Inversão gravimétrica do relevo da Moho em coordenadas esféricas

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Observatório Nacional - 2015

O problema geofísico

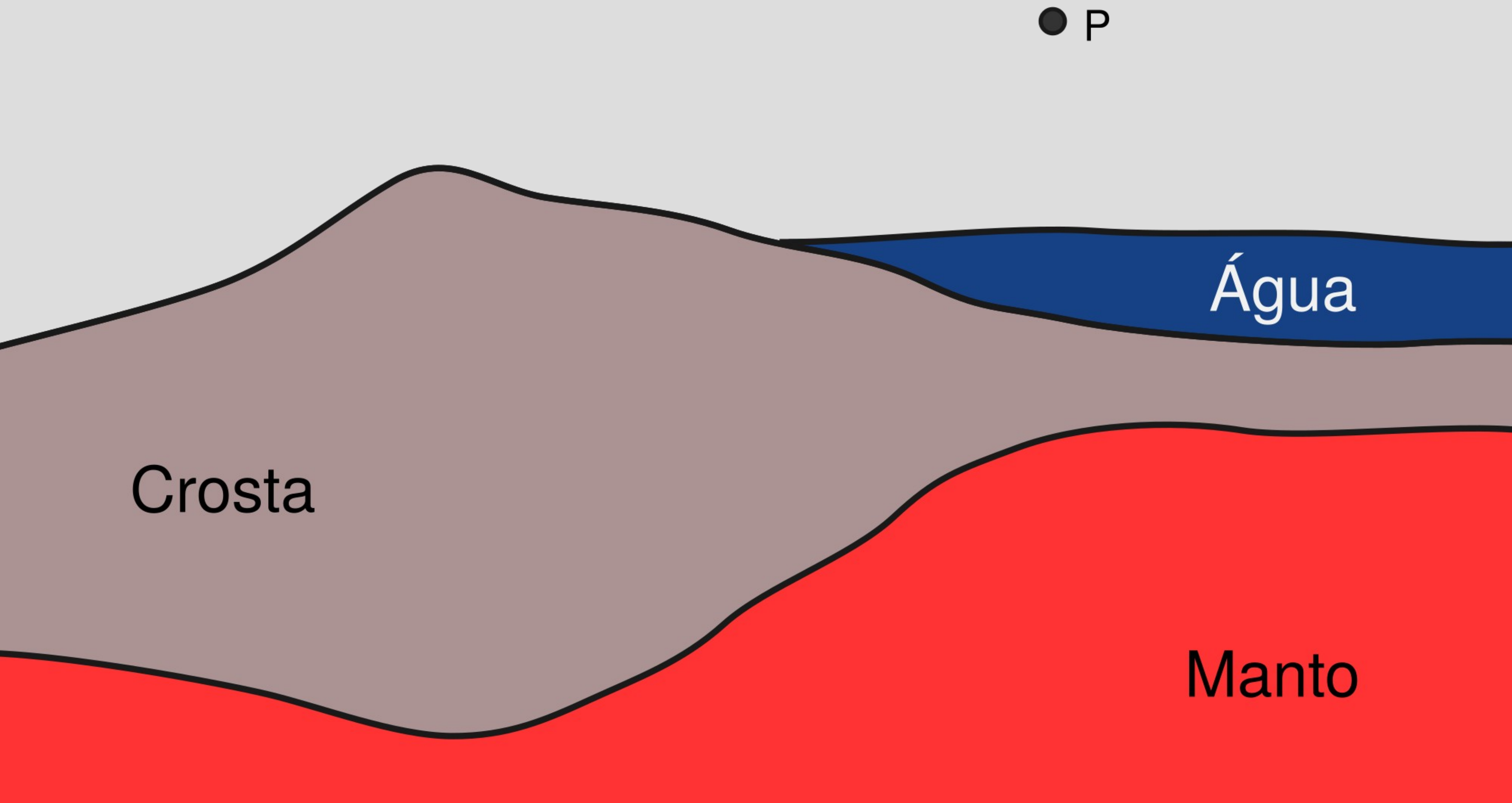
Estimar

um relevo

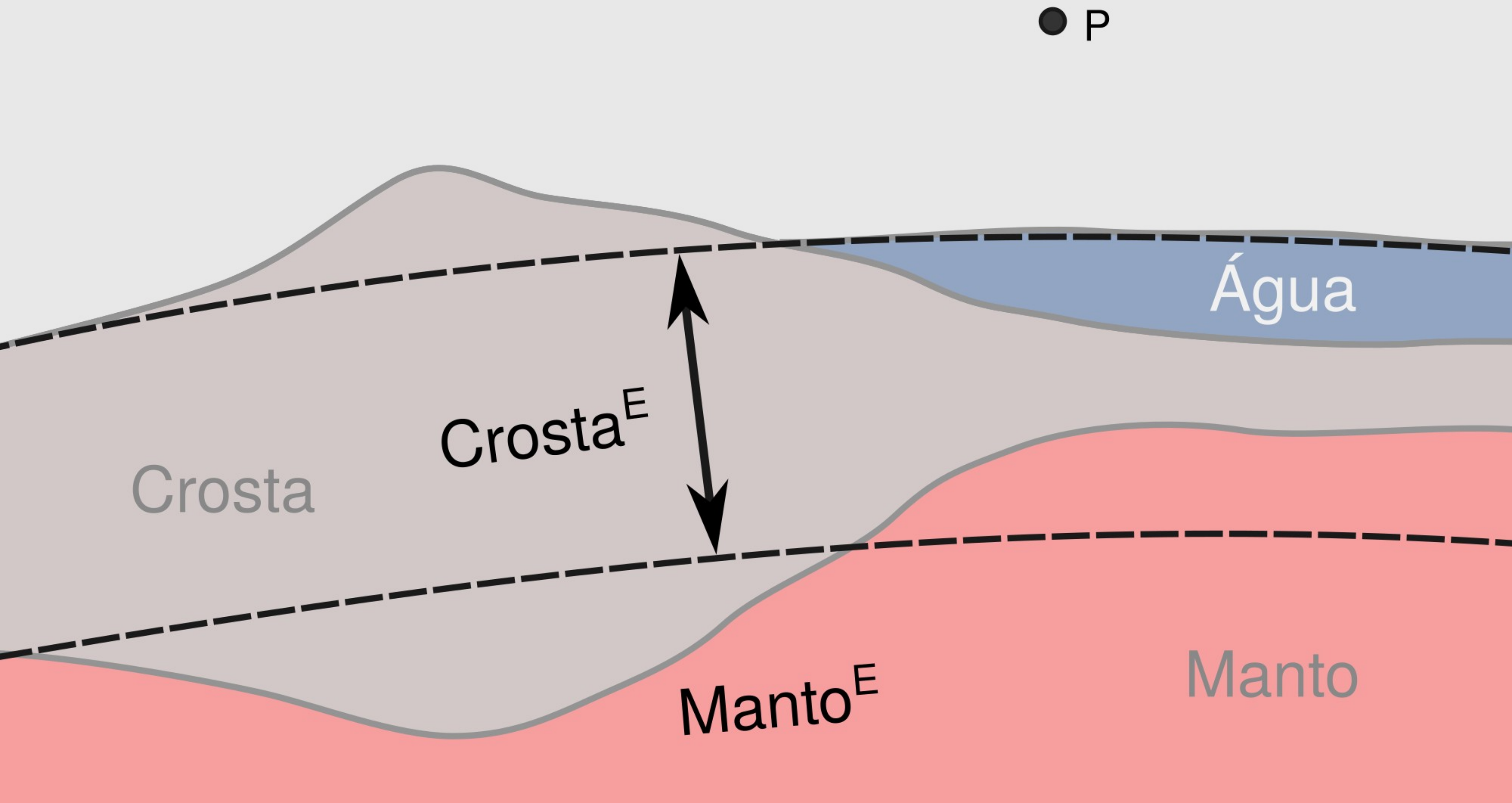
Moho, embasamento

Isolar efeito gravitacional

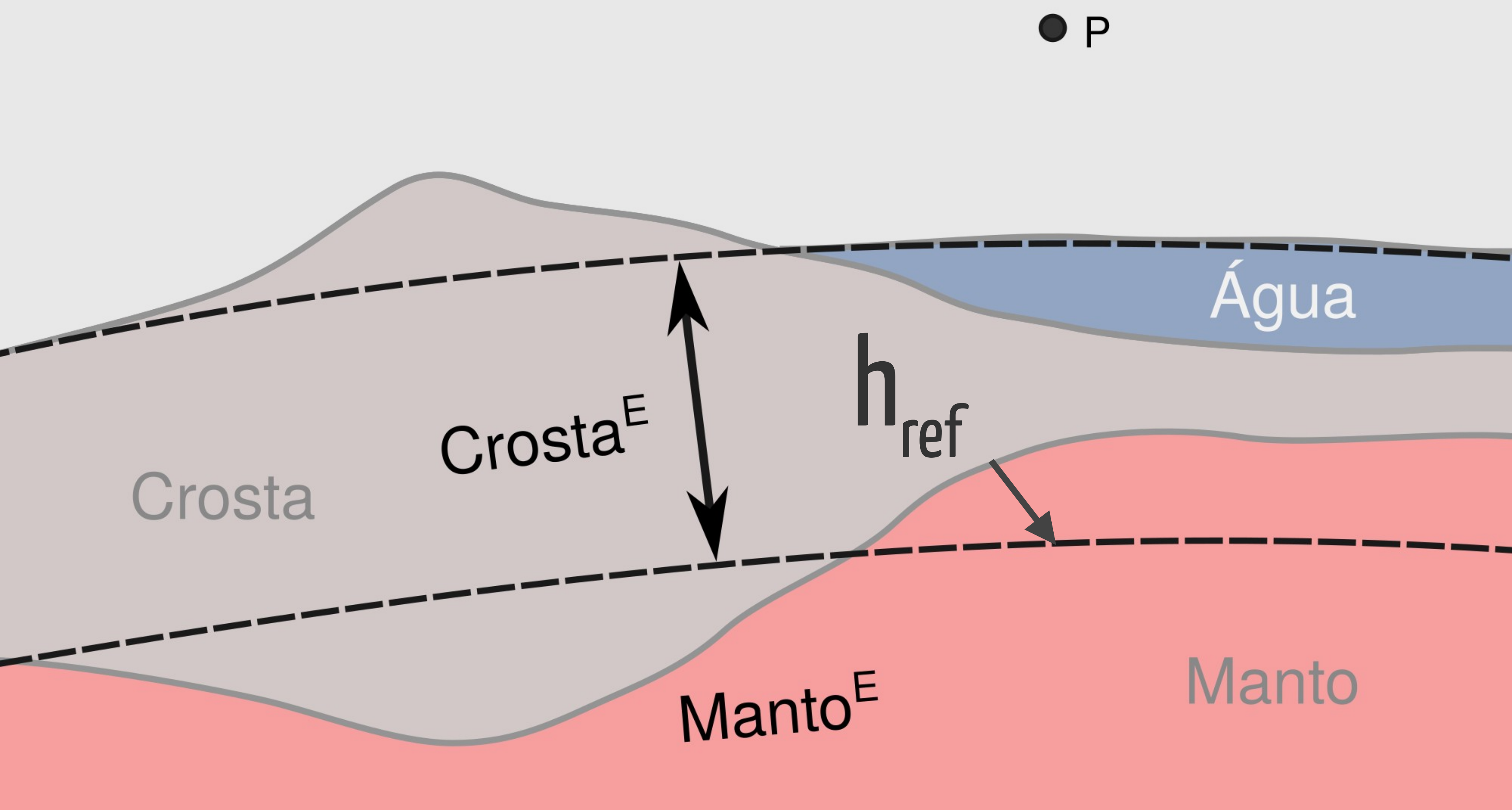
Terra Real



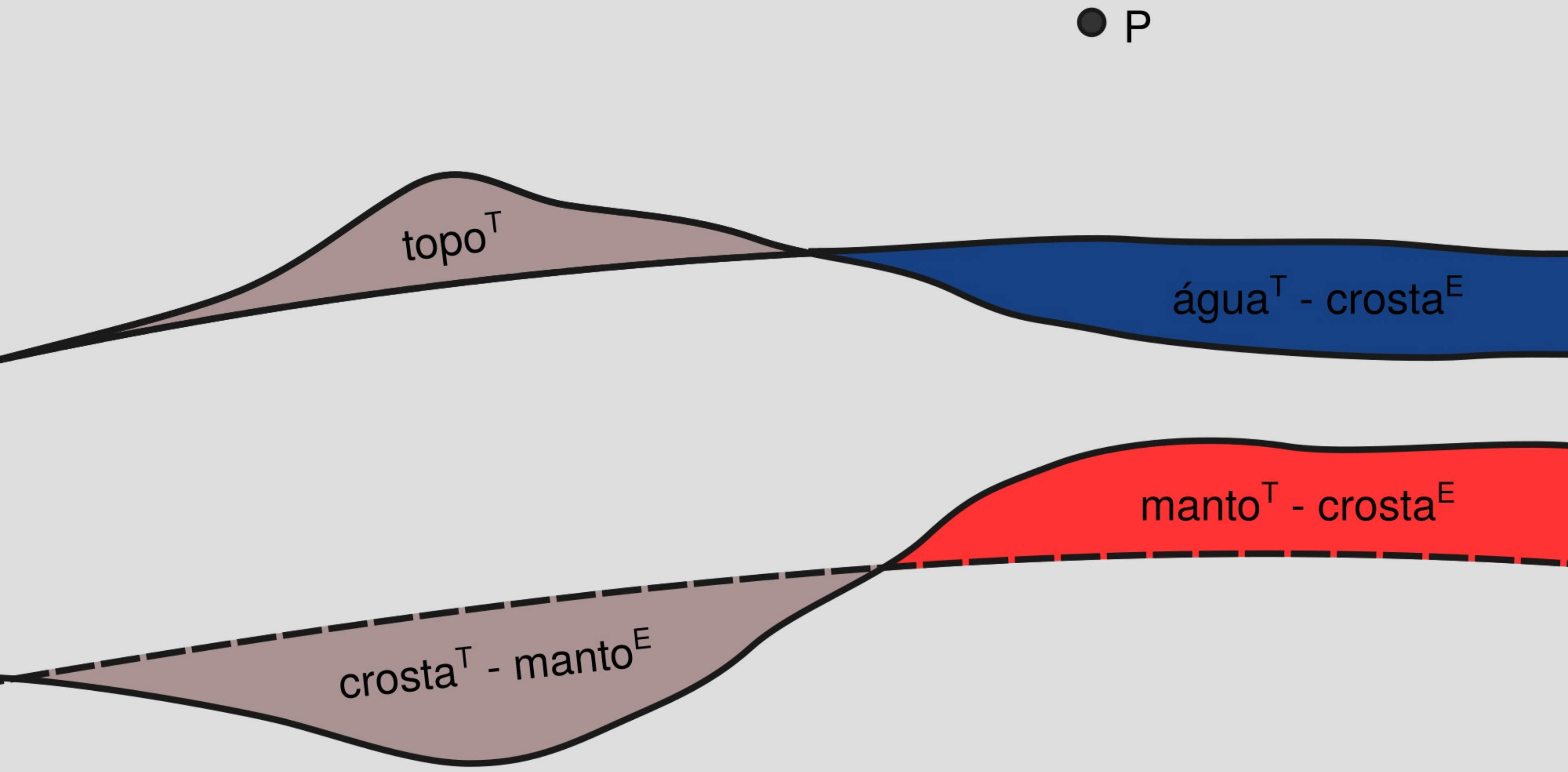
Terra Normal



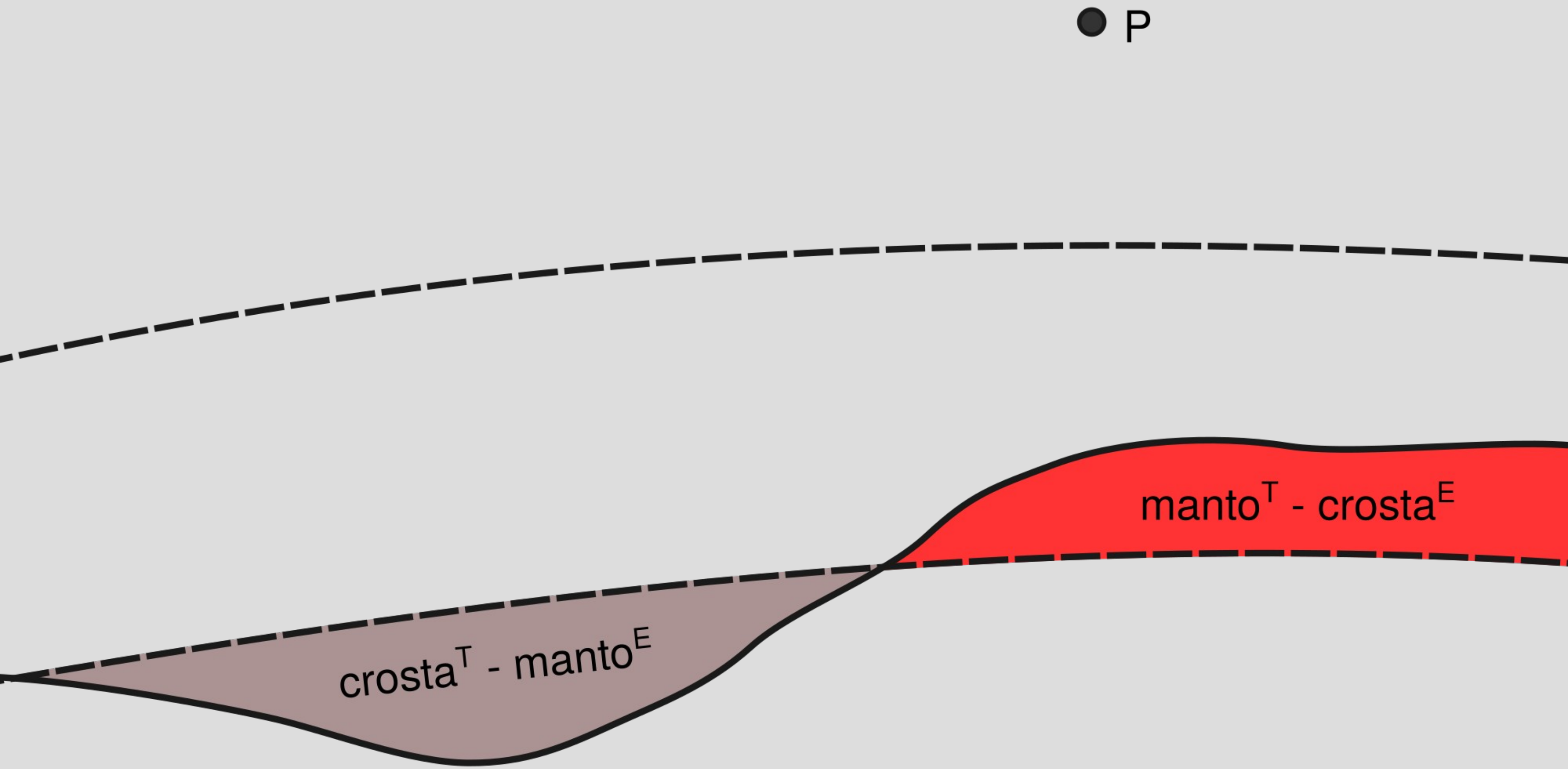
Terra Normal



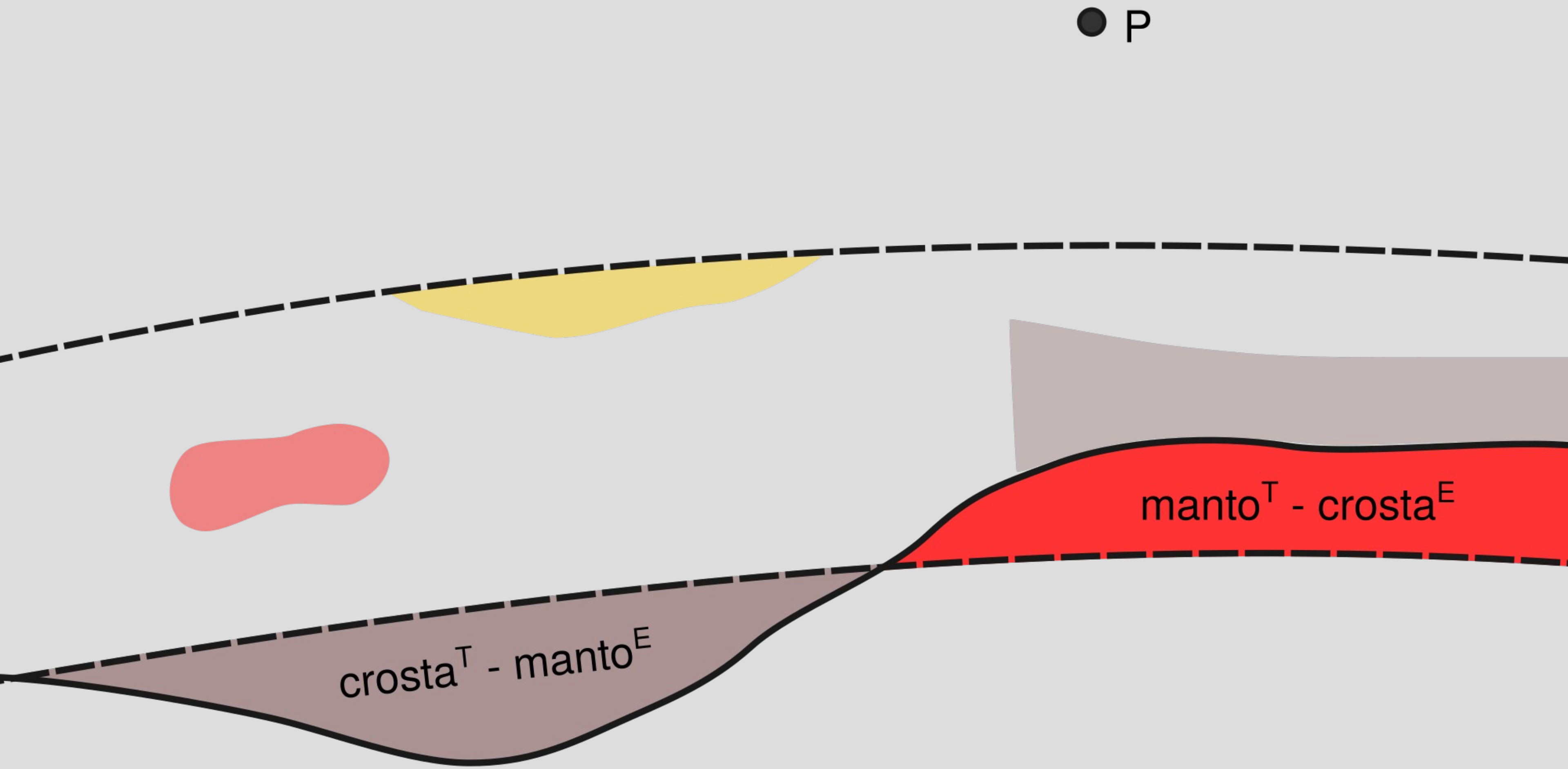
Distúrbio da gravidade



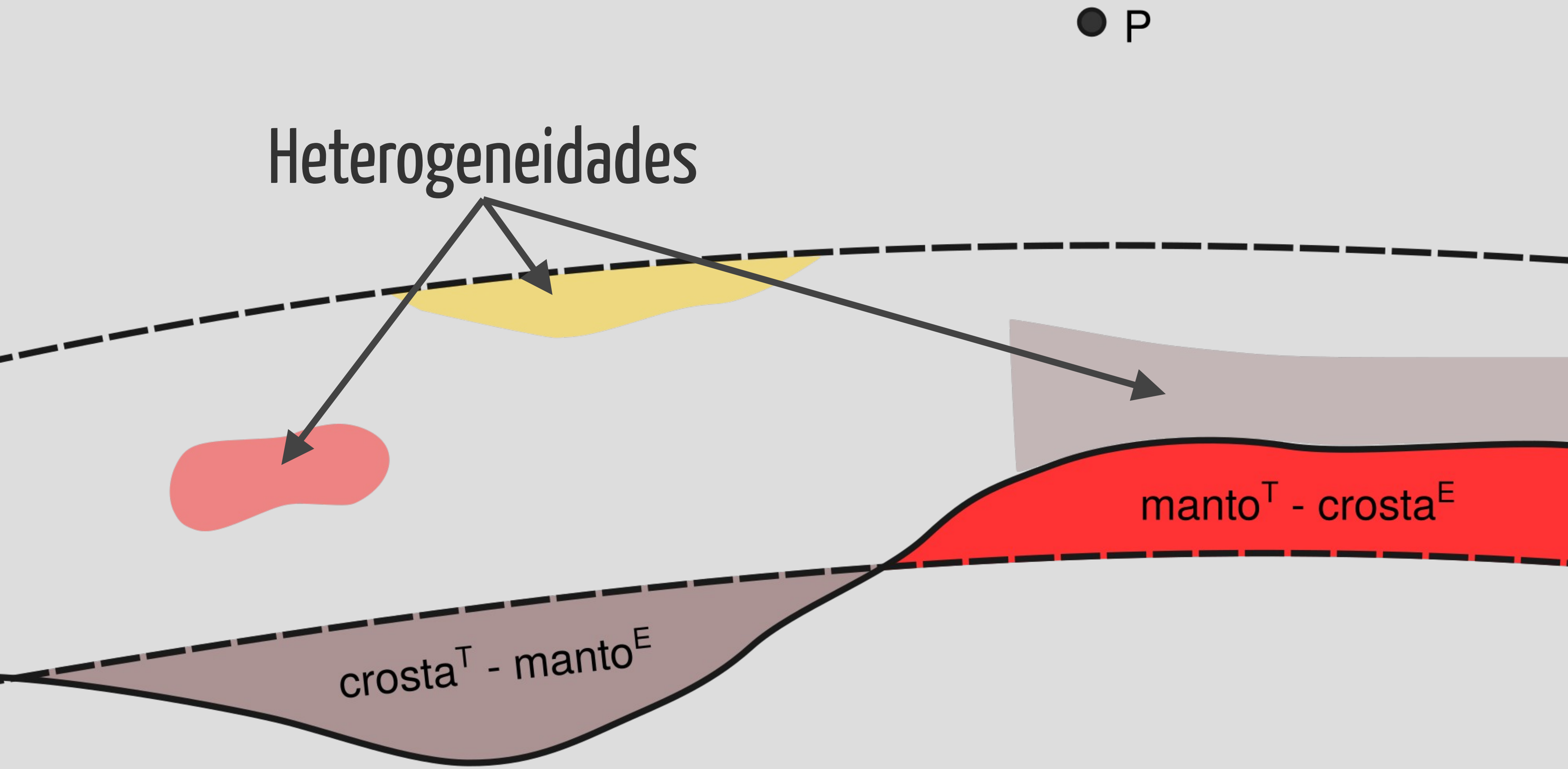
Anomalia Bouguer



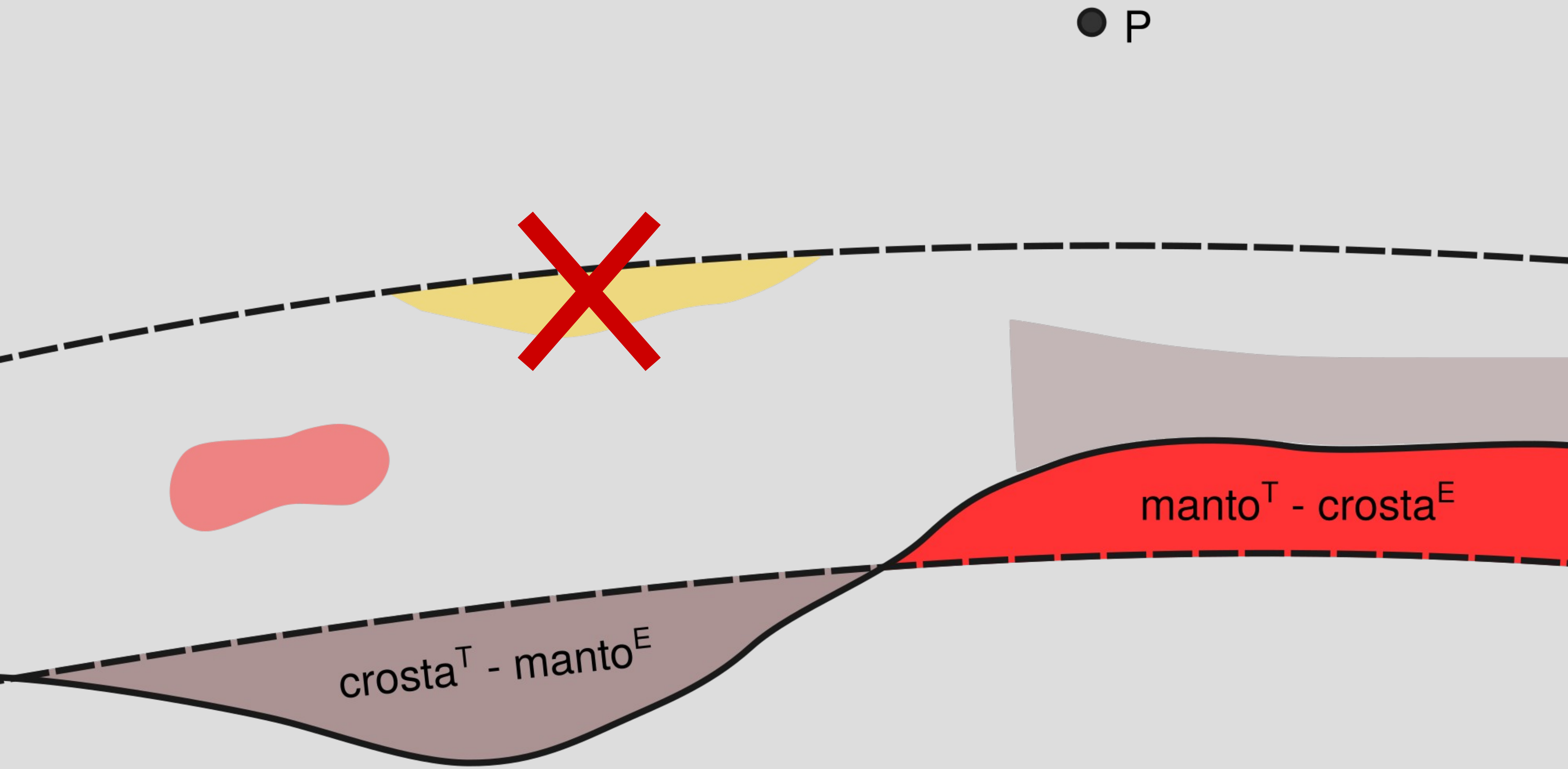
Anomalia Bouguer



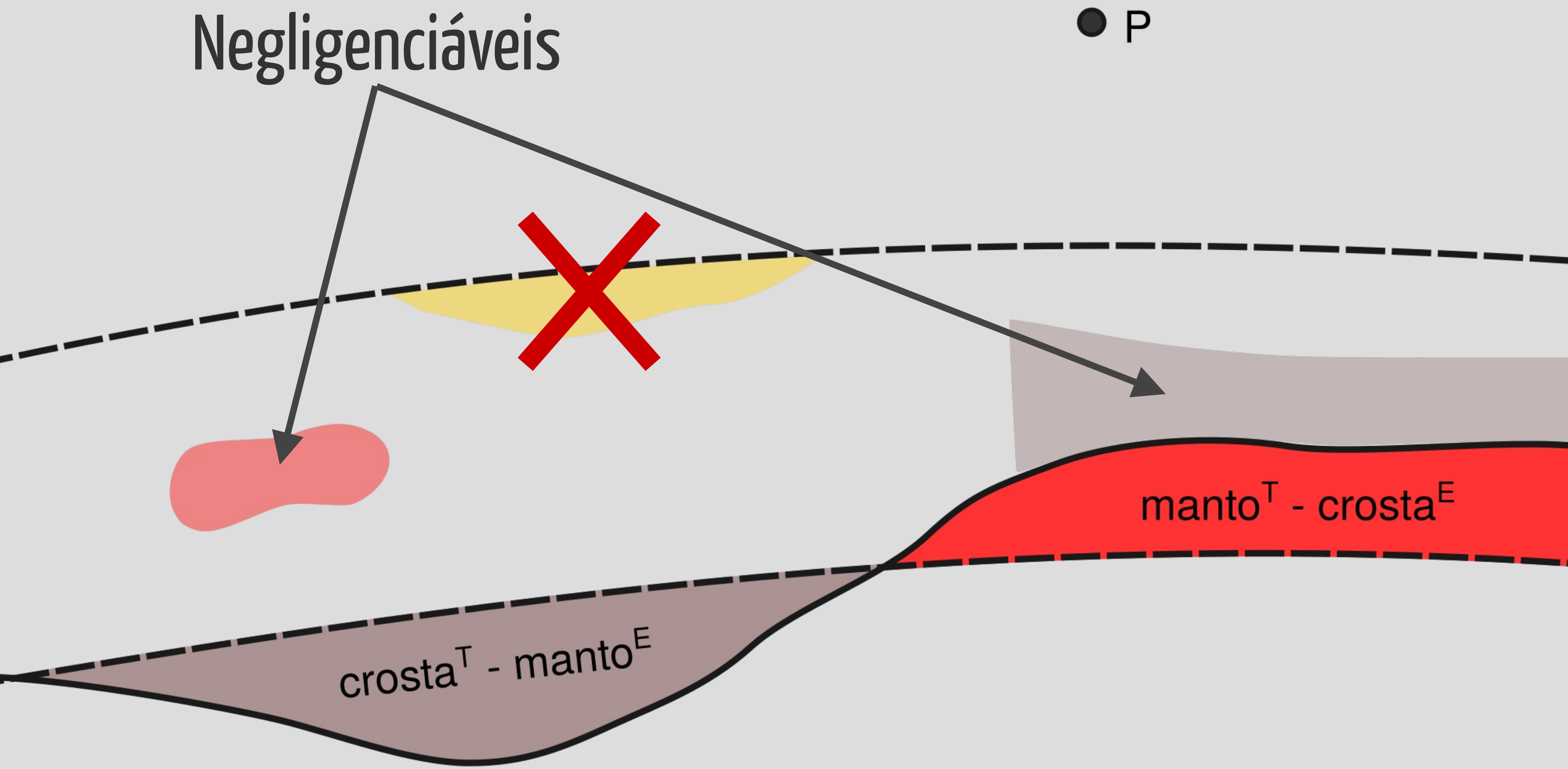
Anomalia Bouguer



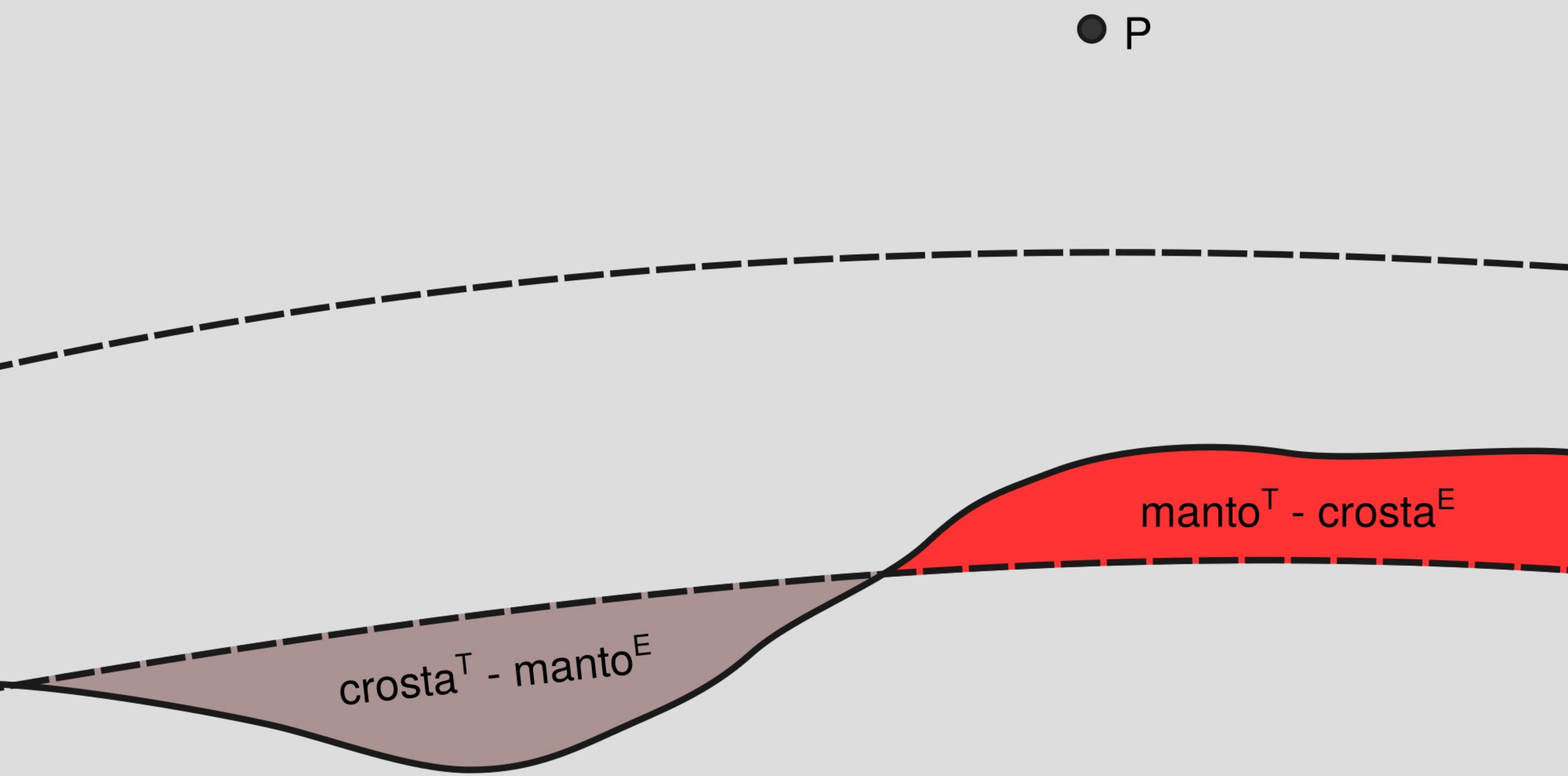
Anomalia Bouguer



Anomalia Bouguer

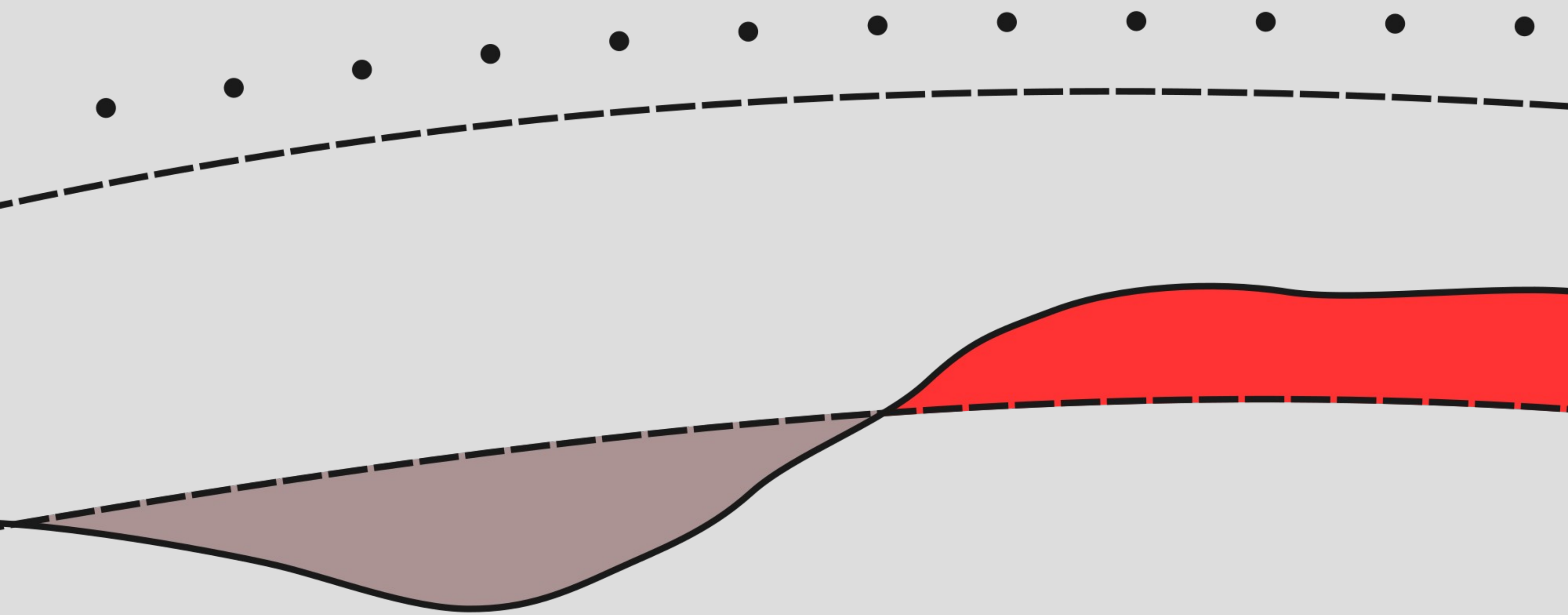


Bouguer ➡ Moho

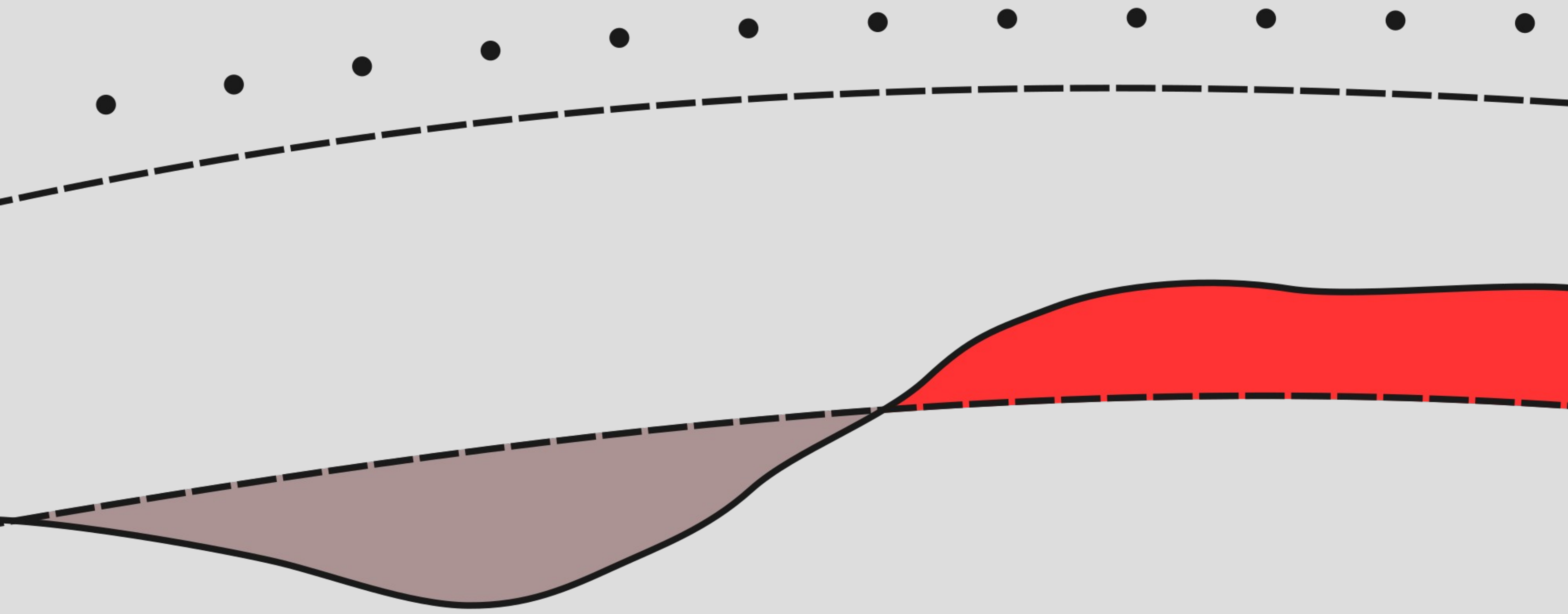


Parametrização

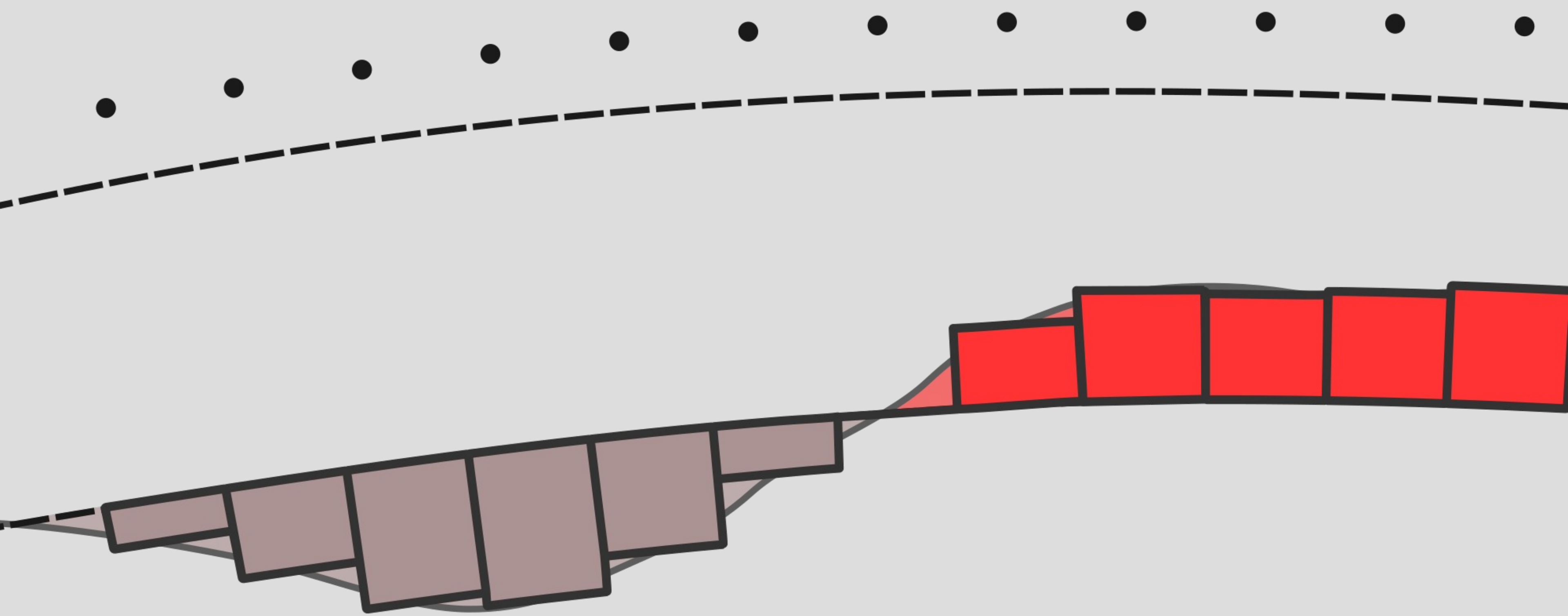
Grid de observações



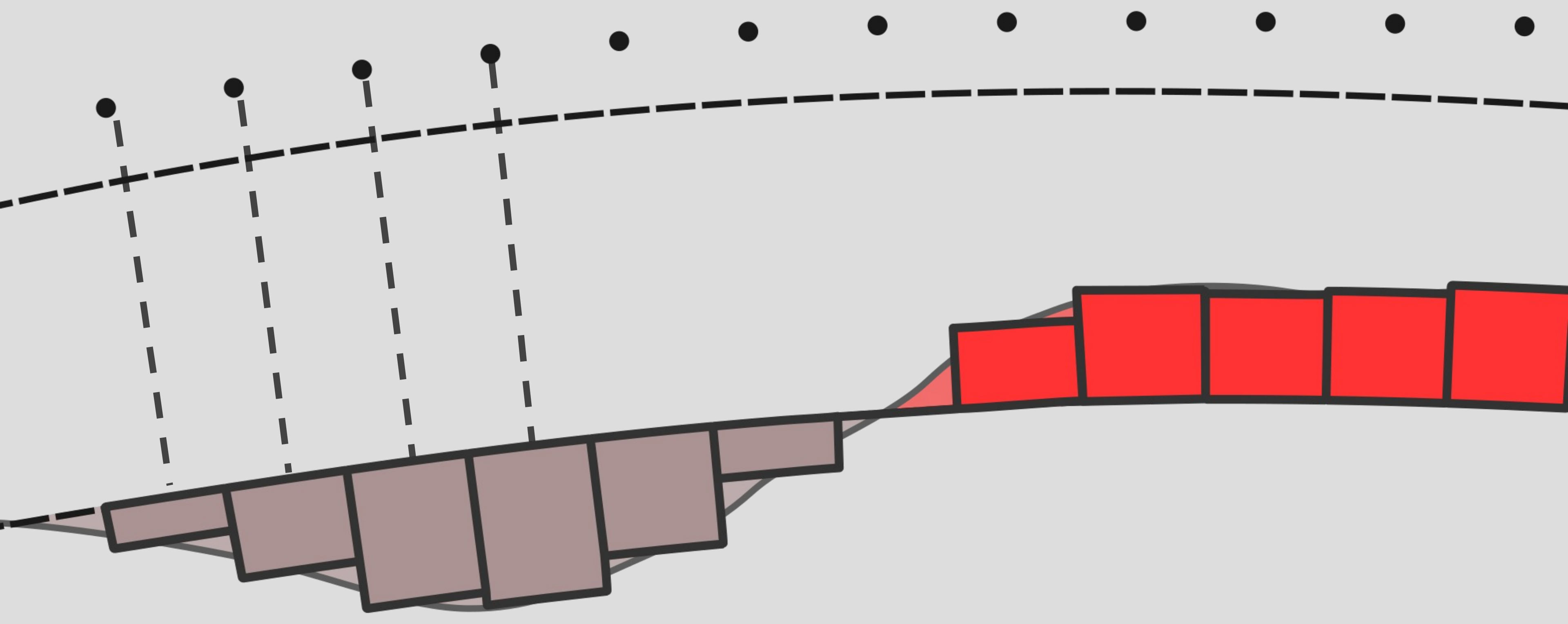
$\overline{d}^o$



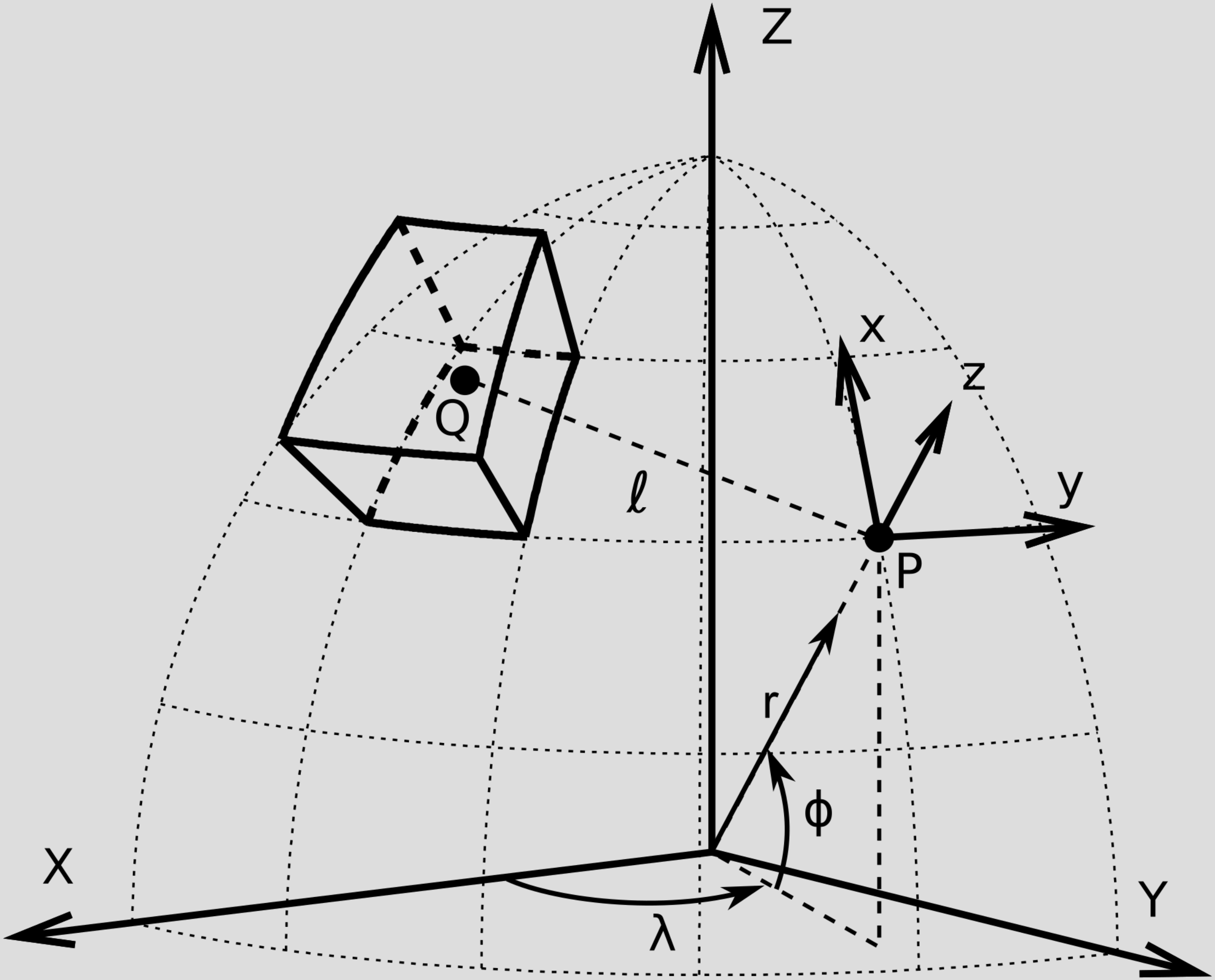
1 tesseroide para cada



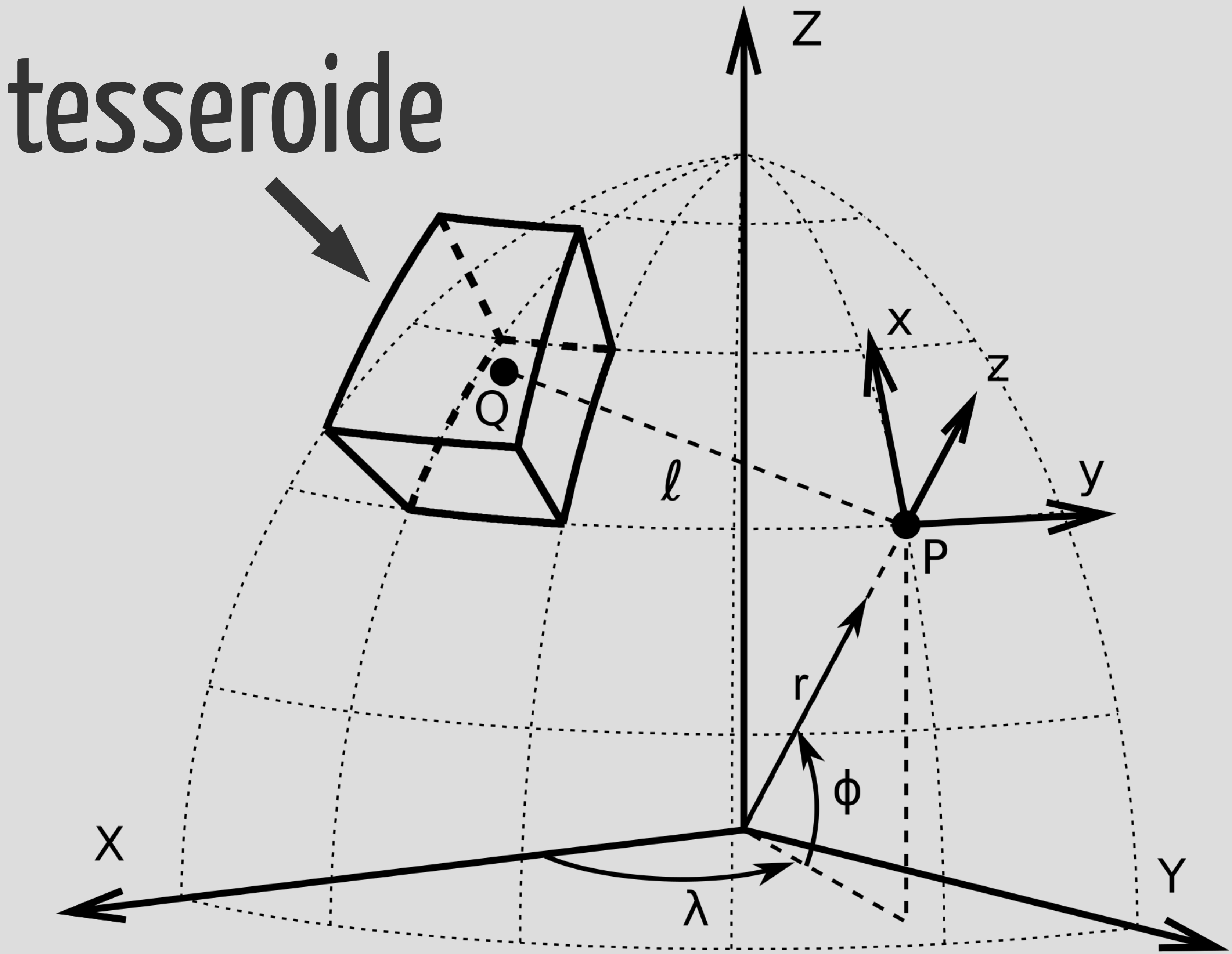
1 tesseroide para cada



(

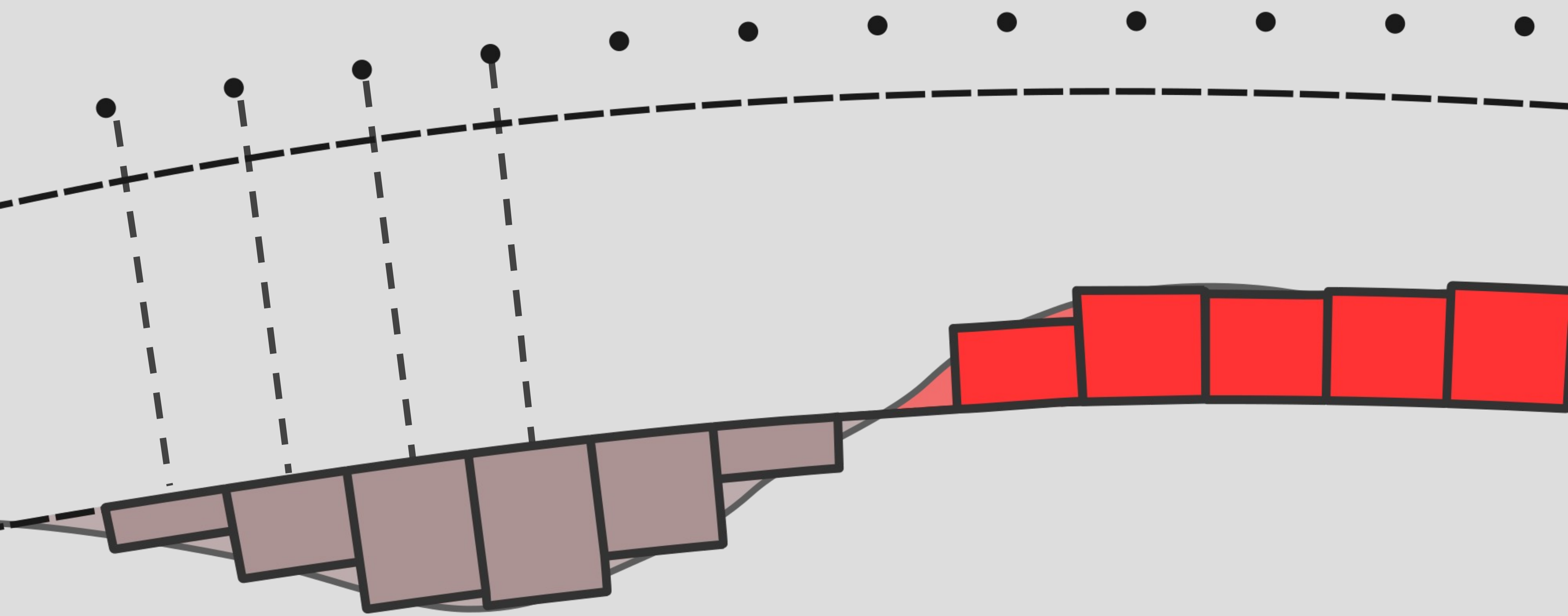


Z
x
z
y
Q
ℓ
P
r
ϕ
X
λ
Y

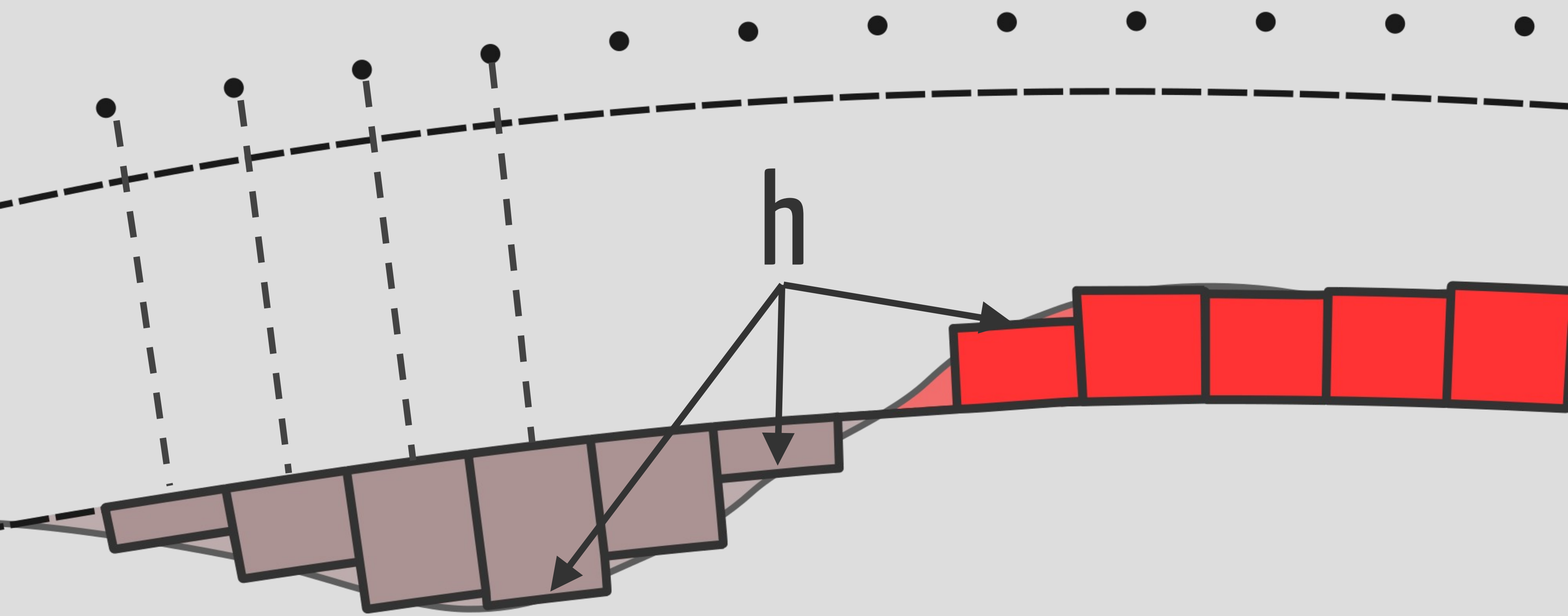

tesseroide
Z
X
Y
x
y
z
Q
P
ℓ
r
ϕ
λ

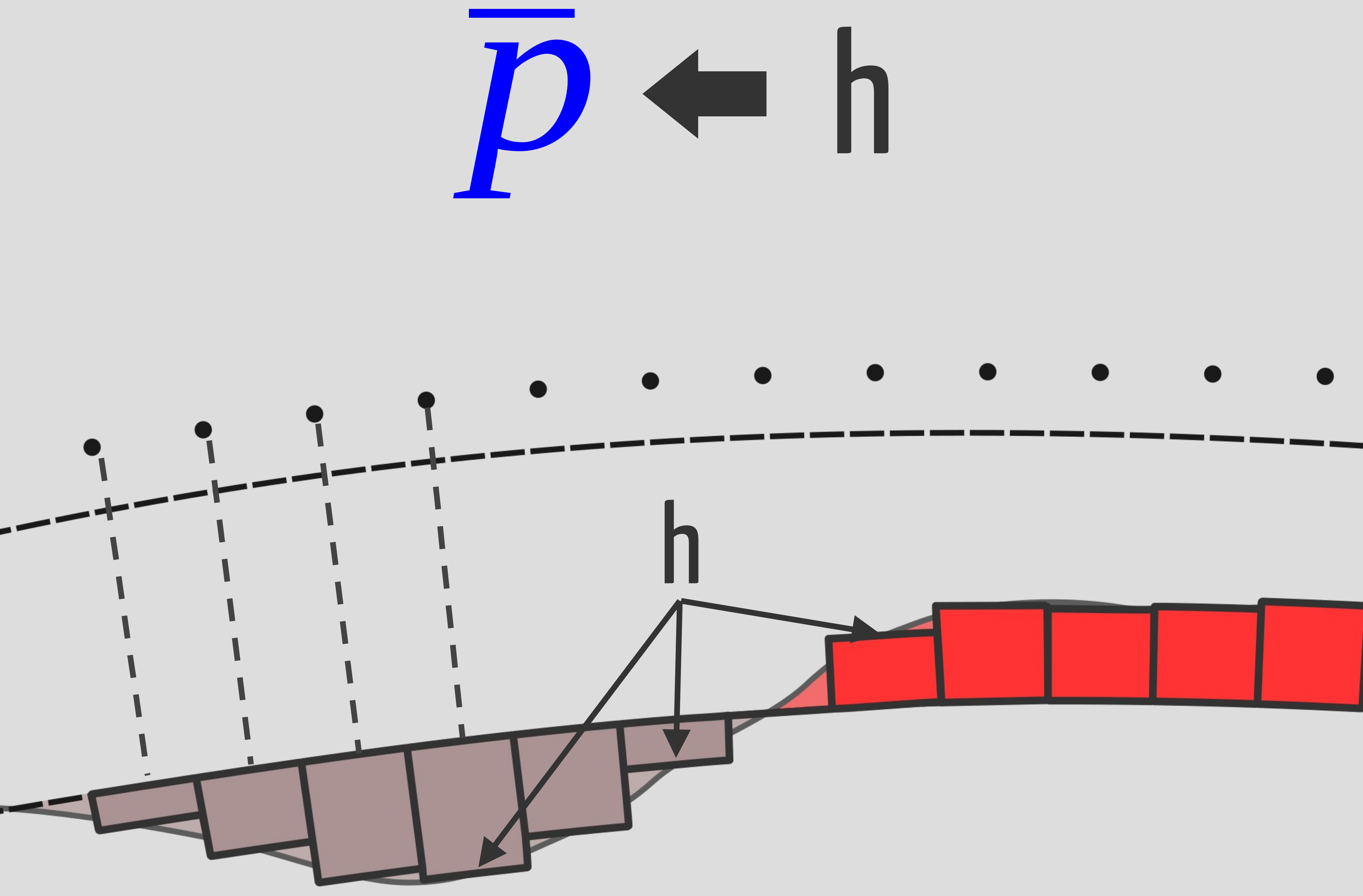
)

1 tesseroide para cada



parâmetros = h





$\bar{p}$
h
h

Estimar $\overline{p}$

a partir de $\overline{d^{o}}$

Inversão não-linear

Resíduos

$$\bar{r} = \bar{d}^{o} - \bar{d}(\bar{p})$$

Minimizar

$$\phi\left(\bar{p}\right) = \left\| \bar{r} \right\|_2^2$$

Gauss-Newton

$$\Delta \bar{p} = \left(\bar{\bar{A}}^T \bar{\bar{A}} \right)^{-1} \bar{\bar{A}}^T \left[\bar{d}^o - \bar{d} \left(\bar{p}^k \right) \right]$$

Gauss-Newton

$$\Delta \bar{p} = (\bar{\bar{A}}^T \bar{\bar{A}})^{-1} \bar{\bar{A}}^T [\bar{d}^o - \bar{d}(\bar{p}^k)]$$

$$\boxed{A_{ij} = \frac{\partial d_i}{\partial p_j} \quad \underline{\text{Jacobiana}}}$$

Gauss-Newton

$$\Delta \bar{p} = \left[\left(A^T A \right)^{-1} A^T \left(\bar{d} - \bar{d}\left(\bar{p}^k \right) \right) \right]$$

mal posto

$$\boxed{A_{ij} = \frac{\partial d_i}{\partial p_j} \quad \underline{\text{Jacobiana}}}$$

