CS 171: Intro to ML and DM

Christian Shelton

UC Riverside

Slide Set 12: Decision Trees II

Slides from CS 171

- From UC Riverside
  - CS 171: Introduction to Machine Learning and Data Mining
  - Professor Christian Shelton
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    - Elements of Statistical Learning (Hastie, et al.)
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    - Machine Learning: A Probabilistic Perspective (Murphy)
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Building a toy decision tree

x_0	x_1	x_2	y
cold	2	red	1
cold	2.3	green	1
hot	4.5	red	0
hot	6.5	green	1
cold	3	blue	0
hot	3	blue	1
cold	2	blue	0
cold	2.2	blue	1
hot	3	red	1
cold	1.5	red	1

$\boxed{y = 1}$

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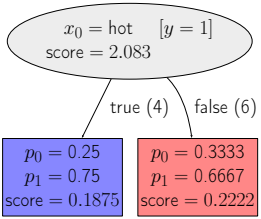

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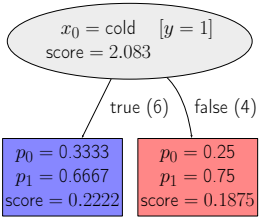

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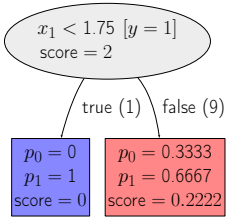

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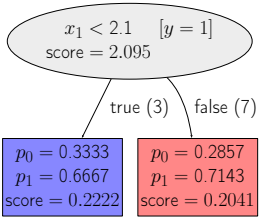

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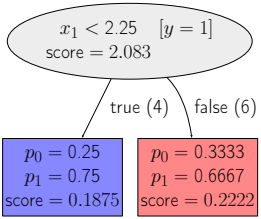

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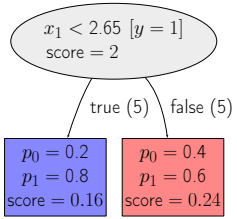

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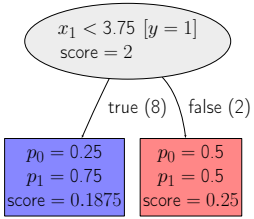

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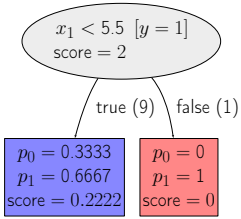

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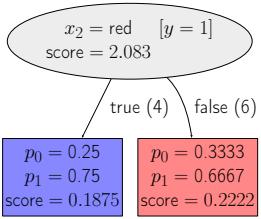

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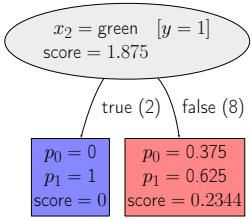

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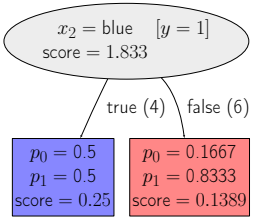

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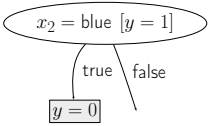

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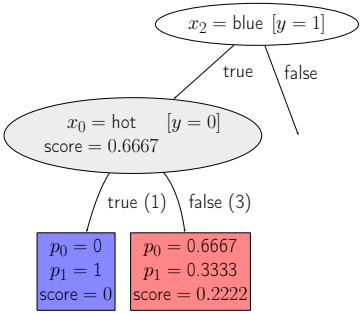

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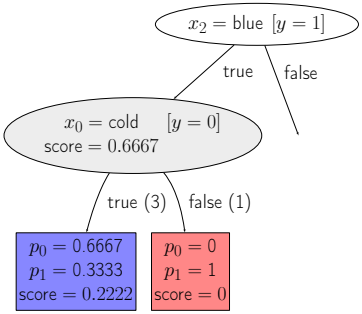

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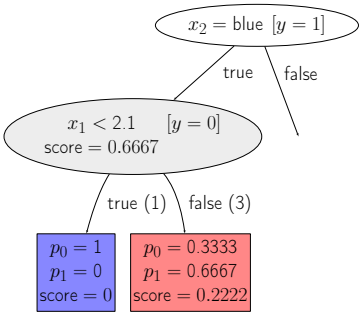

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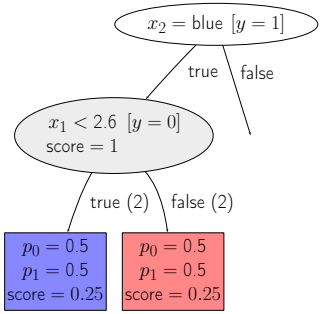

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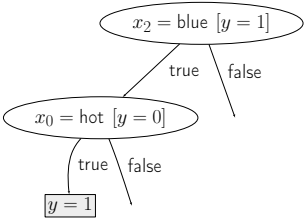

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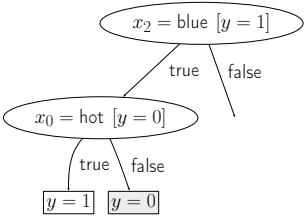

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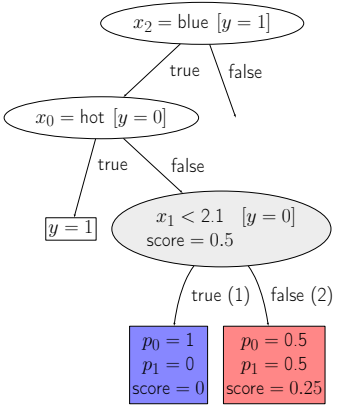

