Robust 3D gravity gradient inversion by planting anomalous densities

Leonardo Uieda

Valéria C. F. Barbosa

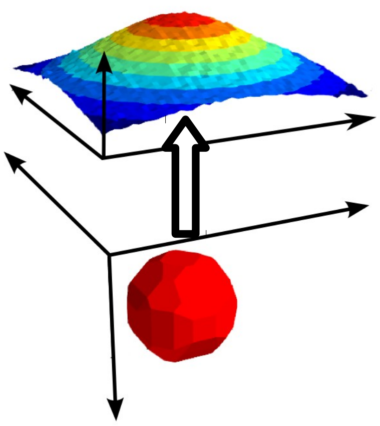
Observatório Nacional

2011

Outline

Outline

Forward Problem



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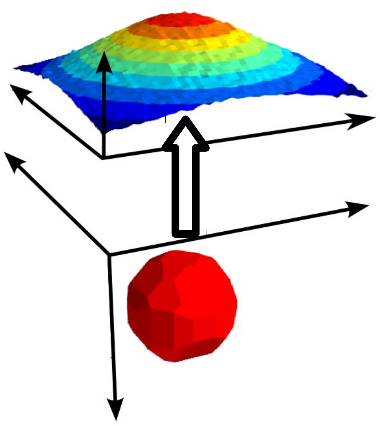
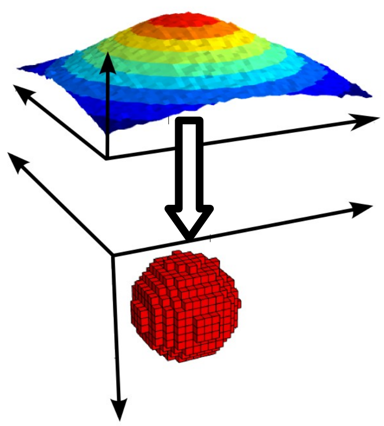


Inverse Problem



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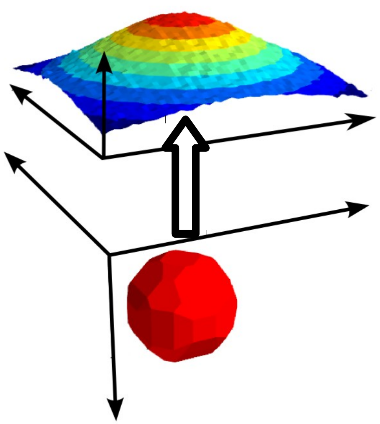
Forward Problem



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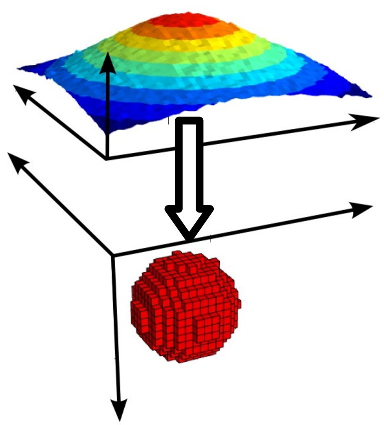
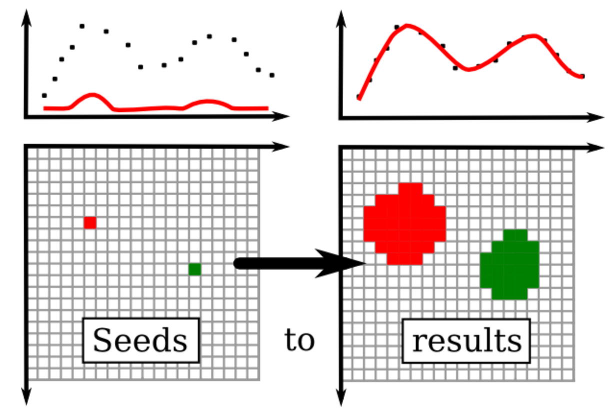


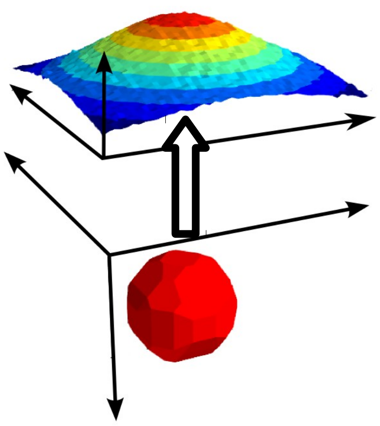
Planting Algorithm



Inspired by René (1986)

Outline

Forward Problem



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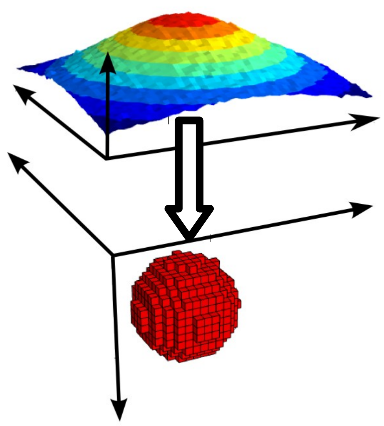
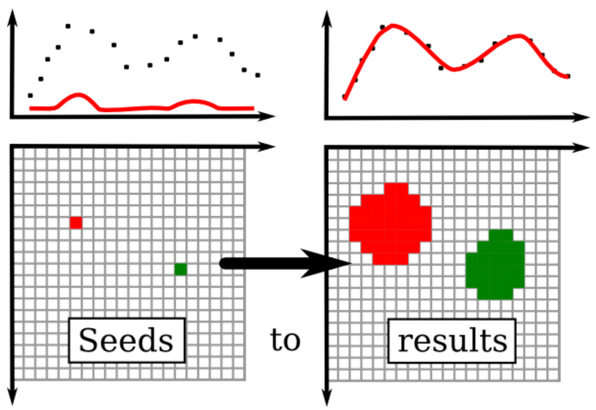
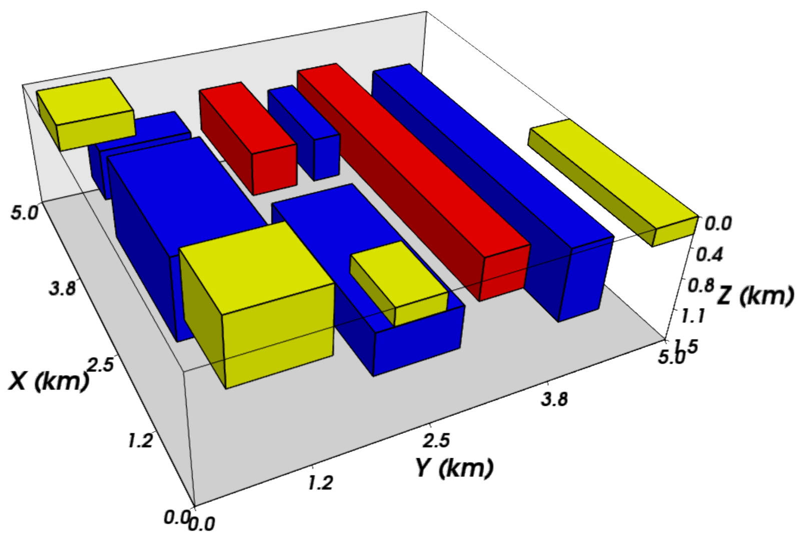


Planting Algorithm



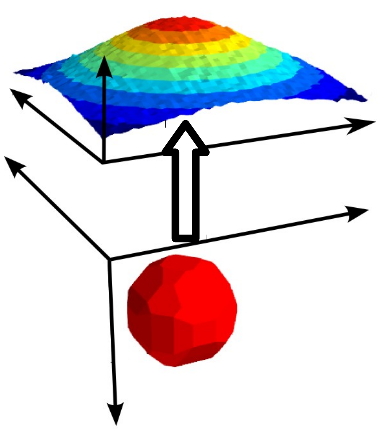
Inspired by René (1986)

Synthetic Data

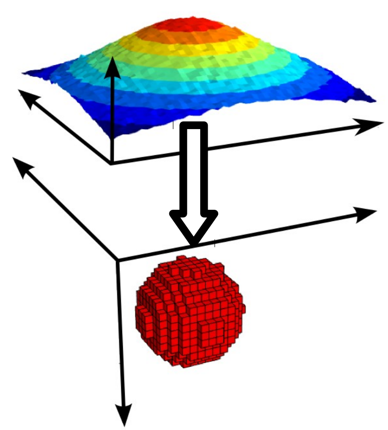


Outline

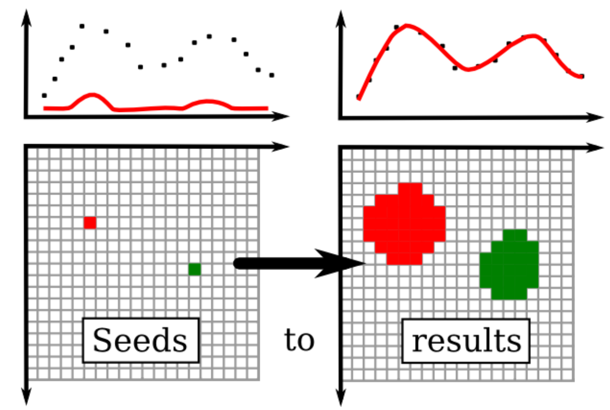
Forward Problem



Inverse Problem

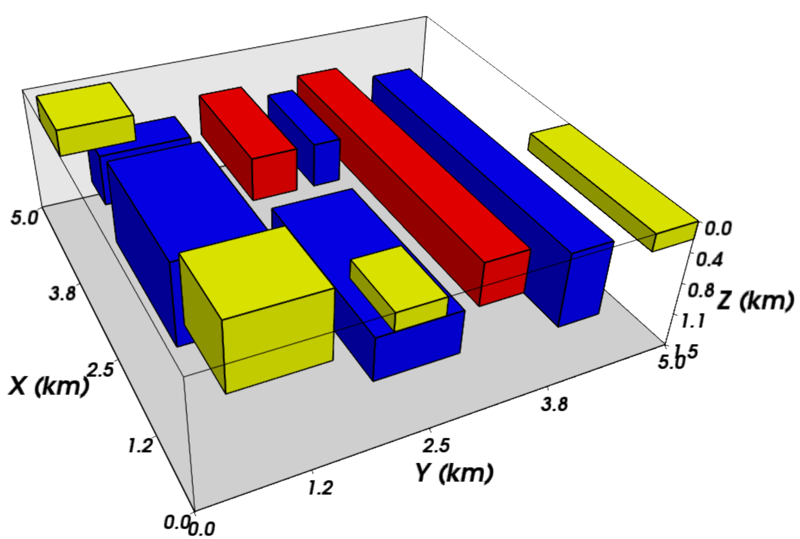


Planting Algorithm

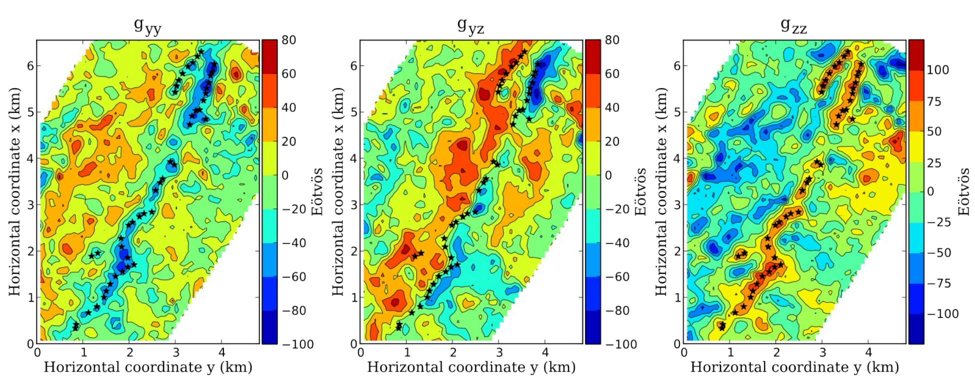


Inspired by René (1986)

Synthetic Data

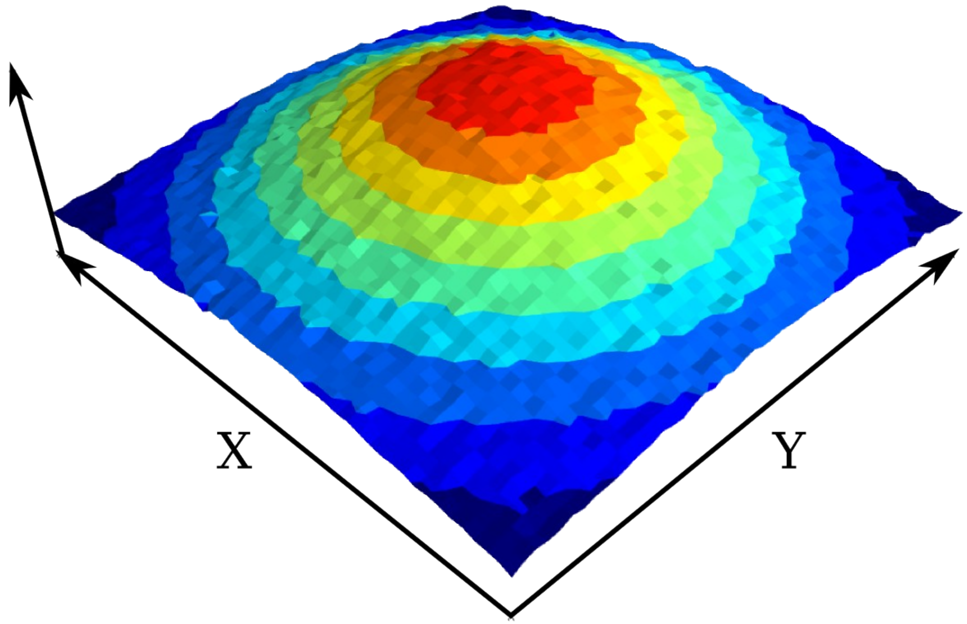


Real Data



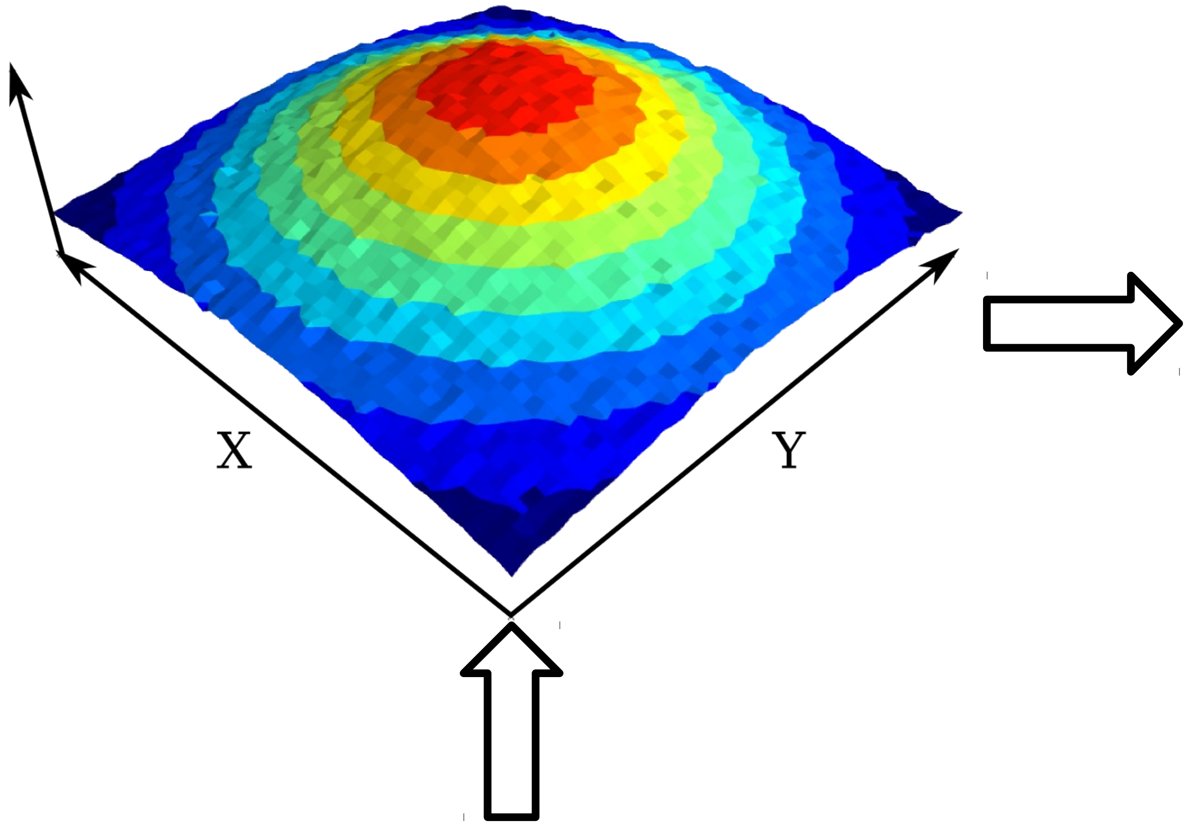
Quadrilátero Ferrífero

Forward problem



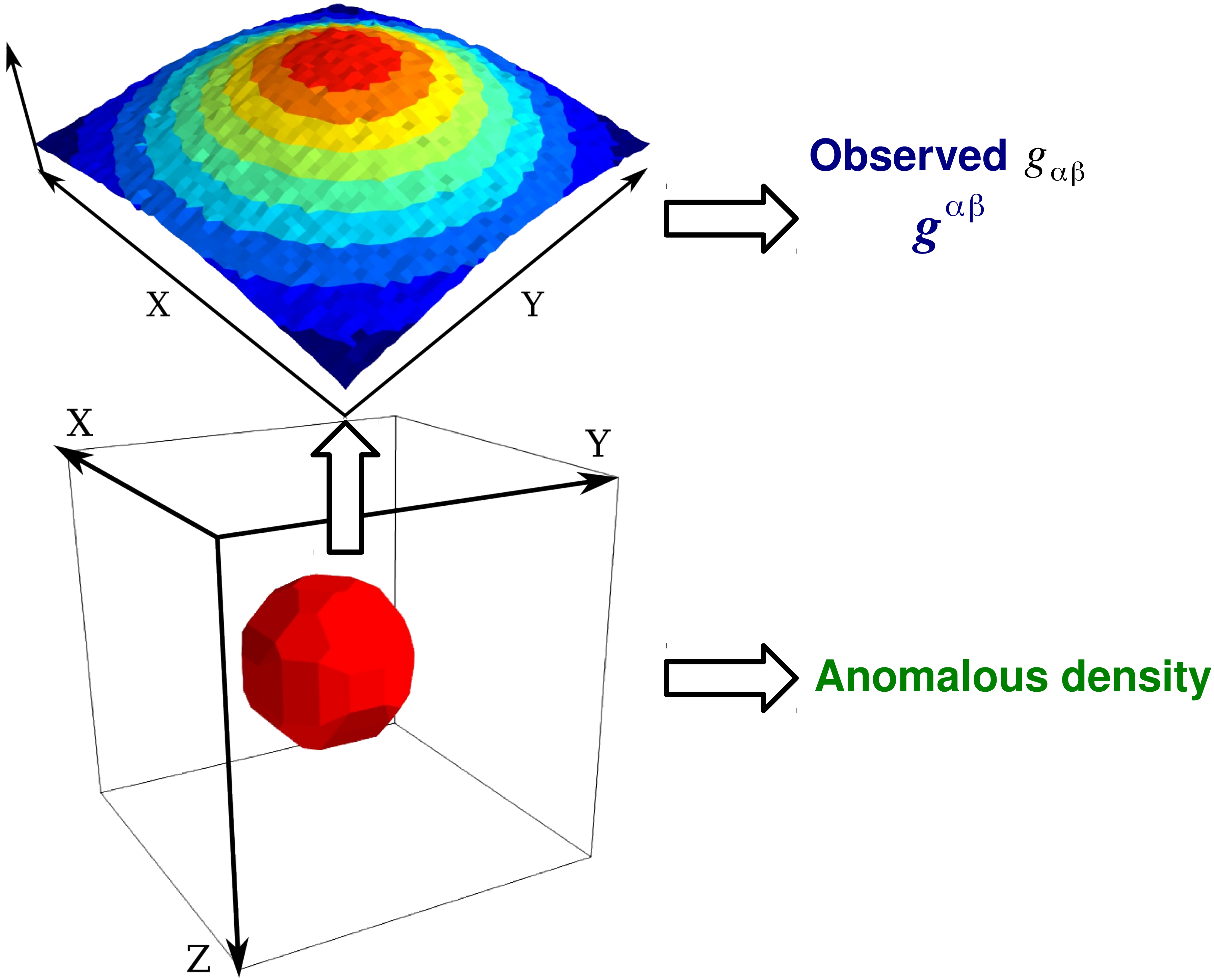
Observed $g_{\alpha\beta}$

$\boldsymbol{g}^{\alpha\beta}$

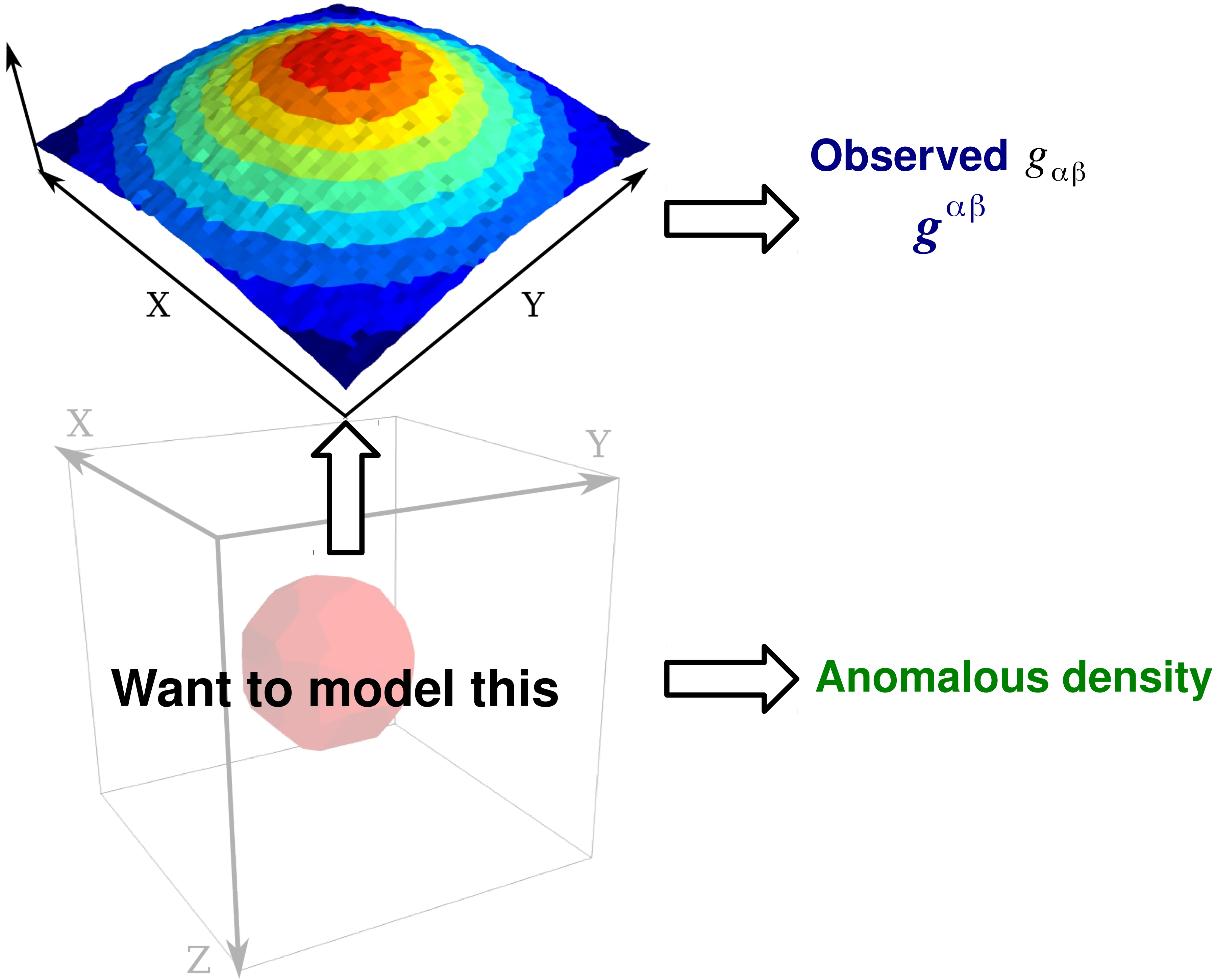


Observed $g_{\alpha\beta}$

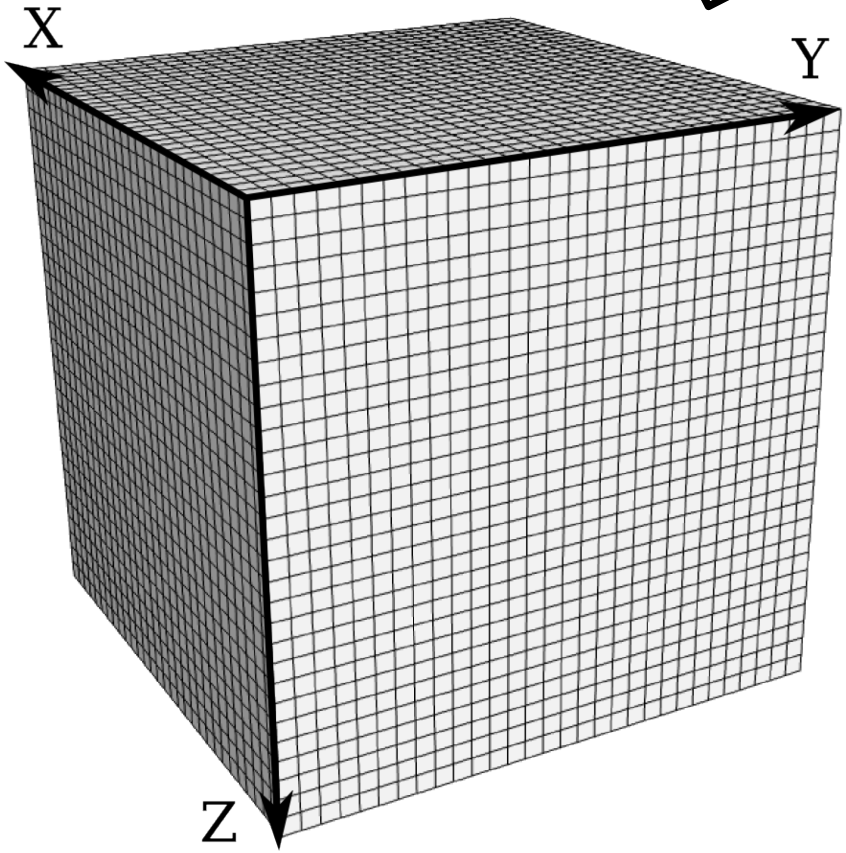
$g^{\alpha\beta}$



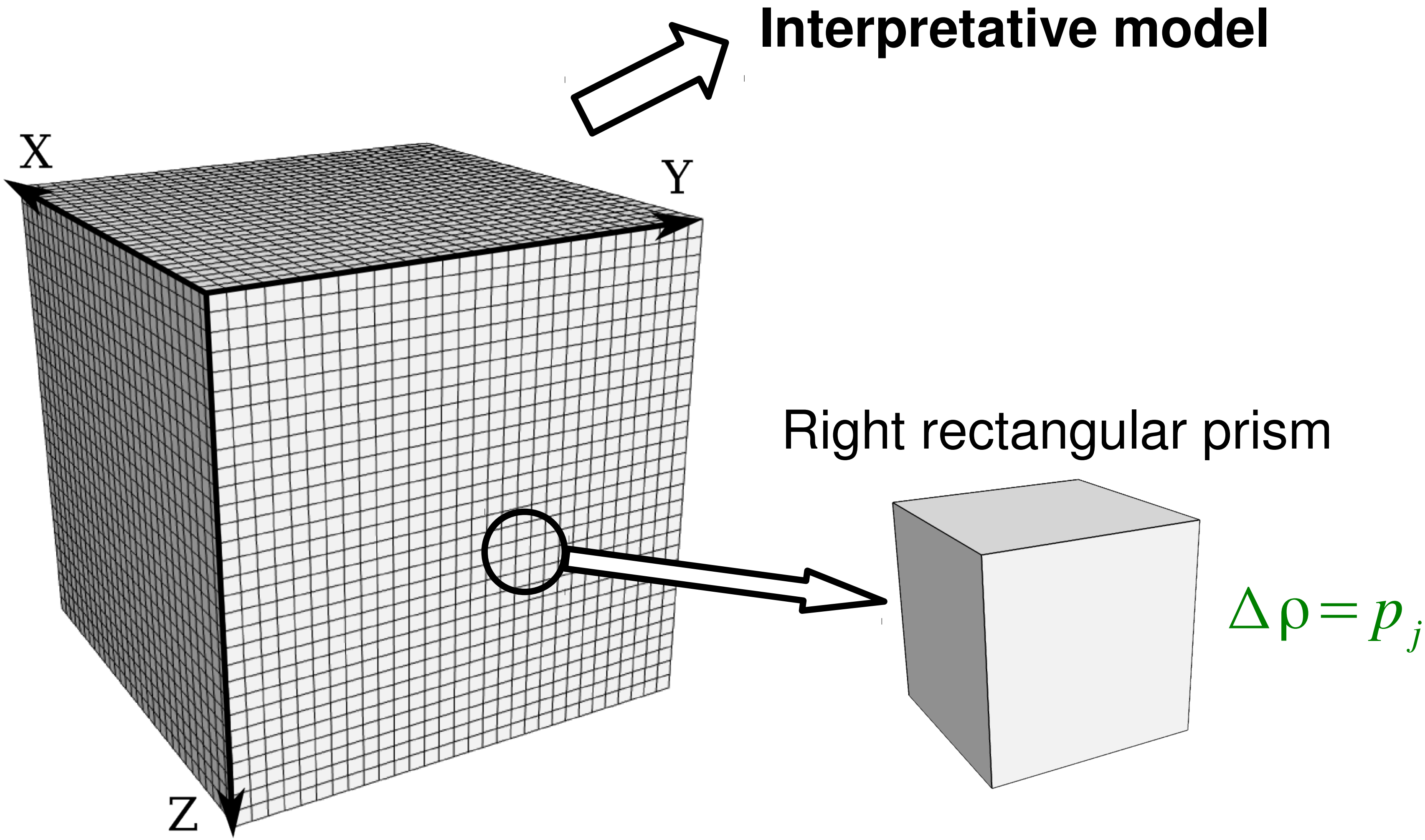
X
Y
Observed $g_{\alpha\beta}$
$\boldsymbol{g}^{\alpha\beta}$
X
Y
Z
Anomalous density



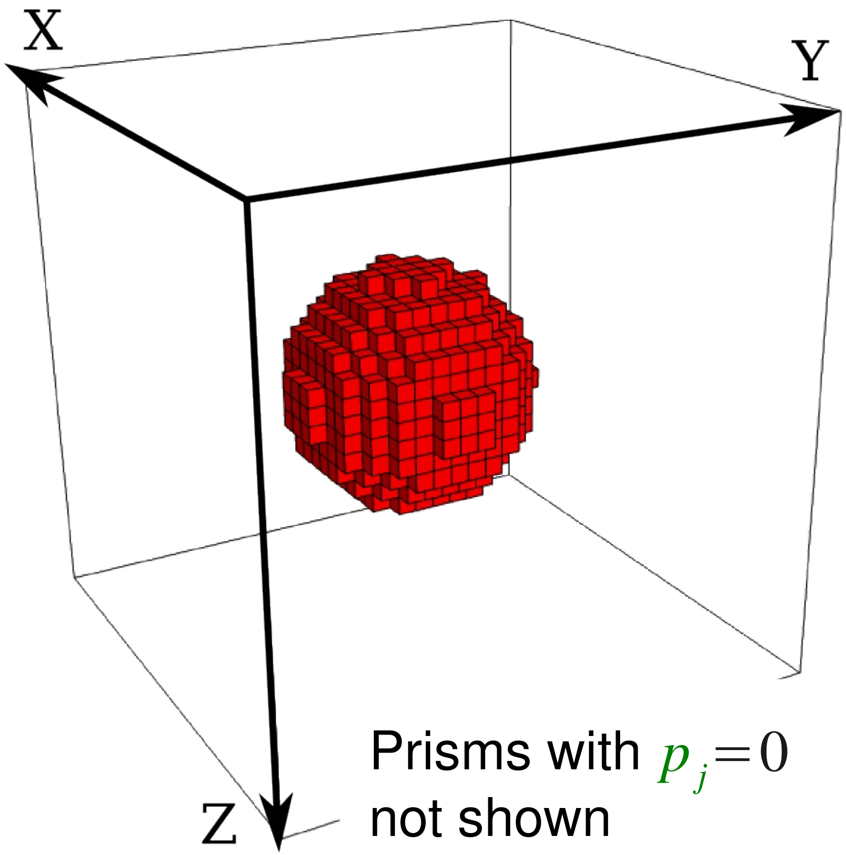
X
Y
Observed $g_{\alpha\beta}$
$\boldsymbol{g}^{\alpha\beta}$
X
Y
Want to model this
Anomalous density
Z

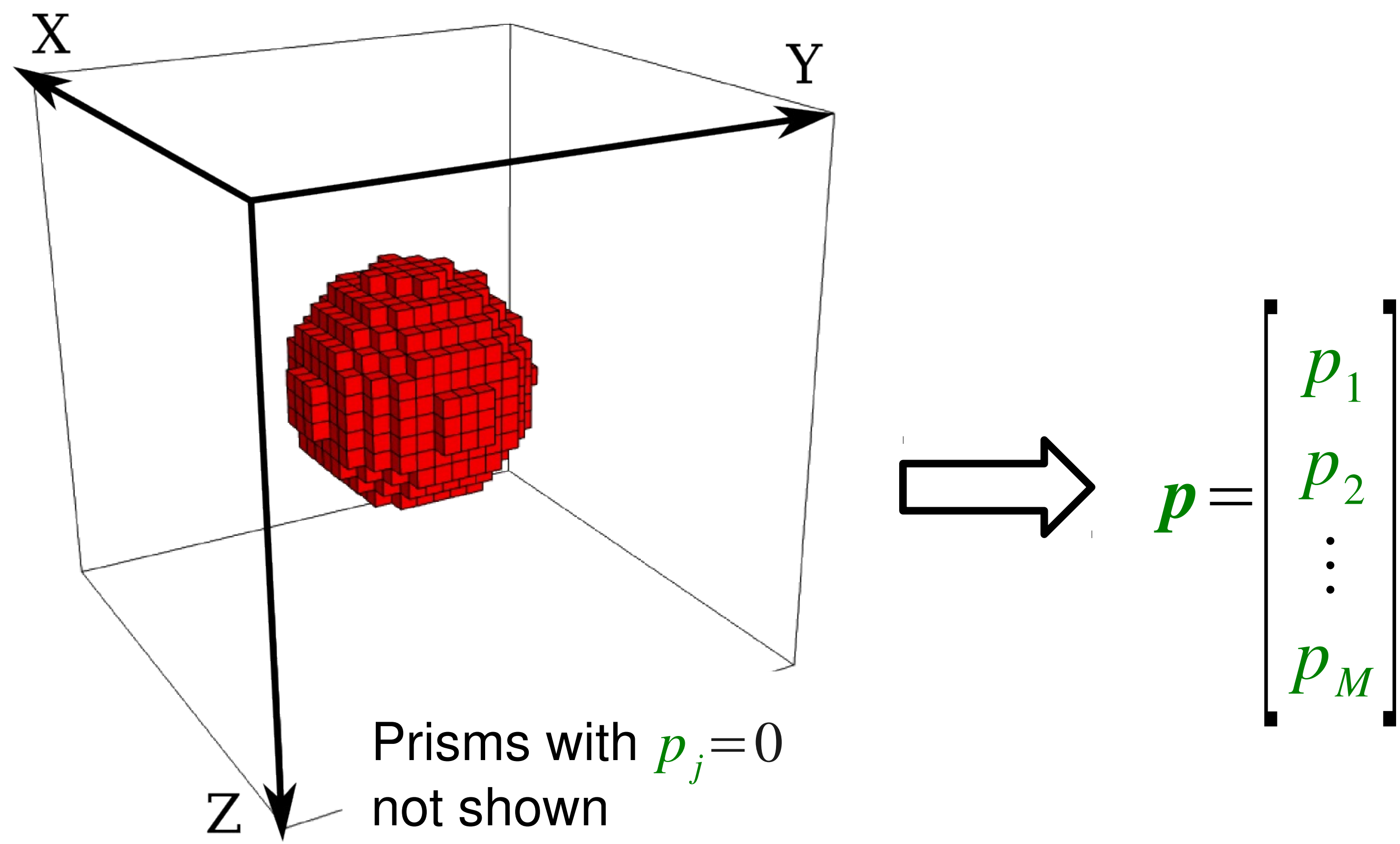


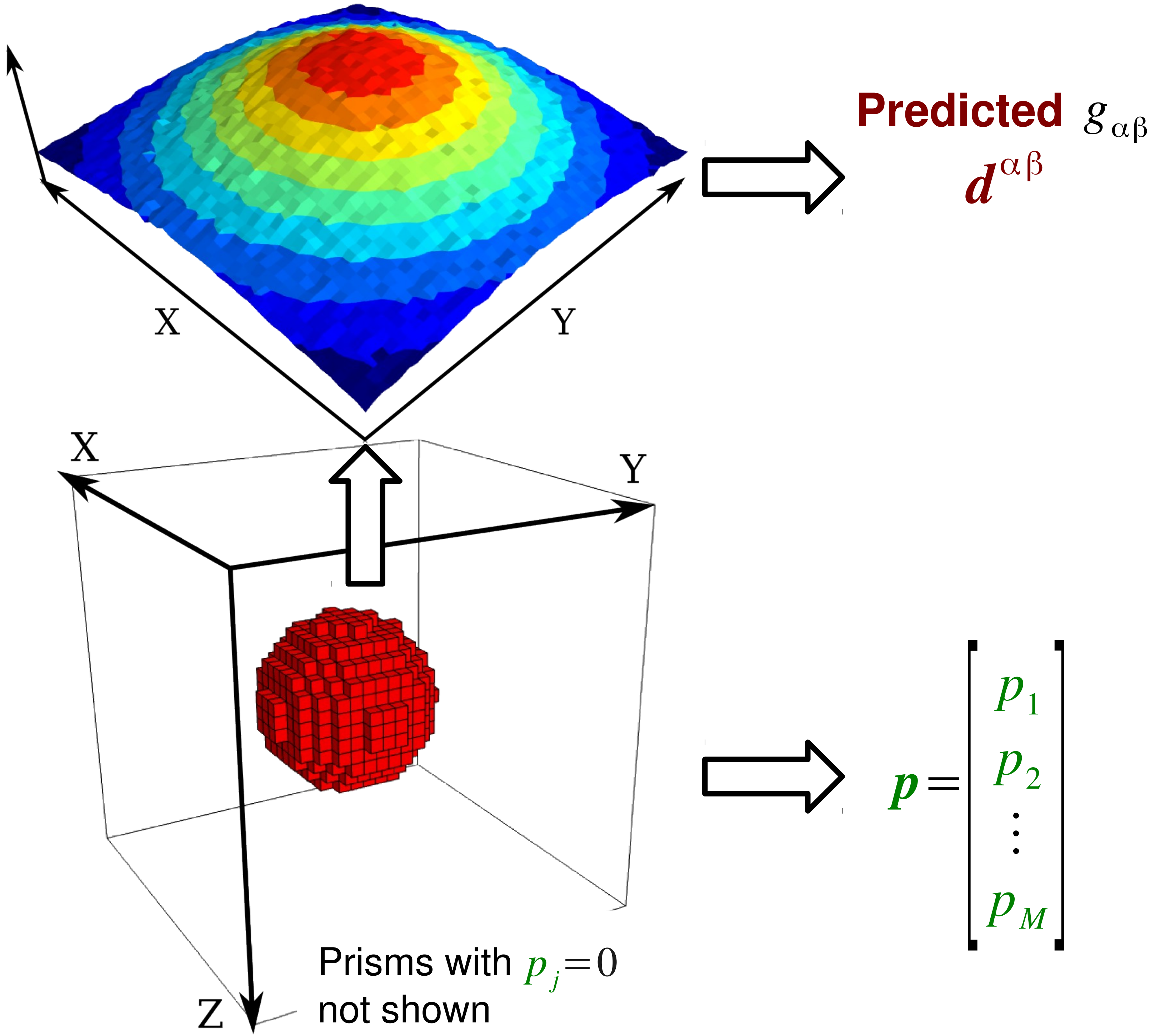
Interpretative model

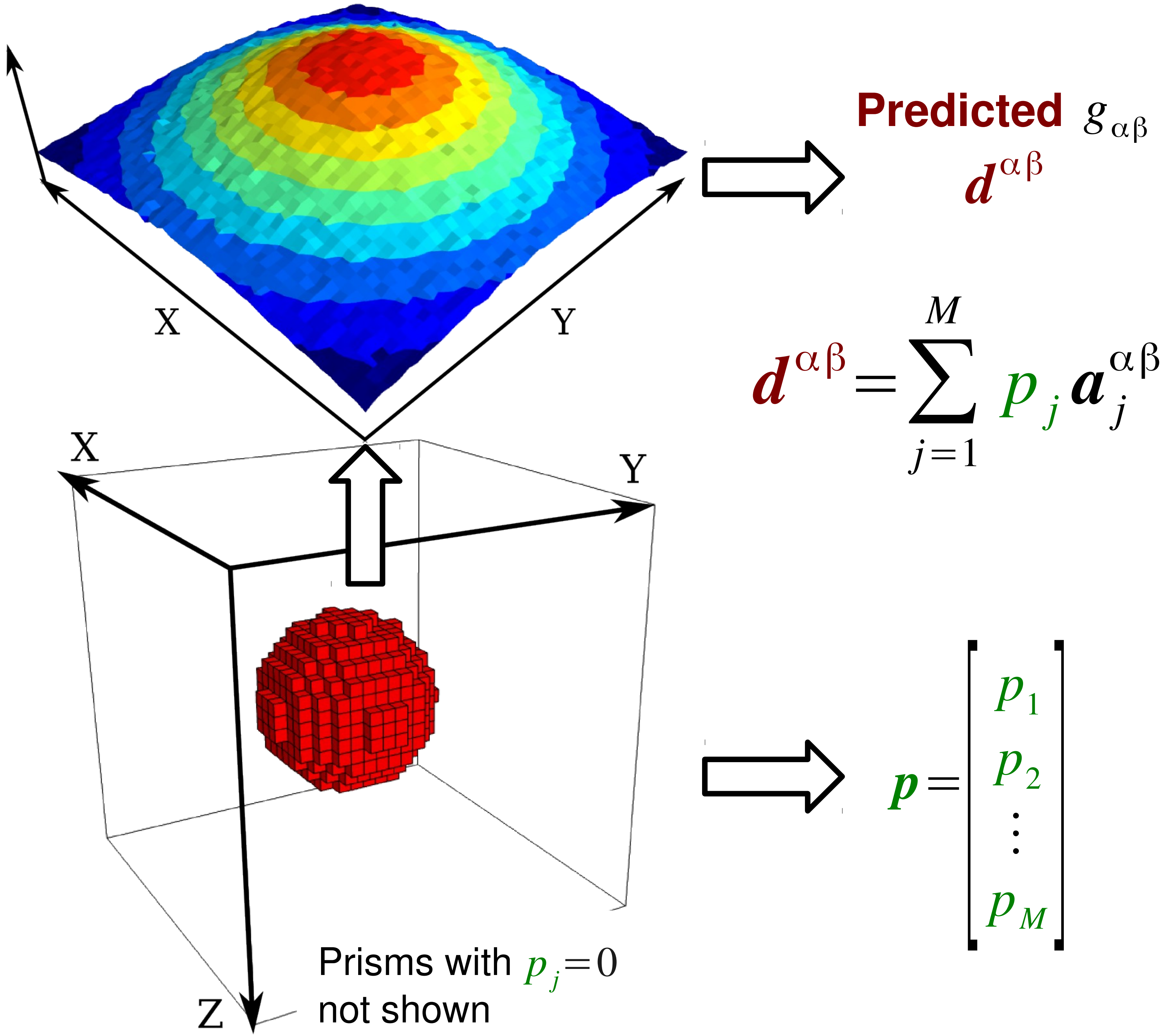


Interpretative model
X
Y
Z
Right rectangular prism
$\Delta\rho = p_j$

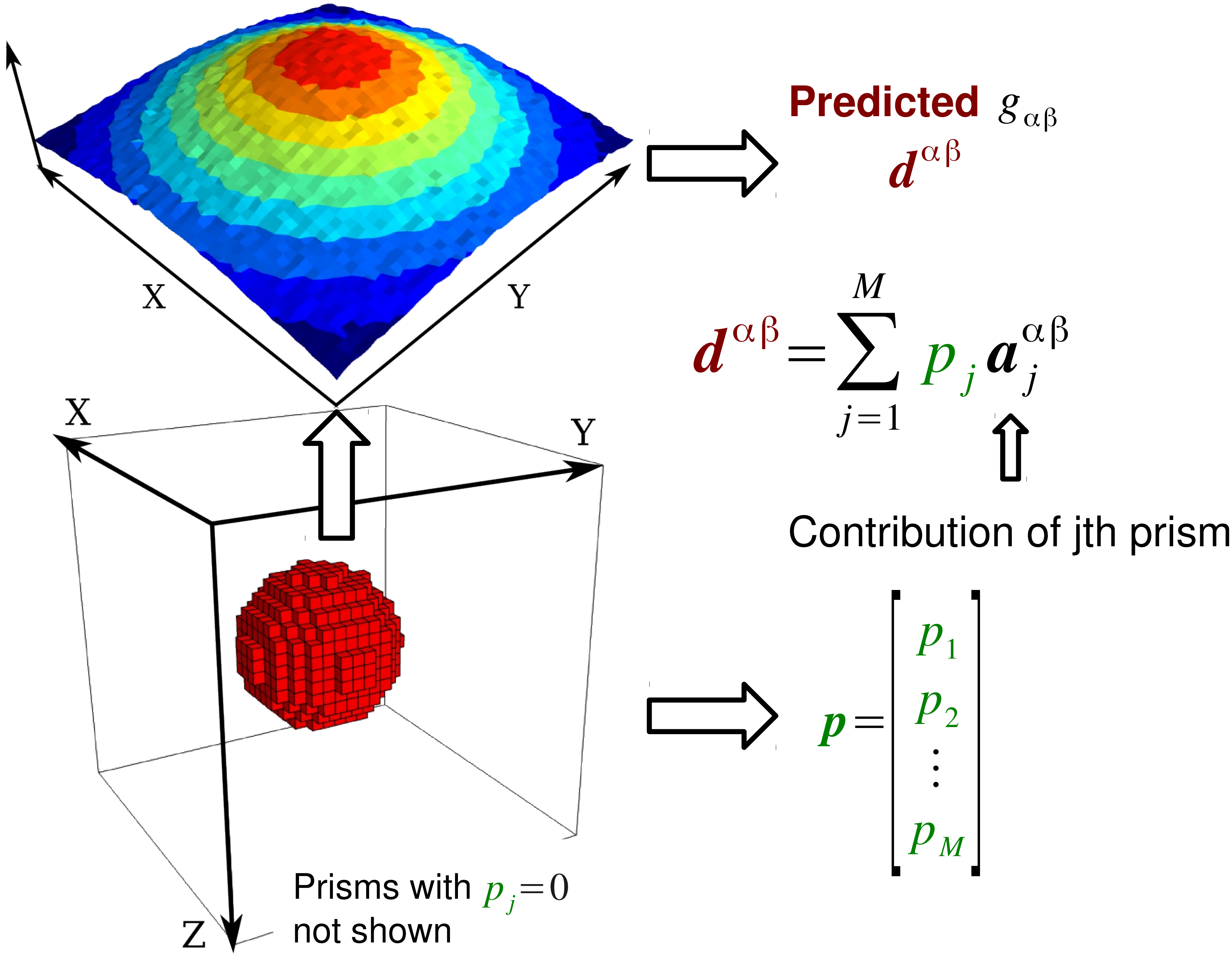

X
Y
Z
Prisms with $p_j = 0$
not shown


X
Y
Z
Prisms with $p_j = 0$
not shown
$\boldsymbol{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix}$


