CS 171: Intro to ML and DM

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UC Riverside

Slide Set 5: Cross Validation

Slides from CS 171

- From UC Riverside
  - CS 171: Introduction to Machine Learning and Data Mining
  - Professor Christian Shelton
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    - Elements of Statistical Learning (Hastie, et al.)
    - Pattern Recognition and Machine Learning (Bishop)
    - An Introduction to Machine Learning (Kubat)
    - Machine Learning: A Probabilistic Perspective (Murphy)
  - For use only by enrolled students in the course

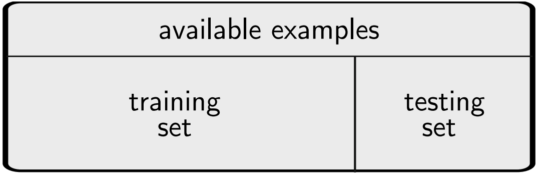
Cross-Validation for Regularization

So, how to pick λ?

Cross-Validation for Regularization

So, how to pick λ?
Cross validation! (or n-fold cross validation)

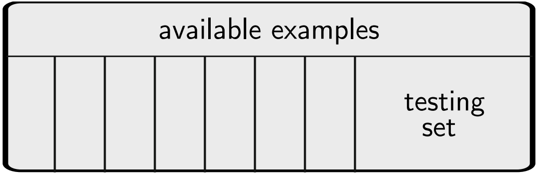
Cross Validation



Cross Validation

available examples		
training set	validation set	testing set

Cross Validation



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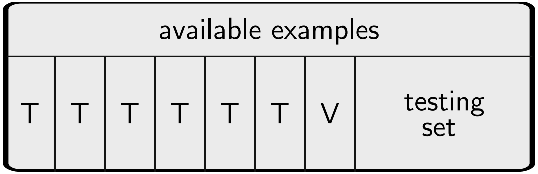

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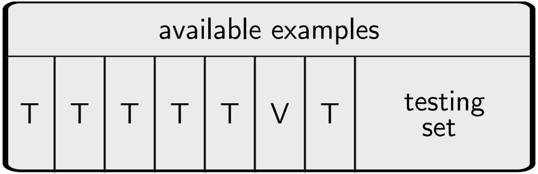

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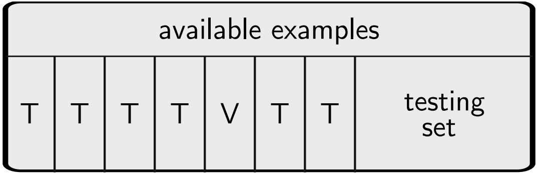

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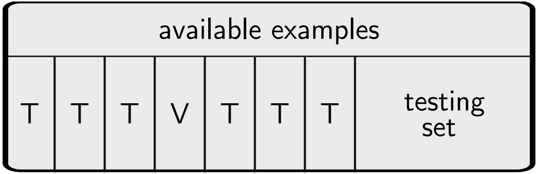

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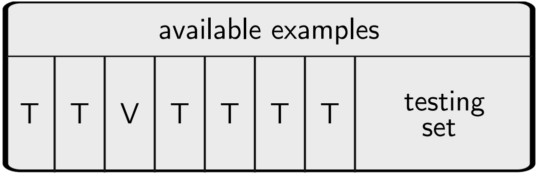

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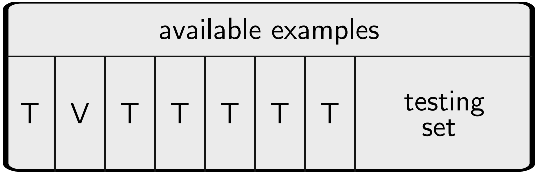

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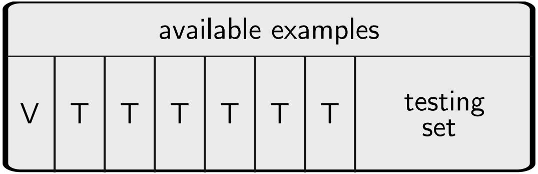