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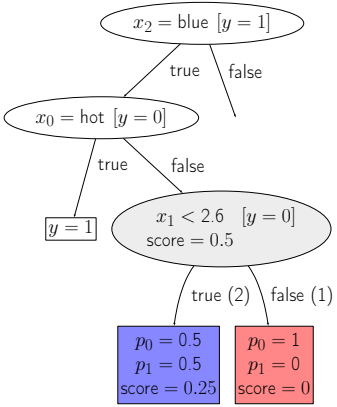

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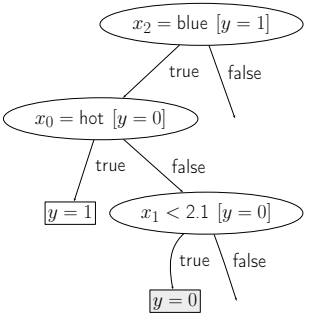

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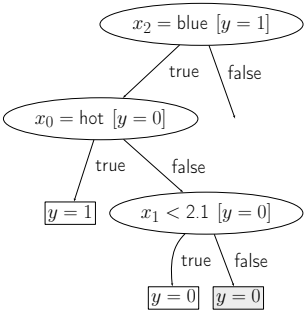

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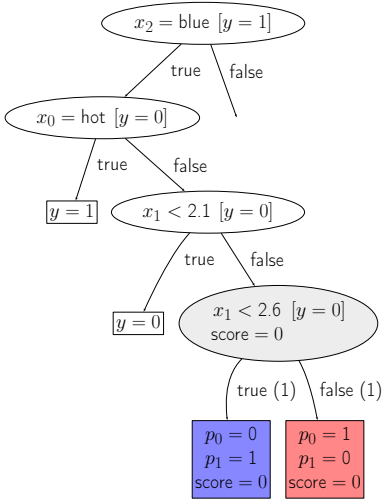

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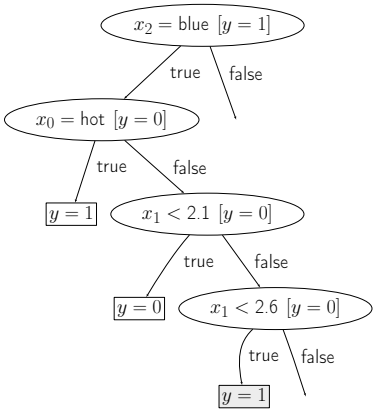

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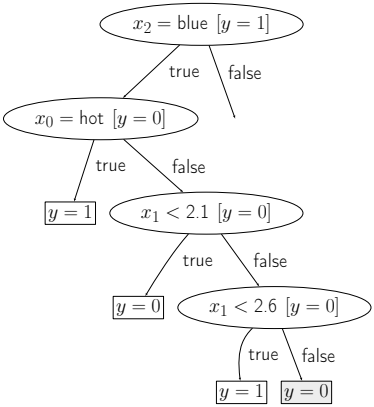

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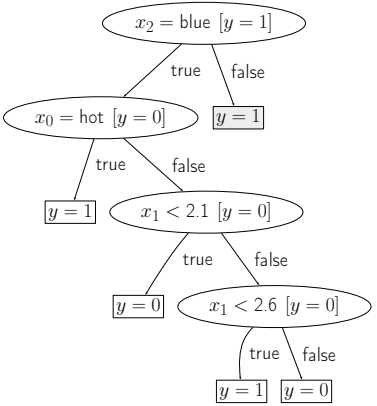

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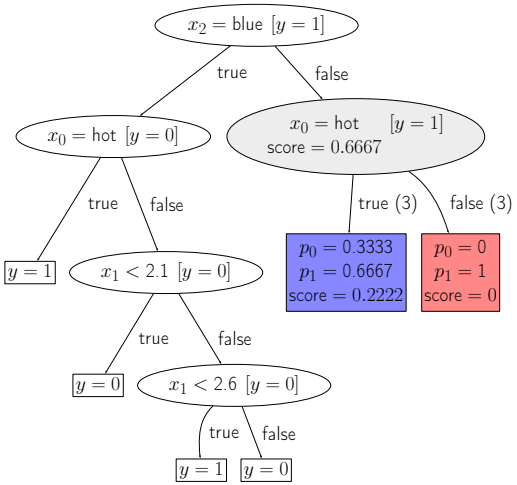

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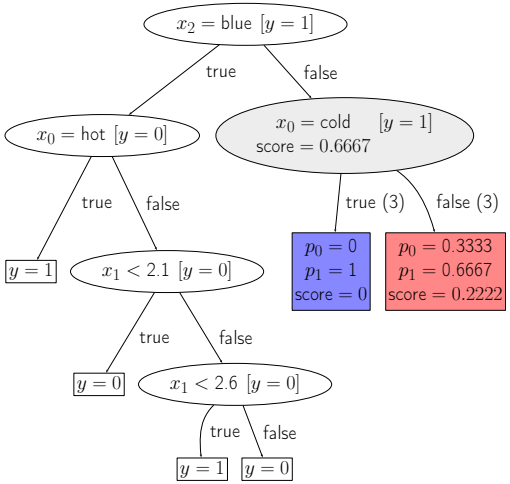
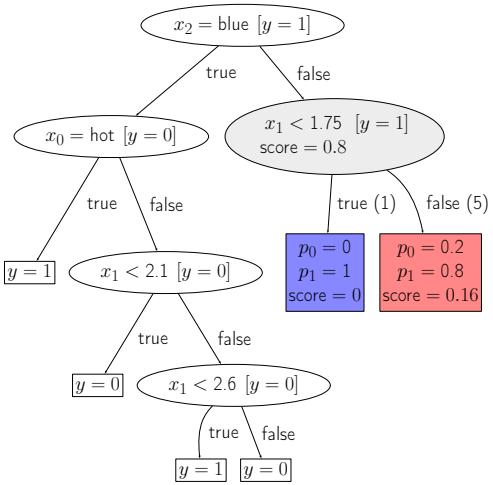