Predicted $g_{\alpha\beta}$
$\boldsymbol{d}^{\alpha\beta}$
X
Y
X
Y
Z
$\boldsymbol{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix}$
Prisms with $p_j = 0$ not shown



Predicted $g_{\alpha\beta}$
$\boldsymbol{d}^{\alpha\beta}$
X
Y
$\boldsymbol{d}^{\alpha\beta} = \sum_{j=1}^{M} p_j \boldsymbol{a}_j^{\alpha\beta}$
X
Y
Z
$\boldsymbol{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix}$
Prisms with $p_j = 0$ not shown



X
Y
Predicted $g_{\alpha\beta}$
$d^{\alpha\beta}$
$d^{\alpha\beta} = \sum_{j=1}^{M} p_j a_j^{\alpha\beta}$
X
Y
Contribution of jth prism
$p = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix}$
Z
Prisms with $p_j = 0$
not shown

More components:

$$\boldsymbol{d}^{xx}$$

$$\boldsymbol{d}^{xy}$$

$$\boldsymbol{d}^{xz}$$

$$\boldsymbol{d}^{yy}$$

$$\boldsymbol{d}^{yz}$$

$$\boldsymbol{d}^{zz}$$

More components:

$$d^{xx}$$

$$d^{xy}$$

$$d^{xz} \Rightarrow d$$

$$d^{yy}$$

$$d^{yz}$$

$$d^{zz}$$

More components:

$$\boldsymbol{d}^{xx}$$
$$\boldsymbol{d}^{xy}$$
$$\boldsymbol{d}^{xz}$$
$$\boldsymbol{d}^{yy}$$
$$\boldsymbol{d}^{yz}$$
$$\boldsymbol{d}^{zz}$$

$$\Rightarrow \quad \boldsymbol{d} = \sum_{j=1}^{M} p_j \boldsymbol{a}_j$$

More components:

$$\boldsymbol{d}^{xx}$$
$$\boldsymbol{d}^{xy}$$
$$\boldsymbol{d}^{xz}$$
$$\boldsymbol{d}^{yy}$$
$$\boldsymbol{d}^{yz}$$
$$\boldsymbol{d}^{zz}$$

$$\Rightarrow \quad \boldsymbol{d} = \sum_{j=1}^{M} p_j \boldsymbol{a}_j = \boldsymbol{A}\,\boldsymbol{p}$$

More components:

$\boldsymbol{d}^{xx}$

$\boldsymbol{d}^{xy}$

$\boldsymbol{d}^{xz}$

$\boldsymbol{d}^{yy}$

$\boldsymbol{d}^{yz}$

$\boldsymbol{d}^{zz}$

$$\Rightarrow \quad \boldsymbol{d} = \sum_{j=1}^{M} p_j \boldsymbol{a}_j = \boldsymbol{A}\,\boldsymbol{p}$$

$\Uparrow$

Jacobian (sensitivity) matrix

More components:

$$\boldsymbol{d}^{xx}$$
$$\boldsymbol{d}^{xy}$$
$$\boldsymbol{d}^{xz}$$
$$\boldsymbol{d}^{yy}$$
$$\boldsymbol{d}^{yz}$$
$$\boldsymbol{d}^{zz}$$

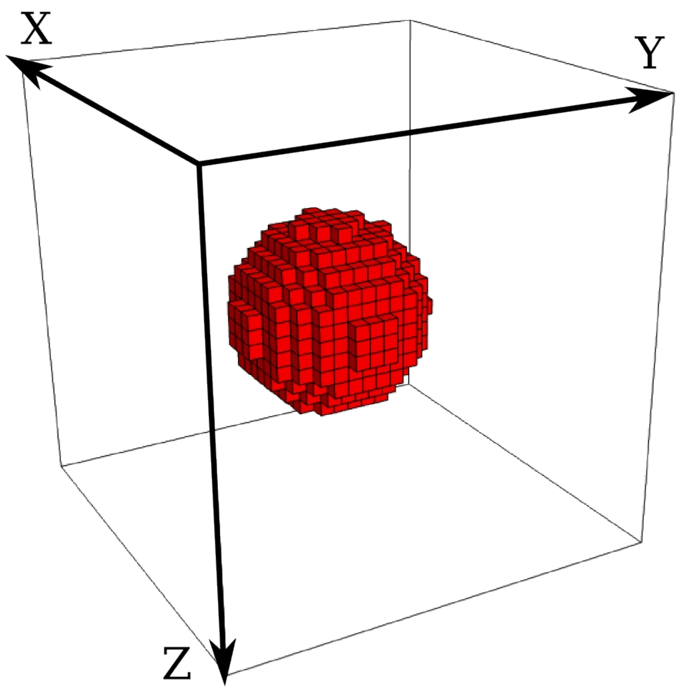
$$\Rightarrow \quad \boldsymbol{d} = \sum_{j=1}^{M} p_j \boldsymbol{a}_j = \boldsymbol{A}\,\boldsymbol{p}$$

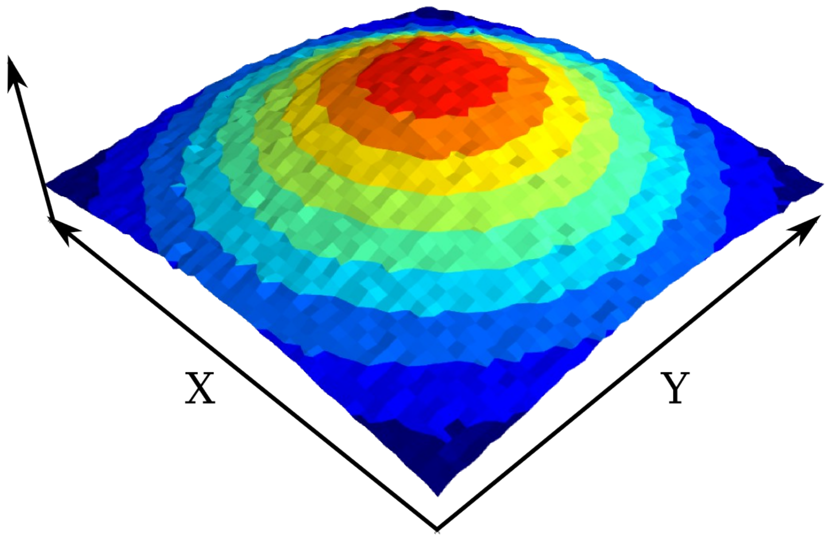
$\Uparrow$

Column vector of $\boldsymbol{A}$

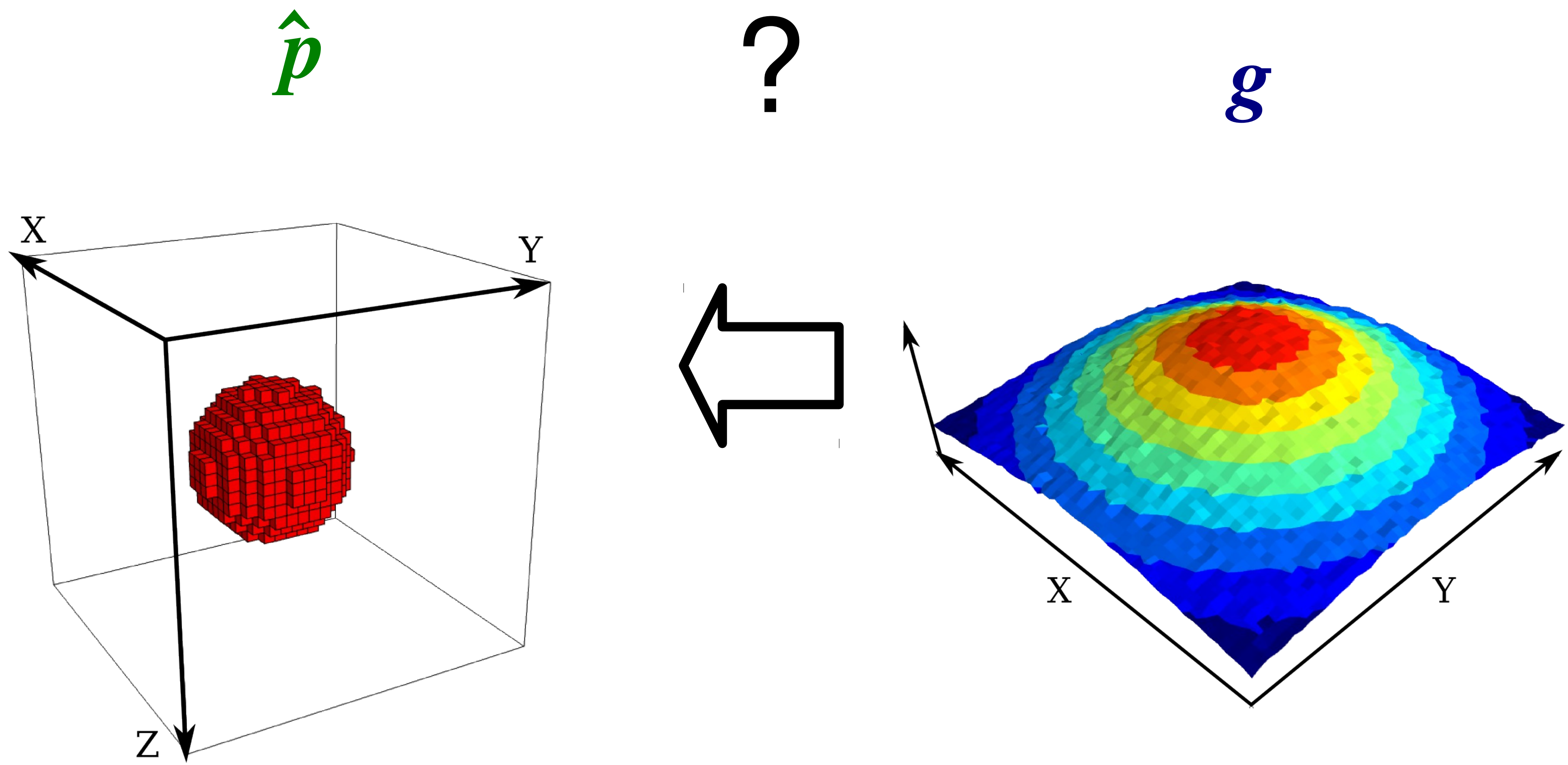
Forward problem:

$$\boldsymbol{p} \qquad \boldsymbol{d} = \sum_{j=1}^{M} p_j \boldsymbol{a}_j \qquad \boldsymbol{d}$$





Inverse problem:



Inverse problem

Minimize difference between $\boldsymbol{g}$ and $\boldsymbol{d}$

Residual vector $\Rightarrow$ $\boldsymbol{r} = \boldsymbol{g} - \boldsymbol{d}$

Minimize difference between $\boldsymbol{g}$ and $\boldsymbol{d}$

Residual vector $\Rightarrow$ $\boldsymbol{r} = \boldsymbol{g} - \boldsymbol{d}$

Data-misfit function:

$$\phi(\boldsymbol{p}) = \|\boldsymbol{r}\|_2 = \left(\sum_{i=1}^{N} (g_i - d_i)^2 \right)^{\frac{1}{2}}$$

Minimize difference between $\boldsymbol{g}$ and $\boldsymbol{d}$

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$\Uparrow$

ℓ_2-norm of **r**

Minimize difference between $\boldsymbol{g}$ and $\boldsymbol{d}$

Residual vector $\Rightarrow$ $\boldsymbol{r} = \boldsymbol{g} - \boldsymbol{d}$

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$\Uparrow$

ℓ_2-norm of **r**

Least-squares fit

Minimize difference between $\boldsymbol{g}$ and $\boldsymbol{d}$

Residual vector $\Rightarrow$ $\boldsymbol{r} = \boldsymbol{g} - \boldsymbol{d}$

Data-misfit function:

$$\phi(\boldsymbol{p}) = \|\boldsymbol{r}\|_2 = \left(\sum_{i=1}^{N} (g_i - d_i)^2 \right)^{\frac{1}{2}}$$

$\Uparrow$

ℓ_2-norm of **r**

Least-squares fit

$$\phi(\boldsymbol{p}) = \|\boldsymbol{r}\|_1 = \sum_{i=1}^{N} |g_i - d_i|$$

$\Uparrow$

ℓ_1-norm of **r**

Minimize difference between $\boldsymbol{g}$ and $\boldsymbol{d}$

Residual vector $\Rightarrow$ $\boldsymbol{r} = \boldsymbol{g} - \boldsymbol{d}$

Data-misfit function:

$$\phi(\boldsymbol{p}) = \|\boldsymbol{r}\|_2 = \left(\sum_{i=1}^{N} (g_i - d_i)^2\right)^{\frac{1}{2}}$$

$\Uparrow$

ℓ_2-norm of **r**

Least-squares fit

$$\phi(\boldsymbol{p}) = \|\boldsymbol{r}\|_1 = \sum_{i=1}^{N} |g_i - d_i|$$

$\Uparrow$

ℓ_1-norm of **r**

Robust fit

ill-posed problem

non-existent

non-unique

non-stable

ill-posed problem $\xrightarrow{\text{constraints}}$

non-existent

non-unique

non-stable

ill-posed problem $\xrightarrow{\text{constraints}}$ **well-posed problem**

non-existent	exist
non-unique	unique
non-stable	stable

Constraints:

1. Compact

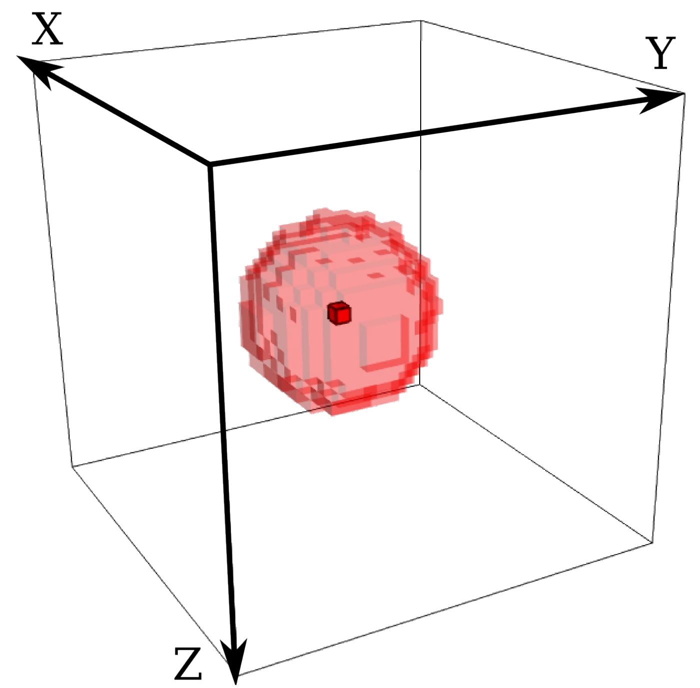
Constraints:

1. Compact $\Rightarrow$ no holes inside

Constraints:

1. Compact ⇨ no holes inside

2. Concentrated around “seeds”

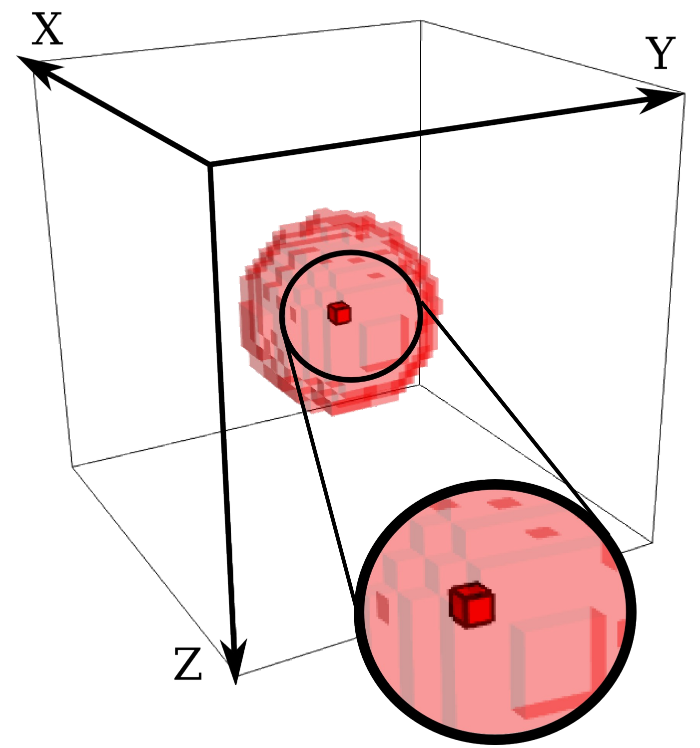


Constraints:

1. Compact ⇨ no holes inside

2. Concentrated around "seeds"

- **User-specified** prisms
- **Given** density contrasts ρ_s
- Any # of ≠ density contrasts



Constraints:

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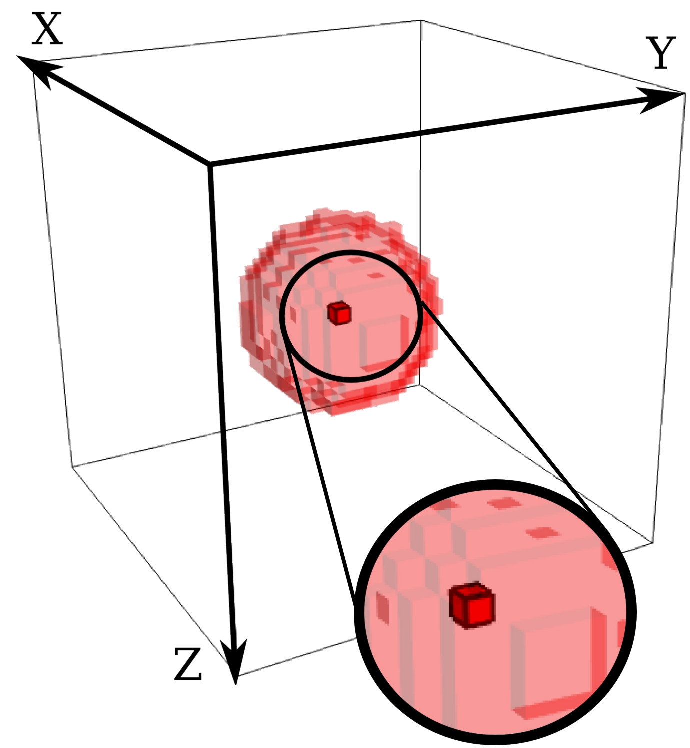
3. Only $p_j = 0$ or $p_j = \rho_s$



Constraints:

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   - **User-specified** prisms
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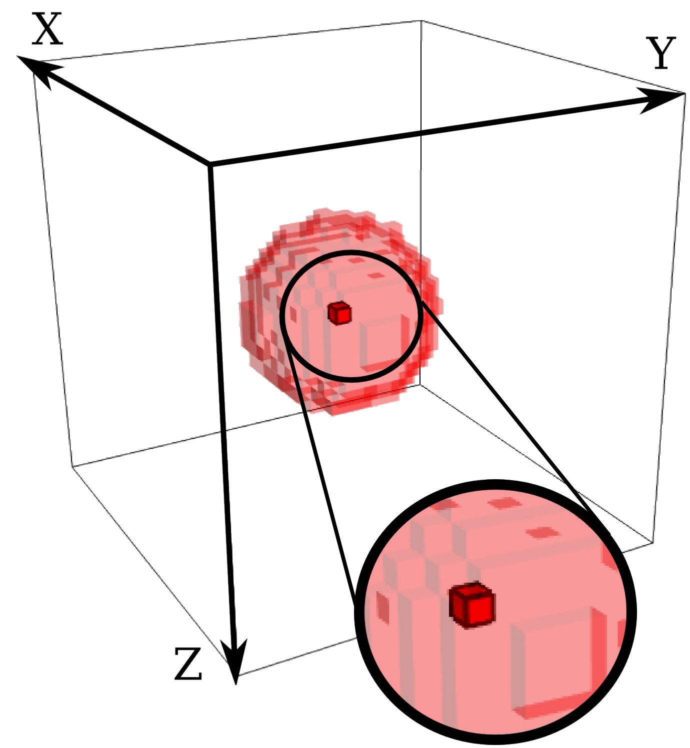
3. Only $p_j = 0$ or $p_j = \rho_s$

4. $p_j = \rho_s$ of closest seed



Well-posed problem: Minimize **goal function**

$$\Gamma(\boldsymbol{p}) = \phi(\boldsymbol{p}) + \mu\,\theta(\boldsymbol{p})$$

Well-posed problem: Minimize **goal function**

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$$\Uparrow$$

Data-misfit function

Well-posed problem: Minimize **goal function**

$$\Gamma(\boldsymbol{p}) = \phi(\boldsymbol{p}) + \mu\,\theta(\boldsymbol{p})$$

$\Uparrow$

Regularizing parameter

(Tradeoff between fit and regularization)

Well-posed problem: Minimize **goal function**

$$\Gamma(\boldsymbol{p}) = \phi(\boldsymbol{p}) + \mu\,\theta(\boldsymbol{p})$$

$\Uparrow$

Regularizing function

$$\theta(\boldsymbol{p}) = \sum_{j=1}^{M} \frac{p_j}{p_j + \epsilon}\, l_j^{\beta}$$

Well-posed problem: Minimize **goal function**

$$\Gamma(\boldsymbol{p}) = \phi(\boldsymbol{p}) + \mu\,\theta(\boldsymbol{p})$$

Regularizing function

Similar to
Silva Dias et al. (2009)

$$\theta(\boldsymbol{p}) = \sum_{j=1}^{M} \frac{p_j}{p_j + \epsilon}\, l_j^{\beta}$$

Well-posed problem: Minimize **goal function**

$$\Gamma(\boldsymbol{p}) = \phi(\boldsymbol{p}) + \mu\,\theta(\boldsymbol{p})$$

Regularizing function

Similar to
Silva Dias et al. (2009)

$$\theta(\boldsymbol{p}) = \sum_{j=1}^{M} \frac{p_j}{p_j + \epsilon} l_j^{\beta}$$

Distance between
jth prism and seed

Well-posed problem: Minimize **goal function**

$$\Gamma(\boldsymbol{p}) = \phi(\boldsymbol{p}) + \mu\,\theta(\boldsymbol{p})$$

Regularizing function

Similar to
Silva Dias et al. (2009)

$$\theta(\boldsymbol{p}) = \sum_{j=1}^{M} \frac{p_j}{p_j + \epsilon} l_j^{\beta}$$

Distance between
jth prism and seed

Imposes:

- Compactness
- Concentration around seeds

Constraints:

1. Compact

2. Concentrated around “seeds”

$\Rightarrow$ Regularization

3. Only $p_j = 0$ or $p_j = \rho_s$

4. $p_j = \rho_s$ of closest seed

Constraints:

1. Compact

2. Concentrated around “seeds”

$\Rightarrow$ Regularization

3. Only $p_j = 0$ or $p_j = \rho_s$

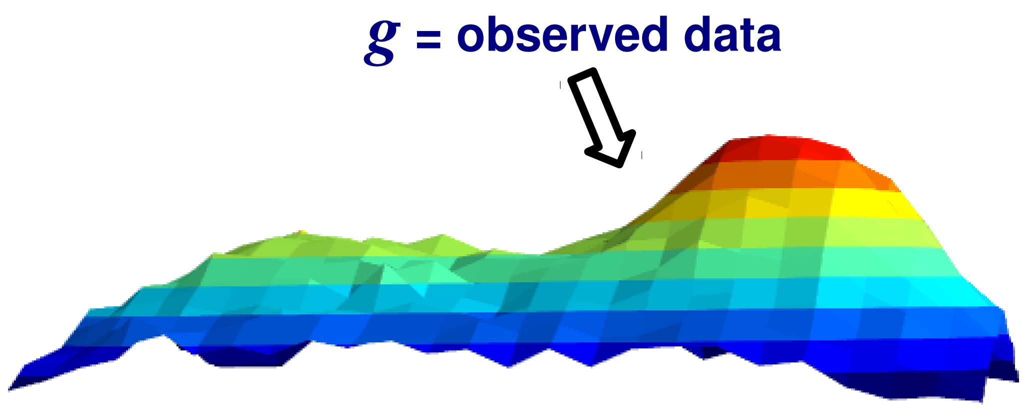
4. $p_j = \rho_s$ of closest seed

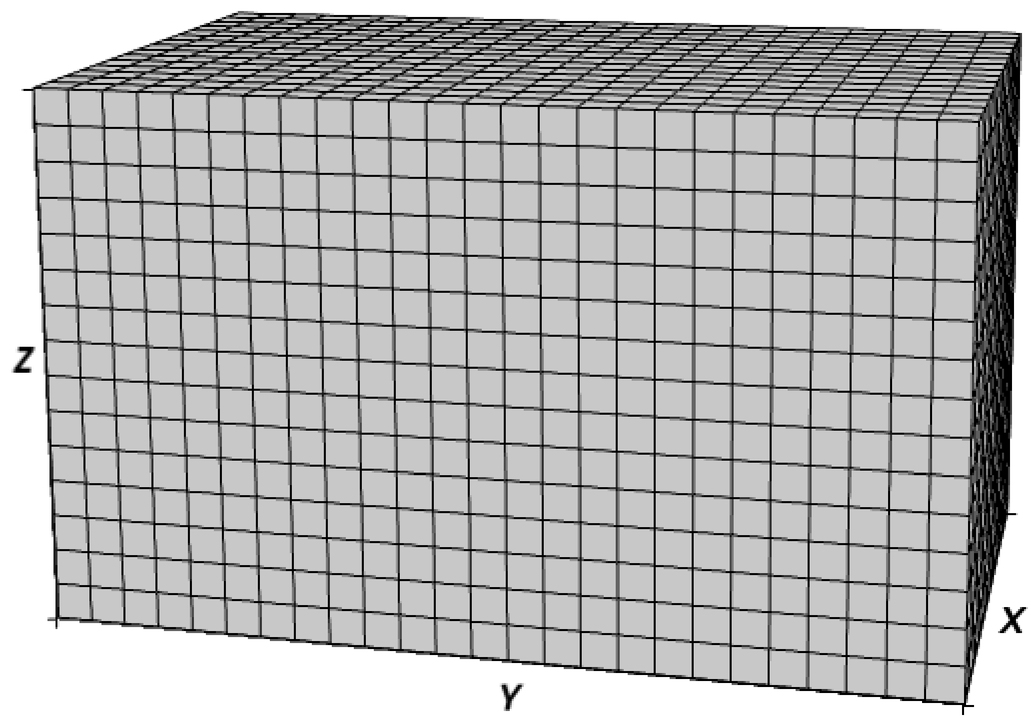
$\Rightarrow$ Algorithm

Based on René (1986)

Planting Algorithm

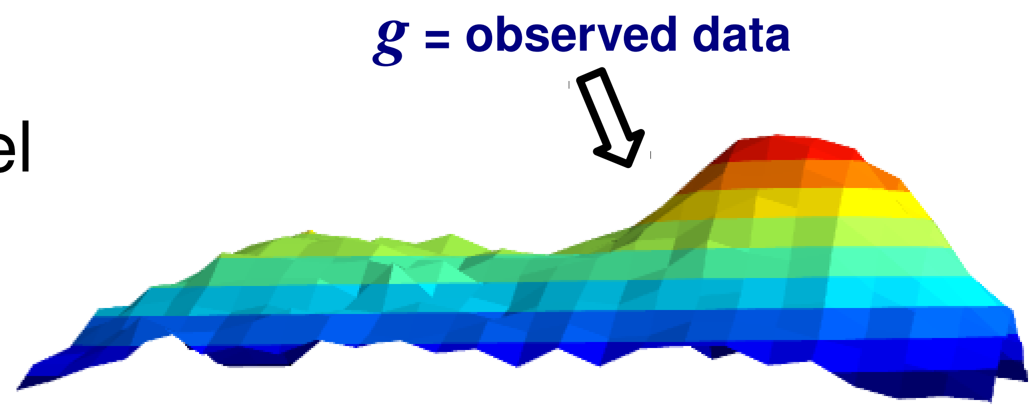
Setup:

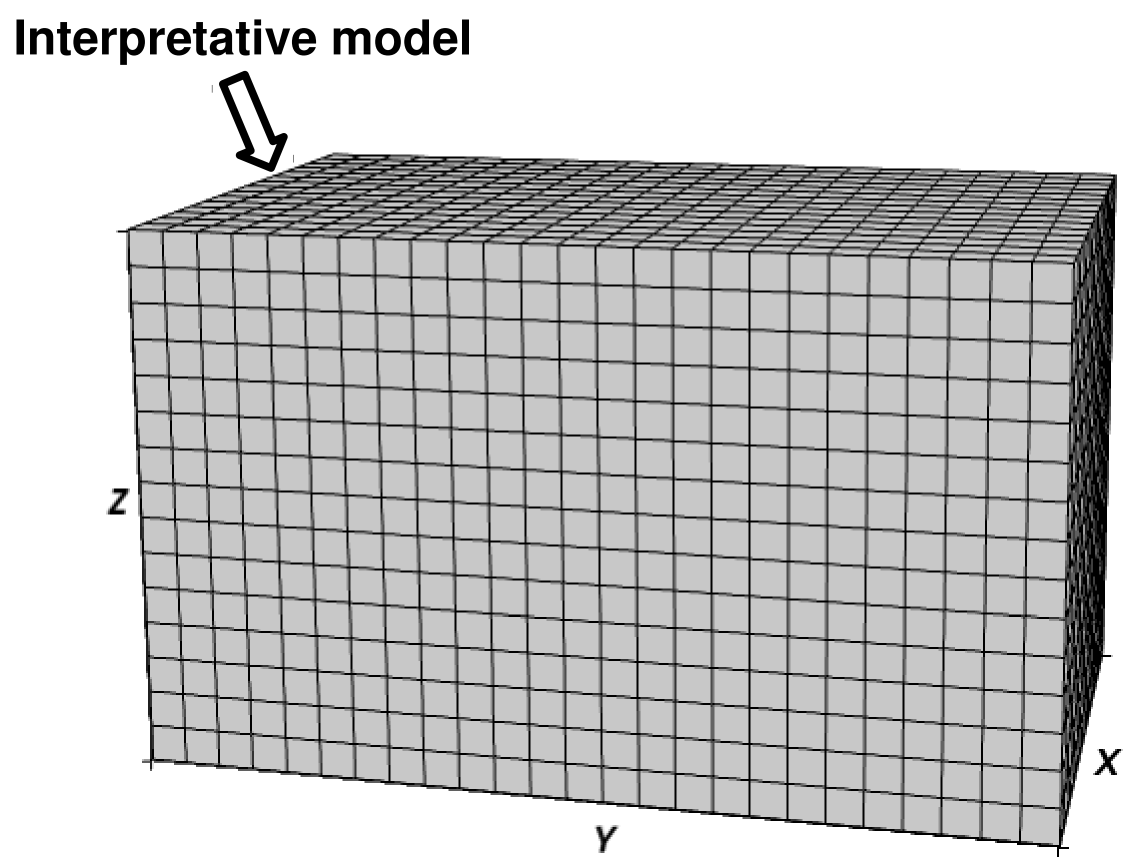




Setup:

Define interpretative model

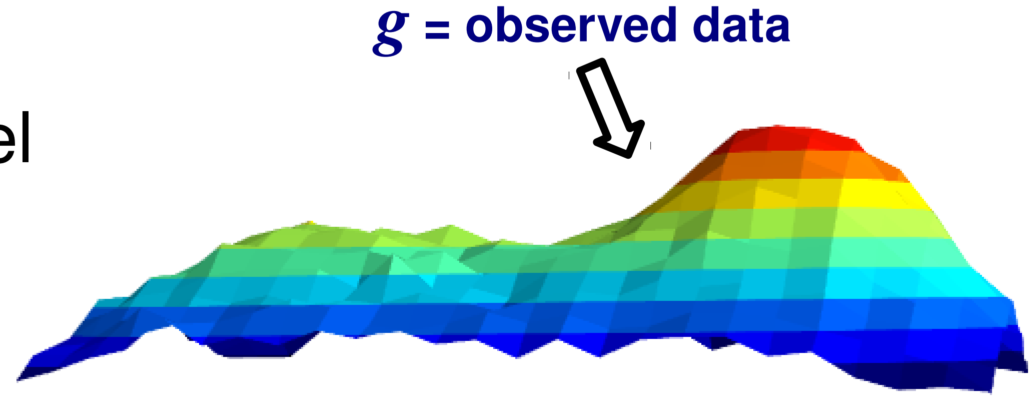


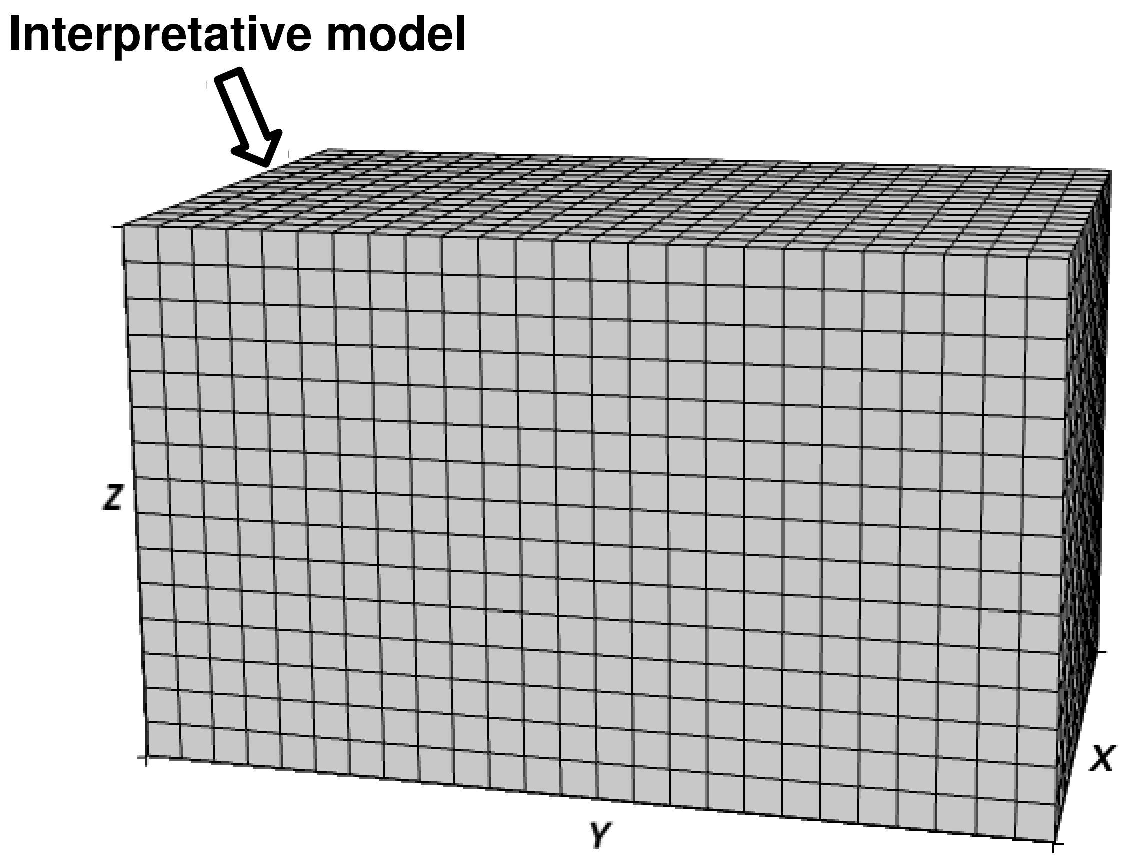


Setup:

Define interpretative model

All parameters zero



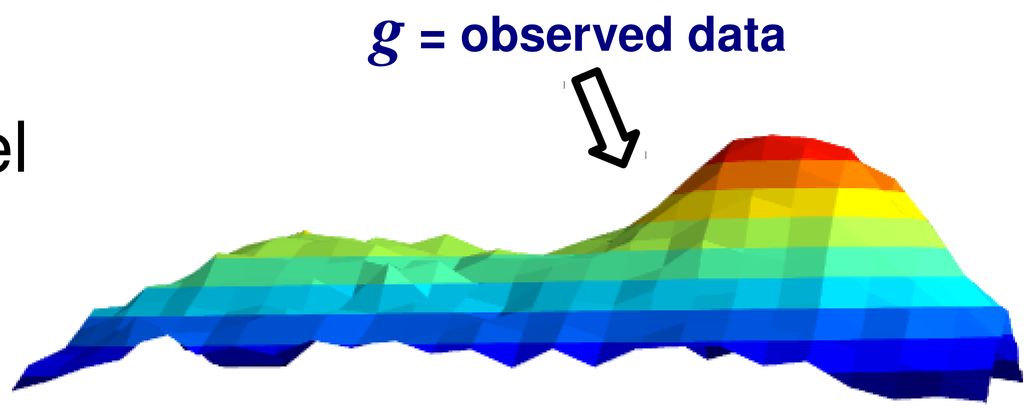


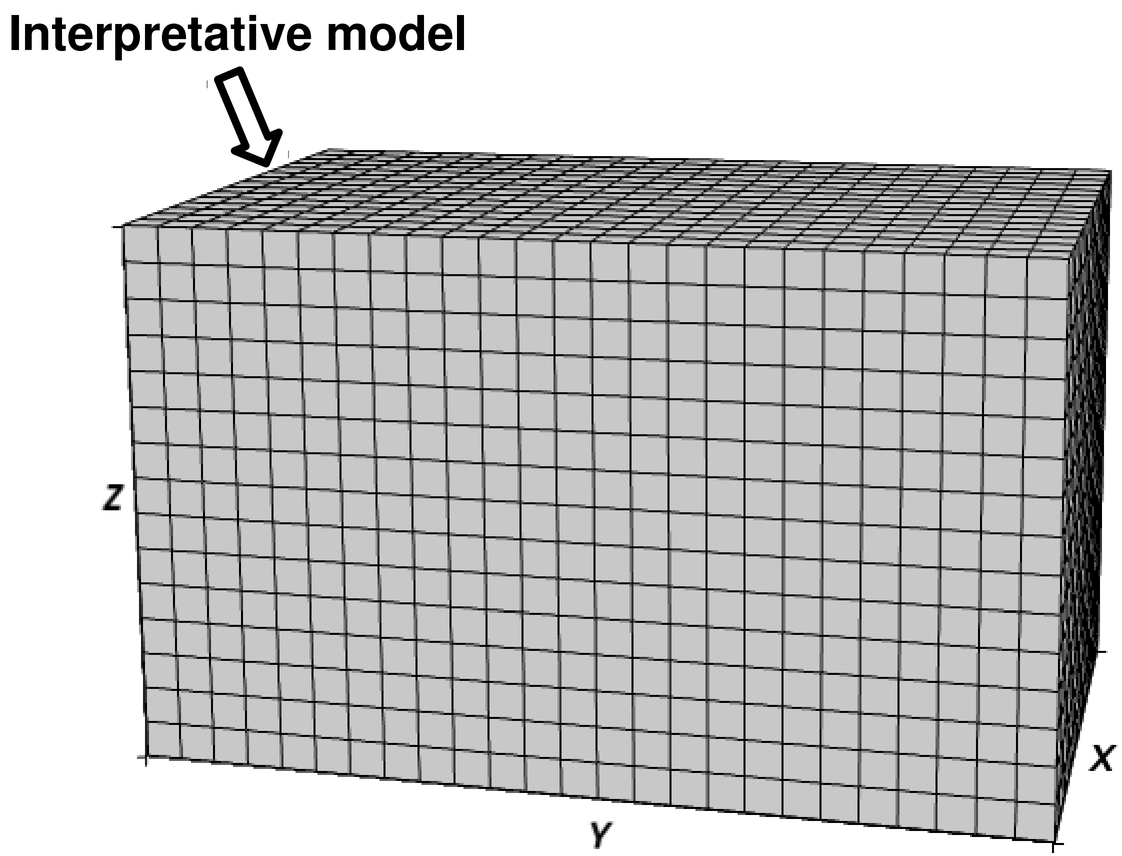
Setup:

Define interpretative model

All parameters zero

N_S seeds





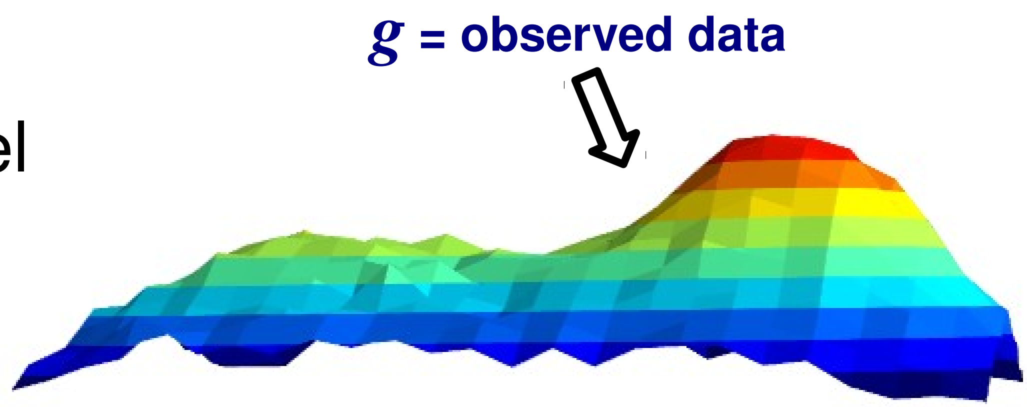
Setup:

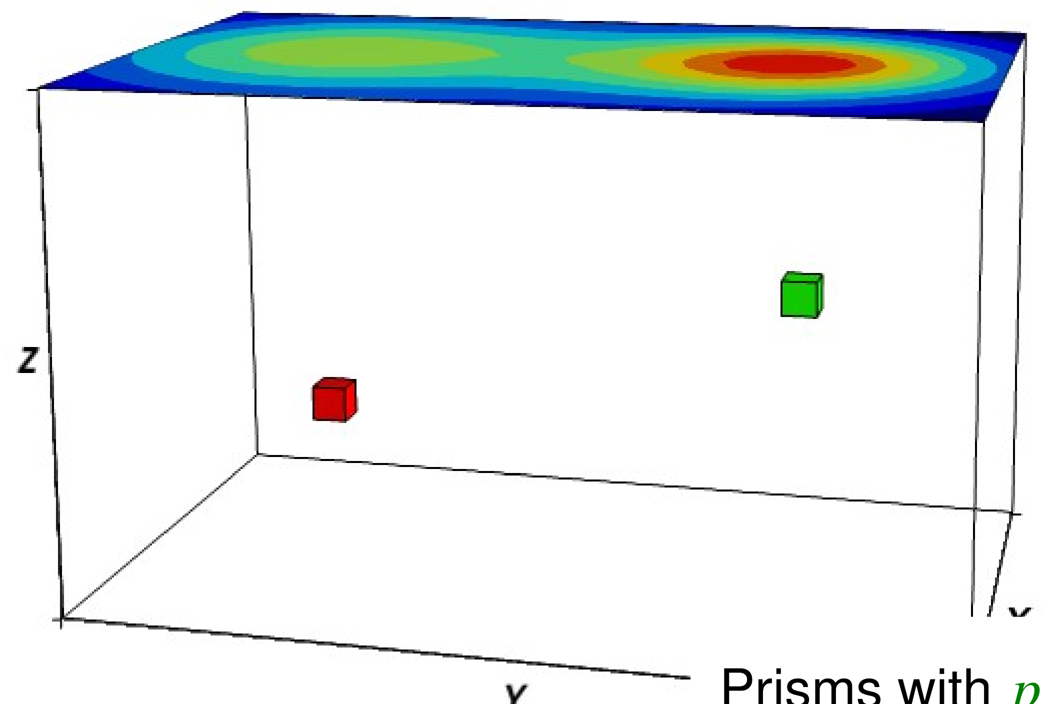
Define interpretative model

All parameters zero

N_S seeds

Include seeds





Prisms with $p_j = 0$ not shown

Setup:

Define interpretative model

All parameters zero

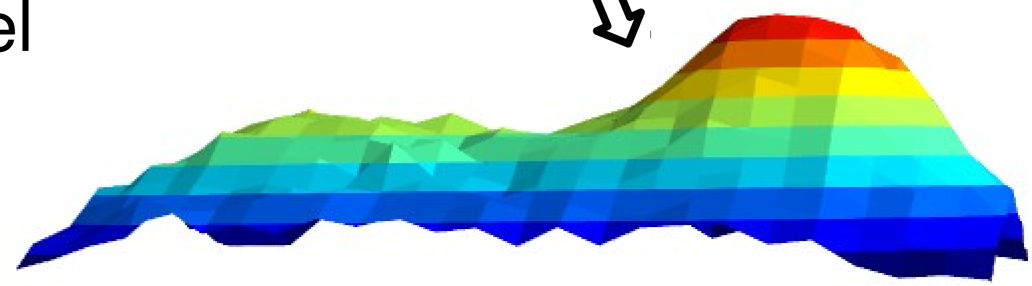
N_s seeds

Include seeds

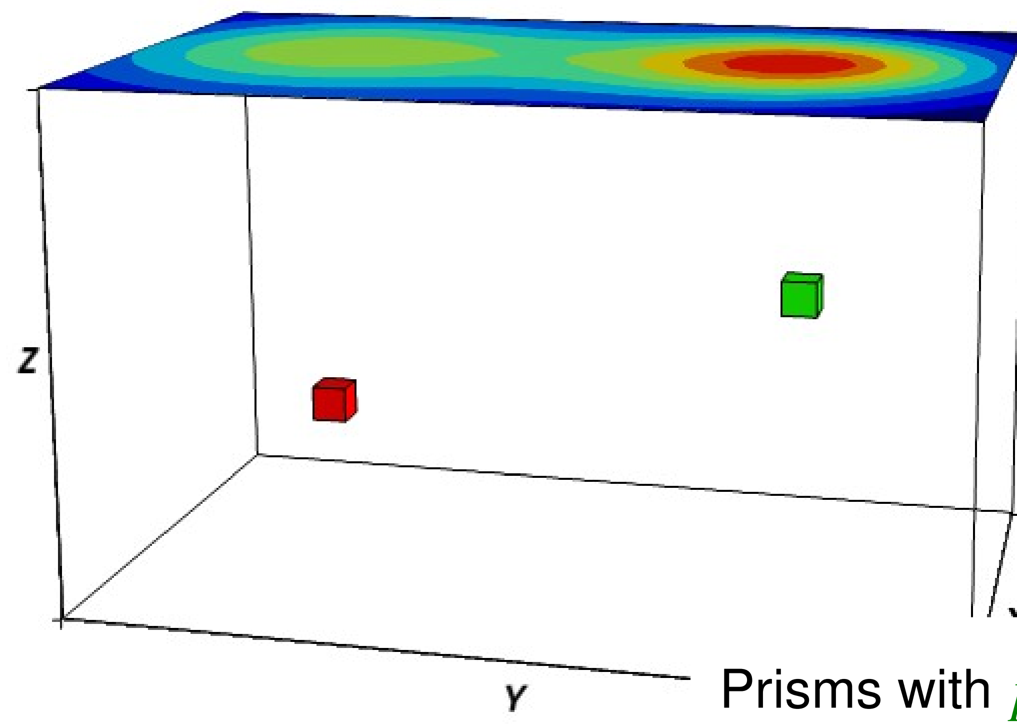
Compute initial residuals

$$\boldsymbol{r}^{(0)} = \boldsymbol{g} - \boldsymbol{d}^{(0)}$$

$\boldsymbol{g}$ = observed data



$\boldsymbol{d}$ = predicted data



Prisms with $p_j = 0$
not shown

Setup:

Define interpretative model

All parameters zero

N_S seeds

Include seeds

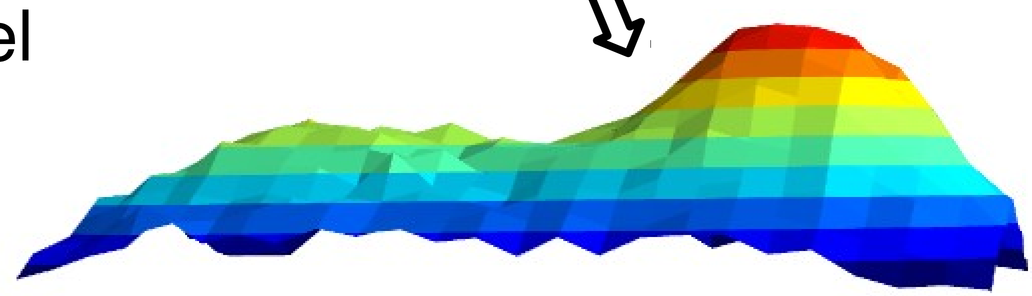
Compute initial residuals

$$\boldsymbol{r}^{(0)} = \boldsymbol{g} - \boldsymbol{d}^{(0)}$$

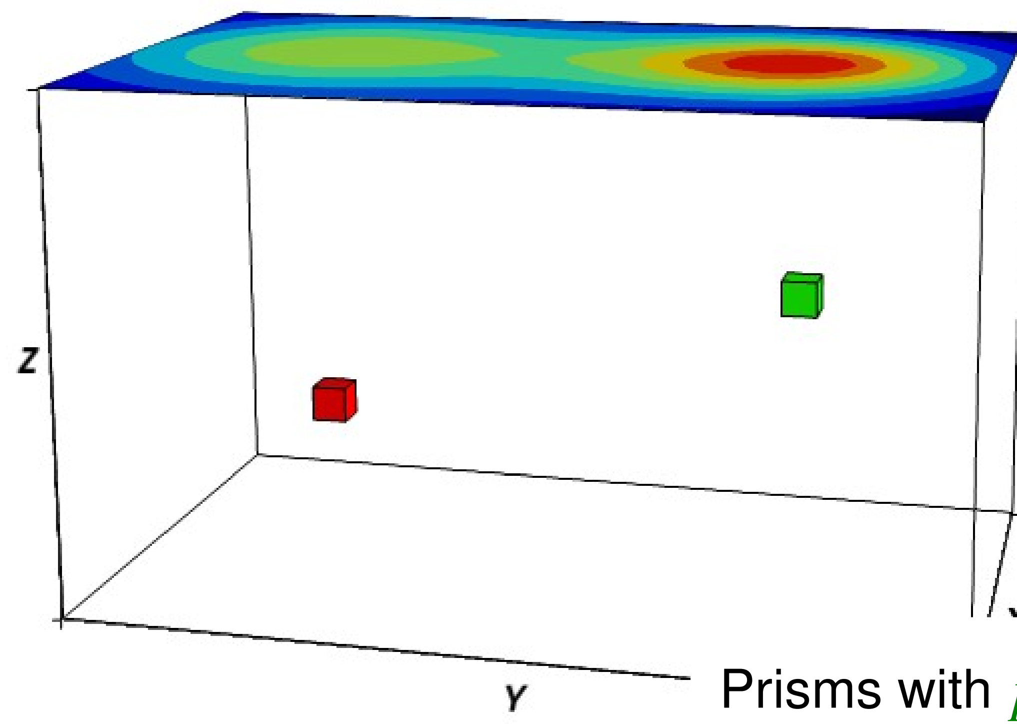
$\Downarrow$

Predicted by seeds

$\boldsymbol{g}$ = **observed data**



$\boldsymbol{d}$ = **predicted data**



Prisms with $p_j = 0$ not shown

Setup:

Define interpretative model

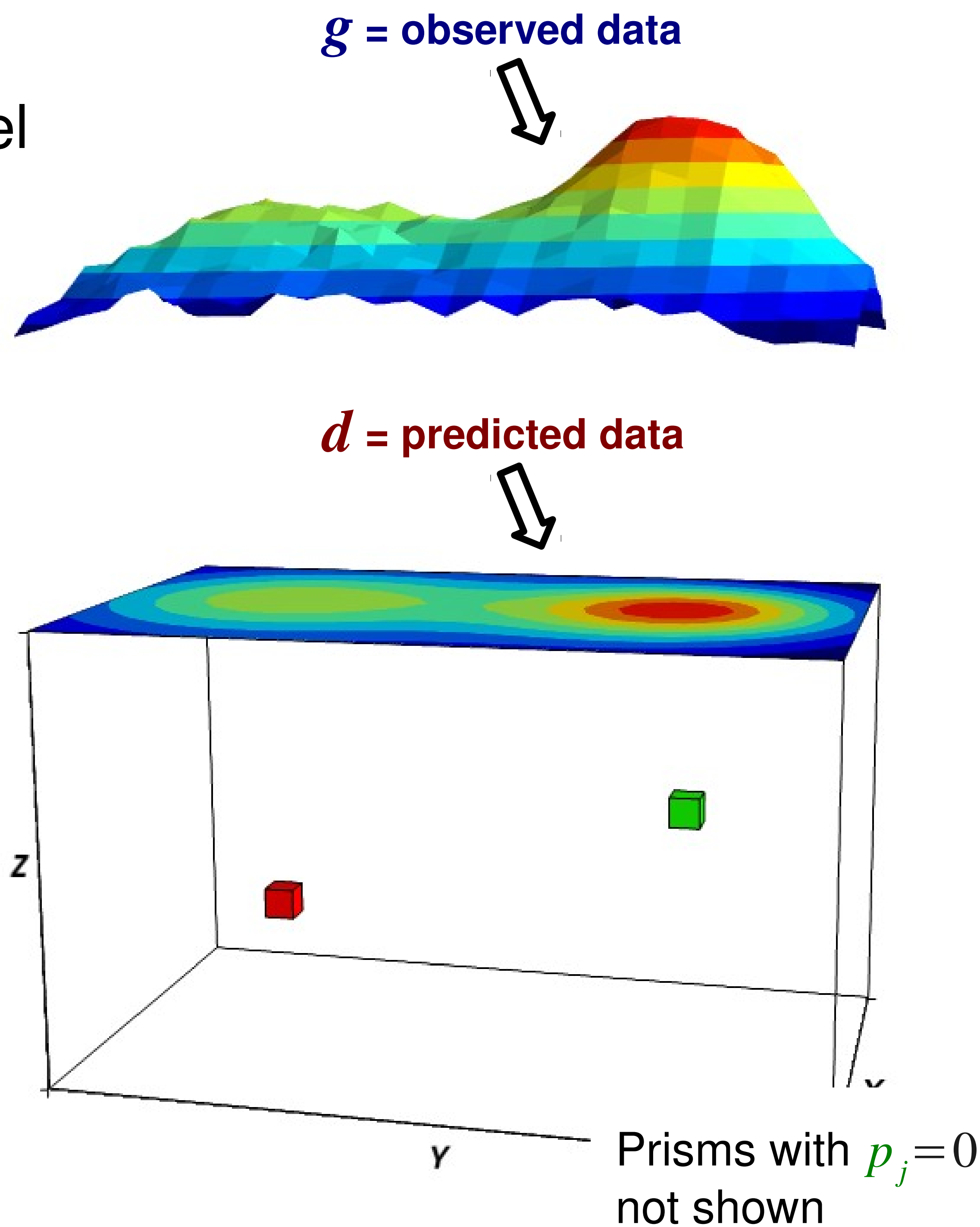
All parameters zero

N_s seeds

Include seeds

Compute initial residuals

$$\boldsymbol{r}^{(0)} = \boldsymbol{g} - \left(\sum_{s=1}^{N_s} \rho_s \boldsymbol{a}_{j_s} \right)$$



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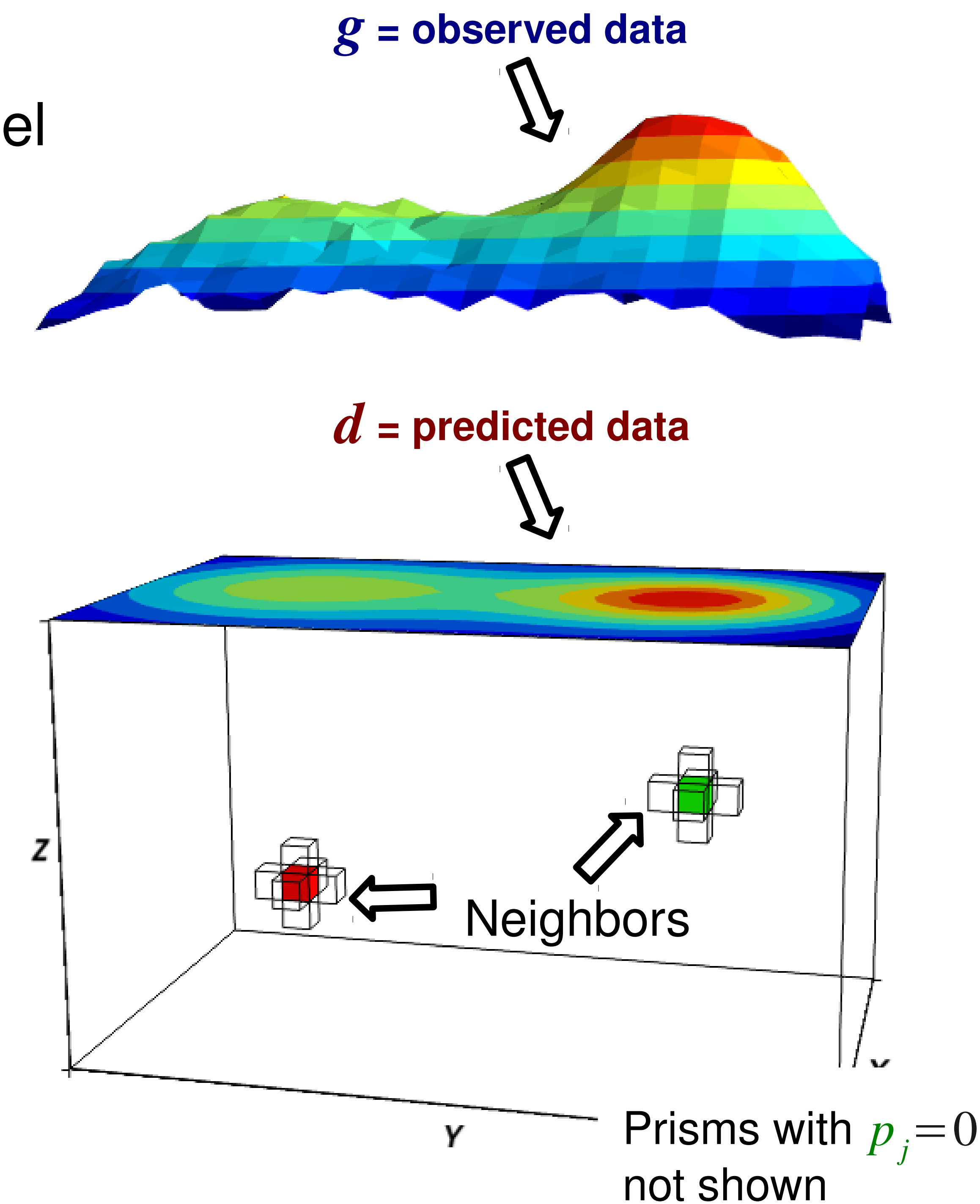
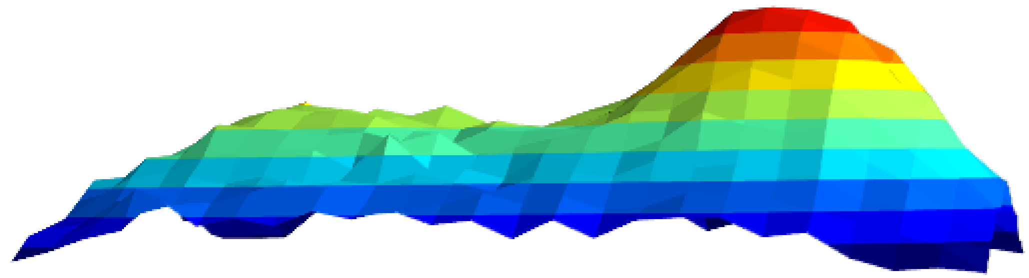
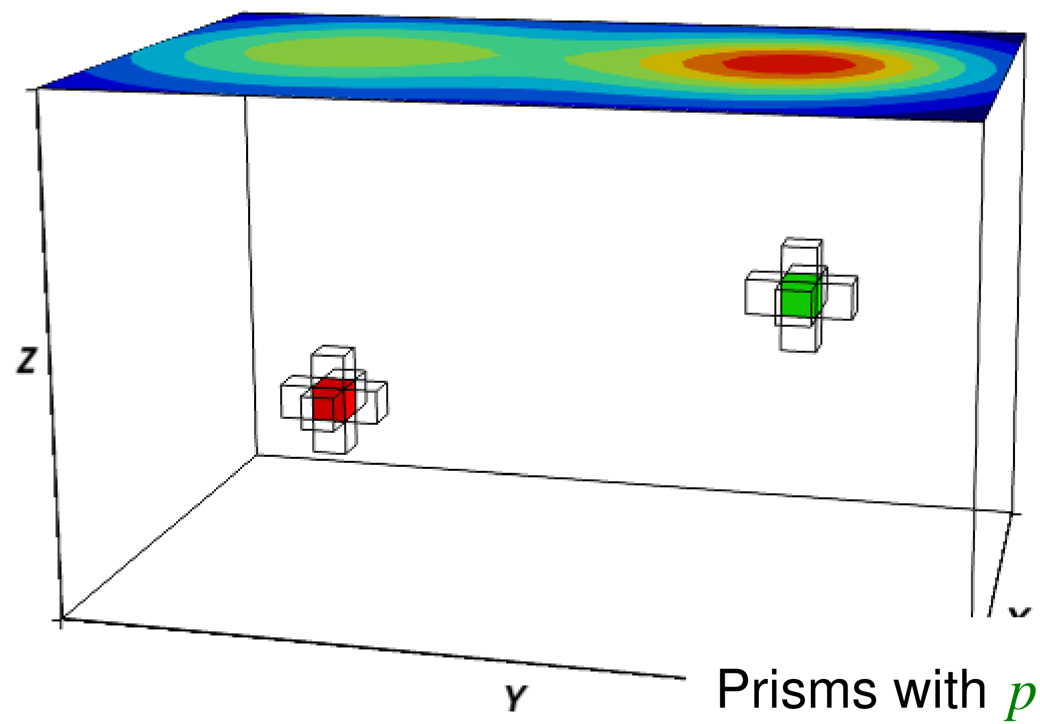
Find neighbors of seeds



Growth:

Try accretion to sth seed:





Prisms with $p_j = 0$ not shown

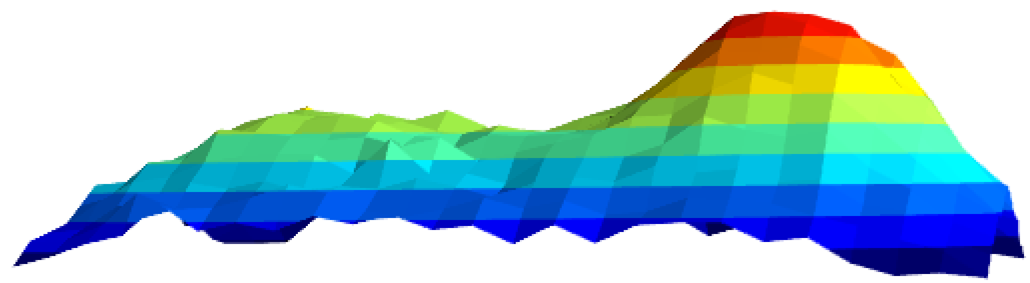
Growth:

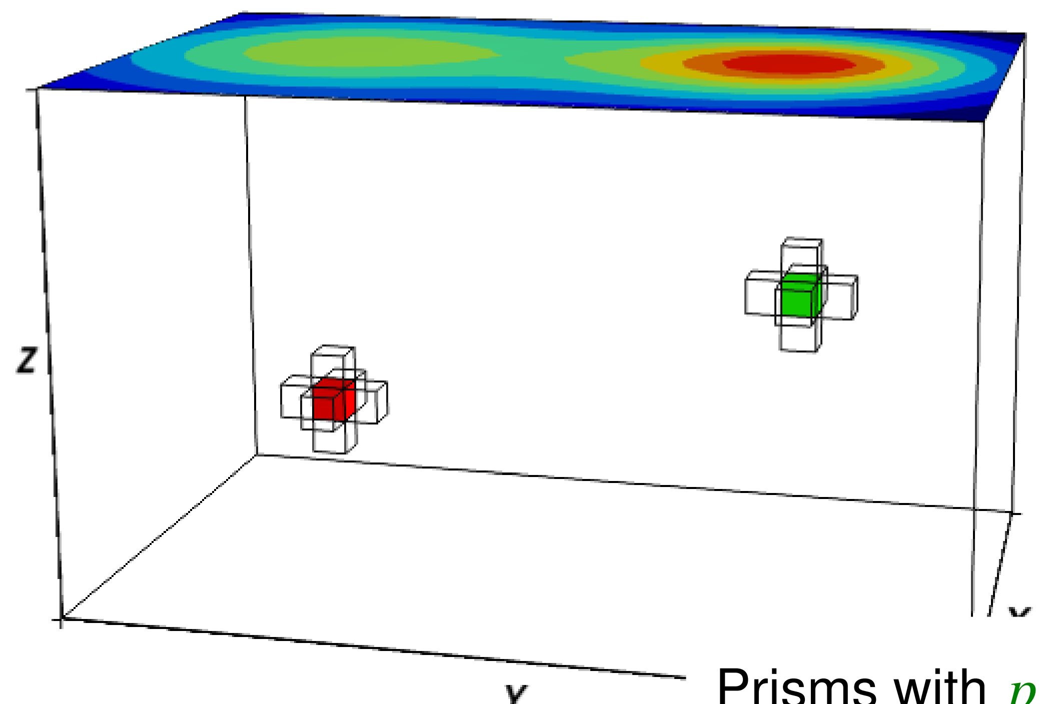
Try accretion to sth seed:

Choose neighbor:

1. **Reduce** data misfit

2. **Smallest** goal function





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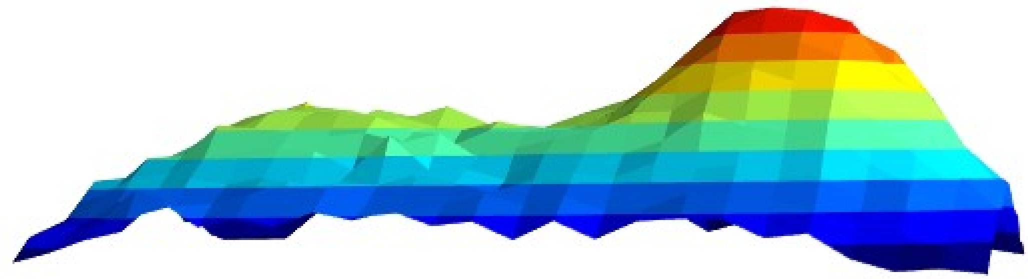
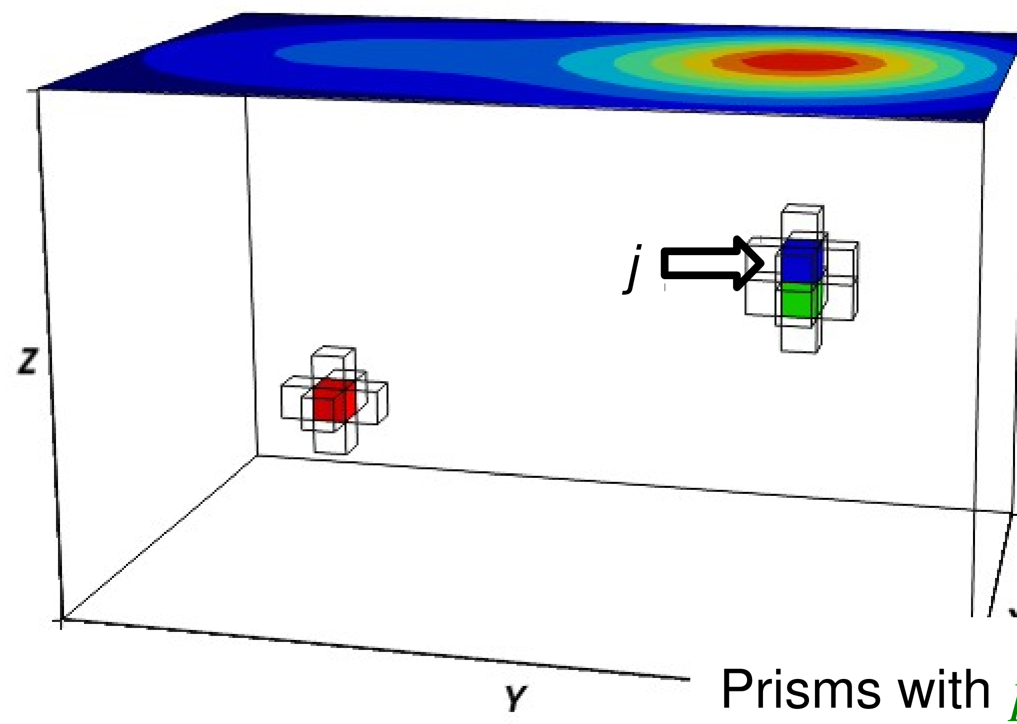


1. **Reduce** data misfit

2. **Smallest** goal function

j = chosen $\Longrightarrow$ $p_j = \rho_s$ **(New elements)**



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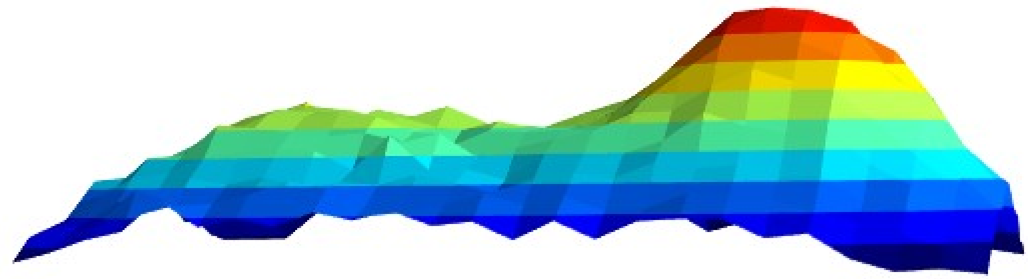

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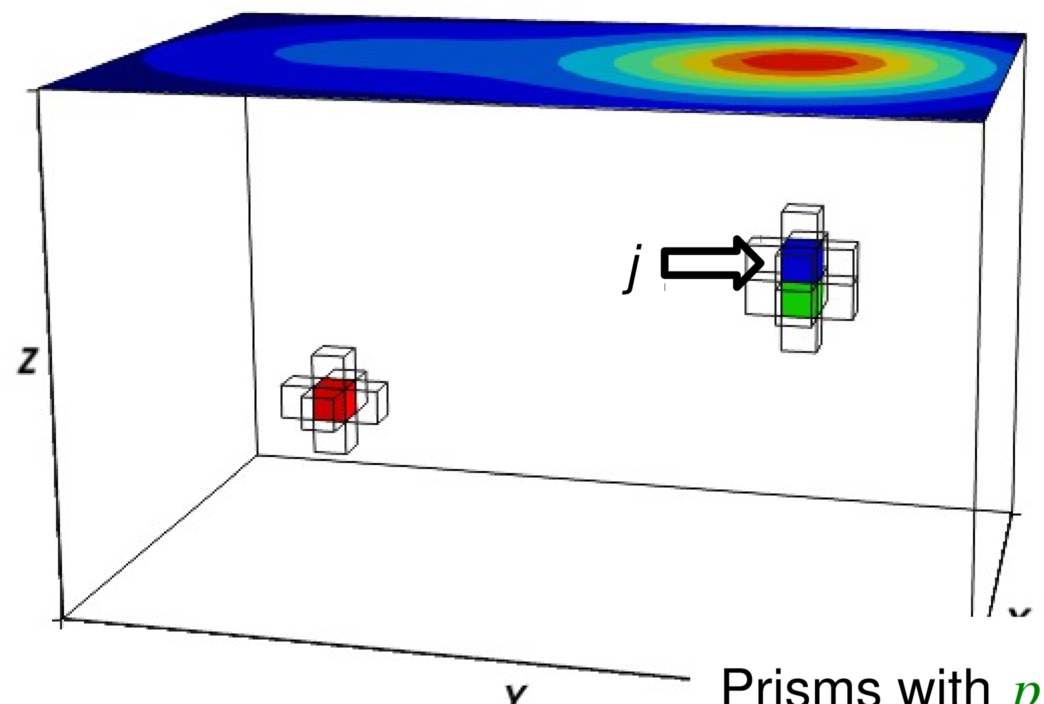
Update residuals

$$\boldsymbol{r}^{(new)} = \boldsymbol{r}^{(old)} - p_j \boldsymbol{a}_j$$



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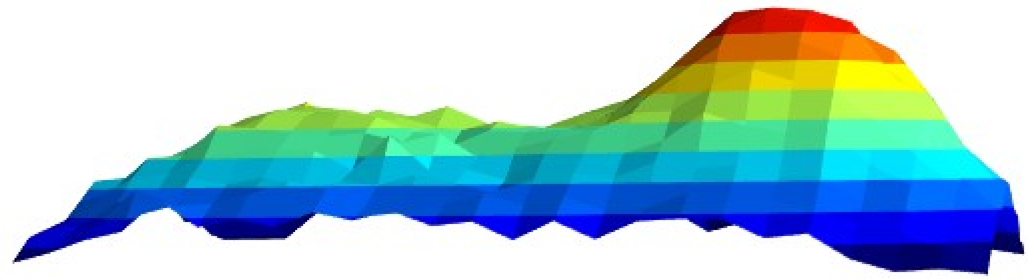

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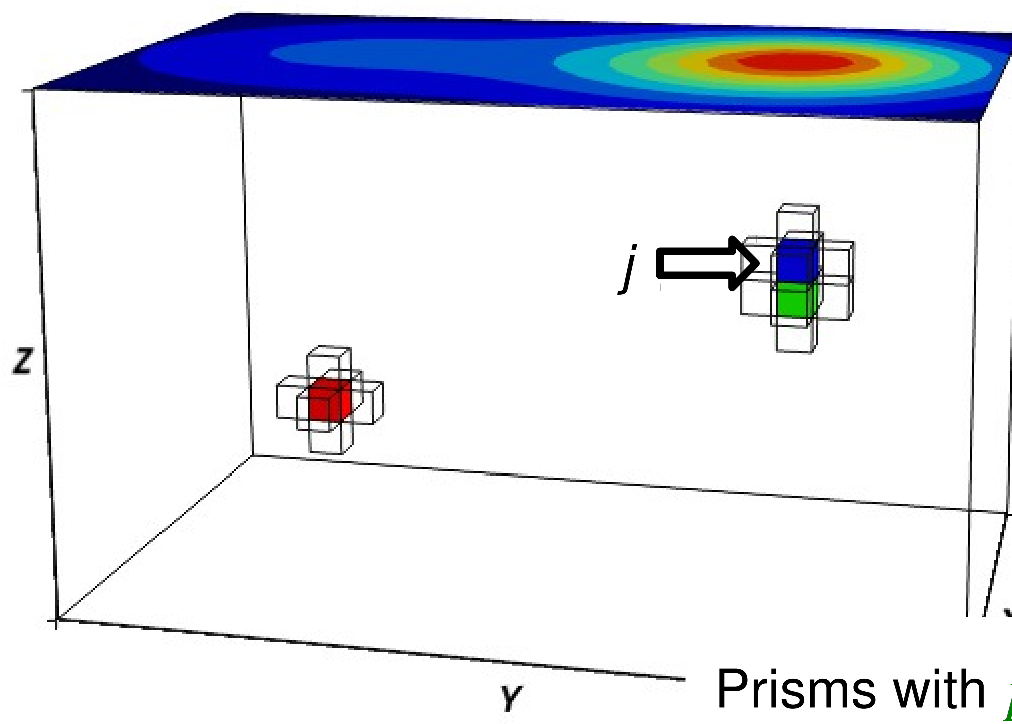
$\Downarrow$

Contribution of j



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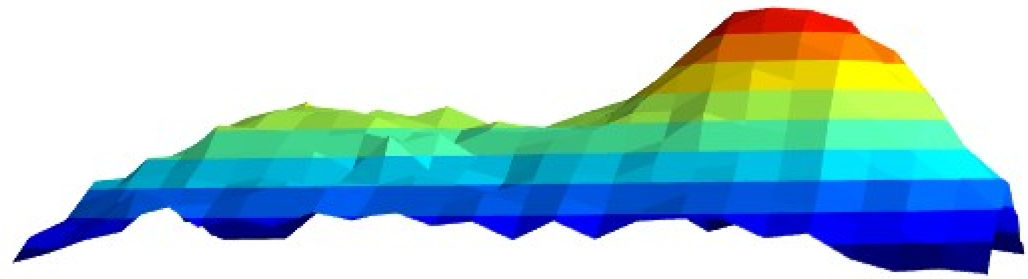

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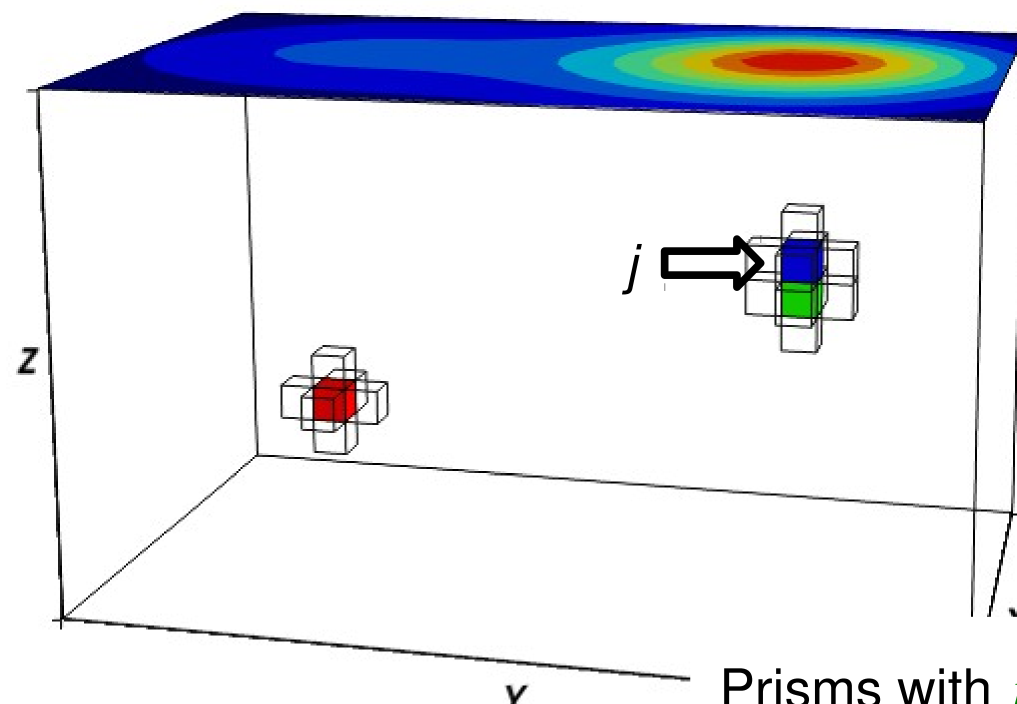
None found = no accretion