Regularização

Suavidade

$$\theta(\bar{p}) = \left\| \bar{\bar{R}}\, \bar{p} \right\|_2^2$$

Função objetivo

$$\Gamma\left(\bar{p}\right) = \phi + \mu\,\theta$$

Função objetivo

ajuste

$$\Gamma(\bar{p}) = \phi + \mu\theta$$

Função objetivo

ajuste

$$\Gamma(\bar{p}) = \phi + \mu\,\theta$$

regularização

Função objetivo

$$\Gamma(\bar{p}) = \overset{\text{ajuste}}{\phi} + \overset{\text{balanço}}{\mu}\,\underset{\text{regularização}}{\theta}$$

Gauss-Newton

$$\Delta\bar{p} = \left(\bar{\bar{A}}^T \bar{\bar{A}} + \mu \bar{\bar{R}}^T \bar{\bar{R}}\right)^{-1} \left[\bar{\bar{A}}^T \bar{r}^k - \mu \bar{\bar{R}}^T \bar{\bar{R}} \bar{p}^k \right]$$

Gauss-Newton

$$\Delta \bar{p} = (\bar{A}^T \bar{A} + \mu \bar{R}^T \bar{R})^{-1} [$$

custoso

(computacionalmente)

1. Construir $\bar{\bar{A}}$

1. Construir $\bar{\bar{A}}$

2. Sistema linear

1. Construir $\bar{\bar{A}}$

2. Sistema linear

3. Calcular $\bar{r}$

Bott

(1960)

$$\Delta \bar{p} = \frac{\bar{r}}{2 \pi G \Delta \rho}$$

$$\Delta\bar{\ }p = \frac{\bar{r}}{\boxed{2\pi G \Delta\rho}}$$

$$\frac{\partial \, \textit{platô de Bouguer}}{\partial h}$$

1. Construir $\bar{\bar{A}}$

2. Sistema linear

3. Calcular $\bar{r}$

1. ~~Construir~~ $\bar{\bar{A}}$

2. ~~Sistema~~ linear

3. Calcular $\bar{r}$

rápido

pouca memória

converge

instável

regularização

“empírico”

Silva et al.

(2014)

Bott

Bott

caso particular

Gauss-Newton

$$\Delta\bar{p} = \frac{\bar{r}}{2\pi G \Delta\rho}$$

caso particular

$$\Delta\bar{p} = \left(\bar{\bar{A}}^{T}\bar{\bar{A}}\right)^{-1}\bar{\bar{A}}^{T}\bar{r}$$

$$\overline{\Delta p} = \frac{\bar{r}}{\boxed{2\pi G \Delta\rho}}$$

caso particular

$$\overline{\Delta p} = \boxed{\left(\bar{\bar{A}}^{T}\,\bar{\bar{A}}\right)^{-1}\bar{\bar{A}}^{T}}\,\bar{r}$$

$$\Delta \bar{p} = \frac{\bar{r}}{\boxed{2\pi G \Delta\rho}}$$

$$\boxed{\begin{array}{l} A_{ii} = 2\pi G \Delta\rho \\ A_{ij} = 0 \ \ para\ i \neq j \end{array}}$$

caso particular

$$\Delta \bar{p} = \boxed{\left(\bar{\bar{A}}^{T} \bar{\bar{A}}\right)^{-1} \bar{\bar{A}}^{T}} \bar{r}$$

$$\bar{\bar{A}}$$

$$\Downarrow$$

$$\begin{pmatrix} 2\pi G\Delta\rho & 0 & \cdots & 0 \\ 0 & 2\pi G\Delta\rho & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 2\pi G\Delta\rho \end{pmatrix}$$

