CS 171: Intro to ML and DM

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UC Riverside

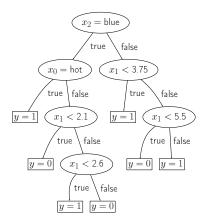
Slide Set 13: Decision Trees III



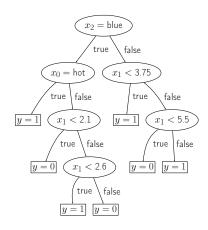
Slides from CS 171

- From UC Riverside
 - CS 171: Introduction to Machine Learning and Data Mining
 - Professor Christian Shelton
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 - ► Elements of Statistical Learning (Hastie, et al.)
 - ► Pattern Recognition and Machine Learning (Bishop)
 - An Introduction to Machine Learning (Kubat)
 - Machine Learning: A Probabilistic Perspective (Murphy)
 - ► For use only by enrolled students in the course

Decision Tree as a Rule Set



Decision Tree as a Rule Set



$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \to y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 < 2.1) \to y = 0$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \to y = 1$$

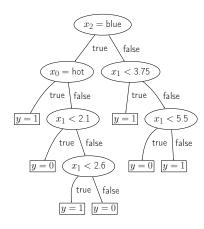
$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 \geq 2.6) \to y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \to y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \to y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 \leq 5.5) \to y = 1$$

Decision Tree as a Rule Set



$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \to y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 < 2.1) \to y = 0$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \to y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \to y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \to y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \to y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) \to y = 1$$

 x_2 cold 2 red cold 2.3 green 1 hot 1 red 1 hot 4.5 red cold 1.5 blue 0 hot 6.5 green 1 cold 3 blue 0 hot 3 blue 1 cold 3.1 blue 1 cold 5.3 green 0 cold 2 blue 0 cold 2.2 blue 1 hot 3 red cold 1.5 red cold 1.8 blue 1 cold 1.8 blue 1

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1 \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 < 2.1) &\to y = 0 \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) &\to y = 1 \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) &\to y = 1 \end{split}$$

cold 2 red 1 cold 2.3 green 1 hot 4.5 red 0 cold 1.5 blue 0 hot 6.5 green 1 cold 3 blue 0 hot 3 blue 1 cold 3.1 blue 1 cold 5.3 green 0 cold 2 blue 0 cold 2.2 blue 1 hot 3 red 1 cold 1.5 red 1 cold 1.8 blue 1 cold 1.8 blue 1

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 < 2.1) \rightarrow y = 0$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) \rightarrow y = 1$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 4/5]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 < 2.1) \rightarrow y = 0$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) \rightarrow y = 1$$

```
cold 2 red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 5/8]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 < 2.1) \rightarrow y = 0$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) \rightarrow y = 1$$

cold 2 red 1 cold 2.3 green 1 hot 4.5 red 0 cold 1.5 blue 0 hot 6.5 green 1 cold 3 blue 0 hot 3 blue 1 cold 3.1 blue 1 cold 5.3 green 0 cold 2 blue 0 cold 2.2 blue 1 hot 3 red 1 cold 1.5 red 1 cold 1.8 blue 1 cold 1.8 blue 1

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 < 2.1) \rightarrow y = 0$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) \rightarrow y = 1$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 < 2.1) \rightarrow y = 0[\mathsf{acc} = 2/4]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

```
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 < 2.1) \rightarrow y = 0[\mathsf{acc} = 2/6]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 > 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 < 2.1) \rightarrow y = 0[\mathsf{acc} = 2/4]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 < 2.1) \rightarrow y = 0[\mathsf{acc} = 3/7]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 > 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_1 < 2.1) \rightarrow y = 0[\mathsf{acc} = 2/4]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \leq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) \rightarrow y = 1$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_1 < 2.1) \rightarrow y = 0[\mathsf{acc} = 2/4]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \leq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) \rightarrow y = 1$$

```
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \!\!\rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_1 < 2.1) \!\!\rightarrow y = 0[\mathsf{acc} = 2/7]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \!\!\rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \!\!\rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \!\!\rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \!\!\rightarrow y = 0$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_1 < 2.1) \rightarrow y = 0[\mathsf{acc} = 3/8]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_1 < 2.1) \rightarrow y = 0[\mathsf{acc} = 2/4]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \leq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) \rightarrow y = 1$$

```
cold 2 red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_1 < 2.1) \rightarrow y = 0[\mathsf{acc} = 2/4]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

```
cold 2
       red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) &\to y = 0 \end{split}$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_1 < 2.1) \rightarrow y = 0[\mathsf{acc} = 2/4]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

```
cold 2 red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_1 < 2.1) \rightarrow y = 0[\mathsf{acc} = 2/4]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1[\mathsf{acc} = 3/5]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

```
cold 2 red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_1 < 2.1) \rightarrow y = 0[\mathsf{acc} = 2/4]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.1 \land x_1 < 2.6) \rightarrow y = 1[\mathsf{acc} = 2/3]$$

$$(x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) \rightarrow y = 0$$

$$(x_2 \neq \mathsf{blue} \land x_1 < 3.75) \rightarrow y = 1$$

$$(x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) \rightarrow y = 0$$