$\Downarrow$

Variable sizes



Prisms with $p_j = 0$ not shown

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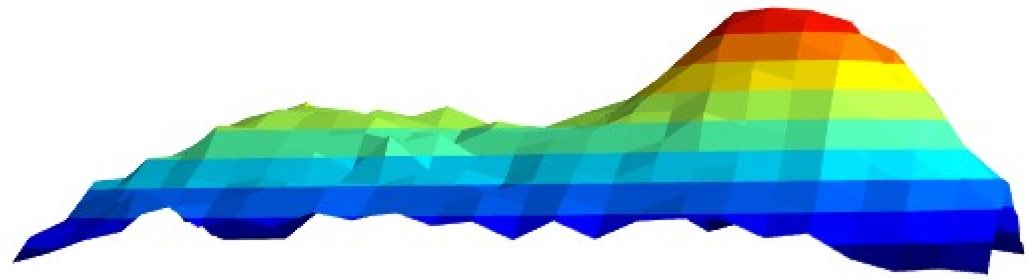

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N_s

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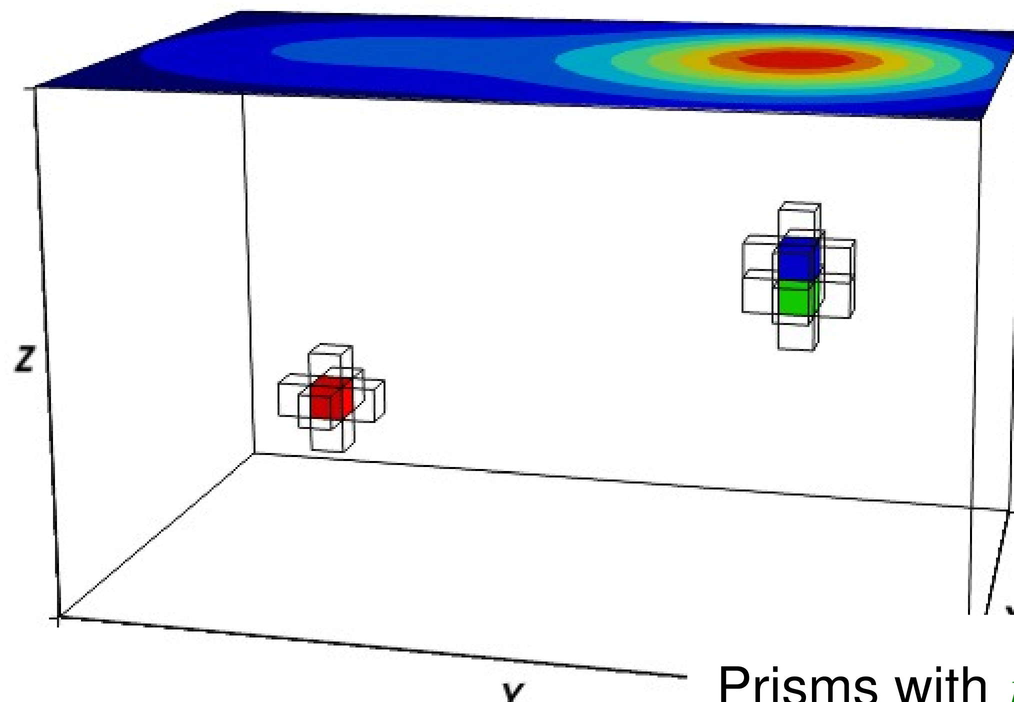

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N_s

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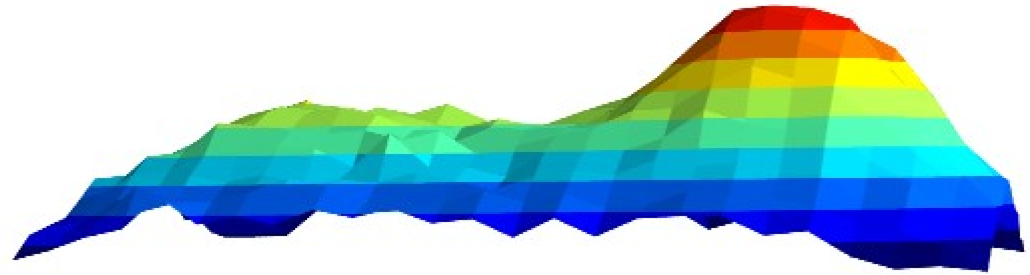

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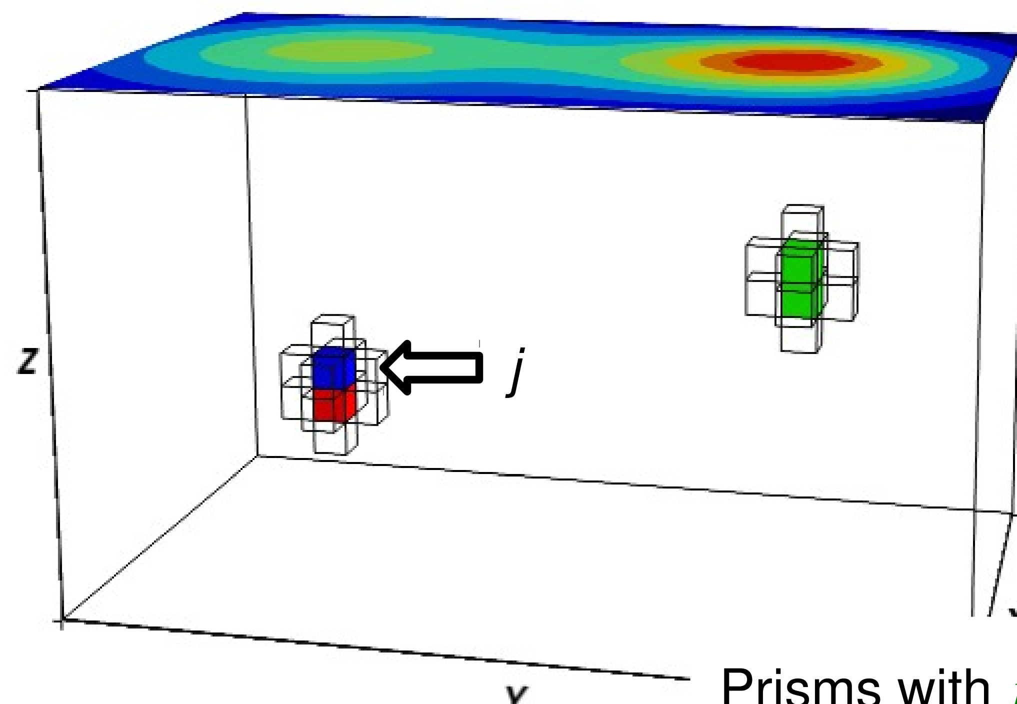

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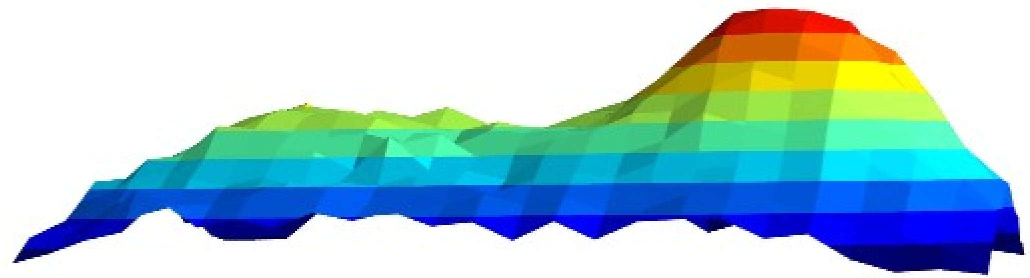

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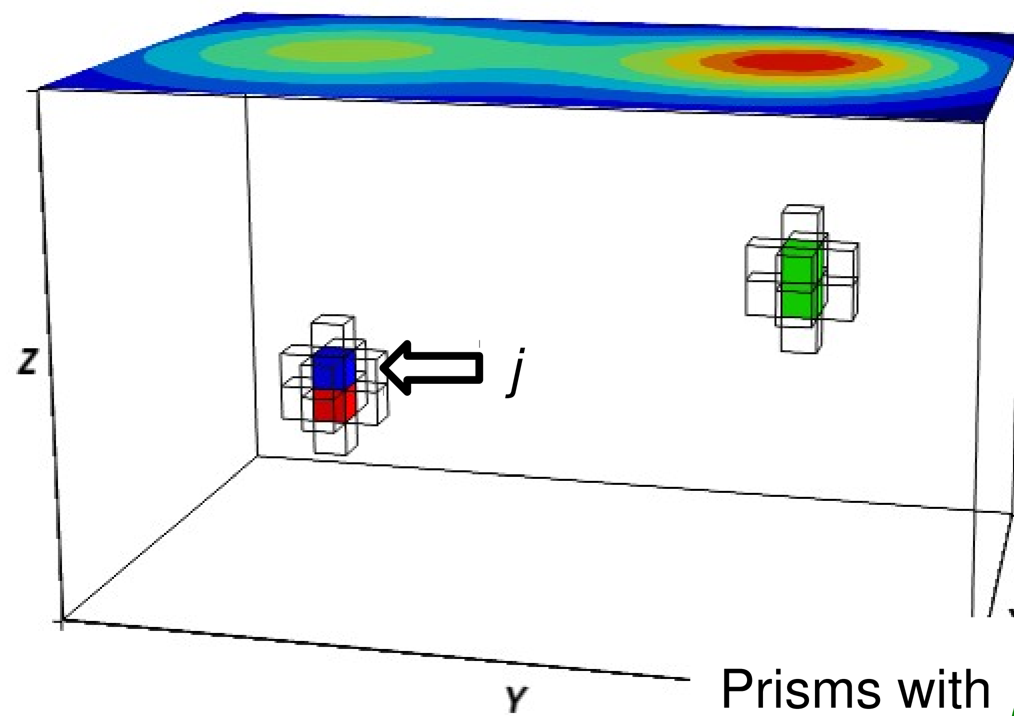
At least **one** seed grow?



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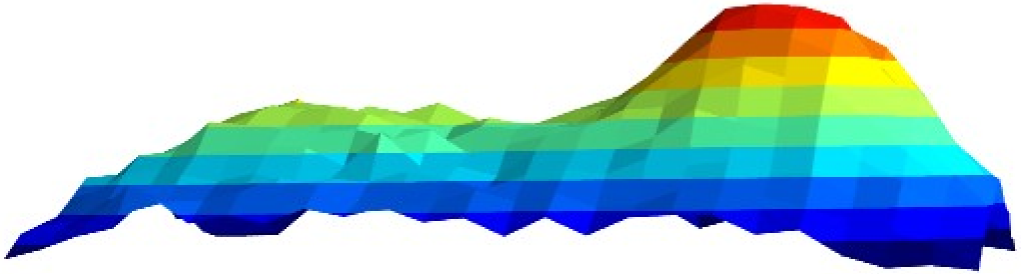

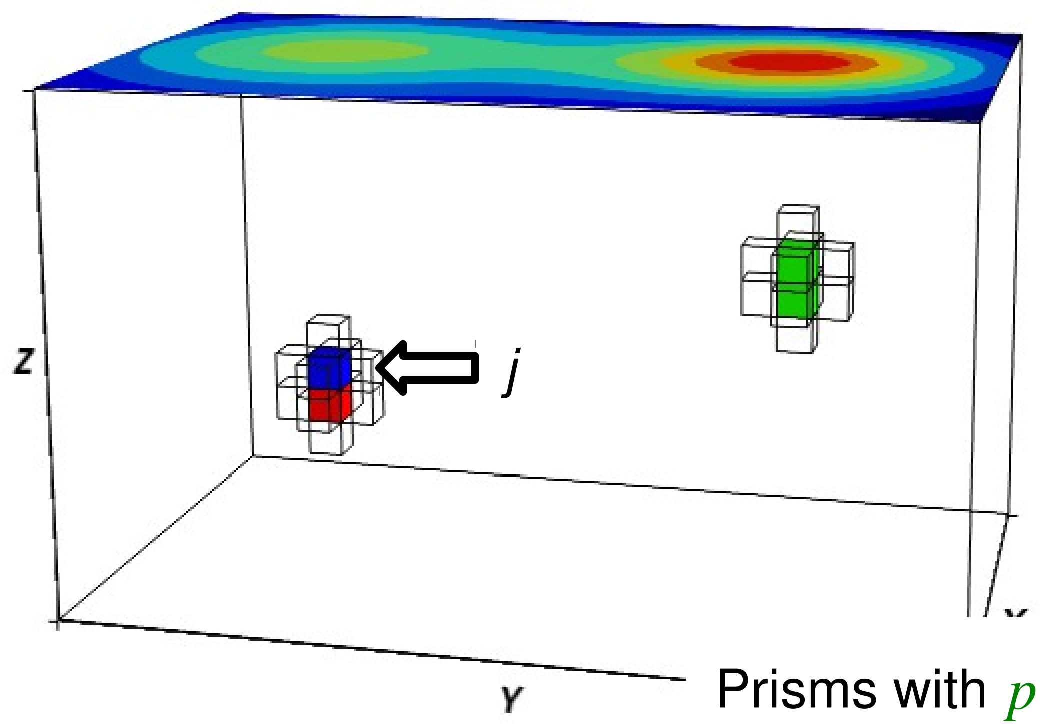

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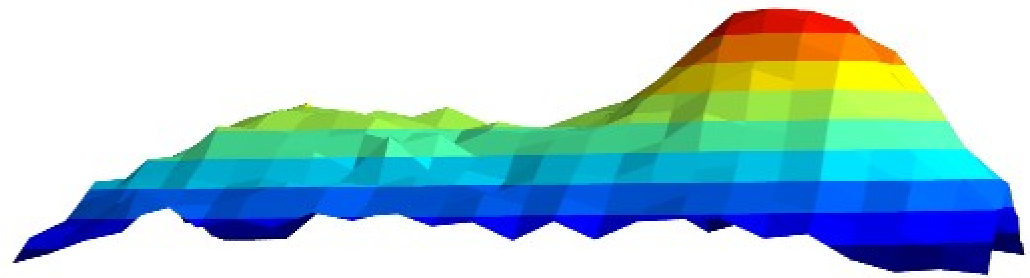

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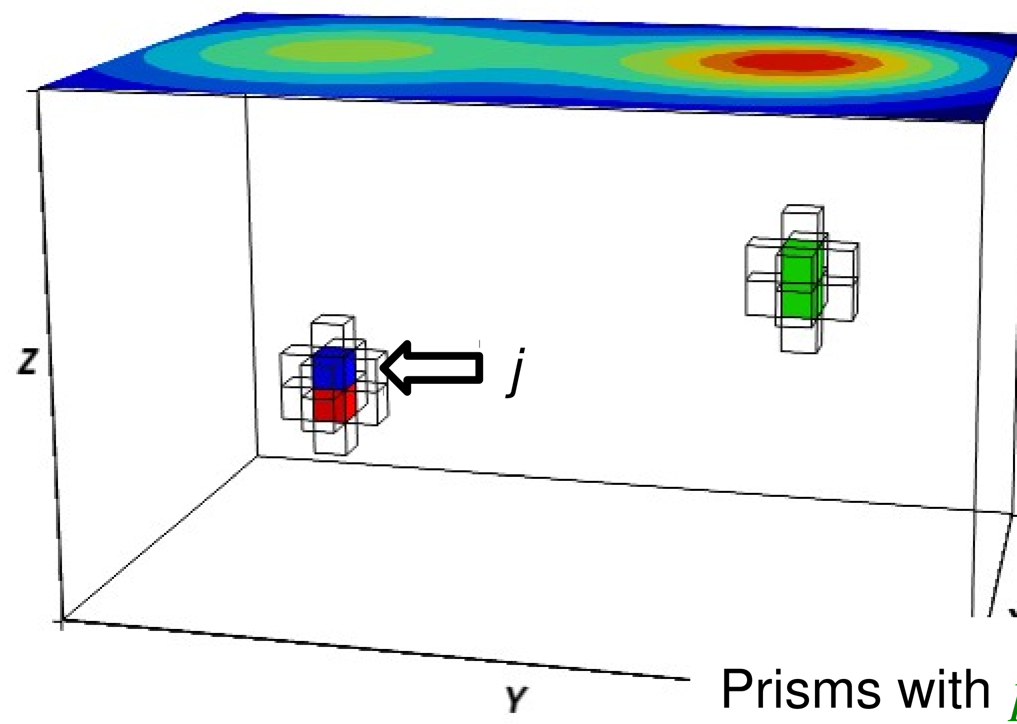
At least **one** seed grow?

Yes

No

Done!



Prisms with $p_j = 0$ not shown

Advantages:

- Compact & non-smooth
- Any number of sources
- Any number of different density contrasts
- No large equation system
- Search limited to neighbors

Remember equations:

Initial residual

$$\boldsymbol{r}^{(0)} = \boldsymbol{g} - \left(\sum_{s=1}^{N_s} \rho_s \boldsymbol{a}_{j_s} \right)$$

Update residual vector

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Only need some **columns** of **A**

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Calculate only when needed

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Calculate only when needed & delete after update

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Calculate only when needed & delete after update

$$\Downarrow$$

Lazy evaluation

Advantages:

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- Compact & non-smooth
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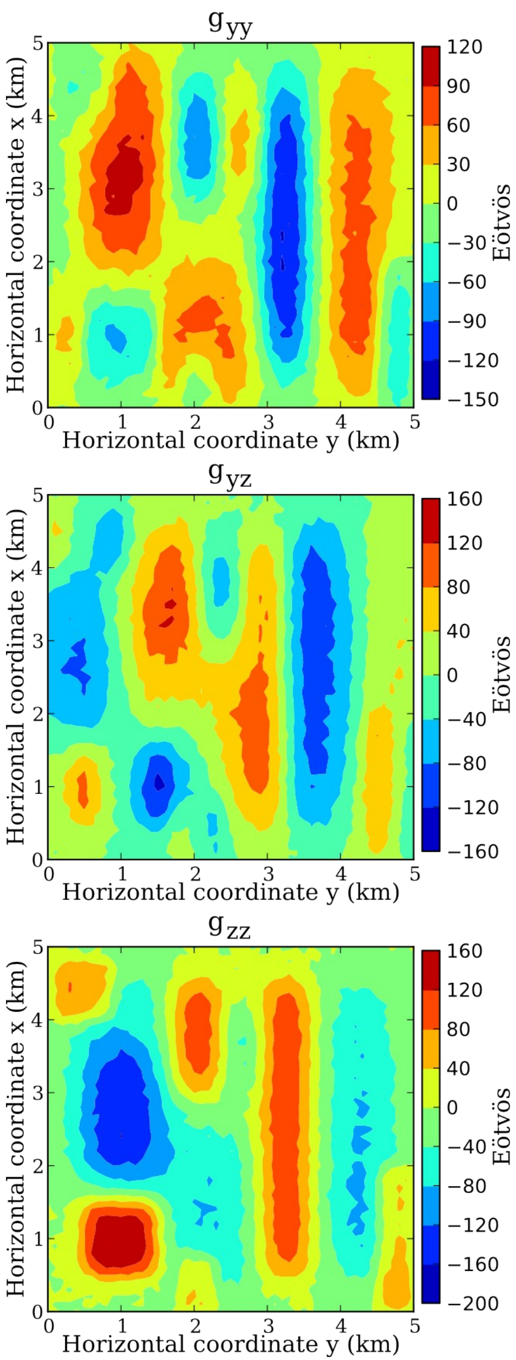
Fast inversion + low memory usage

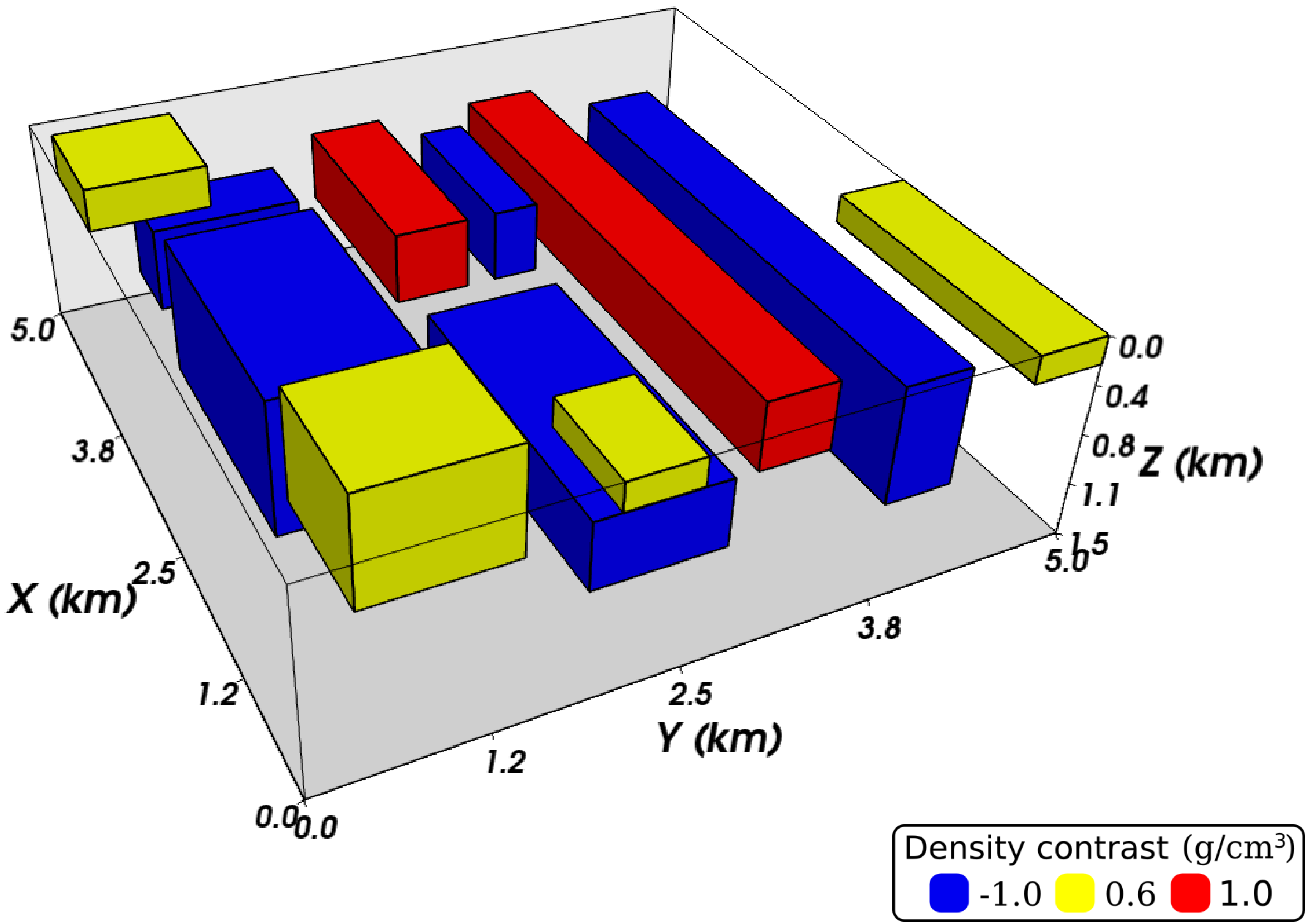
Synthetic Data



Data set:

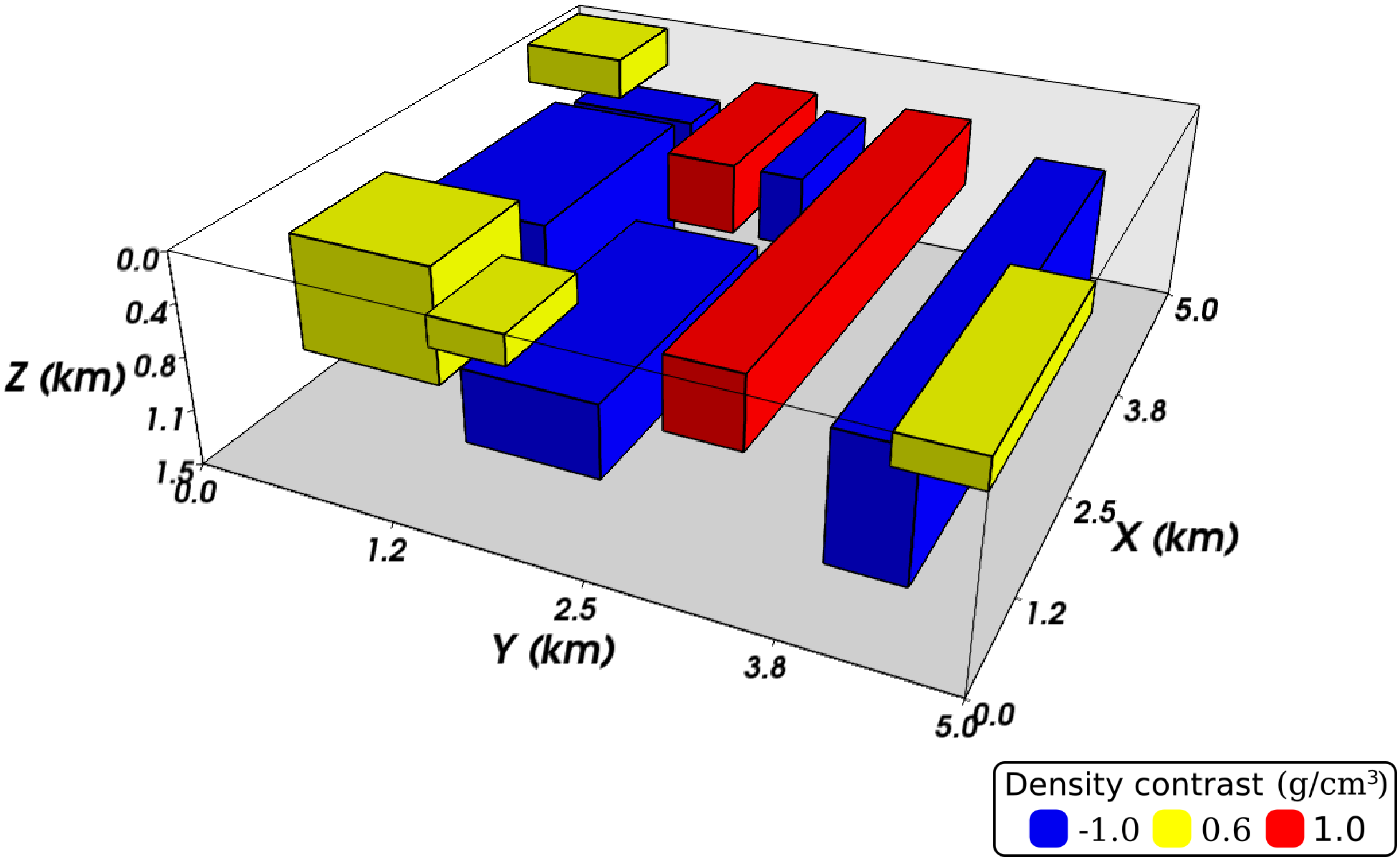
- 3 components
- 51 x 51 points
- 2601 points/component
- 7803 measurements
- 5 Eötvös noise

Model:

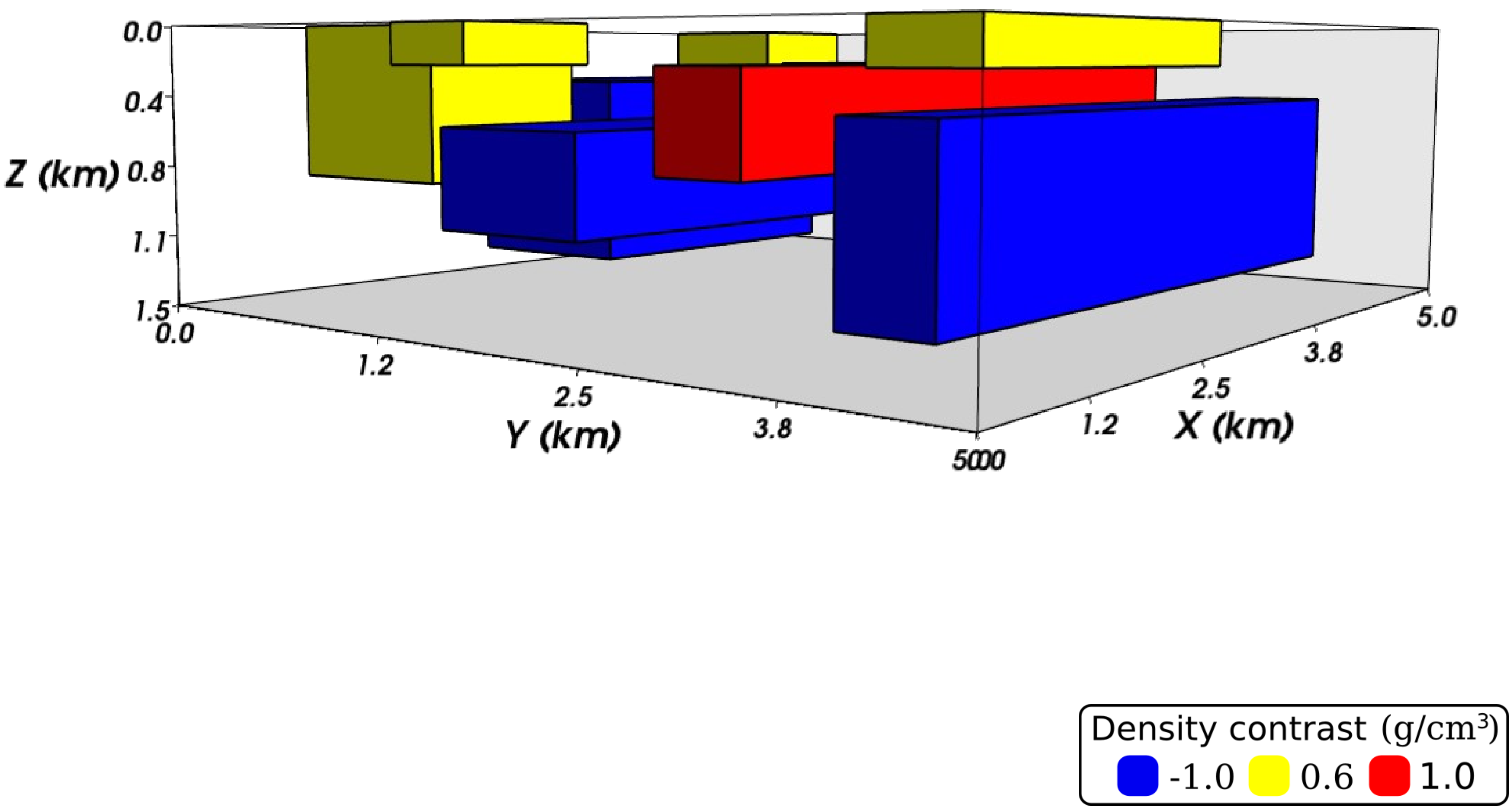


5.0
3.8
2.5
1.2
0.0
X (km)
0.0
1.2
2.5
3.8
5.0
Y (km)
0.0
0.4
0.8
1.1
1.5
Z (km)
Density contrast (g/cm³)
-1.0
0.6
1.0

Model: • 11 prisms



Model: • 11 prisms • 4 outcropping



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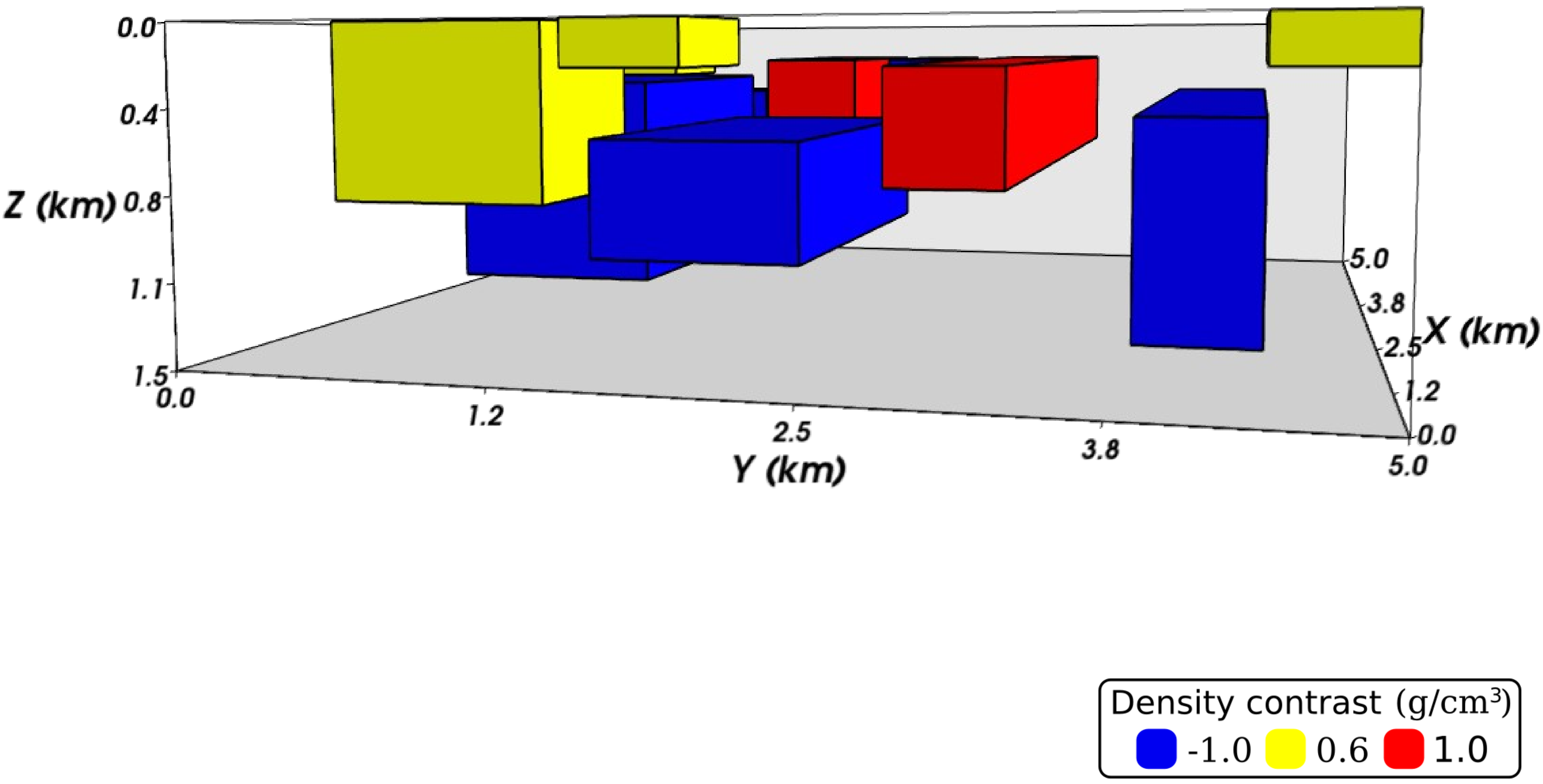

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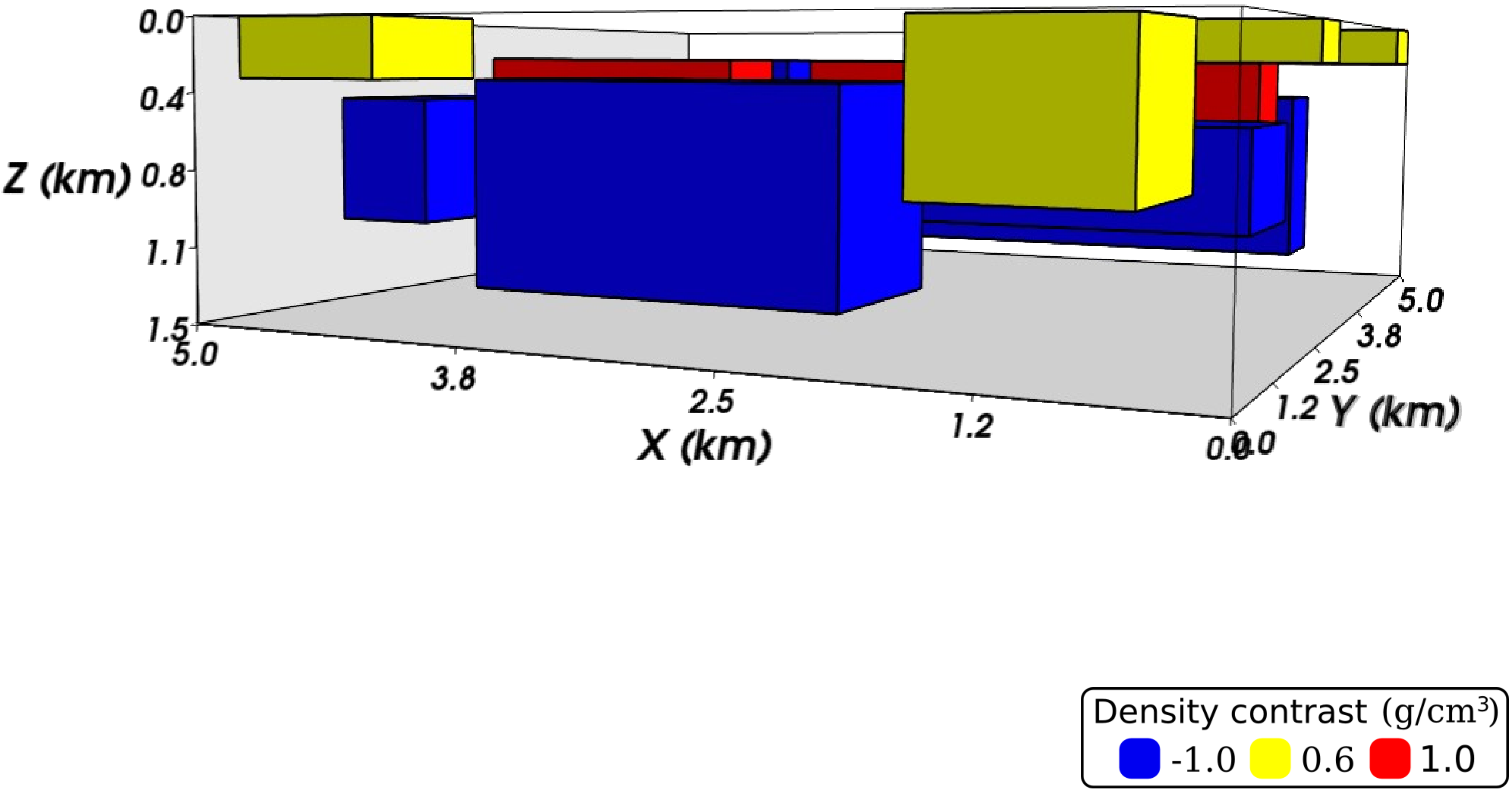
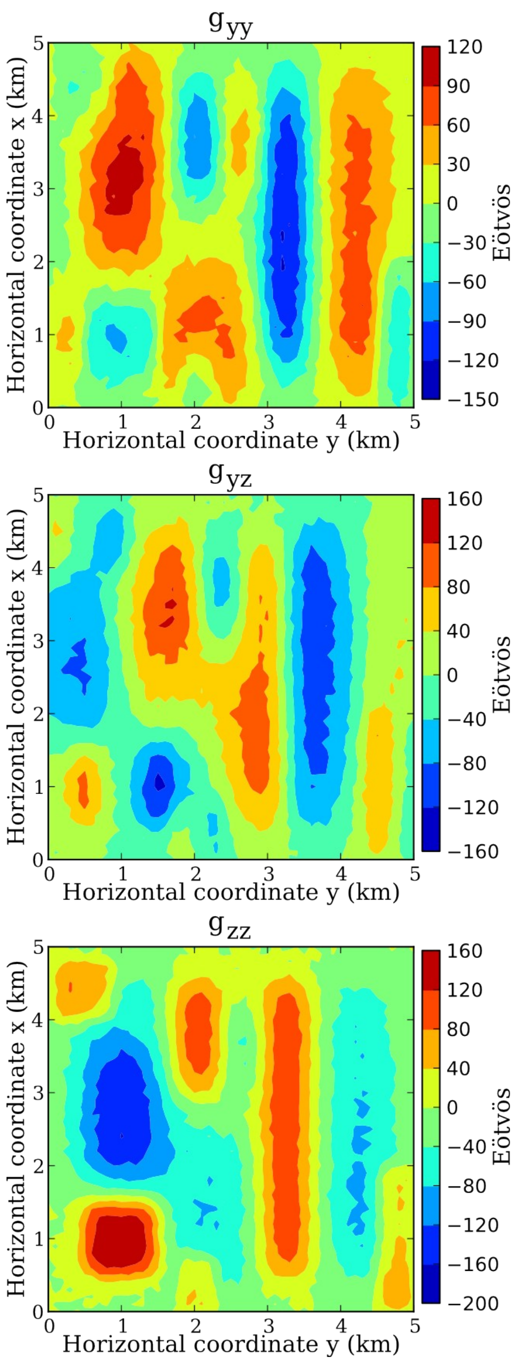
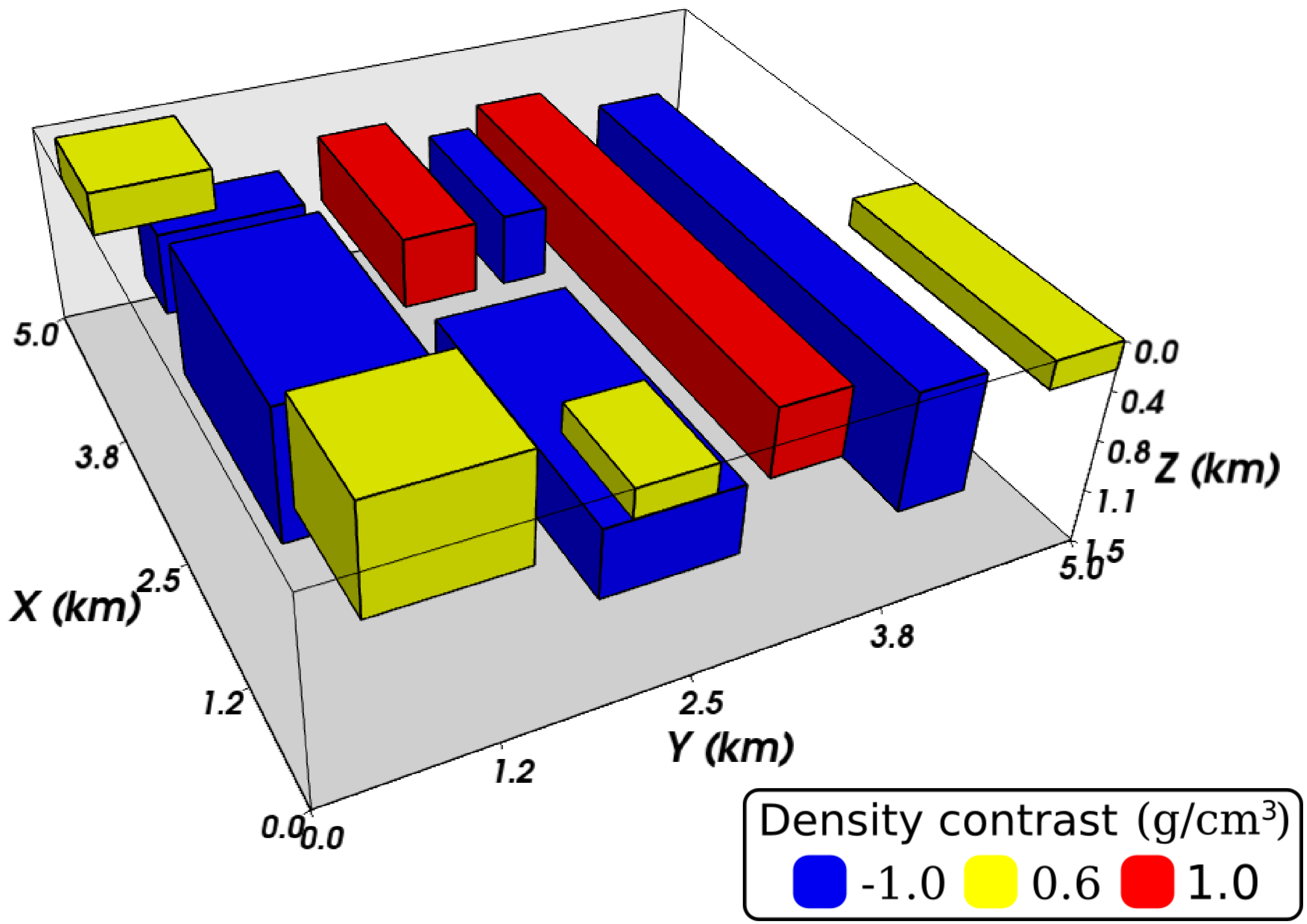
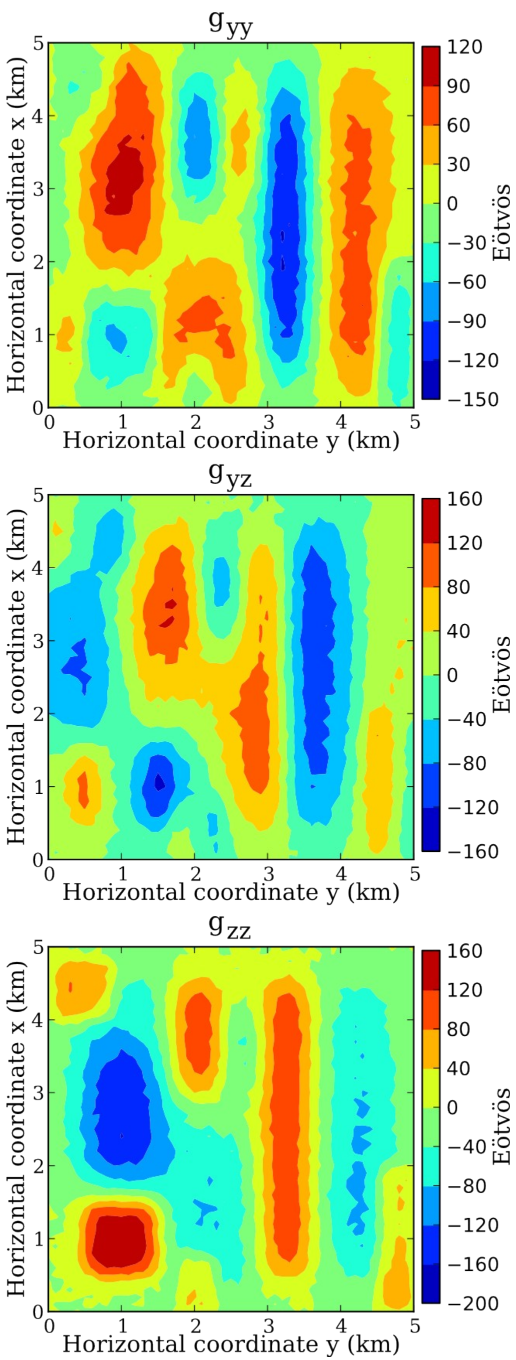


