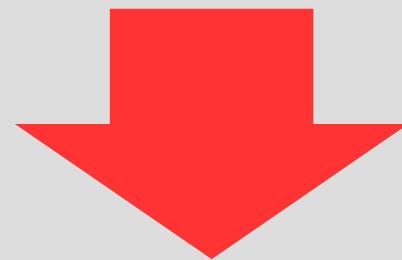
$\bar{d}^o$

$\bar{d}^o$



$\hat{\bar{p}}$

$$\varphi = ||\bar{d}^o - \bar{d}||^2$$



$\hat{\bar{p}}$

$$\varphi = \|\bar{d}^o - \bar{d}\|^2$$

minimizar



$\hat{\bar{p}}$

$$\varphi = \|\bar{d}^o - \bar{d}\|^2$$

função  
do ajuste  
(misfit)

minimizar



$\hat{p}$

específico

genérico

específico

genérico

$$y_i = a x_i + b$$

$\bar{\bar{A}}$

específico

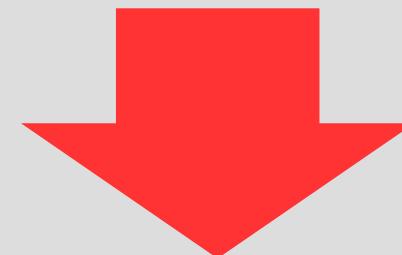
$$y_i = a x_i + b$$

=  
 $\bar{A}$

genérico

$$\varphi = \|\bar{d}^o - \bar{d}\|^2$$

minimizar

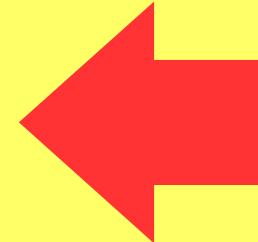


específico

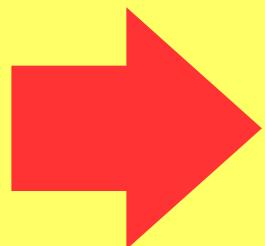
$$y_i = a x_i + b$$

=  
Ā

usuário  
implementa



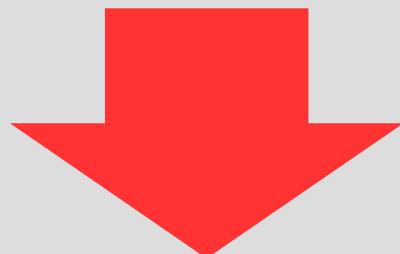
fatiando.inversion



genérico

$$\varphi = \|\bar{d}^o - \bar{d}\|^2$$

minimizar



```
from fatiando.inversion import Misfit

class Regressao(Misfit):
    def __init__(self, x, y):
        Misfit.__init__(self, data=y, nparams=2,
                        islinear=True)
        self.x = x

    def predicted(self, p):
        a, b = p
        return a*self.x + b

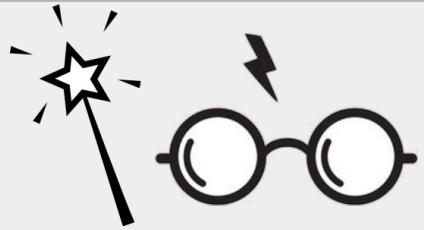
    def jacobian(self, p):
        A = np.empty((self.ndata,
                      self.nparams))
        A[:, 0] = self.x
        A[:, 1] = 1
        return A
```

```
from fatiando.inversion import Misfit
```

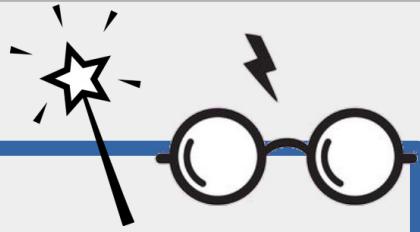
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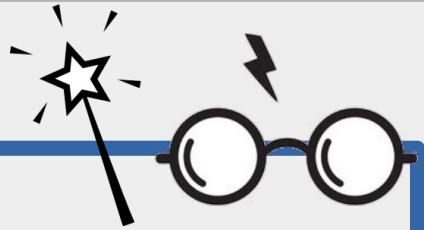


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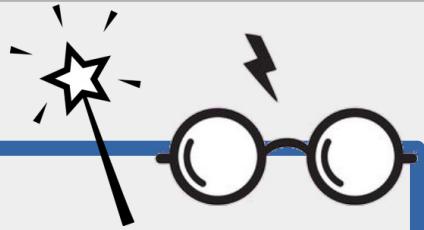
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```

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def predicted(self, p):
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```

$$y_i = a x_i + b$$

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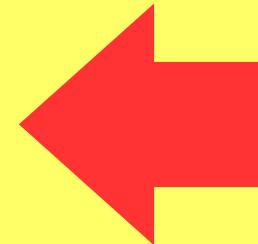
$$\bar{A}$$

específico

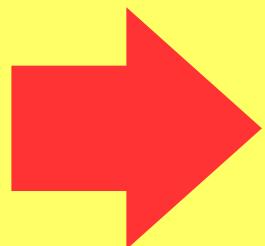
$$y_i = a x_i + b$$

=  
Ā

usuário  
implementa



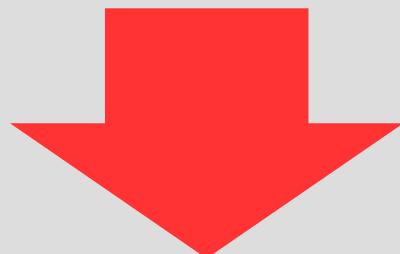
fatiando.inversion



genérico

$$\varphi = \|\bar{d}^o - \bar{d}\|^2$$

minimizar



In: reg = Regressao(x, yo)  
reg.fit()  
print(reg.estimate\_)

Out: [ 20.395431 4980.22844991 ]

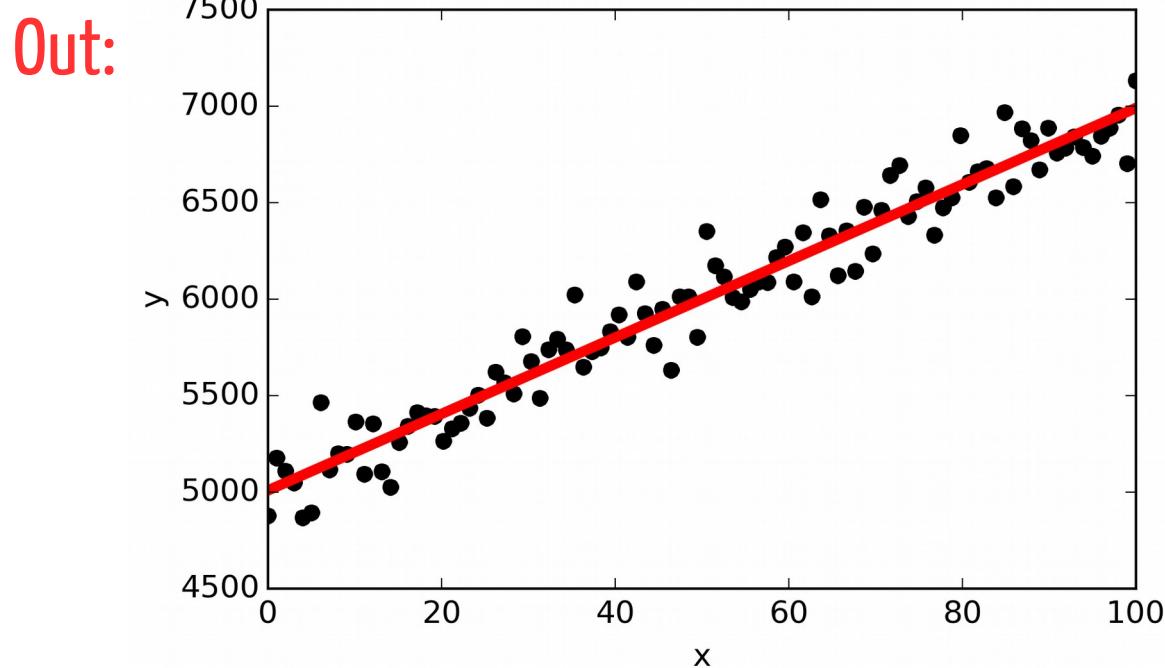
In: reg = Regressao(x, yo)  
reg.fit()  
print(reg.estimate\_)

Out: [ 20.395431 4980.22844991]  
 $a$                      $b$

```
In: reg = Regressao(x, yo)  
reg.fit()  
print(reg.estimate_)
```

Out: [ 20.395431 4980.22844991]  
 $a$                             $b$

```
In: plt.plot(x, yo, 'ok')  
plt.plot(x, reg.predicted(), '-r')
```



In: `reg.config('newton', initial=[1, 1]).fit()  
print(reg.estimate_)`

Out: [ 20.395431 4980.22844991]

In: `reg.config('newton', initial=[1, 1]).fit()  
print(reg.estimate_)`

Out: [ 20.395431 4980.22844991]

In: `reg.config('levmarq', initial=[0, 0]).fit()  
print(reg.estimate_)`

Out: [ 20.39562568 4980.21718193]

```
In: reg.config('newton', initial=[1, 1]).fit()  
print(reg.estimate_)
```

```
Out: [ 20.395431 4980.22844991]
```

```
In: reg.config('levmarq', initial=[0, 0]).fit()  
print(reg.estimate_)
```

```
Out: [ 20.39562568 4980.21718193]
```

```
In: reg.config('acor',  
             bounds=[0, 1000, 0, 10e5]).fit()  
print(reg.estimate_)
```

```
Out: [ 20.39543101 4980.22844959]
```

$$\text{minimizar } \varphi(\bar{p}) = \|\bar{d}^o - \bar{d}\|^2$$

minimizar

instável

$$\|\bar{d}^o - \bar{d}\|^2$$

minimizar

instável

$$\|\bar{d}^o - \bar{d}\|^2$$

minimizar



$$\Gamma(\bar{p}) = \varphi(\bar{p}) + \mu \theta(\bar{p})$$

minimizar

instável

$\|\bar{d}^o - \bar{d}\|^2$

minimizar



função

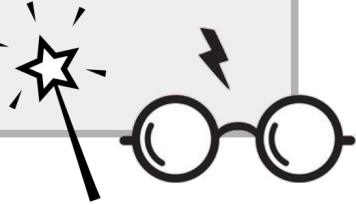
regularizadora

$$\Gamma(\bar{p}) = \varphi(\bar{p}) + \mu \theta(\bar{p})$$

escalar

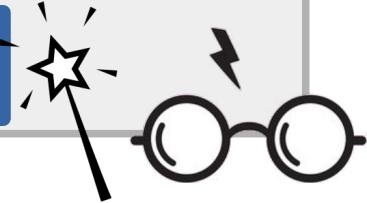
In: `from fatiando.inversion import Damping  
phi = Regressao(x, yo)  
gamma = phi + 10e-7*Damping(nparams=2)`

```
In: from fatiando.inversion import Damping  
phi = Regressao(x, vo)  
gamma = phi + 10e-7*Damping(nparams=2)
```



$$\Gamma(\bar{p}) = \varphi(\bar{p}) + \mu \theta(\bar{p})$$

```
In: from fatiando.inversion import Damping  
phi = Regressao(x, vo)  
gamma = phi + 10e-7*Damping(nparams=2)
```

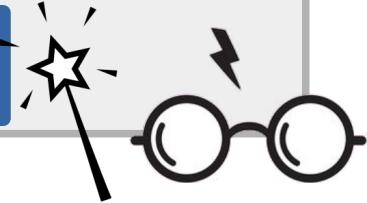


$$\Gamma(\bar{p}) = \varphi(\bar{p}) + \mu \theta(\bar{p})$$

```
In: gamma.fit()  
print(gamma.estimate_)
```

Out: [ 20.395431 4980.22844991]

```
In: from fatiando.inversion import Damping  
phi = Rearessao(x, vo)  
gamma = phi + 10e-7*Damping(nparams=2)
```



$$\Gamma(\bar{p}) = \varphi(\bar{p}) + \mu \theta(\bar{p})$$

```
In: gamma.fit()  
print(gamma.estimate_)
```

Out: [ 20.395431 4980.22844991]

```
In: gamma.config('acor',  
                 bounds=[0, 1000, 0, 10e5]).fit()  
print(gamma.estimate_)
```

Out: [ 20.14654754 4992.15040076]

# conclusão



# fatiando a terra



## biblioteca: funções e classes



# fatiando a terra

- ↳ biblioteca: funções e classes
- ↳ modelagem, processamento, visualização



# fatiando a terra

- ↳ biblioteca: funções e classes
- ↳ modelagem, processamento, visualização
- ↳ inversão: fatiando.inversion



# fatiando a terra

- ↳ biblioteca: funções e classes
- ↳ modelagem, processamento, visualização
- ↳ inversão: fatiando.inversion
- ↳ Simples → Complexos (reais)



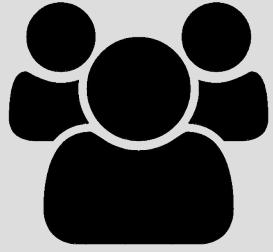
# fatiando a terra

- ↳ biblioteca: funções e classes
- ↳ modelagem, processamento, visualização
- ↳ inversão: fatiando.inversion
  - ↳ Simples → Complexos (reais)

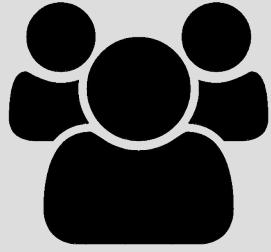
Caching

Matrizes esparsas

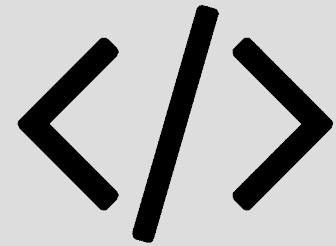
BLAS, LAPACK, MKL



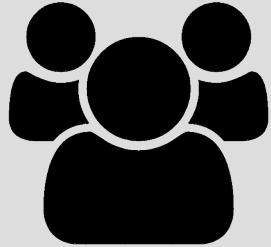
# Usuários: downloads, citações



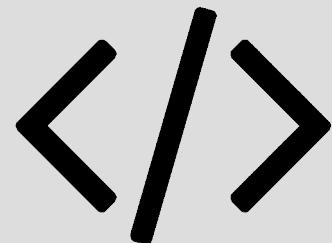
Usuários: downloads, citações



Colaboradores: locais e externos



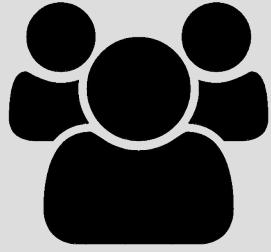
Usuários: downloads, citações



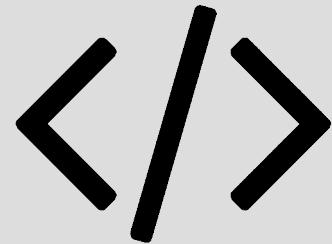
Colaboradores: locais e externos



Open-source: BSD license



Usuários: downloads, citações



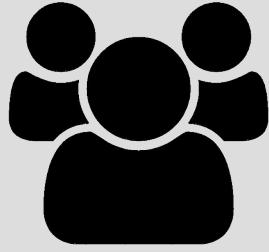
Colaboradores: locais e externos



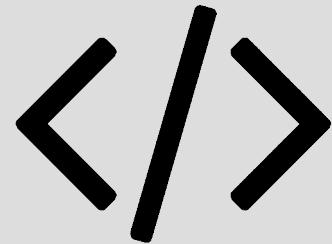
Open-source: BSD license



fatiando.org



Usuários: downloads, citações



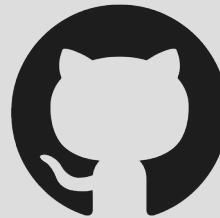
Colaboradores: locais e externos



Open-source: BSD license



fatiando.org



[github.com/fatiando](https://github.com/fatiando)

fatiando.org



modelagem direta  
aprox. esférica



fatiando a terra  
modelagem direta  
otimização  
regularização, etc

# Novo método

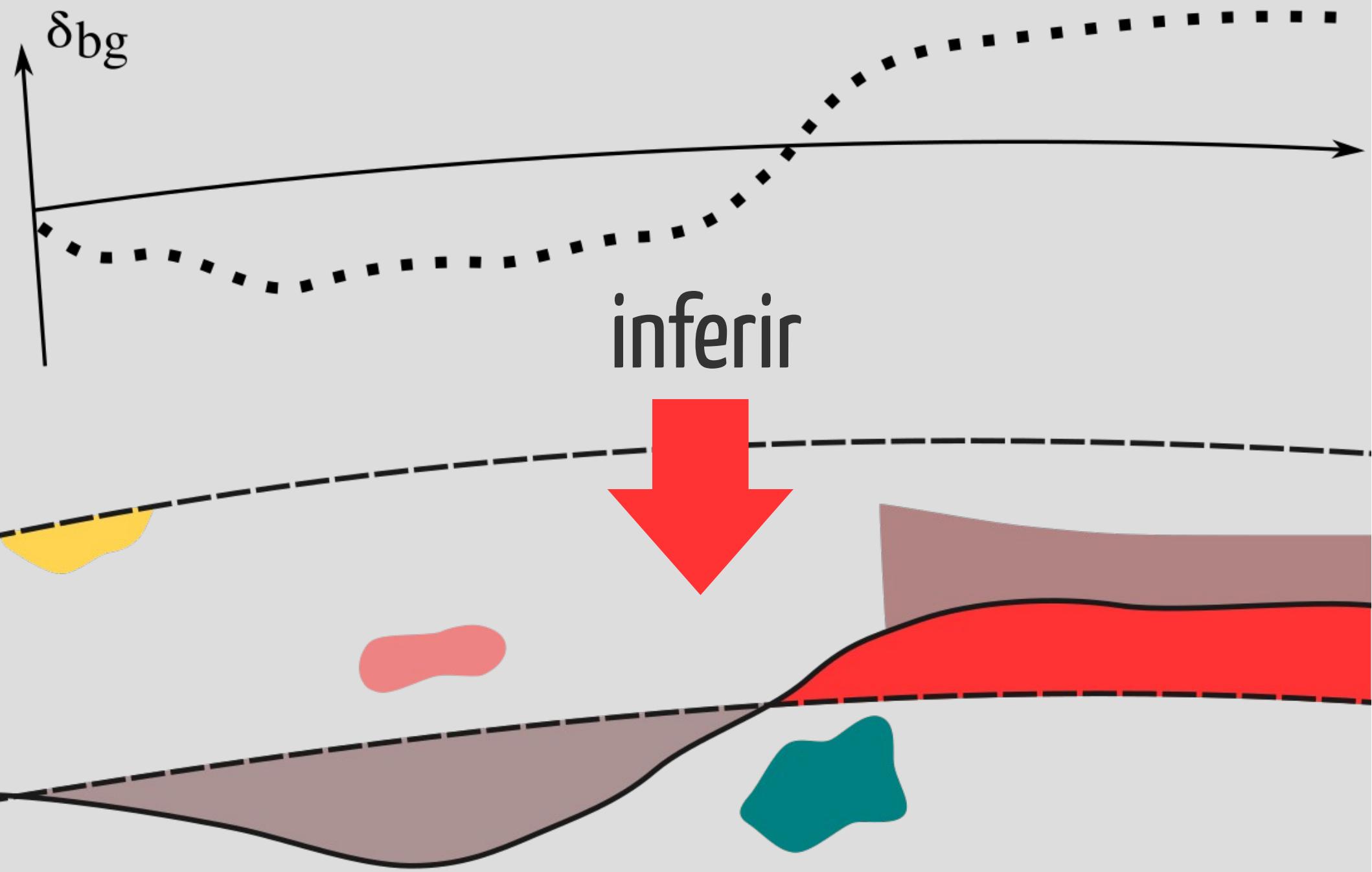
Introdução  
Tesseroids  
Fatiando a Terra  
Inversão Moho  
Conclusão

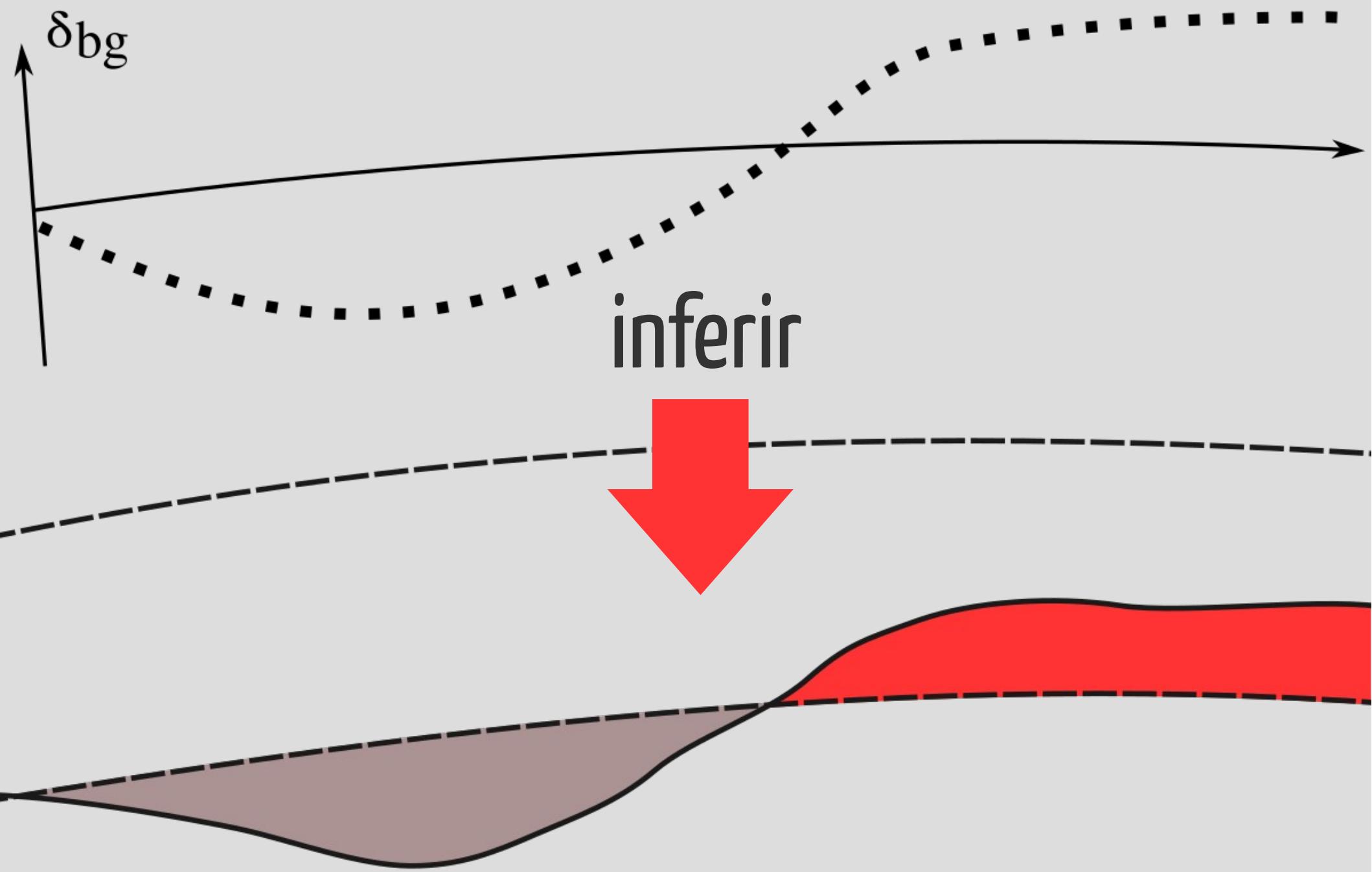
*rápida*

Inversão não-linear ✓  
em

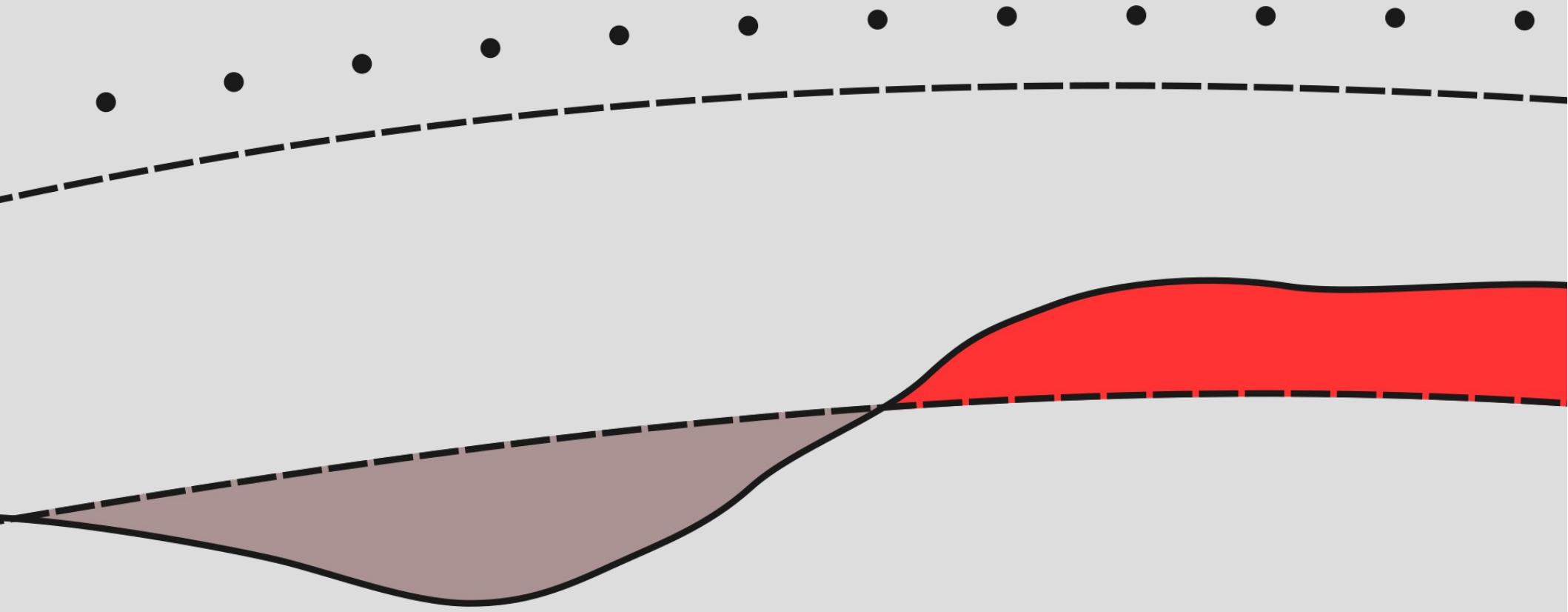
# coordenadas esféricas

com aplicação na Moho da América do Sul

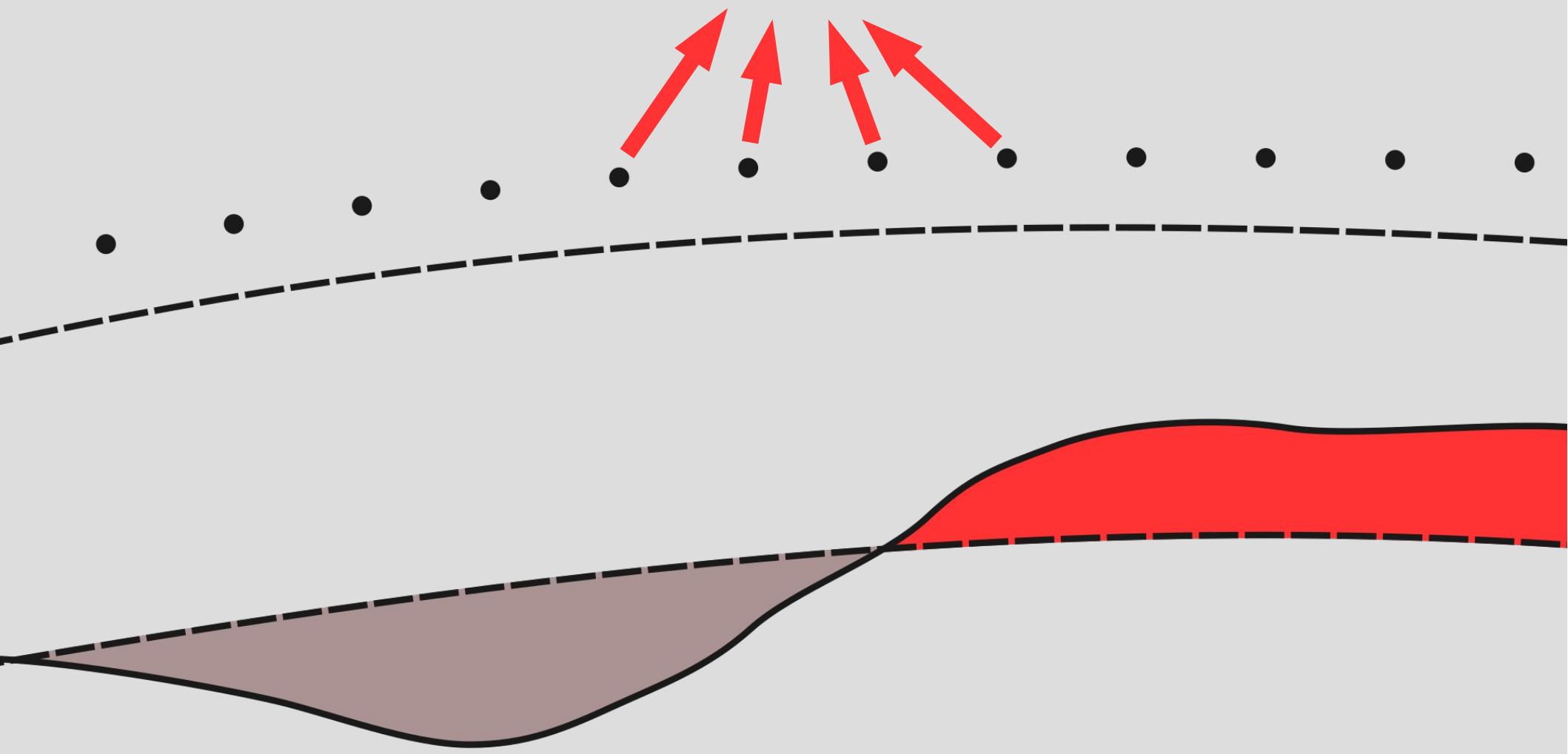




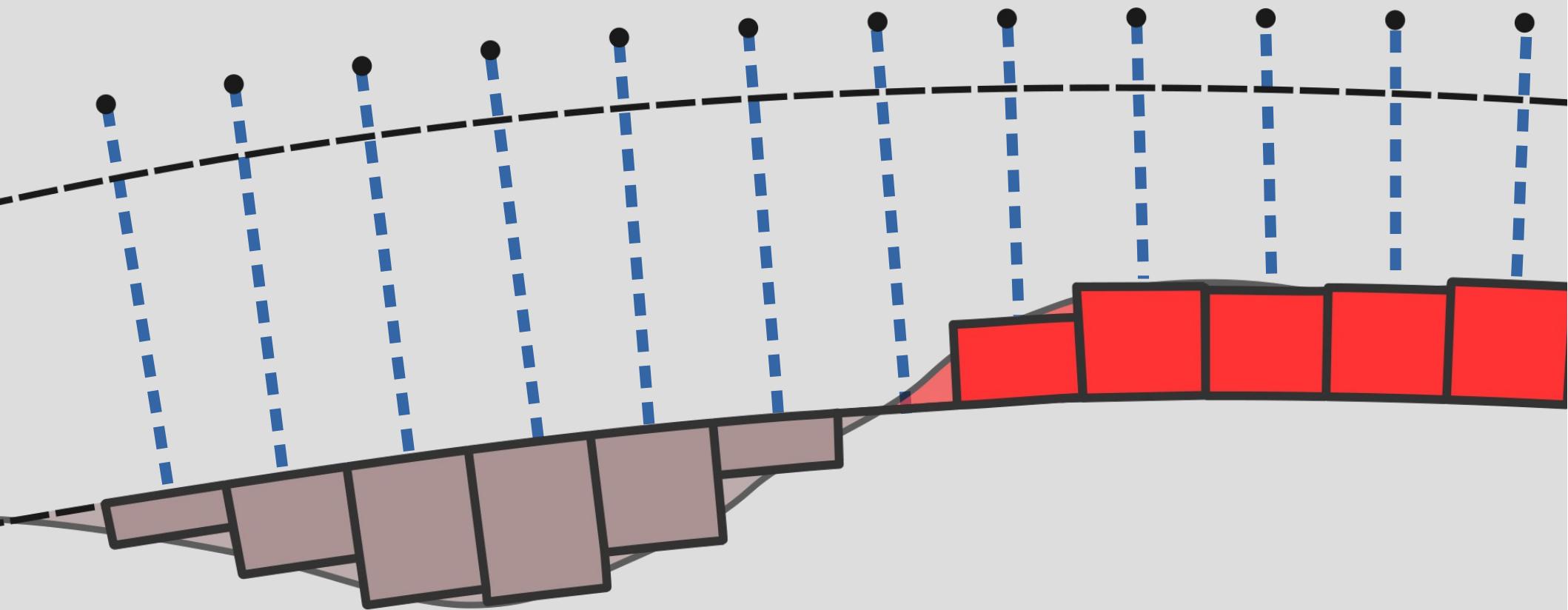
# Grid de observações



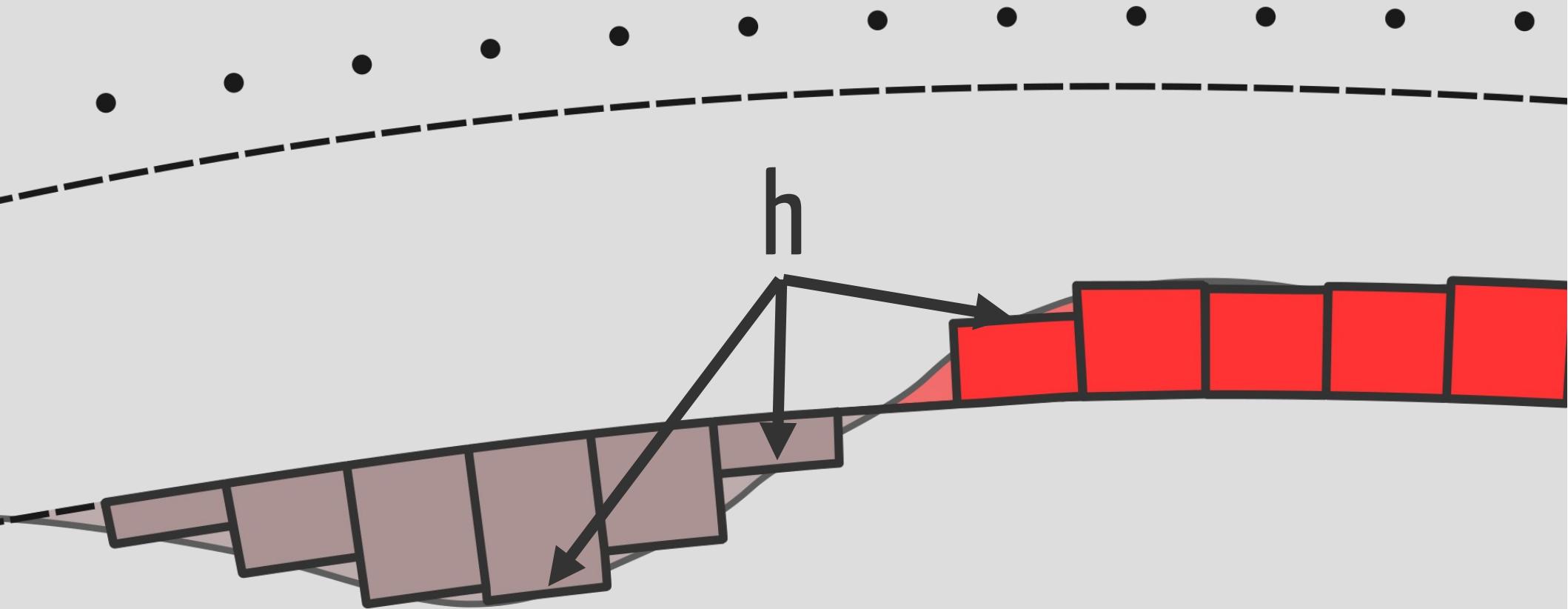
$\bar{d}^o$

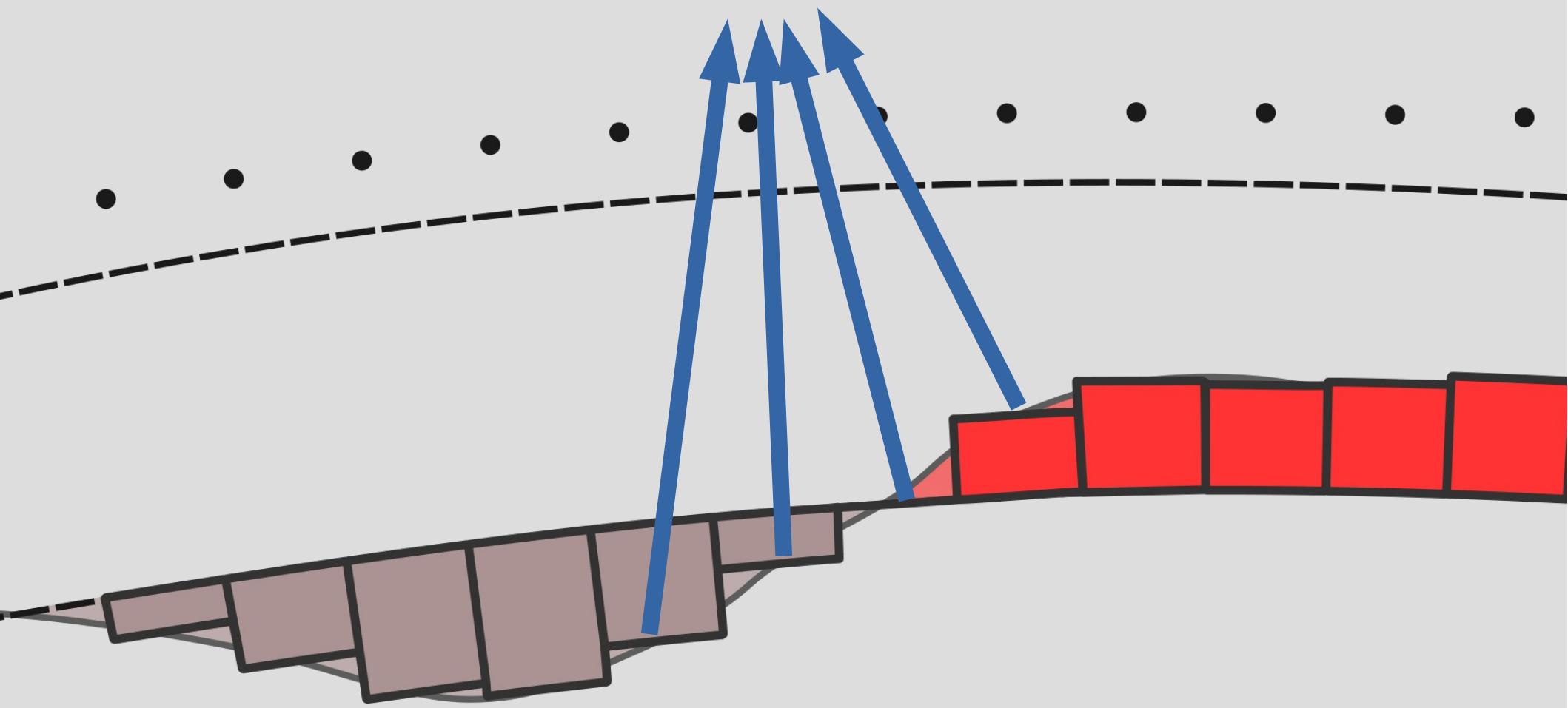


# 1 tesseróide para cada



# parâmetros = h



$\bar{p}$ 

Estimar  $\bar{p}$

a partir de  $\bar{d}^o$

inversão

não-linear

$$d_i = f_i(\bar{p})$$

$$d_i = f_i(\bar{p})$$



modelagem direta  
(tesseroides)