Building a toy decision tree

x_0	x_1	x_2	y
cold	2	red	1
cold	2.3	green	1
hot	4.5	red	0
hot	6.5	green	1
cold	3	blue	0
hot	3	blue	1
cold	2	blue	0
cold	2.2	blue	1
hot	3	red	1
cold	1.5	red	1



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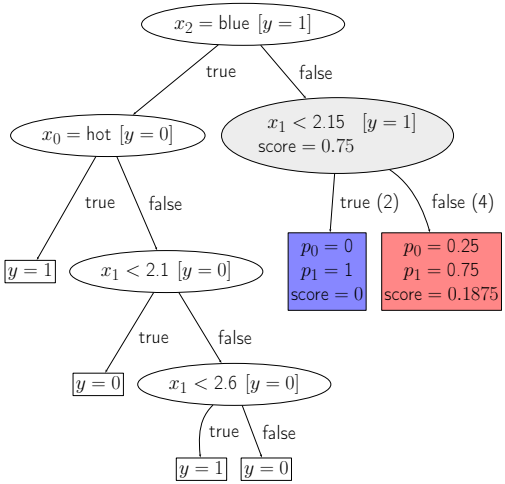
cold	2	red	1
cold	2.3	green	1
hot	4.5	red	0
hot	6.5	green	1
cold	3	blue	0
hot	3	blue	1
cold	2	blue	0
cold	2.2	blue	1
hot	3	red	1
cold	1.5	red	1



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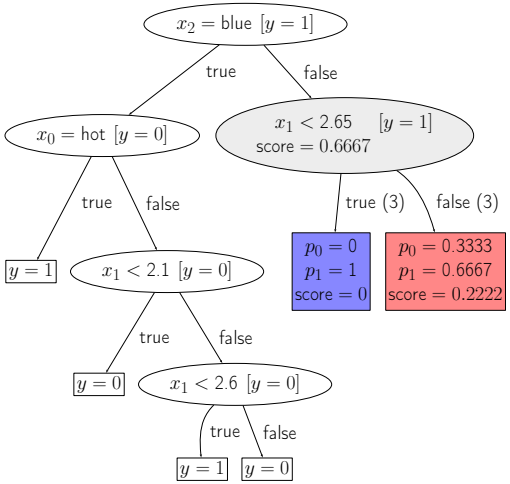
x_0	x_1	x_2	y
cold	2	red	1
cold	2.3	green	1
hot	4.5	red	0
hot	6.5	green	1
cold	3	blue	0
hot	3	blue	1
cold	2	blue	0
cold	2.2	blue	1
hot	3	red	1
cold	1.5	red	1



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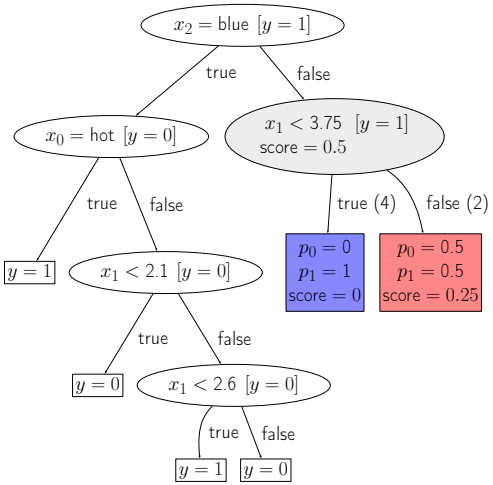
cold	3	blue	0
hot	3	blue	1
cold	2	blue	0
cold	2.2	blue	1
hot	3	red	1
cold	1.5	red	1



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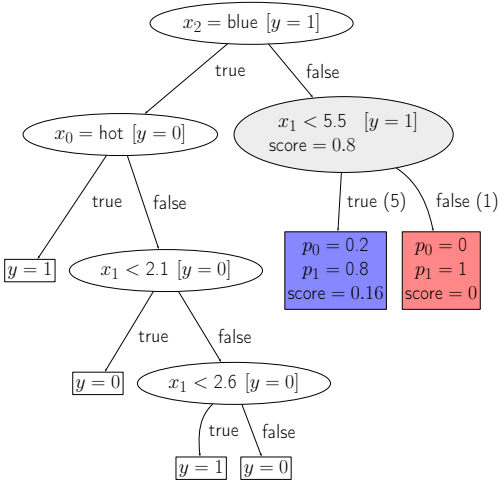
cold	3	blue	0
hot	3	blue	1
cold	2	blue	0
cold	2.2	blue	1
hot	3	red	1
cold	1.5	red	1



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cold	3	blue	0
hot	3	blue	1
cold	2	blue	0
cold	2.2	blue	1
hot	3	red	1
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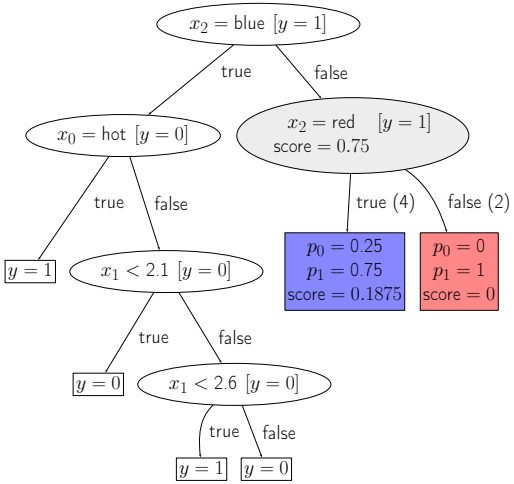