Cross Validation

Cross Validation:

- Split Training data into two parts: Train and Validation
- For each version (different k), train on train and check performance on validate
- Pick the version that does best on validation set

n-fold Cross Validation:

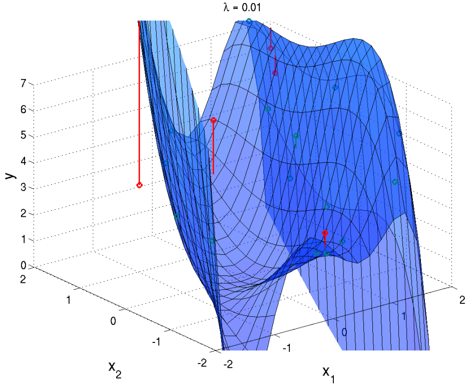
- Do cross validation n times (n different train/validation splits)
- Pick version (value of k) that does best on average across these n different trails
- Most common: partition data into n equal-sized sets and use each one as validation once (and other $n-1$ sets as train)

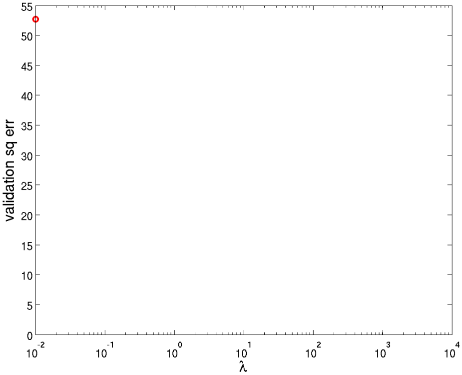
Leave-one-out Cross Validation (LOO CV):

- n-fold Cross Validation where $n = m$

Cross-Validation for Regularization

5th order polynomial





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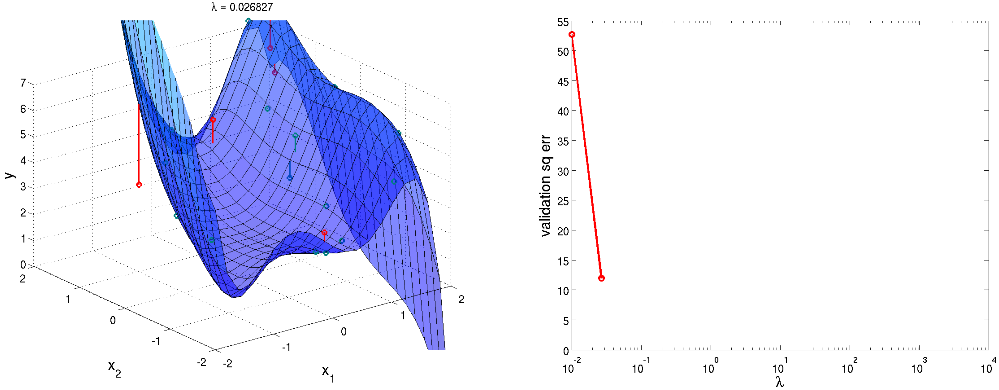

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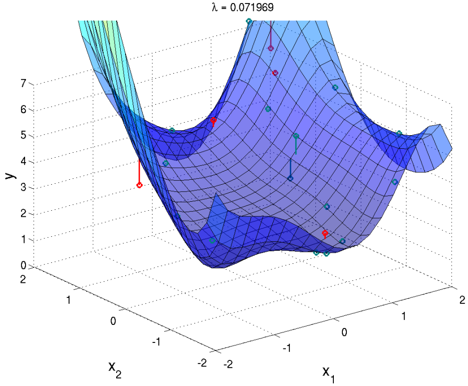