# Resíduos

$$\bar{r} = \bar{d}^o - \bar{d}$$

# Minimizar

$$\phi(\bar{p}) = \|\bar{r}\|^2$$

# Gauss-Newton

$$\bar{p}^0$$

$$\bar{p}^0 \rightarrow \bar{p}^1 = \bar{p}^0 + \Delta^- p^0$$

$$\bar{p}^0 \rightarrow \bar{p}^1 = \bar{p}^0 + \Delta^- p^0$$

$$\left(\bar{\bar{A}}^{kT}\bar{\bar{A}}^k\right)\Delta^- p^k = \bar{\bar{A}}^{kT}\left[\bar{d}^o - \bar{d}\left(\bar{p}^k\right)\right]$$

$$\bar{p}^0 \rightarrow \bar{p}^1 = \bar{p}^0 + \Delta^- p^0$$

$$(\bar{\bar{A}}^{kT} \bar{\bar{A}}^k) \Delta^- p^k = \bar{\bar{A}}^{kT} [\bar{d}^o - \bar{d}(\bar{p}^k)]$$

$$A_{\textcolor{red}{i} \textcolor{blue}{j}} = \frac{\partial f_i}{\partial p_j} \quad \underline{\text{Jacobian}}$$

$$\bar{p}^0 \rightarrow \bar{p}^1 = \bar{p}^0 + \Delta^- p^0$$

**mal posto** ( $\bar{p}^k$ )

$$A_{ij} = \frac{\partial f_i}{\partial p_j}$$

Jacobiana

# Regularização

# Suavidade

$$\theta(\bar{p}) = \|\bar{\bar{R}}\bar{p}\|^2$$

# Função objetivo

$$\Gamma(\bar{p}) = \phi + \mu \theta$$

# Função objetivo

ajuste


$$\Gamma(\bar{p}) = \phi + \mu \theta$$

# Função objetivo

$$\Gamma(\bar{p}) = \phi + \mu \theta$$

ajuste →

regularização ↑

# Função objetivo

$$\Gamma(\bar{p}) = \phi + \mu \theta$$

ajuste                  balanço                  regularização

```
graph TD; A[ajuste] --> B["Γ(̄p) = φ + μ θ"]; C[balanço] --> D["Γ(̄p) = φ + μ θ"]; E[regularização] --> F["Γ(̄p) = φ + μ θ"];
```

# Gauss-Newton

$$\Delta \bar{p} = (\bar{\bar{A}}^T \bar{\bar{A}} + \mu \bar{\bar{R}}^T \bar{\bar{R}})^{-1} [$$
  
$$\bar{\bar{A}}^T \bar{r}^k - \mu \bar{\bar{R}}^T \bar{\bar{R}} \bar{p}^k ]$$

# Gauss-Newton

# **custoso**

**(computacionalmente)**

1. Construir  $\bar{A}$

1. Construir  $\bar{A}$

2. Sistema linear

1. Construir  $\bar{A}$

2. Sistema linear

3. Calcular  $\bar{r}$

Bott

(1960)

$$\Delta \bar{p} = \frac{\bar{r}}{2\pi G \Delta \rho}$$

$$\Delta p = \frac{\bar{r}}{2\pi G \Delta \rho}$$

*platô de Bouguer*

*h*

1. Construir  $\bar{A}$

2. Sistema linear

3. Calcular  $\bar{r}$

~~1. Construir  $\bar{A}$~~

~~2. Sistema linear~~

3. Calcular  $\bar{r}$

rápido

pouca memória

converge

instável

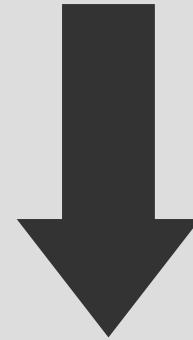
regularização

Silva et al.

(2014)

# Bott

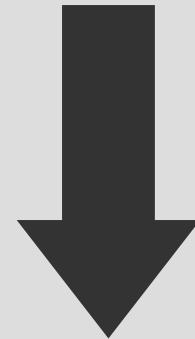
# Bott



caso  
particular

# Gauss-Newton

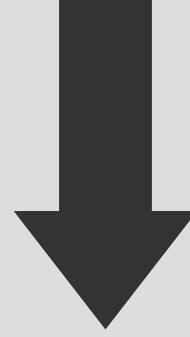
$$\Delta^- p = \frac{\bar{r}}{2\pi G \Delta \rho}$$



caso  
particular

$$\Delta^- p = (\bar{\bar{A}}^T \bar{\bar{A}})^{-1} \bar{\bar{A}}^T \bar{r}$$

$$\bar{\Delta p} = \frac{\bar{r}}{2\pi G \Delta \rho}$$



caso  
particular

$$\bar{\Delta p} = (\bar{\bar{A}}^T \bar{\bar{A}})^{-1} \bar{\bar{A}}^T \bar{r}$$

$$\bar{\Delta p} = \frac{\bar{r}}{2\pi G \Delta \rho}$$

$$A_{ii} = 2\pi G \Delta \rho$$

$$A_{ij} = 0 \text{ para } i \neq j$$

caso  
particular

$$\bar{\Delta p} = (\bar{\bar{A}}^T \bar{\bar{A}})^{-1} \bar{\bar{A}}^T \bar{r}$$

$\tilde{A}$ 

$$\begin{pmatrix} 2\pi G \Delta \rho & 0 & \cdots & 0 \\ 0 & 2\pi G \Delta \rho & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2\pi G \Delta \rho \end{pmatrix}$$

nesse

trabalho

# Gauss-Newton

$$\bar{\Delta p} = (\bar{\bar{A}}^T \bar{\bar{A}})^{-1} \bar{\bar{A}}^T [\bar{d}^o - \bar{d}(\bar{p}^k)]$$

# Gauss-Newton

$$\bar{\Delta p} = (\bar{\bar{A}}^T \bar{\bar{A}})^{-1} \bar{\bar{A}}^T [\bar{d}^o - \bar{d}(\bar{p}^k)]$$

  
tesseroides

# Gauss-Newton

$$\bar{\Delta} \bar{p} = (\tilde{A}^T \tilde{A})^{-1} \tilde{A}^T [\bar{d}^o - \bar{d}(\bar{p}^k)]$$

$$\tilde{A}_{ii} = 2\pi G \Delta \rho_i$$

tesseroides

regularização

suavidade

$$\Delta \bar{p} = (\tilde{A}^T \tilde{A} + \mu \bar{\bar{R}}^T \bar{\bar{R}})^{-1} [$$

$$\tilde{A}^T \bar{r}^k - \mu \bar{\bar{R}}^T \bar{\bar{R}} \bar{p}^k ]$$

esparsas

$$\Delta \bar{p} = (\boxed{\tilde{A}^T \tilde{A}} + \mu \boxed{\bar{\bar{R}}^T \bar{\bar{R}}})^{-1} [$$

$$\tilde{A}^T \bar{r}^k - \mu \bar{\bar{R}}^T \bar{\bar{R}} \bar{p}^k ]$$

esparsas

$$\Delta \bar{p} = (\tilde{A}^T \tilde{A} + \mu \bar{\bar{R}}^T \bar{\bar{R}})^{-1} [$$

$$\tilde{A}^T \bar{r}^k - \mu \bar{\bar{R}}^T \bar{\bar{R}} \bar{p}^k ]$$



~99.8% tempo de computação

1. Construir  $\bar{A}$

2. Sistema linear

3. Calcular  $\bar{r}$

~~1. Construir  $\bar{A}$~~   
Bott

2. Sistema linear

3. Calcular  $\bar{r}$

~~1. Construir  $\bar{A}$~~   
Bott

~~2. Sistema linear~~  
matrizes esparsas

3. Calcular  $\bar{r}$

rápido

pouca memória

converge

instável

regularização

instável

regularização

# implementação



python™



fatiando a terra

modeling and inversion in geophysics

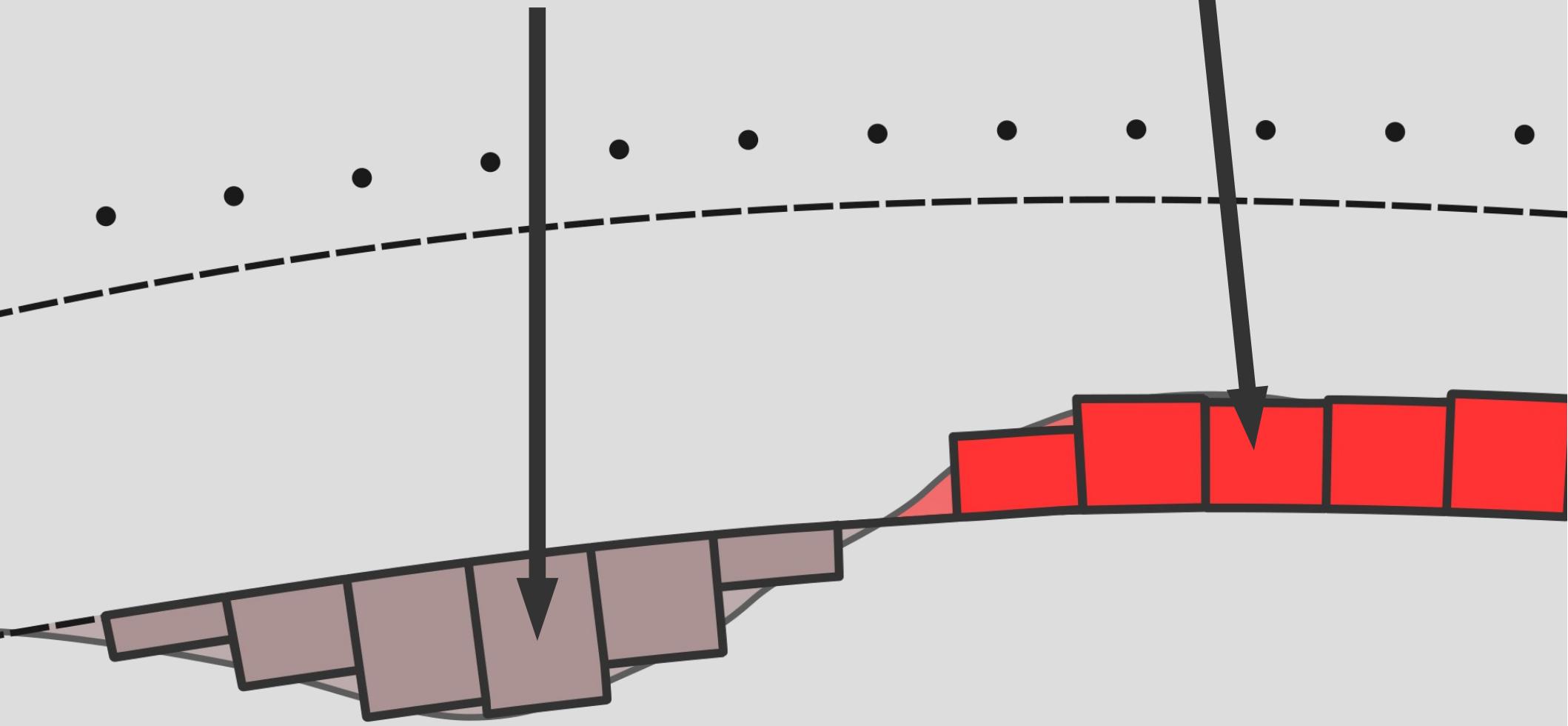
scipy (matrices esparsas)

estimar

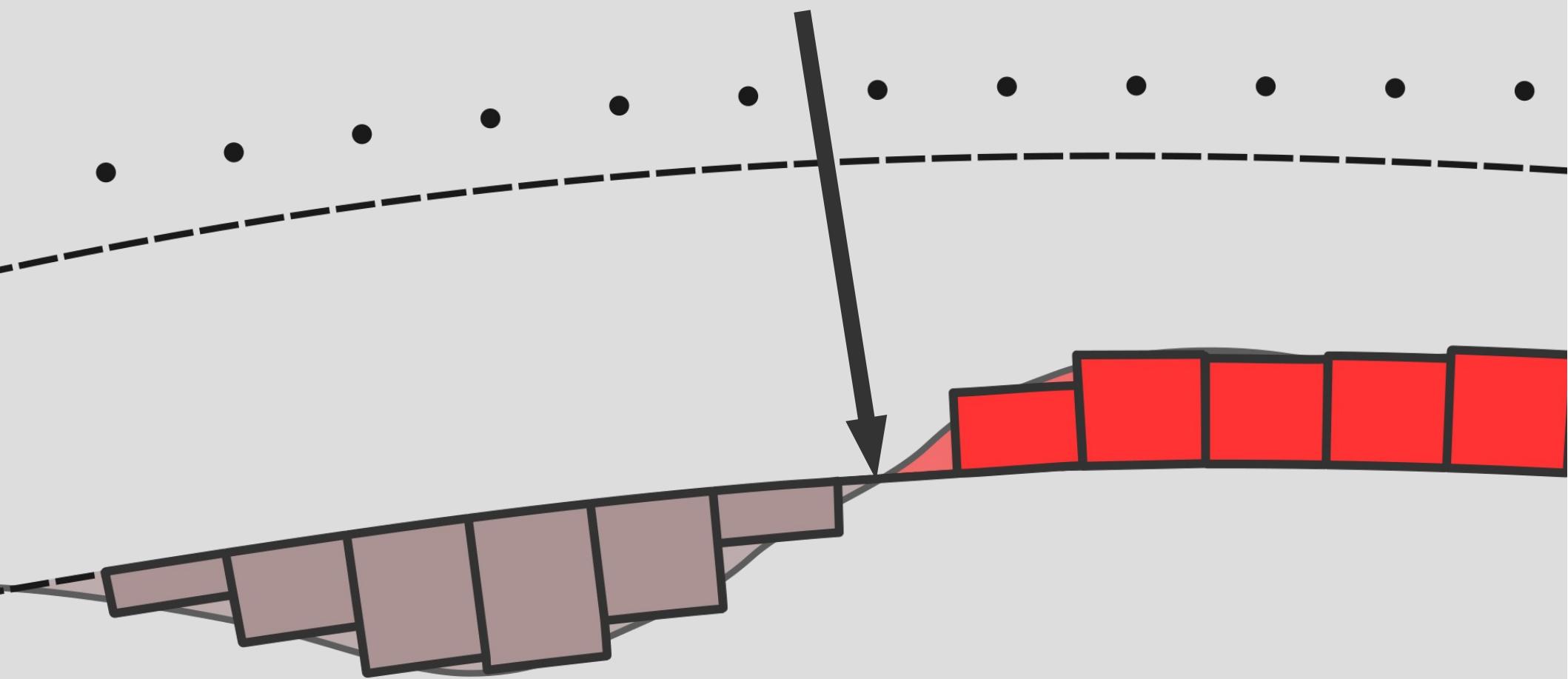
hiperparâmetros

$-\Delta\rho$

$+\Delta\rho$



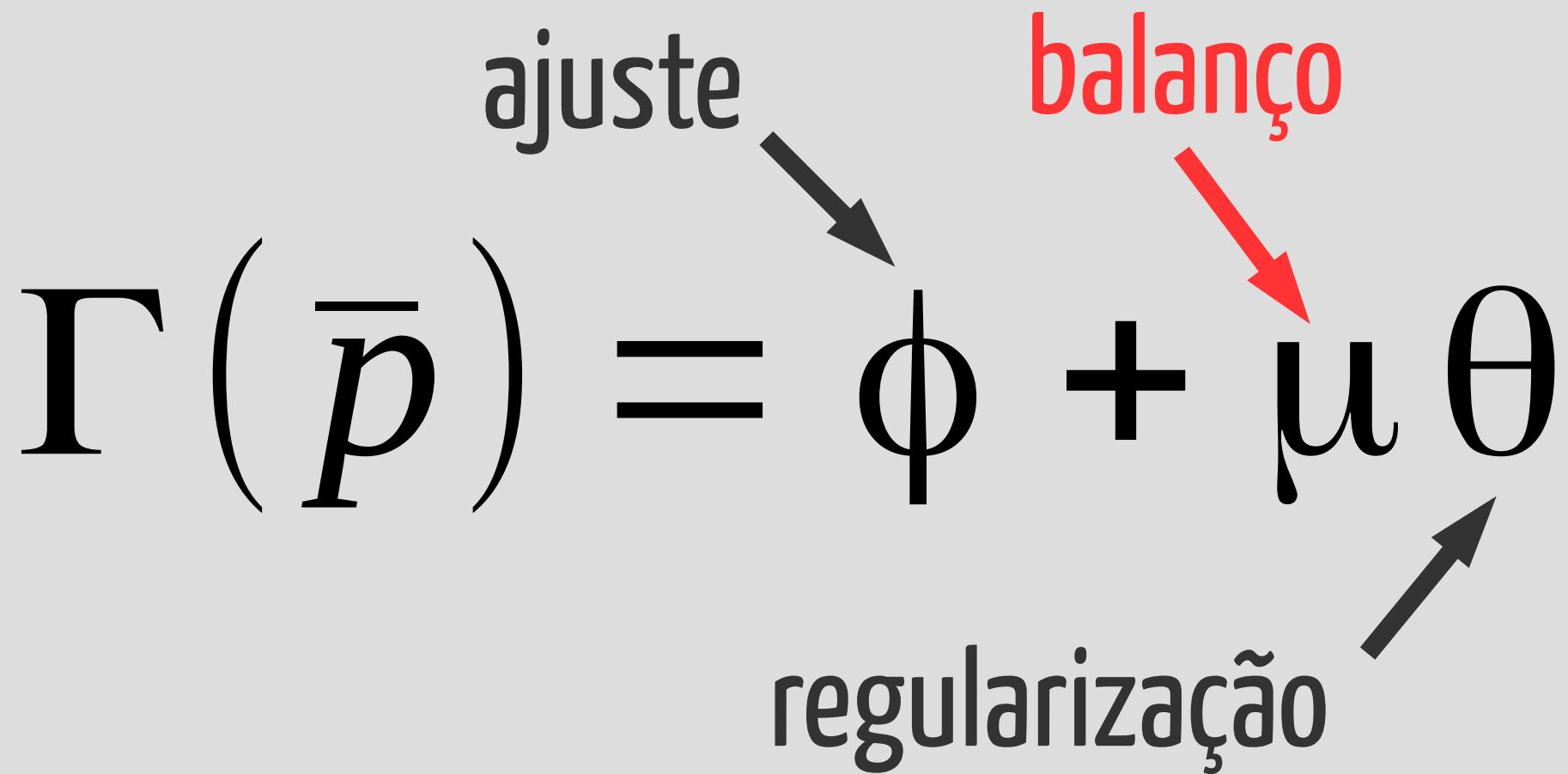
$z_{\text{ref}}$



# Função objetivo

$$\Gamma(\bar{p}) = \phi + \mu \theta$$

ajuste      balanço      regularização



$\mu$

$\Delta \rho$

$z_{\text{ref}}$

# validação cruzada

validação cruzada

$\mu$

validação

$\mu$

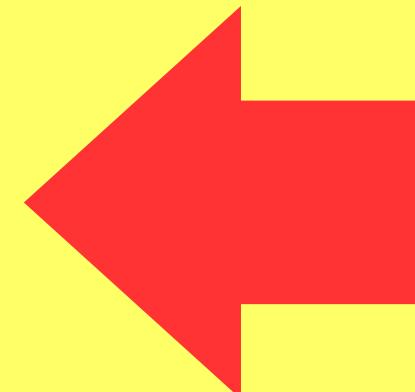
cruzada

$\Delta \rho$

$z_{\text{ref}}$

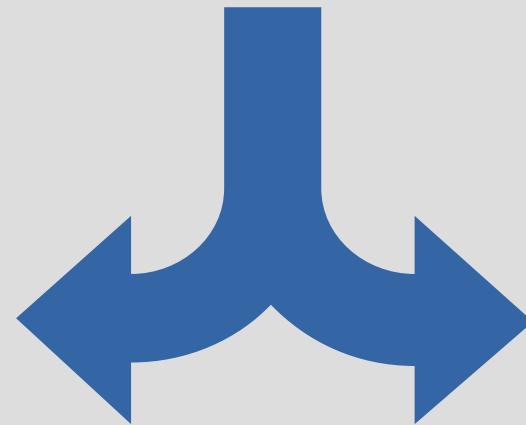
# validação

$\mu$



# Separar os dados

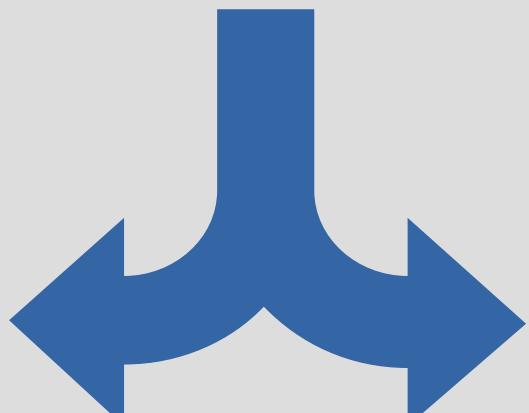
$\bar{d}^o$



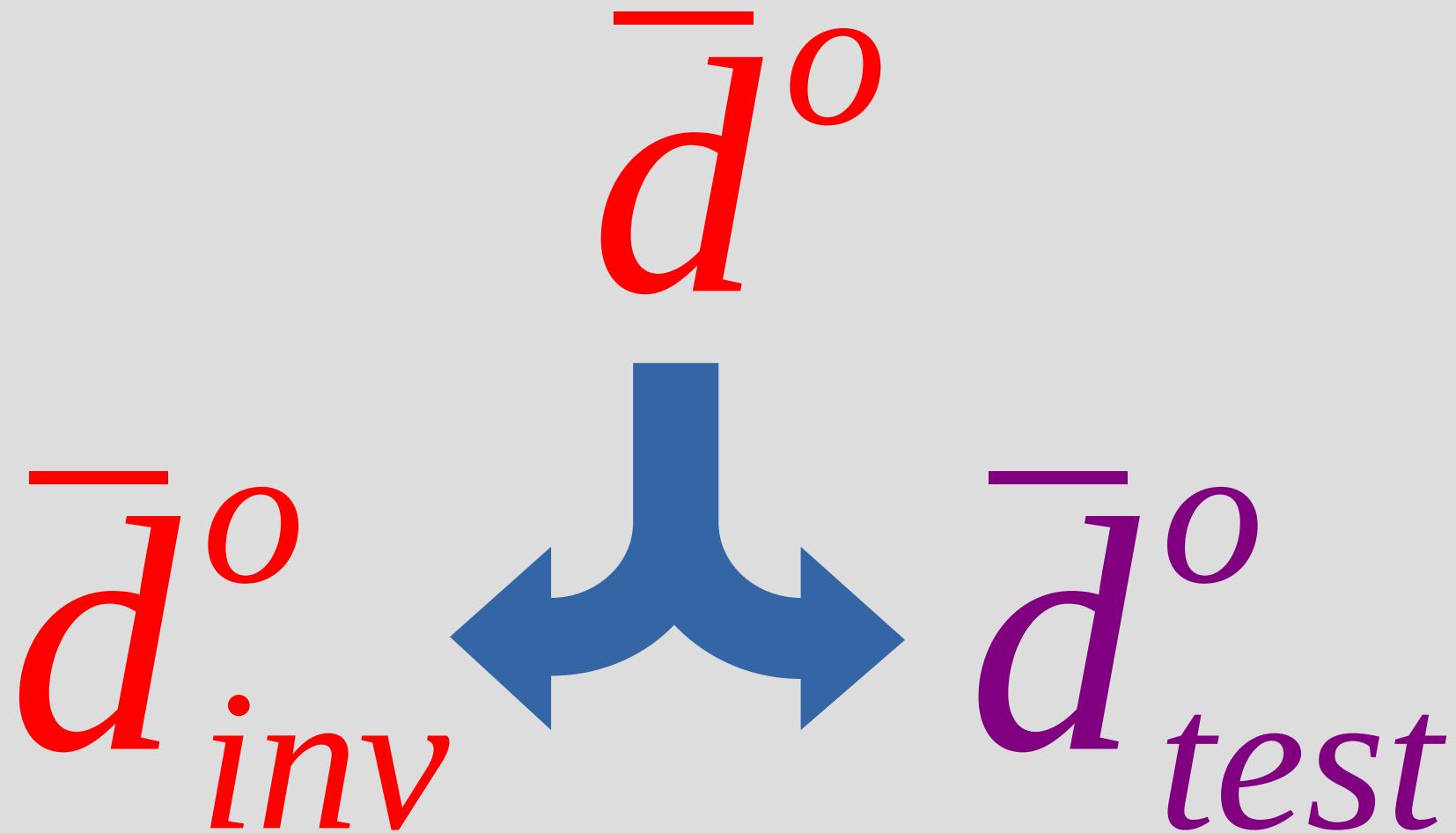
# Separar os dados

$\bar{d}^o$

$\bar{d}_{inv}^o$



# Separar os dados



para  $\mu_n$  em  $[\mu_1, \dots, \mu_N]$ :

para  $\mu_n$  em  $[\mu_1, \dots, \mu_N]$ :

inversão:  $\bar{d}_{inv}^o \rightarrow \hat{\bar{p}}^n$

para  $\mu_n$  em  $[\mu_1, \dots, \mu_N]$ :

inversão:  $\bar{d}_{inv}^o \rightarrow \hat{\bar{p}}^n$

$\hat{\bar{p}}^n \rightarrow$  prever  $\bar{d}_{test}^n$

para  $\mu_n$  em  $[\mu_1, \dots, \mu_N]$ :

inversão:  $\bar{d}_{inv}^o \rightarrow \hat{\bar{p}}^n$

$\hat{\bar{p}}^n \rightarrow$  prever  $\bar{d}_{test}^n$

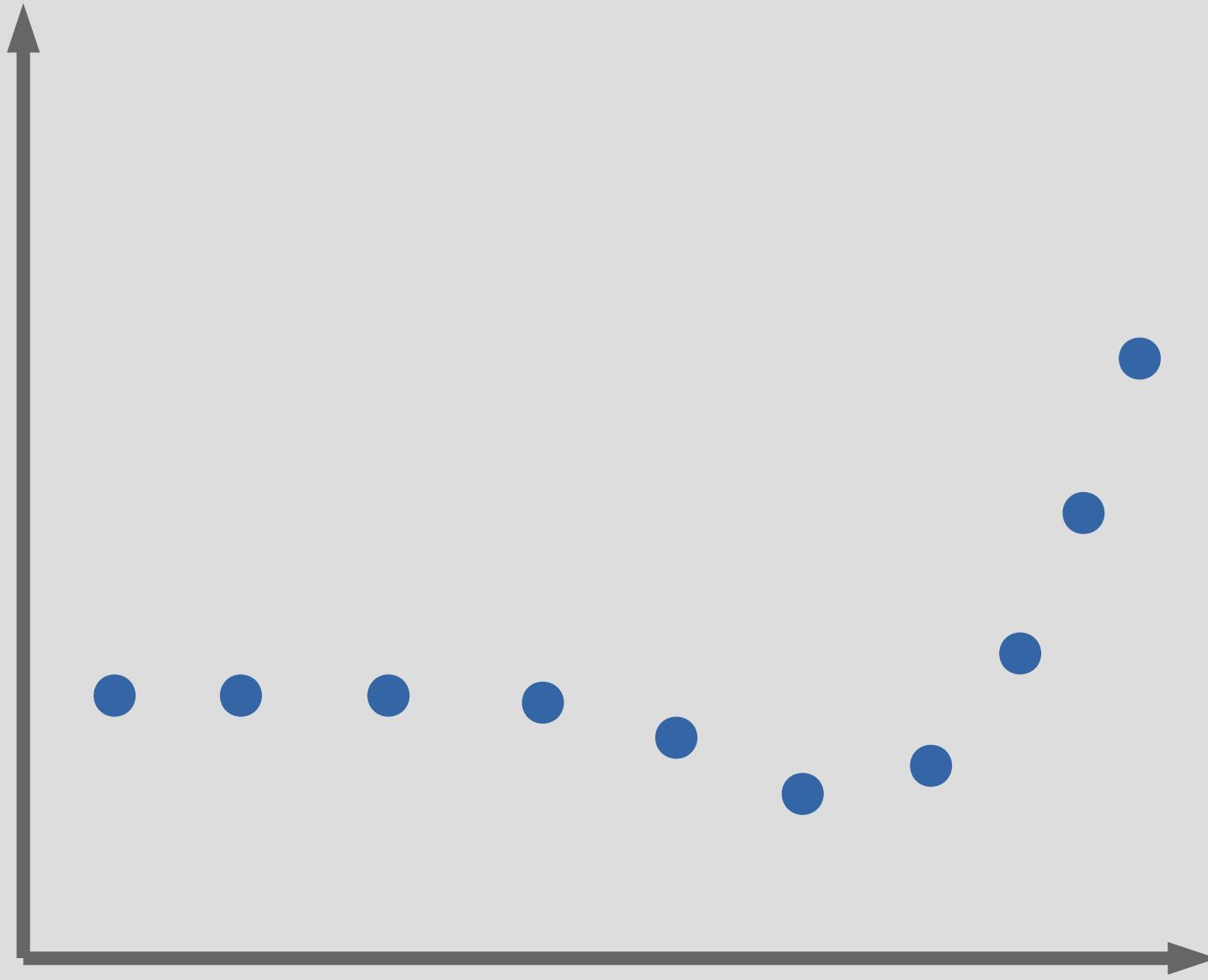
$$MSE_n = \frac{\|\bar{d}_{test}^o - \bar{d}_{test}^n\|^2}{N_{test}}$$