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hot	3	blue	1
cold	2	blue	0
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hot	3	red	1
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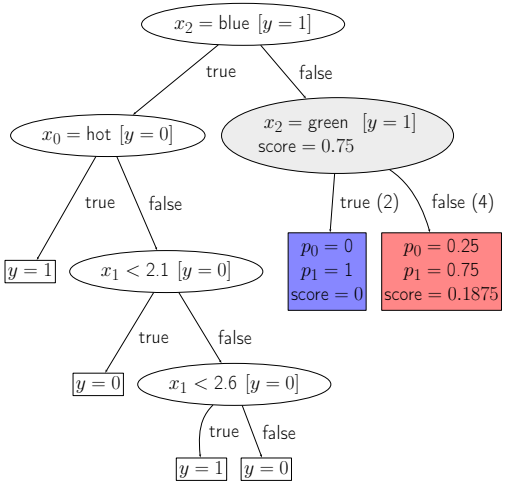

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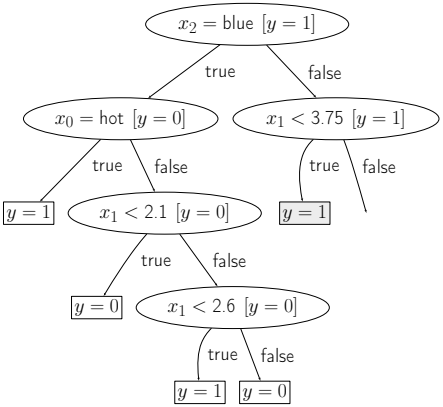

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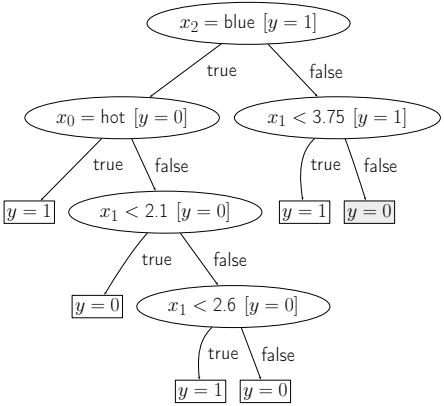

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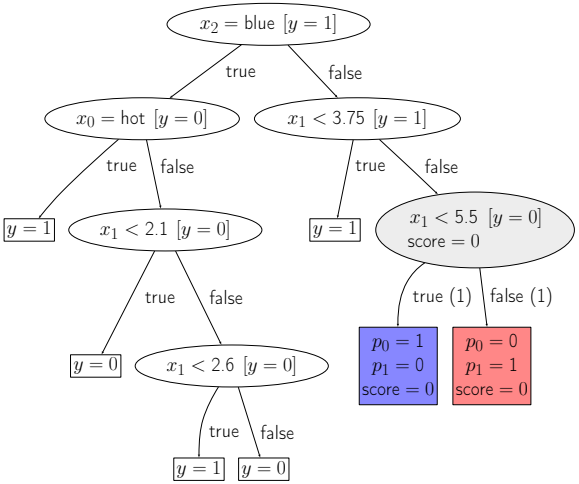

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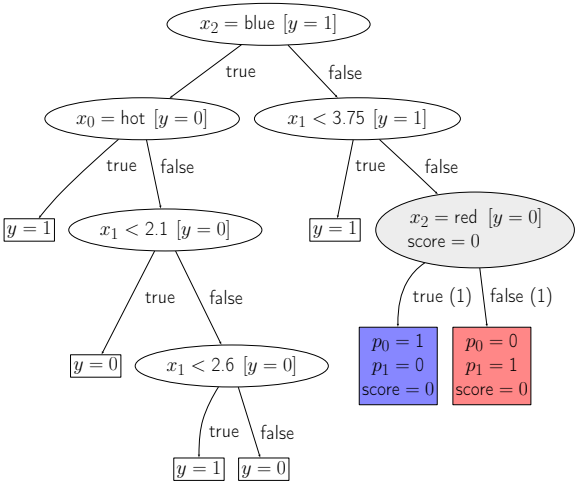

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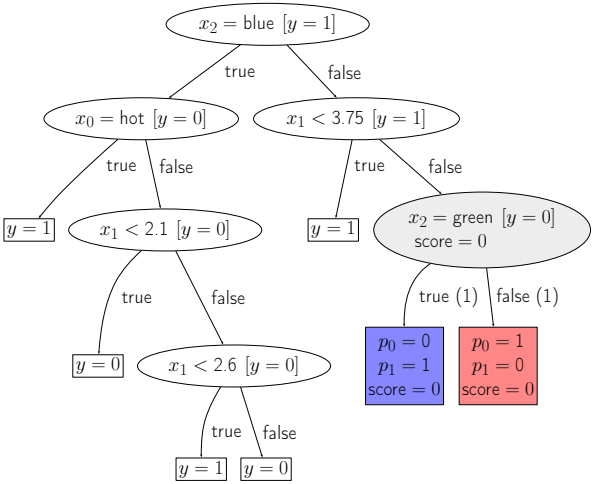