Generalizar

$$\Delta \bar{p} = \frac{\bar{r}}{b^k}$$

Generalizar

$$\Delta \bar{p} = \frac{\bar{r}}{b^k}$$

Regular passo (~ Marquardt)

sem matrizes
regular passo
média móvel

Nesse

trabalho

Gauss-Newton

$$\Delta \bar{p} = (\bar{\bar{A}}^T \bar{\bar{A}})^{-1} \bar{\bar{A}}^T [\bar{d}^o - \bar{d}(\bar{p}^k)]$$

Gauss-Newton

$$\Delta \bar{p} = \left(\bar{\bar{A}}^T \bar{\bar{A}}\right)^{-1} \bar{\bar{A}}^T \left[\bar{d}^o - \bar{d}\left(\bar{p}^k\right)\right]$$

tesseroides

Gauss-Newton

$$\Delta\bar{p} = (\bar{\bar{A}}^T \bar{\bar{A}})^{-1} \bar{\bar{A}}^T [\bar{d}^o - \bar{d}(\bar{p}^k)]$$

$$A_{ii} = 2\pi G \Delta\rho_i$$

tesseroides

Gauss-Newton

$$\Delta\bar{p} = \left(\bar{\bar{A}}^T \bar{\bar{A}}\right)^{-1} \bar{\bar{A}}^T \left[\bar{d}^o - \bar{d}\left(\bar{p}^k\right)\right]$$

$$A_{ii} = 2\pi G \Delta\rho_i$$

tesseroides

por tesseroide

Por que?

$$A_{ii} = 2\pi G \Delta\rho_i$$

tesseroide ⟶ ∞

tesseroide $\longrightarrow$ ∞

$$g \longrightarrow 2\pi G \Delta\rho h$$

tesseroide $\longrightarrow$ ∞

$$g \longrightarrow 2\pi G \Delta\rho h$$

$$A_{ii} = \frac{\partial g}{\partial h} \longrightarrow 2\pi G \Delta\rho$$

tesseroide

de 0.5°

tesseroide

de 0.5°

≈ 55 km

Regularização

suavidade

$$\Delta\bar{p} = \left(\bar{\bar{A}}^T \bar{\bar{A}} + \mu \bar{\bar{R}}^T \bar{\bar{R}}\right)^{-1} \left[\bar{\bar{A}}^T \bar{r}^k - \mu \bar{\bar{R}}^T \bar{\bar{R}} \bar{p}^k \right]$$

esparsas

$$\Delta\bar{p} = \left(\boxed{\bar{\bar{A}}^T \bar{\bar{A}}} + \mu \boxed{\bar{\bar{R}}^T \bar{\bar{R}}}\right)^{-1} \left[\bar{\bar{A}}^T \bar{r}^k - \mu \bar{\bar{R}}^T \bar{\bar{R}} \bar{p}^k\right]$$

esparsas

$$\Delta\bar{p} = \left(\boxed{\bar{\bar{A}}^T \bar{\bar{A}}} + \mu \boxed{\bar{\bar{R}}^T \bar{\bar{R}}} \right)^{-1} \left[\bar{\bar{A}}^T \bar{r}^k - \mu \bar{\bar{R}}^T \bar{\bar{R}} \bar{p}^k \right]$$

~99.9% tempo de computação

1. Construir $\bar{\bar{A}}$

2. Sistema linear

3. Calcular $\bar{r}$

1. ~~Construir~~ $\bar{\bar{A}}$

Bott

2. Sistema linear

3. Calcular $\bar{r}$

1. Construir $\bar{\bar{A}}$

Bott

2. Sistema linear

matrizes esparsas

3. Calcular $\bar{r}$

rápido

pouca memória

converge

instável

regularização

“empírico”