MSE

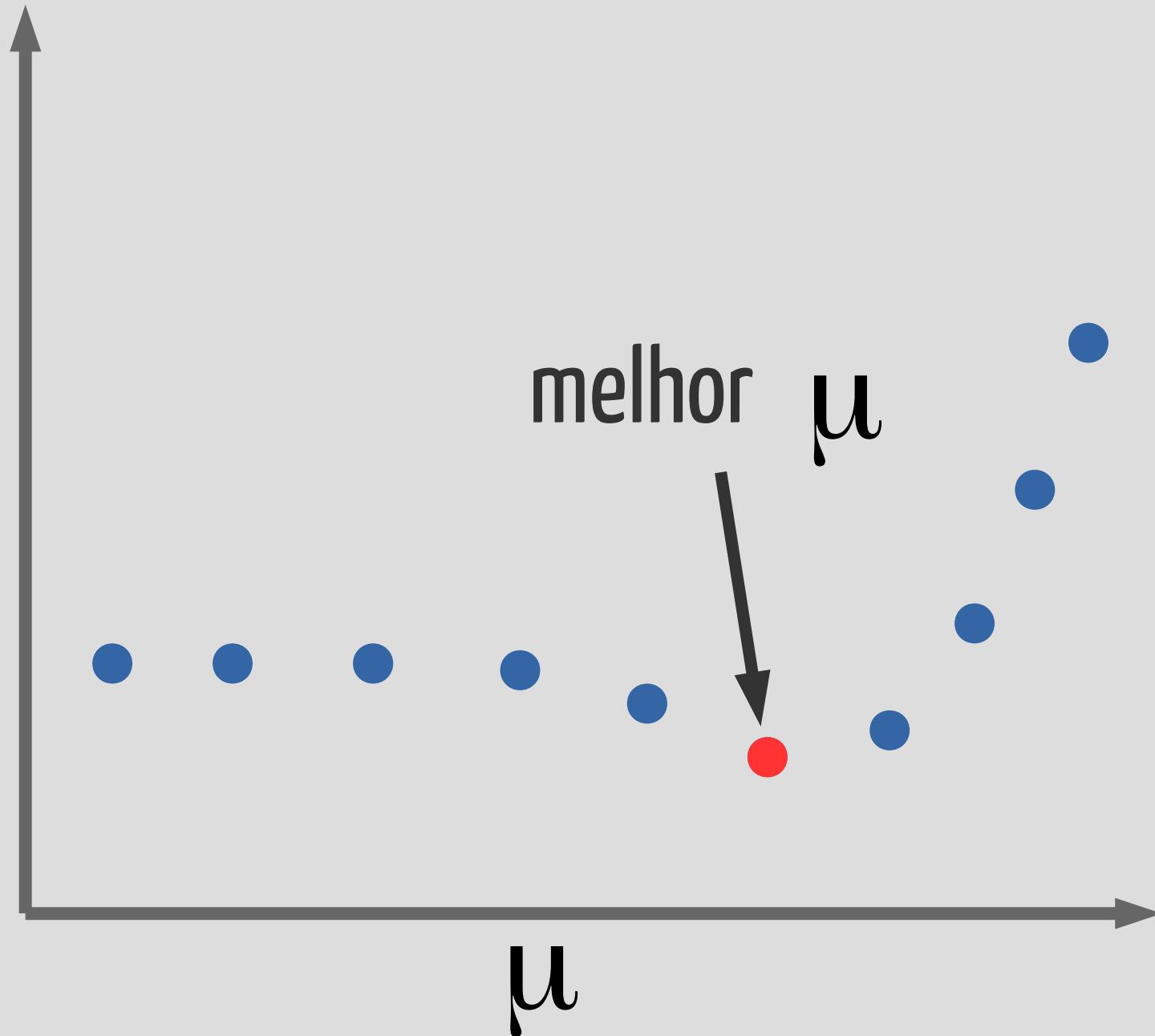
$\mu$

MSE

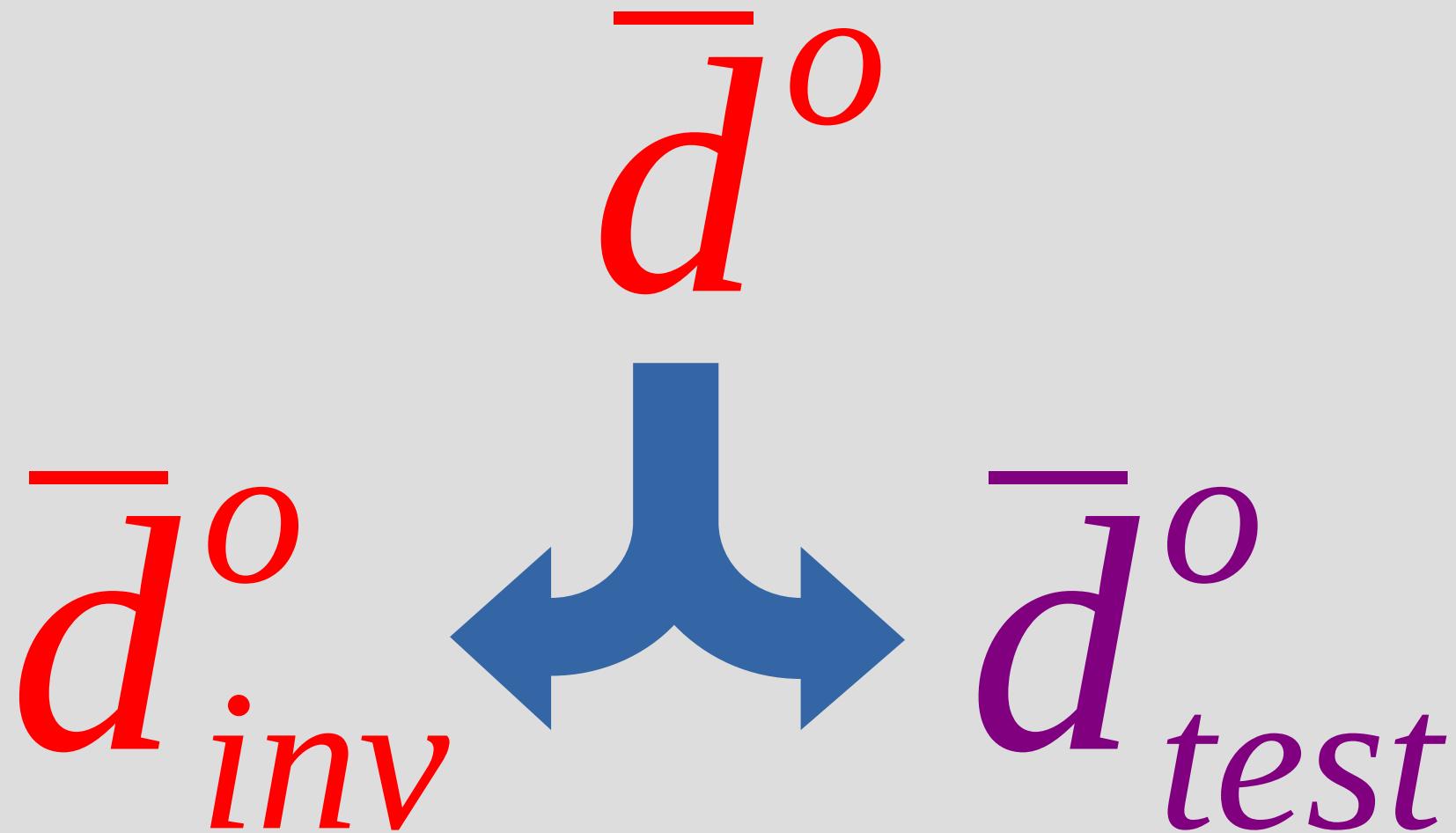
$\mu$



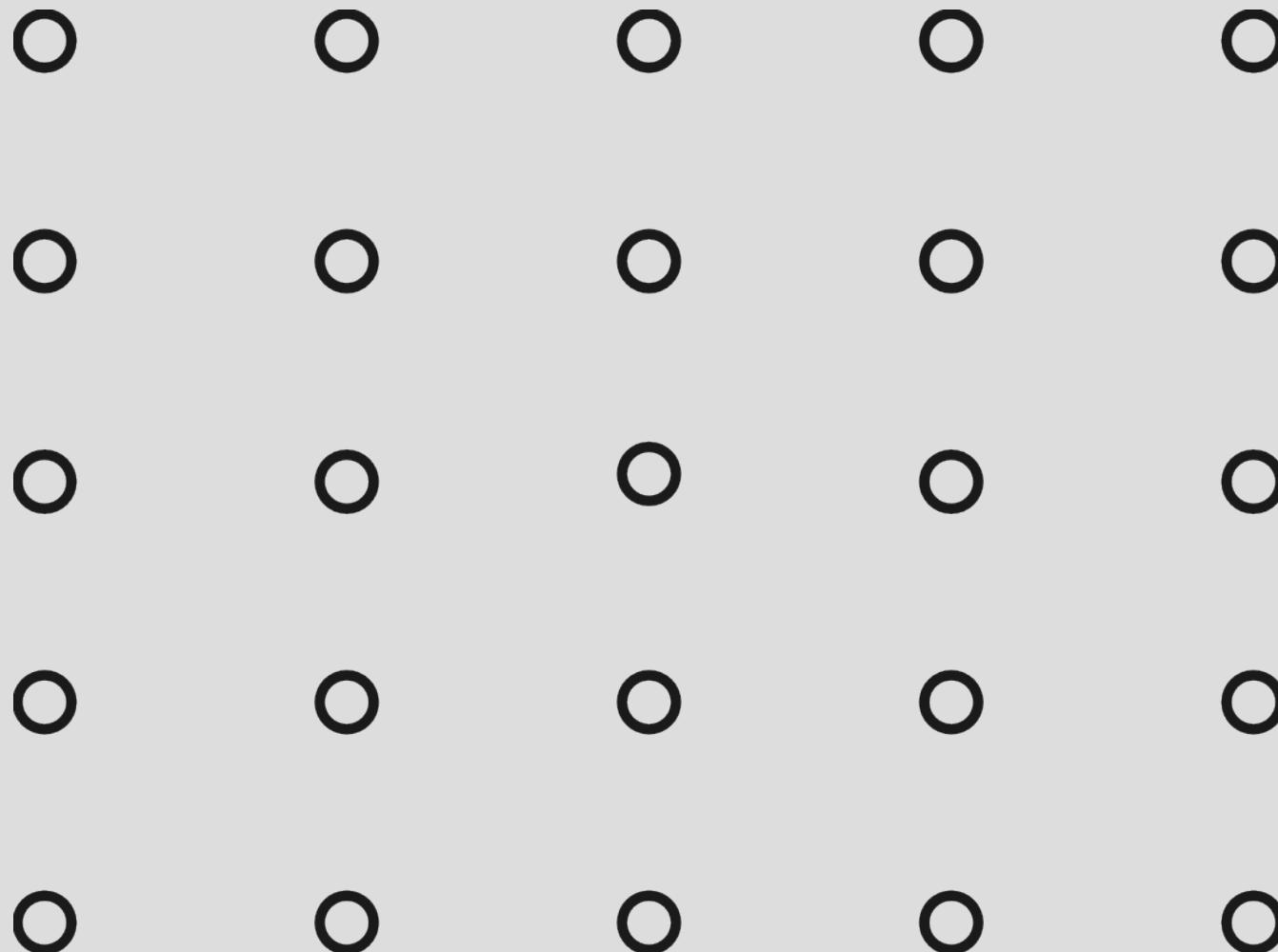
MSE



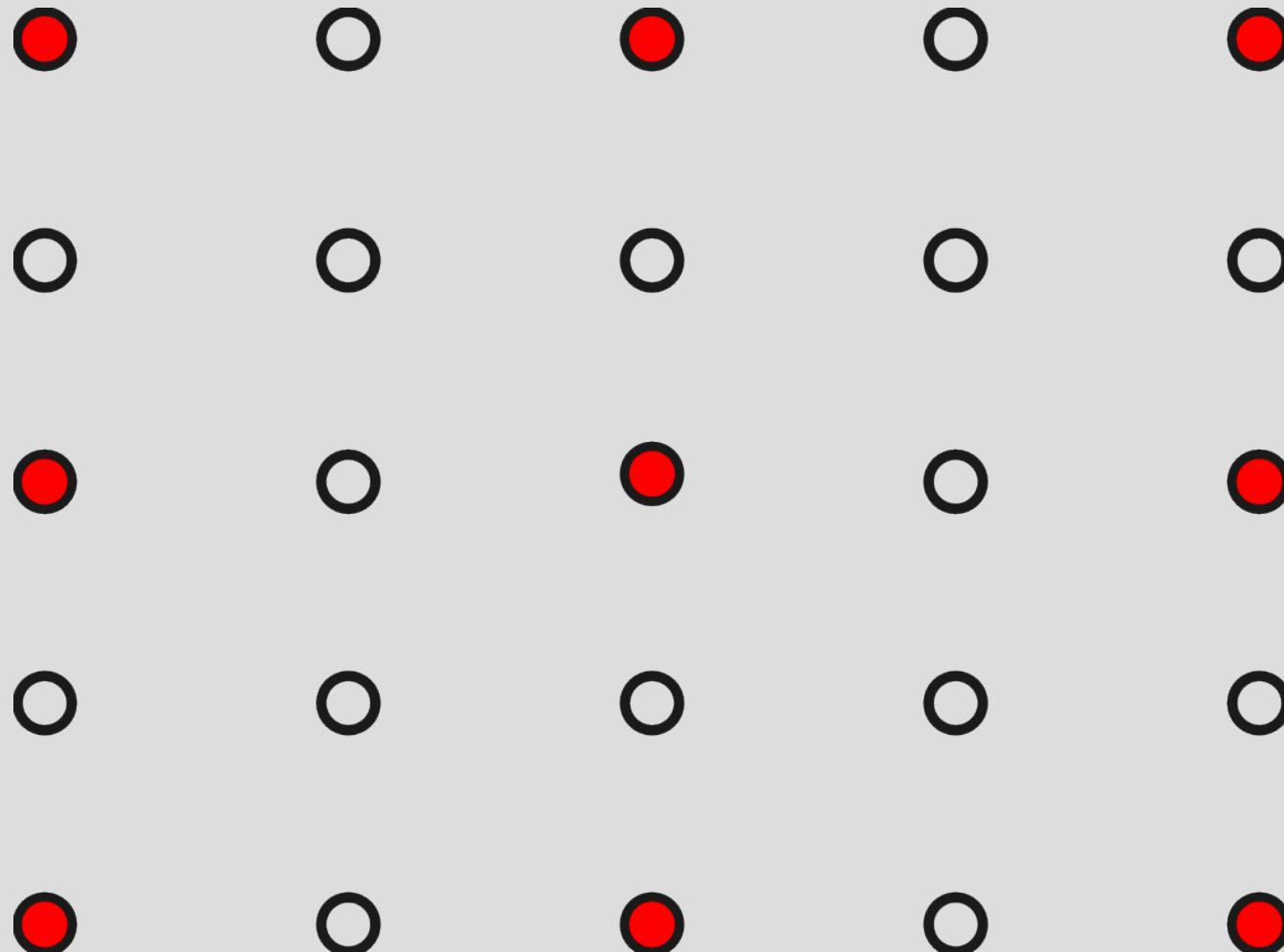
# Separar os dados



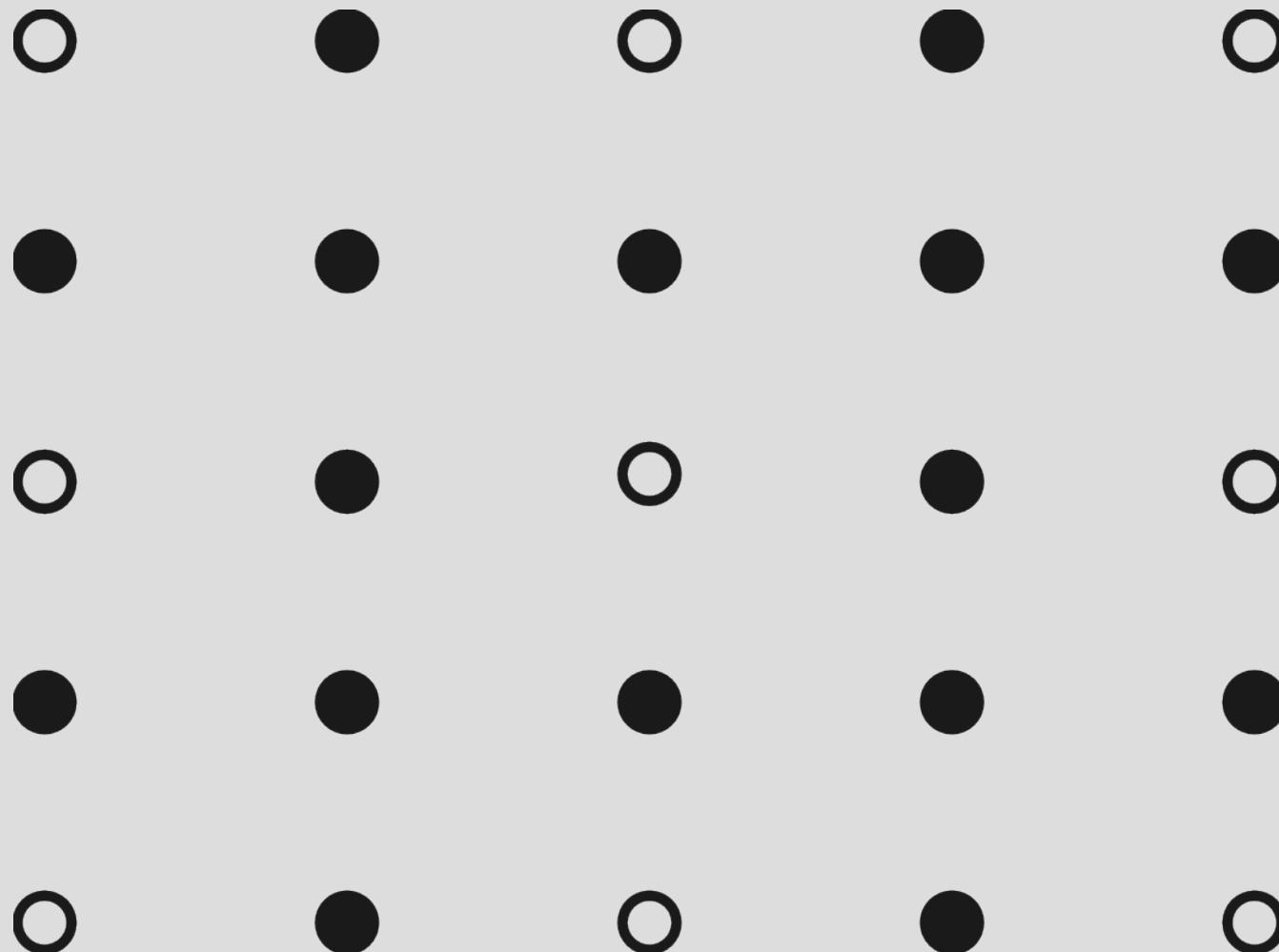
# dado completo



dado inversão

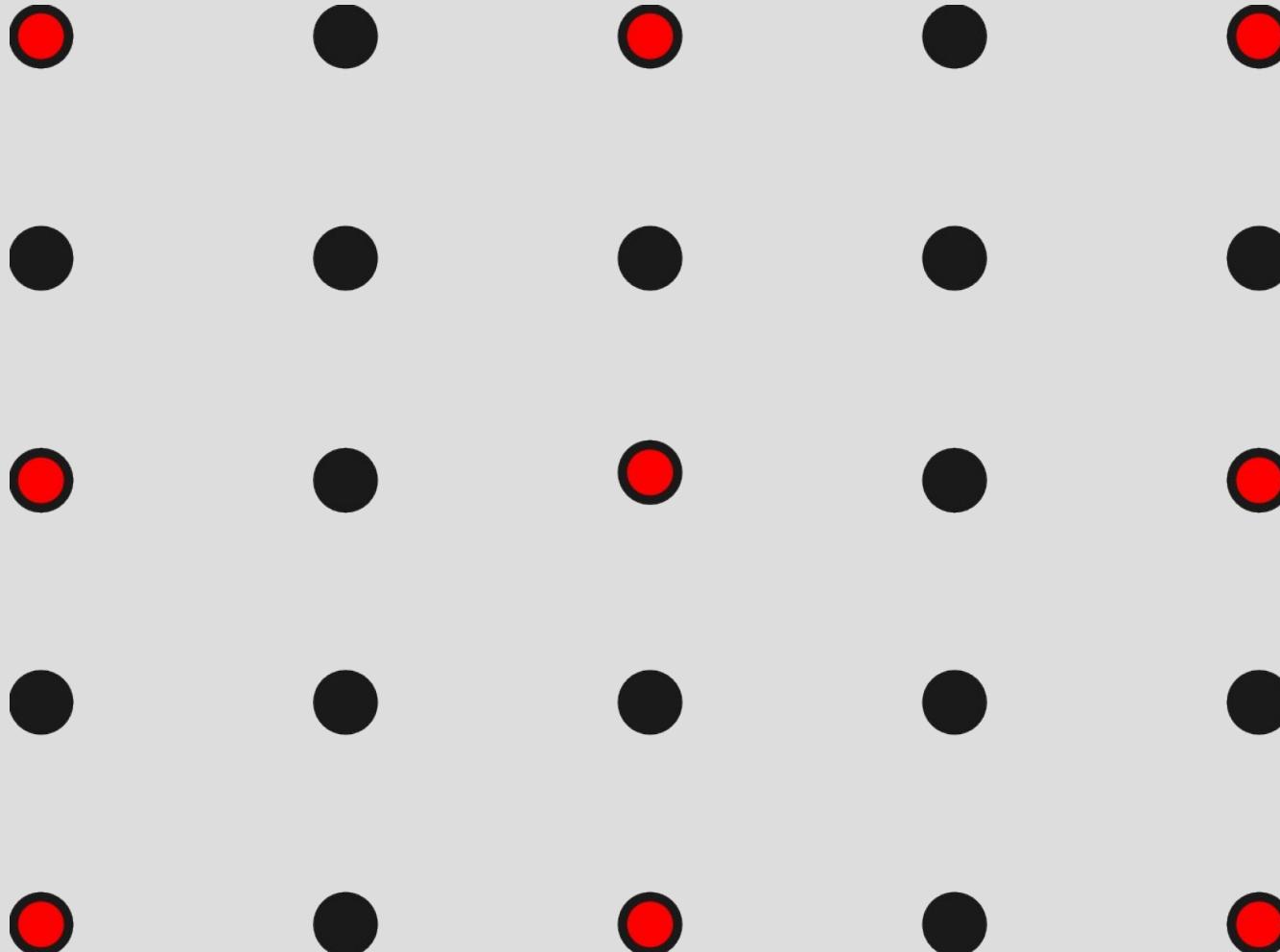


# dado teste



dado teste

dado inversão



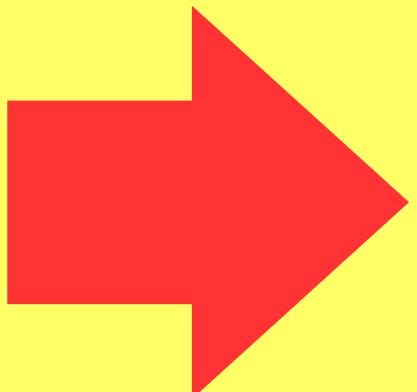
validação

$\mu$

cruzada

$\Delta \rho$

$z_{\text{ref}}$

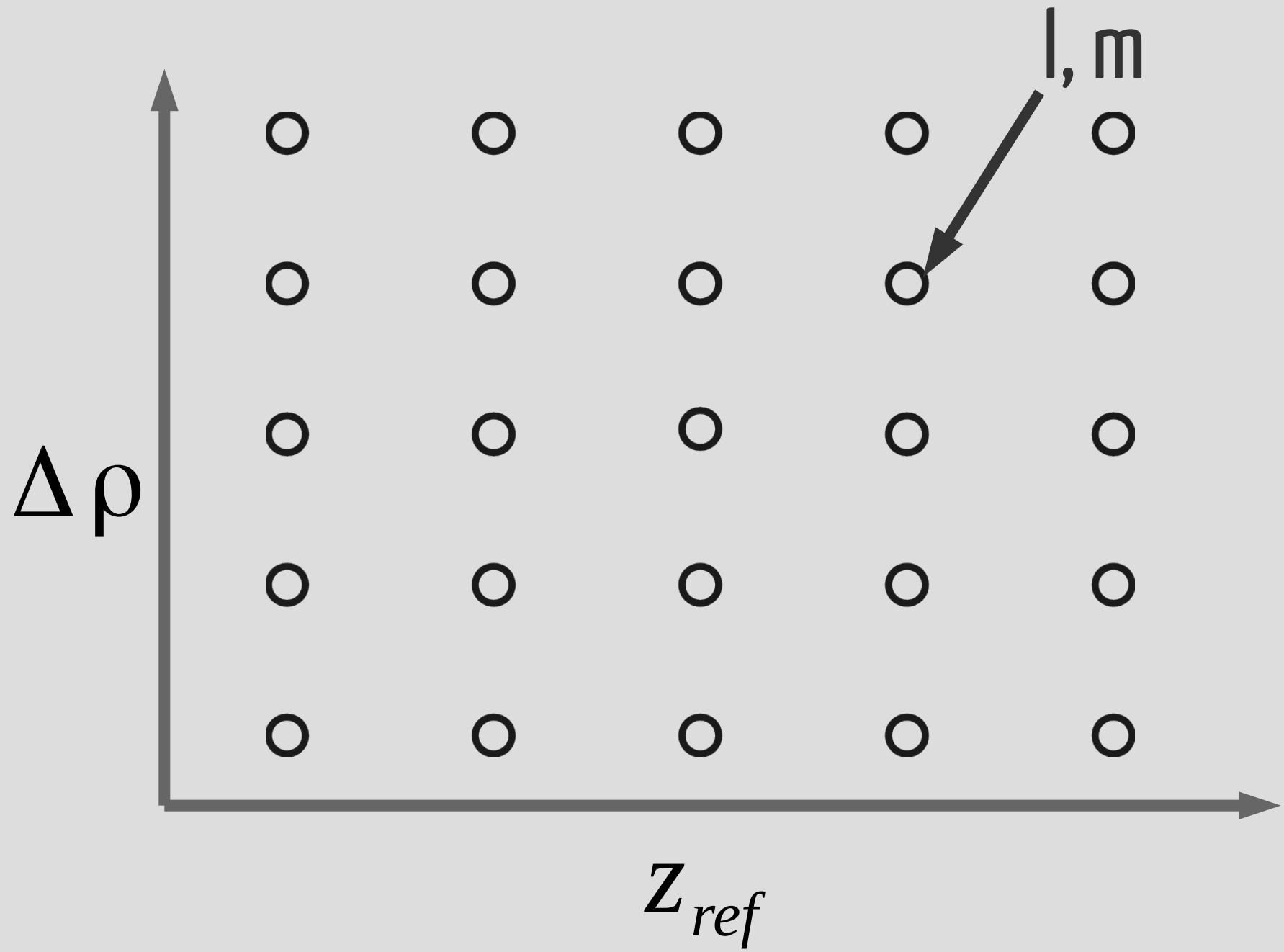


cruzada

 $\Delta \rho$ 

$z_{\text{ref}}$

estimativas  $\bar{z}_s^o$   
pontuais



para cada  $z_{ref,l}$  e  $\Delta \rho_m$ :

para cada  $z_{ref,l}$  e  $\Delta \rho_m$ :

inversão:  $\bar{d}_{inv}^o \rightarrow \hat{\bar{p}}^{l,m}$

para cada  $z_{ref,l}$  e  $\Delta \rho_m$ :

inversão:  $\bar{d}_{inv}^o \rightarrow \hat{\bar{p}}^{l,m}$

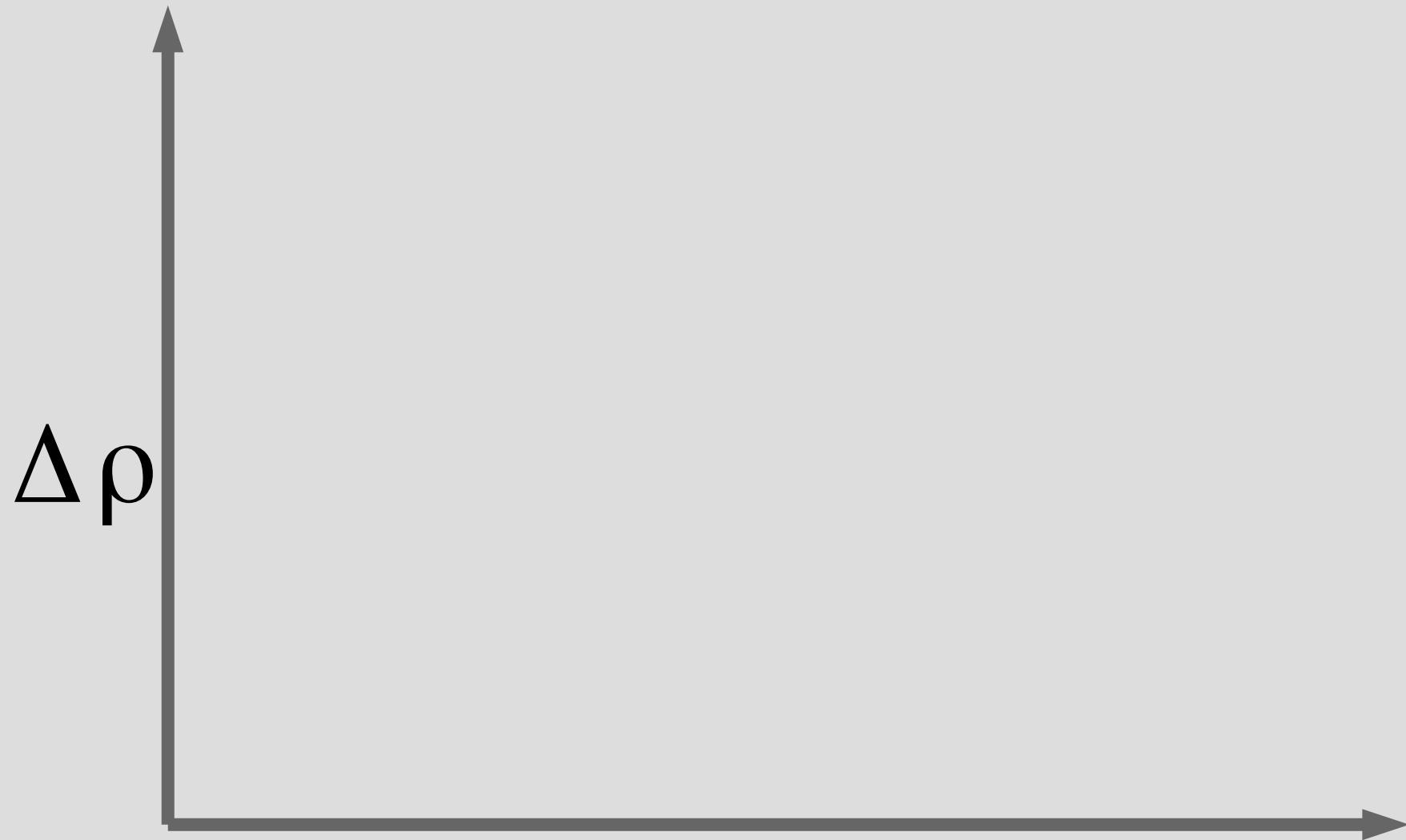
interpolar  $\hat{\bar{p}}^{l,m} \rightarrow \bar{z}_s^{l,m}$

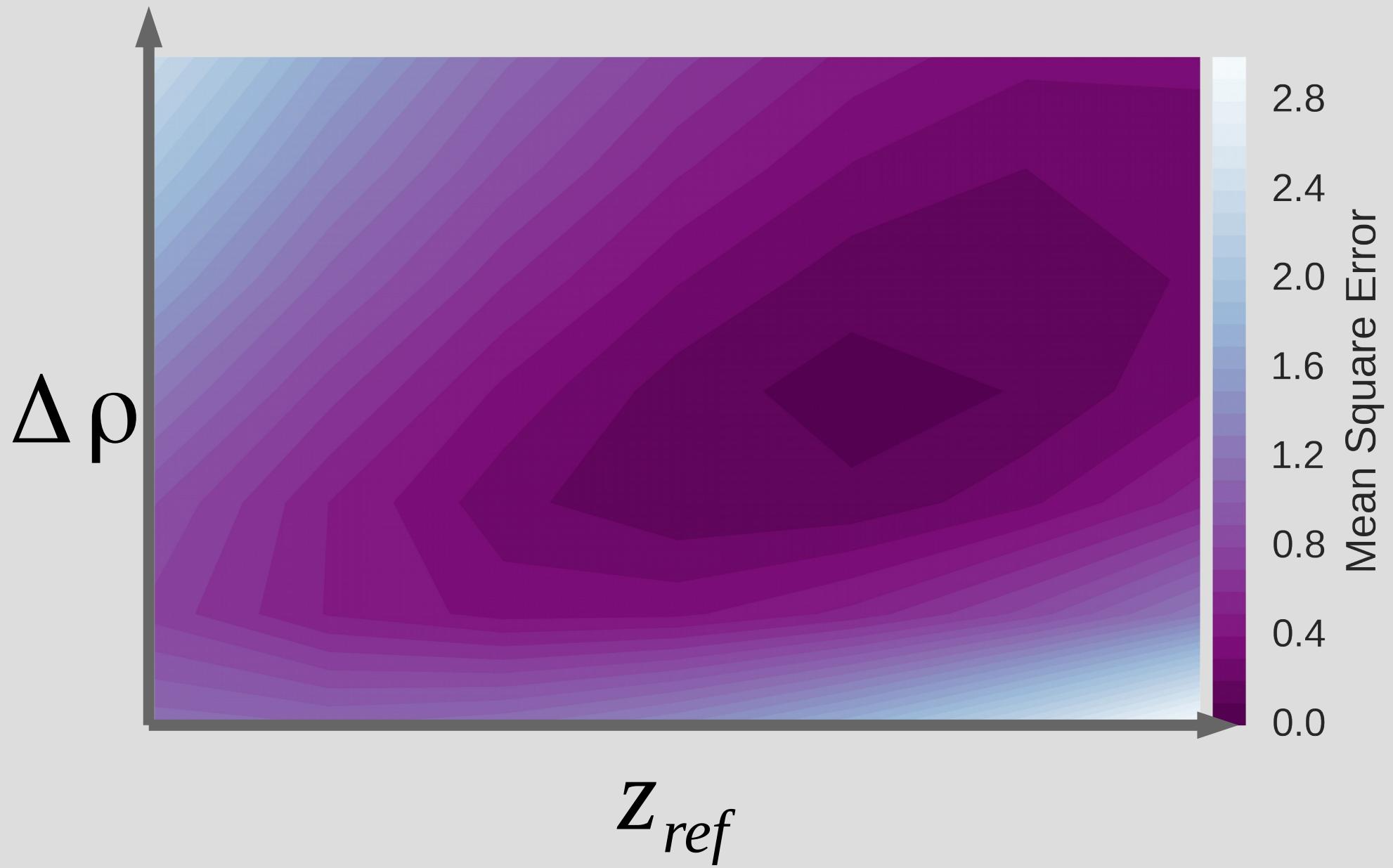
para cada  $z_{ref,l}$  e  $\Delta \rho_m$ :

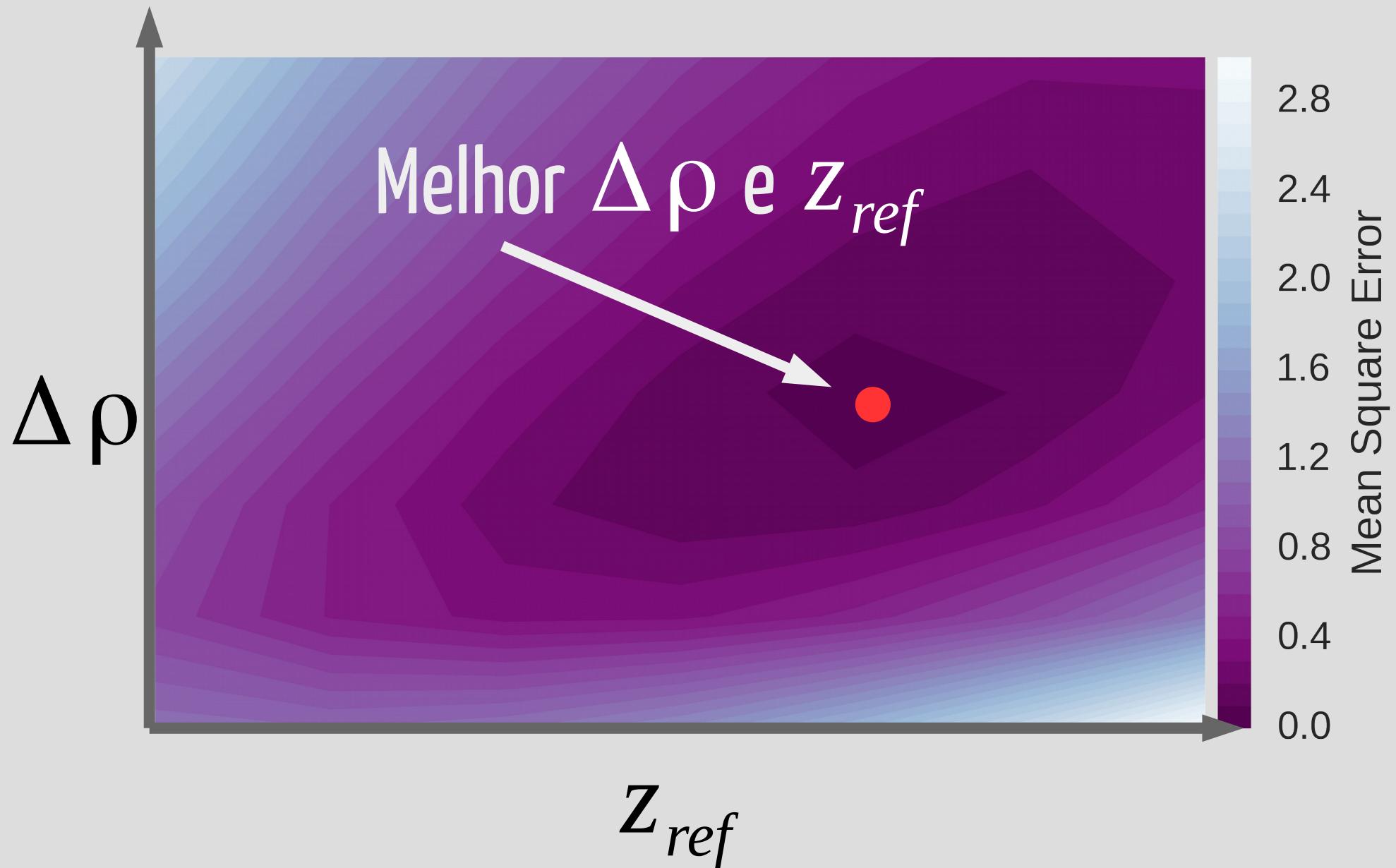
inversão:  $\bar{d}_{inv}^o \rightarrow \hat{\bar{p}}^{l,m}$