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cold	2.2	blue	1
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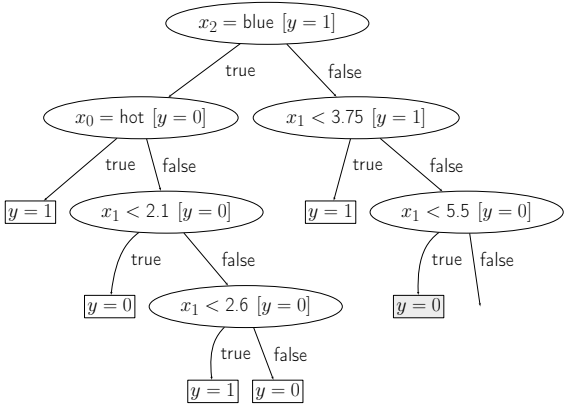

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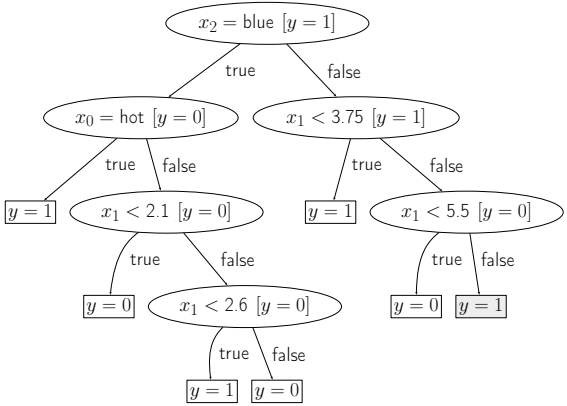

Decision Tree Pruning

Consider (recursively, bottom-up) replacing each subtree with a leaf

- Cost-complexity pruning: Prune if it improves $L + \alpha\|\mathcal{T}\|$ (α chosen by cross-validation)
- Chi-squared pruning: use statistical test to check if test is correlated with label
- Reduced error pruning: Prune if doing so does not change or improves **pruning set** (like a validation set) **error**

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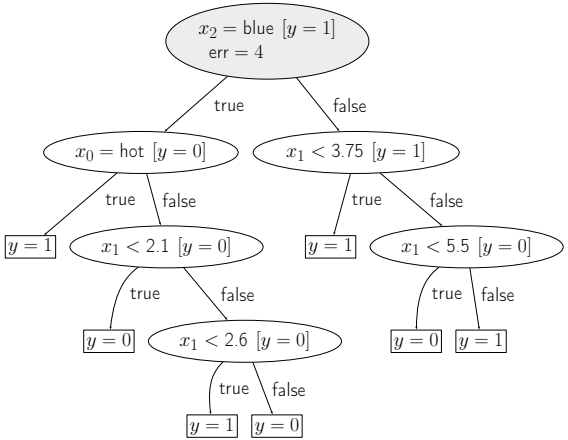
x_0	x_1	x_2	y
cold	3.5	blue	1
cold	1.5	blue	0
cold	2	blue	0
hot	3.5	red	0
cold	5.9	green	1
cold	2.2	blue	0
hot	3	blue	1
cold	3	blue	1



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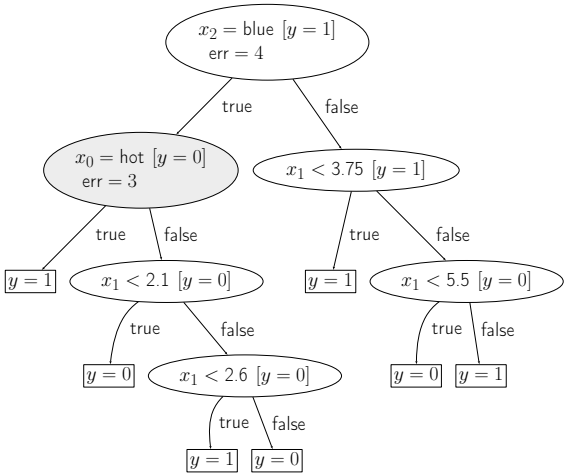
x_0	x_1	x_2	y
cold	3.5	blue	1
cold	1.5	blue	0
cold	2	blue	0
hot	3.5	red	0
cold	5.9	green	1
cold	2.2	blue	0
hot	3	blue	1
cold	3	blue	1



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cold	3.5	blue	1
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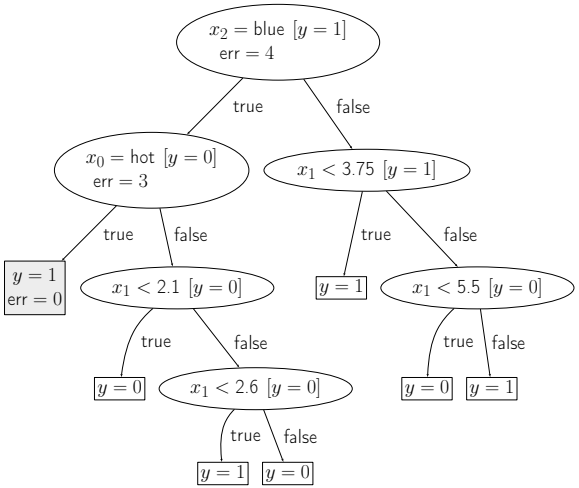