```
cold 2 red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\rightarrow y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\rightarrow y = 0[\mathsf{acc} = 2/4] \\ (x_2 = \mathsf{blue} \land x_1 \geq 2.1 \land x_1 < 2.6) &\rightarrow y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\rightarrow y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\rightarrow y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) &\rightarrow y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) &\rightarrow y = 1 \end{split}$$

```
cold 2 red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\rightarrow y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\rightarrow y = 0[\mathsf{acc} = 2/4] \\ (x_2 = \mathsf{blue} \land x_1 \geq 2.1 \land x_1 < 2.6) &\rightarrow y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\rightarrow y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\rightarrow y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) &\rightarrow y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) &\rightarrow y = 1 \end{split}$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_2 = \mathsf{blue} \land x_1 \geq 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) &\to y = 1 \end{split}$$

```
cold 2 red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\rightarrow y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\rightarrow y = 0[\mathsf{acc} = 2/4] \\ (x_2 = \mathsf{blue} \land x_1 \geq 2.1 \land x_1 < 2.6) &\rightarrow y = 1[\mathsf{acc} = 3/5] \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\rightarrow y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\rightarrow y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) &\rightarrow y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) &\rightarrow y = 1 \end{split}$$

```
cold 2 red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_2 = \mathsf{blue} \land x_1 \geq 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 3/4] \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) &\to y = 1 \end{split}$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \wedge x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \wedge x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \geq 2.1 \wedge x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_2 = \mathsf{blue} \wedge x_0 \neq \mathsf{hot} \wedge x_1 \geq 2.6) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \wedge x_1 < 3.75) &\to y = 1 \\ (x_2 \neq \mathsf{blue} \wedge x_1 \geq 3.75 \wedge x_1 < 5.5) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \wedge x_1 \geq 5.5) &\to y = 1 \end{split}$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \wedge x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \wedge x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \geq 2.1 \wedge x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_2 = \mathsf{blue} \wedge x_0 \neq \mathsf{hot} \wedge x_1 \geq 2.6) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \wedge x_1 < 3.75) &\to y = 1 \\ (x_2 \neq \mathsf{blue} \wedge x_1 \geq 3.75 \wedge x_1 < 5.5) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \wedge x_1 \geq 5.5) &\to y = 1 \end{split}$$

```
x_2
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\rightarrow y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\rightarrow y = 0[\mathsf{acc} = 2/4] \\ (x_1 \geq 2.1 \land x_1 < 2.6) &\rightarrow y = 1[\mathsf{acc} = 7/9] \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\rightarrow y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\rightarrow y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) &\rightarrow y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) &\rightarrow y = 1 \end{split}$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 6/9] \\ (x_2 = \mathsf{blue} \land x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0 \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{split}$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\rightarrow y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\rightarrow y = 0[\mathsf{acc} = 2/4] \\ (x_1 \geq 2.1 \land x_1 < 2.6) &\rightarrow y = 1[\mathsf{acc} = 2/2] \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\rightarrow y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\rightarrow y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) &\rightarrow y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) &\rightarrow y = 1 \end{split}$$

```
cold 2 red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_2 = \mathsf{blue} \land x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 1/2] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{split}$$

```
cold 2 red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \geq 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) &\to y = 1 \end{aligned}$$

cold 2 red 1 cold 2.3 green 1 hot 4.5 red 0 cold 1.5 blue 0 hot 6.5 green 1 cold 3 blue 0 hot 3 blue 1 cold 3.1 blue 1 cold 5.3 green 0 cold 2 blue 0 cold 2.2 blue 1 hot 3 red 1 cold 1.5 red 1 cold 1.8 blue 1 cold 1.8 blue 1

$$\begin{aligned} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \geq 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\to y = 0[\mathsf{acc} = 1/3] \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) &\to y = 1 \end{aligned}$$

```
cold 2 red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \geq 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_2 = \mathsf{blue} \land x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\to y = 0[\mathsf{acc} = 3/7] \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) &\to y = 1 \end{aligned}$$

```
red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{split}$$

```
red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{split}$$

```
red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\rightarrow y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\rightarrow y = 0[\mathsf{acc} = 2/4] \\ (x_1 \geq 2.1 \land x_1 < 2.6) &\rightarrow y = 1[\mathsf{acc} = 2/2] \\ (x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\rightarrow y = 0[\mathsf{acc} = 3/7] \\ (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\rightarrow y = 1 \\ (x_2 \neq \mathsf{blue} \land x_1 \leq 3.75 \land x_1 < 5.5) &\rightarrow y = 0 \\ (x_2 \neq \mathsf{blue} \land x_1 \geq 5.5) &\rightarrow y = 1 \end{split}$$

```
x_2
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 4/11] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{split}$$

```
red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{split}$$

```
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 5/5] \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{split}$$

```
x_2
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 10/13] \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{aligned}$$

```
x_2
cold 2
         red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 6/8] \\ (x_2 \ne \mathsf{blue} \land x_1 < 5.5) &\to y = 0 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{split}$$