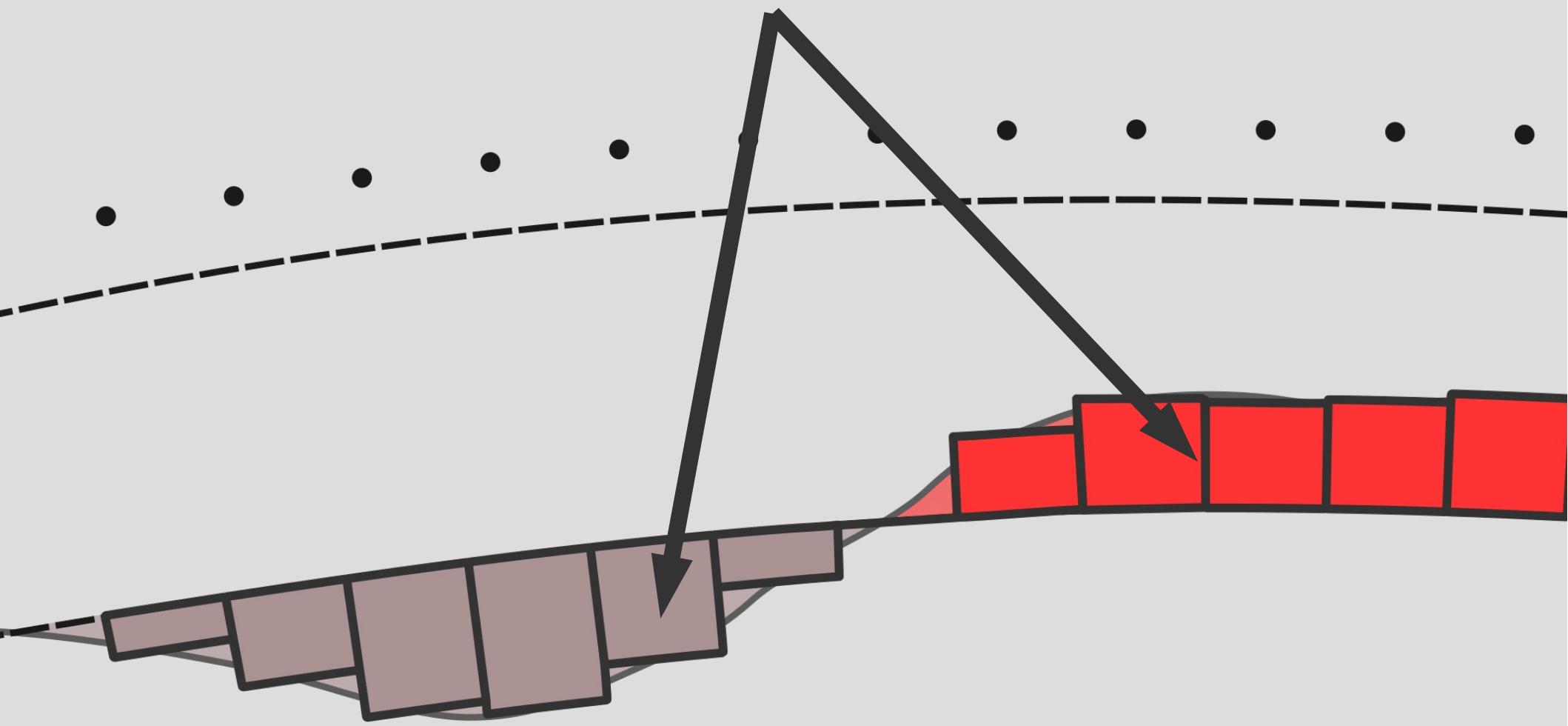
interpolar  $\hat{\bar{p}}^{l,m} \rightarrow \bar{z}_s^{l,m}$

$$MSE_{l,m} = \frac{\|\bar{z}_s^o - \bar{z}_s^{l,m}\|^2}{N_s}$$

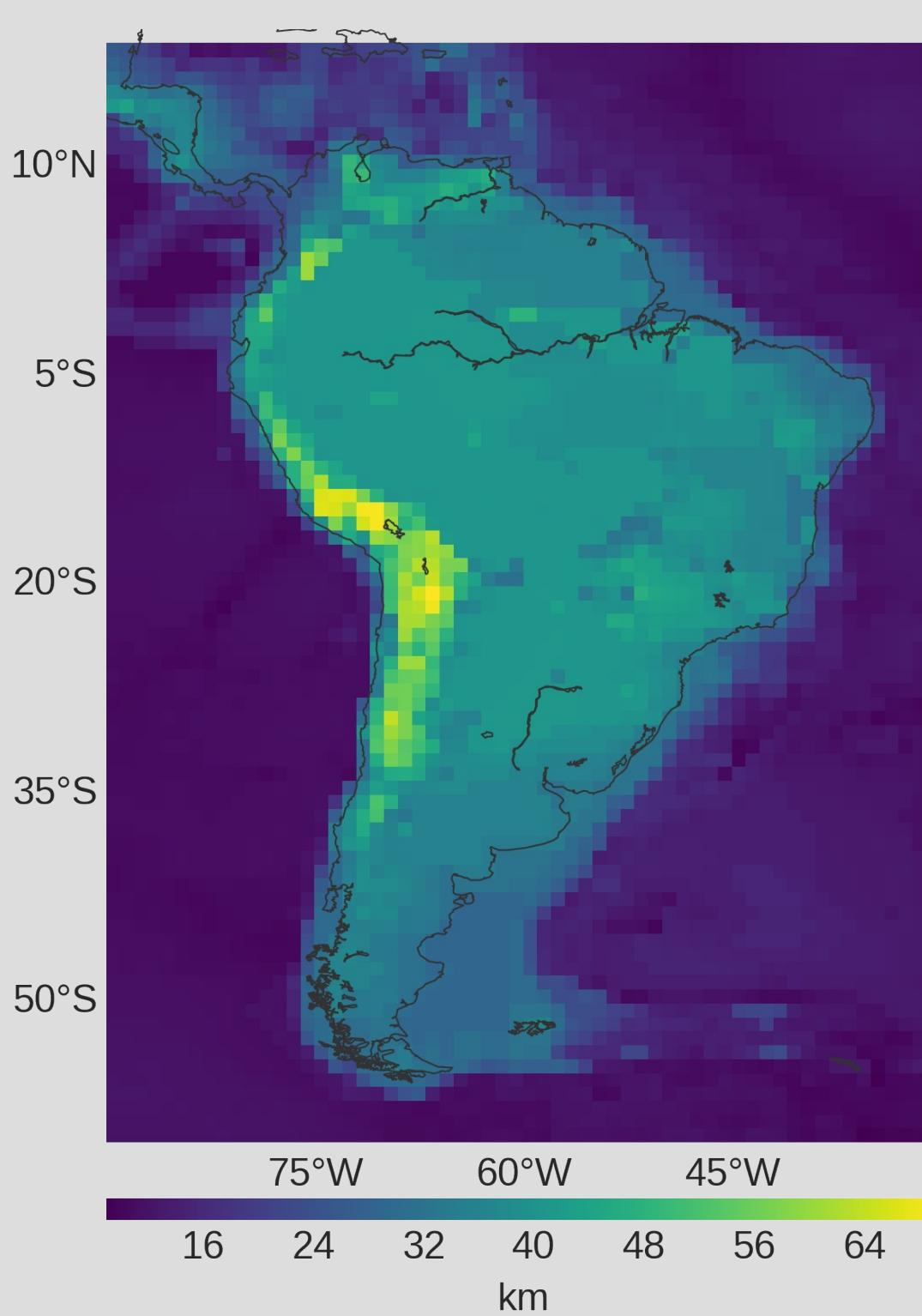






$\hat{\bar{p}}$ 

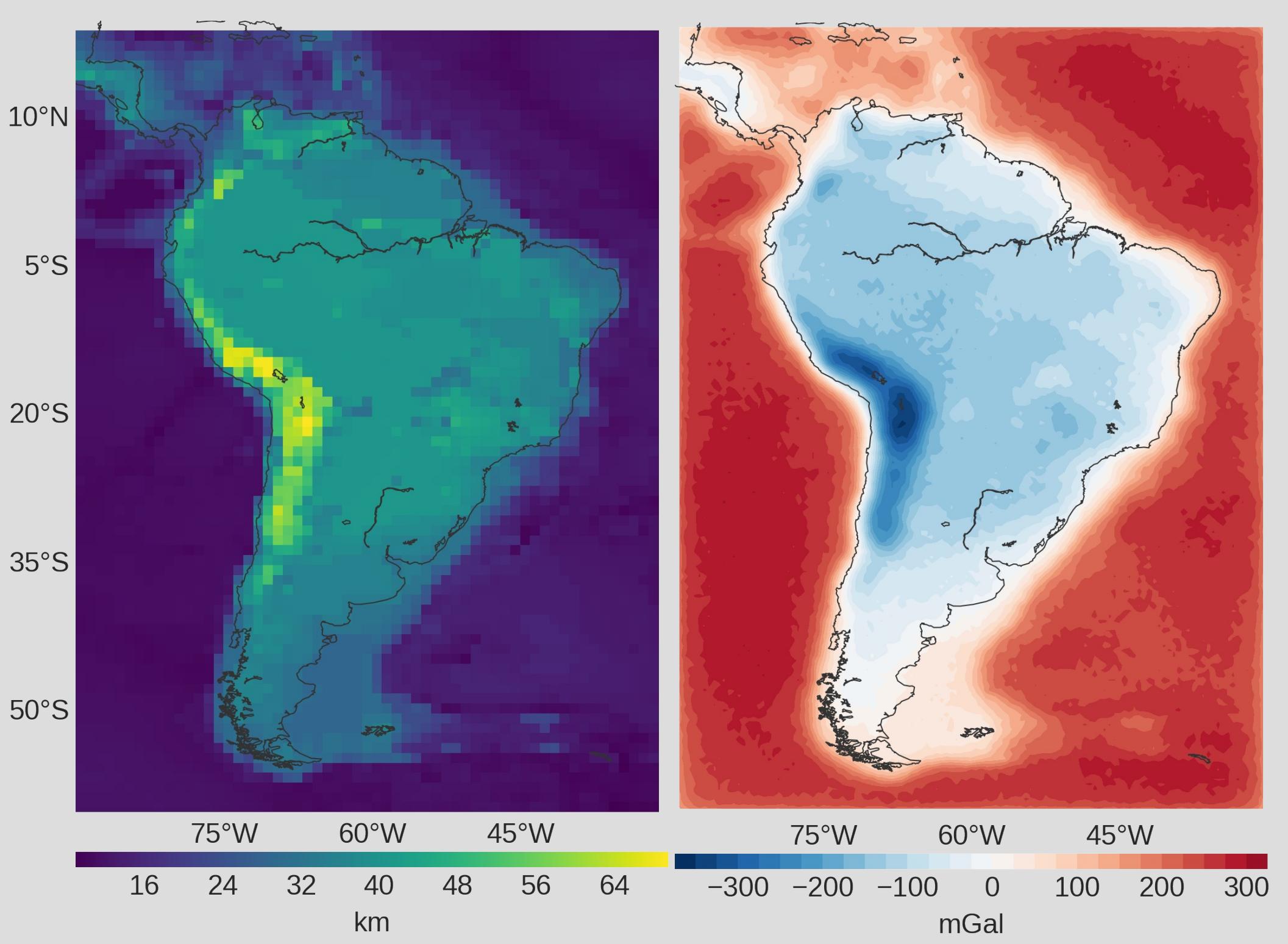
sintético

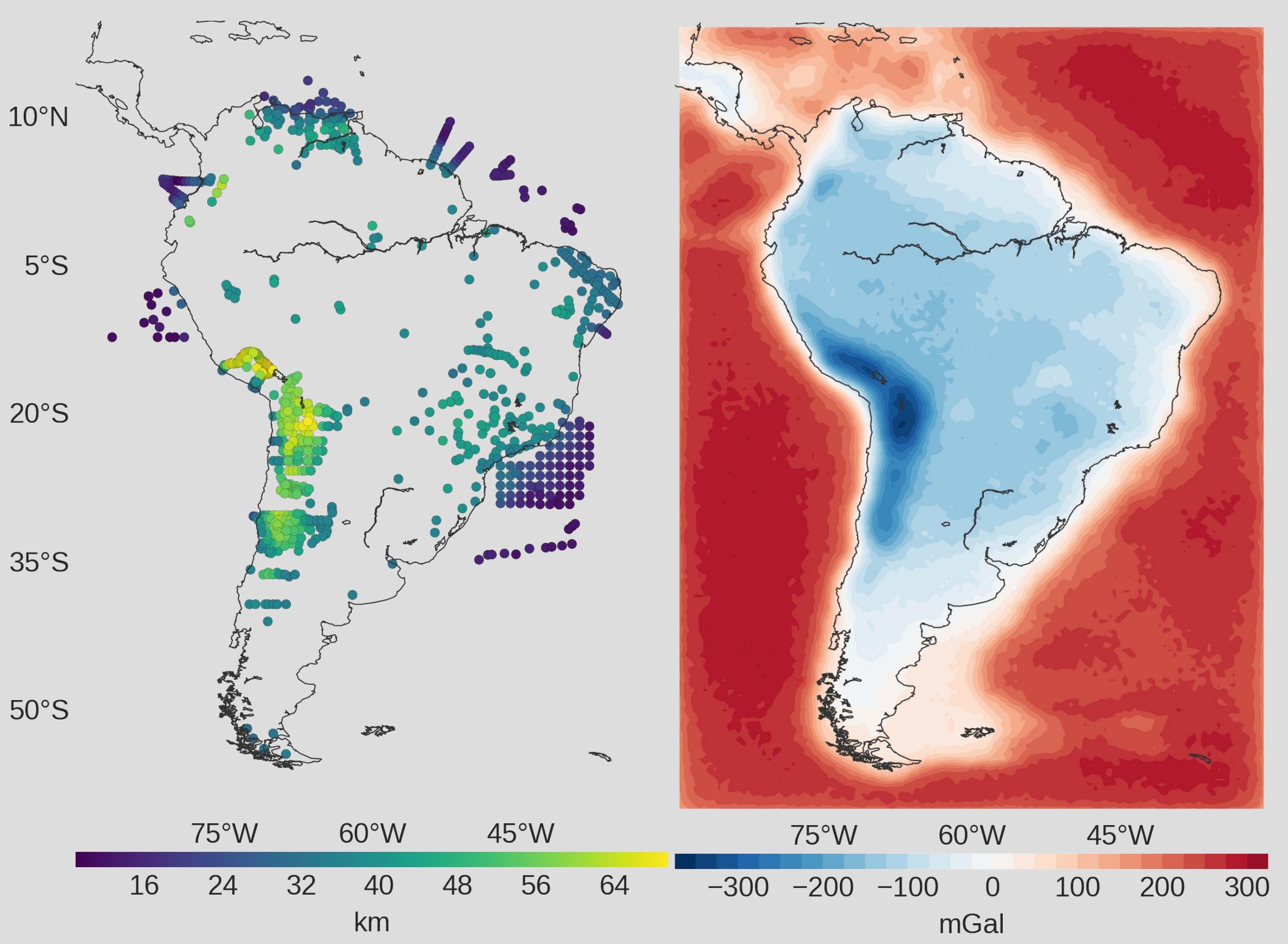


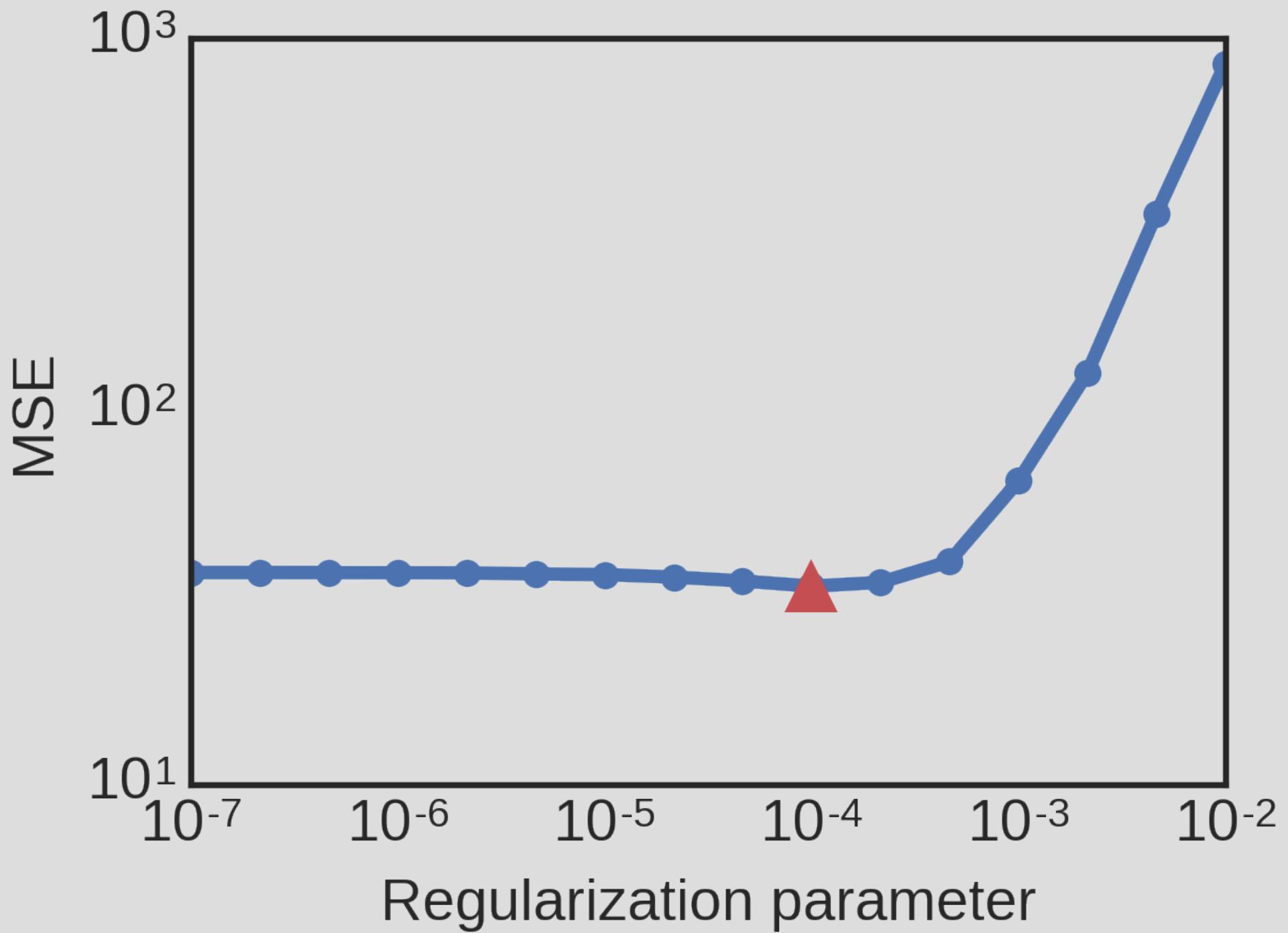
# Moho CRUST1.0

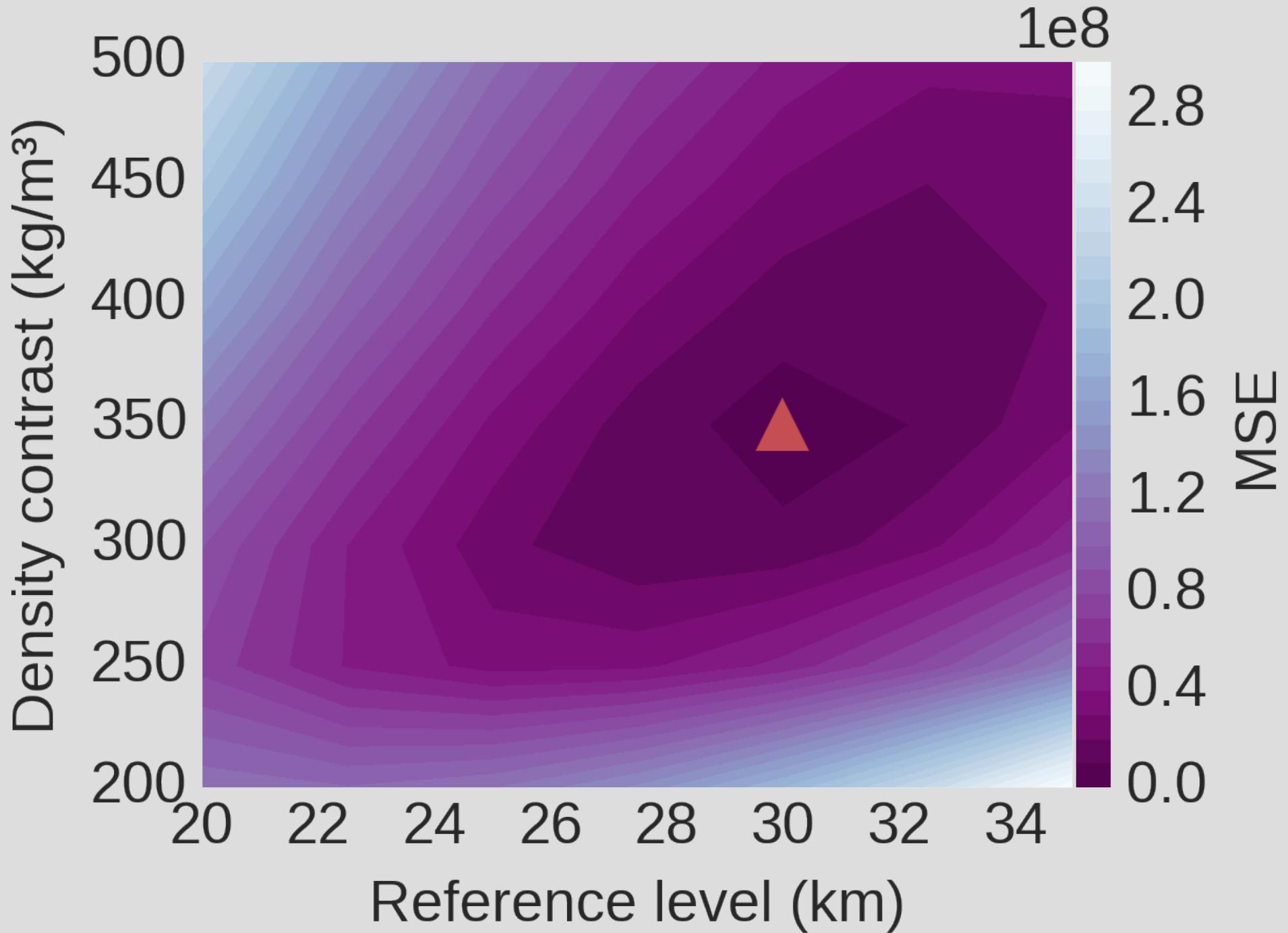
$$\Delta \rho = 350 \text{ } kg \cdot m^{-3}$$