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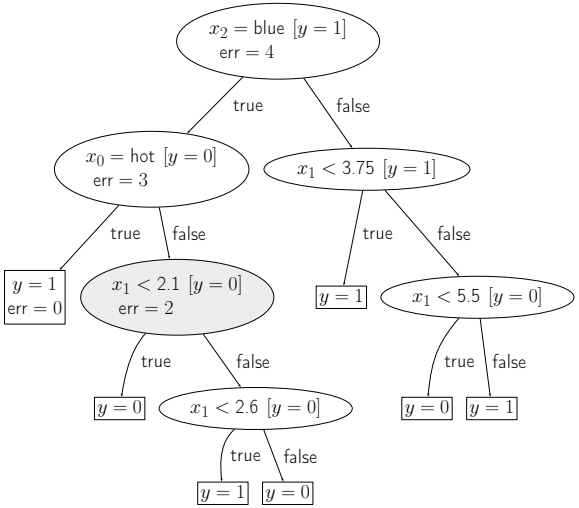

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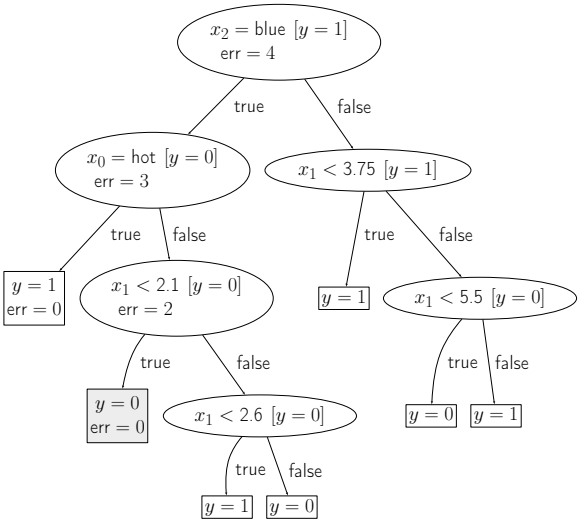

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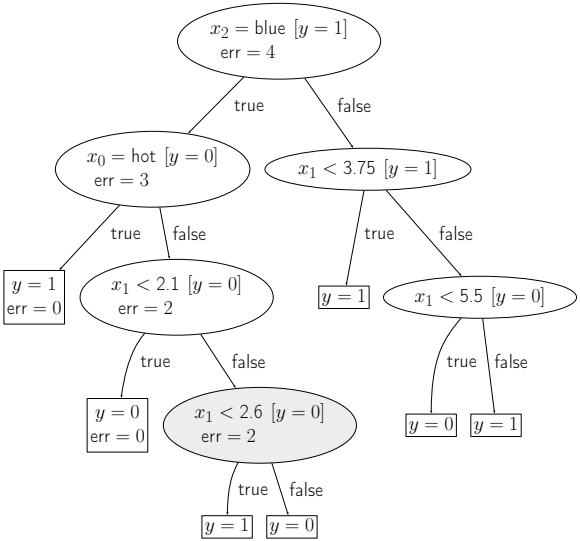

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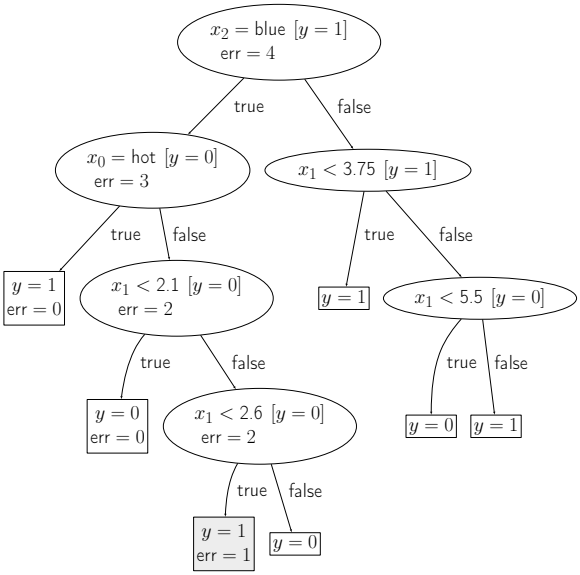

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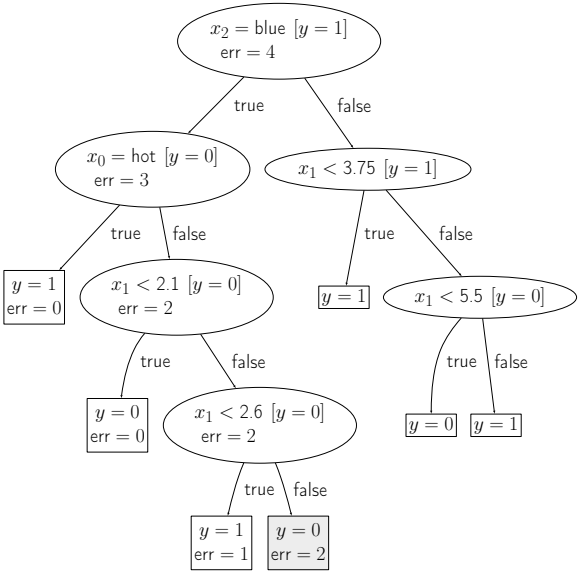

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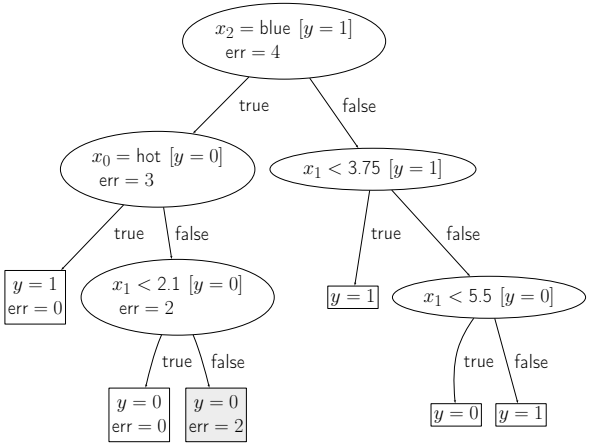