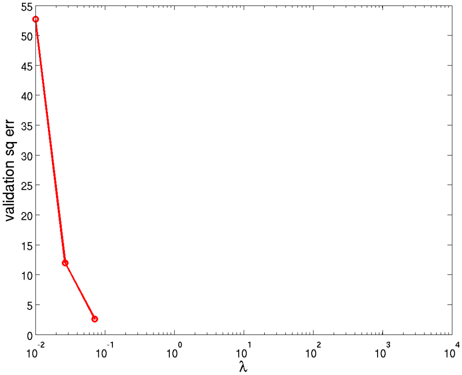

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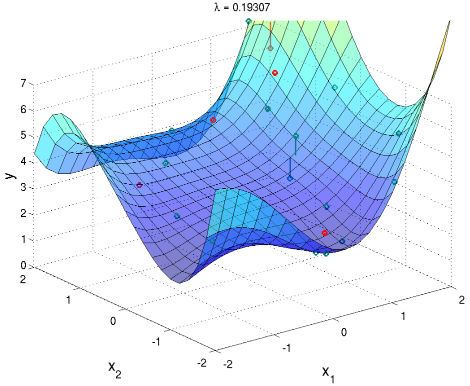

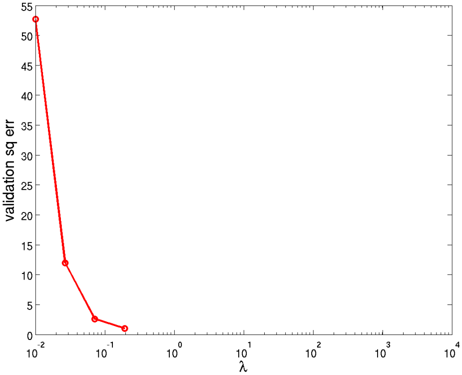

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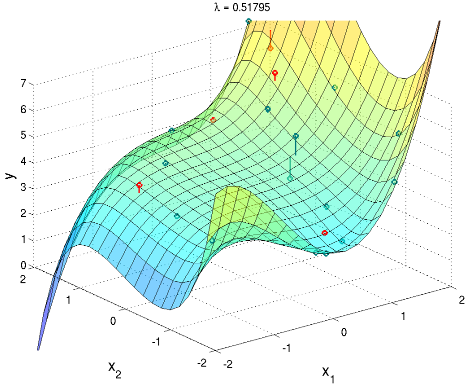

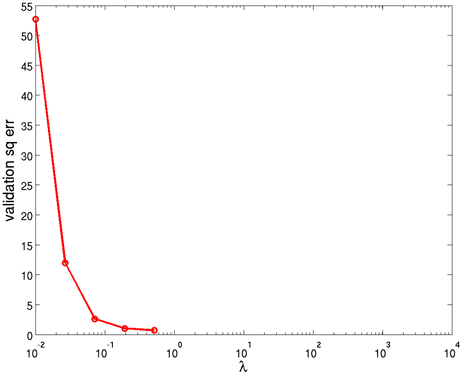

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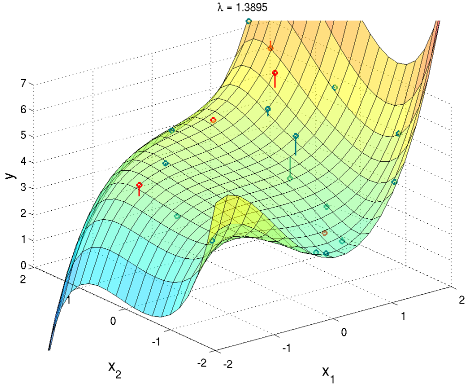