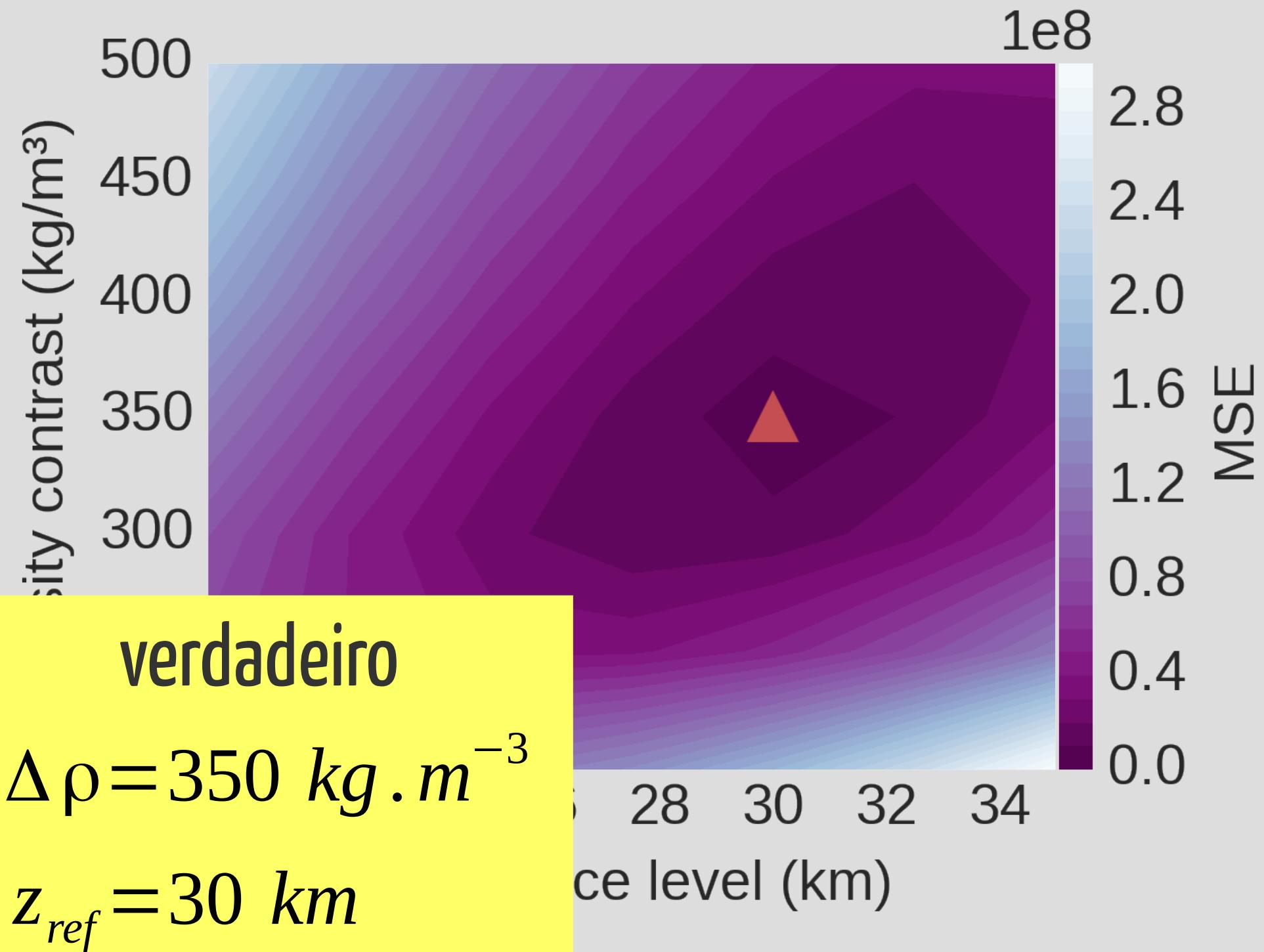
$$z_{ref} = 30 \text{ } km$$

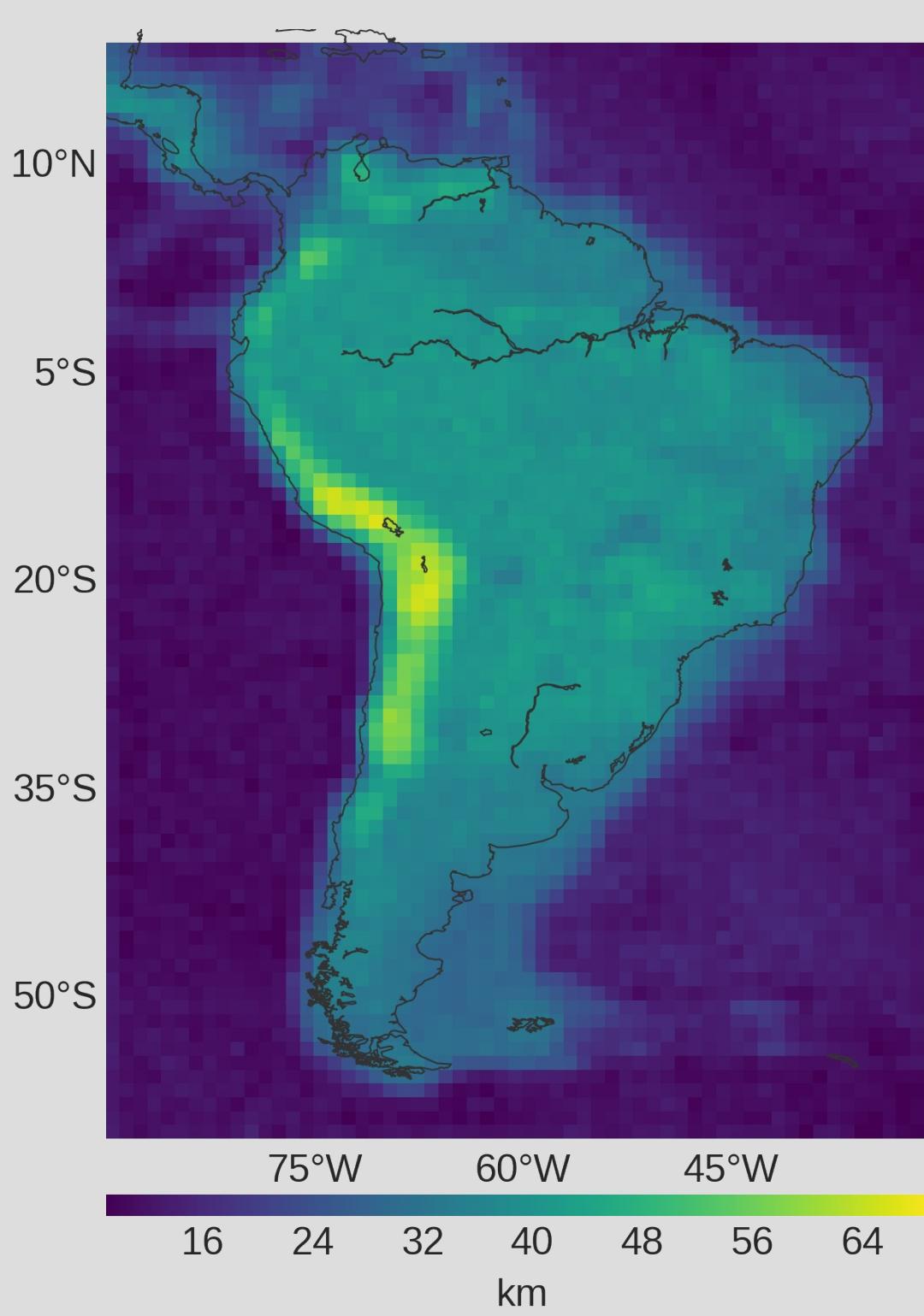


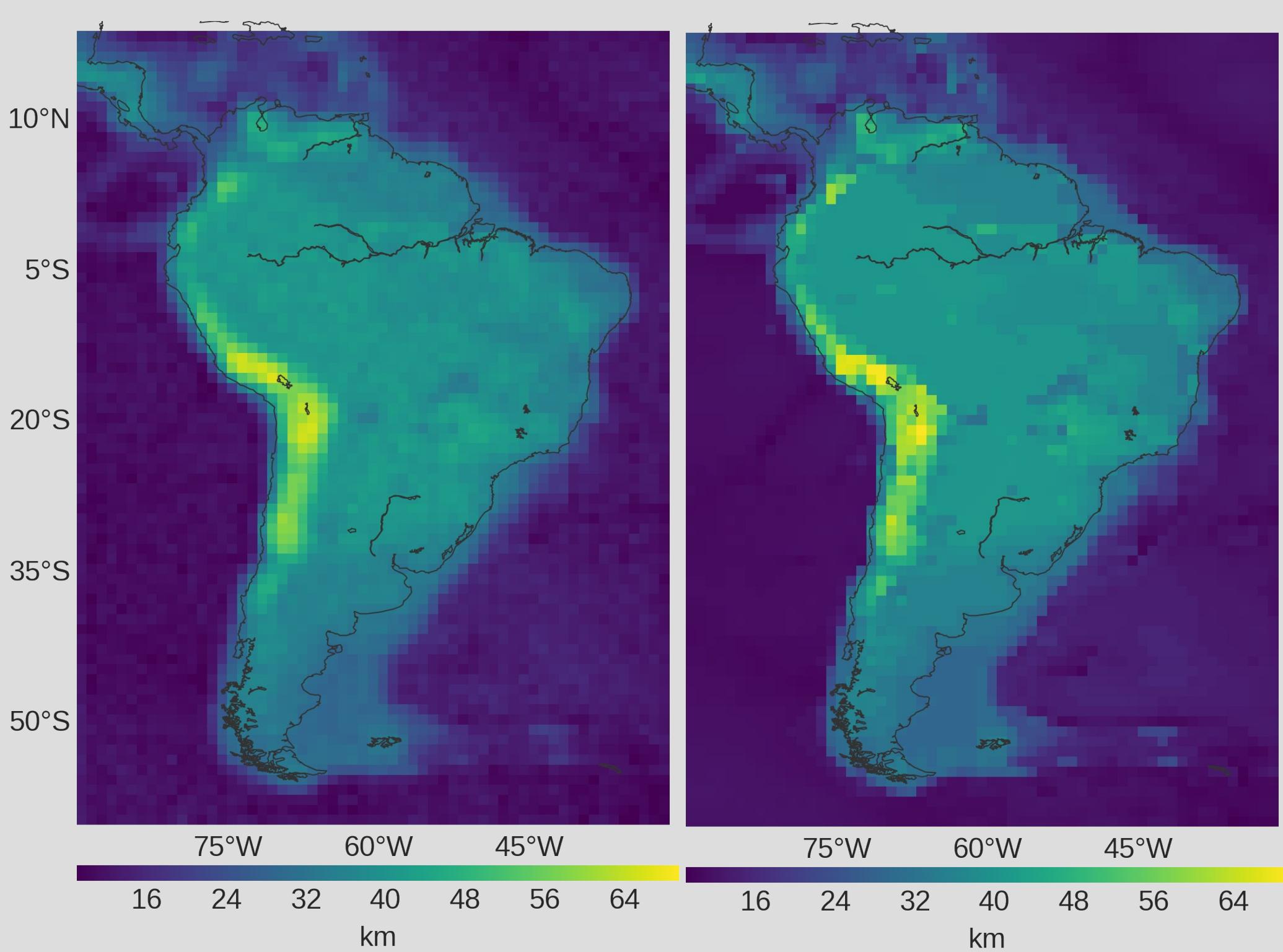


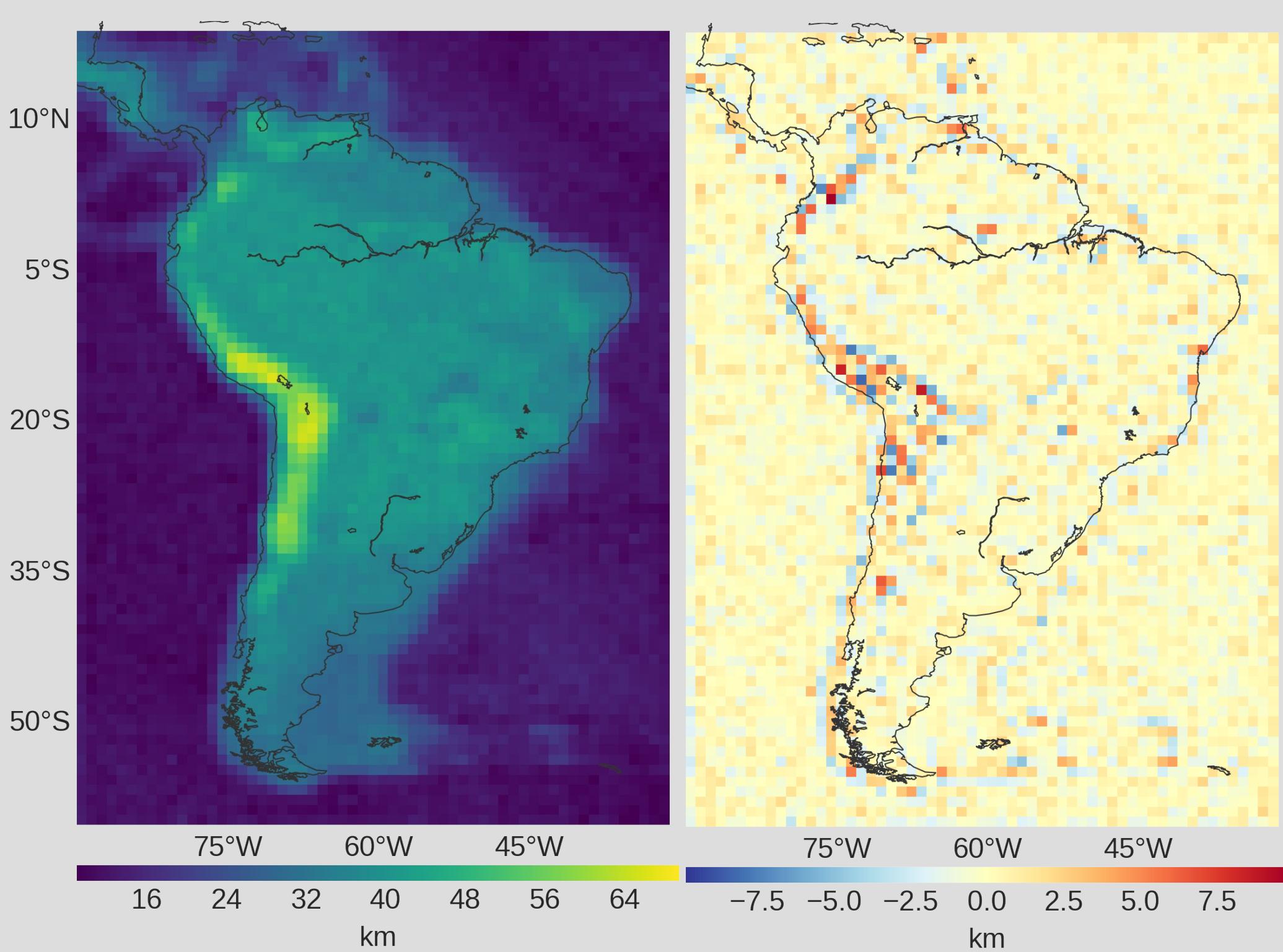


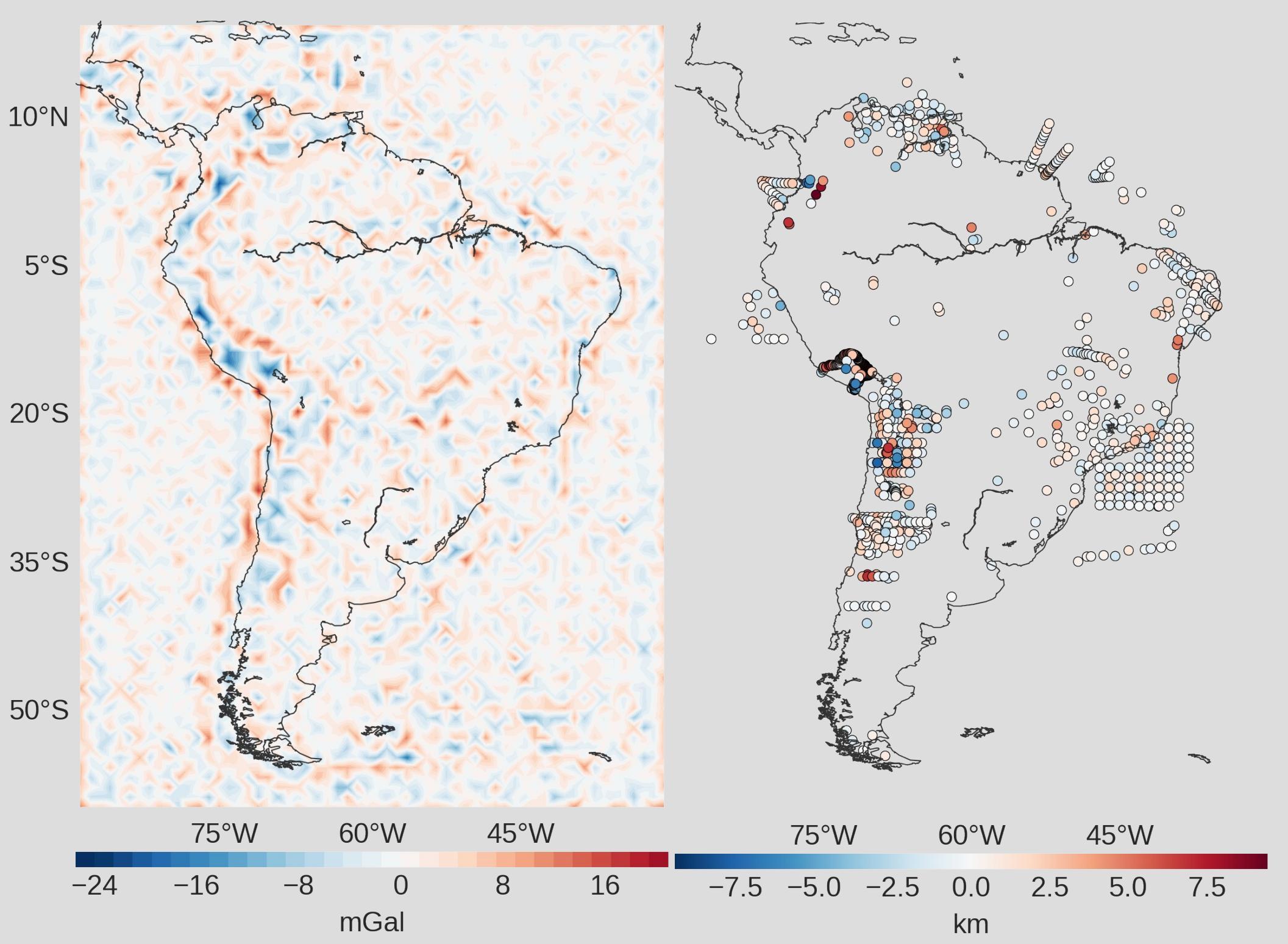








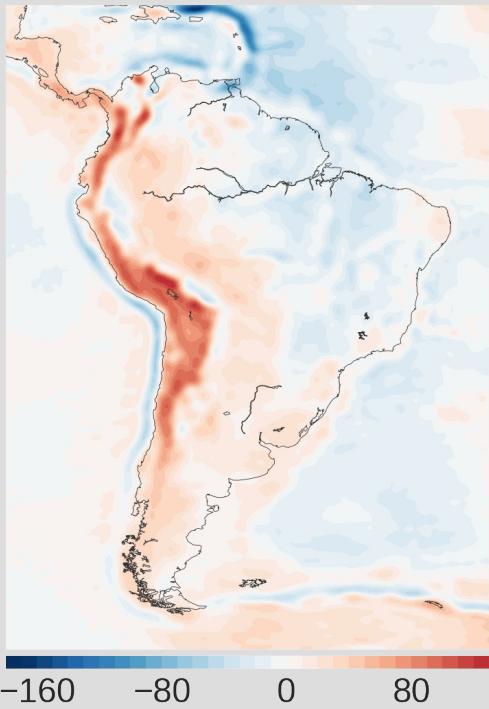




América

do Sul

Gravity disturbance (mGal)



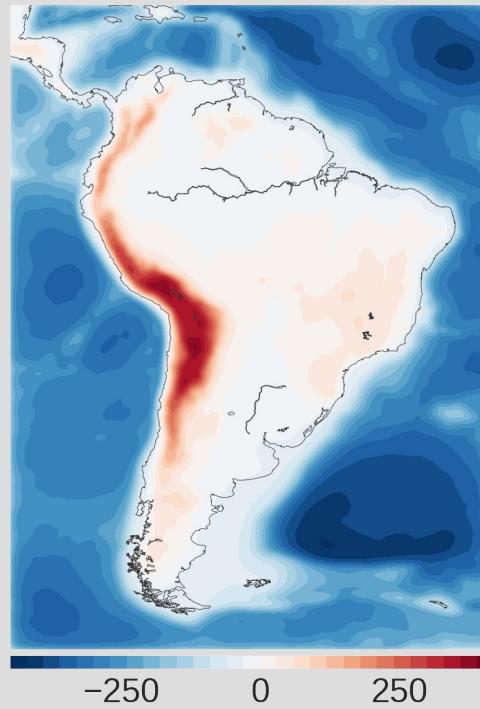
-160 -80 0 80

Topography (km)



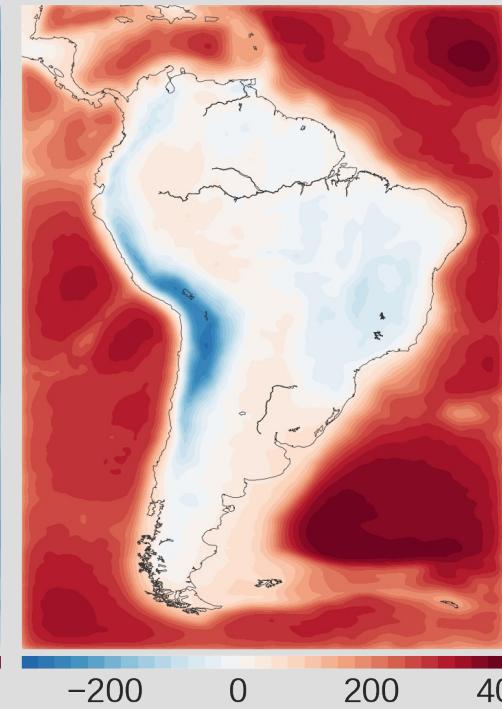
-6 -3 0 3 6

Topographic effect (mGal)



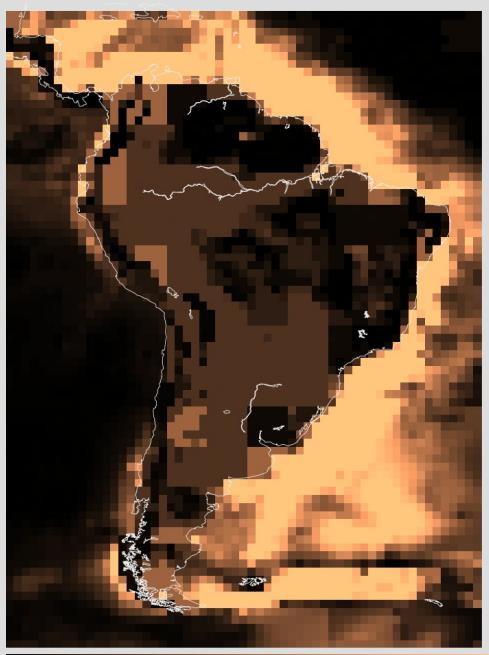
-250 0 250

Bouguer disturbance (mGal)



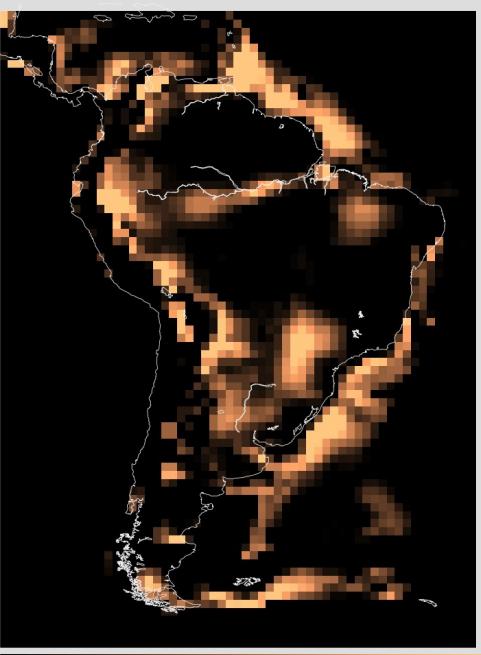
-200 0 200 400

Upper sediments (km)



0.0 0.6 1.2 1.8 0.0

Middle sediments (km)



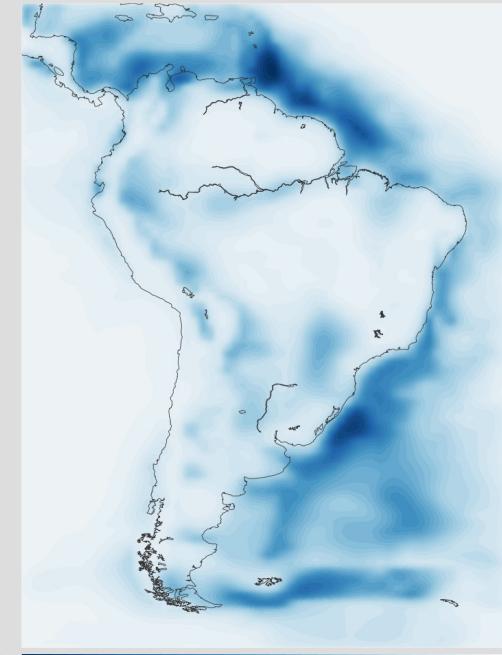
1.5 3.0 0.0

Lower sediments (km)

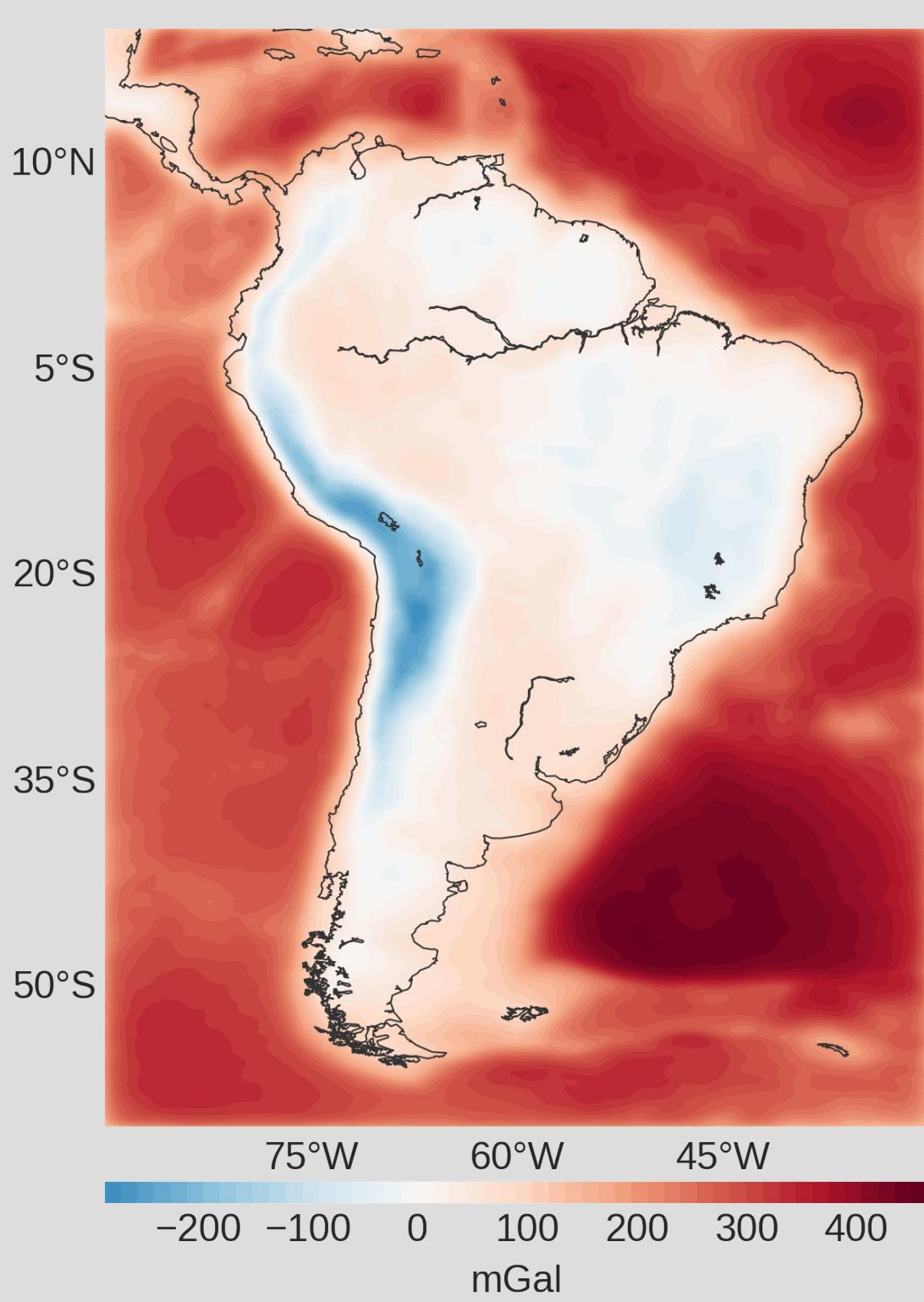


3 6 9 0

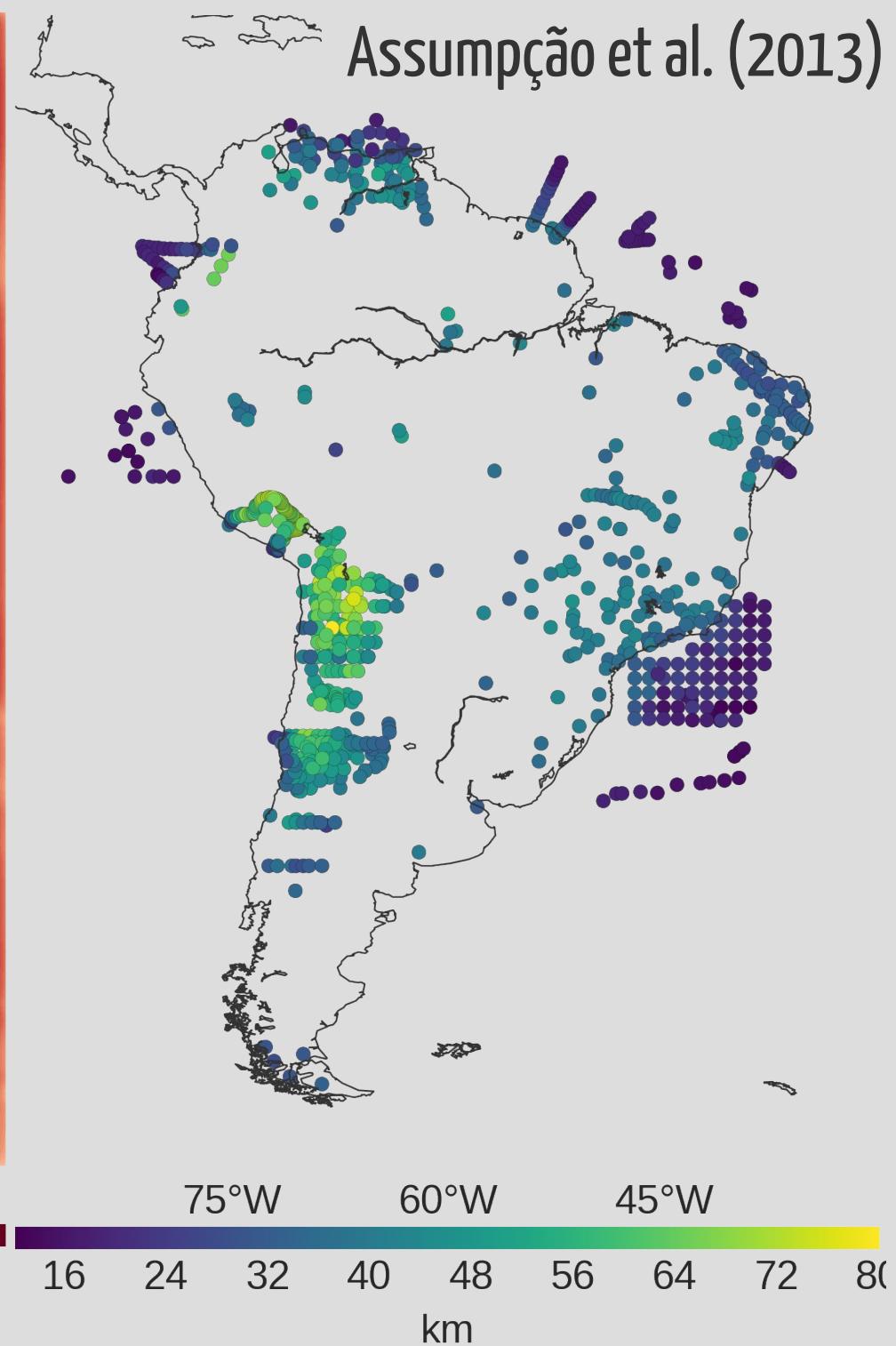
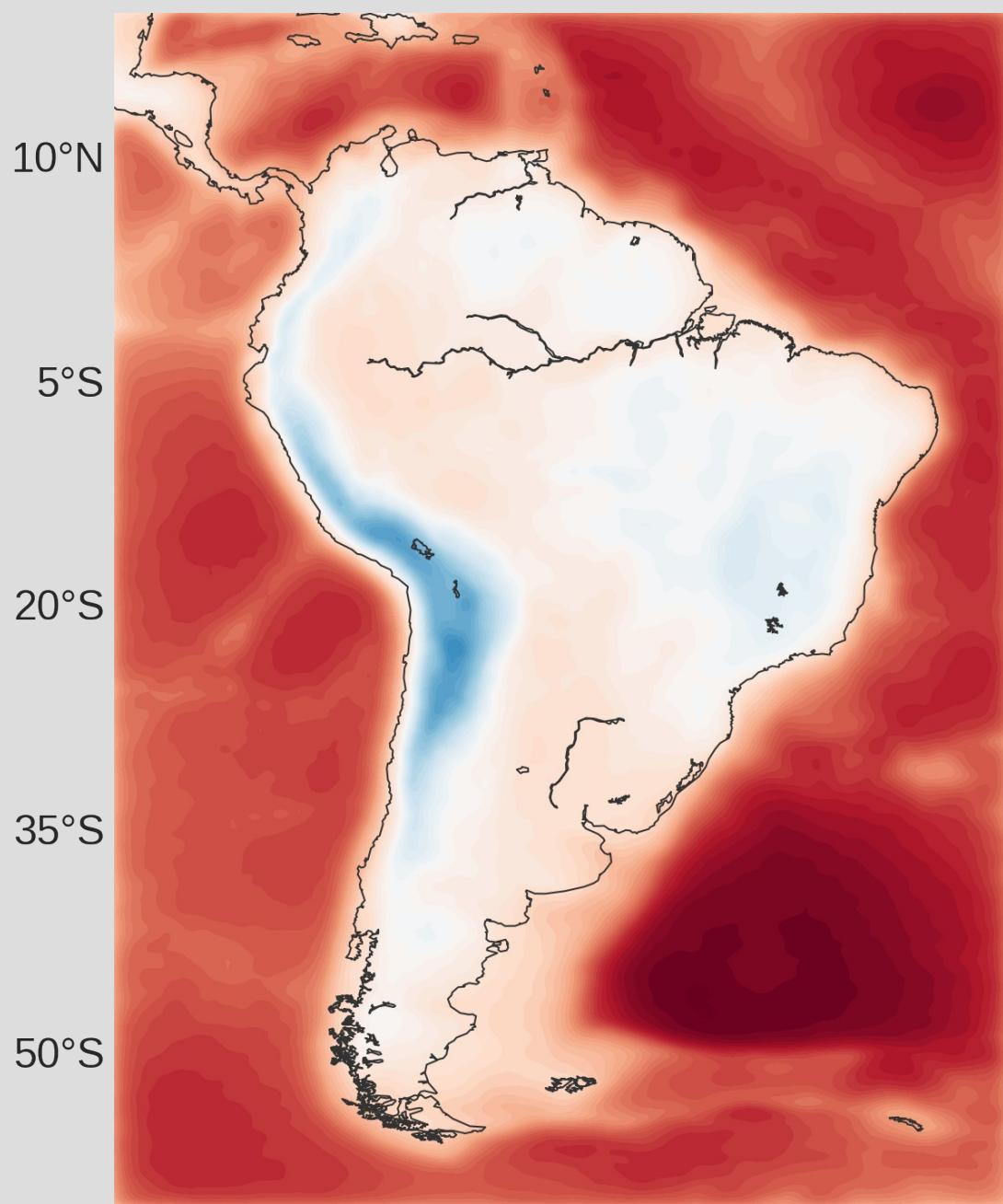
Total sediment effect (mGal)

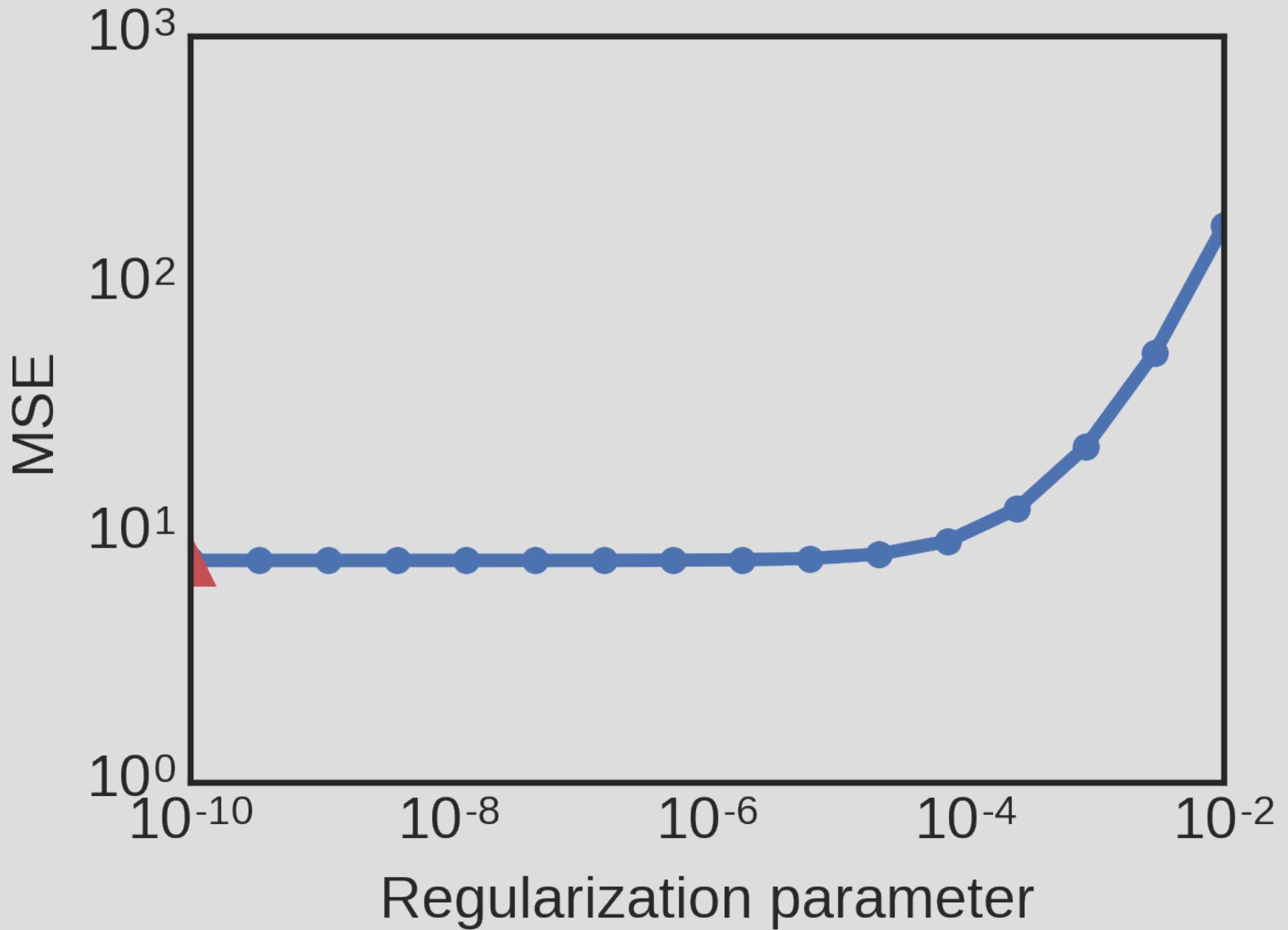


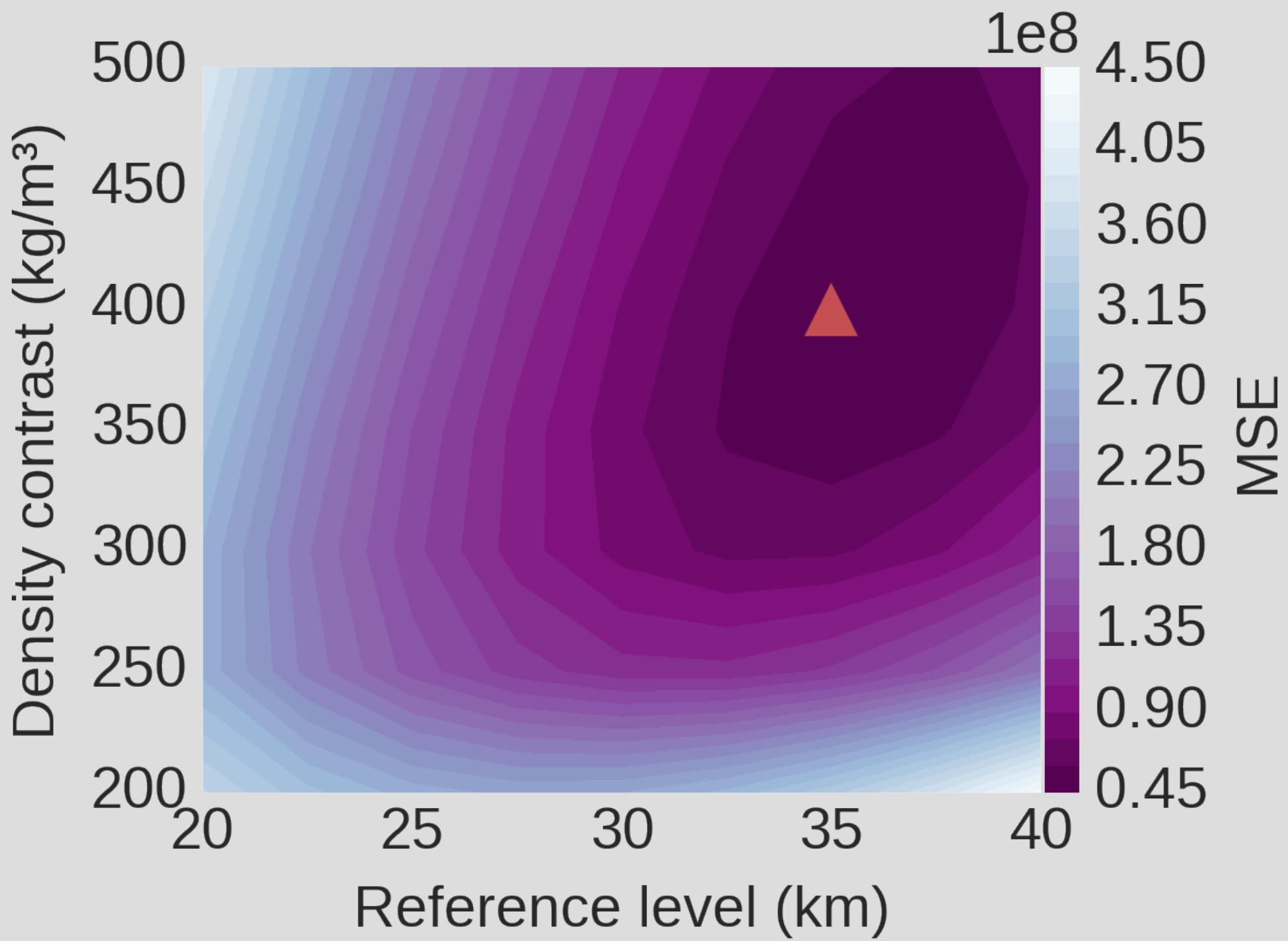
-120 -90 -60 -30

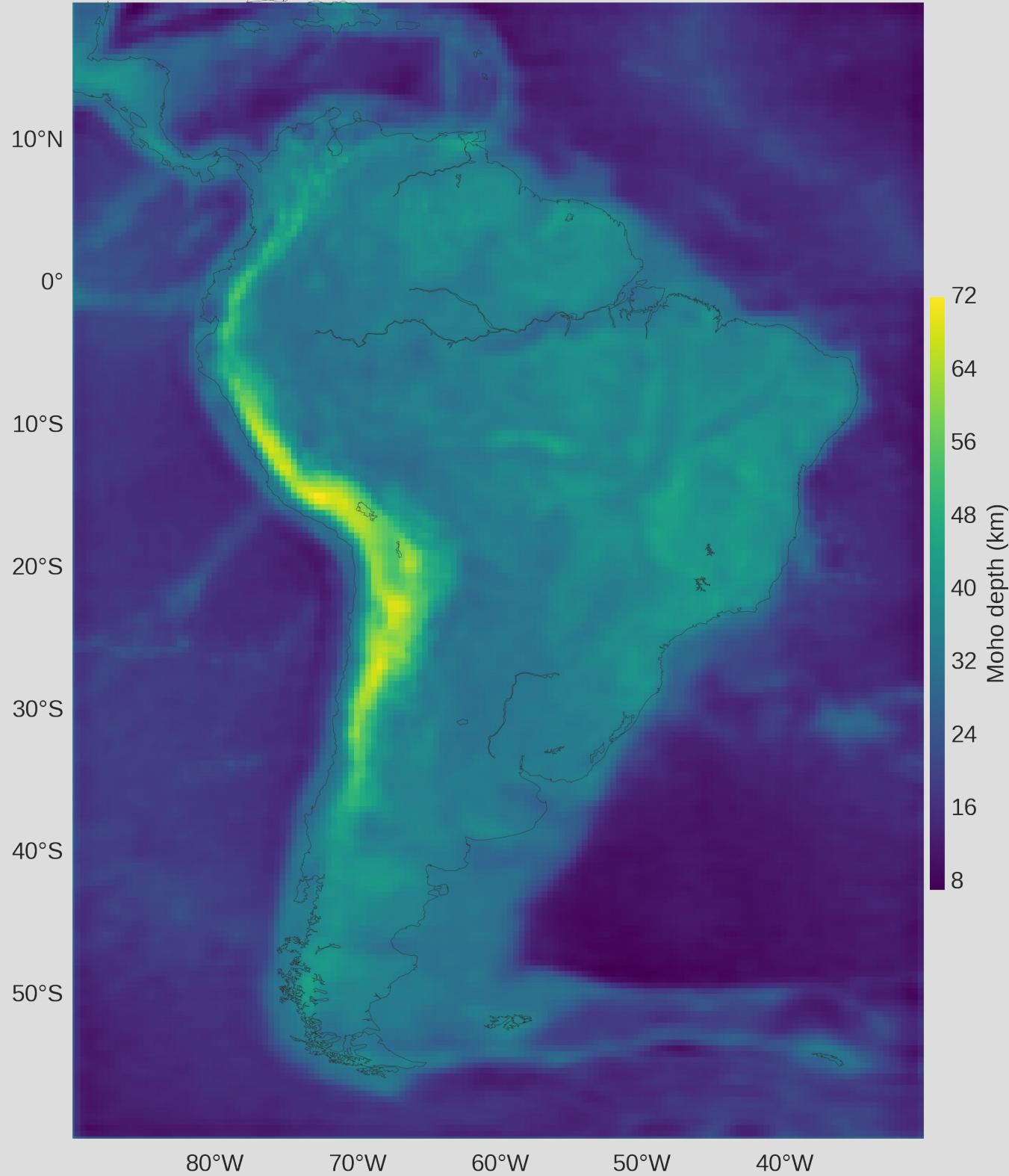


Assumpção et al. (2013)