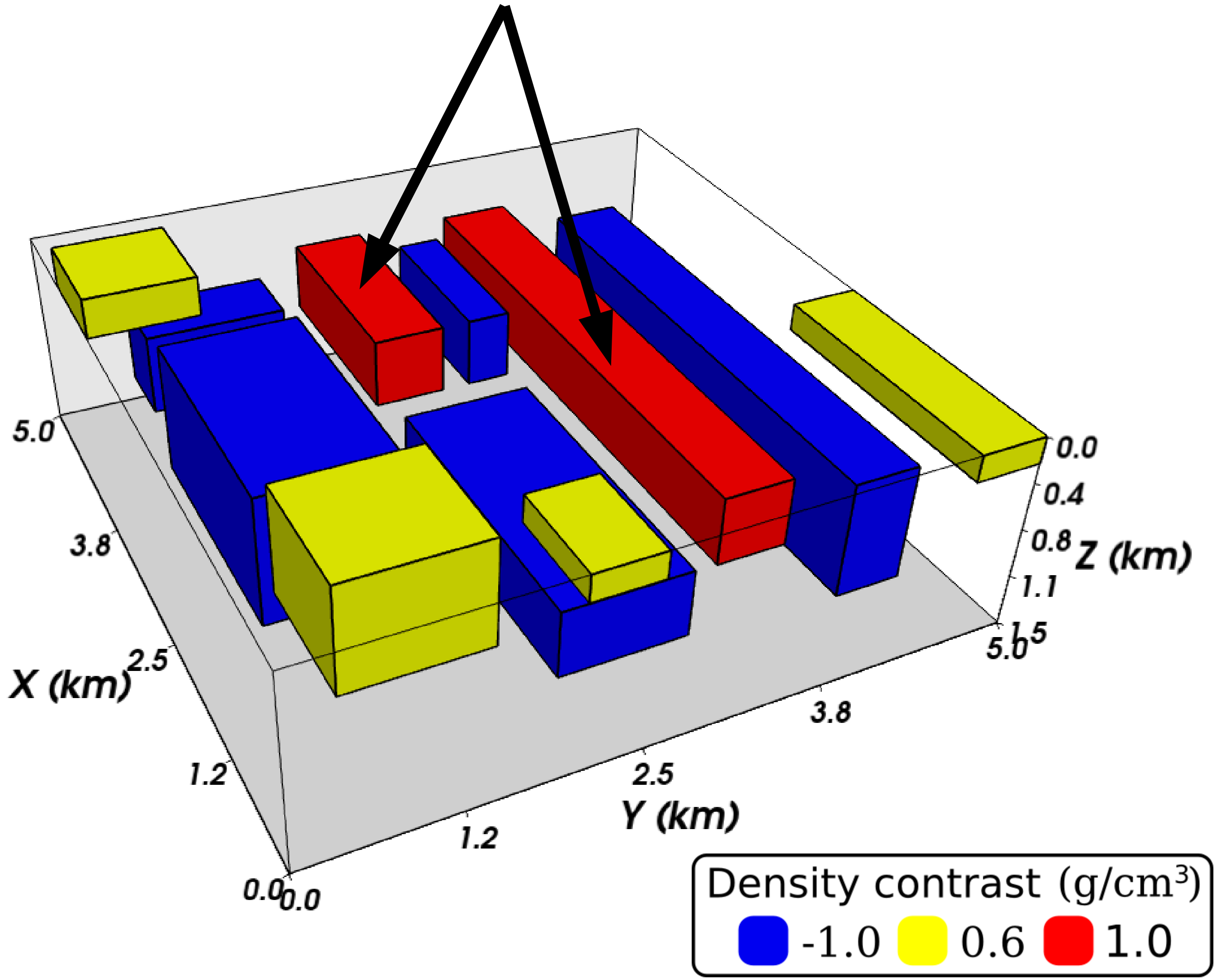
- Strongly interfering effects

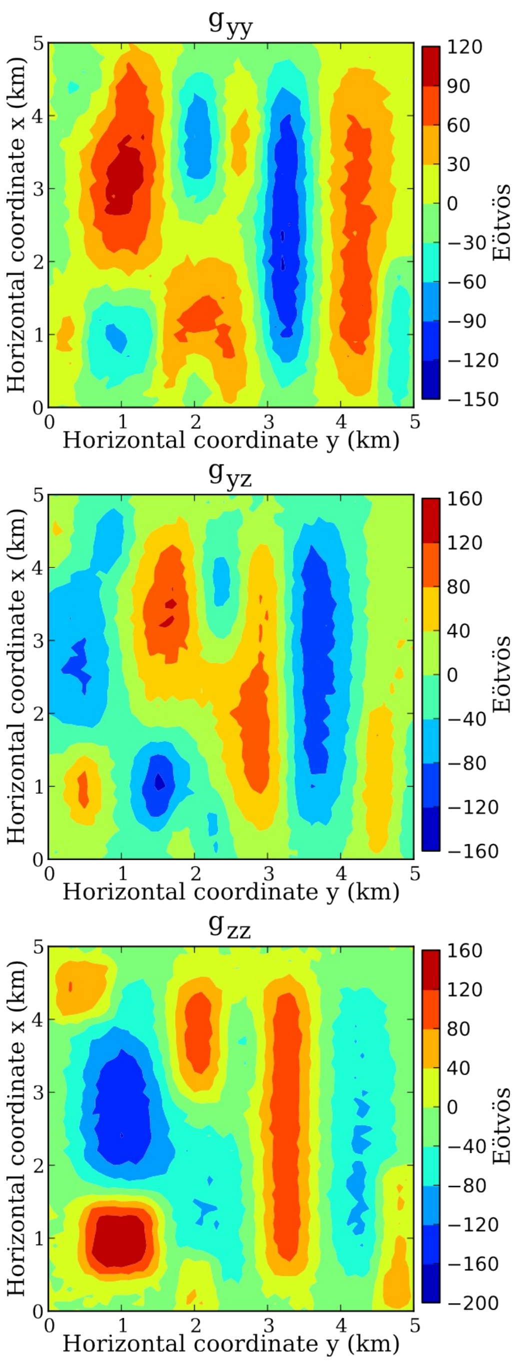




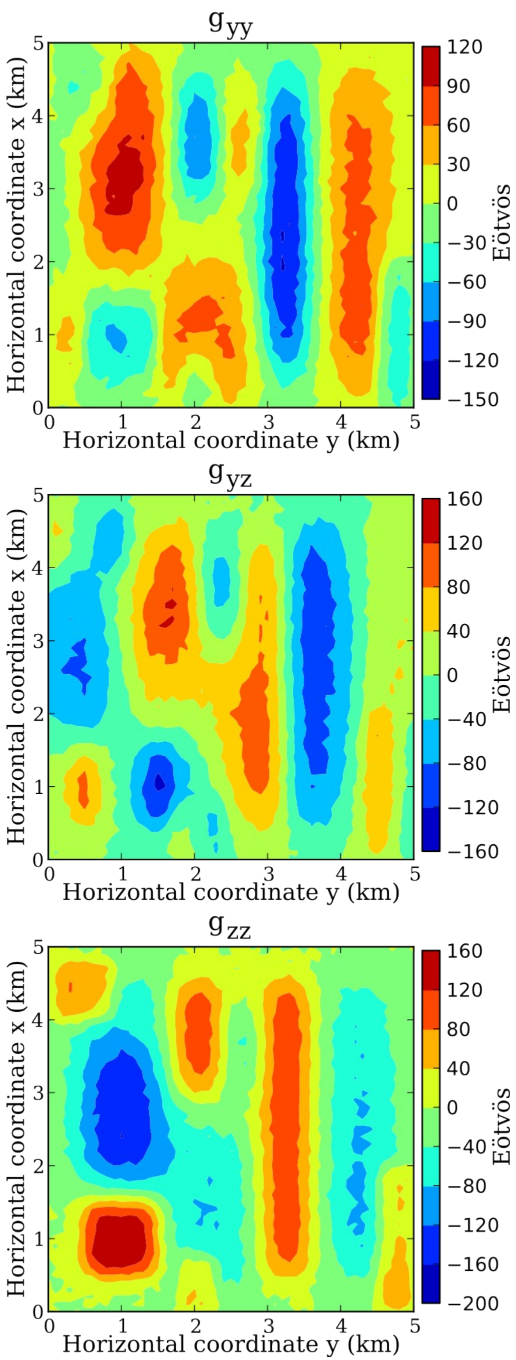


- Strongly interfering effects
- What if only interested in these?

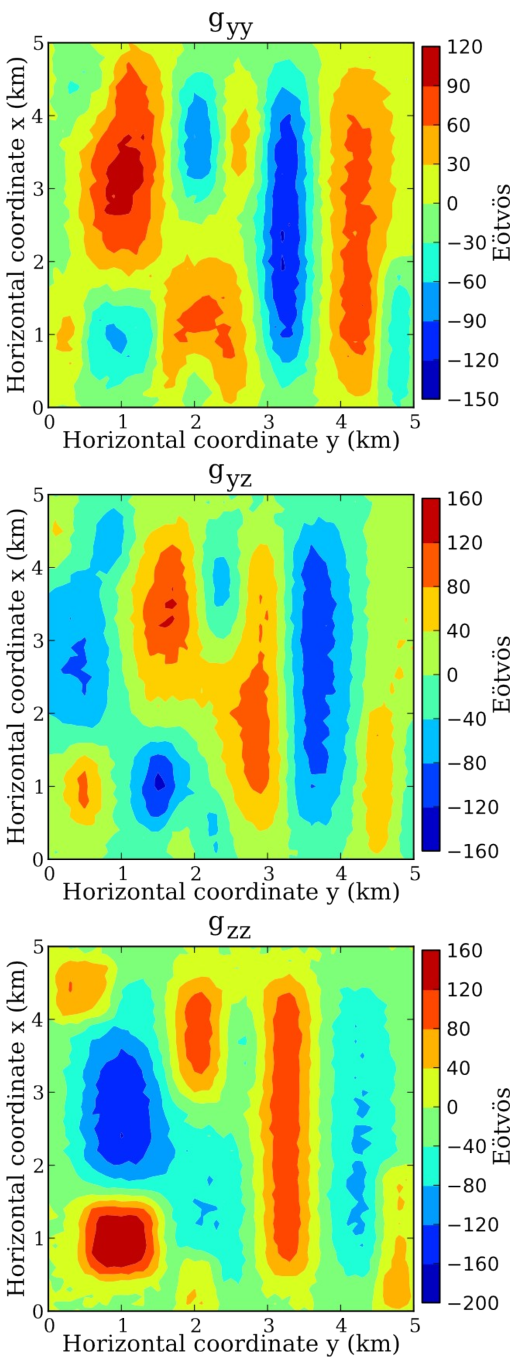




- Common scenario



- Common scenario
- May not have prior information
  - Density contrast
  - Approximate depth



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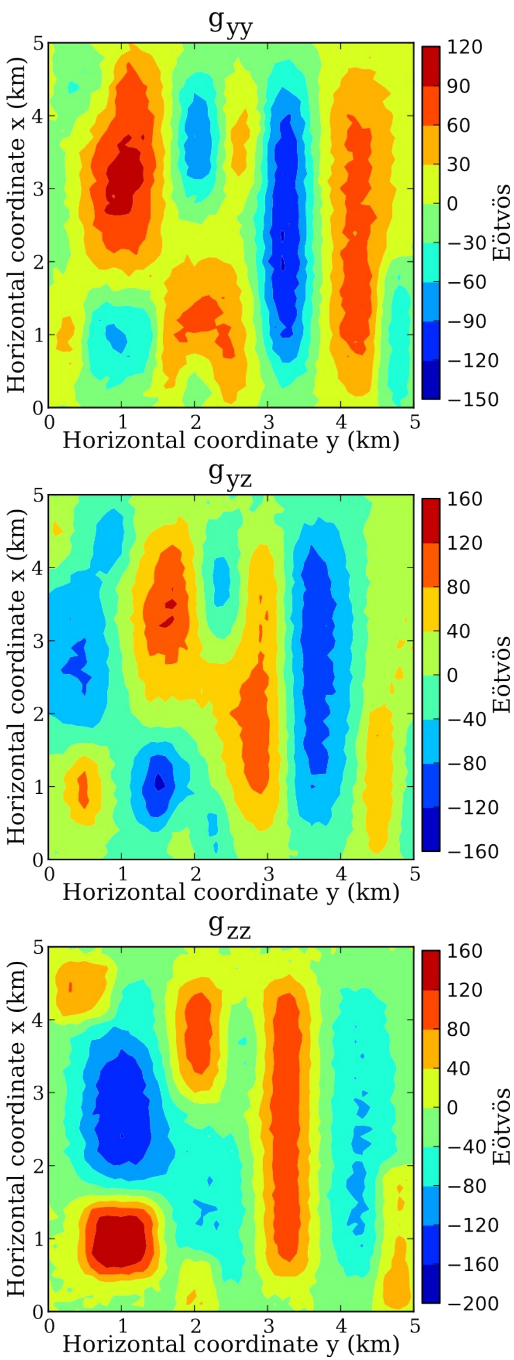
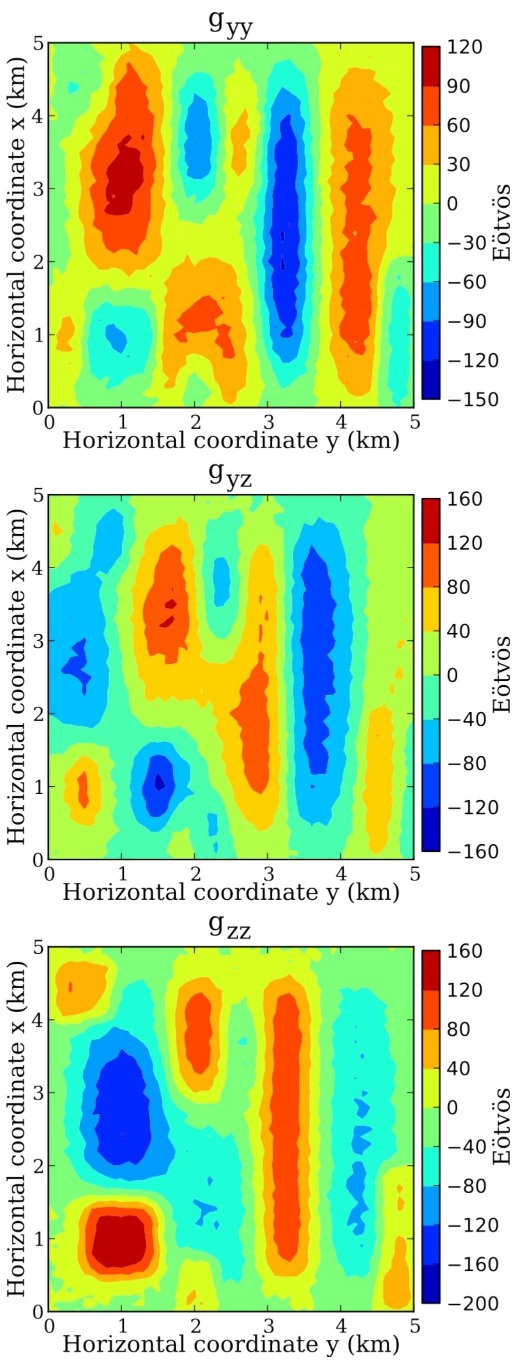
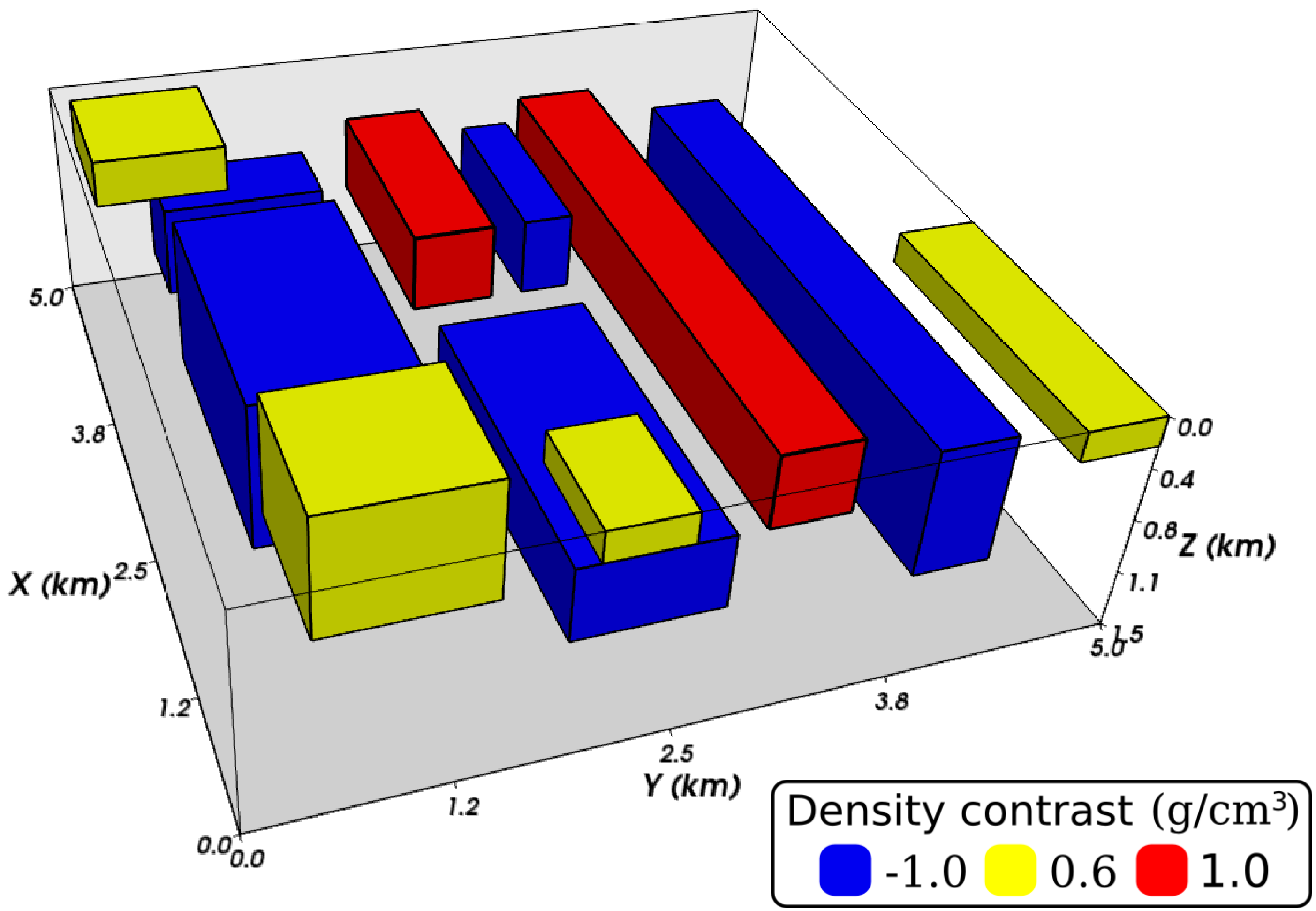
- No way to provide seeds



- Common scenario
- May not have prior information
  - Density contrast
  - Approximate depth
- No way to provide seeds
- Difficult to isolate effect of targets

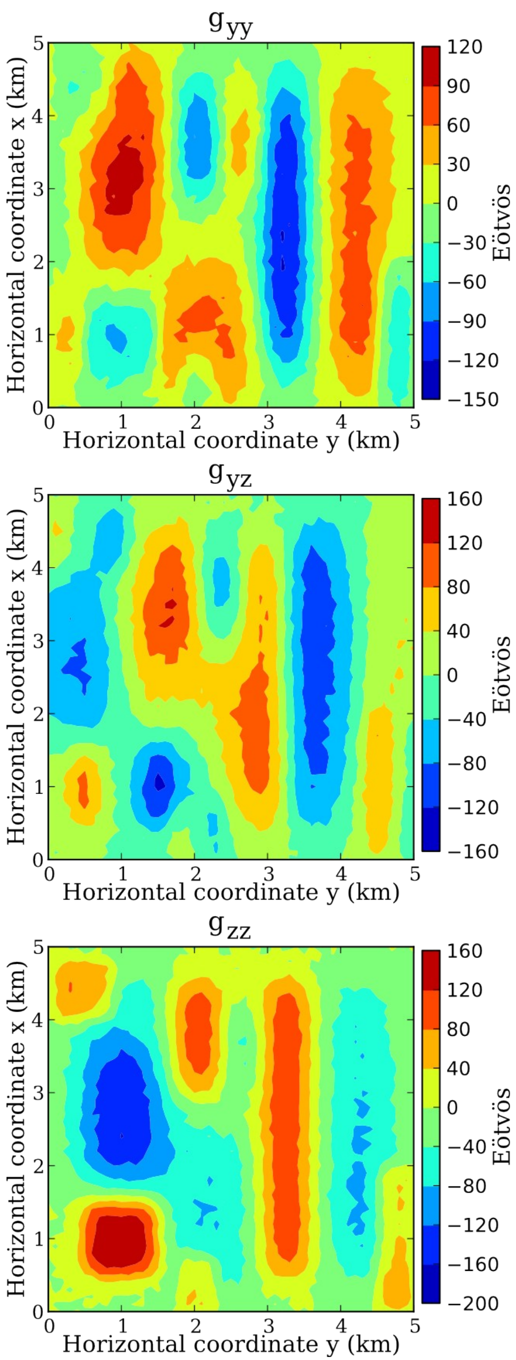
Robust procedure:

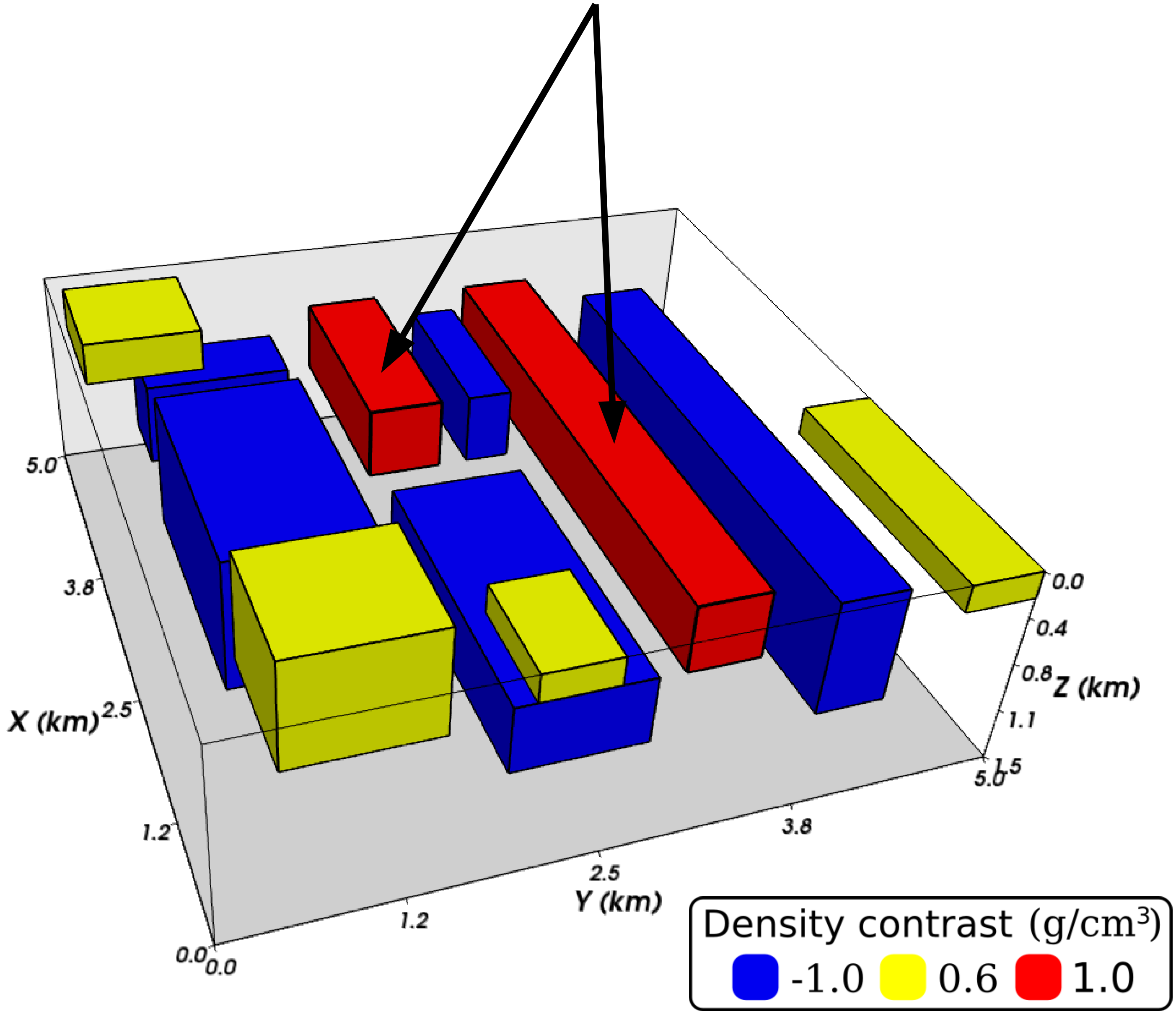




Robust procedure:

- Seeds only for targets





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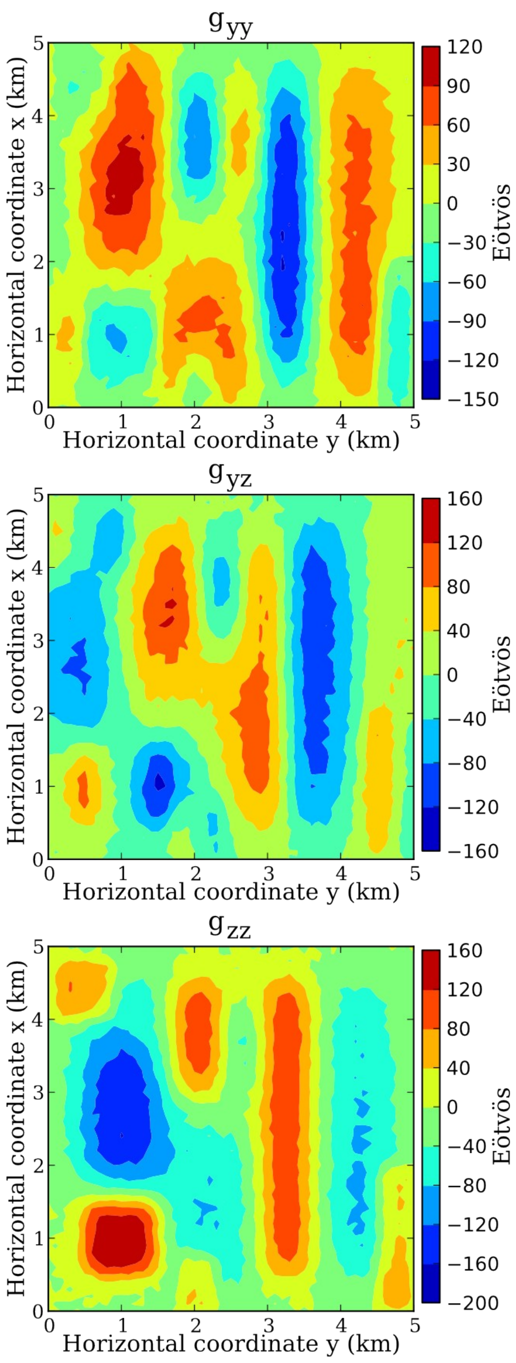

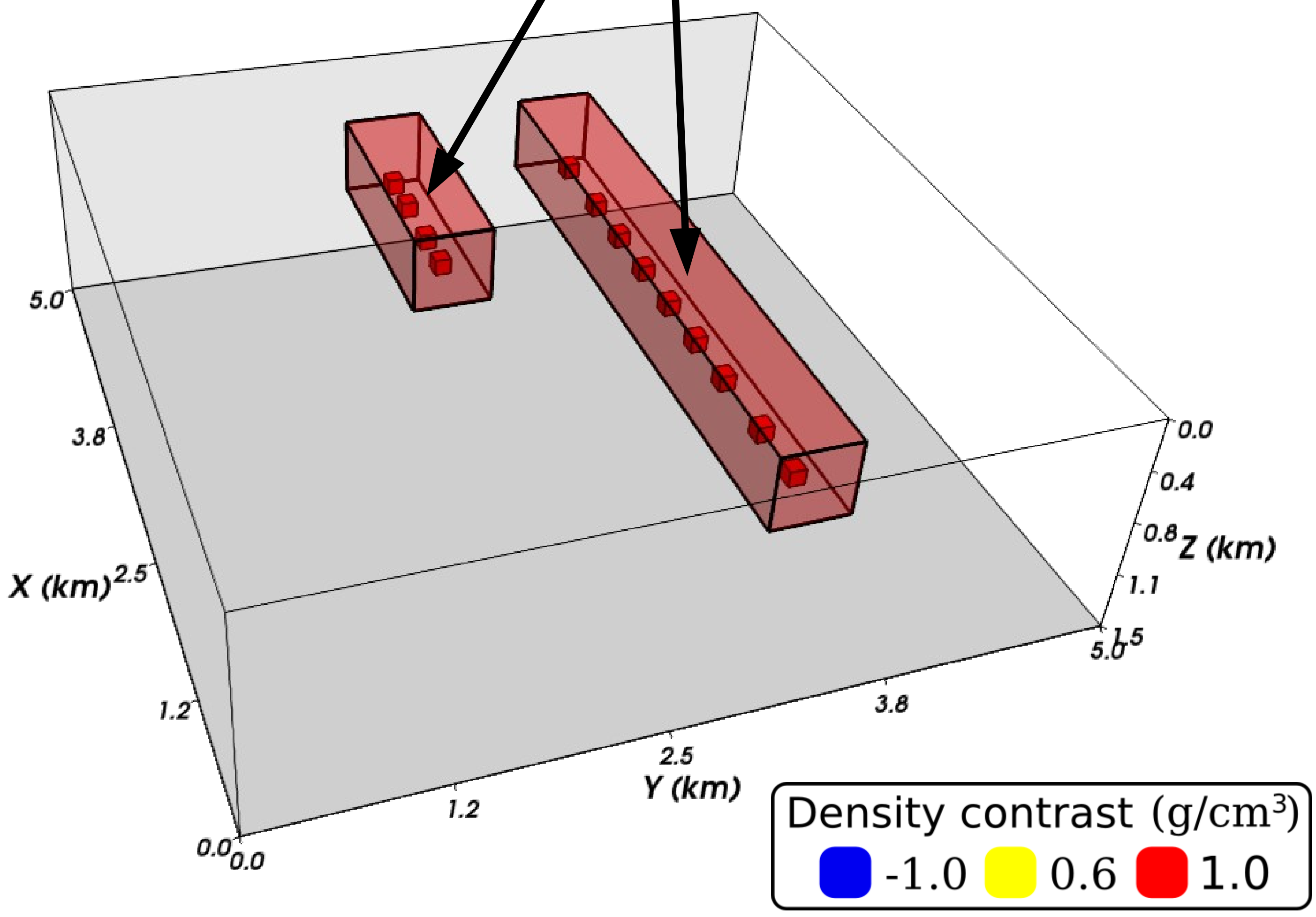

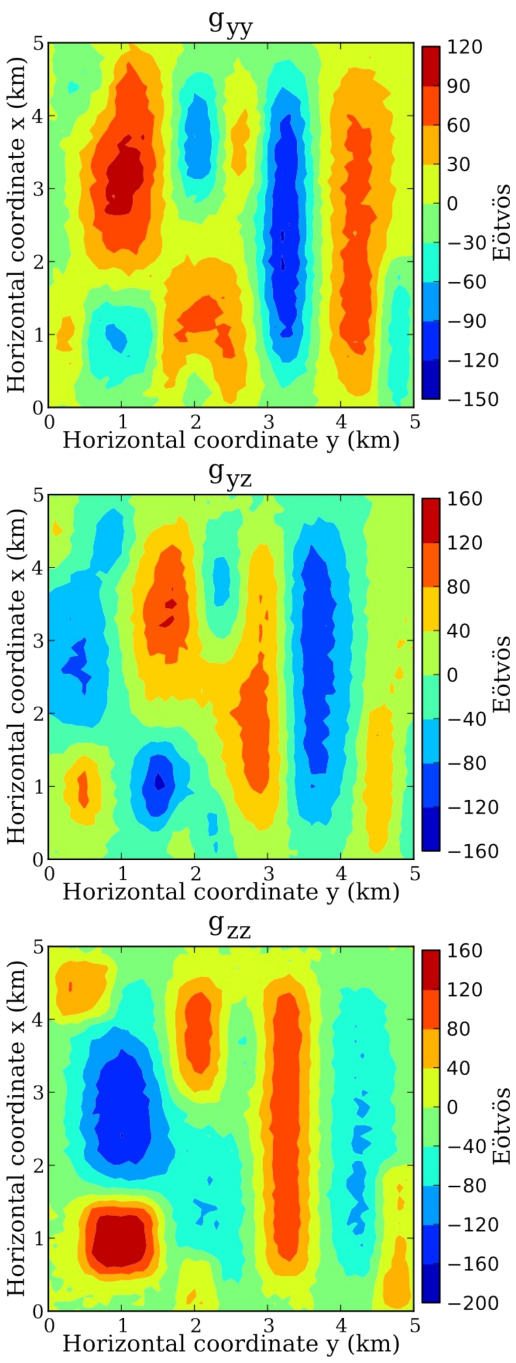

Robust procedure:

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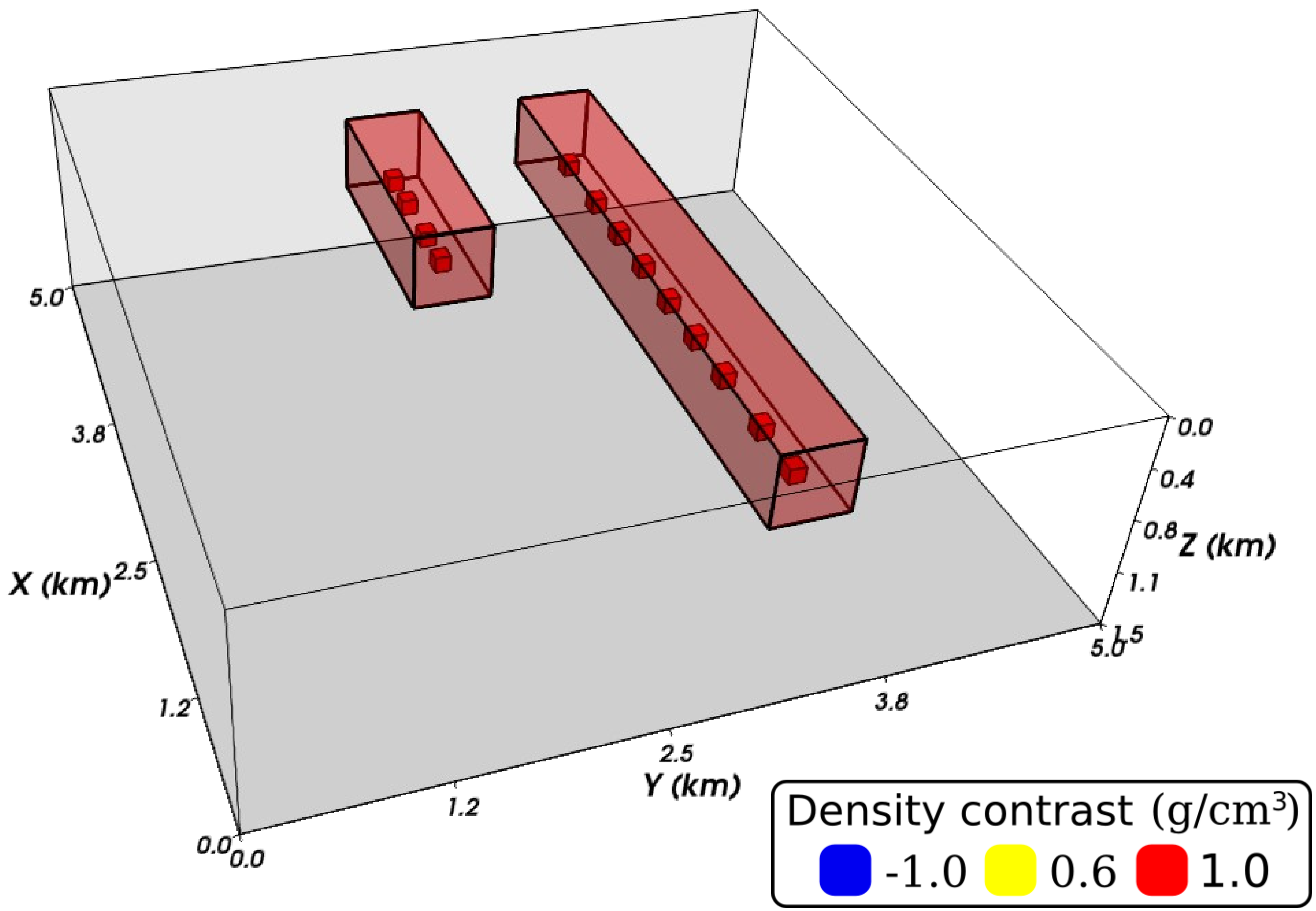