~~instável~~

~~regularização~~

~~“empírico”~~

Implementação

python™

fatiando a terra

modeling and inversion in geophysics

matrizes esparsas

estimar

hyperparâmetros

μ

$\Delta\rho$

h_{ref}

μ regularização

$\Delta\rho$

h_{ref}

μ regularização

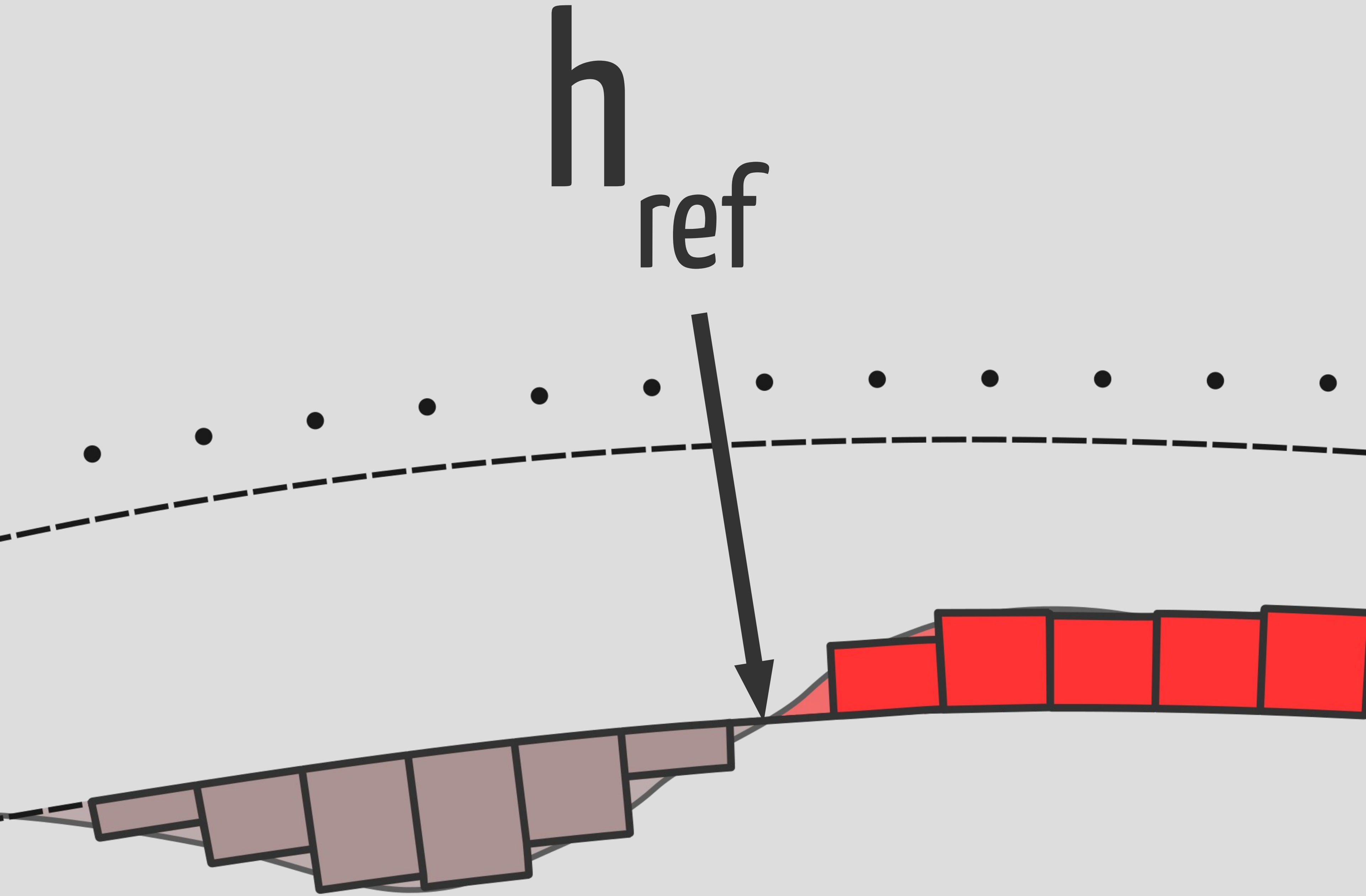
contraste densidade $\Delta\rho$

h_{ref}

μ regularização

contraste densidade $\Delta\rho$

Moho Terra Normal h_{ref}


h_{ref}

validação cruzada

validação | cruzada

μ

validação | cruzada

μ | $\Delta \rho$

h_{ref}

validação

μ

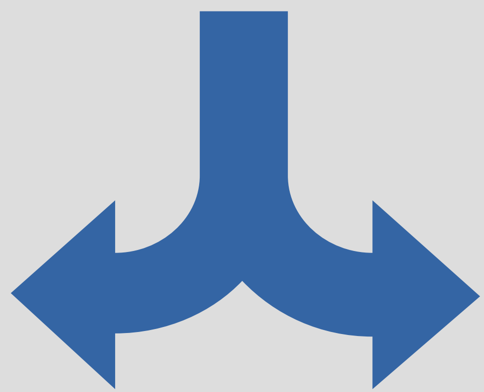
cruzada

$\Delta\rho$

h_{ref}

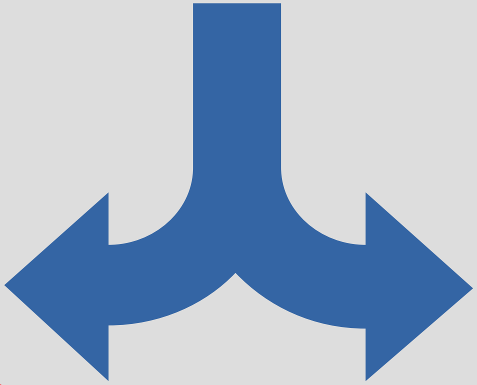
Separar os dados

$$\bar{d}^{o}$$



Separar os dados

$$\bar{d}^{o}$$



$$\bar{d}^{o}_{inv}$$

Separar os dados

$$\bar{d}^o$$

$$\bar{d}^o_{inv} \qquad \bar{d}^o_{test}$$

para μ_i em $[\mu_{1,} \ldots, \mu_m]$:

para μ_i em $[\mu_{1,}, \ldots, \mu_m]$:

inversão: $\bar{d}^o_{inv} \rightarrow \hat{\bar{p}}_i$

para μ_i em $[\mu_{1}, \ldots, \mu_{m}]$:

inversão: $\bar{d}^{o}_{inv} \longrightarrow \hat{\bar{p}}_i$

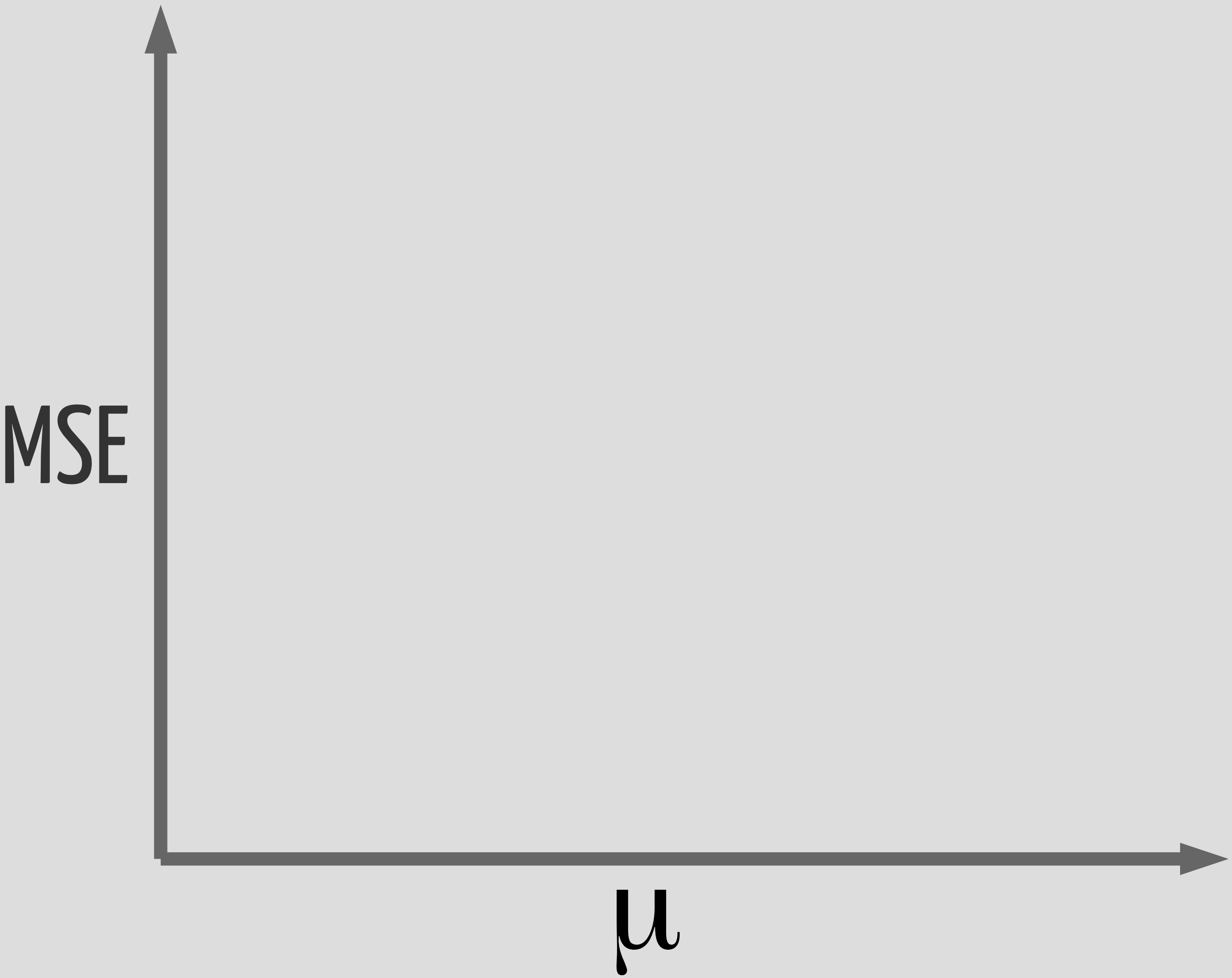
$\hat{\bar{p}}_i \longrightarrow$ prever $\bar{d}_{test}$

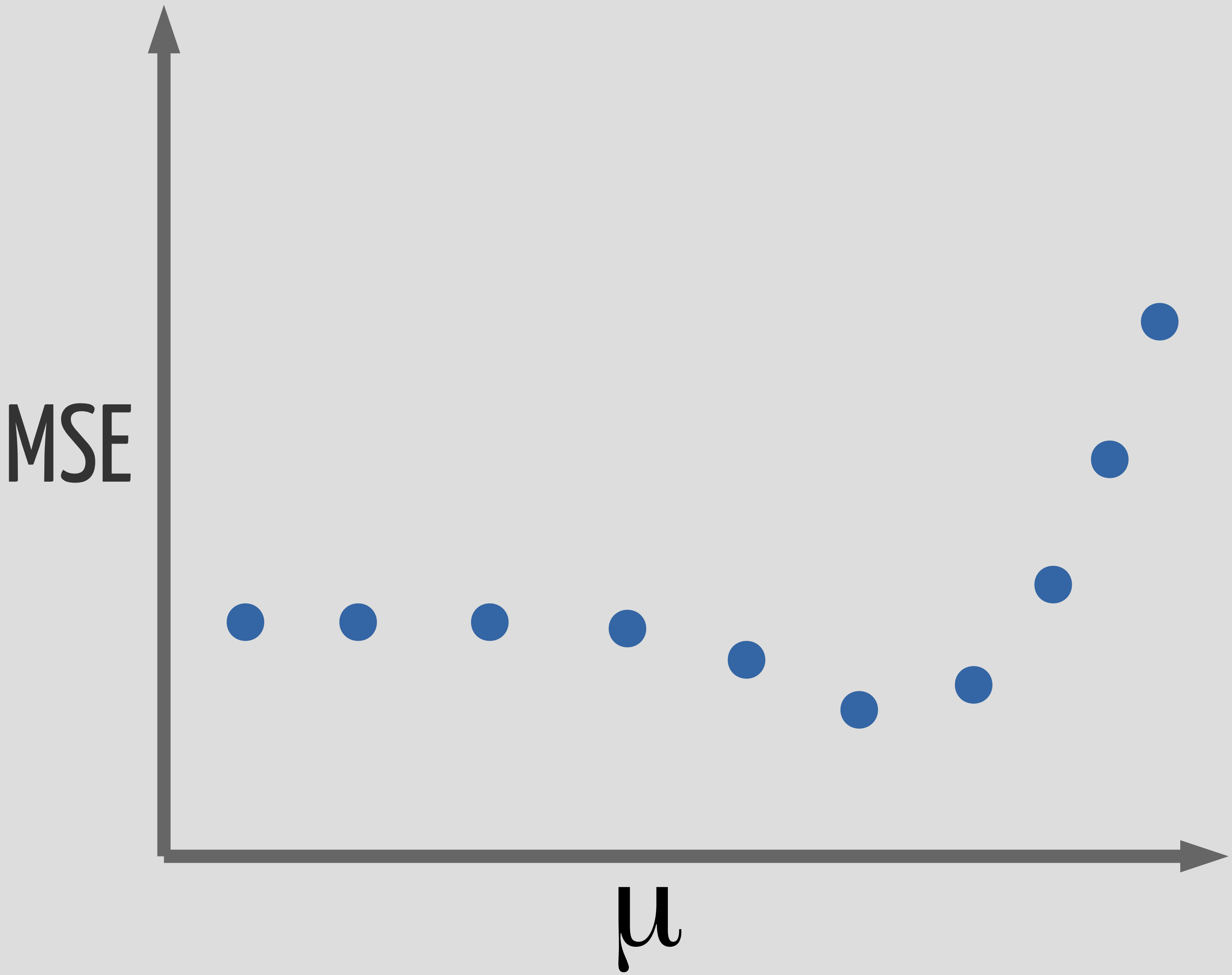
para μ_i em $[\mu_1, \ldots, \mu_m]$:

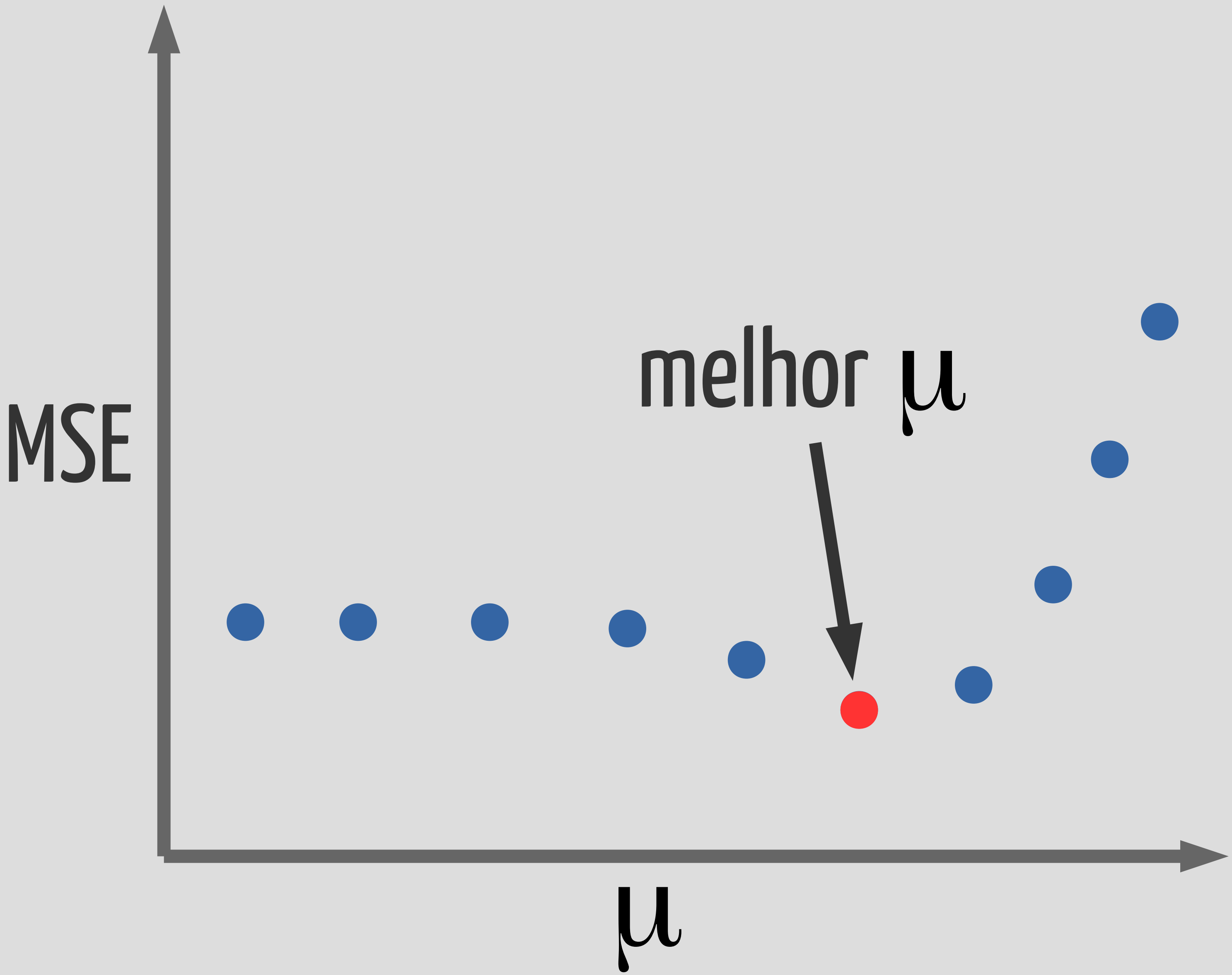
inversão: $\bar{d}^o_{inv} \rightarrow \hat{\bar{p}}_i$