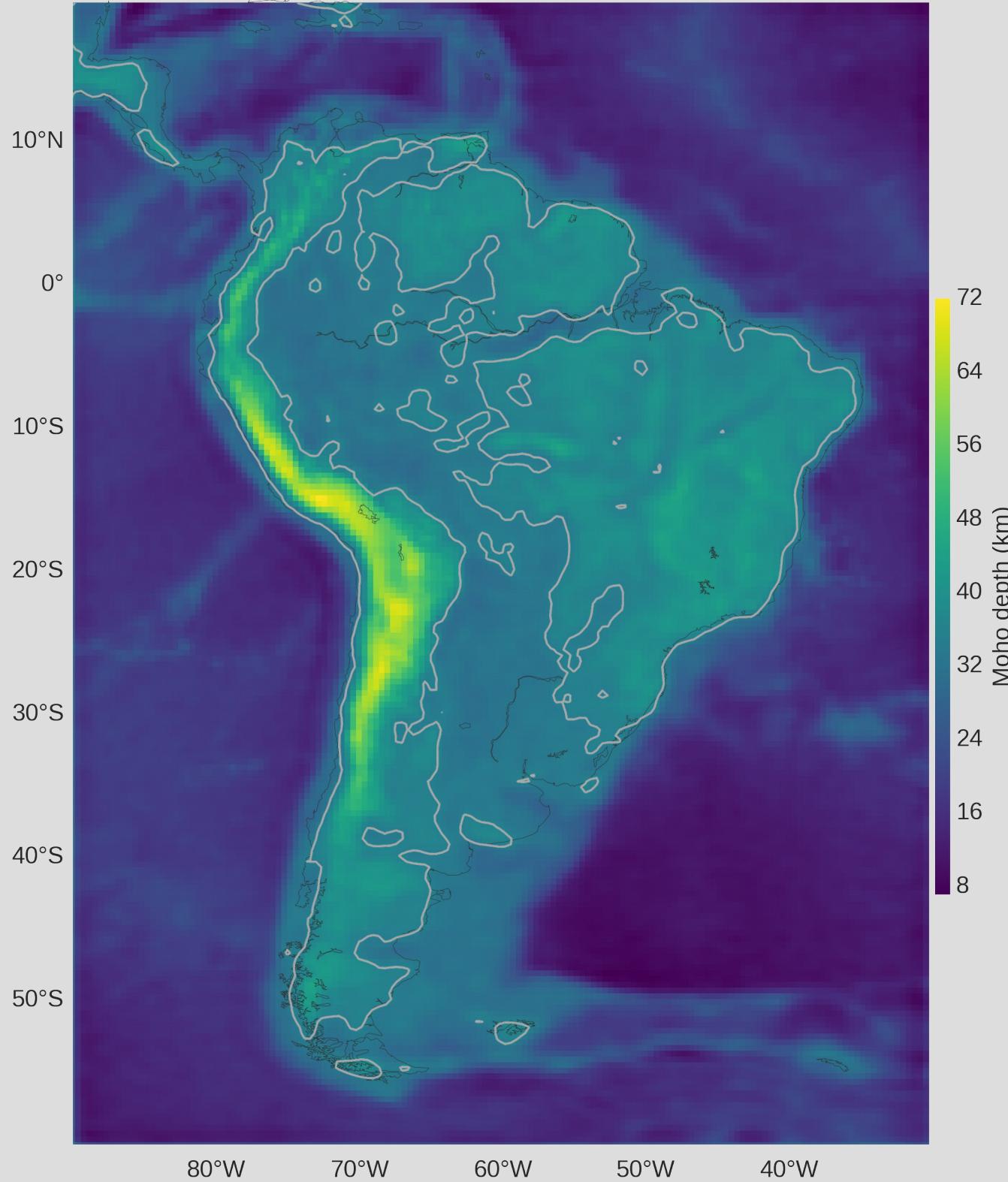




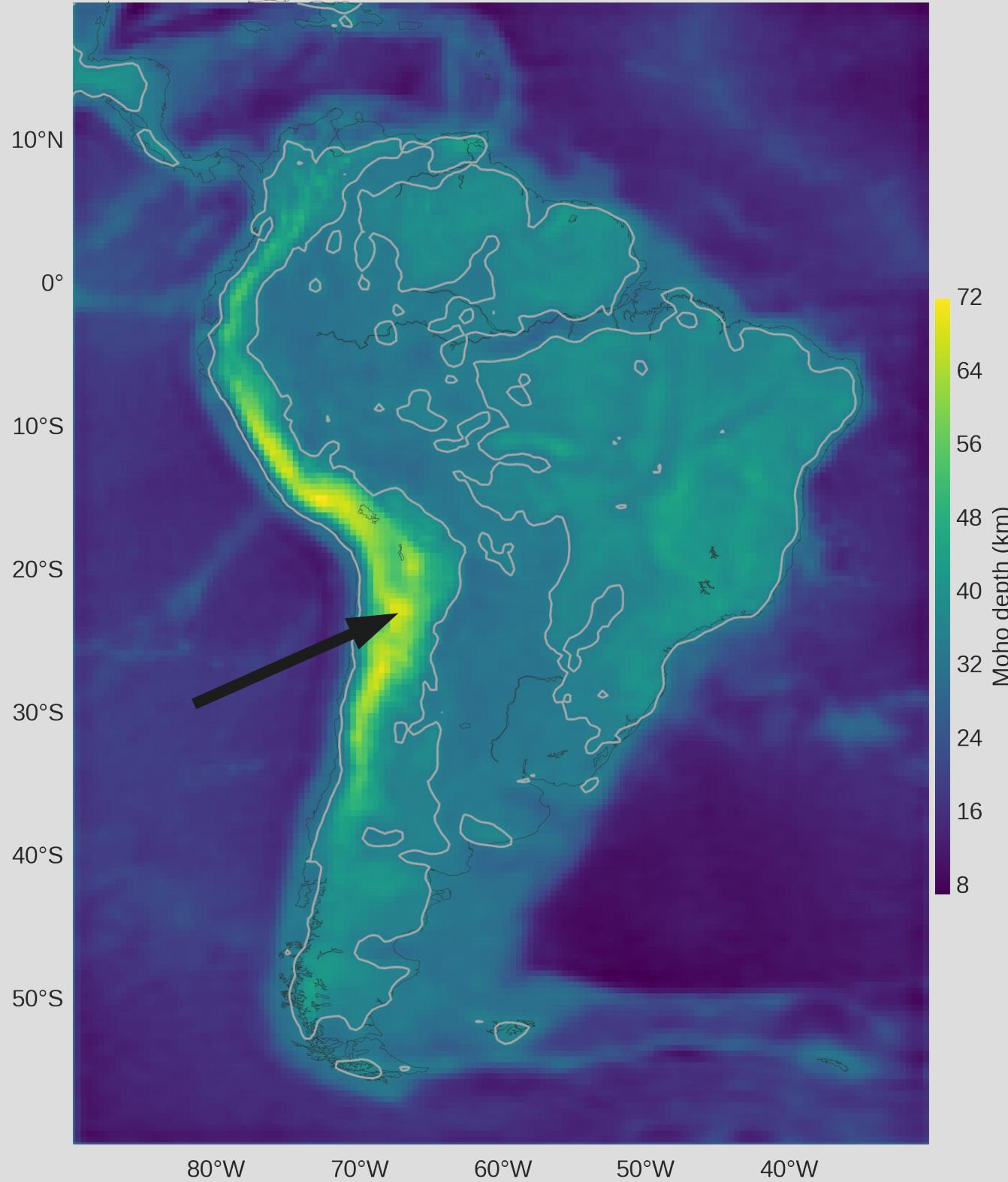




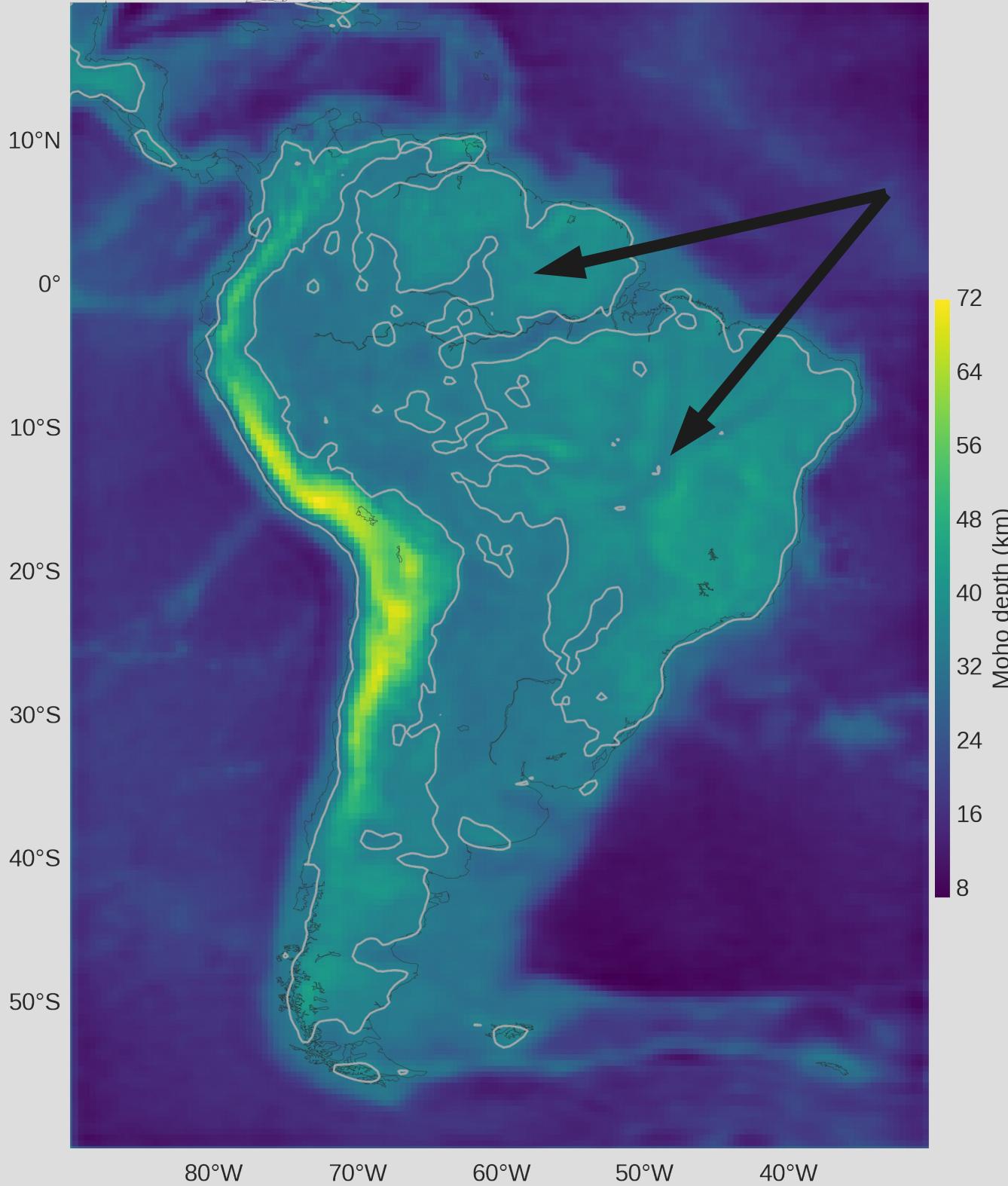
**35 km**



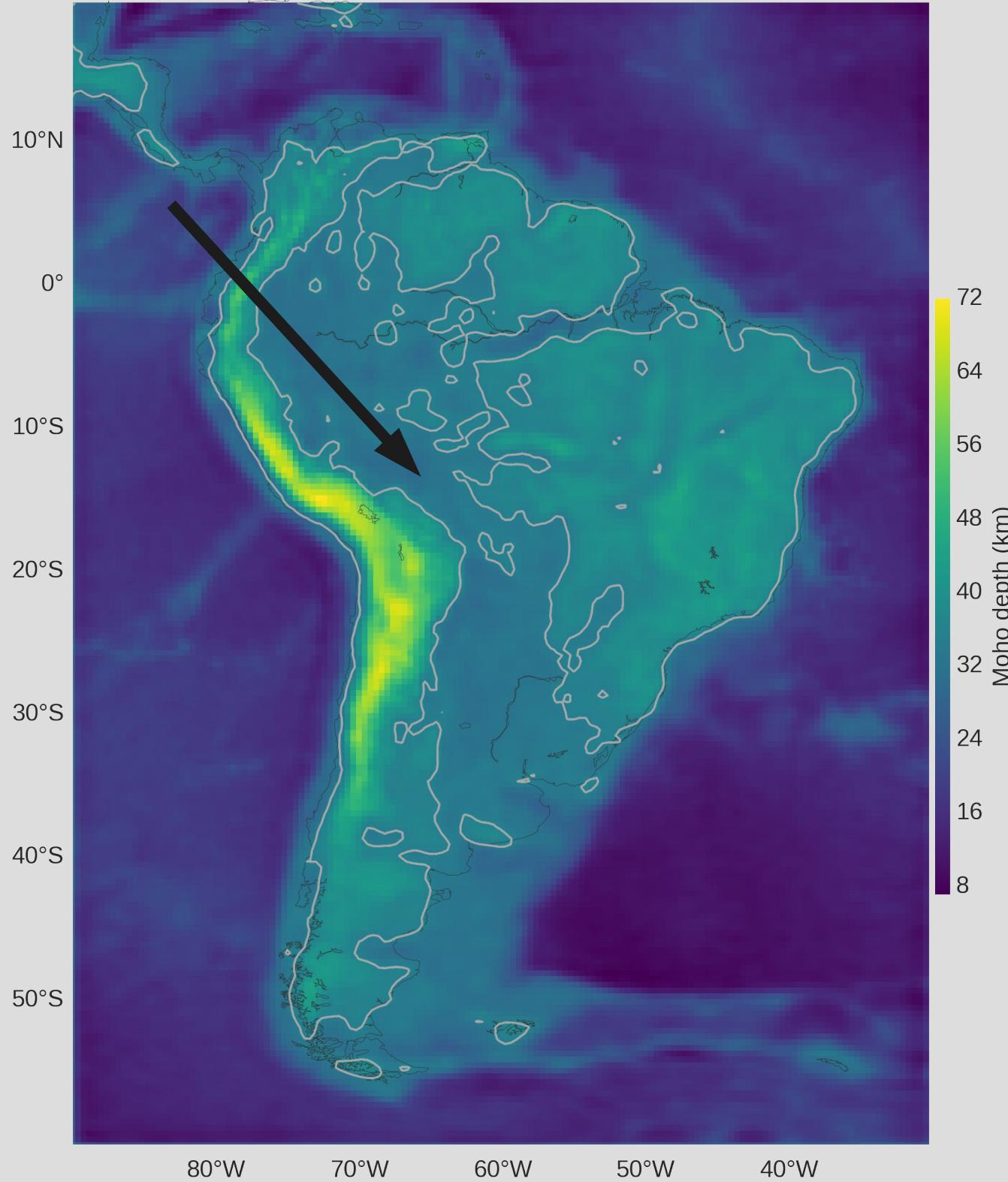
**35 km**



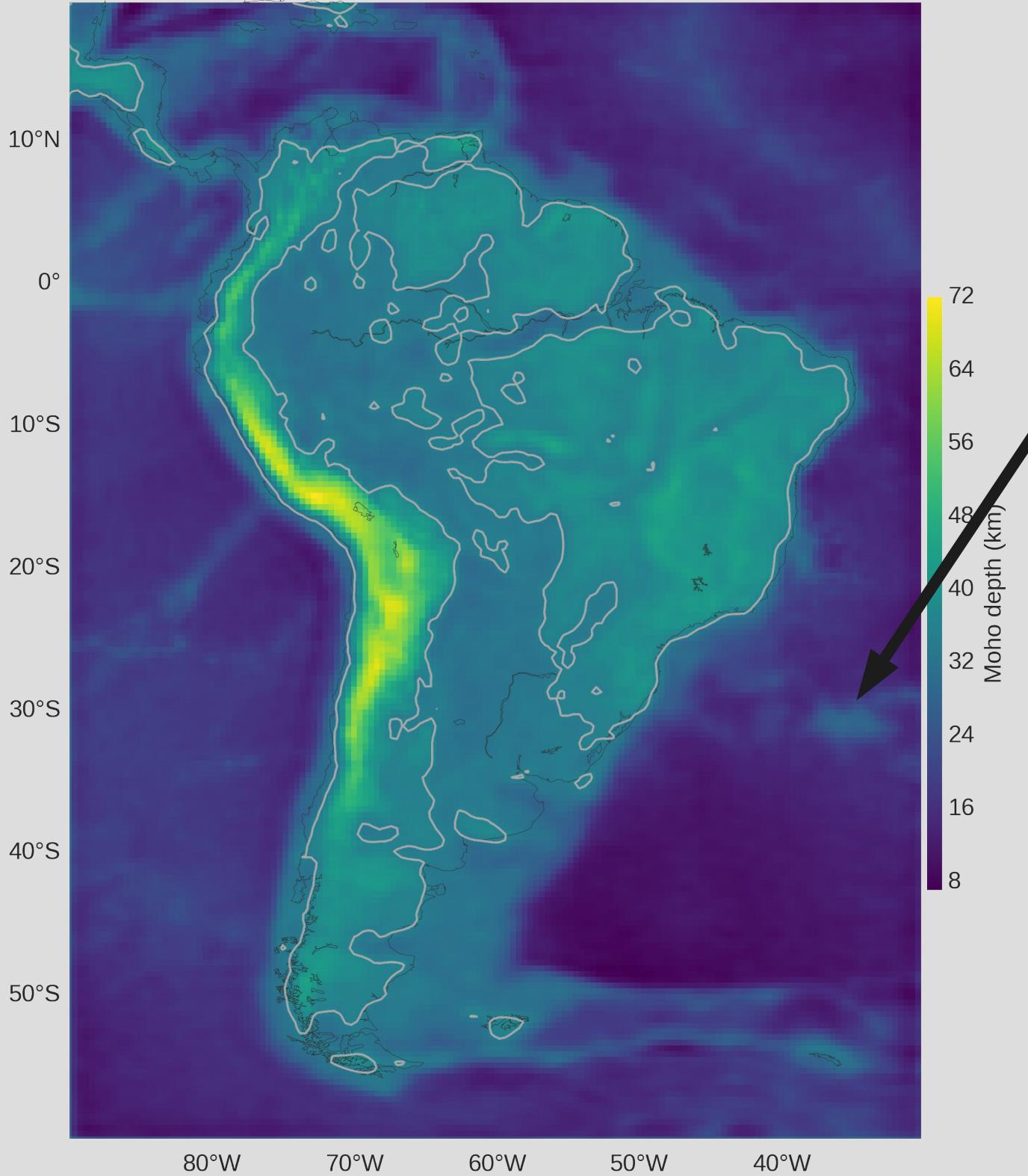
**35 km**

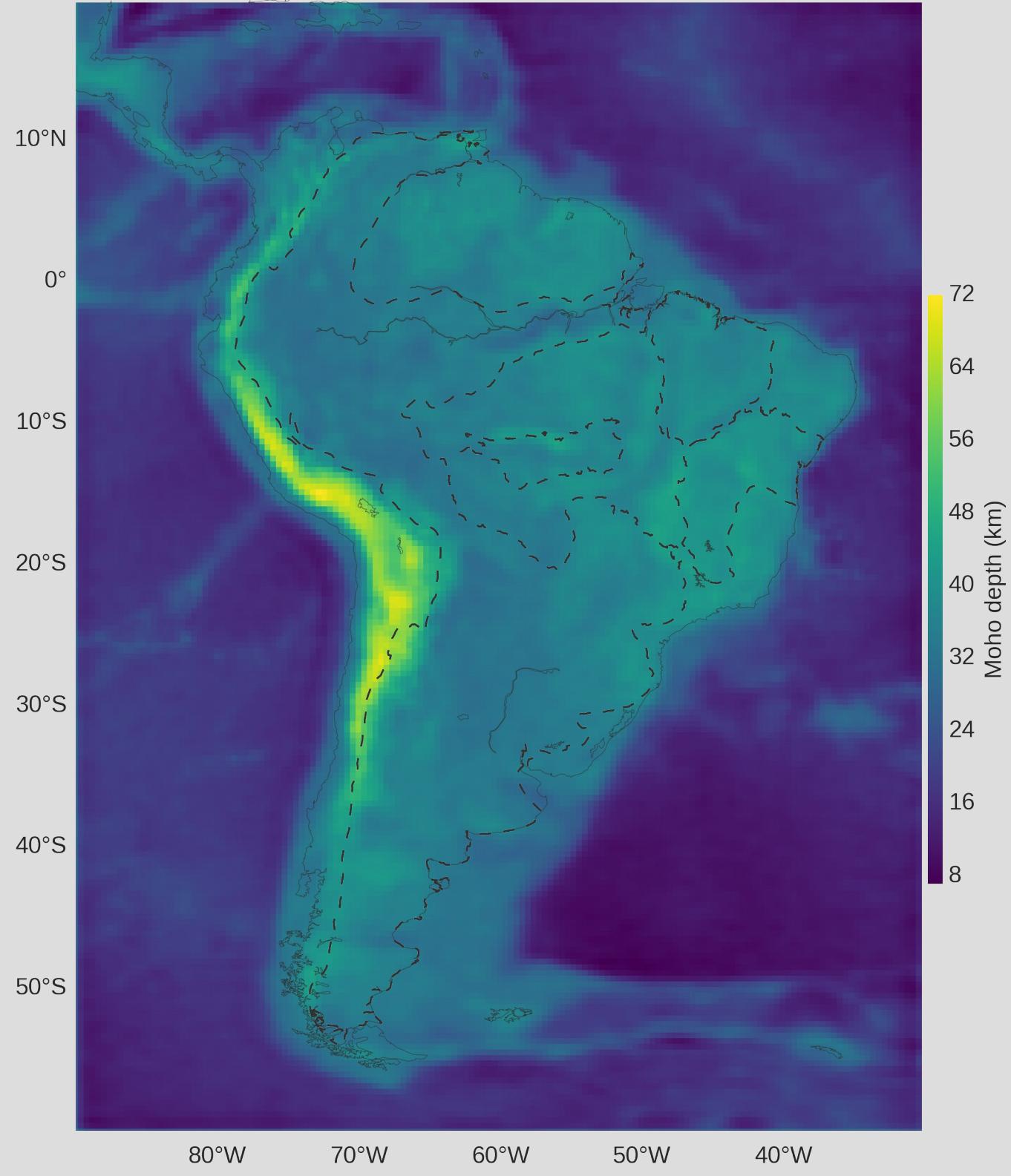


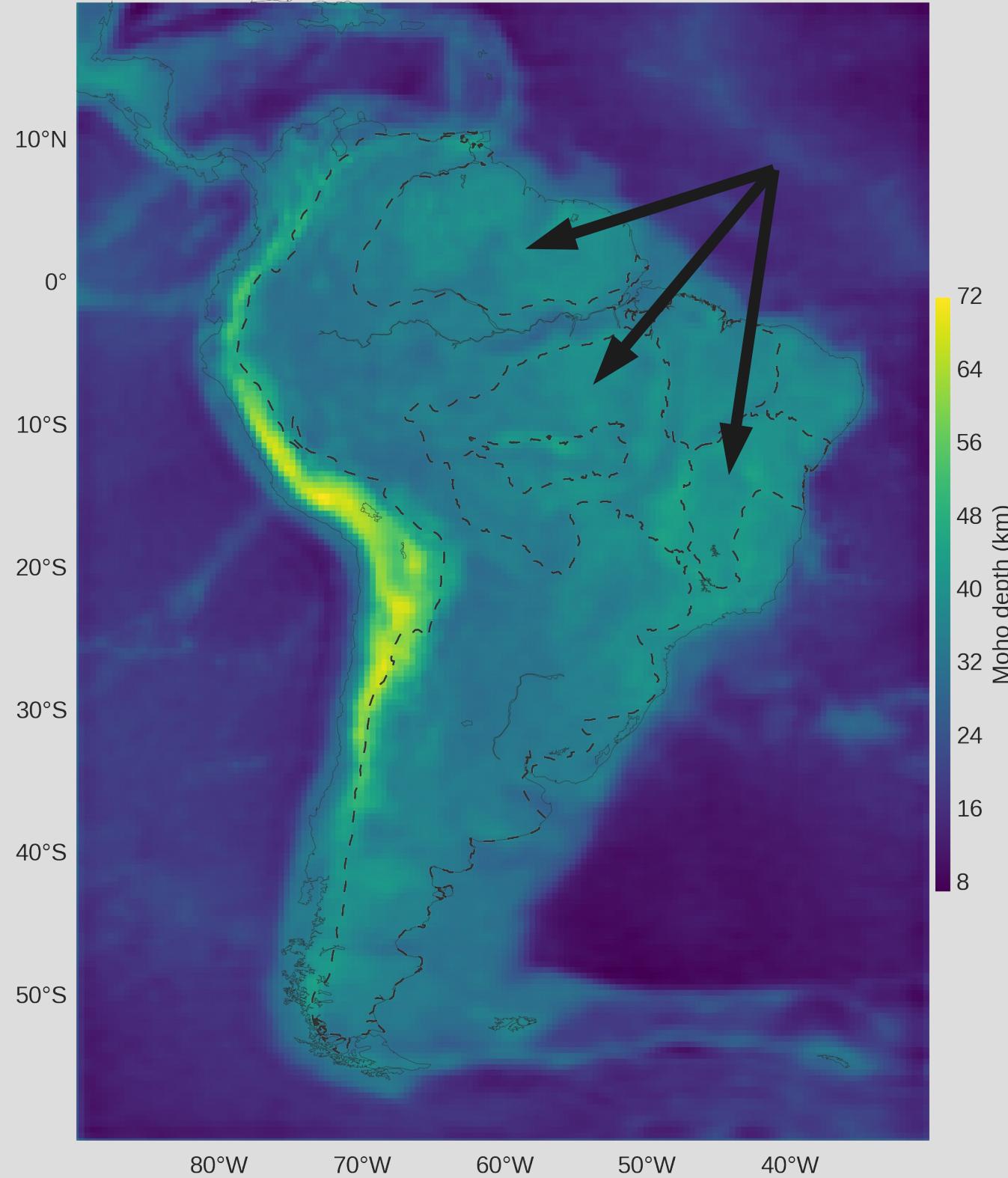
**35 km**

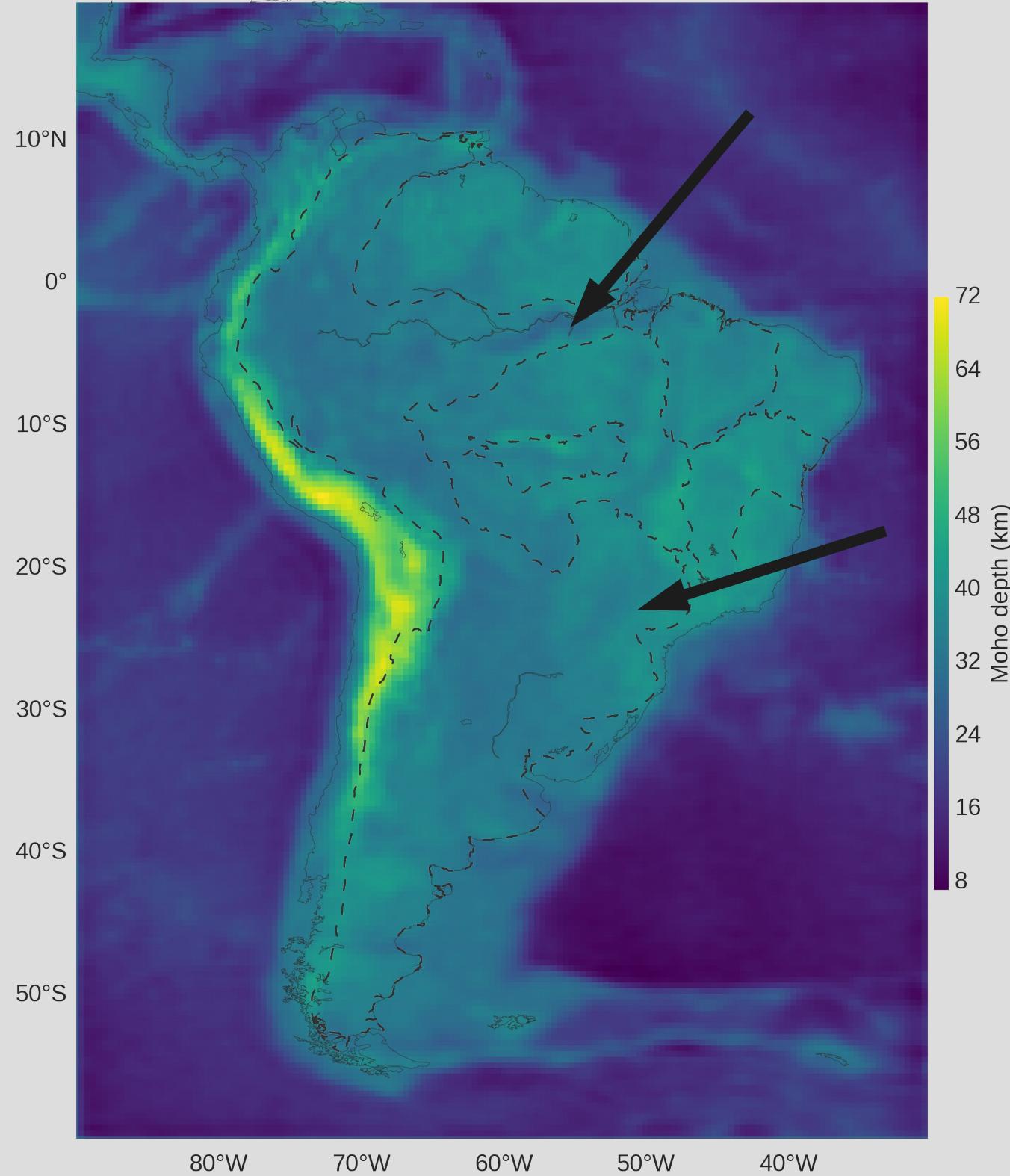


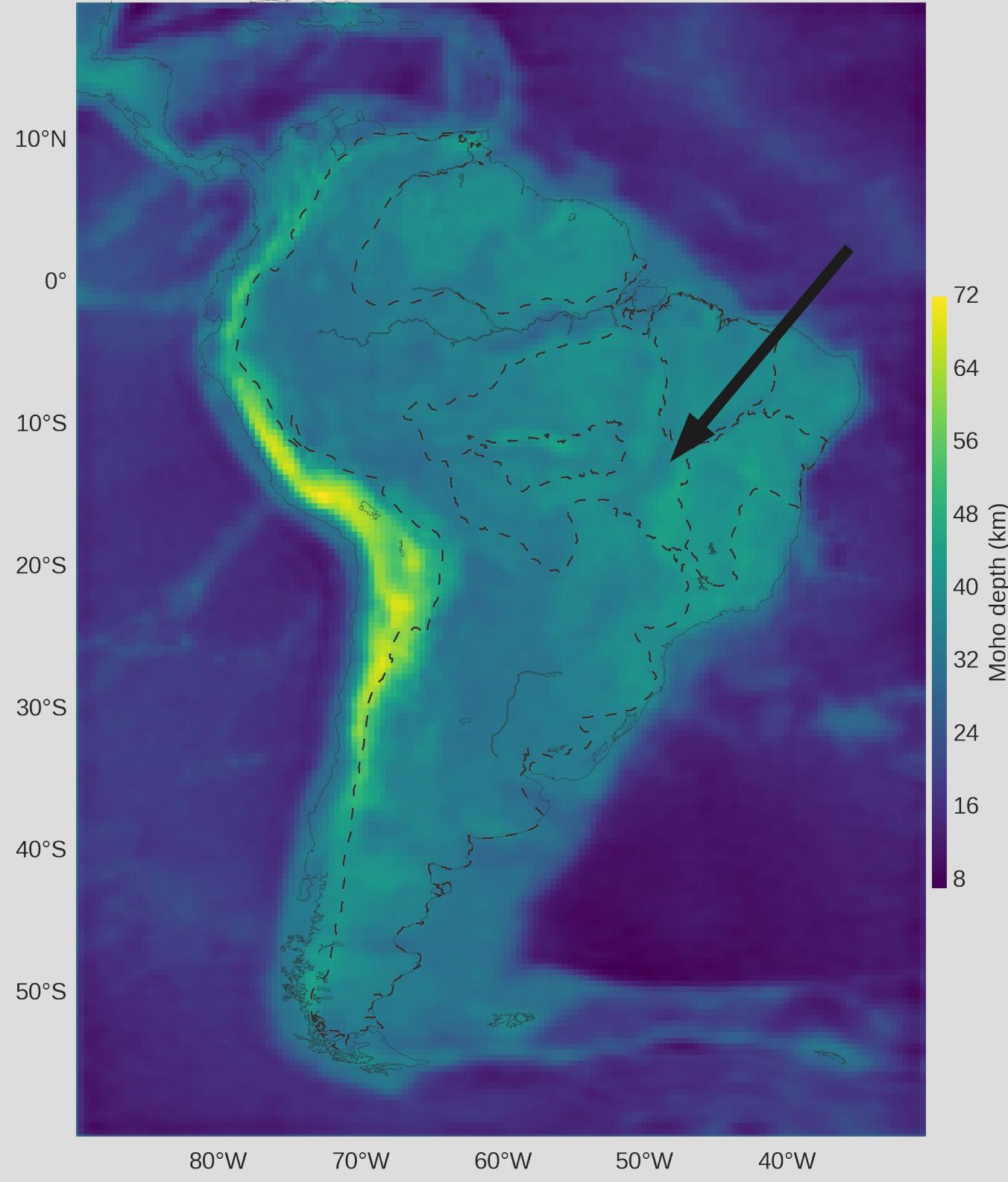