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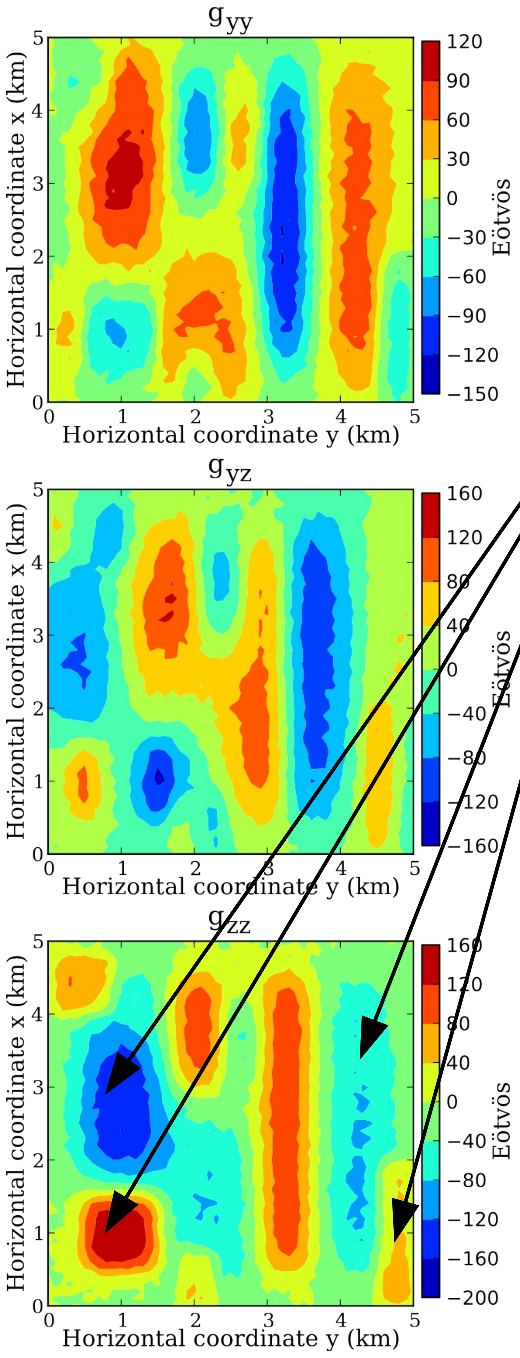
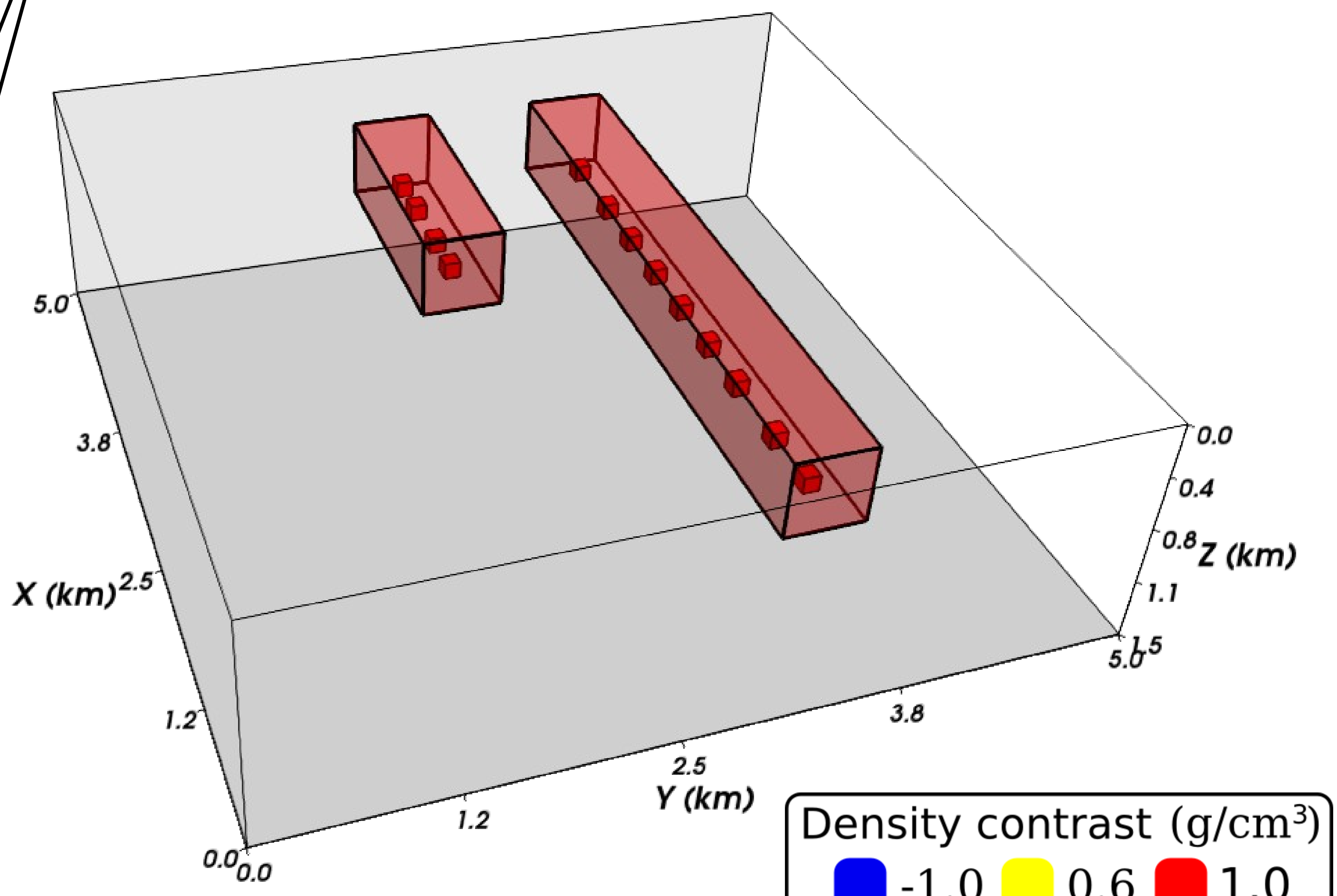
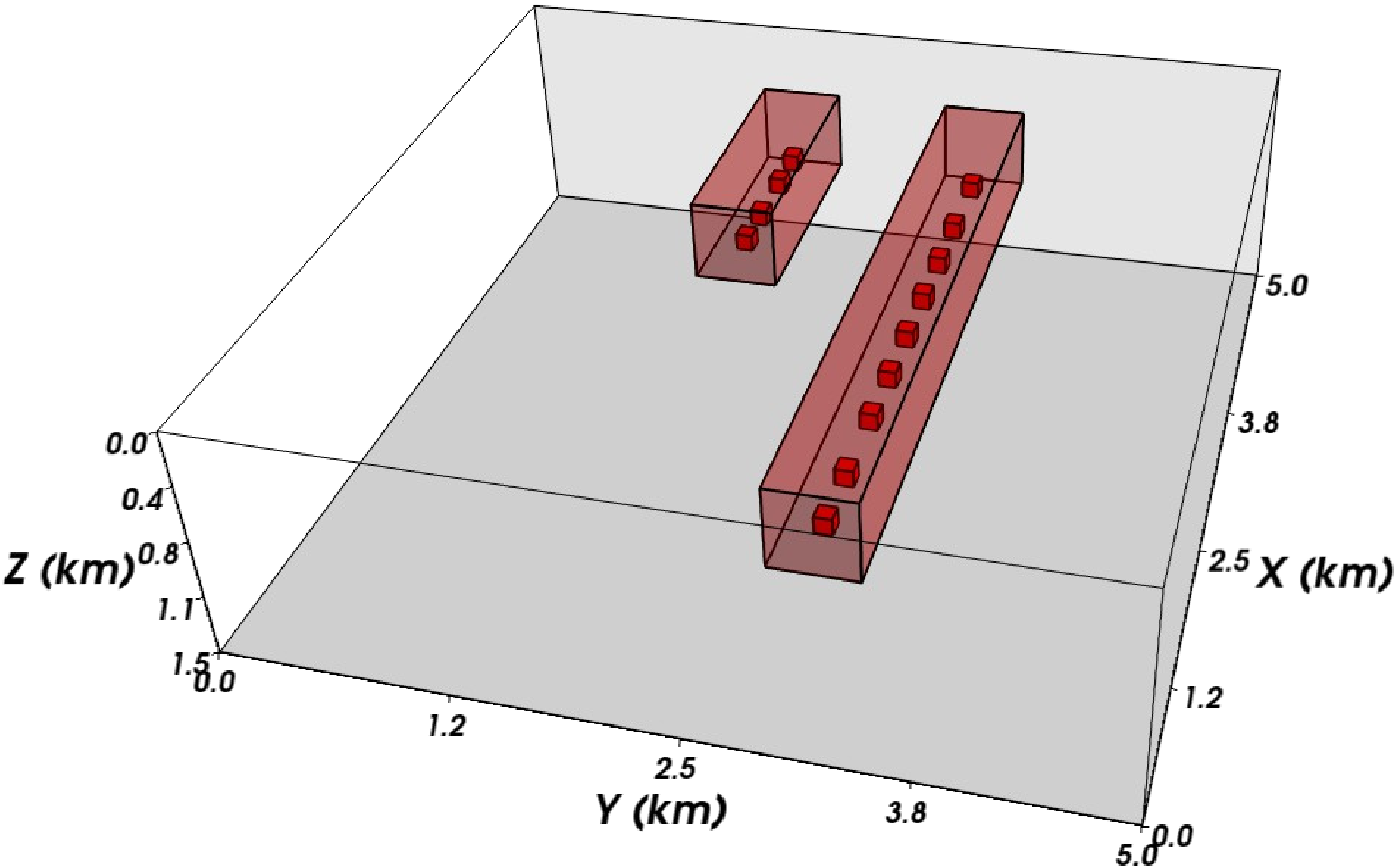




Inversion: • 13 seeds • 7,803 data



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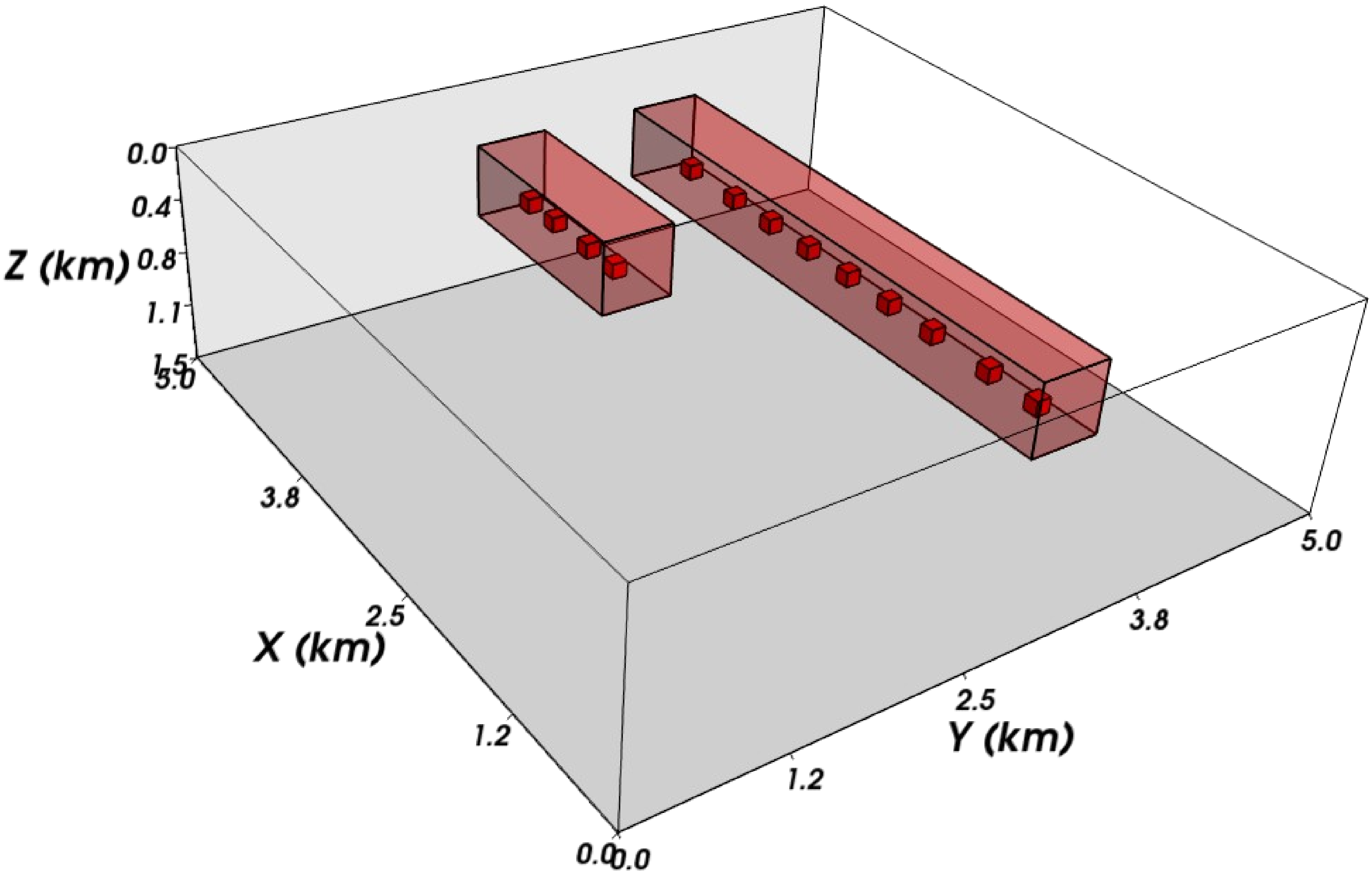
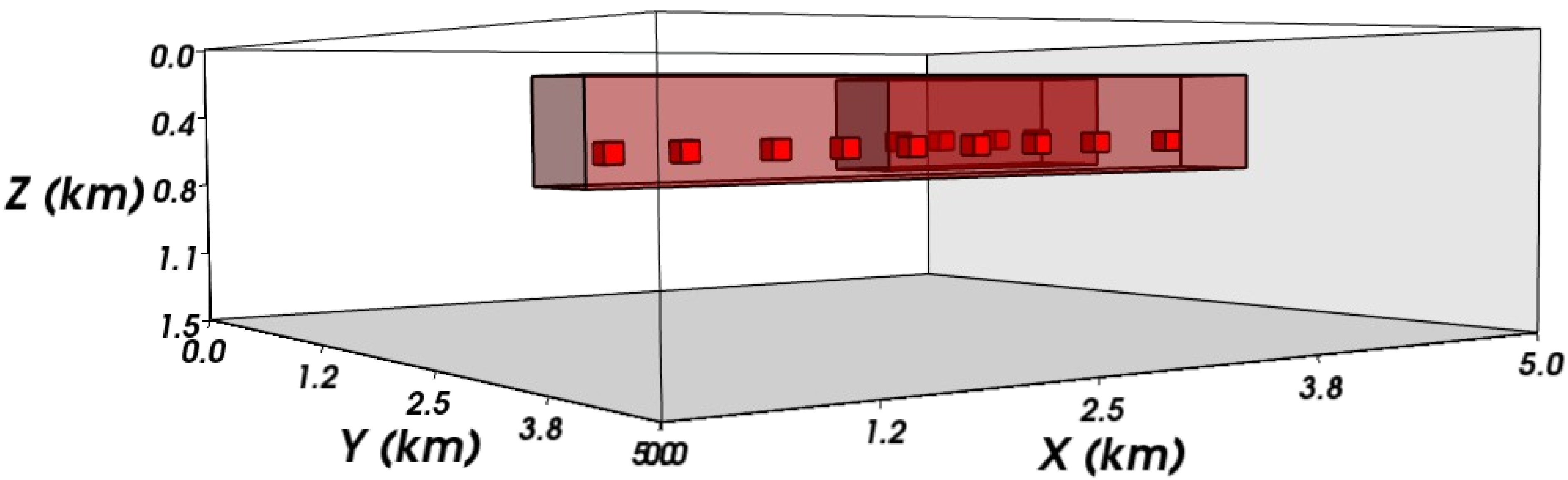
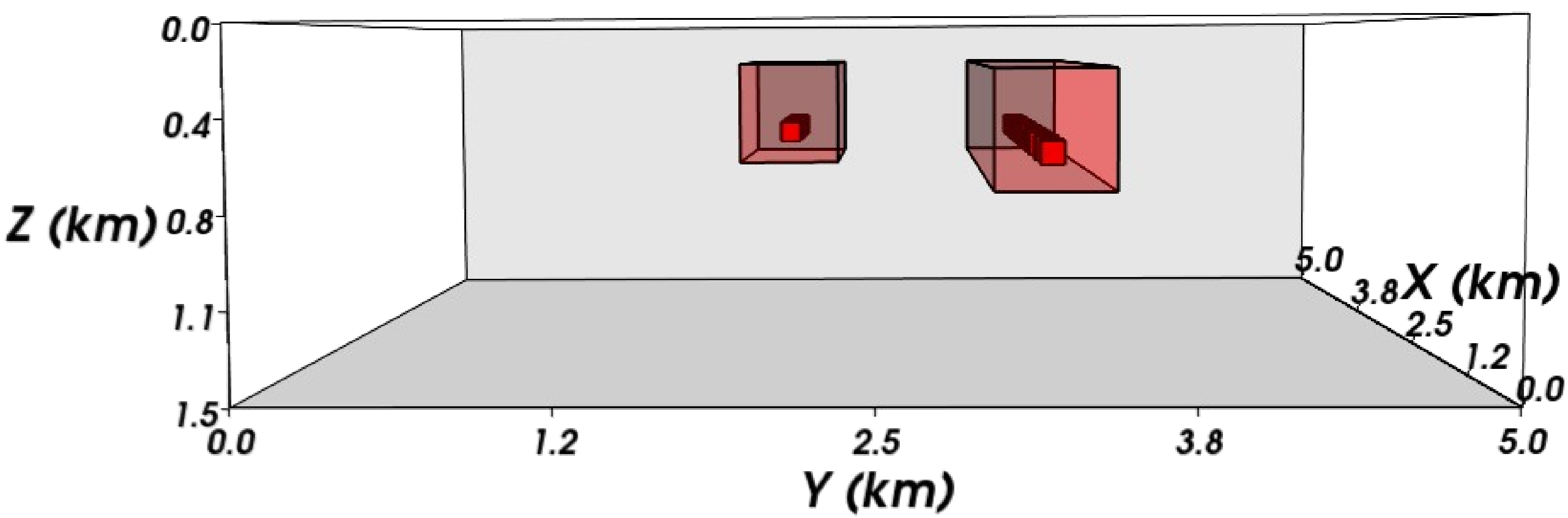


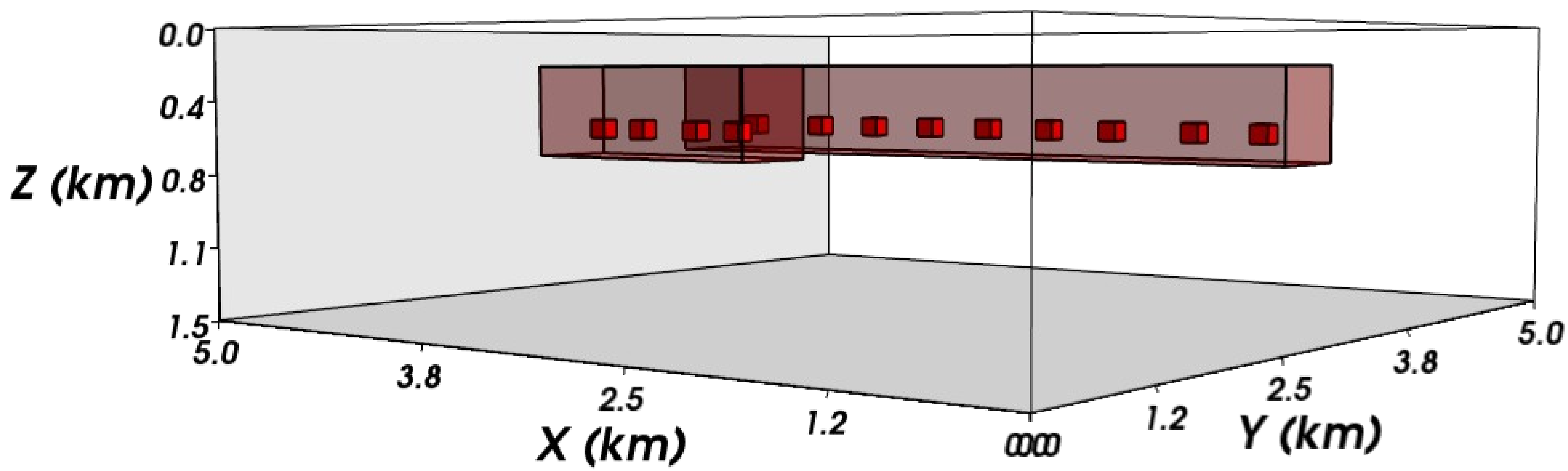
Inversion: • 13 seeds • 7,803 data



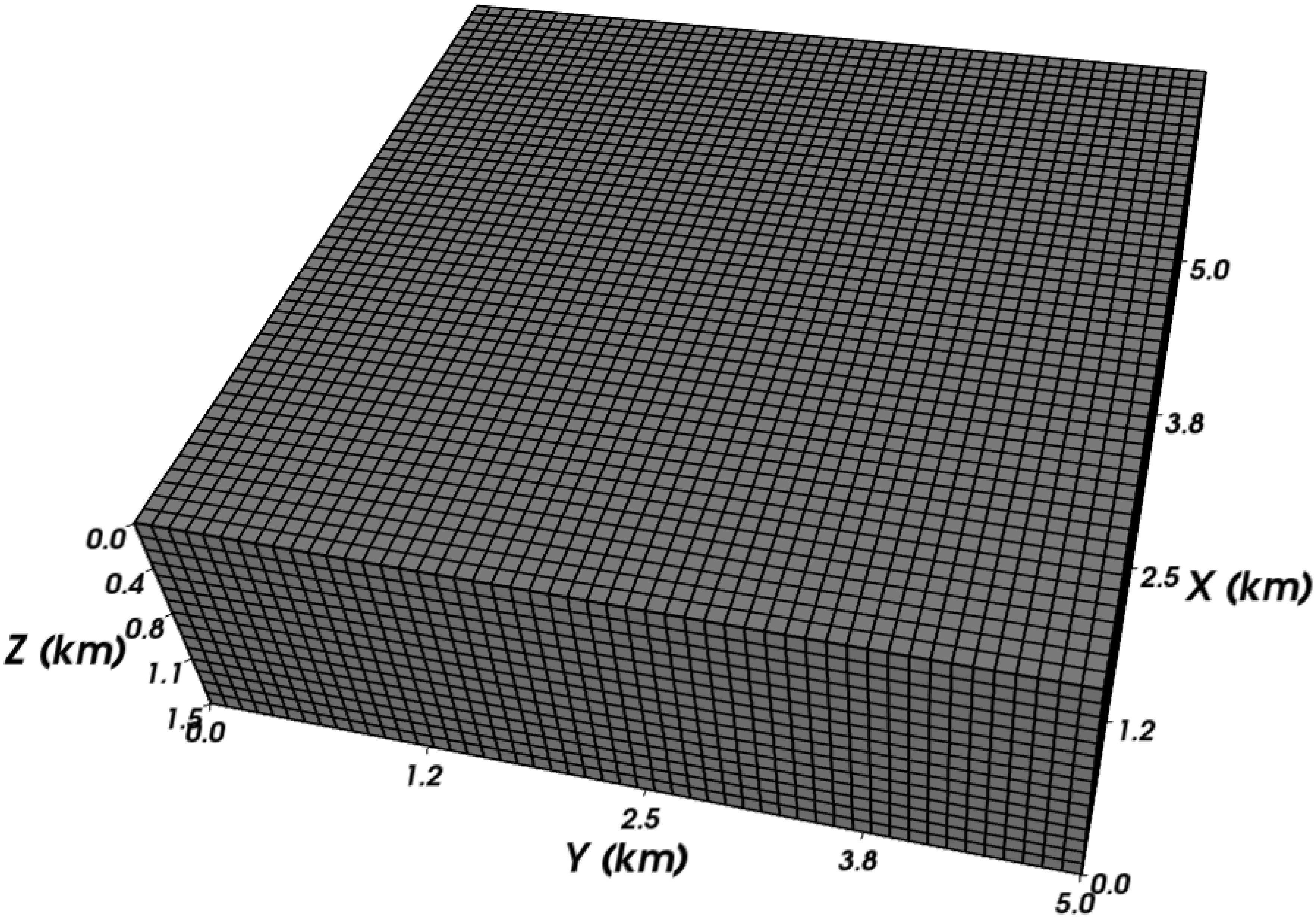
Inversion: • 13 seeds • 7,803 data



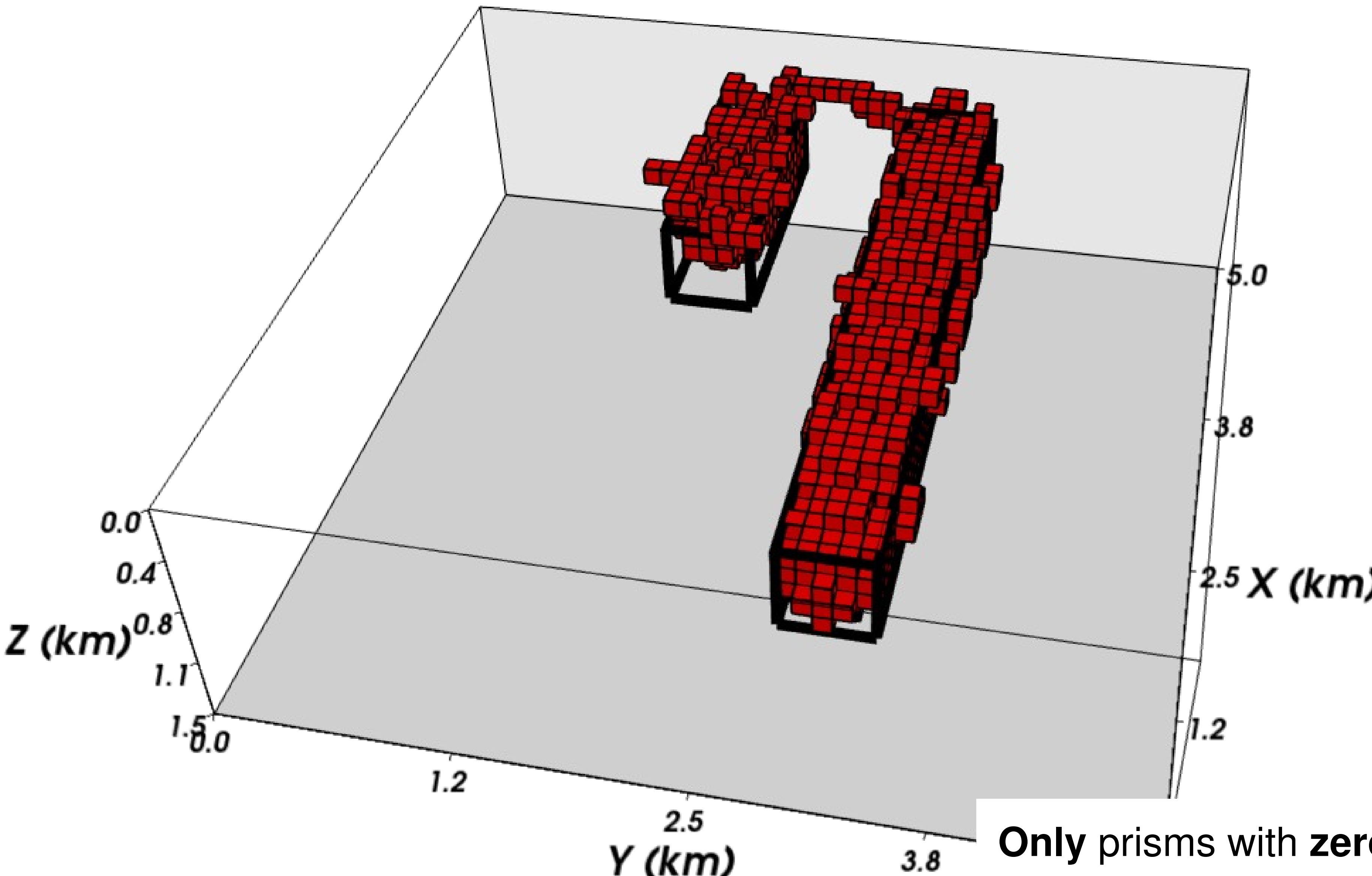
Inversion: • 13 seeds • 7,803 data



Inversion: • 13 seeds • 7,803 data • 37,500 prisms

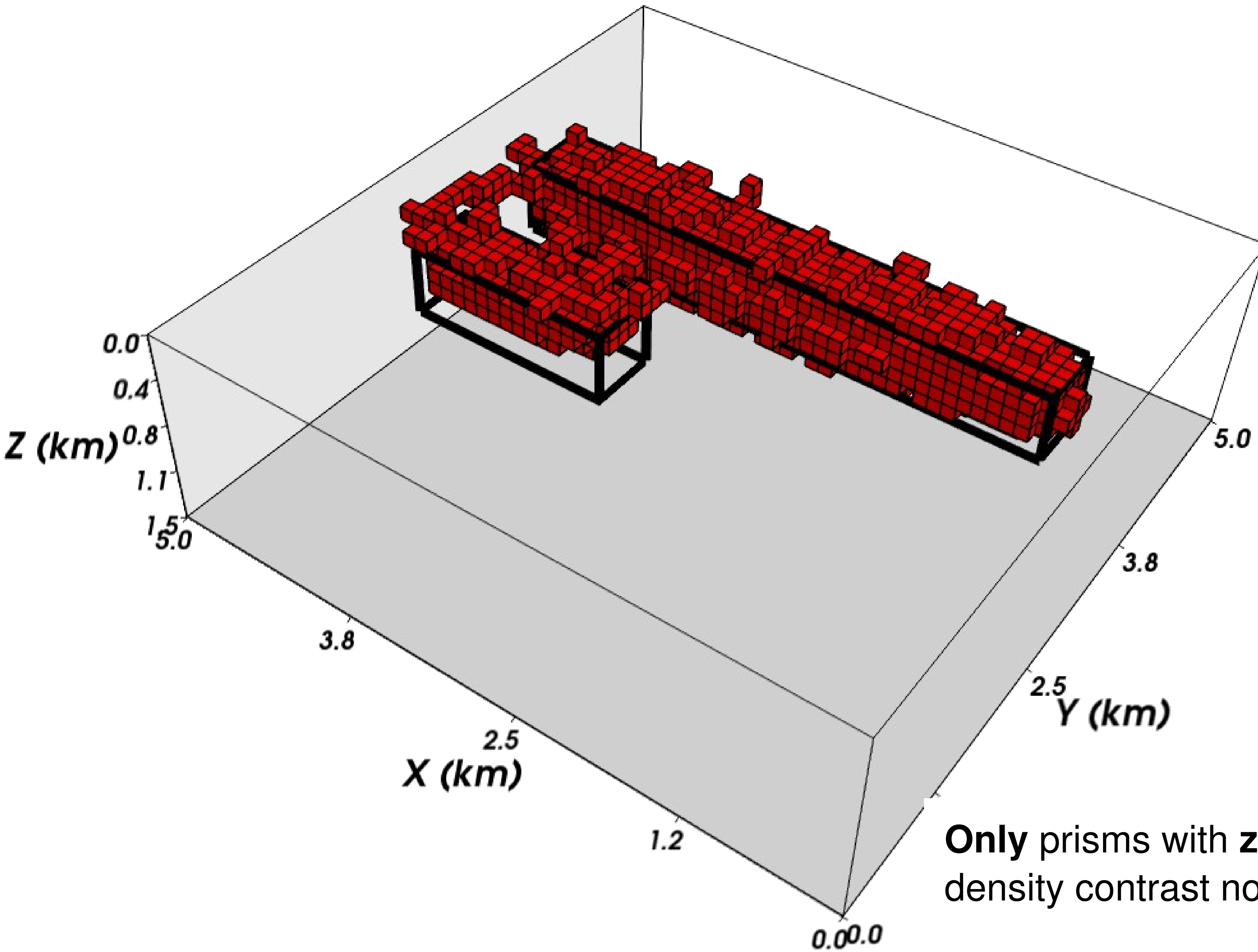


Inversion: • 13 seeds • 7,803 data • 37,500 prisms



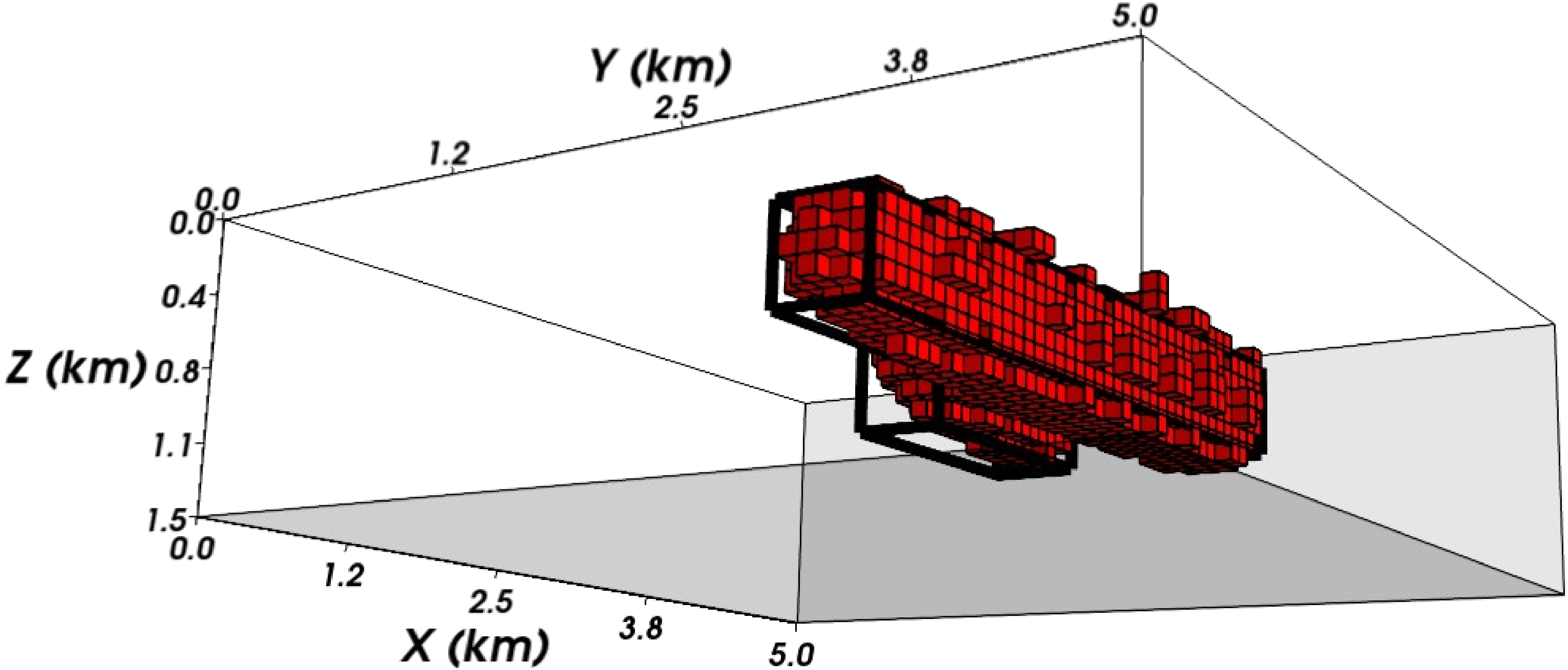
Only prisms with **zero** density contrast not shown

Inversion: • 13 seeds • 7,803 data • 37,500 prisms



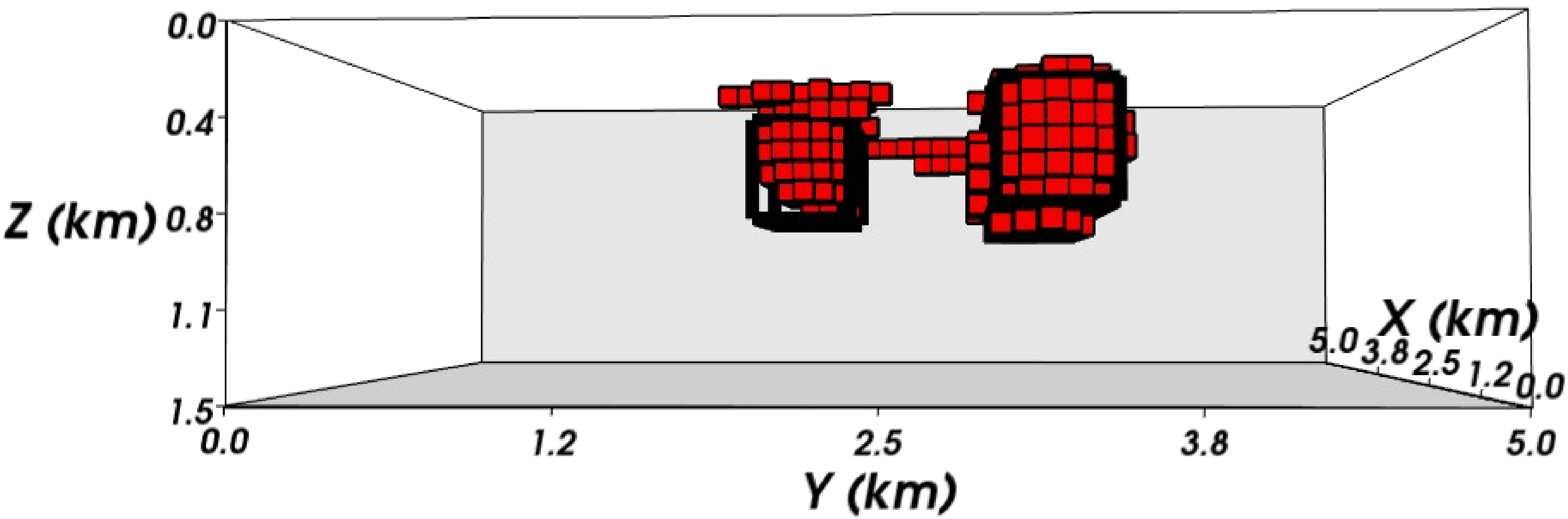
Only prisms with **zero** density contrast not shown

Inversion: • 13 seeds • 7,803 data • 37,500 prisms



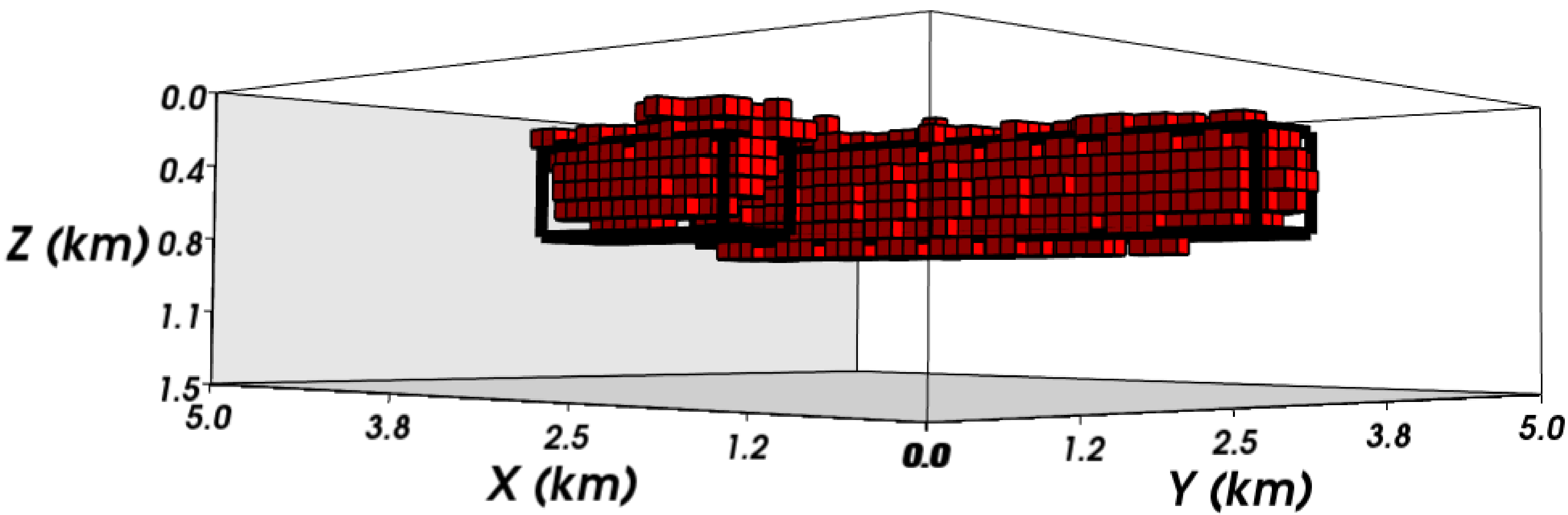
Only prisms with **zero** density contrast not shown