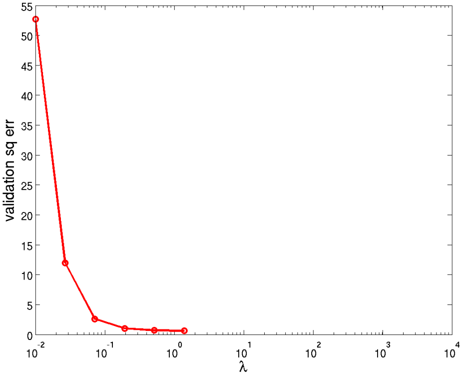

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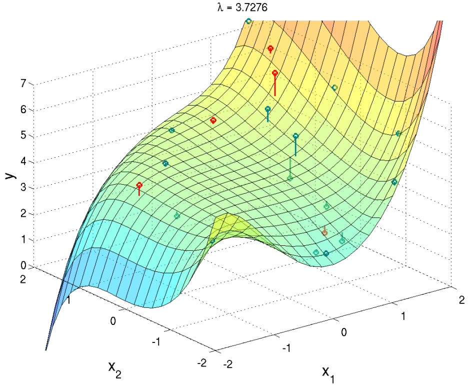

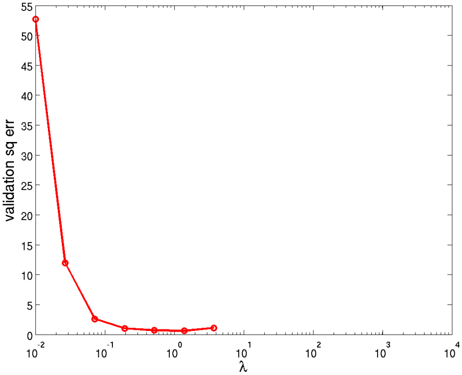

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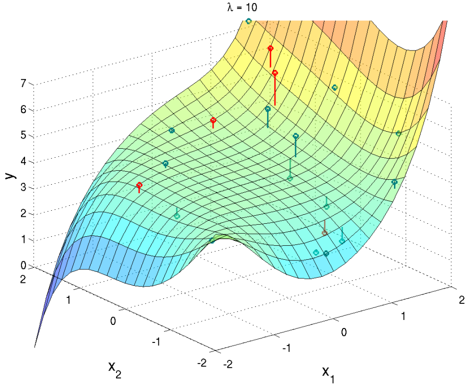

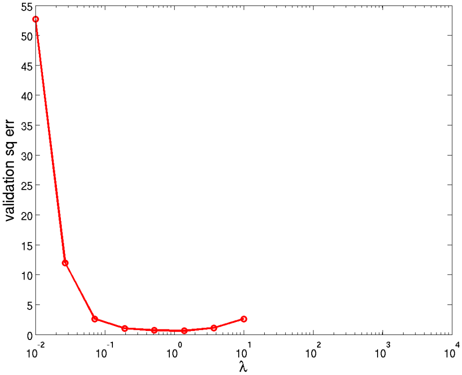

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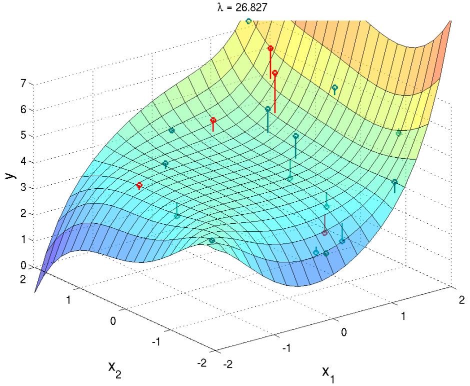