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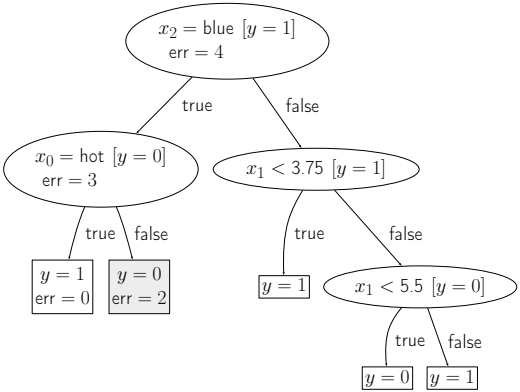

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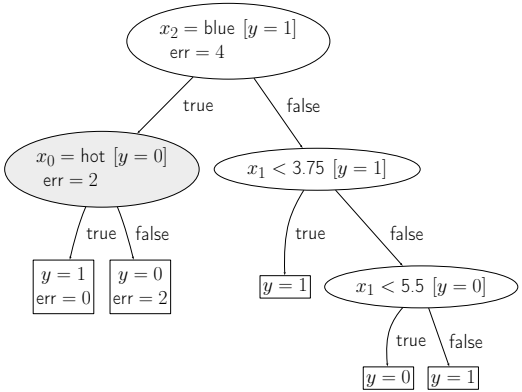

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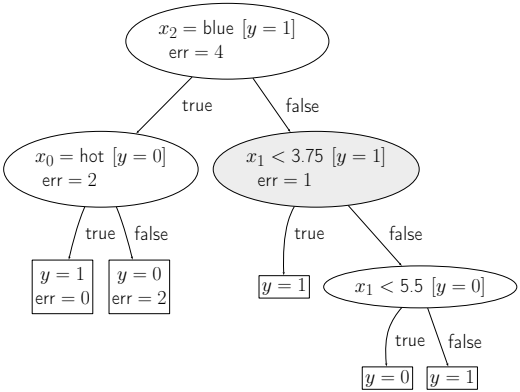

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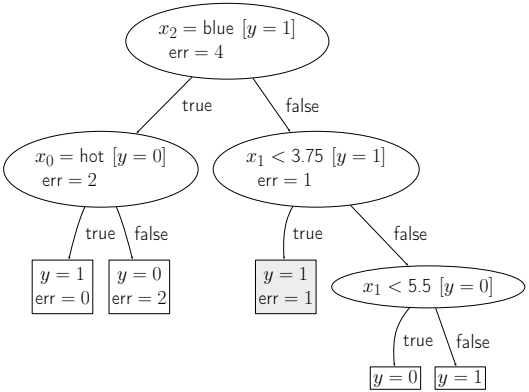

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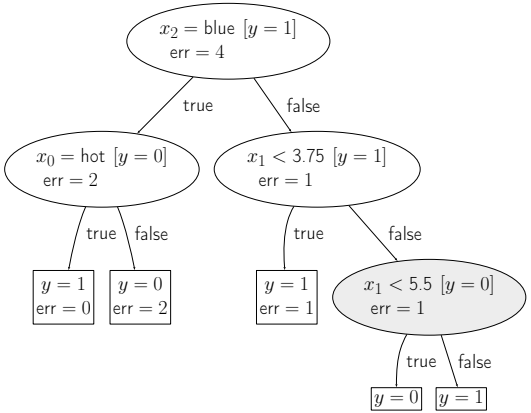
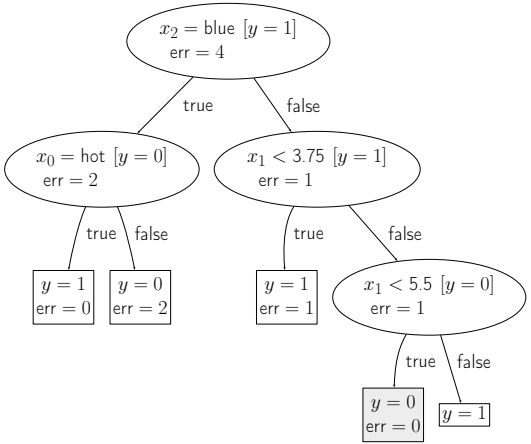


Building a toy decision tree

x_0	x_1	x_2	y
cold	3.5	blue	1
cold	1.5	blue	0
cold	2	blue	0
hot	3.5	red	0
cold	5.9	green	1
cold	2.2	blue	0
hot	3	blue	1
cold	3	blue	1



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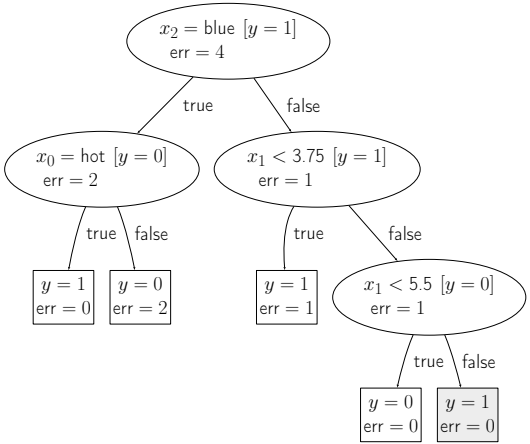