$\hat{\bar{p}}_i \rightarrow$ prever $\bar{d}_{test}$

$$MSE = \frac{\|\bar{d}^o_{test} - \bar{d}_{test}\|^2}{N_{test}}$$


MSE
μ


MSE
μ

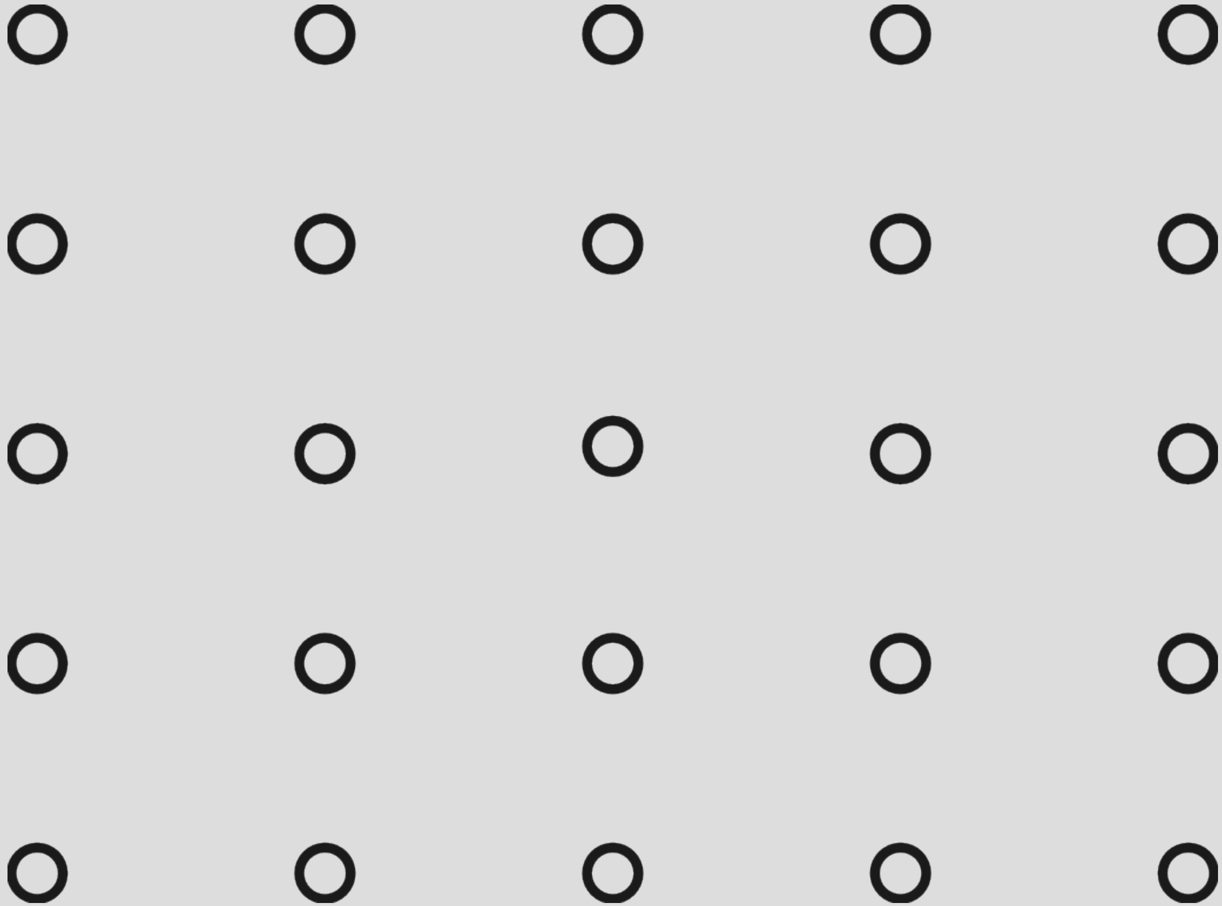

MSE
melhor μ
μ

Separar os dados

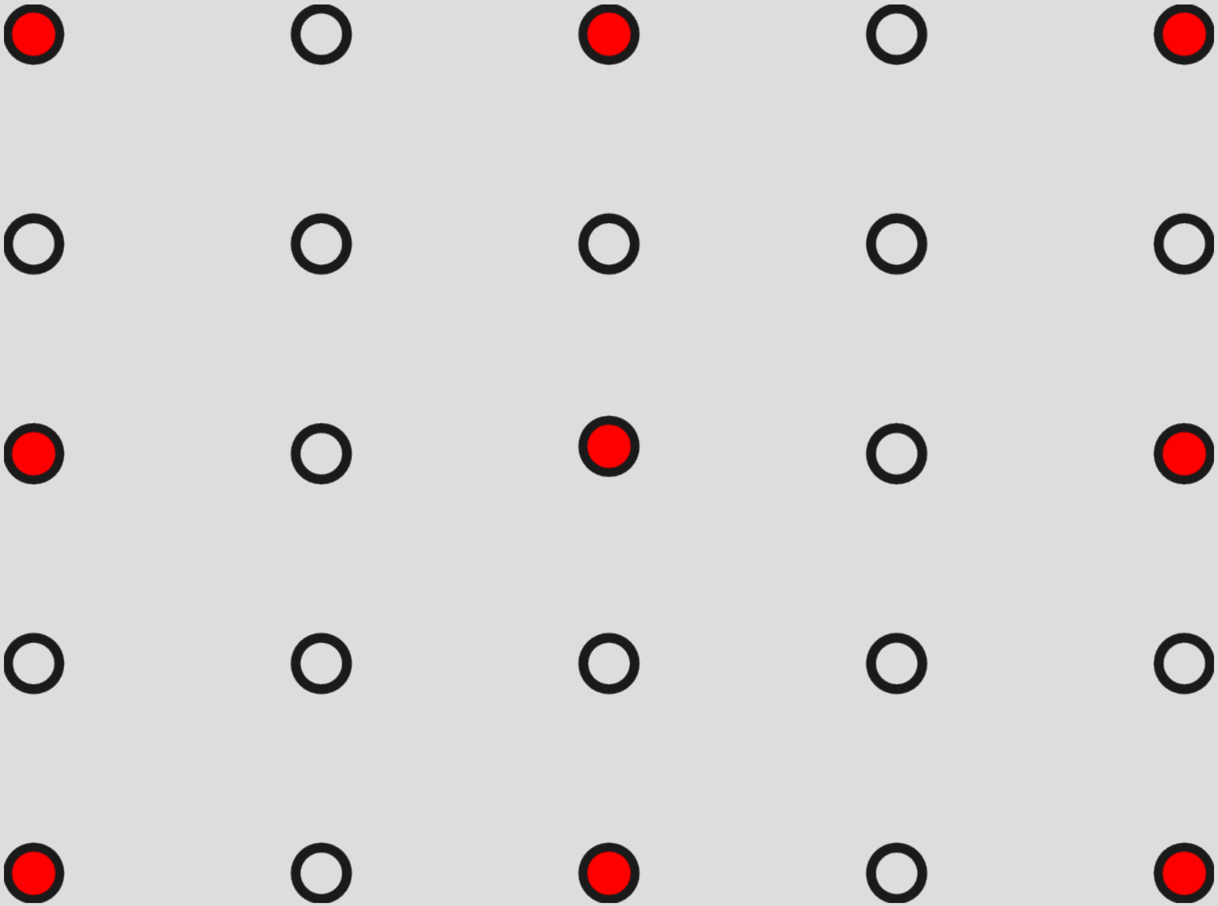
$$\bar{d}^o$$

$$\bar{d}^o_{inv} \qquad \bar{d}^o_{test}$$

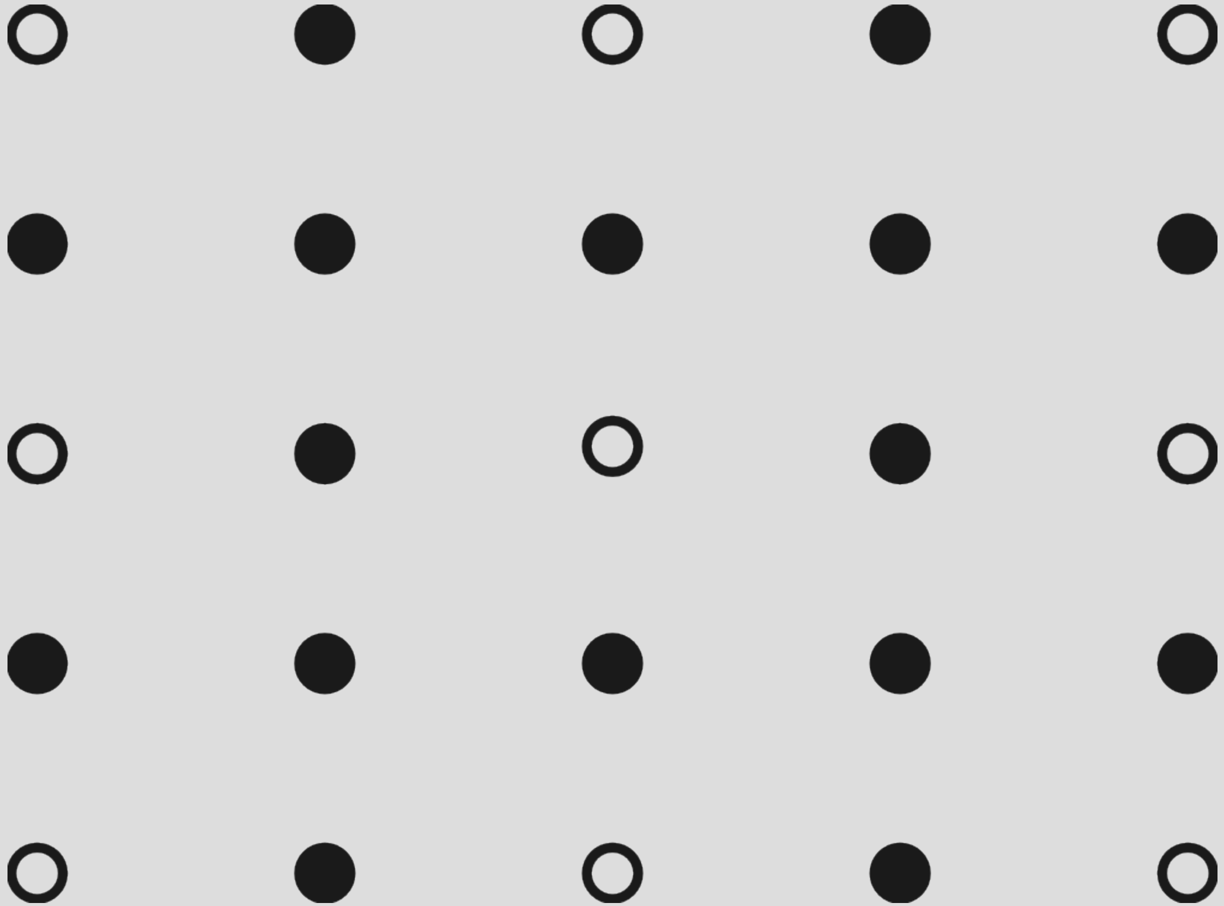
dado completo



dado inversão

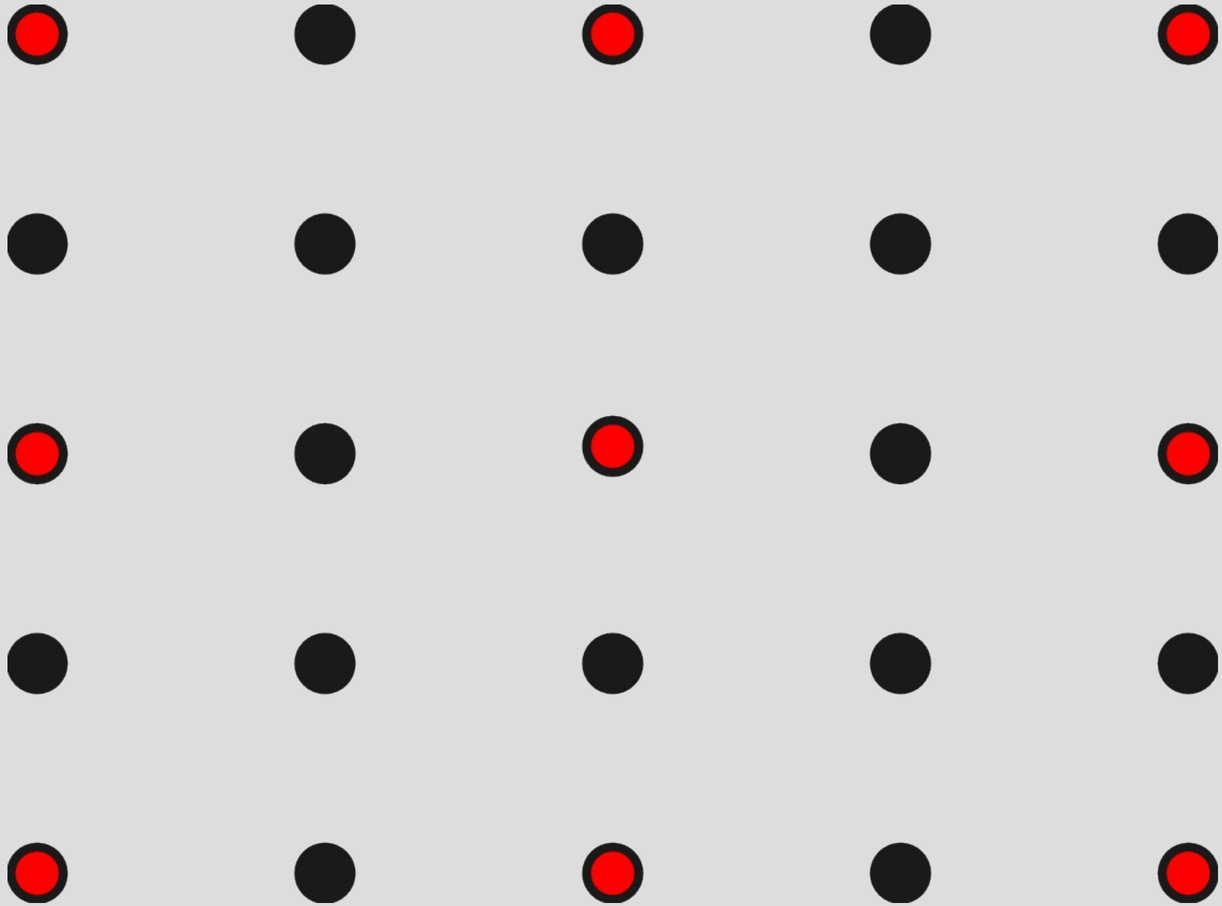


dado teste



dado teste

dado inversão



validação

μ

cruzada

$\Delta\rho$

h_{ref}

vínculos da

sísmica

estimativas pontuais

$$\overline{h}_s^o$$

para $\Delta\rho_i$ e $h_{ref,i}$

para $\Delta\rho_i$ e $h_{ref,i}$

inversão: $\bar{d}^{o}_{inv} \longrightarrow \hat{\bar{p}}_i$

para $\Delta\rho_i$ e $h_{ref,i}$

inversão: $\bar{d}^{o}_{inv} \longrightarrow \hat{\bar{p}}_i$

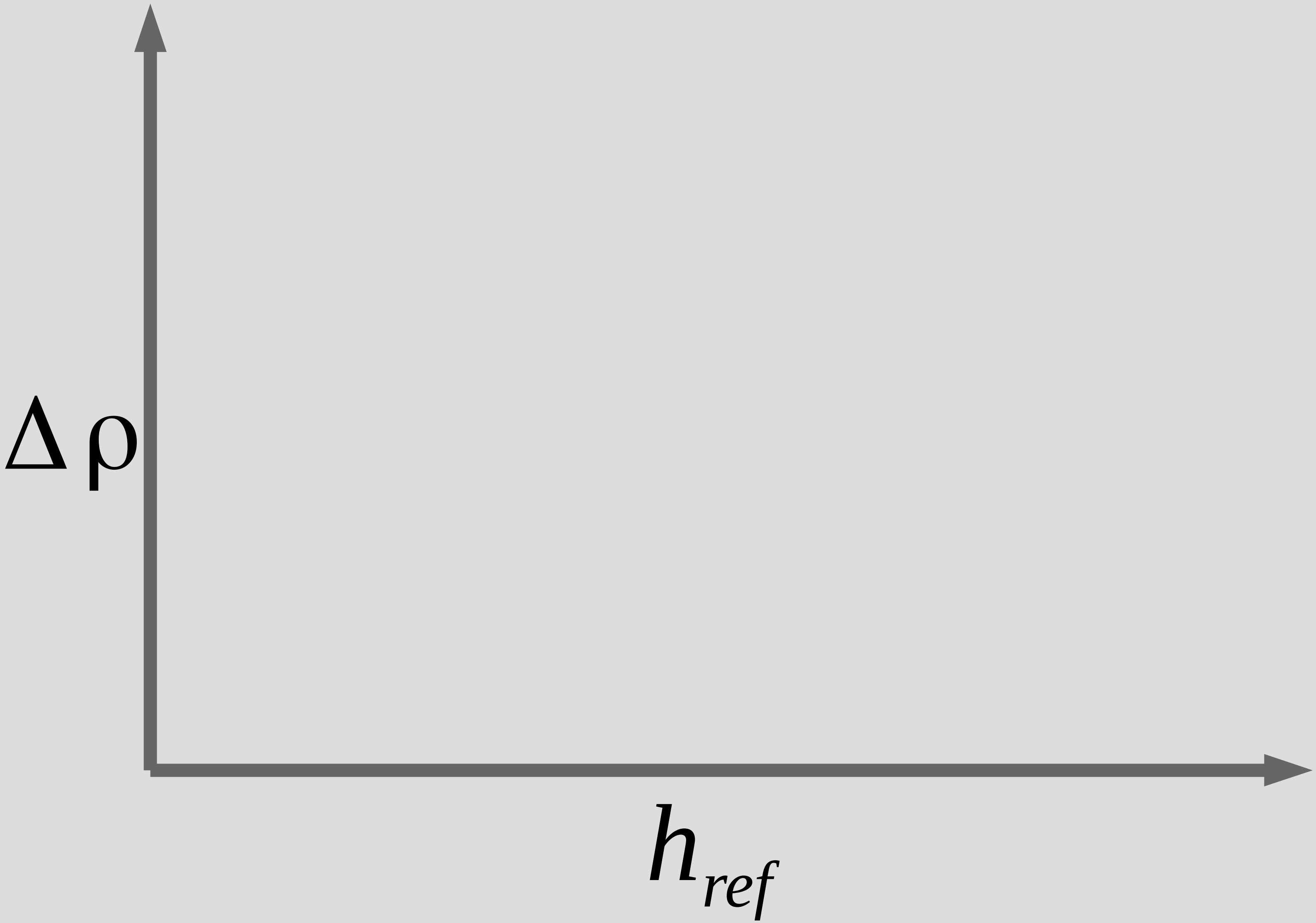
$\hat{\bar{p}}_i \longrightarrow$ interpolar $\bar{h}_s$

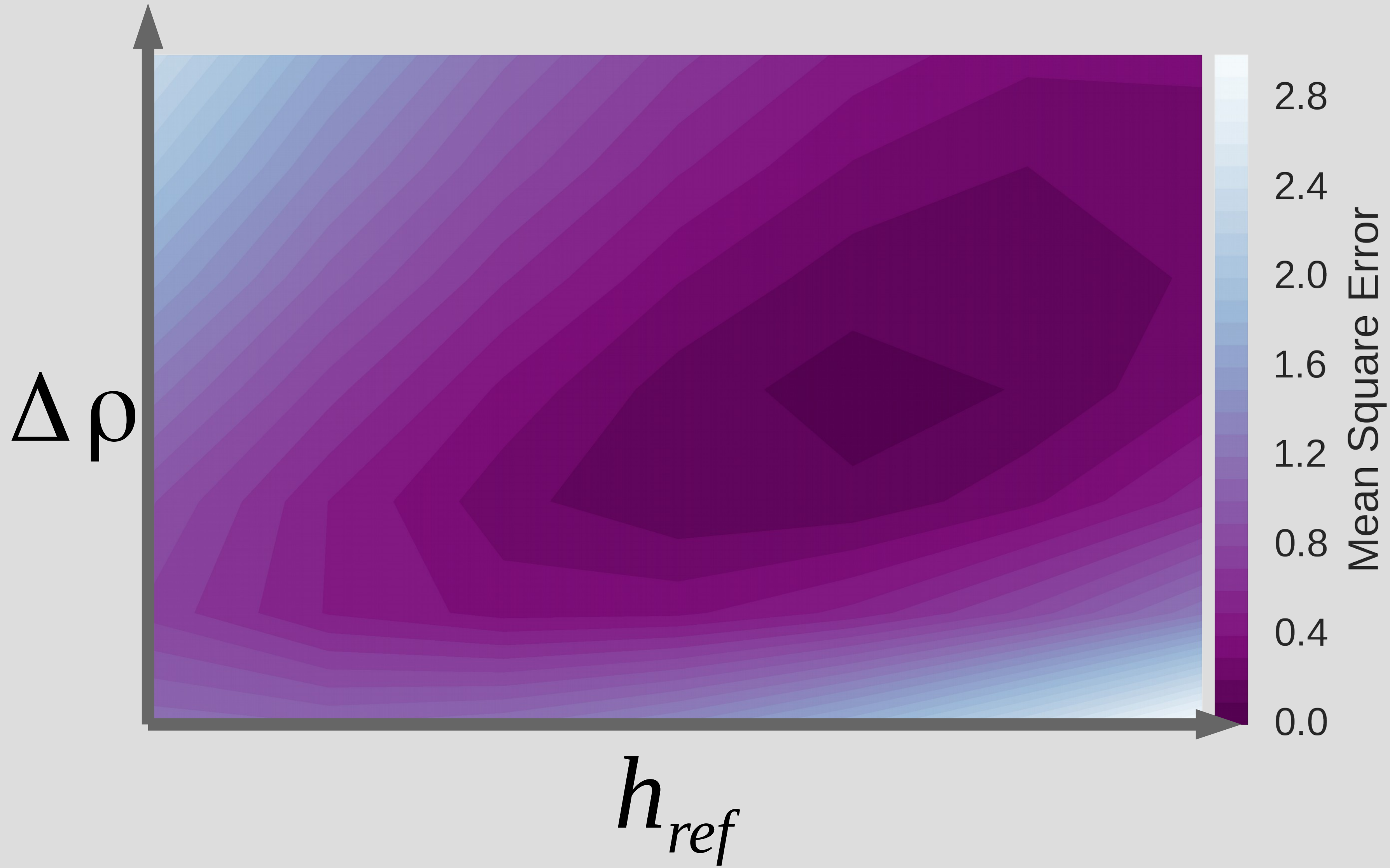
para $\Delta\rho_i$ e $h_{ref,i}$

inversão: $\bar{d}^o_{inv} \rightarrow \hat{\bar{p}}_i$

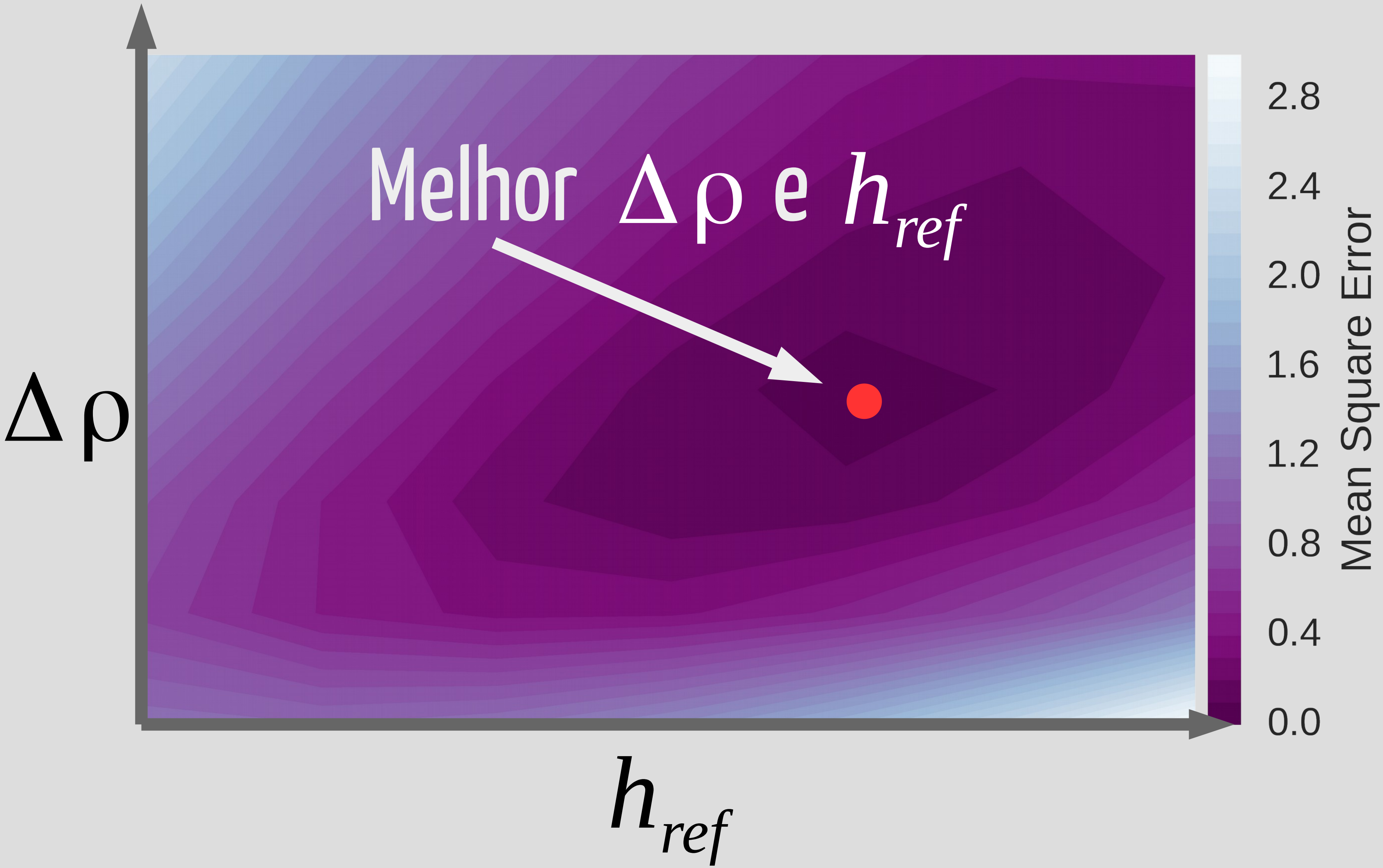
$\hat{\bar{p}}_i \rightarrow$ interpolar $\bar{h}_s$

$$MSE = \frac{\| \bar{h}^o_s - \bar{h}_s \|^2}{N_{test}}$$


Δρ
h_{ref}

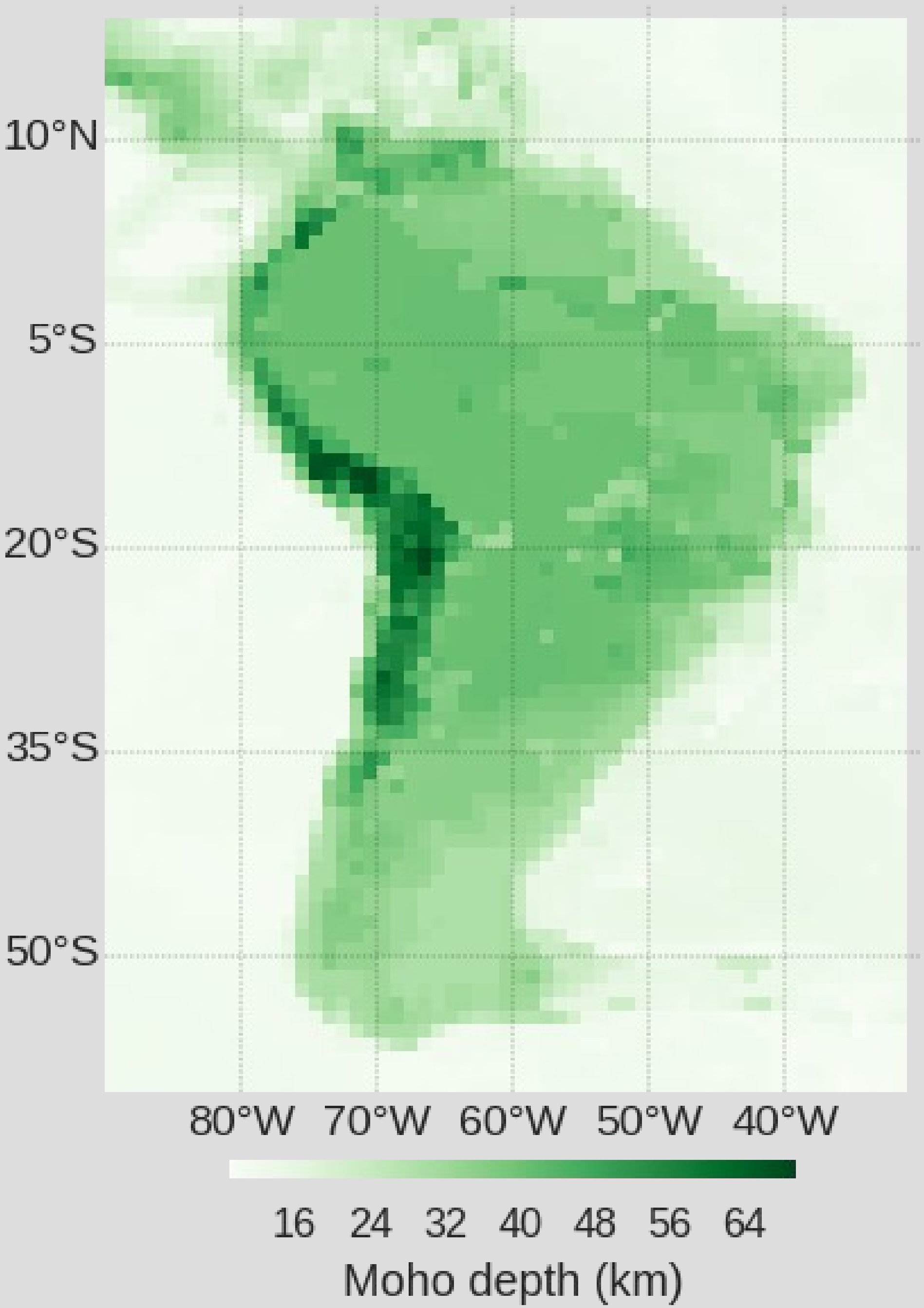


2.8
2.4
2.0
1.6
1.2
0.8
0.4
0.0
Mean Square Error
Δ ρ
h_{ref}

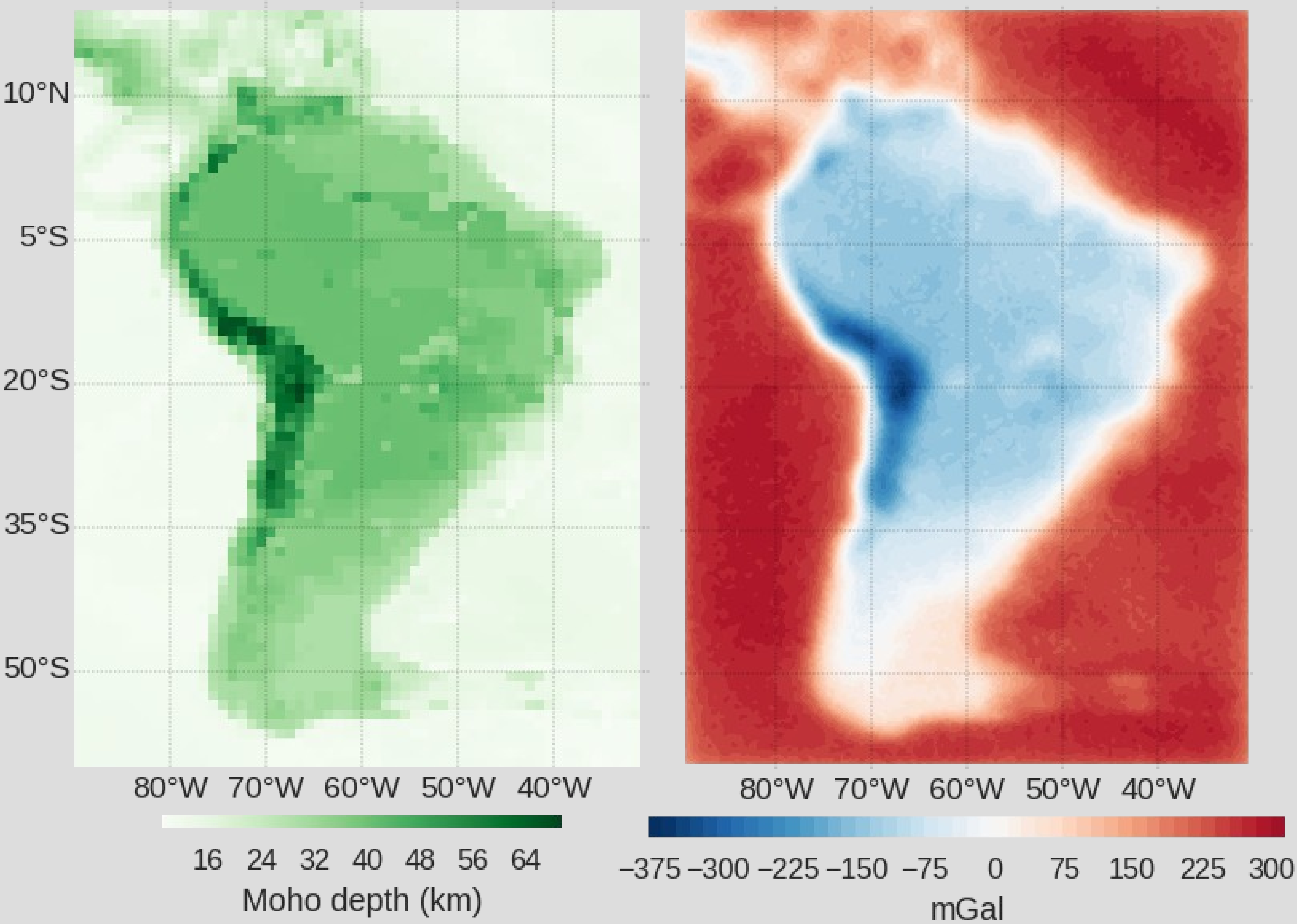


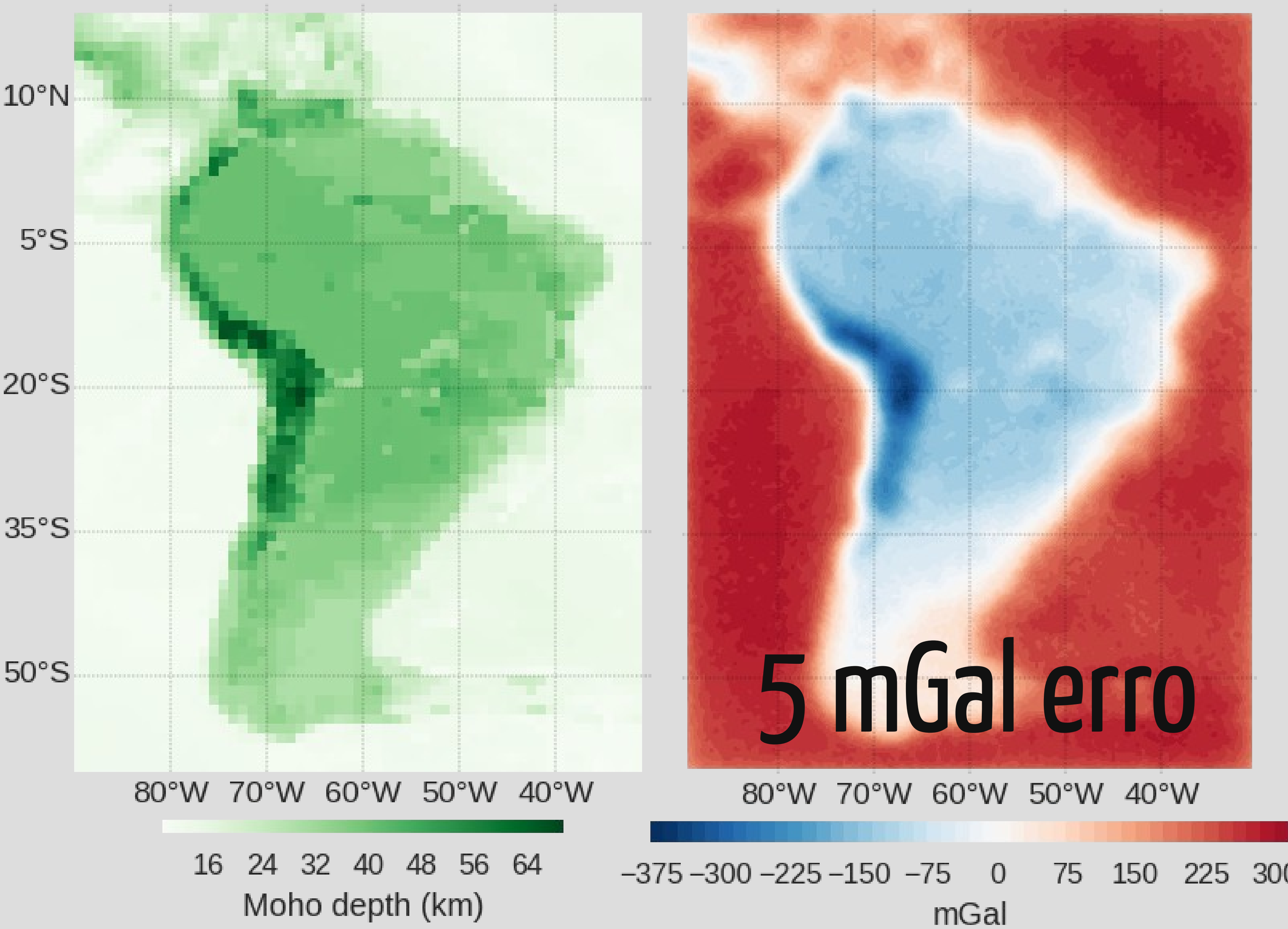
Melhor $\Delta\rho$ e h_{ref}
$\Delta\rho$
h_{ref}
Mean Square Error
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0.0