Inversion: • 13 seeds • 7,803 data • 37,500 prisms



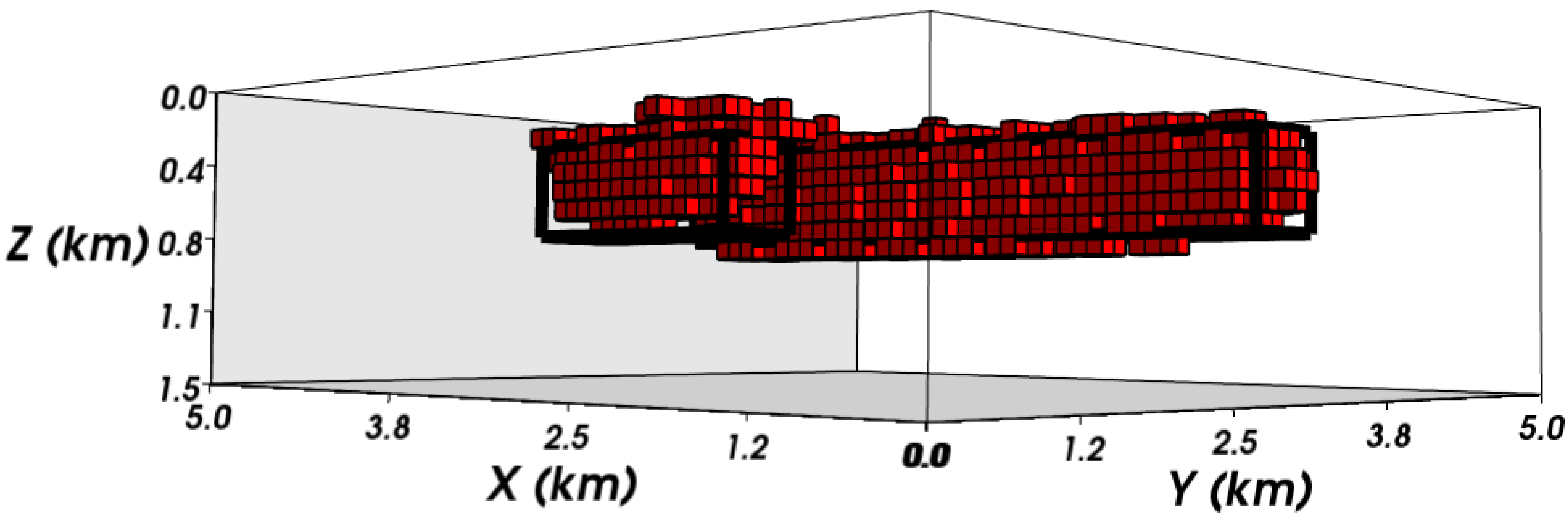
Only prisms with **zero** density contrast not shown

Inversion: • 13 seeds • 7,803 data • 37,500 prisms



Only prisms with **zero** density contrast not shown

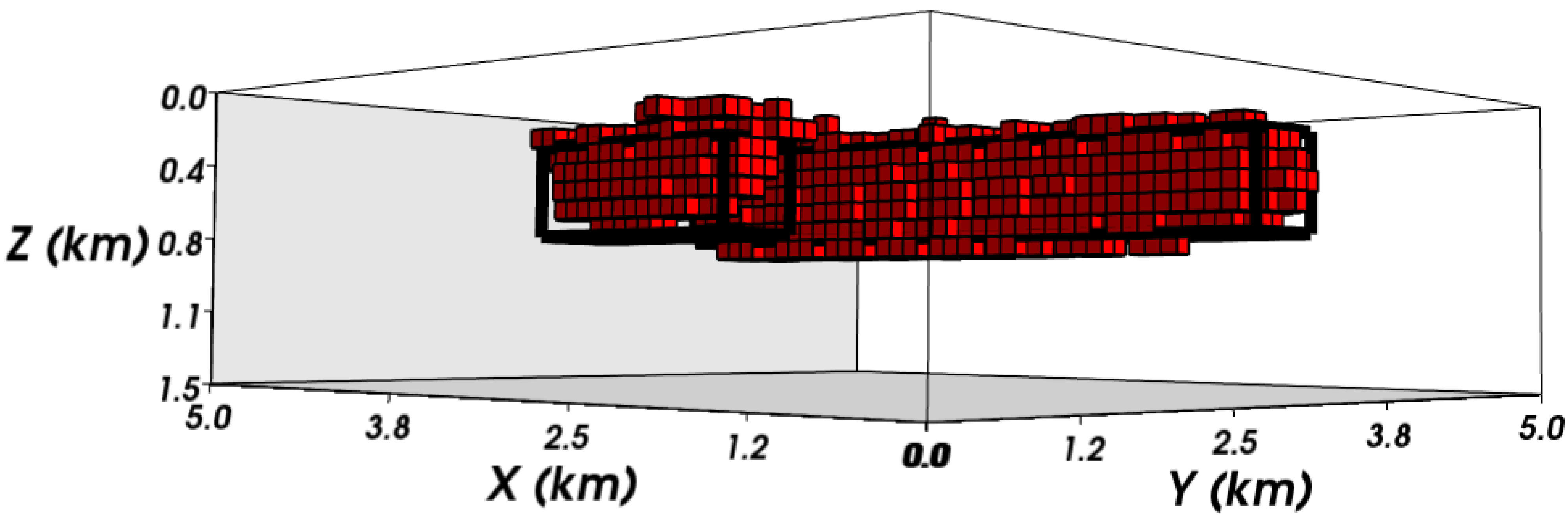
Inversion: • 13 seeds • 7,803 data • 37,500 prisms



• Recover shape of targets

Only prisms with **zero** density contrast not shown

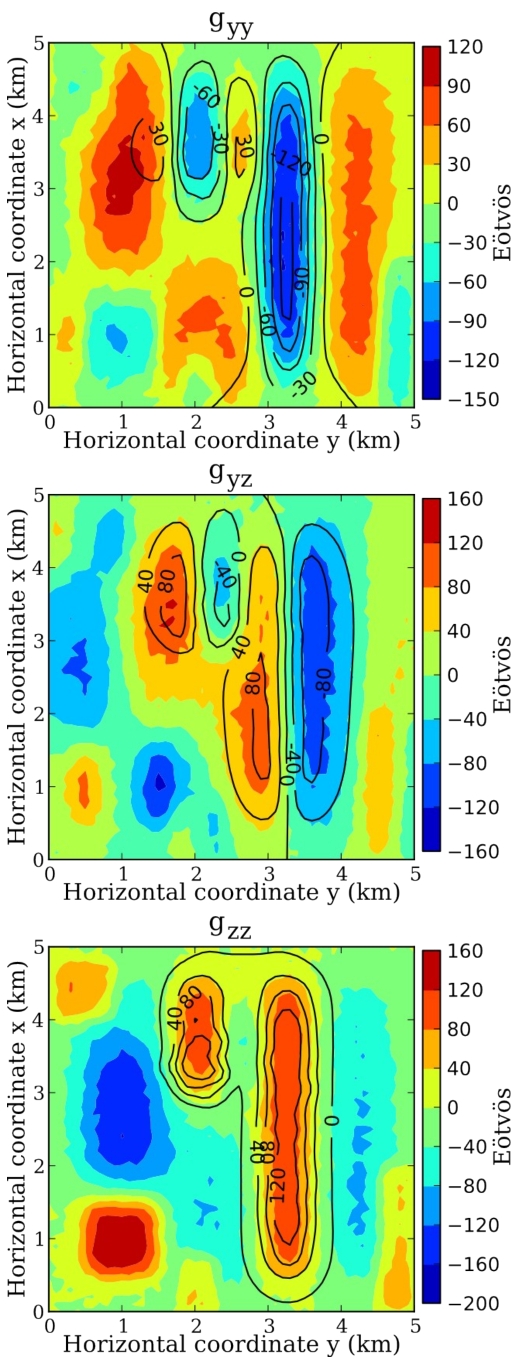
Inversion: • 13 seeds • 7,803 data • 37,500 prisms



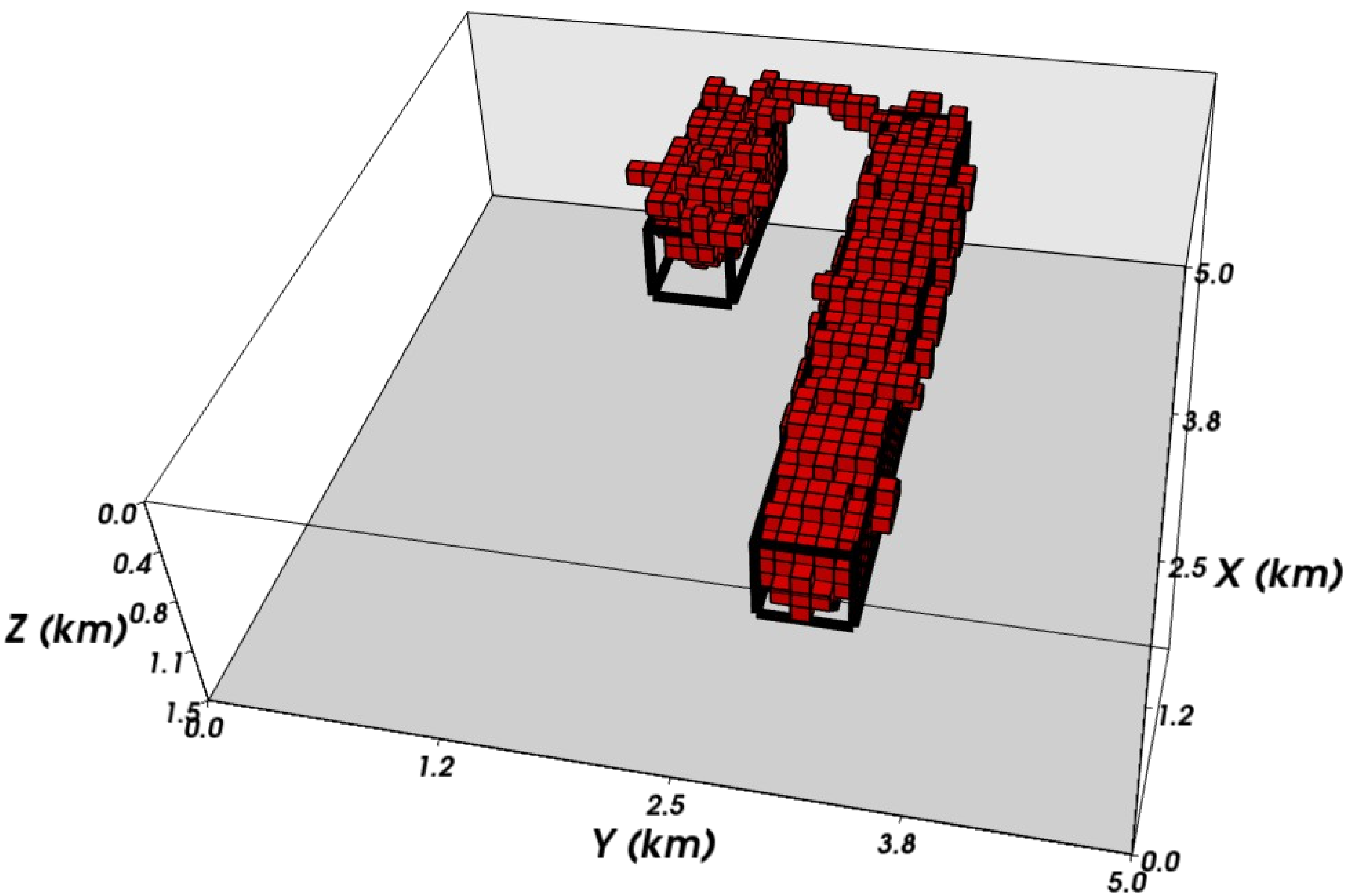
- Recover shape of targets
- Total time = 2.2 minutes (on laptop)

Only prisms with **zero** density contrast not shown

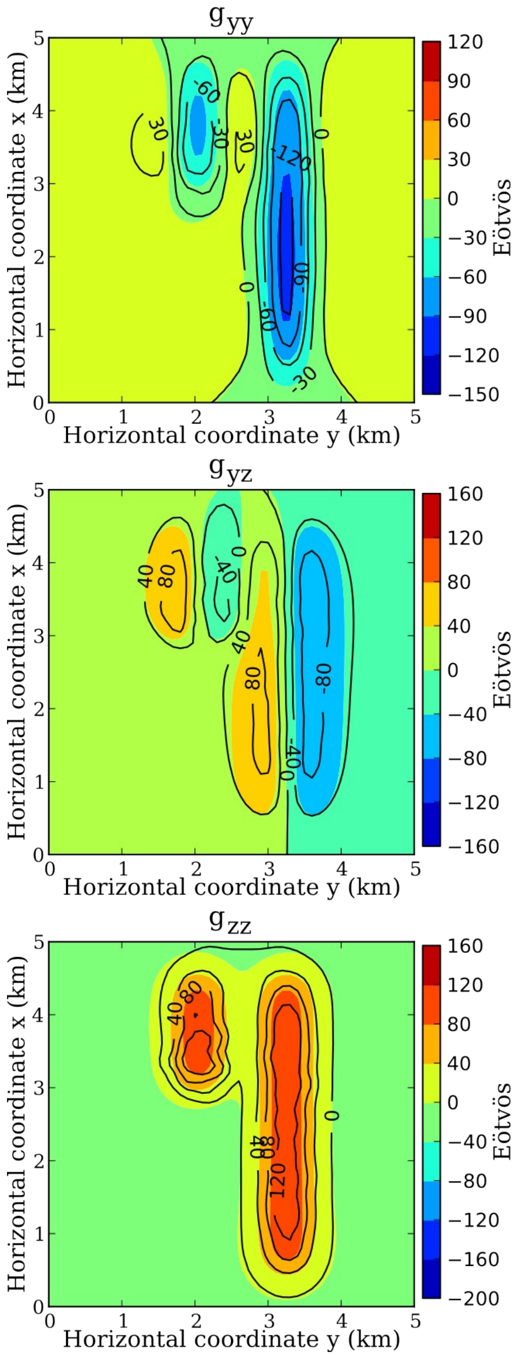
Inversion: • 13 seeds • 7,803 data • 37,500 prisms



Predicted data in contours

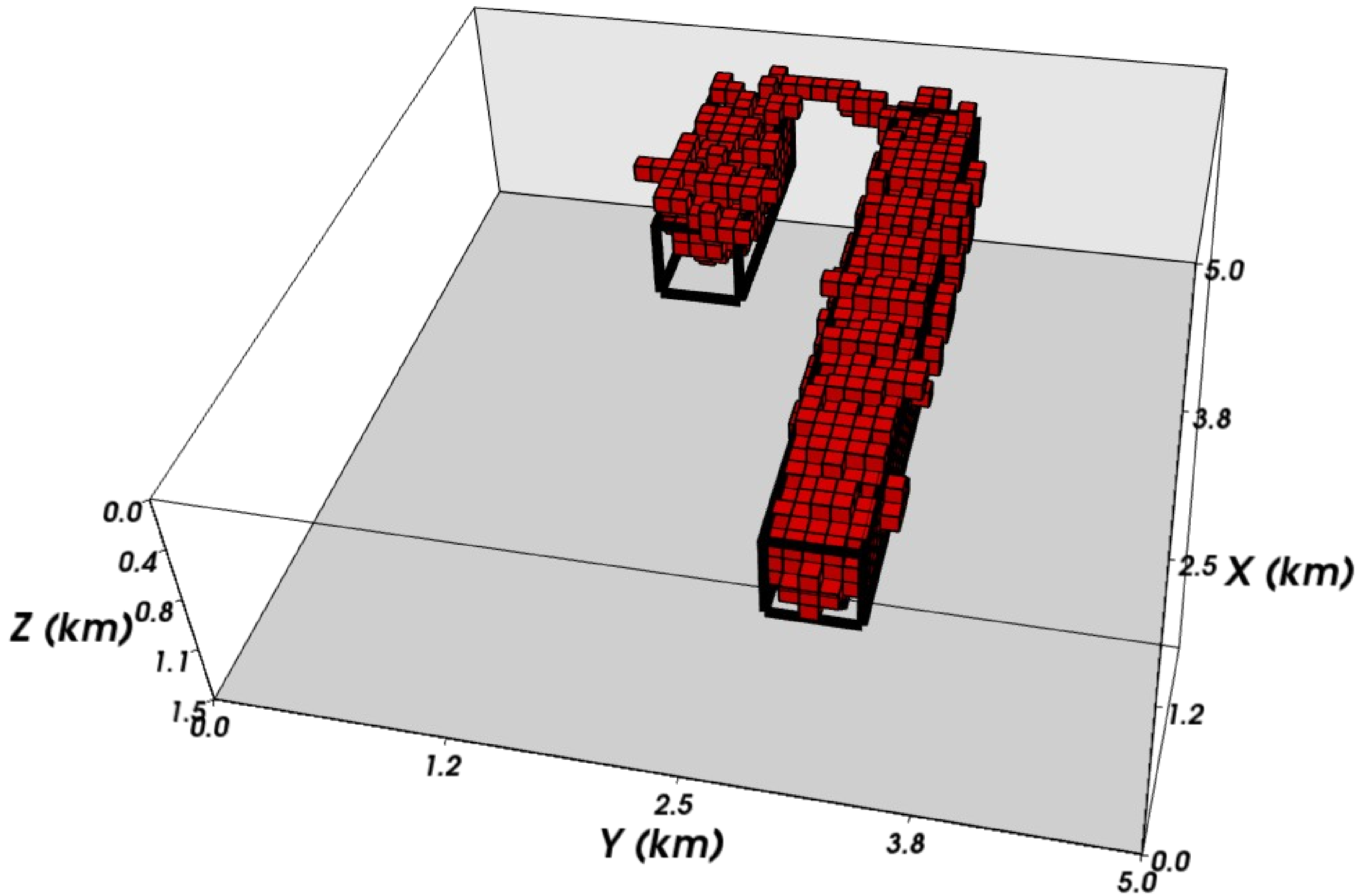


Inversion: • 13 seeds • 7,803 data • 37,500 prisms



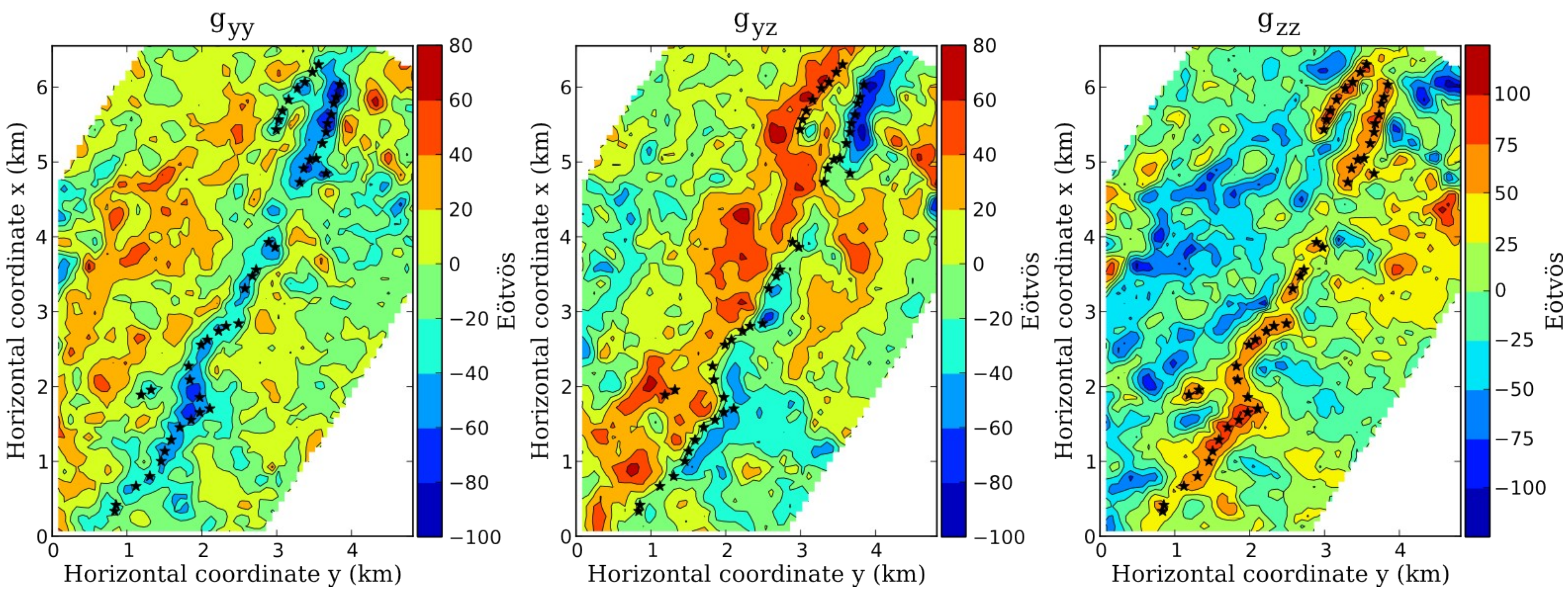
Predicted data in contours

Effect of **true** targeted sources



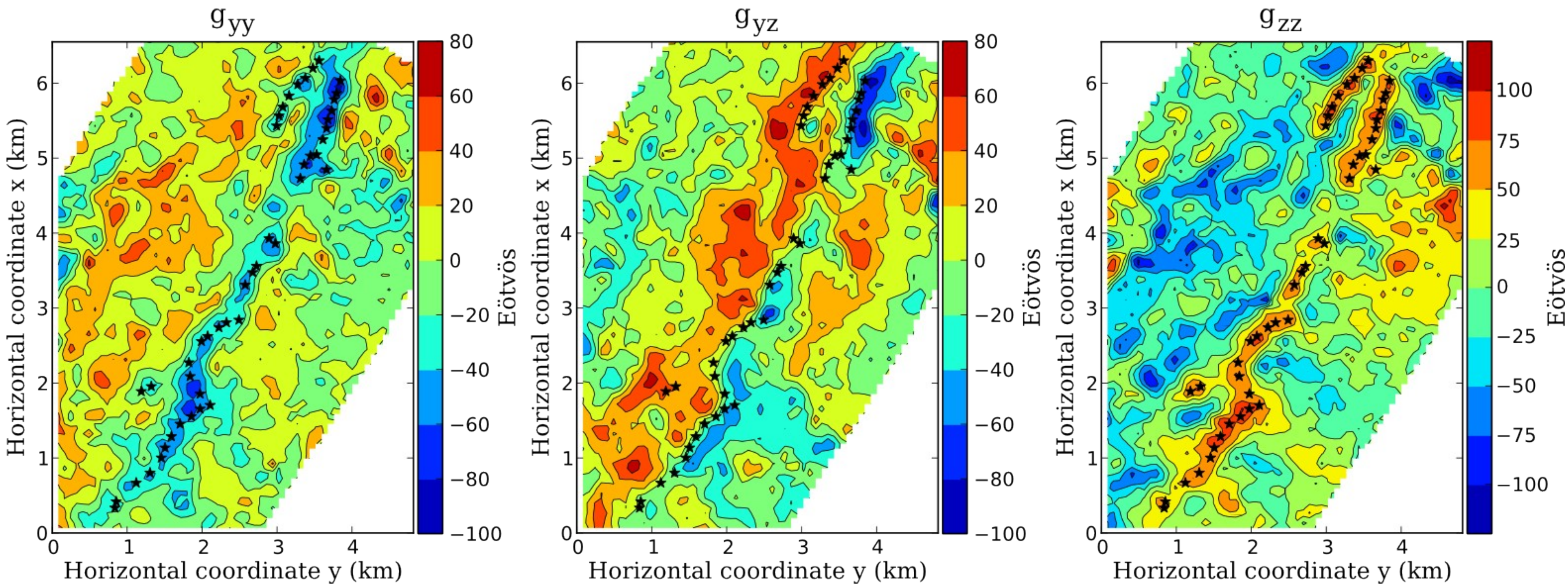
Real Data

Data:



- 3 components
- FTG survey
- Quadrilátero Ferrífero, Brazil

Data:

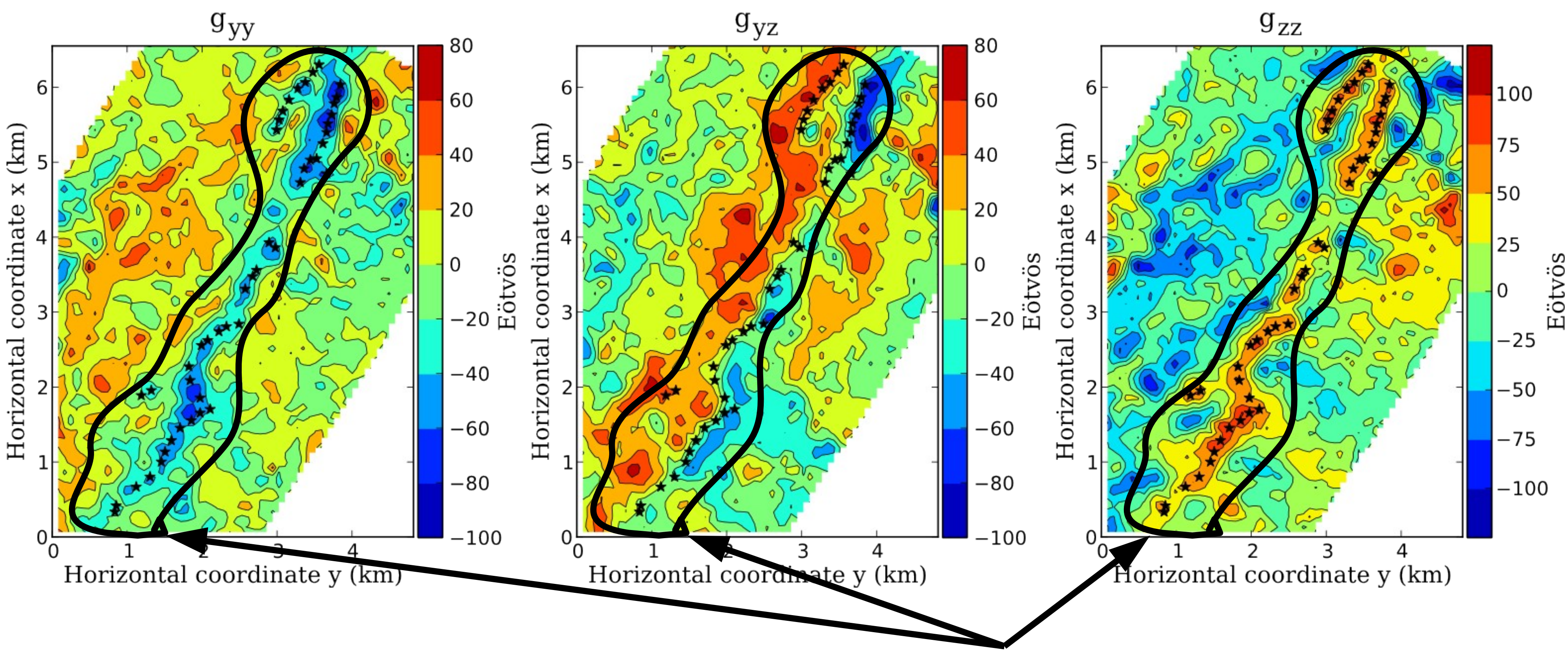


- 3 components
- FTG survey
- Quadrilátero Ferrífero, Brazil

Targets:

- Iron ore bodies
- BIFs of Cauê Formation

Data:

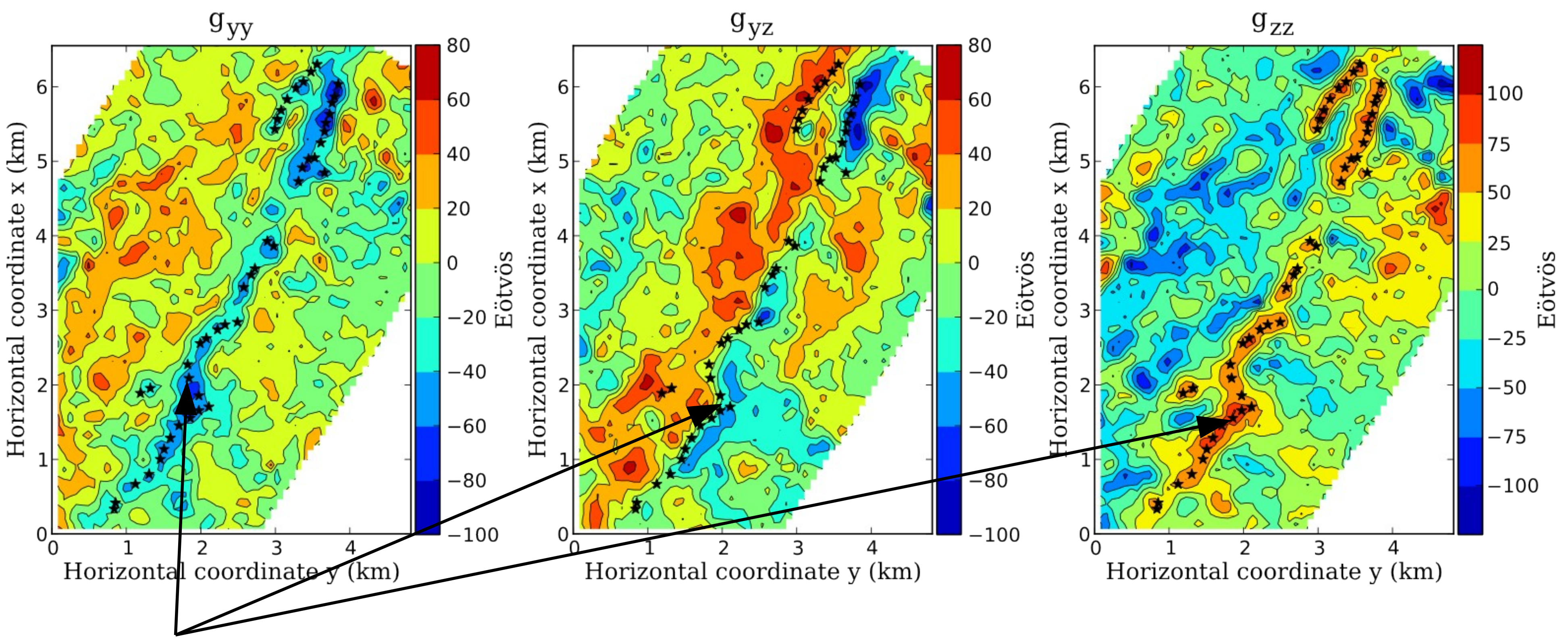


- 3 components
- FTG survey
- Quadrilátero Ferrífero, Brazil

Targets:

- Iron ore bodies
- BIFs of Cauê Formation

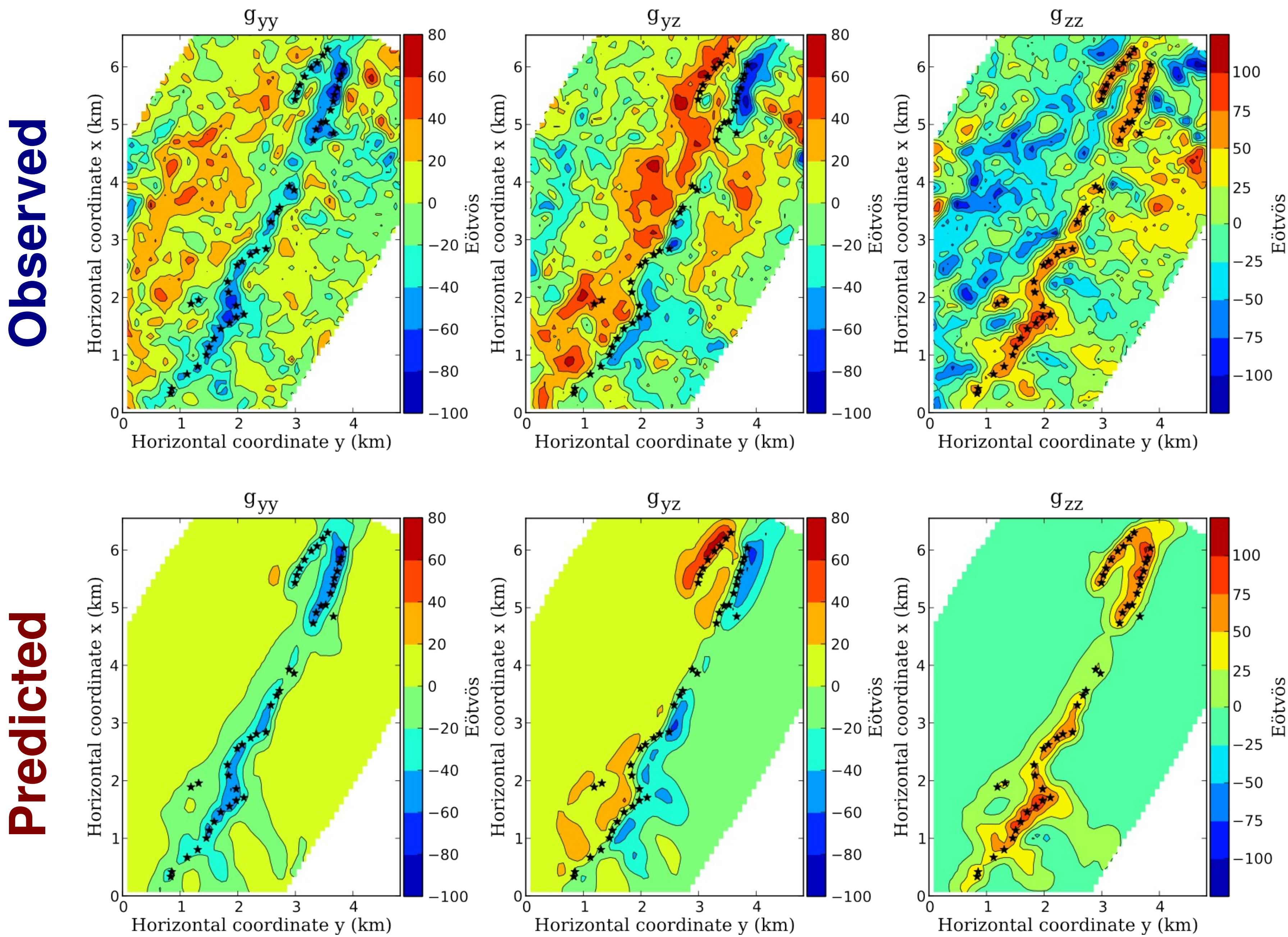
Data:



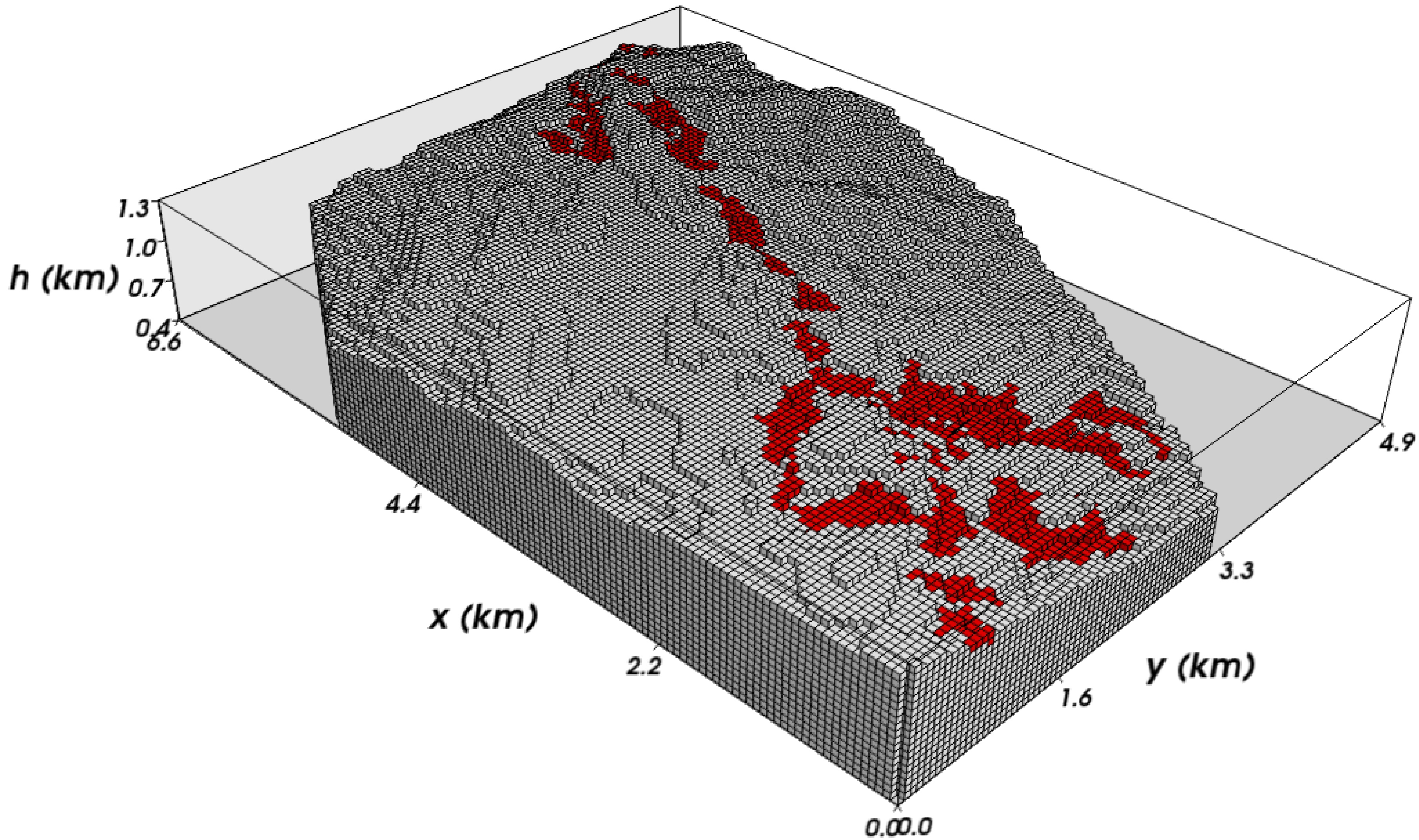
Seeds for iron ore:

- Density contrast 1.0 g/cm^3
- Depth 200 m

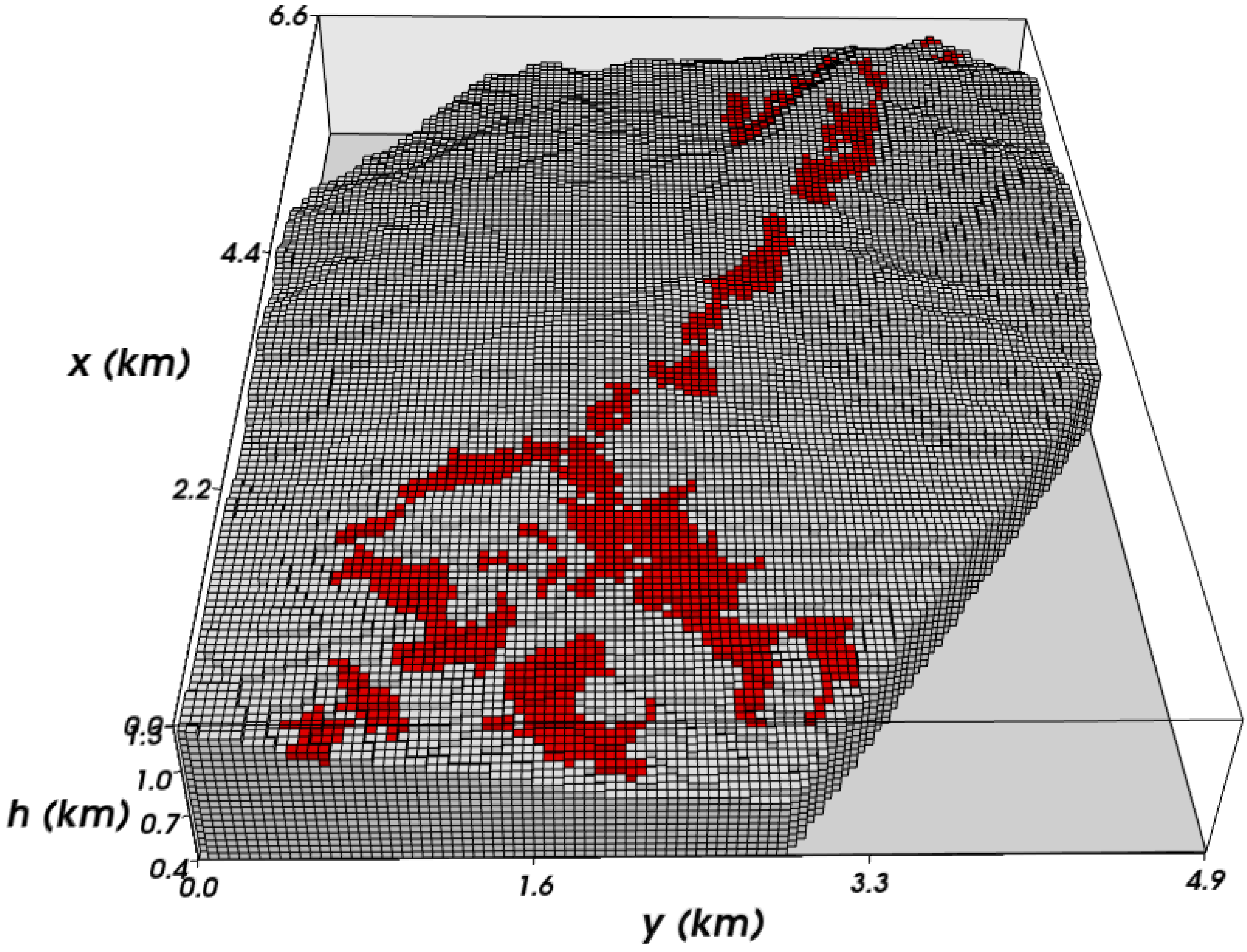
Inversion: • 46 seeds • 13,746 data



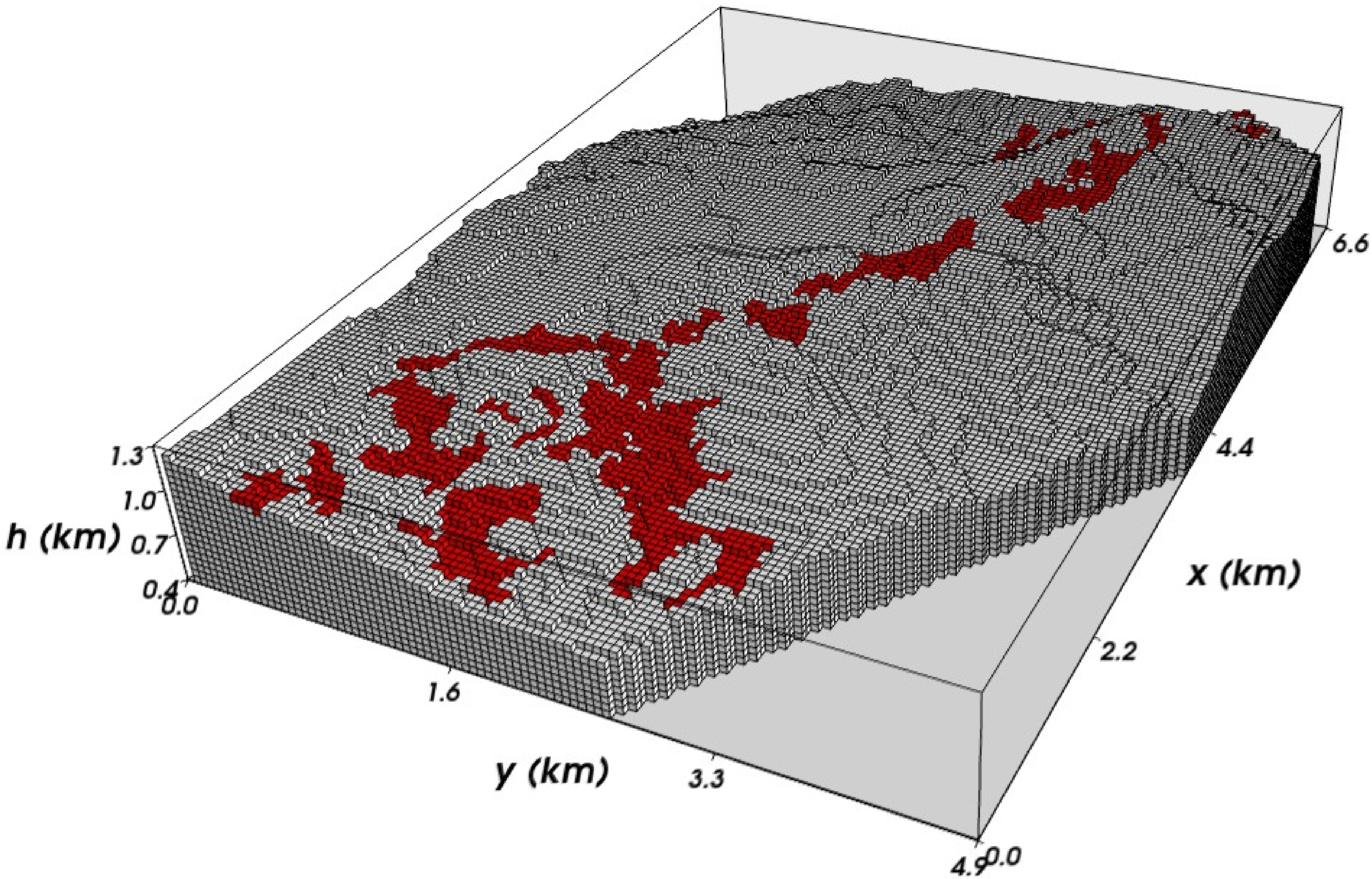
Inversion: • 46 seeds • 13,746 data • 164,892 prisms