**35 km**

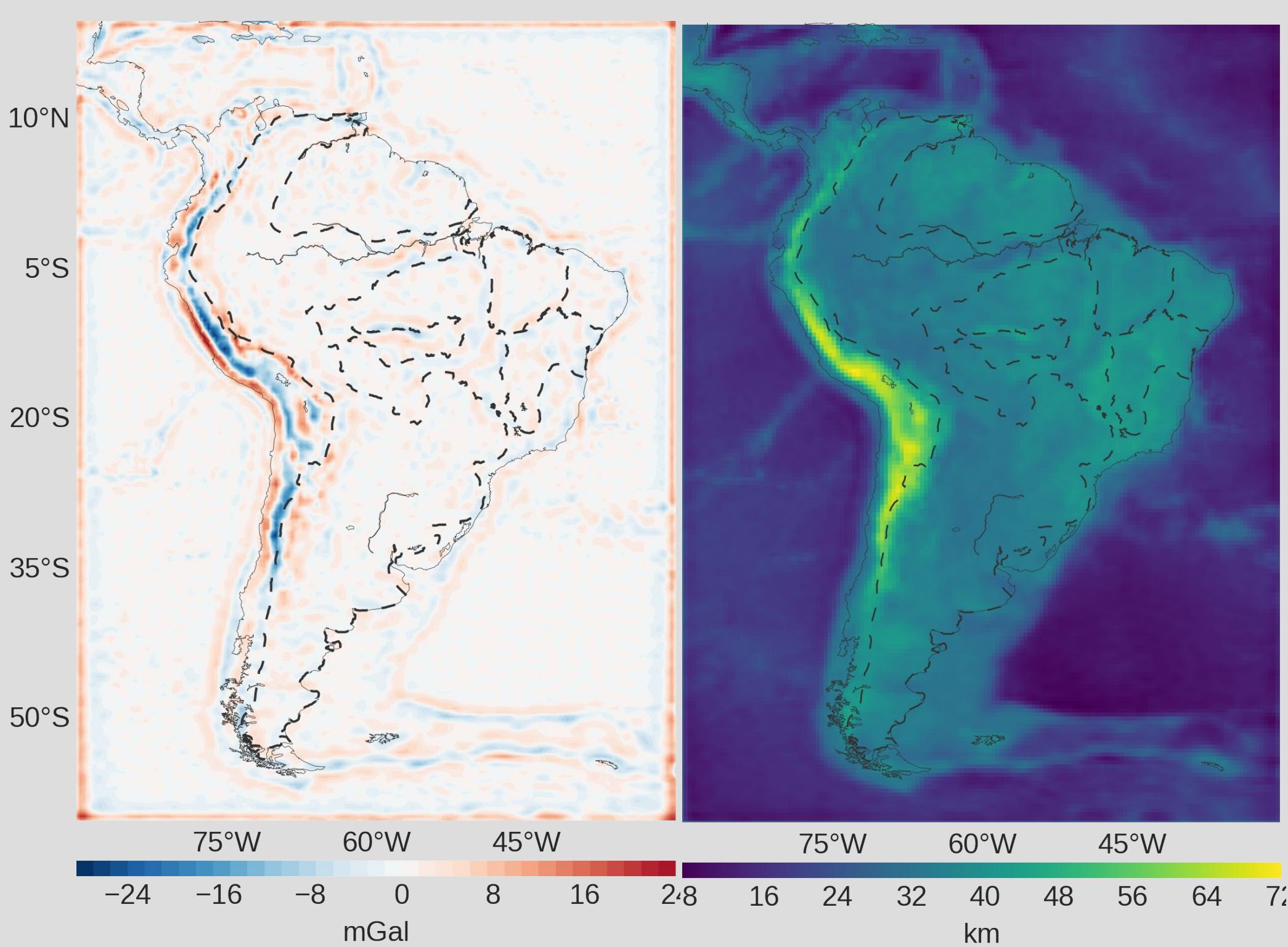


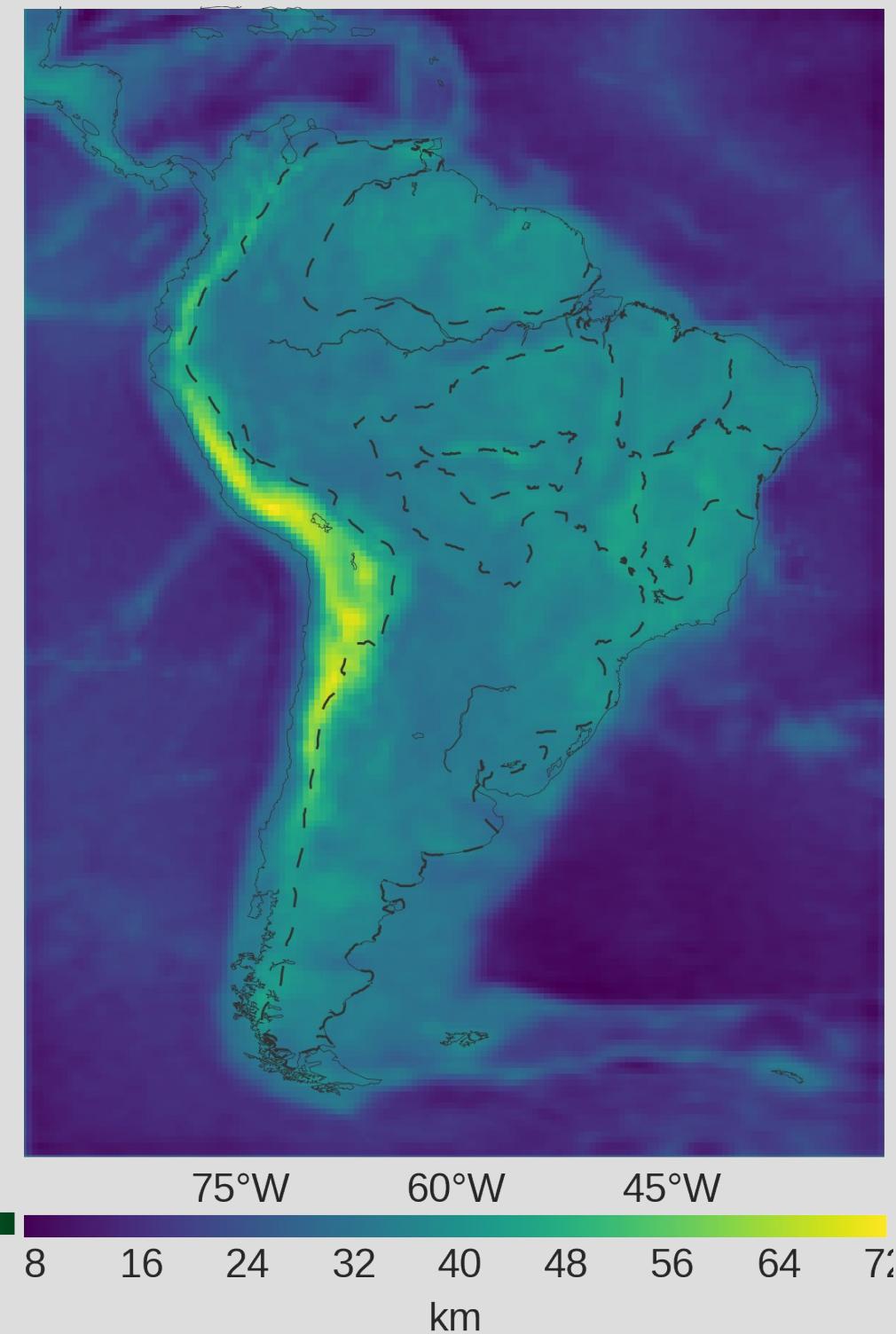
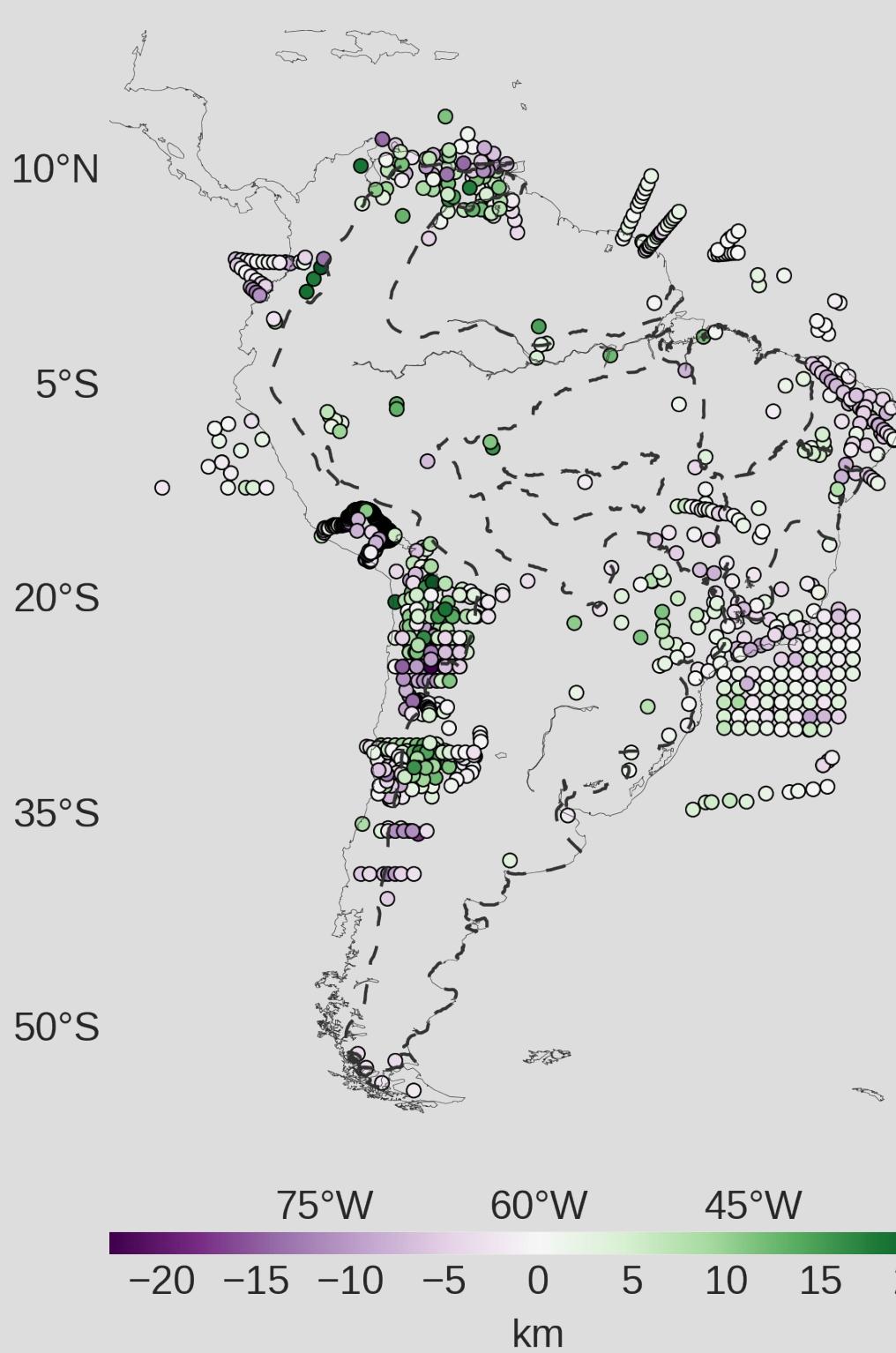


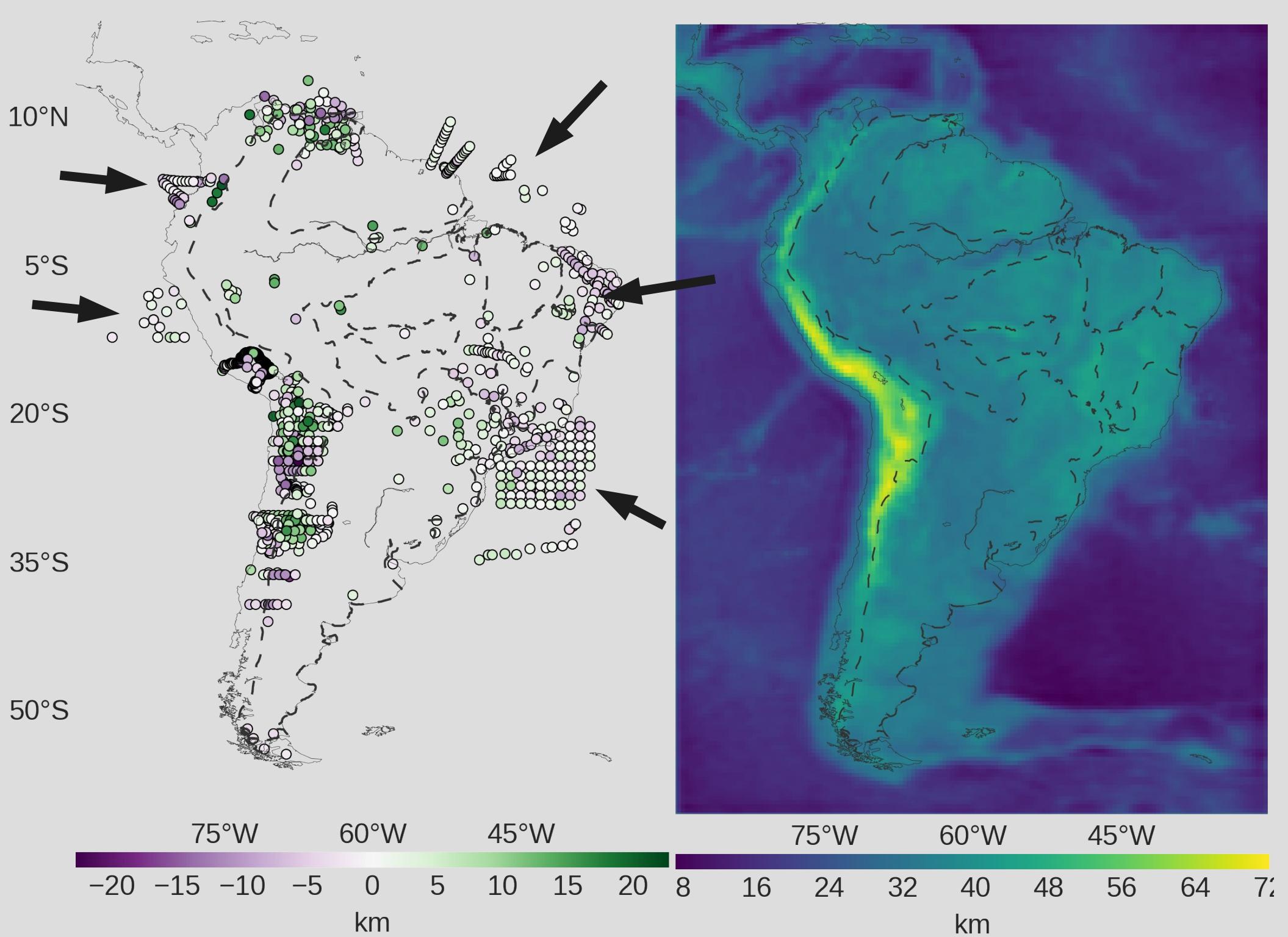


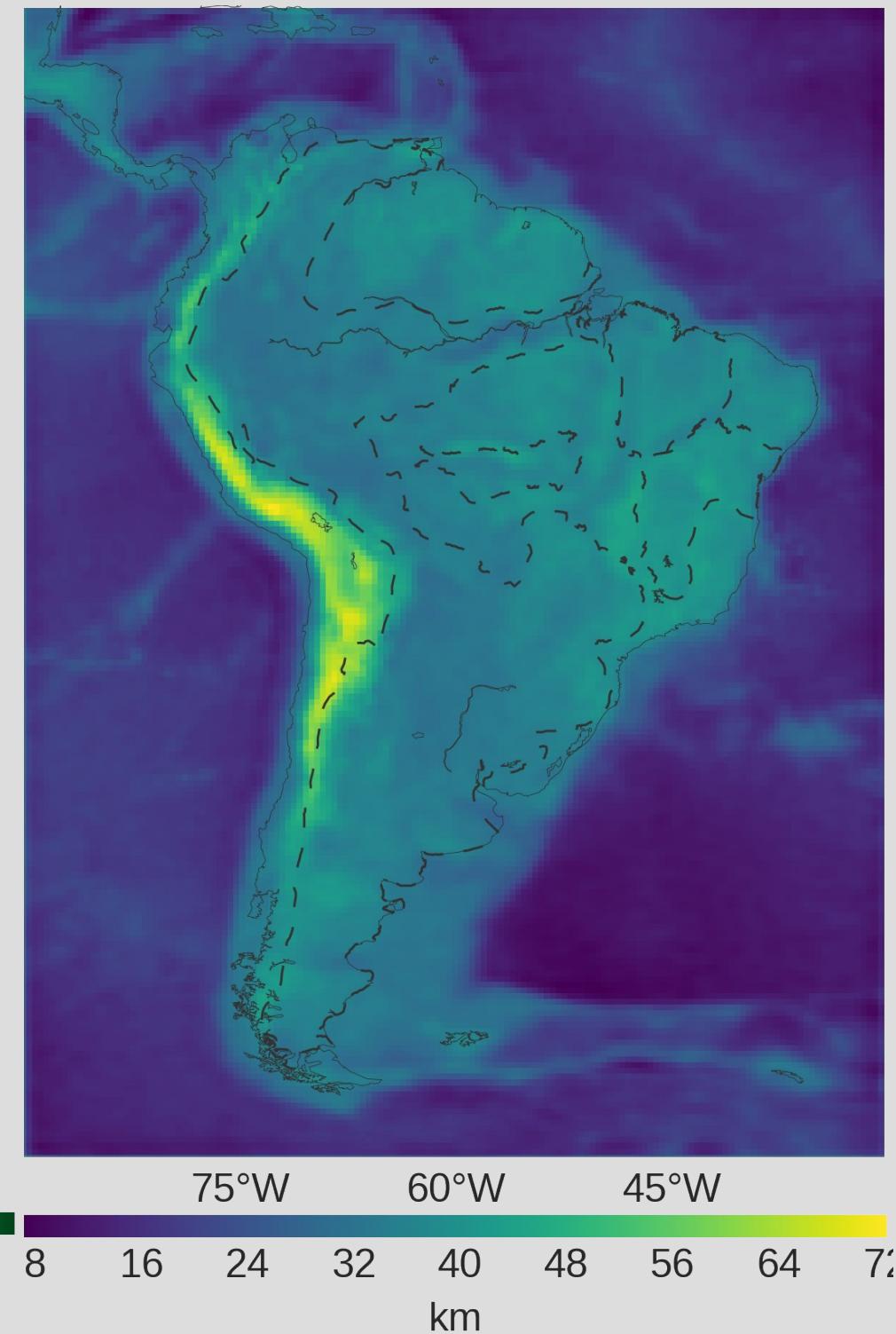
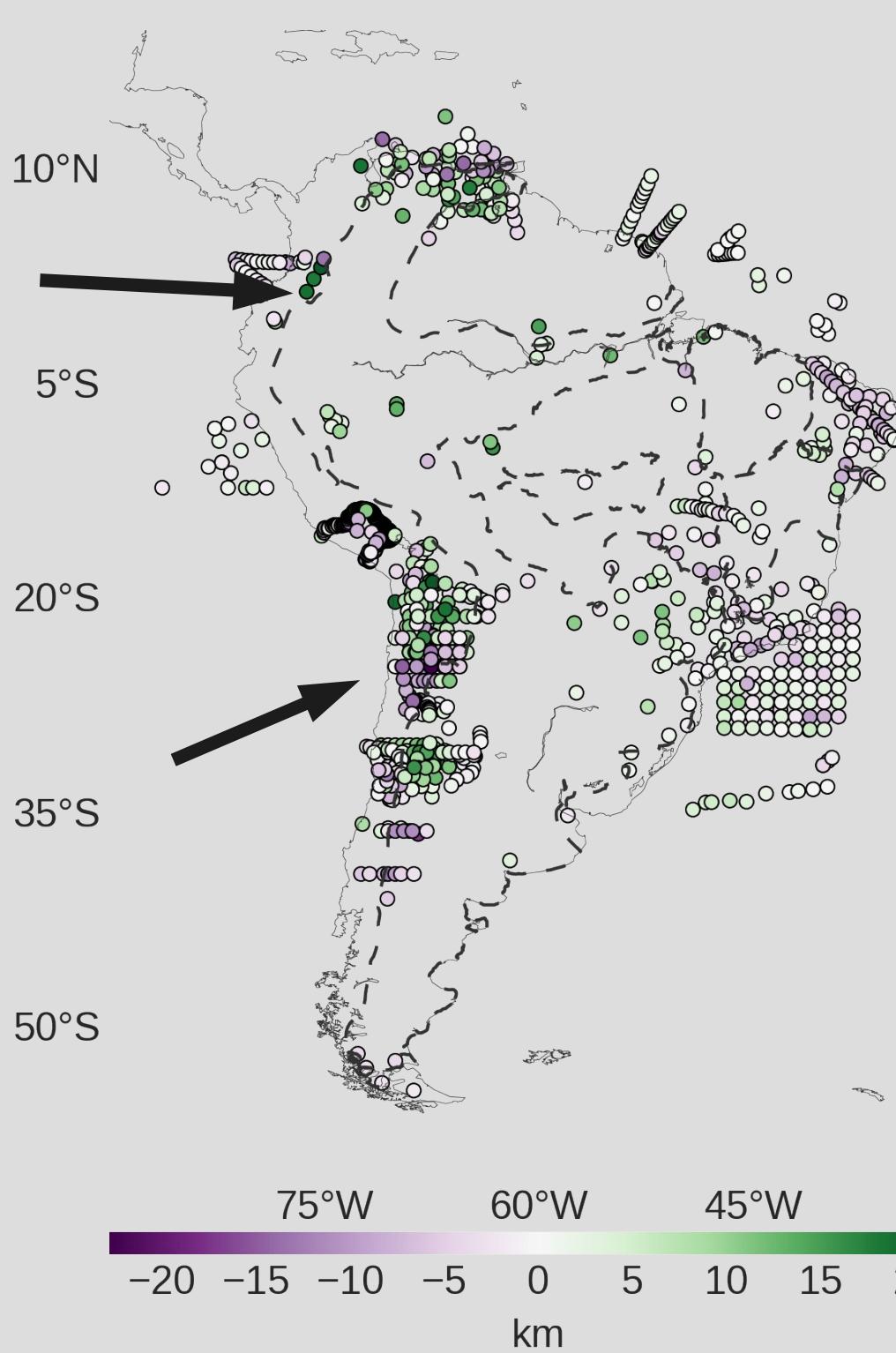


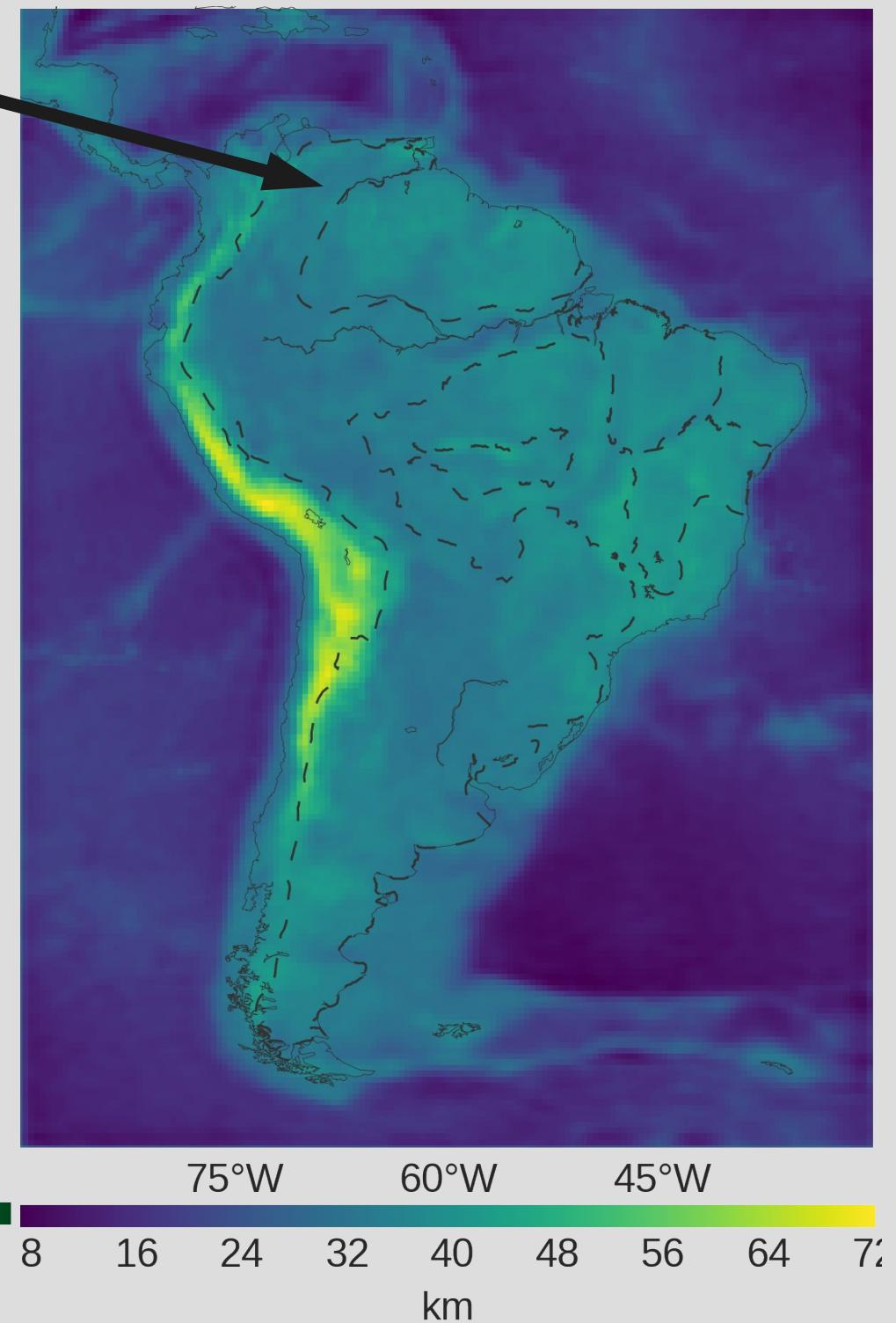
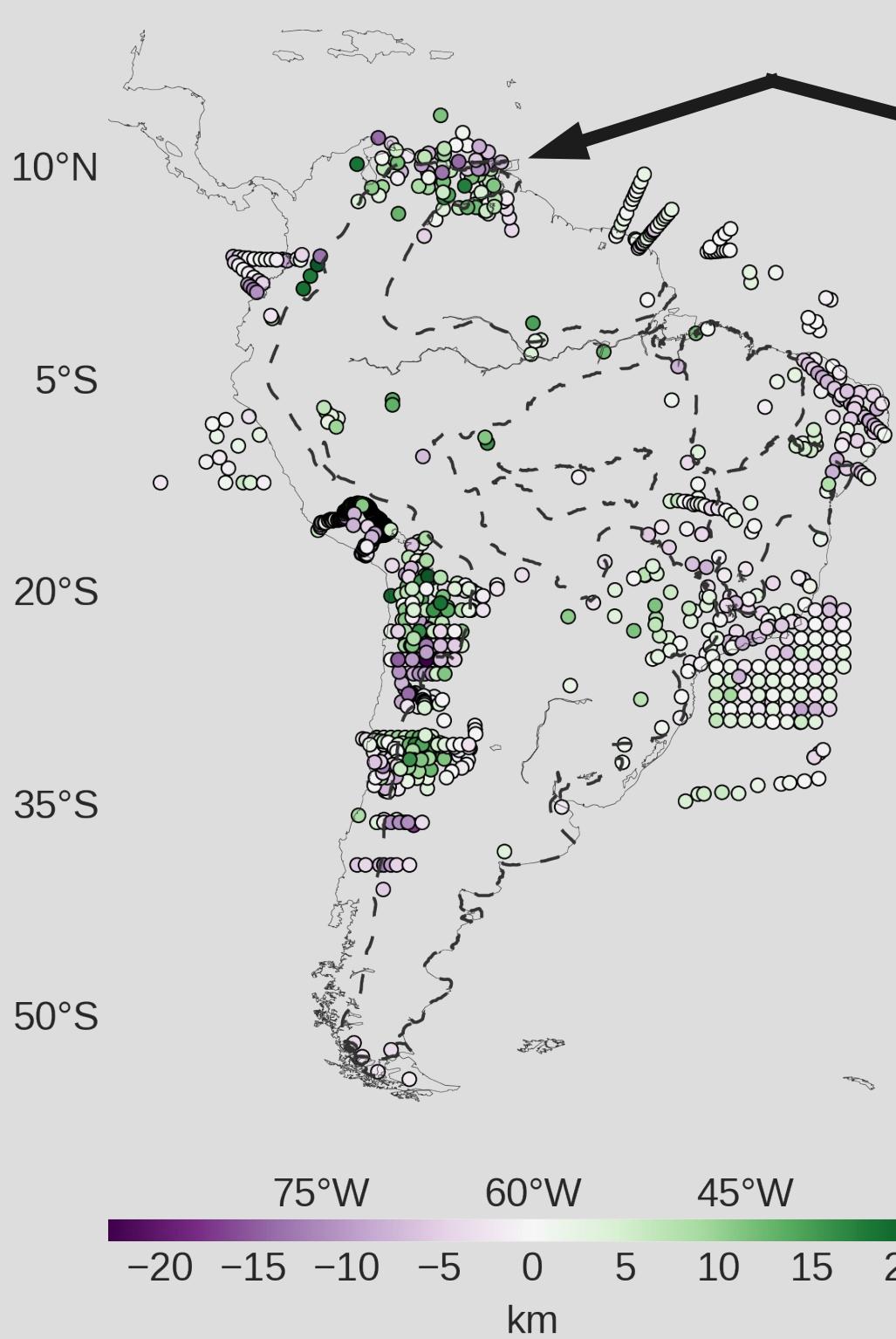


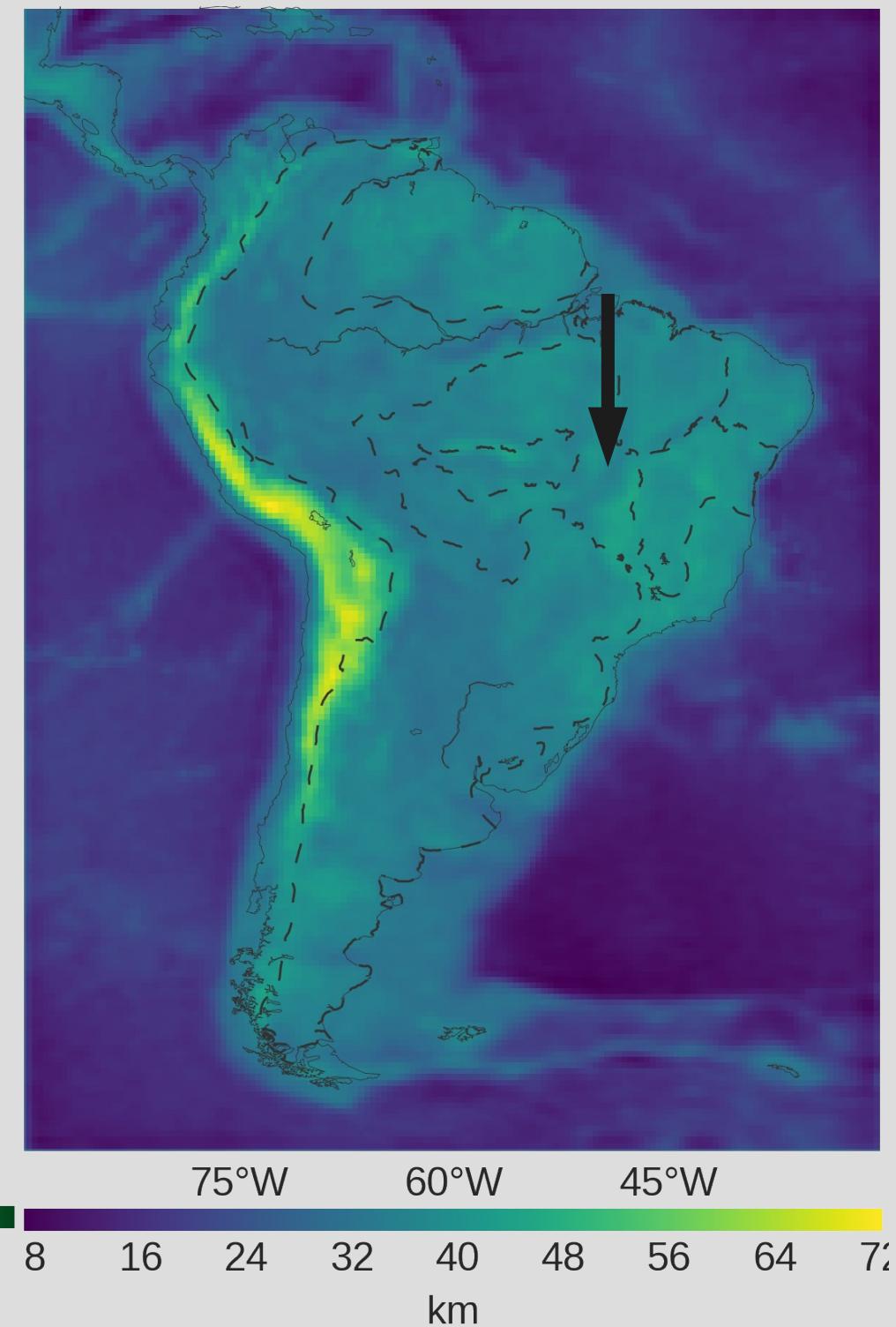
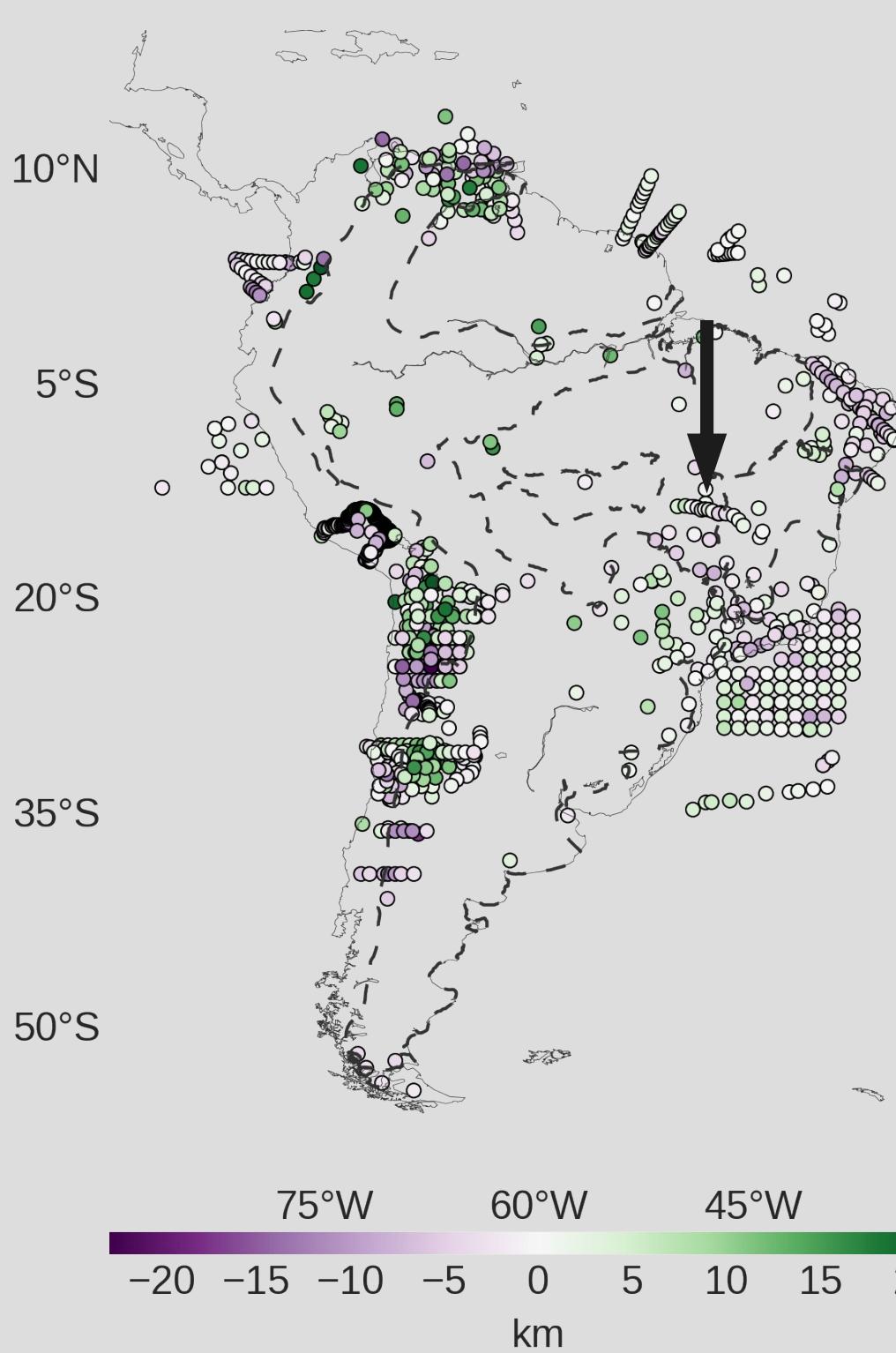