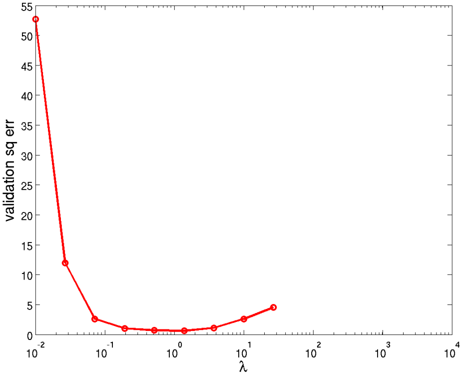

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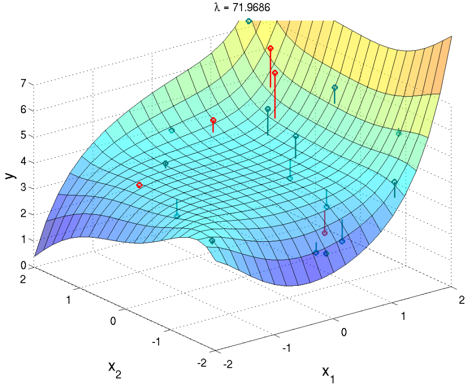

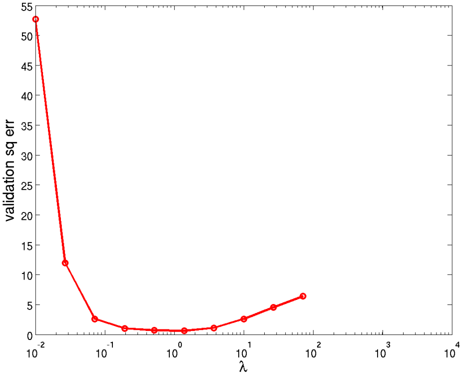

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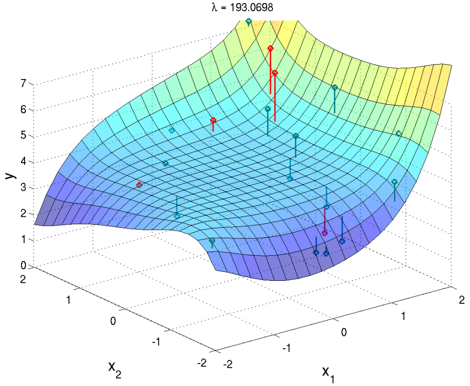

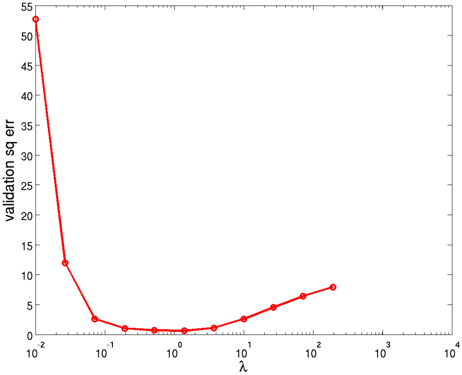

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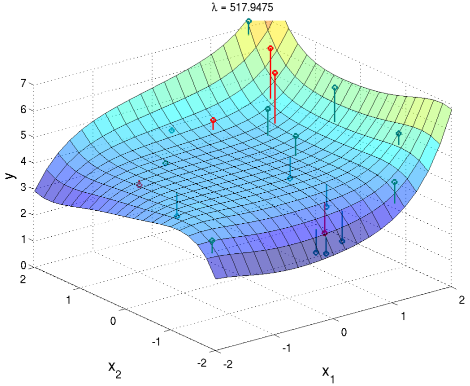

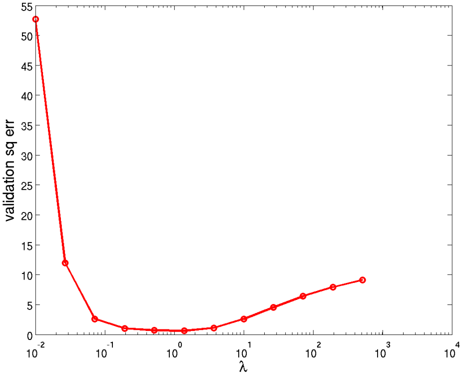

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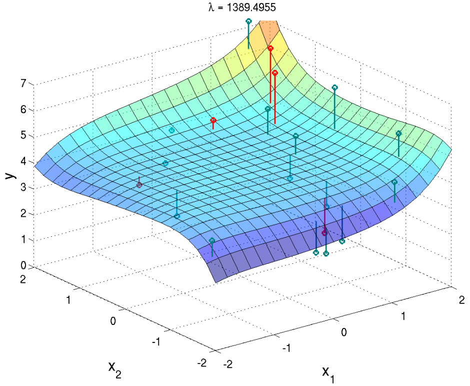