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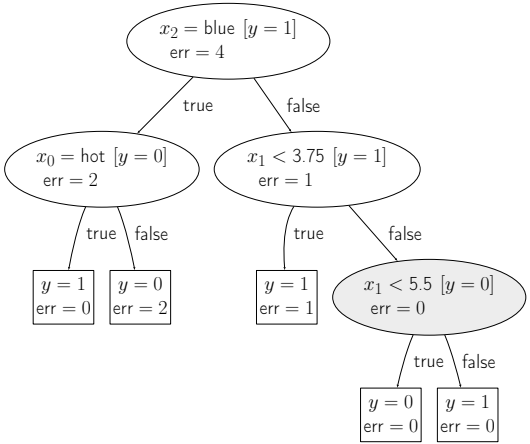

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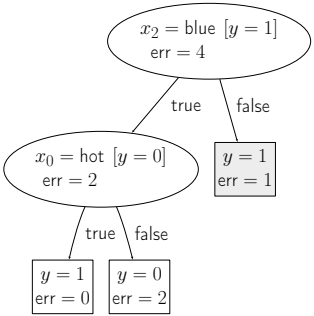

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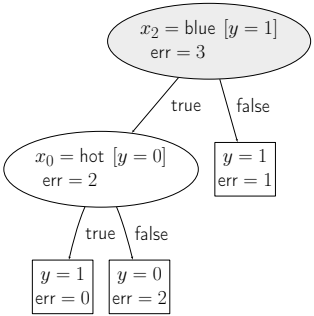
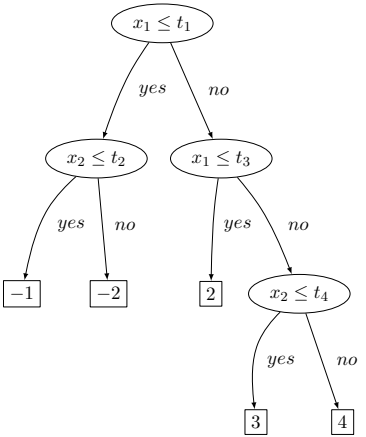
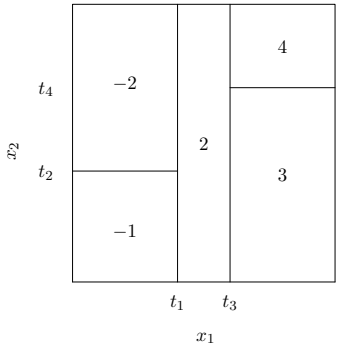


Regression Trees

Decision trees can be turned into **regression** trees:
$f(x)$ represented by a (binary) tree.

- Non-leaves are test
  - Usually uni-variate single-threshold tests
- Leaves are predicted values **(real valued scalar)**





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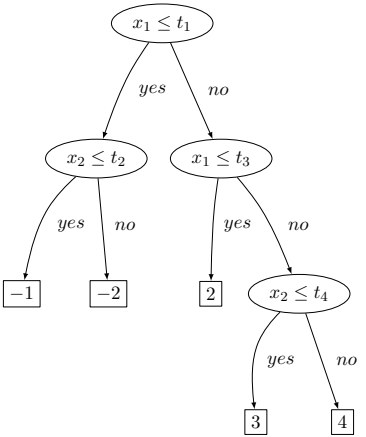

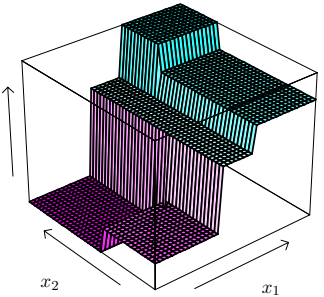

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  Many different splits, but can build up set incrementally.
- Missing values:
  - Use ternary splits (extra "missing" branch)
  - Use surrogate splits (a second split that best mimics the primary split)

Decision Trees

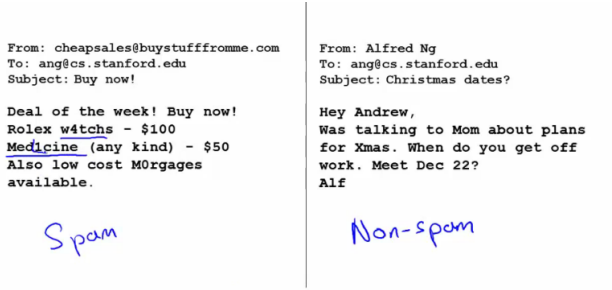
Benefits:

- Interpretable (?)
- Missing values
- Categorical features

Problems:

- High variance
- Not smooth
- Cannot produce linear model

Spam Example



Top image courtesy of Andrew Ng
Bottom example from George Forman of HP Labs

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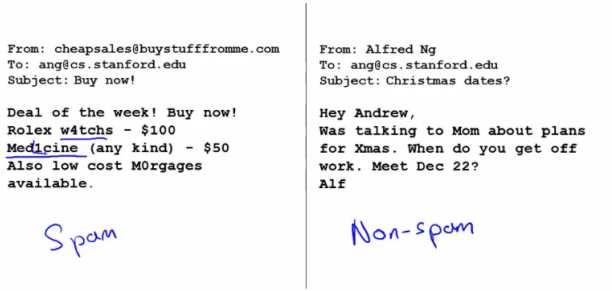

Convert e-mail into features:

- (1) CAPMAX: length of longest uninterrupted sequence of capitals
- (1) CAPAVE: average length of capitals sequences
- (1) CAPTOT: total number of capitals
- (48) % of words that match a particular word, ex: business, address, internet, free, george
- (6) % of characters that match ; ([$ #

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