Resultados sintético

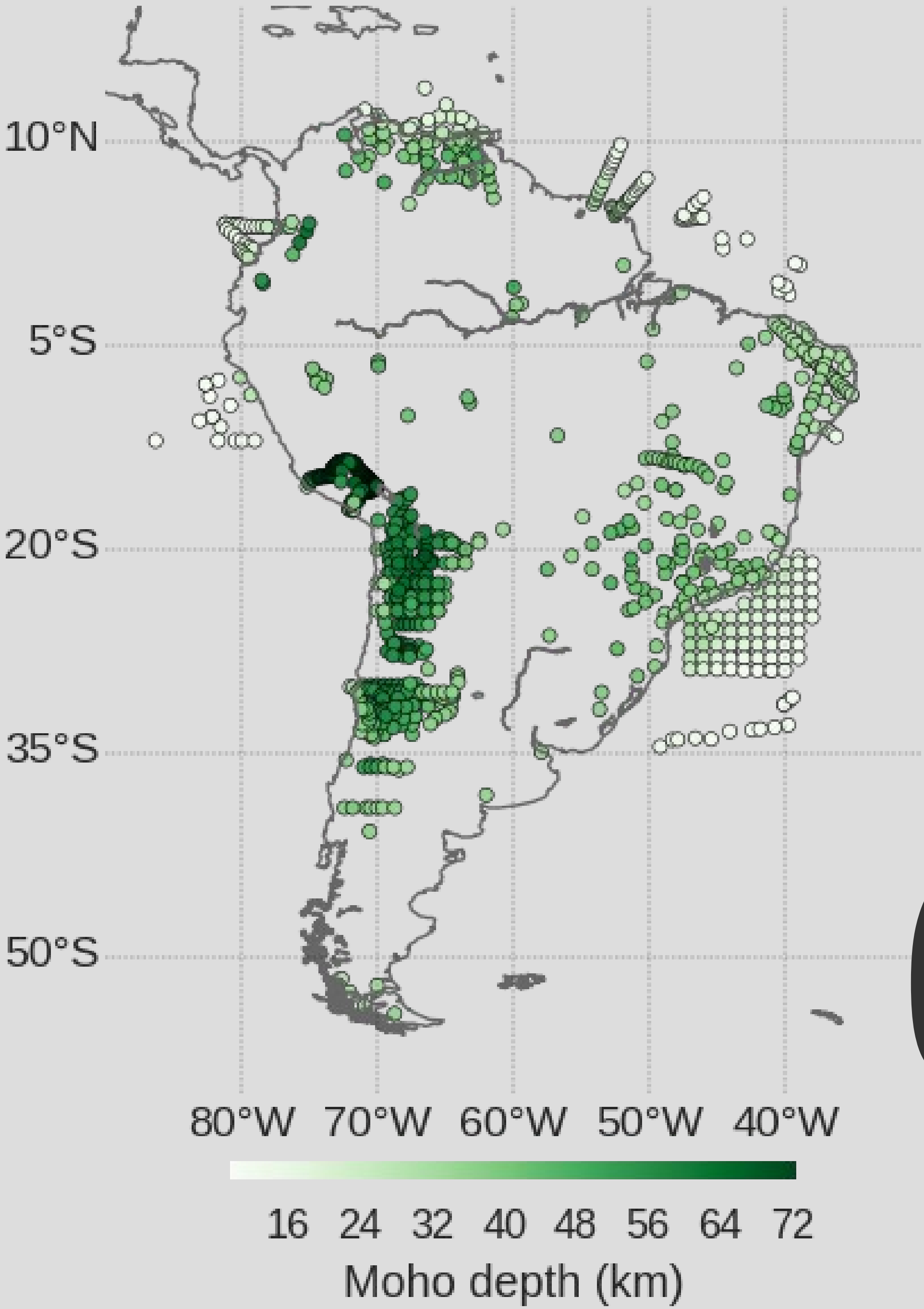


Moho CRUST1.0

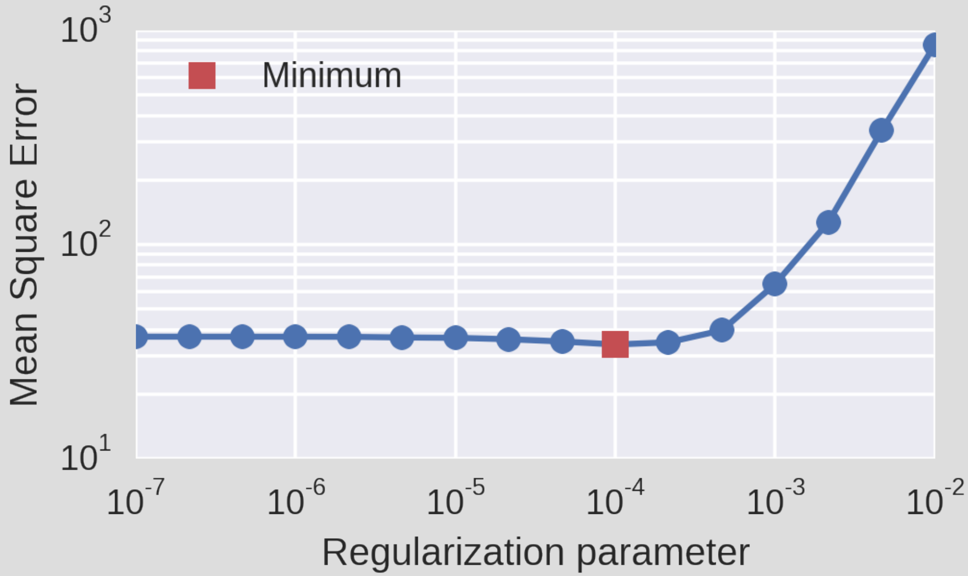

10°N
5°S
20°S
35°S
50°S
80°W
70°W
60°W
50°W
40°W
16
24
32
40
48
56
64
Moho depth (km)
80°W
70°W
60°W
50°W
40°W
−375
−300
−225
−150
−75
0
75
150
225
300
mGal



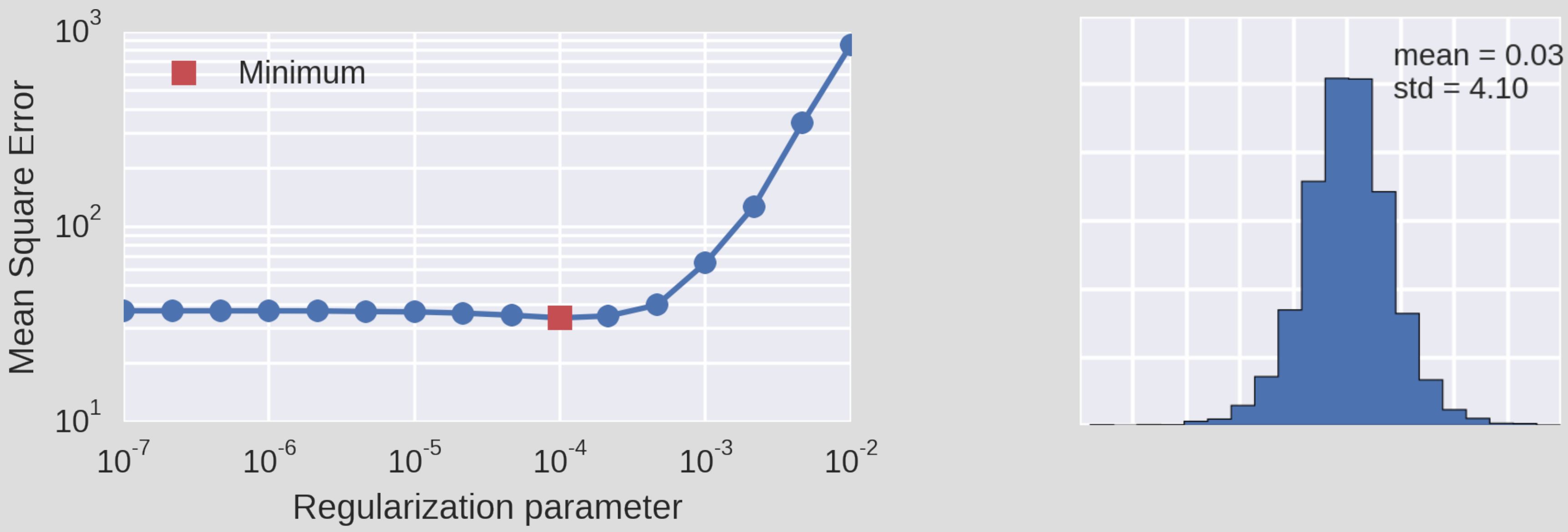
10°N
5°S
20°S
35°S
50°S
80°W 70°W 60°W 50°W 40°W
16 24 32 40 48 56 64
Moho depth (km)
5 mGal erro
80°W 70°W 60°W 50°W 40°W
−375 −300 −225 −150 −75 0 75 150 225 300
mGal

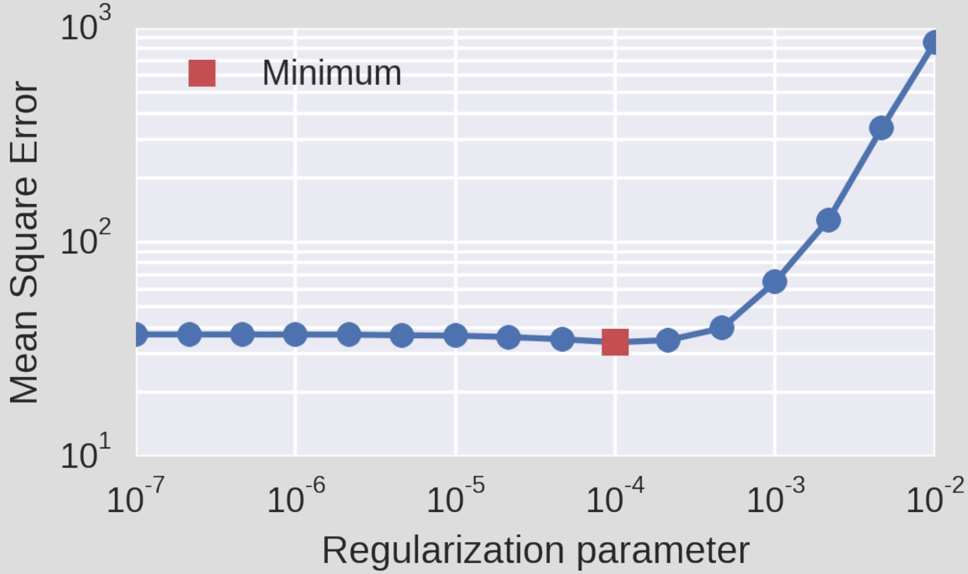


vínculos sísmicos CRUST1.0

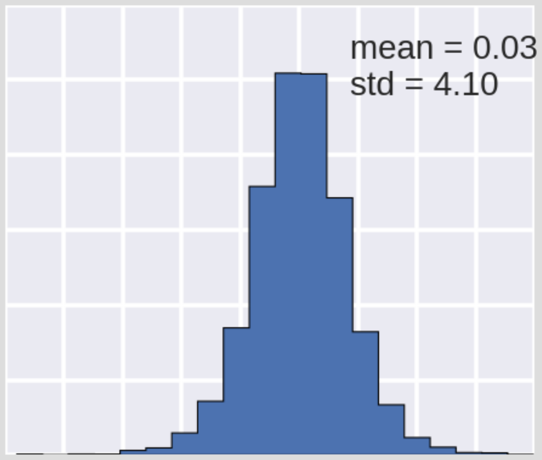


Minimum
Mean Square Error
10^3
10^2
10^1
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
Regularization parameter

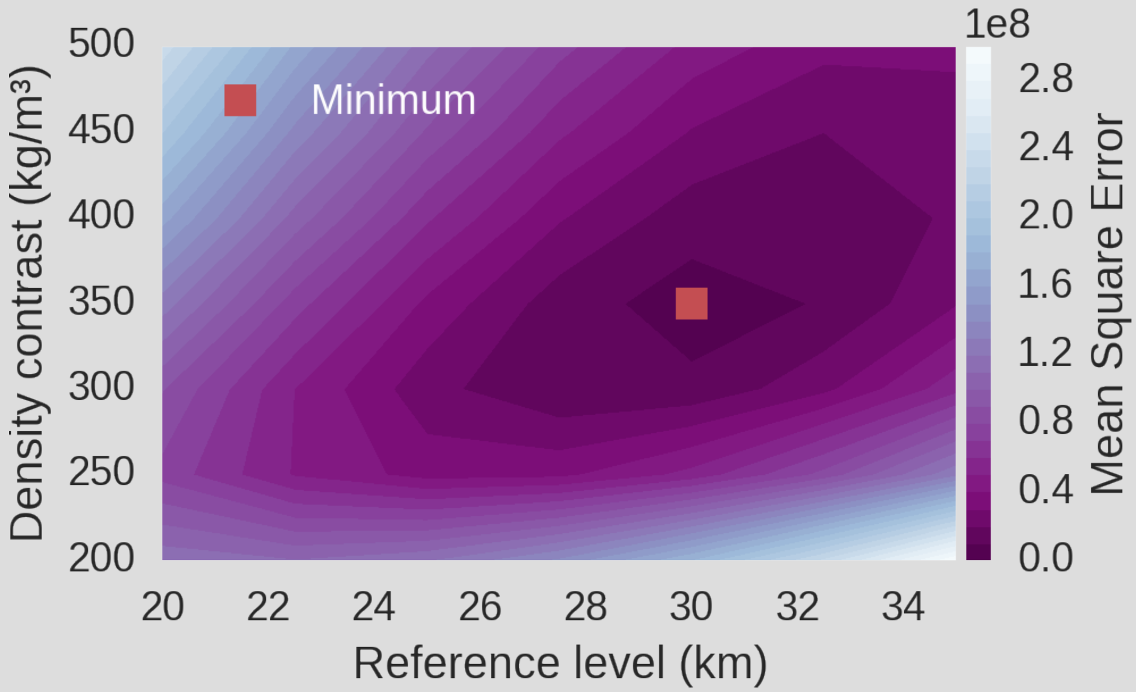

Mean Square Error
Regularization parameter
Minimum
mean = 0.03
std = 4.10



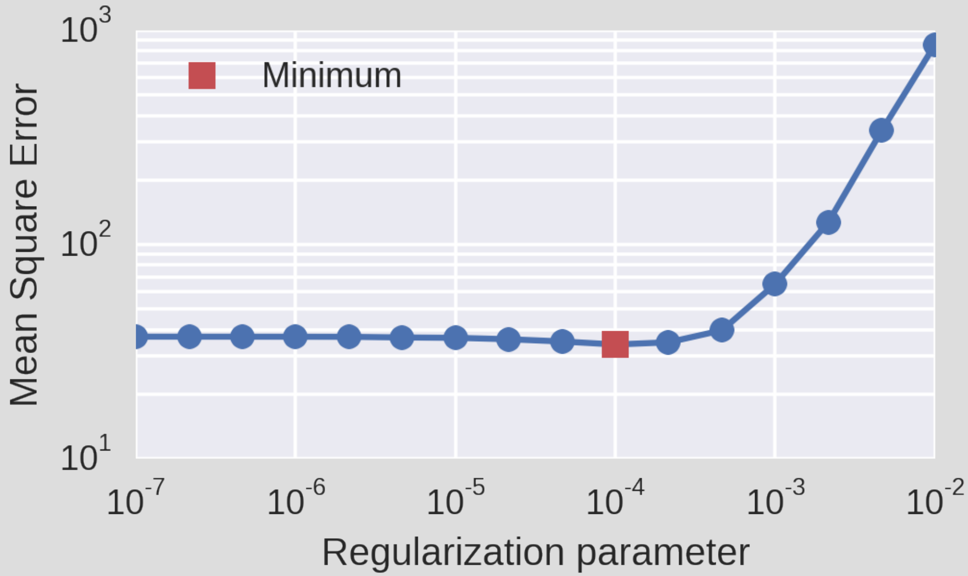
Minimum
Mean Square Error
Regularization parameter
10^{1}
10^{2}
10^{3}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}

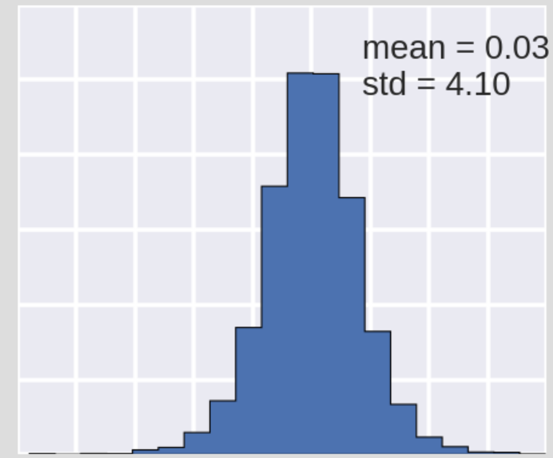


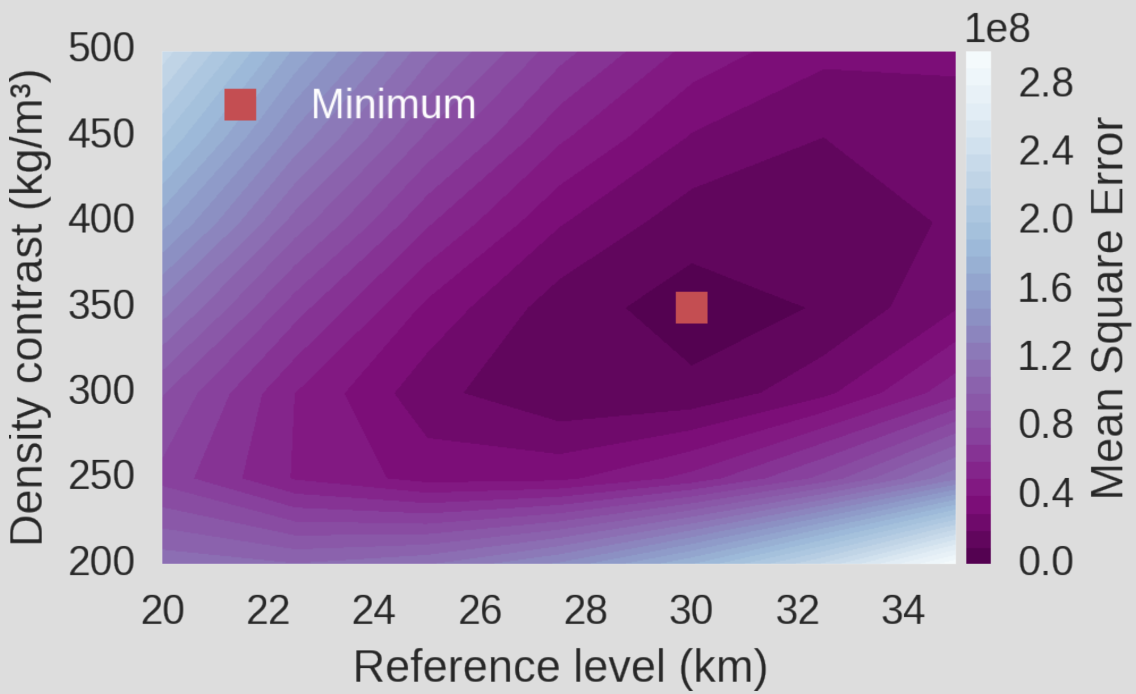
mean = 0.03
std = 4.10



Minimum
Density contrast (kg/m³)
Reference level (km)
Mean Square Error
1e8
500
450
400
350
300
250
200
20
22
24
26
28
30
32
34
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0.0