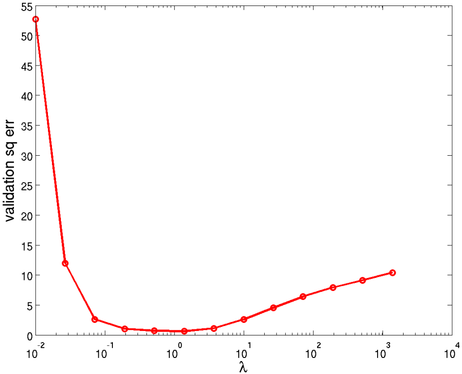

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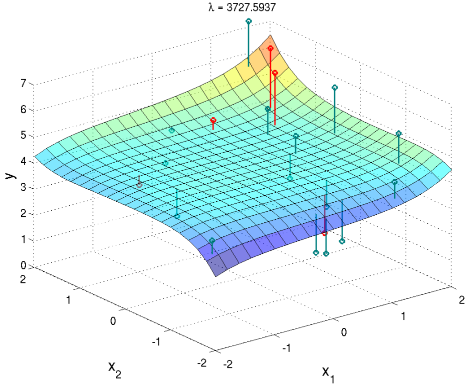

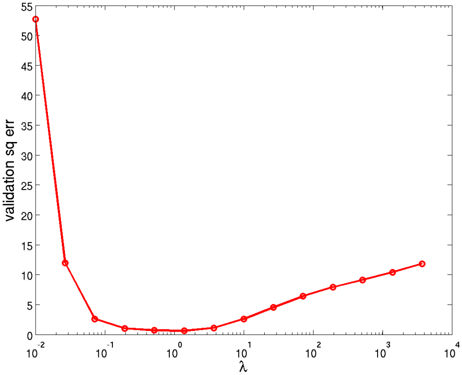

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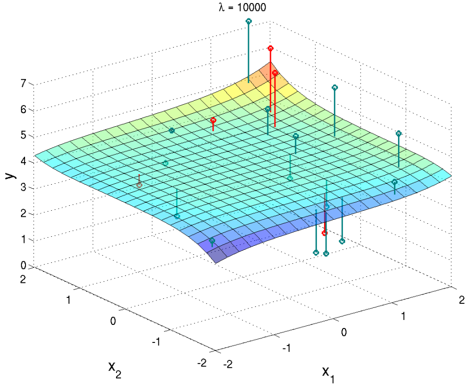

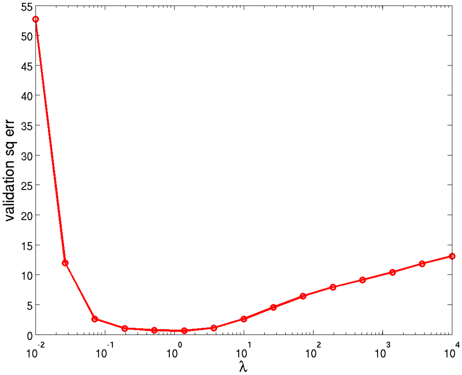

Regularized Least Squares

Algorithm Development:

- Pick form of function to be estimated:

$$f(x) = w^\top x$$

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- Pick per-item loss/cost function:

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$$L = \sum_i l(y_i, f(x_i)) + R(w)$$

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- Divide Training into Training and Validation

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  - If best loss so far, remember w and λ

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  - If best loss so far, remember w and λ
- (Optional) Retrain on Training+Validation with best λ
- Report best w
- (If testing) Check average per-item loss on Testing set