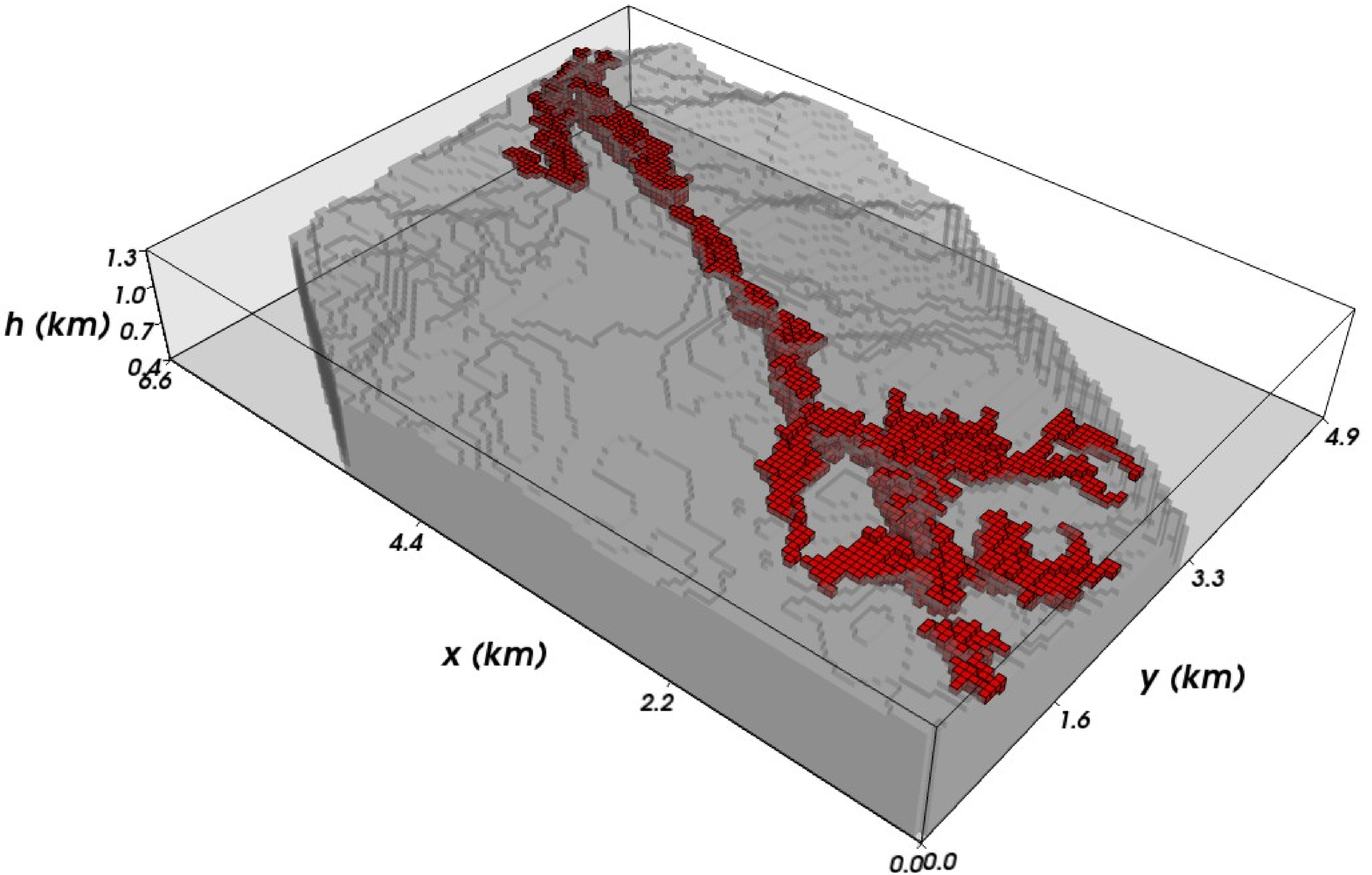
Inversion: • 46 seeds • 13,746 data • 164,892 prisms



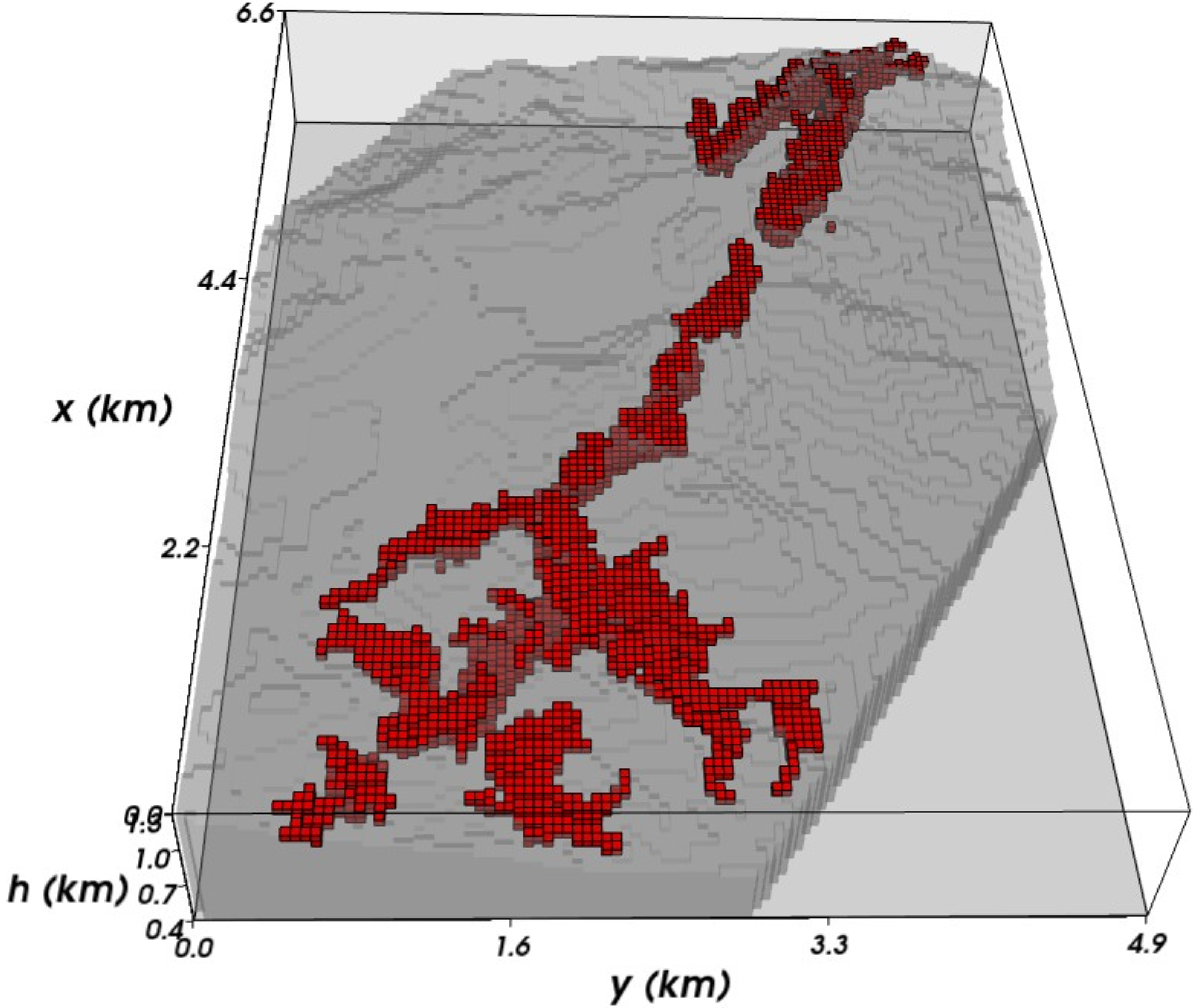
Inversion: • 46 seeds • 13,746 data • 164,892 prisms



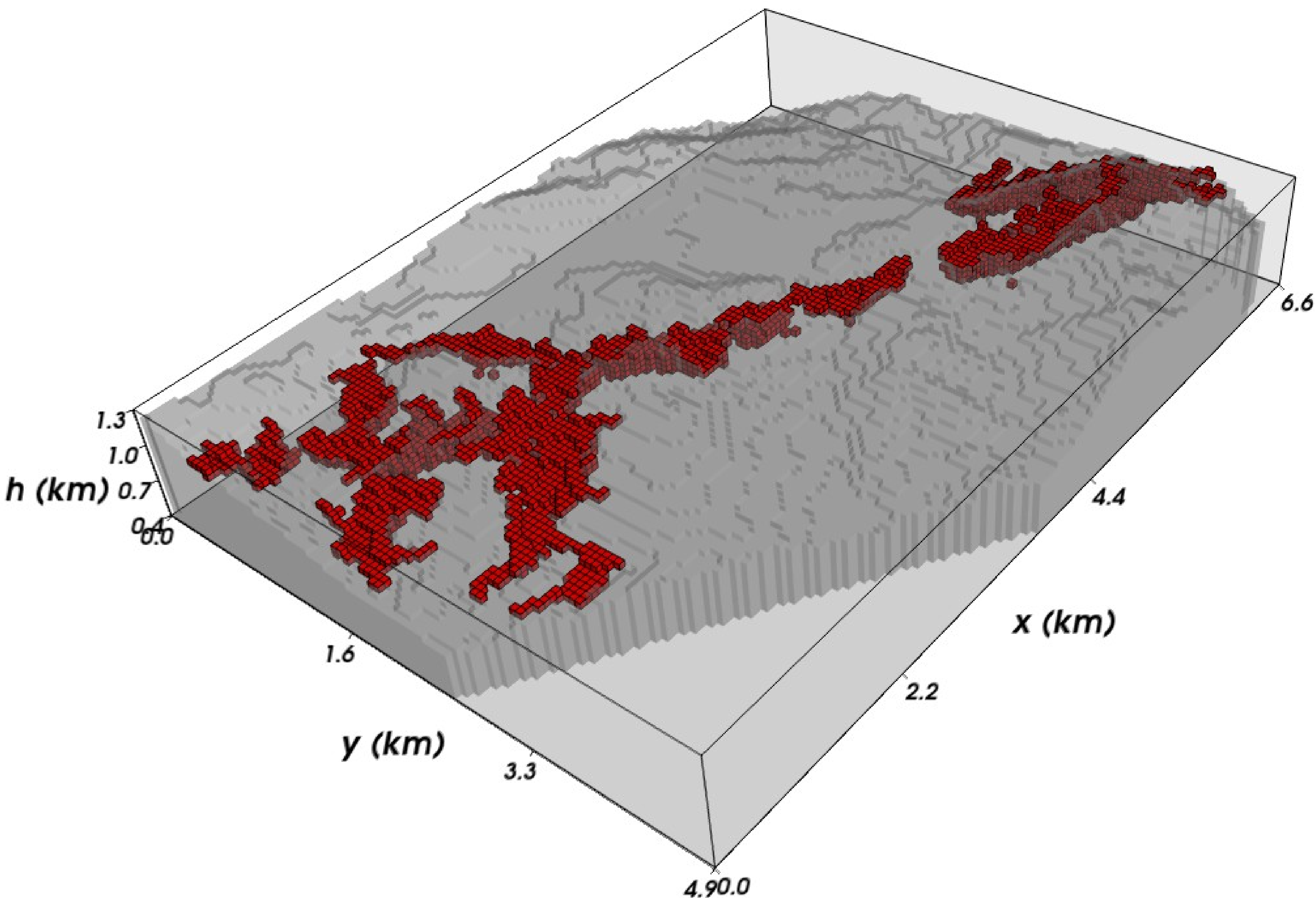
Inversion: • 46 seeds • 13,746 data • 164,892 prisms



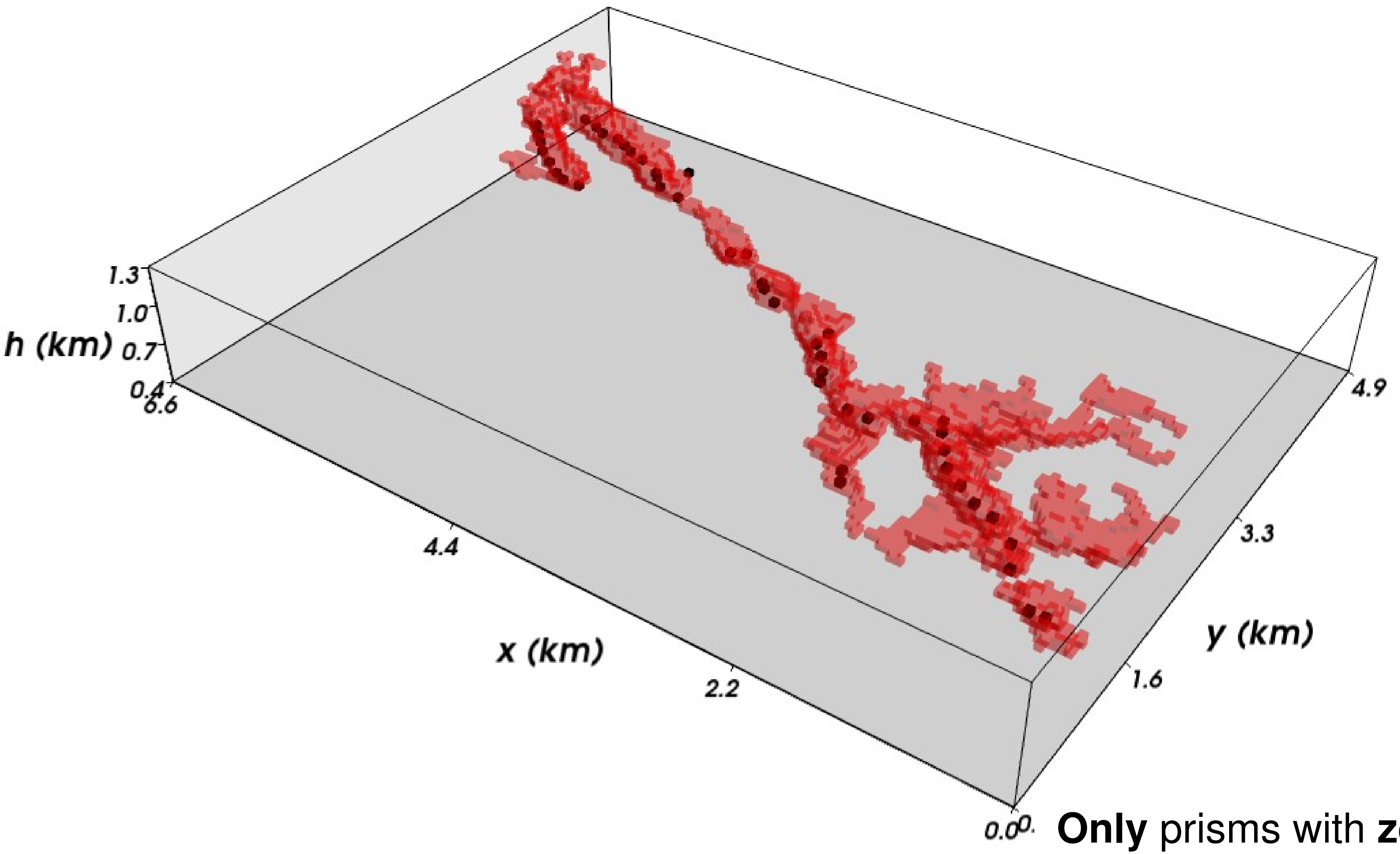
Inversion: • 46 seeds • 13,746 data • 164,892 prisms



Inversion: • 46 seeds • 13,746 data • 164,892 prisms

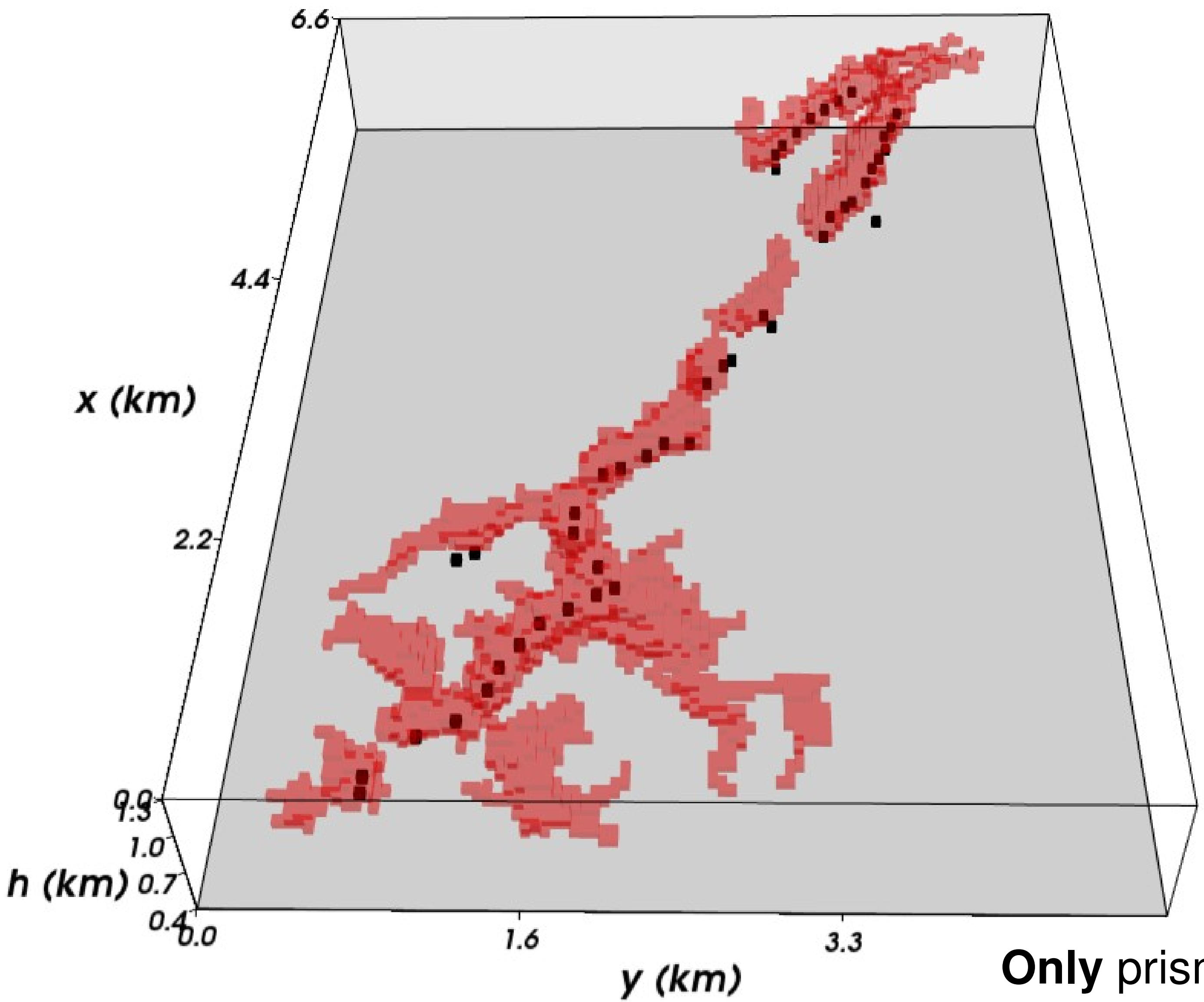


Inversion: • 46 seeds • 13,746 data • 164,892 prisms



Only prisms with **zero** density contrast not shown

Inversion: • 46 seeds • 13,746 data • 164,892 prisms



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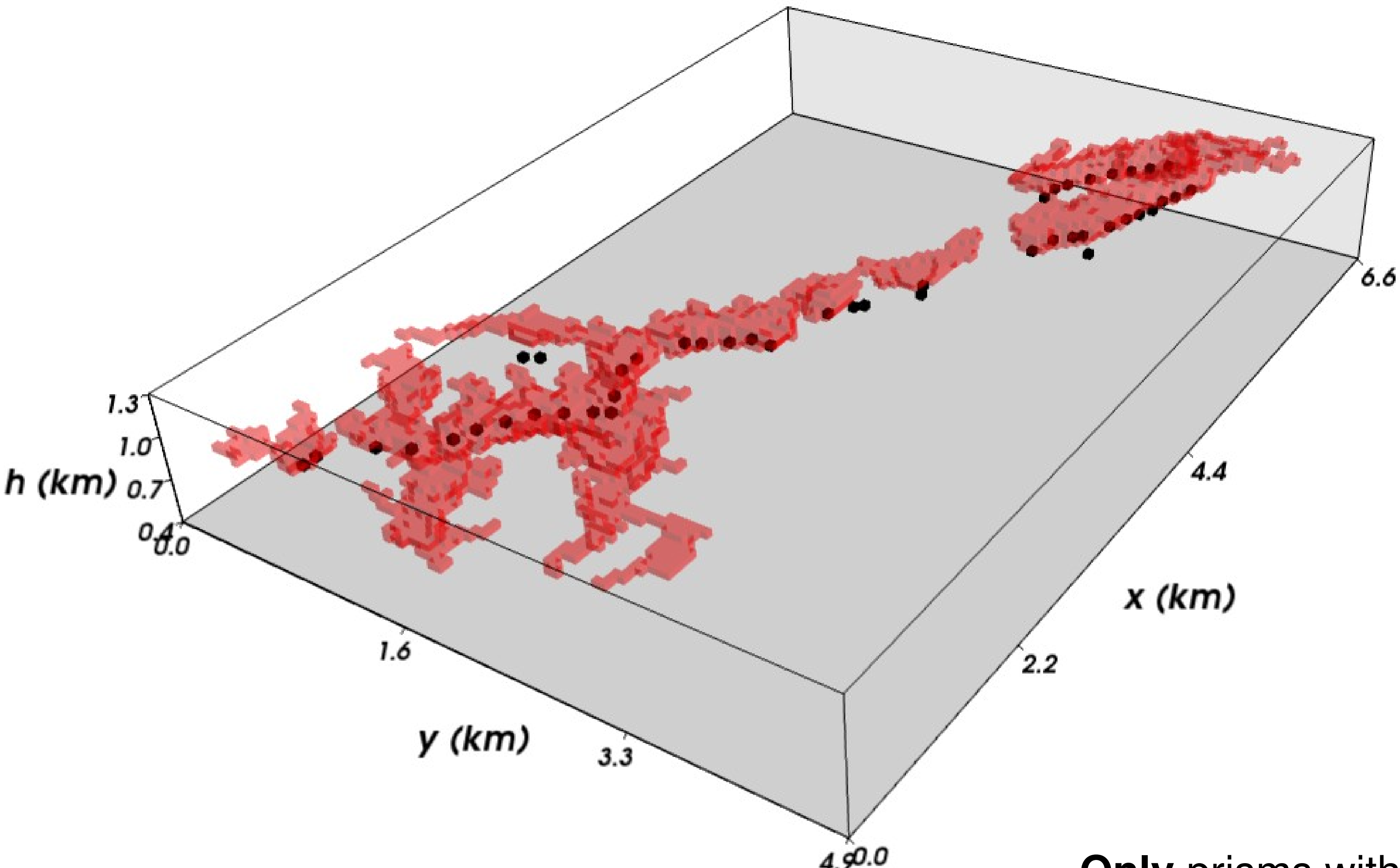
Inversion: • 46 seeds • 13,746 data • 164,892 prisms



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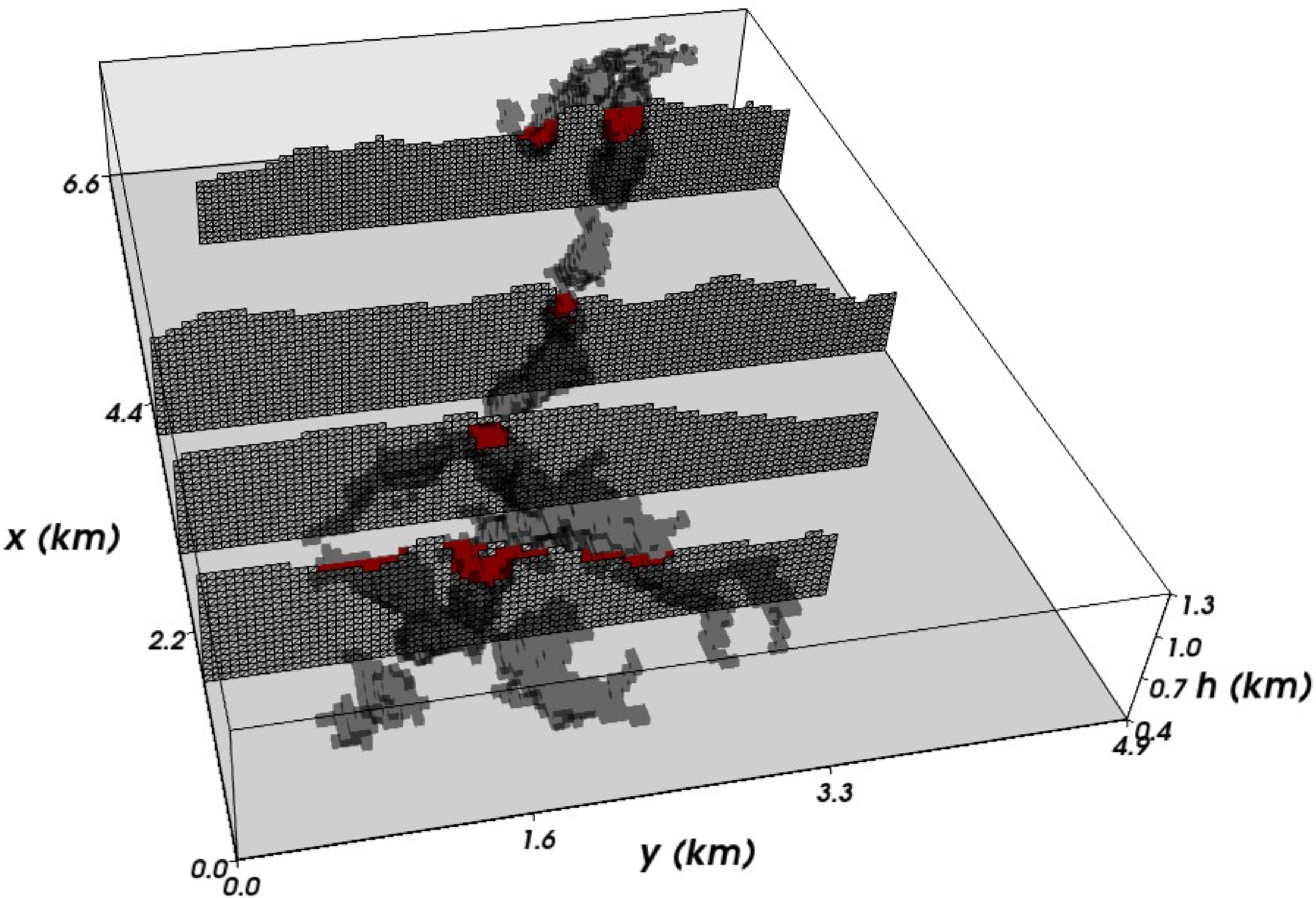
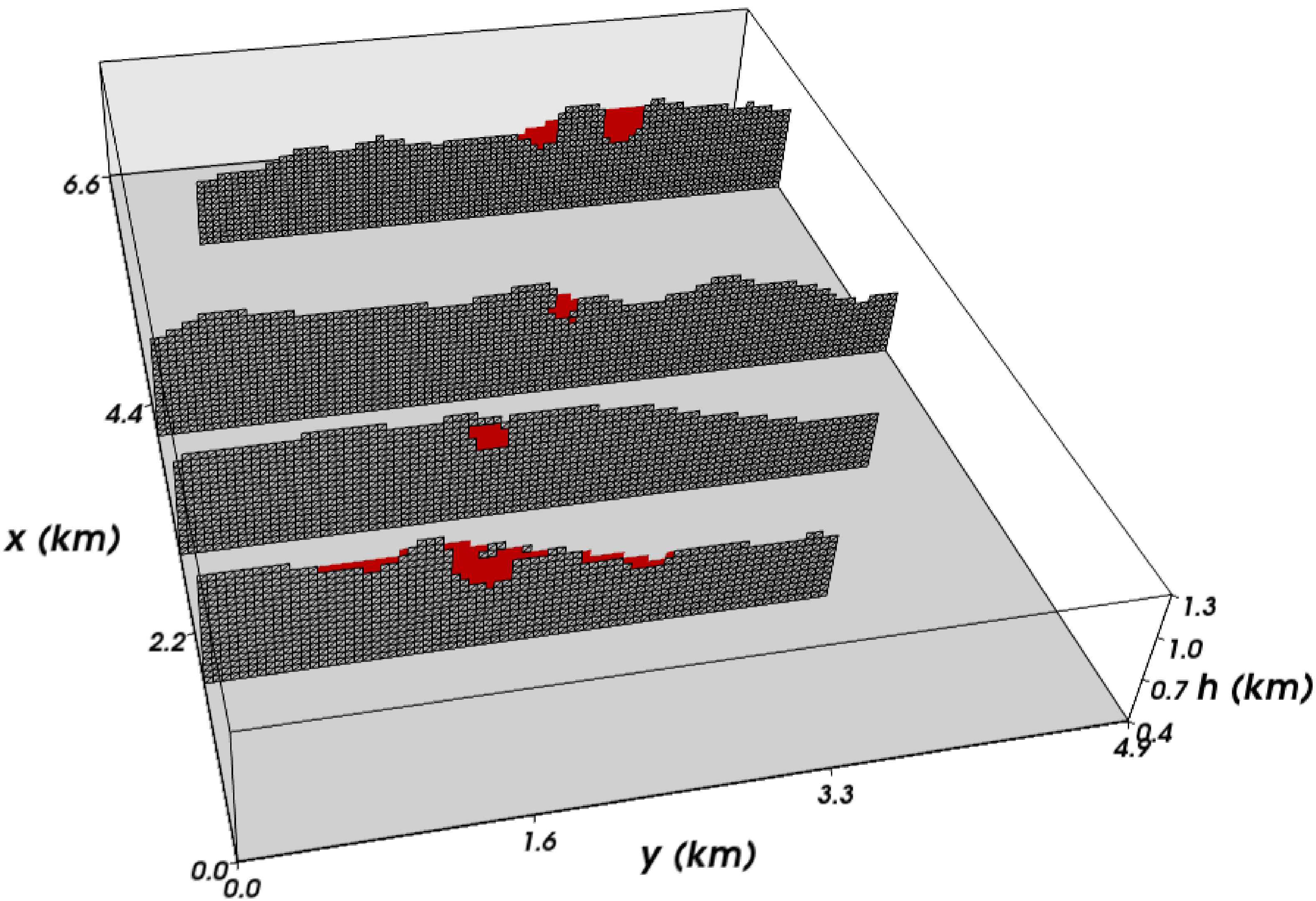
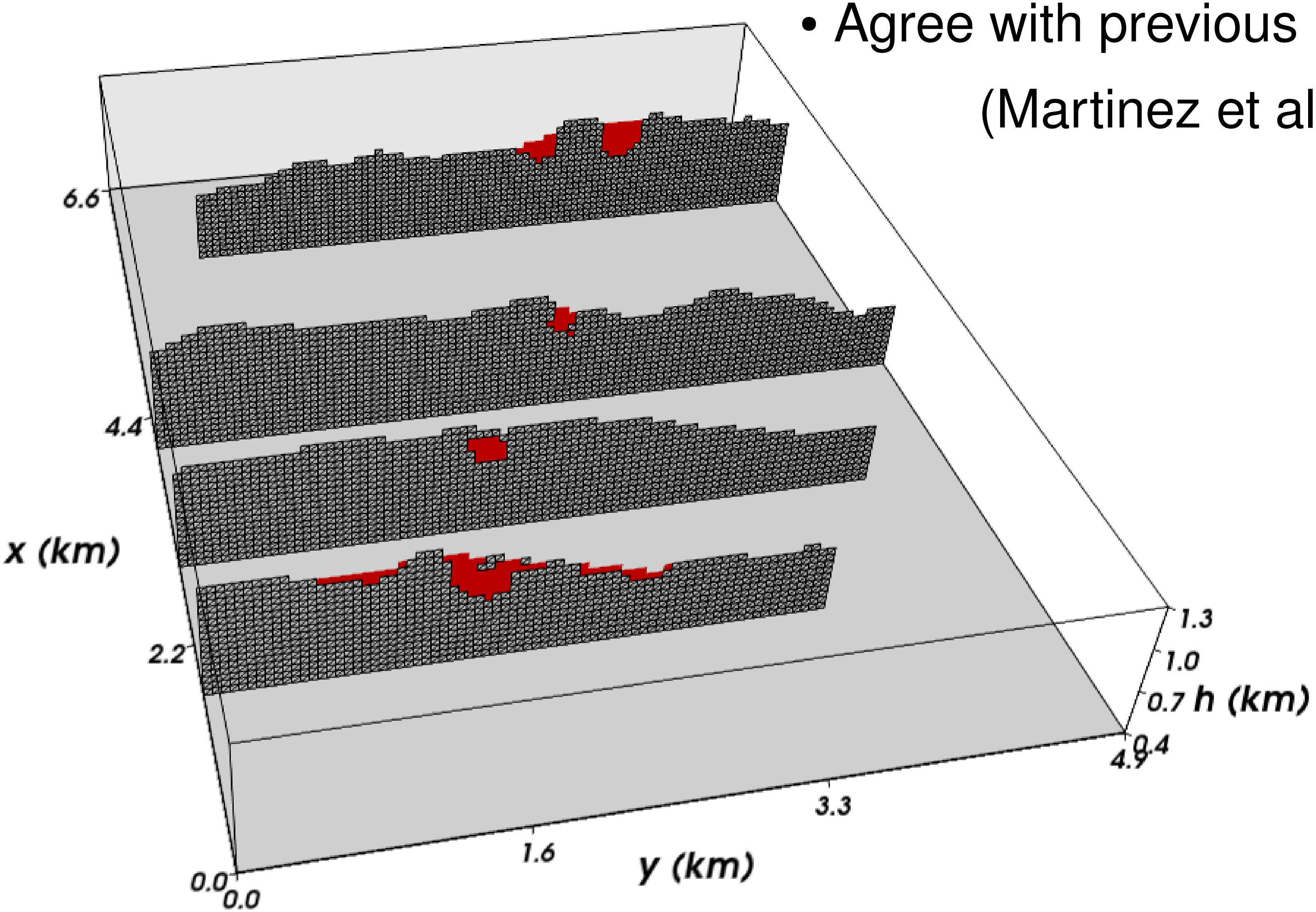


Inversion: • 46 seeds • 13,746 data • 164,892 prisms

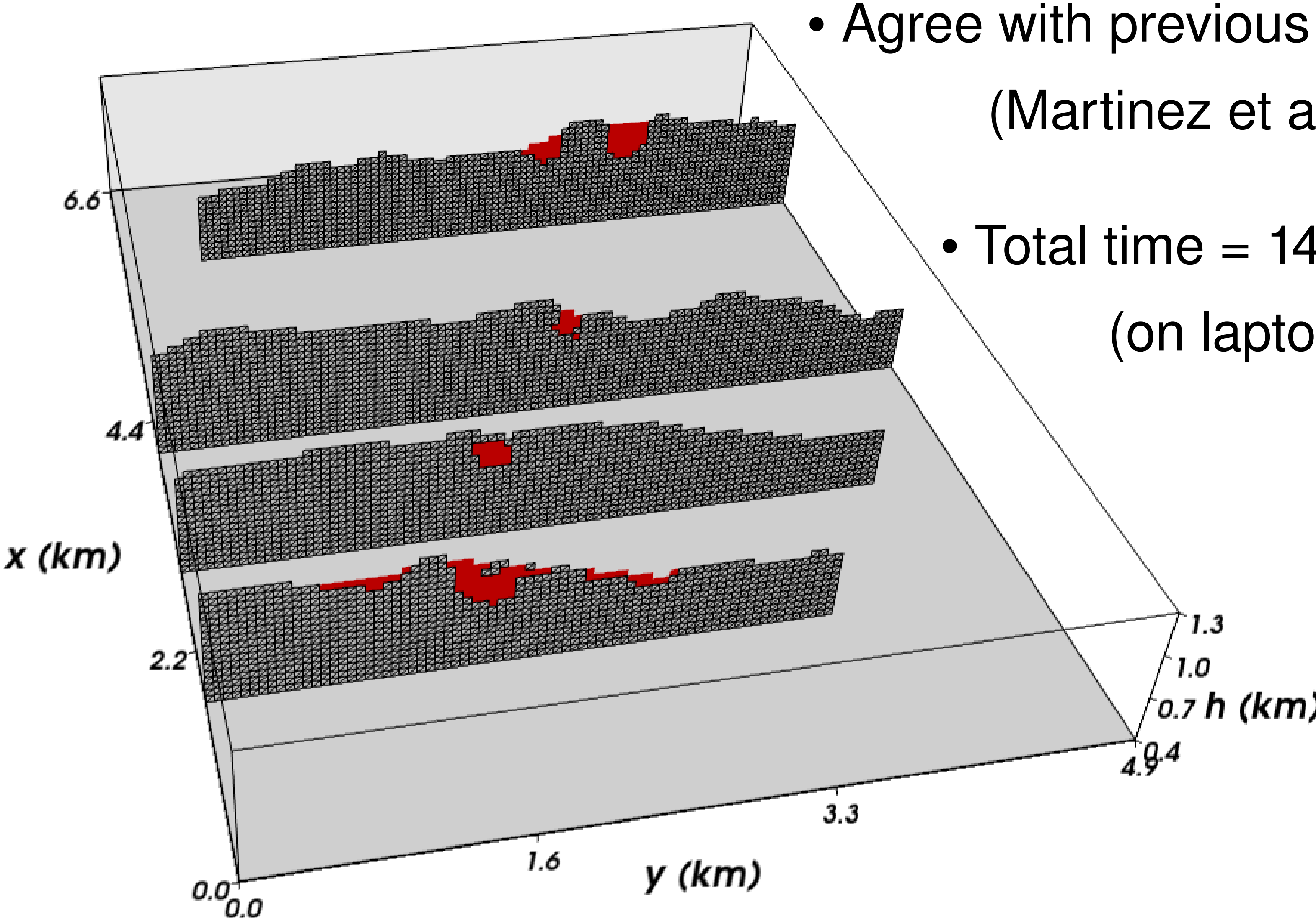


Inversion: • 46 seeds • 13,746 data • 164,892 prisms



• Agree with previous interpretations (Martinez et al., 2010)

Inversion: • 46 seeds • 13,746 data • 164,892 prisms



- Agree with previous interpretations (Martinez et al., 2010)
- Total time = 14 minutes (on laptop)

Conclusions

Conclusions

- New 3D gravity gradient inversion
- Multiple sources
- Interfering gravitational effects
- Non-targeted sources
- No matrix multiplications
- No linear systems
- Lazy evaluation of Jacobian matrix

Conclusions

- Estimates geometry
- Given density contrasts
- Ideal for:
  - Sharp contacts
  - Well-constrained physical properties
    - Ore bodies
    - Intrusive rocks
    - Salt domes

Cronograma

Mestrado

- 2010: Cumprir disciplinas
- 2010-2011: 7 trabalhos em congresso (5 primeiro autor)
- 10/2011: Submeter artigo para *Geophysics*
- 11/2011: Defesa da dissertação de mestrado

Continuação

- Adaptar para gravimetria e magnetometria
- Disponibilizar software Open Source