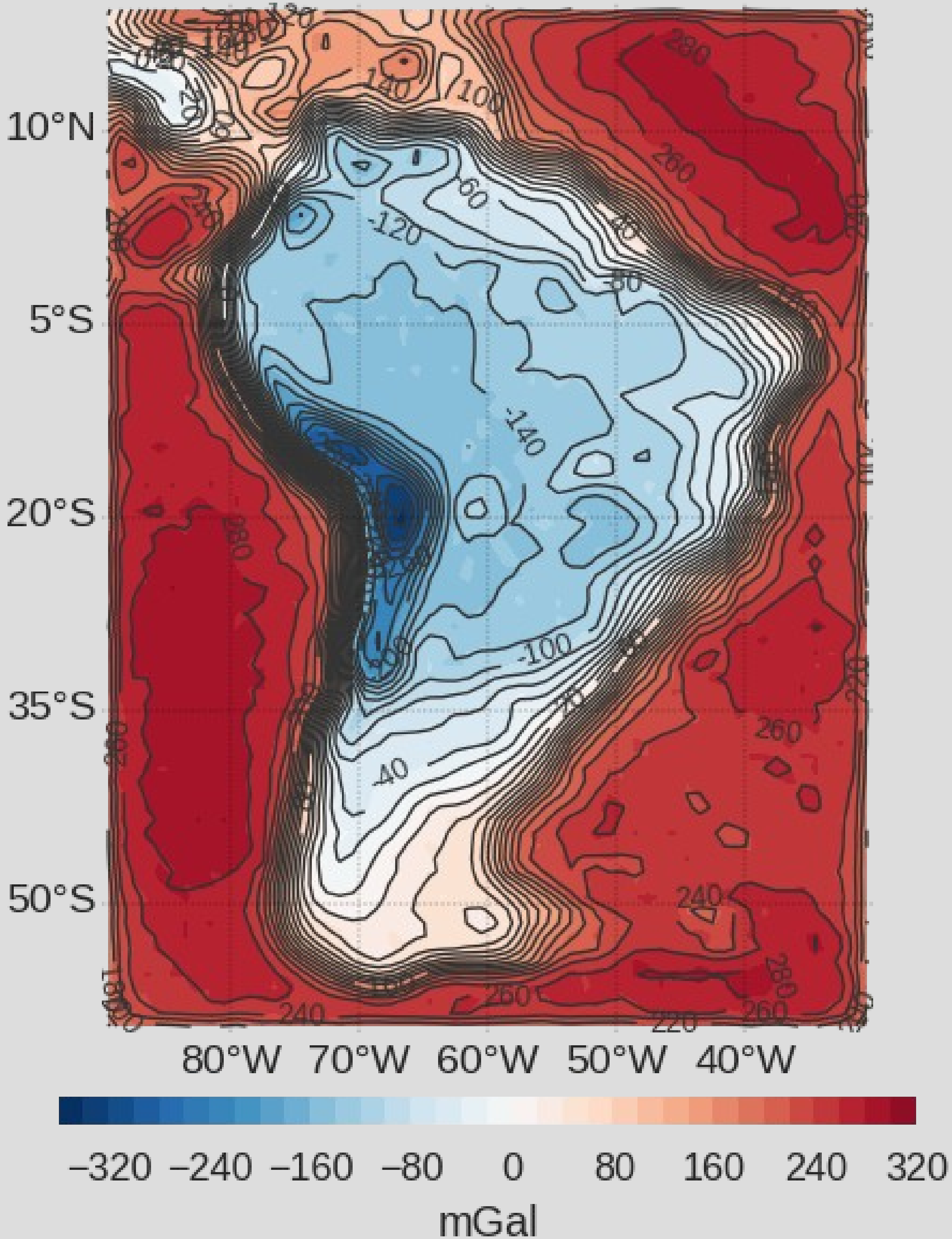




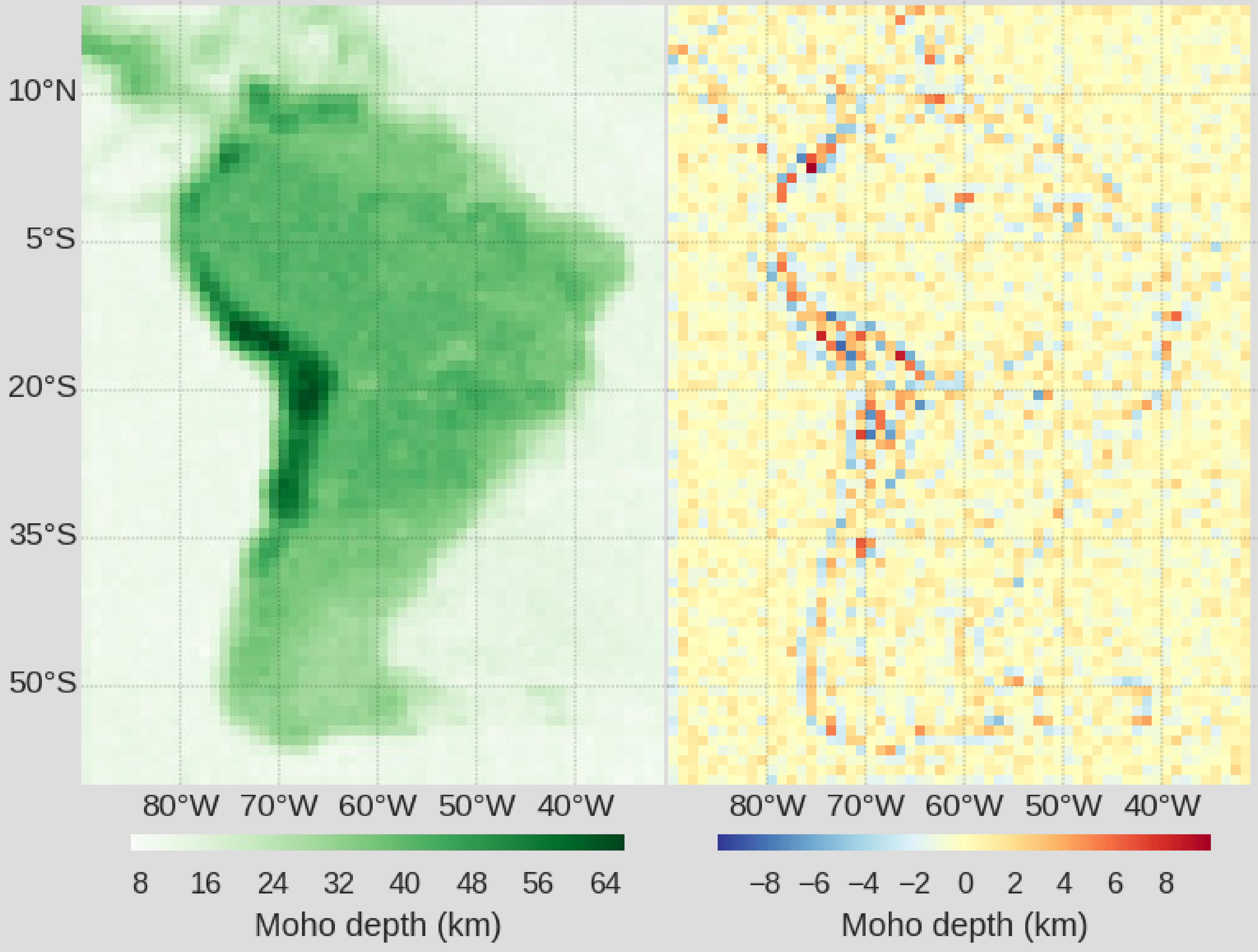
verdadeiros

$$\Delta\rho = 350$$

$$h_{ref} = 30$$

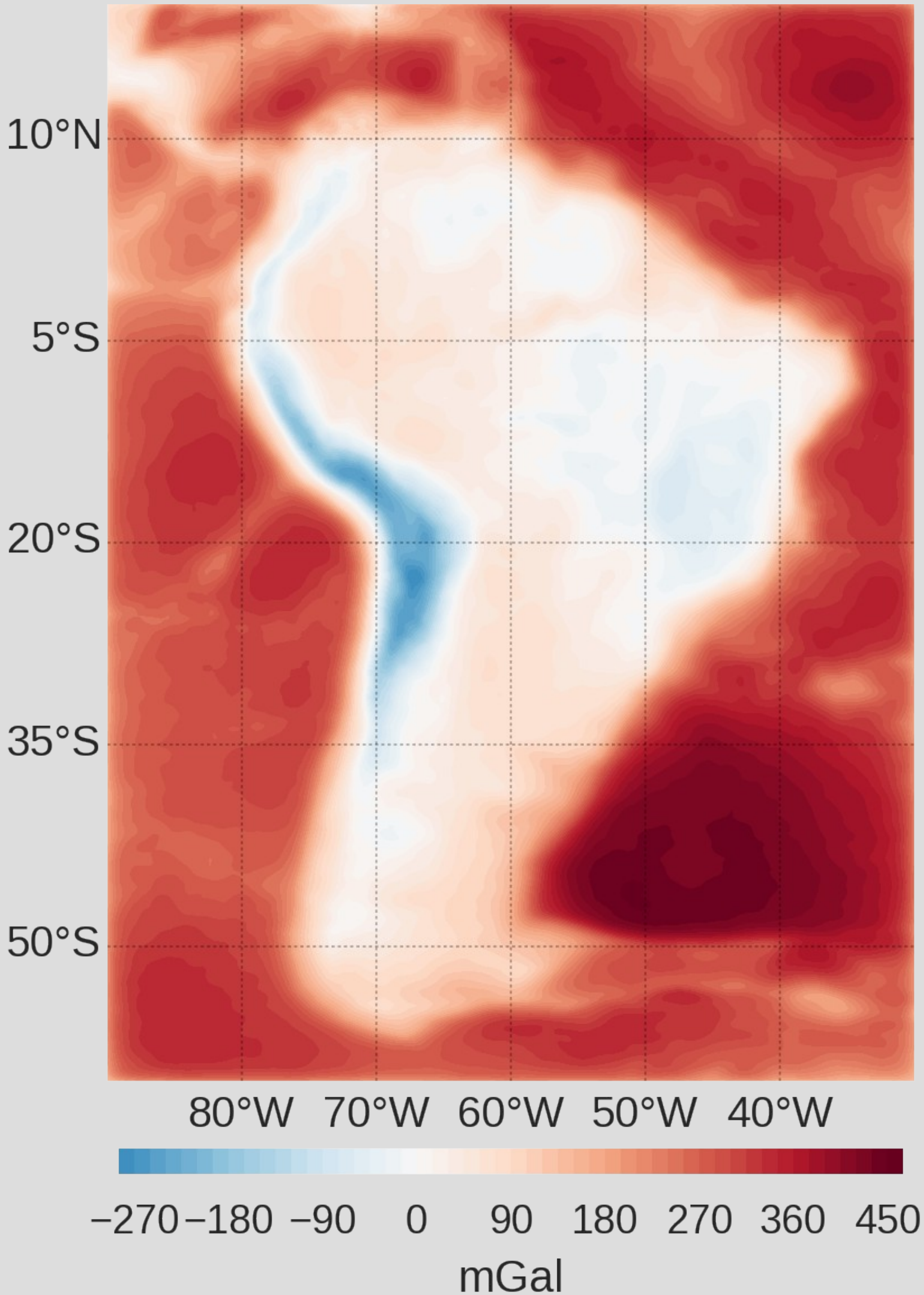


<u>Contorno:</u>
Predito
<u>Cor:</u>
Observado

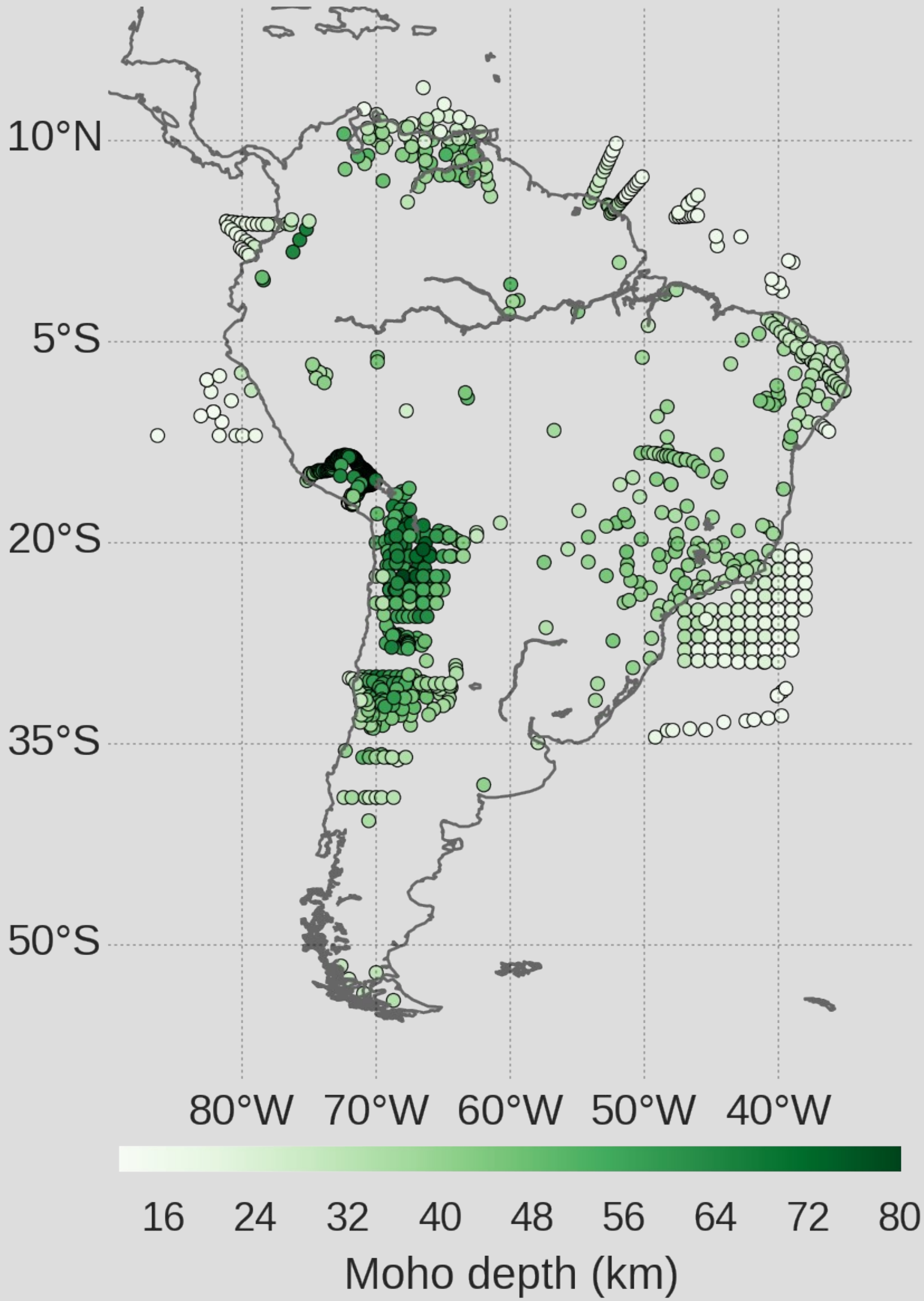

10°N
5°S
20°S
35°S
50°S
80°W
70°W
60°W
50°W
40°W
8
16
24
32
40
48
56
64
Moho depth (km)
−8
−6
−4
−2
0
2
4
6
8
Moho depth (km)

Resultados

Am. do Sul

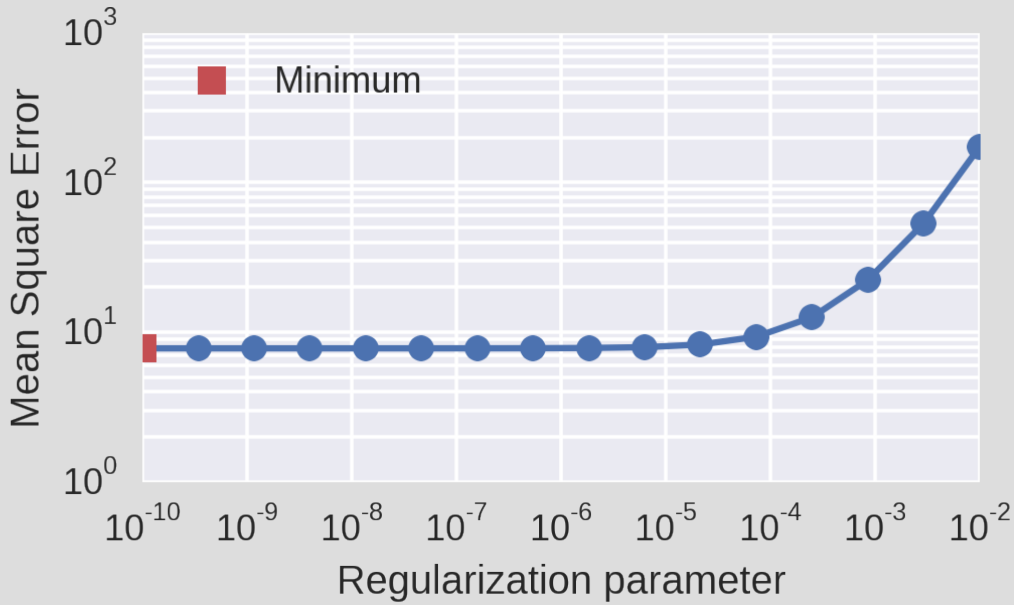


Anomalia <u>Bouguer</u> sem sedimentos

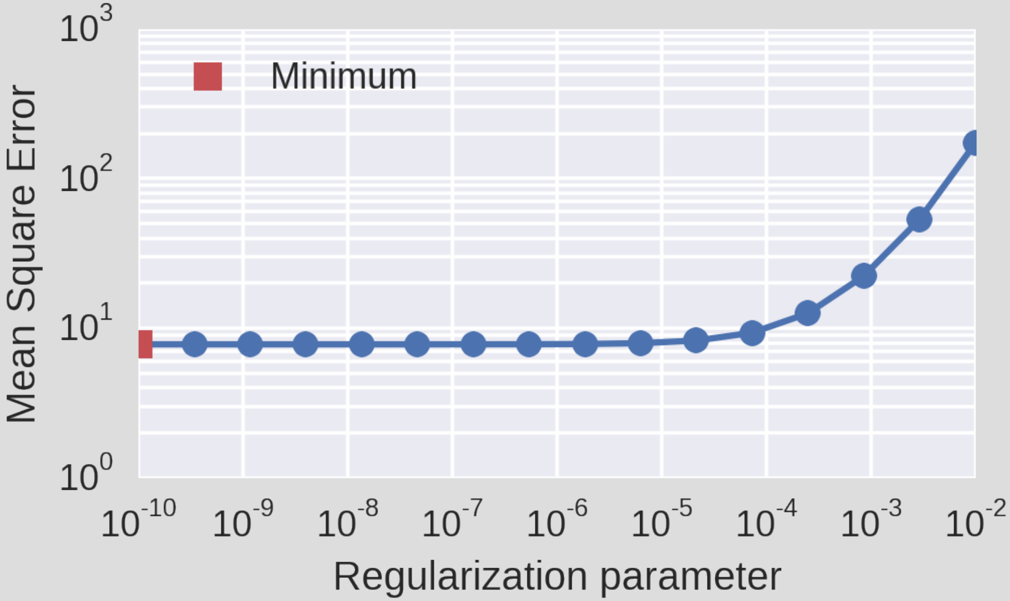


vínculos sísmica

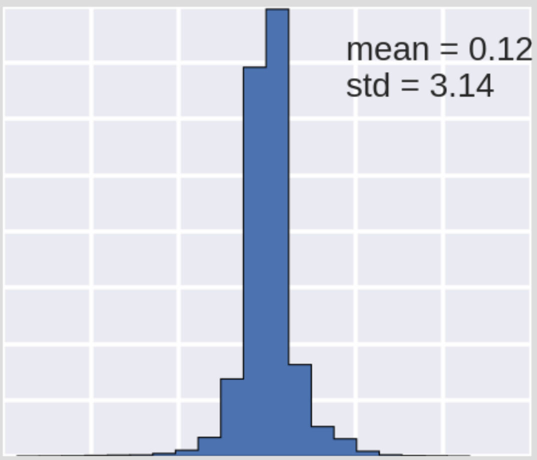
Assumpção et al. (2012)



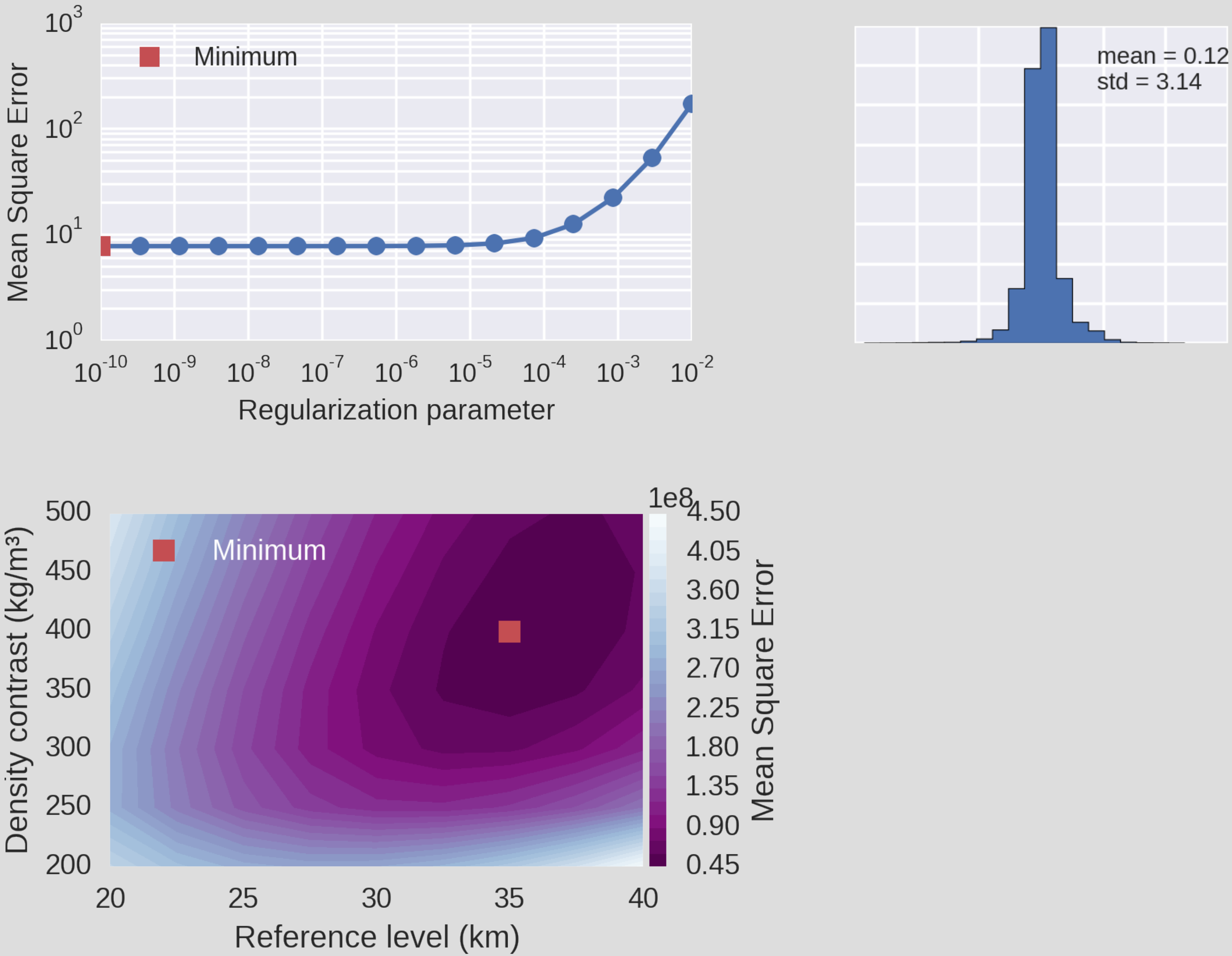
Minimum
Mean Square Error
10^0
10^1
10^2
10^3
10^{-10}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
Regularization parameter



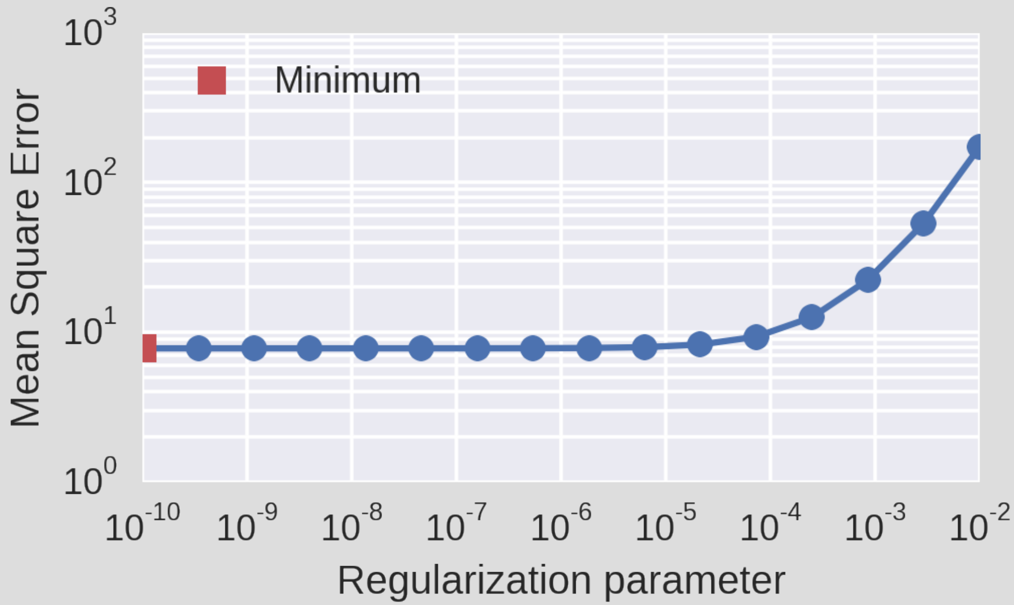
Minimum
Mean Square Error
Regularization parameter
10^0
10^1
10^2
10^3
10^{-10}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}

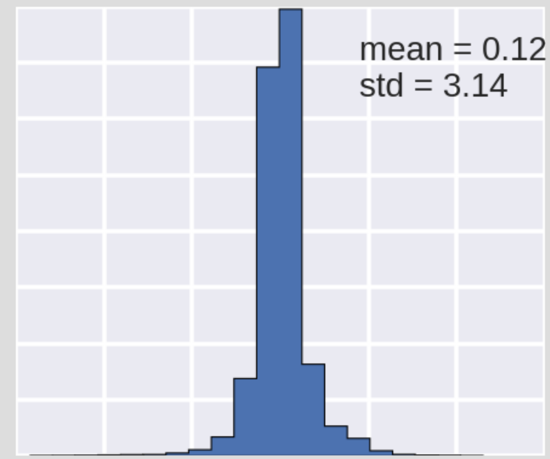


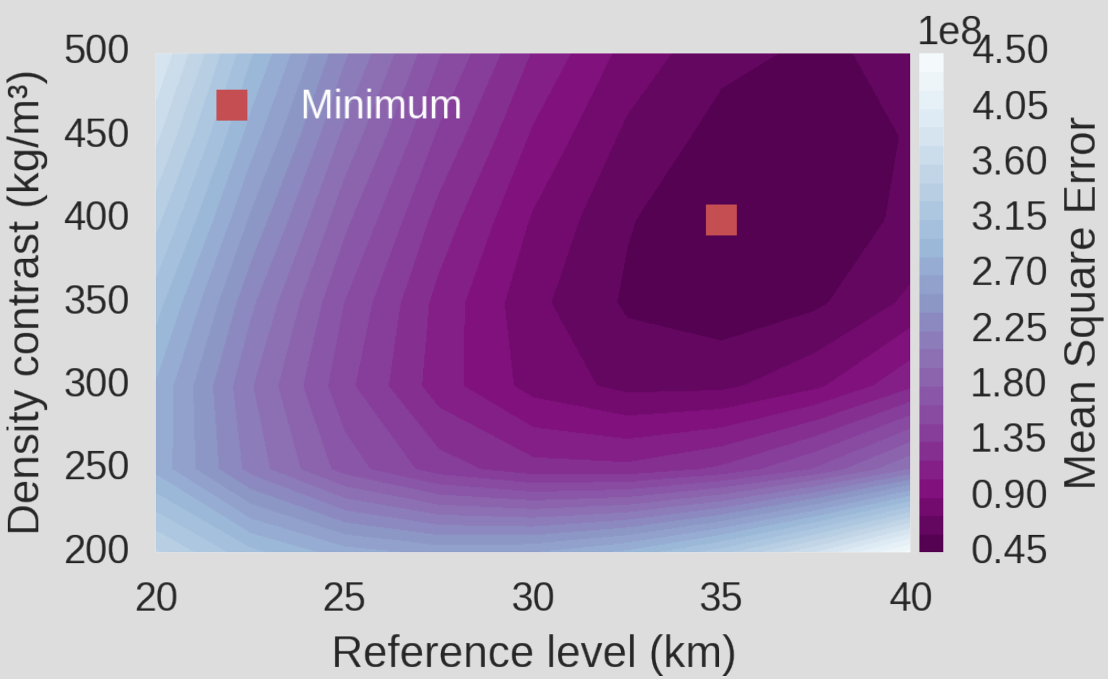
mean = 0.12
std = 3.14



Minimum
Mean Square Error
Regularization parameter
mean = 0.12
std = 3.14
Minimum
Density contrast (kg/m³)
Reference level (km)
1e8
Mean Square Error