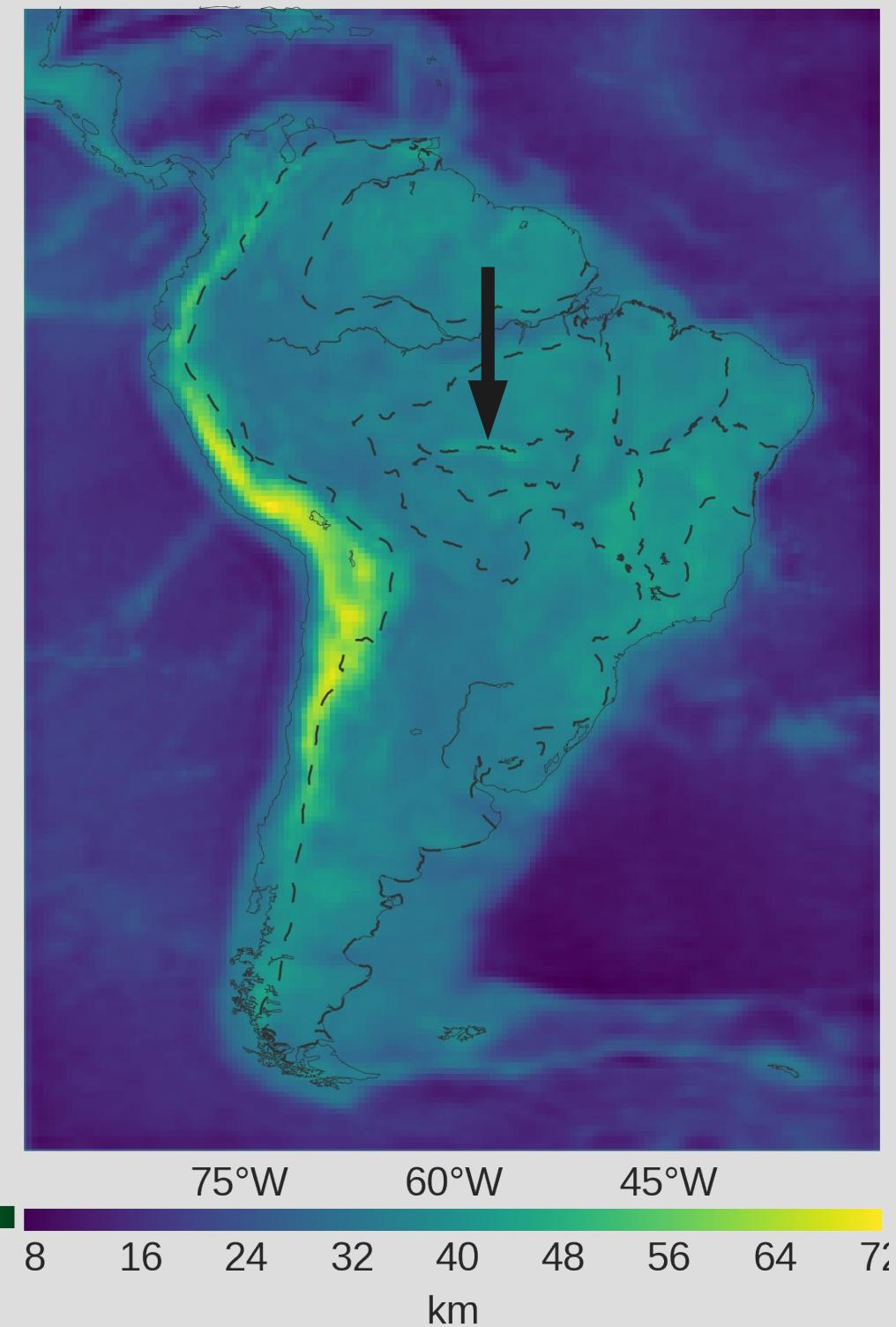
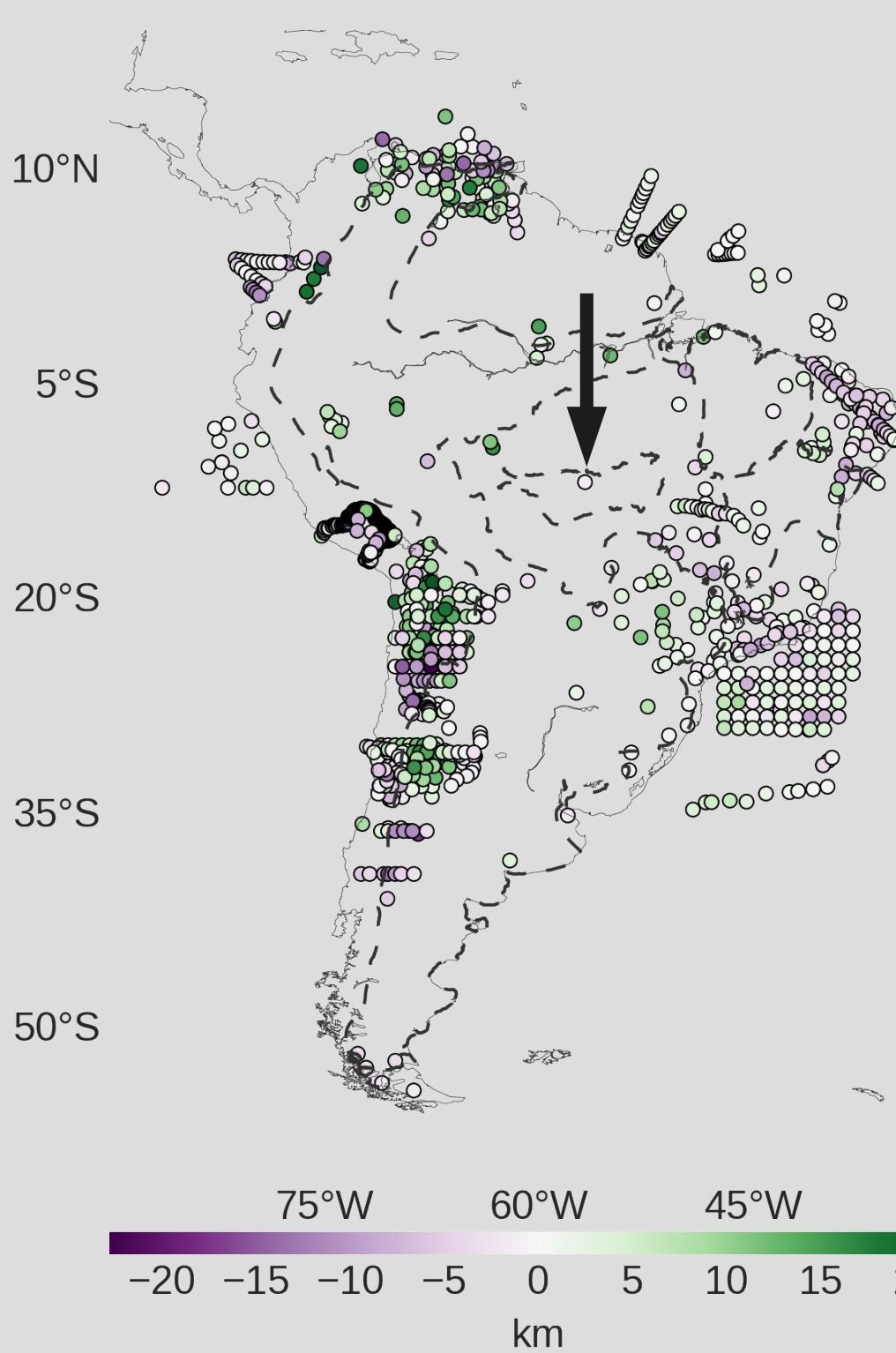


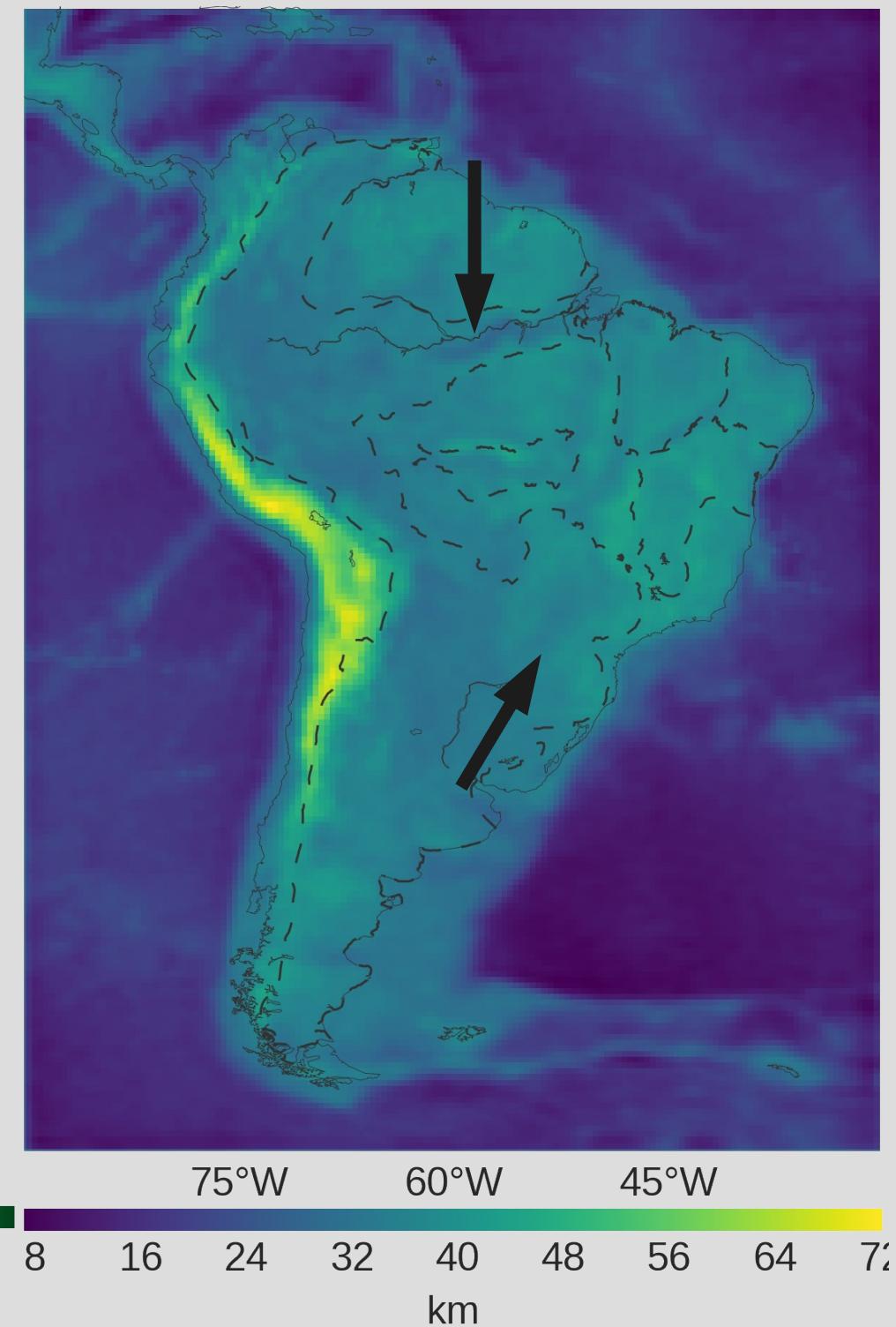
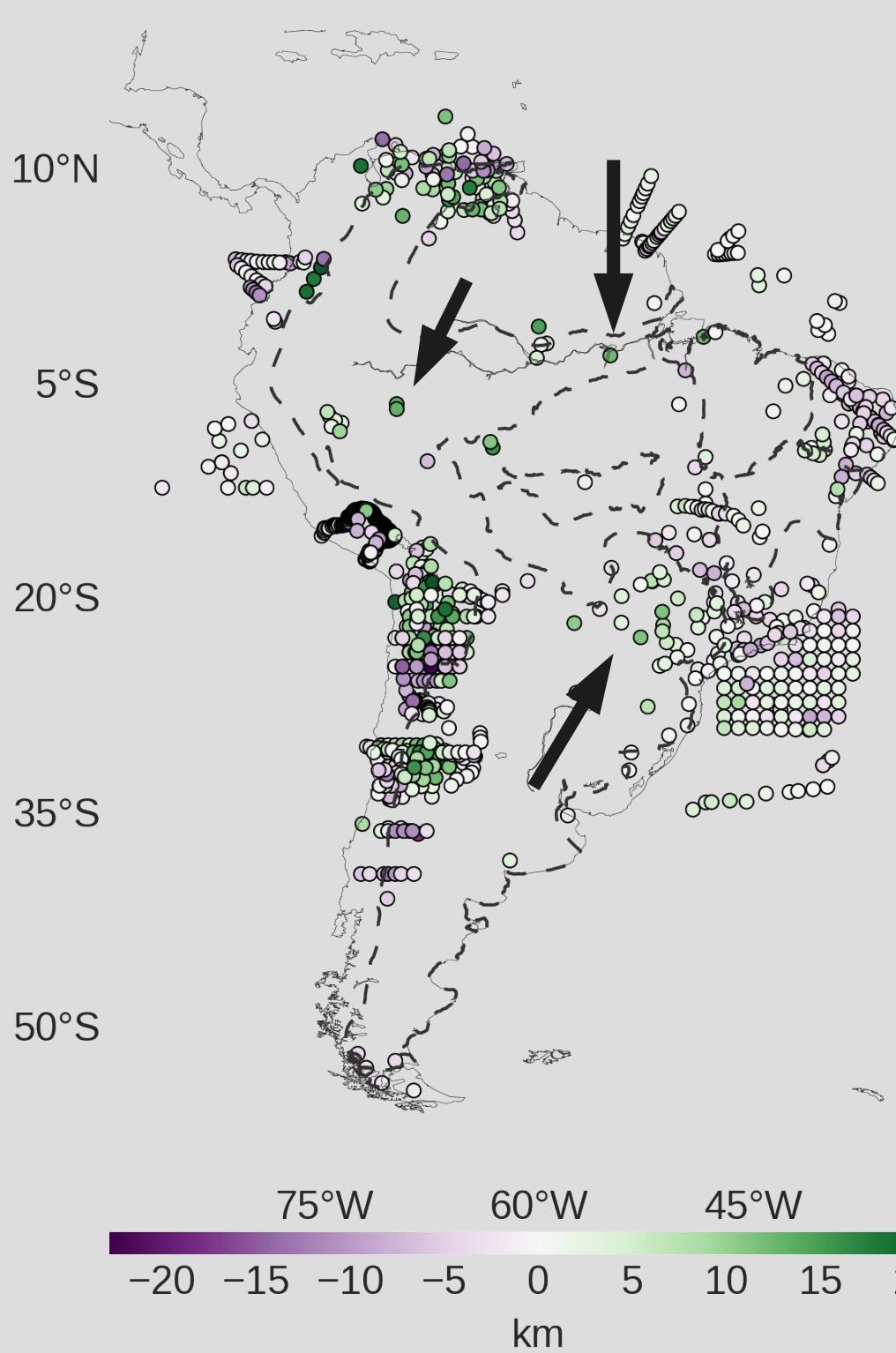


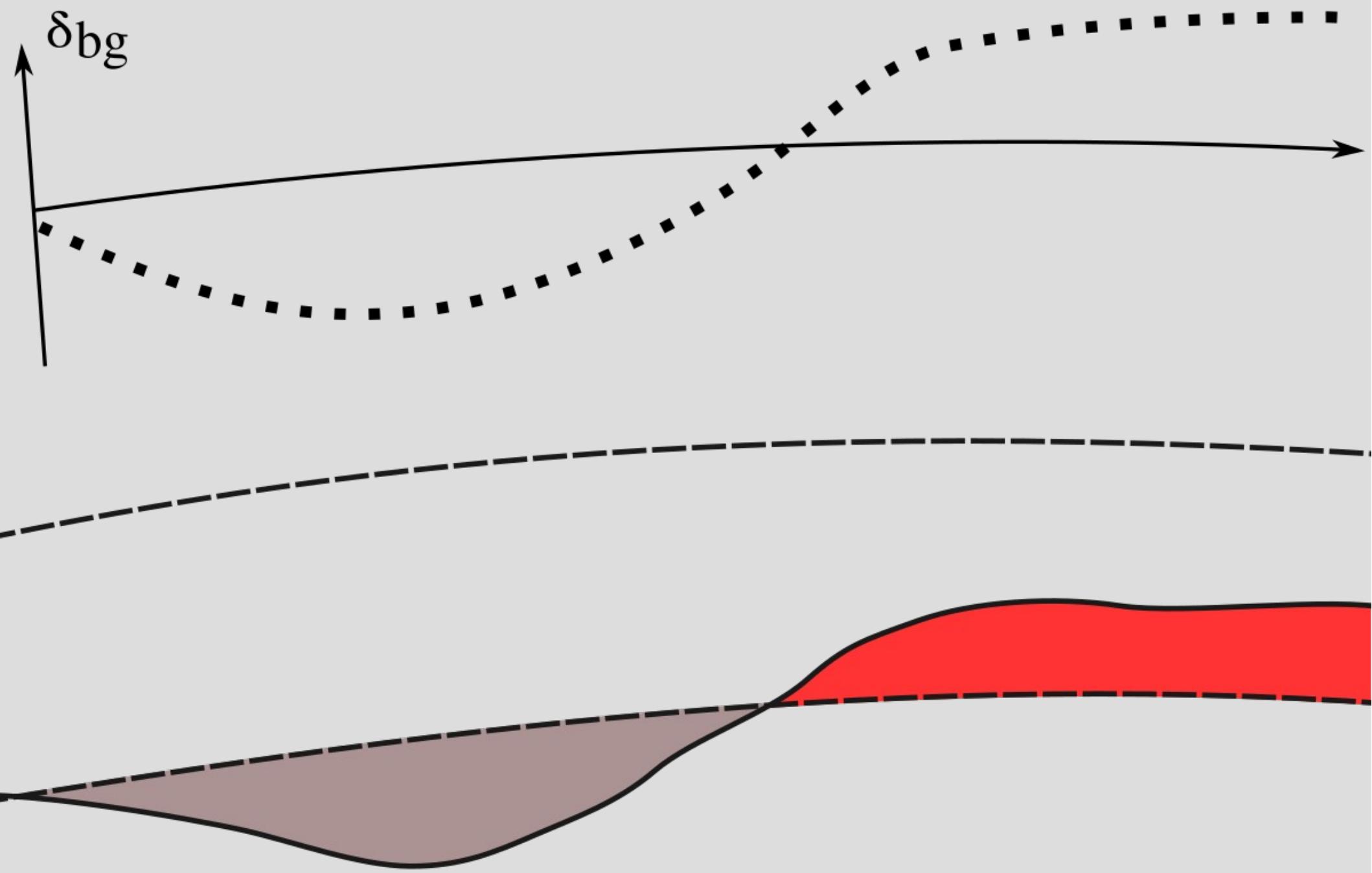


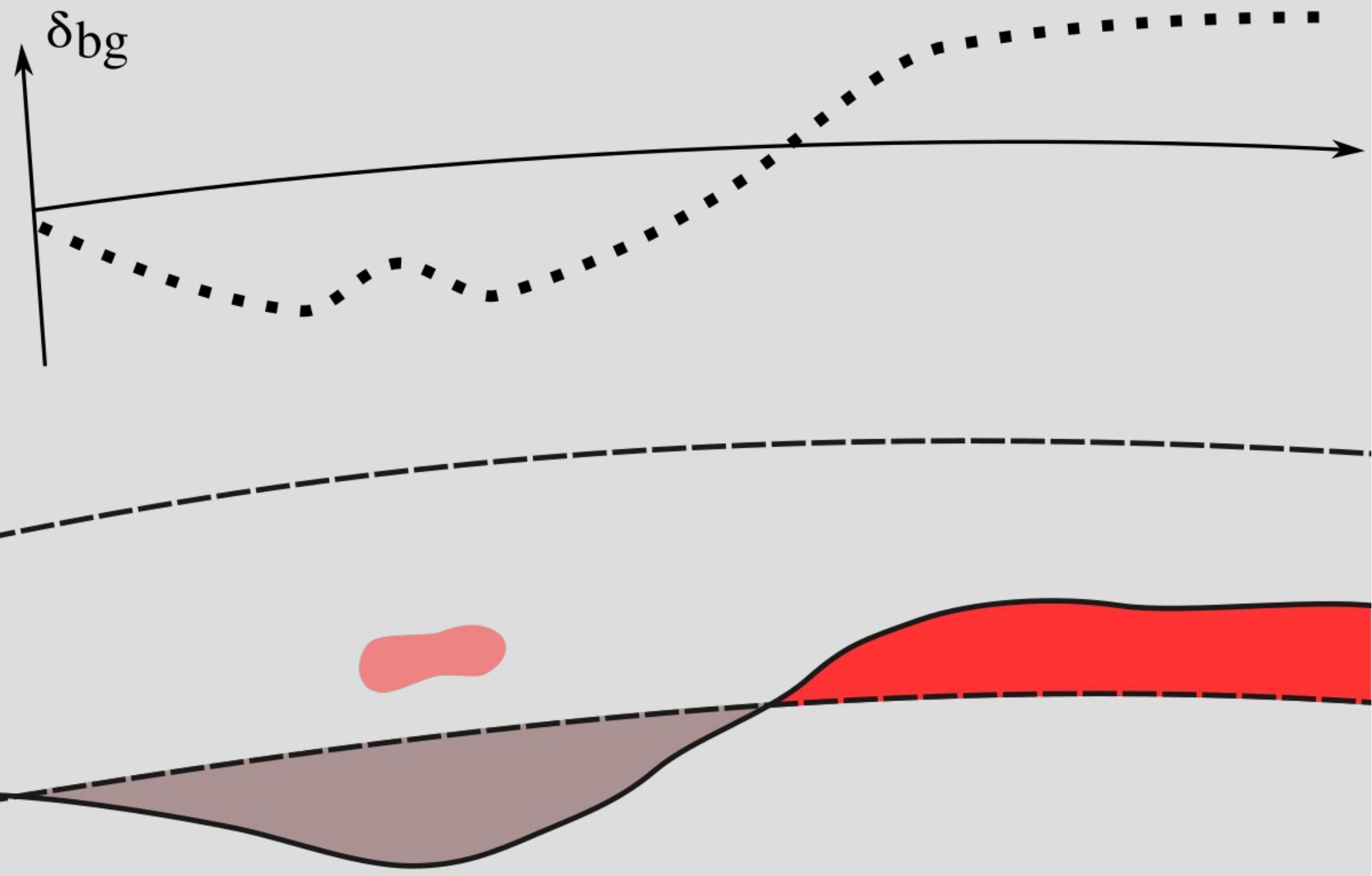


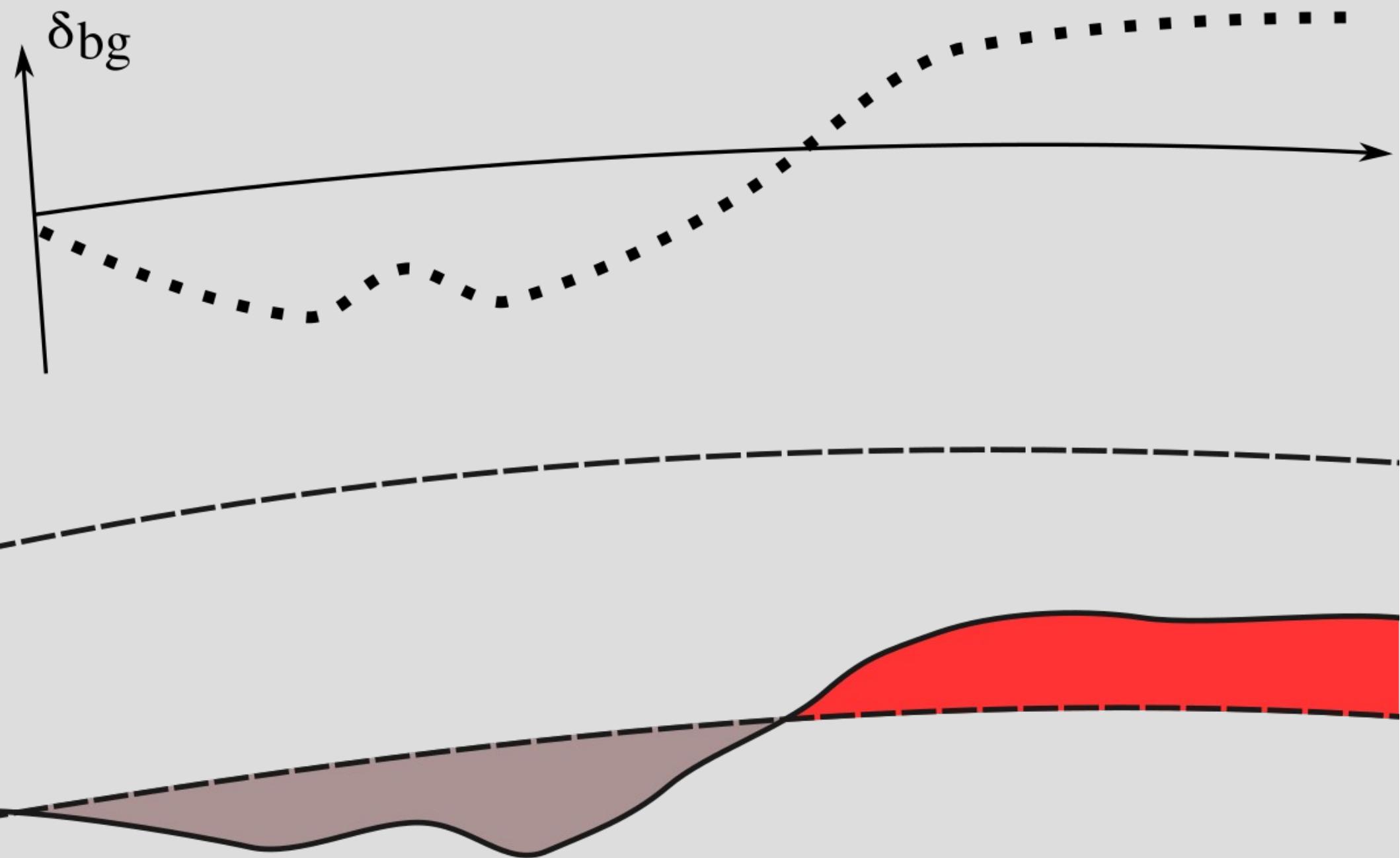












# conclusão

# Novo método

Baseado em Bott (1960) e Silva et al. (2014)

Tesseroides

Gauss-Newton + Regularização

Matrizes esparsas

Validação cruzada →  $\mu$   $\Delta\rho$   $z_{ref}$

# Moho América do Sul

Compatível com soluções anteriores (grav e sismo)

Correções (topo e sedimentos) apropriadas

~6 km desvio padrão com sísmica

Diferença grande concentrada nos Andes

# Moho América do Sul

Compatível com soluções anteriores (grav e sismo)

Correções (topo e sedimentos) apropriadas

~6 km desvio padrão com sísmica

Diferença grande concentrada nos Andes

Resolução pode ser falsa

Depende de correções corretas

Difícil estimar o erro

Introdução

Tesseroids

Fatiando a Terra

Inversão Moho

Conclusão

conclusões

finals

programa A

programa B

Novo método

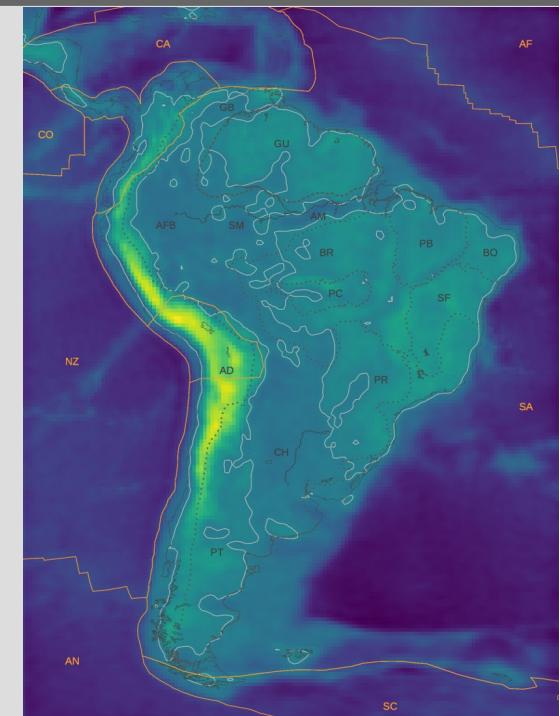
# Tesseroids

modelagem direta  
aprox. esférica



modelagem direta  
otimização  
regularização, etc

método: inversão não-linear rápida  
aplicação: Moho da América do Sul





Algoritmo aprimorado

Quantificação do erro

Usuários



Algoritmo aprimorado

Quantificação do erro

Usuários

Desenvolvedores

Desenvolvimento estagnado



# fatiando a terra

Biblioteca em Python

Vários módulos

Pesquisa + educação

Usuários + desenvolvedores

Além dessa tese



# fatiando a terra

Facilitar a participação:

Guia do desenvolvedor

Guia do usuário

Workshop

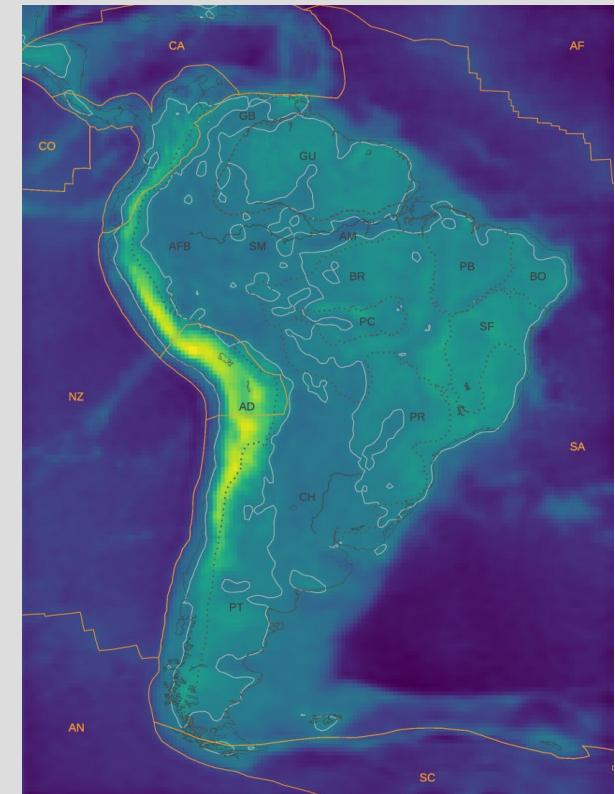
Videos?

# Inversão rápida + Moho Am. do Sul

Método novo + eficiente

Approx. esférica

Python + Fatiando a Terra



# Inversão rápida + Moho Am. do Sul

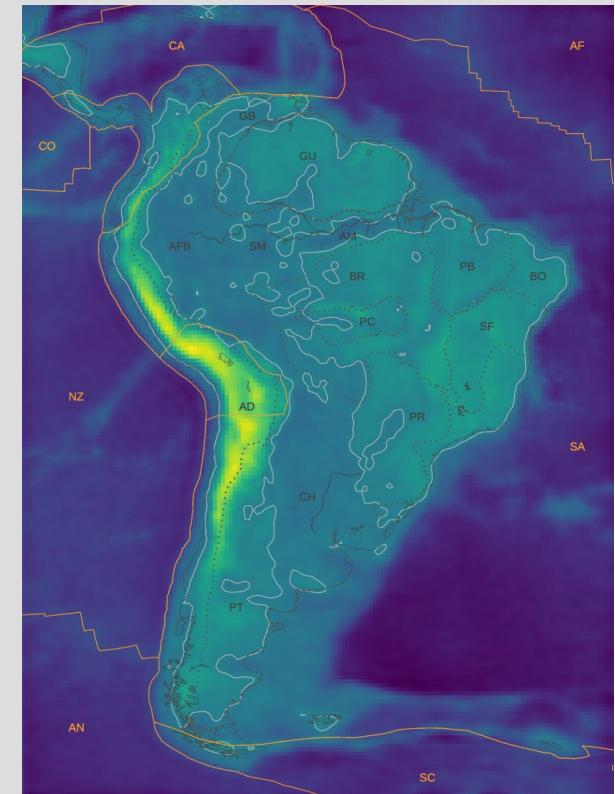
Método novo + eficiente

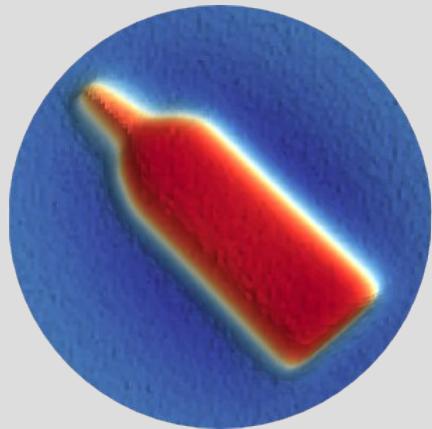
Approx. esférica

Python + Fatiando a Terra

Depende das correções

Incerteza





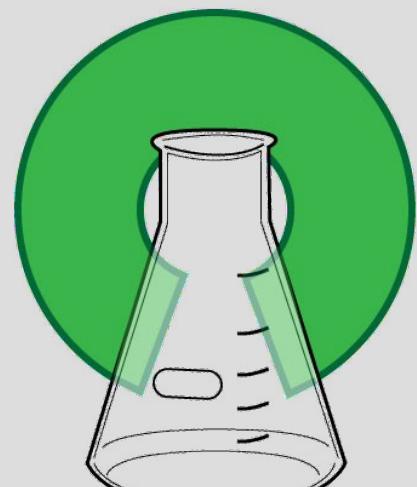
# PINGA

Grupo de problemas inversos em geofísica

código + dados + figuras

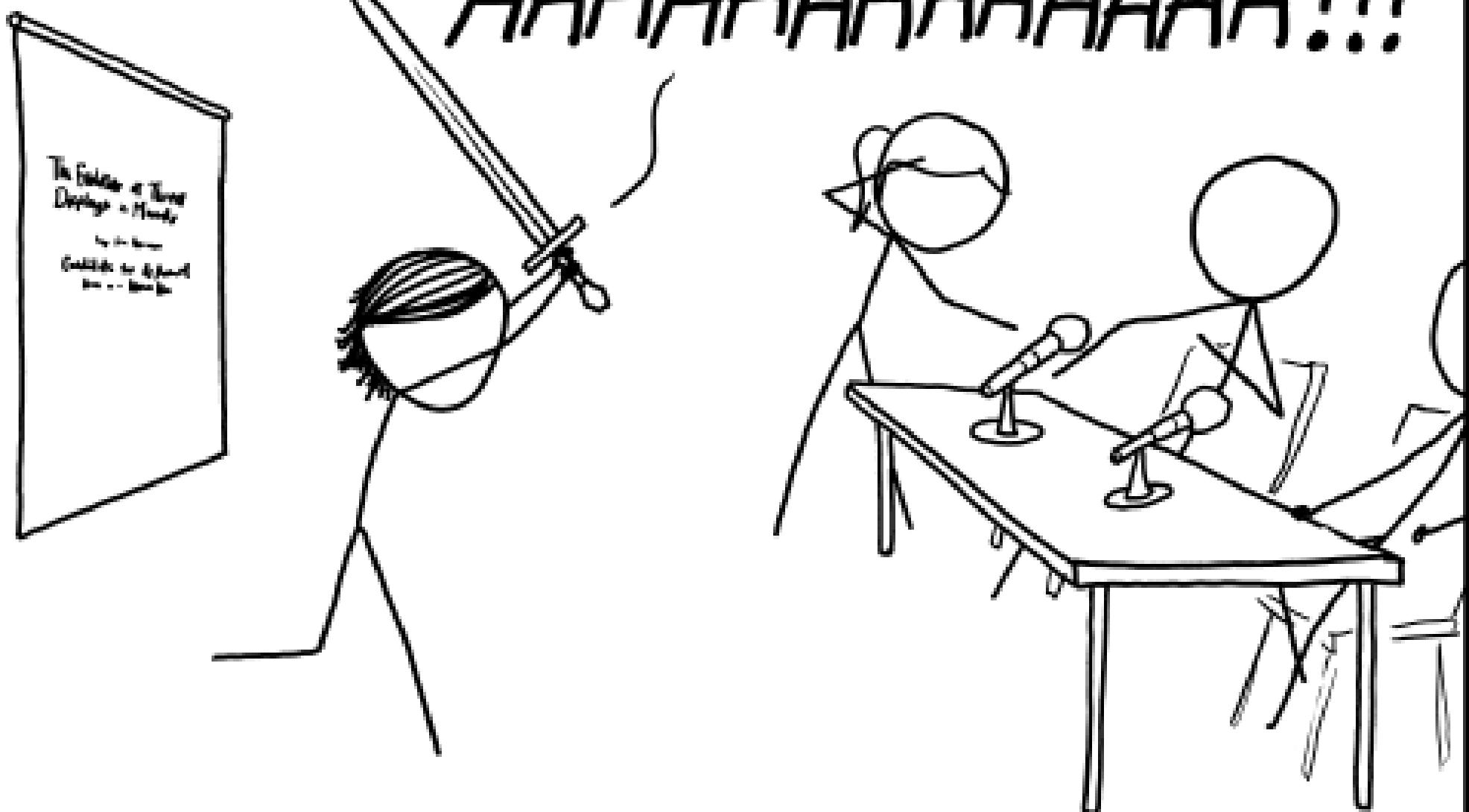
[pinga-lab.org](http://pinga-lab.org)

[leouieda.com](http://leouieda.com)



open science

IN CONCLUSION,  
AAAAAAA!!!!



THE BEST THESIS DEFENSE IS A GOOD THESIS OFFENSE.