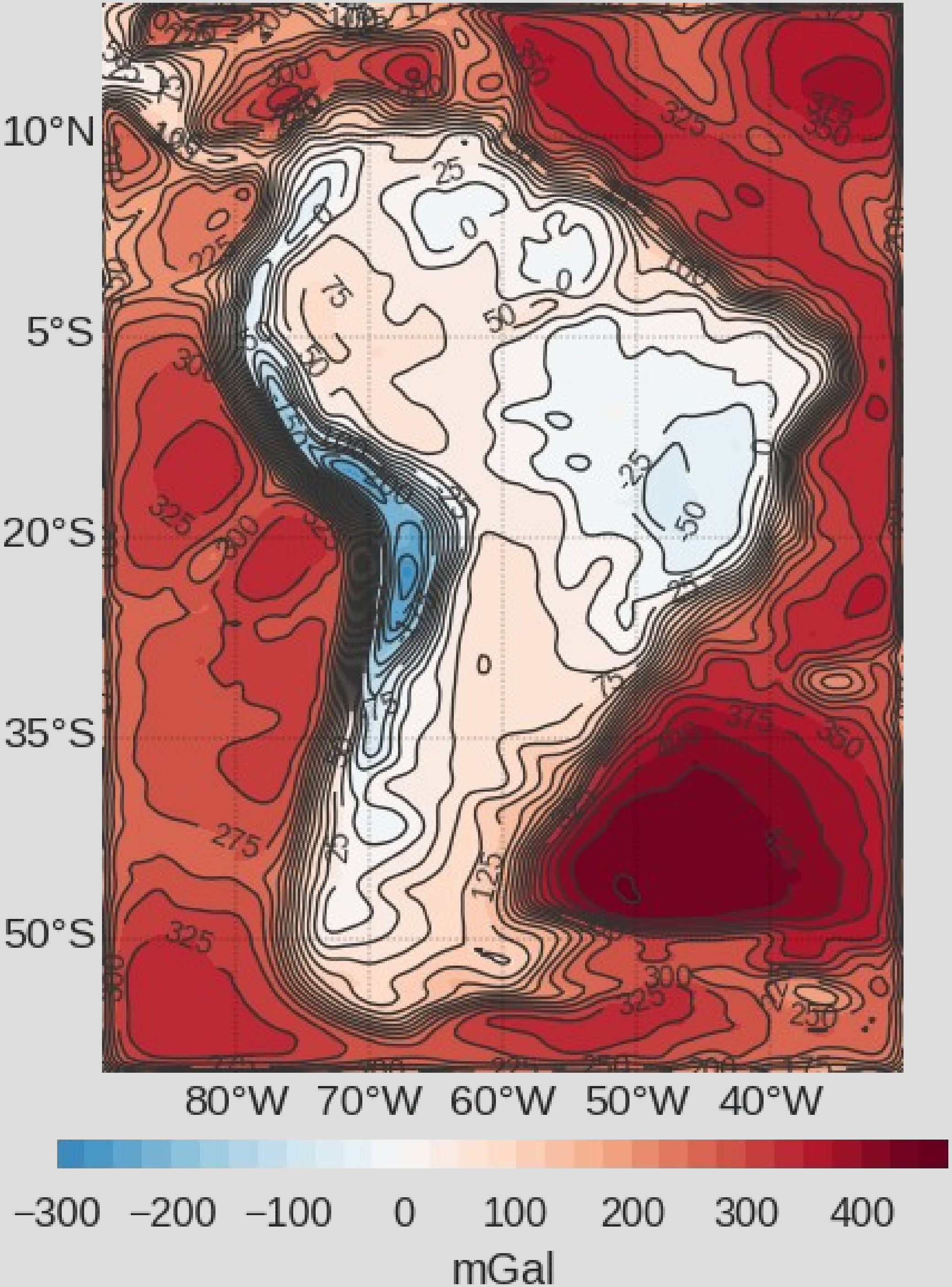




estimados

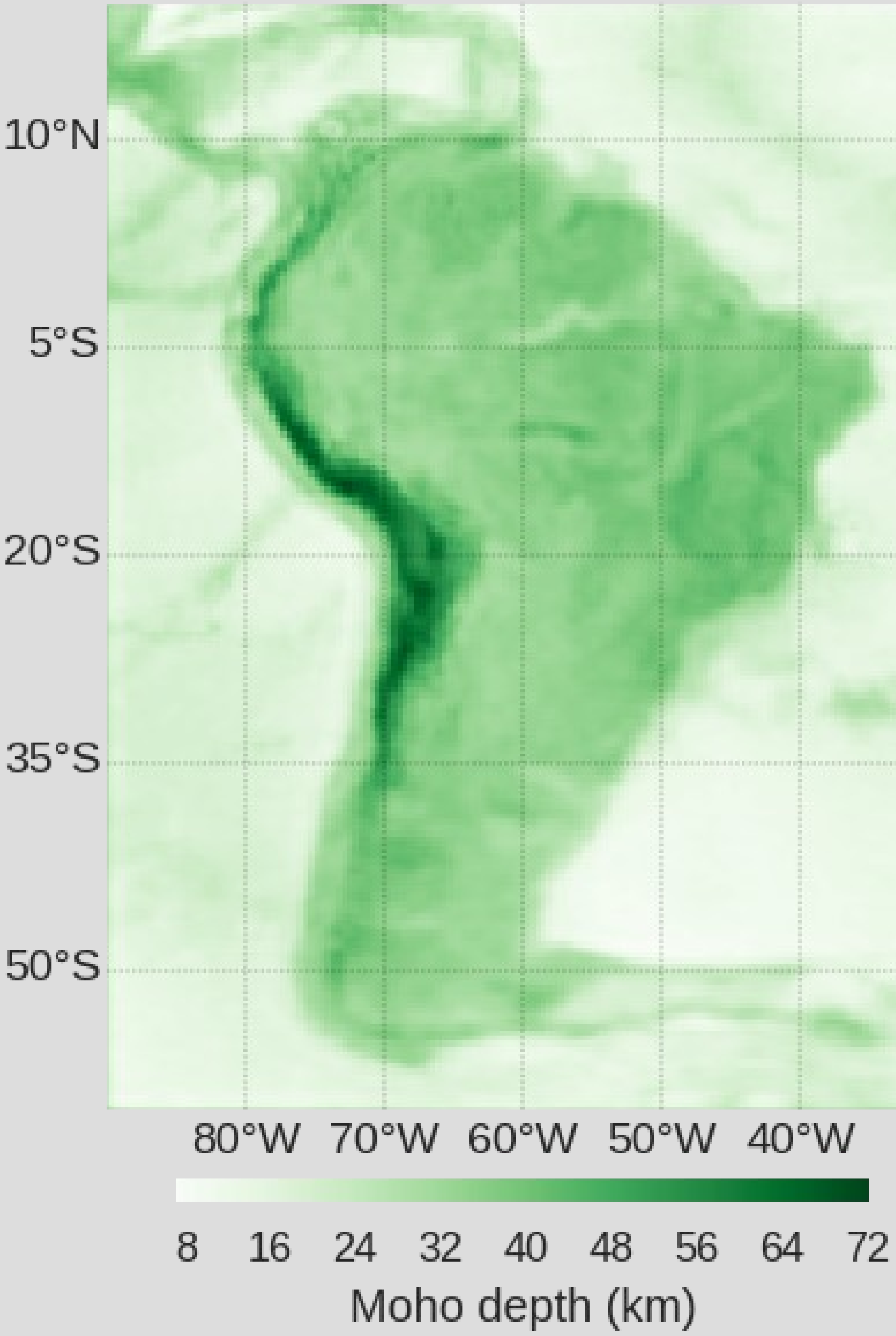
$$\Delta\rho = 400$$

$$h_{ref} = 35$$



Contorno: Predito

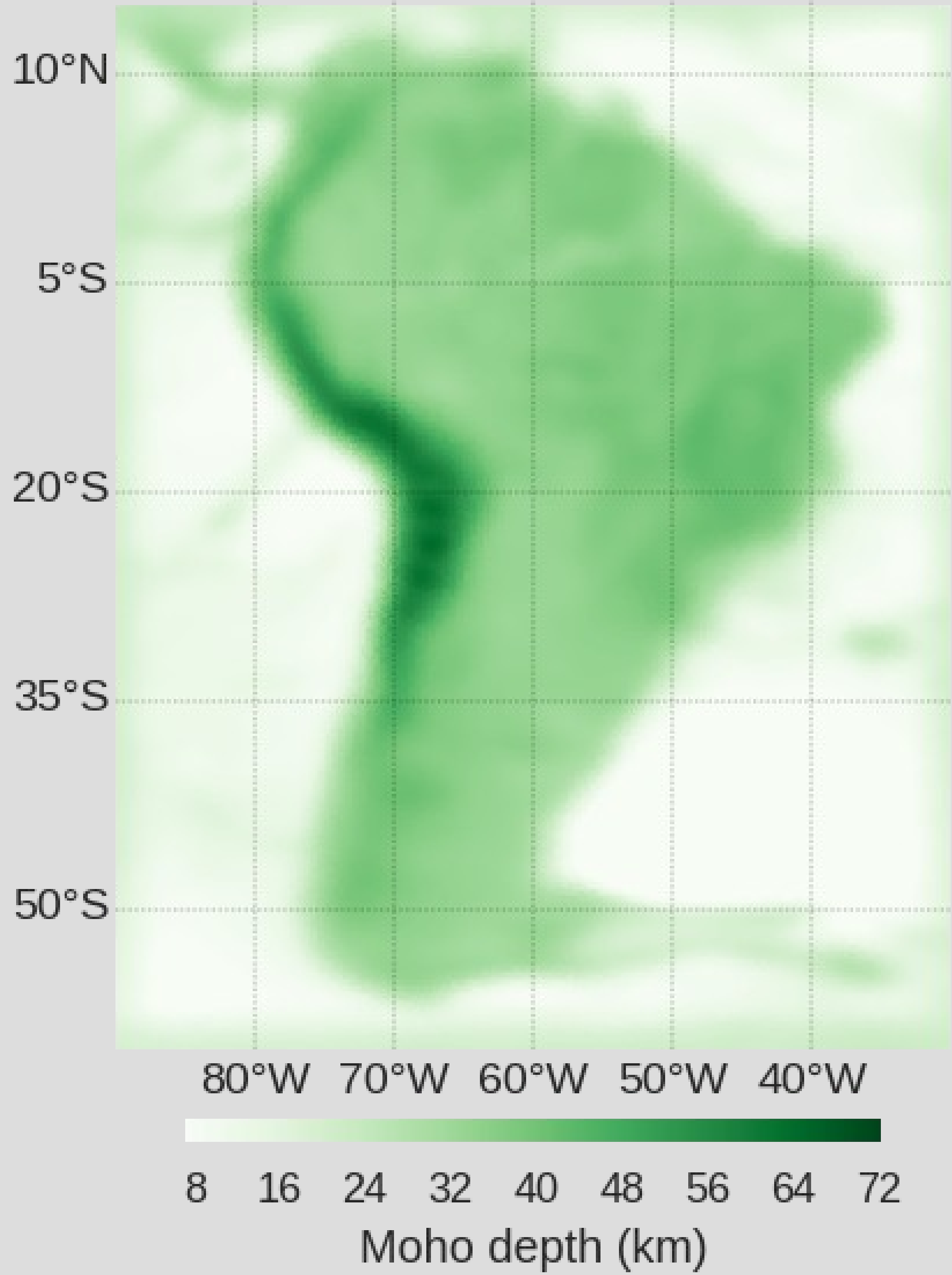
Cor: observado

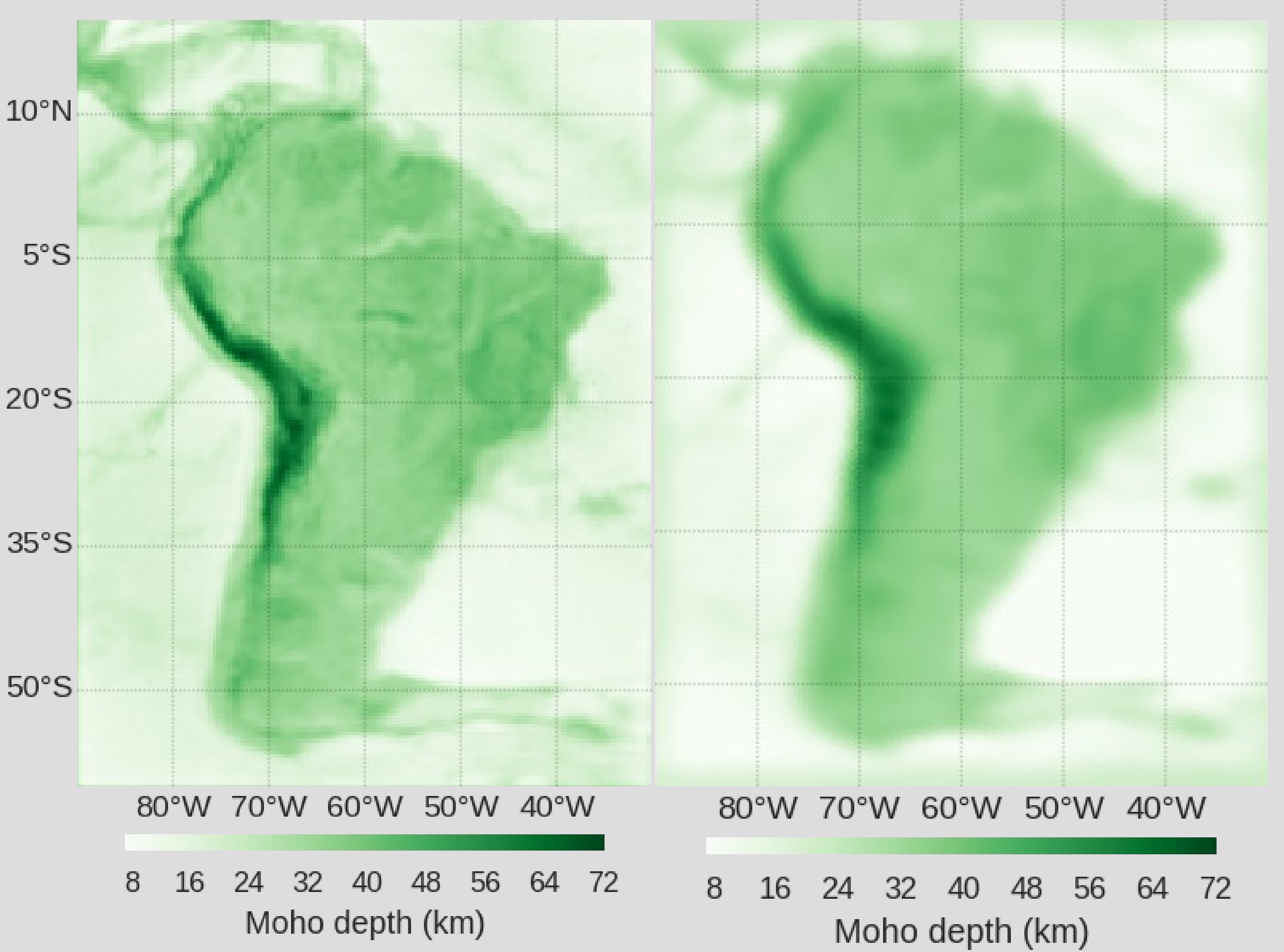


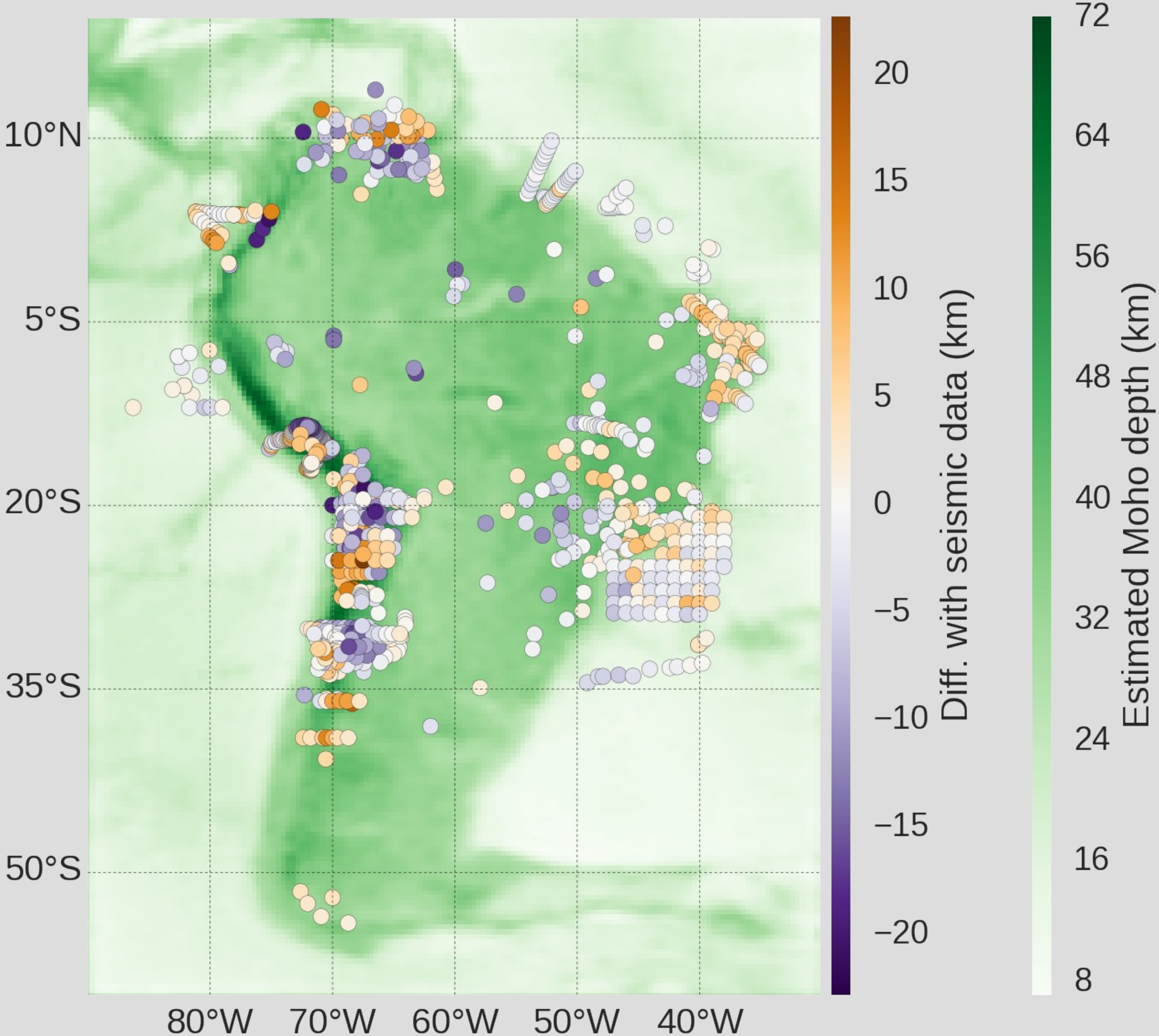
estimativa da Moho

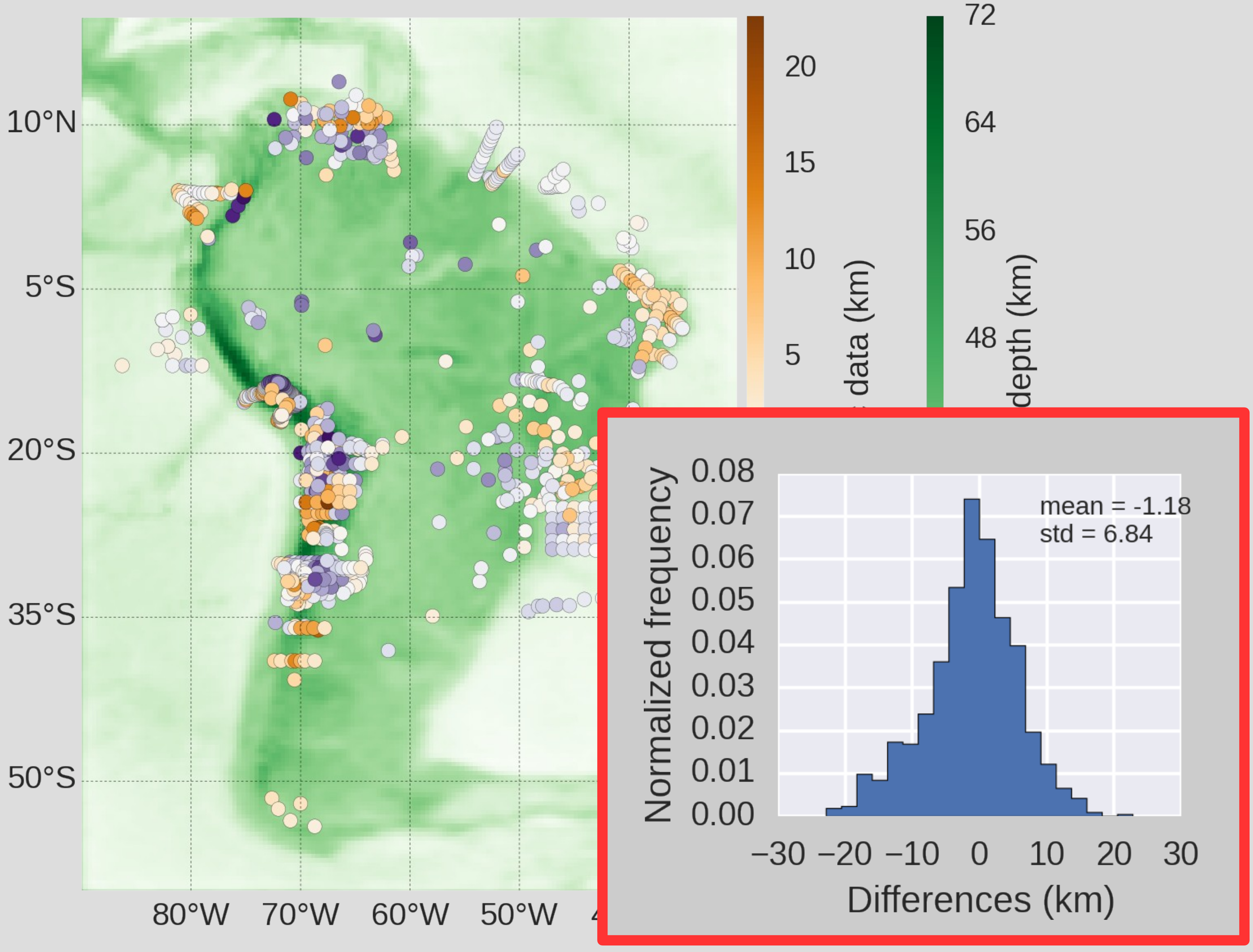
solução anterior

van der Meijde et al. (2013)




10°N
5°S
20°S
35°S
50°S
80°W 70°W 60°W 50°W 40°W
8 16 24 32 40 48 56 64 72
Moho depth (km)
80°W 70°W 60°W 50°W 40°W
8 16 24 32 40 48 56 64 72
Moho depth (km)


10°N
5°S
20°S
35°S
50°S
80°W
70°W
60°W
50°W
40°W
20
15
10
5
0
−5
−10
−15
−20
Diff. with seismic data (km)
72
64
56
48
40
32
24
16
8
Estimated Moho depth (km)



10°N
5°S
20°S
35°S
50°S
80°W
70°W
60°W
50°W
20
15
10
5
data (km)
72
64
56
48
depth (km)
mean = -1.18
std = 6.84
Normalized frequency
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
−30
−20
−10
0
10
20
30
Differences (km)

Conclusões

Novo método

Baseado em Bott (1960) e Silva et al. (2014)

Novo método

Baseado em Bott (1960) e Silva et al. (2014)

Tesseroides

Novo método

Baseado em Bott (1960) e Silva et al. (2014)

Tesseroides

Gauss-Newton + Regularização

Novo método

Baseado em Bott (1960) e Silva et al. (2014)

Tesseroides

Gauss-Newton + Regularização

Matrizes esparsas

Novo método

Baseado em Bott (1960) e Silva et al. (2014)

Tesseroides

Gauss-Newton + Regularização

Matrizes esparsas

Validação cruzada $\longrightarrow$ $\mu \quad \Delta\rho \quad h_{ref}$

Moho América do Sul

Compatível com van der Meijde et al. (2013)

Moho América do Sul

- Compatível com van der Meijde et al. (2013)
- Correções (topo e sedimentos) mais apropriadas

Moho América do Sul

Compatível com van der Meijde et al. (2013)

Correções (topo e sedimentos) mais apropriadas

~6 km stddev com sísmica

Moho América do Sul

- Compatível com van der Meijde et al. (2013)
- Correções (topo e sedimentos) mais apropriadas
- ~6 km stddev com sísmica
- Diferença grande concentrada nos Andes

Moho América do Sul

Compatível com van der Meijde et al. (2013)

Correções (topo e sedimentos) mais apropriadas

~6 km stddev com sísmica

Diferença grande concentrada nos Andes

Resolução maior pode ser falsa

Moho América do Sul

Compatível com van der Meijde et al. (2013)

Correções (topo e sedimentos) mais apropriadas

~6 km stddev com sísmica

Diferença grande concentrada nos Andes

Resolução maior pode ser falsa

Depende de correções corretas

Atividades 2014-2015

artigos

Geophysics | (submetido)

"Tesseroids: forward modeling of gravitational fields in spherical coordinates"

Journal of Applied Geophysics | (submetido)

"How two gravity-gradient inversion methods can be used to reveal different geologic features of ore deposit - a case study from the Quadrilátero Ferrífero (Brazil)"

Nonlinear Processes in Geophysics | doi:10.5194/npg-22-215-2015

"Estimation of the total magnetization direction of approximately spherical bodies"

artigos

1º da tese

Geophysics | (submetido)

"Tesseroids: forward modeling of gravitational fields in spherical coordinates"

Journal of Applied Geophysics | (submetido)

"How two gravity-gradient inversion methods can be used to reveal different geologic features of ore deposit - a case study from the Quadrilátero Ferrífero (Brazil)"

Nonlinear Processes in Geophysics | doi:10.5194/npg-22-215-2015

"Estimation of the total magnetization direction of approximately spherical bodies"

Implementação inversão

Testes sintético

Aplicação América do Sul

Escrita 2º artigo

Submissão 2º artigo

Entrega tese

Defesa

Implementação inversão

Testes sintético

Aplicação América do Sul

feito

Escrita 2º artigo

Submissão 2º artigo

Entrega tese

Defesa

Implementação inversão

Testes sintético

Aplicação América do Sul

→ feito

Escrita 2º artigo

Submissão 2º artigo

Entrega tese

Defesa

Implementação inversão

Testes sintético

Aplicação América do Sul

feito

Escrita 2º artigo

Submissão 2º artigo

Entrega tese

(~ final Out)

Defesa

Implementação inversão

Testes sintético

Aplicação América do Sul

feito

Escrita 2º artigo

Submissão 2º artigo

Entrega tese

(~ final Out)

Defesa

(~ Nov-Dez)

github.com/leouieda/seminario-on-2015

pinga-lab.org

PINGA

Grupo de problemas inversos em geofísica