```
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 5/5] \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 3.75 \land x_1 < 5.5) &\to y = 0 \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{split}$$

```
red
cold 2.3 green 1
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 5/5] \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 3.75 \land x_1 < 5.5) &\to y = 0[\mathsf{acc} = 2/2] \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{split}$$

```
red
cold 2.3 green 1
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 5/5] \\ (x_2 \ne \mathsf{blue} \land x_1 < 5.5) &\to y = 0[\mathsf{acc} = 2/2] \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{aligned}$$

```
x_2
cold 2
         red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 5/5] \\ (x_2 \ne \mathsf{blue} \land x_1 < 5.5) &\to y = 0[\mathsf{acc} = 2/7] \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{split}$$

```
red
cold 2.3 green 1
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \\ (x_1 \ge 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \ne \mathsf{blue} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 5/5] \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 3.75 \land x_1 < 5.5) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) &\to y = 1 \end{split}$$

```
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} &(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \!\!\to y = 1[\mathsf{acc} = 1/1] \\ &(x_2 = \mathsf{blue} \land x_1 < 2.1) \!\!\to y = 0[\mathsf{acc} = 2/4] \\ &(x_1 \ge 2.1 \land x_1 < 2.6) \!\!\to y = 1[\mathsf{acc} = 2/2] \\ &(x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) \!\!\to y = 0[\mathsf{acc} = 2/3] \\ &(x_2 \ne \mathsf{blue} \land x_1 < 3.75) \!\!\to y = 1[\mathsf{acc} = 5/5] \\ &(x_1 \ge 3.75 \land x_1 < 5.5) \!\!\to y = 0[\mathsf{acc} = 2/2] \\ &(x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) \!\!\to y = 1 \end{aligned}$$

```
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} &(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \!\!\to y = 1[\mathsf{acc} = 1/1] \\ &(x_2 = \mathsf{blue} \land x_1 < 2.1) \!\!\to y = 0[\mathsf{acc} = 2/4] \\ &(x_1 \ge 2.1 \land x_1 < 2.6) \!\!\to y = 1[\mathsf{acc} = 2/2] \\ &(x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) \!\!\to y = 0[\mathsf{acc} = 2/3] \\ &(x_2 \ne \mathsf{blue} \land x_1 < 3.75) \!\!\to y = 1[\mathsf{acc} = 5/5] \\ &(x_1 \ge 3.75 \land x_1 < 5.5) \!\!\to y = 0[\mathsf{acc} = 2/2] \\ &(x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) \!\!\to y = 1 \end{aligned}$$

```
x_2
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} &(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \!\!\to y = 1[\mathsf{acc} = 1/1] \\ &(x_2 = \mathsf{blue} \land x_1 < 2.1) \!\!\to y = 0[\mathsf{acc} = 2/4] \\ &(x_1 \ge 2.1 \land x_1 < 2.6) \!\!\to y = 1[\mathsf{acc} = 2/2] \\ &(x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) \!\!\to y = 0[\mathsf{acc} = 2/3] \\ &(x_2 \ne \mathsf{blue} \land x_1 < 3.75) \!\!\to y = 1[\mathsf{acc} = 5/5] \\ &(x_1 \ge 3.75 \land x_1 < 5.5) \!\!\to y = 0[\mathsf{acc} = 5/15] \\ &(x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) \!\!\to y = 1 \end{aligned}$$

```
red 1
cold 2.3 green 1
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} &(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \!\!\to y = 1[\mathsf{acc} = 1/1] \\ &(x_2 = \mathsf{blue} \land x_1 < 2.1) \!\!\to y = 0[\mathsf{acc} = 2/4] \\ &(x_1 \ge 2.1 \land x_1 < 2.6) \!\!\to y = 1[\mathsf{acc} = 2/2] \\ &(x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) \!\!\to y = 0[\mathsf{acc} = 2/3] \\ &(x_2 \ne \mathsf{blue} \land x_1 < 3.75) \!\!\to y = 1[\mathsf{acc} = 5/5] \\ &(x_1 \ge 3.75 \land x_1 < 5.5) \!\!\to y = 0[\mathsf{acc} = 2/3] \\ &(x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) \!\!\to y = 1 \end{aligned}$$

```
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} &(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \!\!\to y = 1[\mathsf{acc} = 1/1] \\ &(x_2 = \mathsf{blue} \land x_1 < 2.1) \!\!\to y = 0[\mathsf{acc} = 2/4] \\ &(x_1 \ge 2.1 \land x_1 < 2.6) \!\!\to y = 1[\mathsf{acc} = 2/2] \\ &(x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) \!\!\to y = 0[\mathsf{acc} = 2/3] \\ &(x_2 \ne \mathsf{blue} \land x_1 < 3.75) \!\!\to y = 1[\mathsf{acc} = 5/5] \\ &(x_1 \ge 3.75 \land x_1 < 5.5) \!\!\to y = 0[\mathsf{acc} = 2/2] \\ &(x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) \!\!\to y = 1 \end{aligned}$$

```
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} &(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \!\!\to y = 1[\mathsf{acc} = 1/1] \\ &(x_2 = \mathsf{blue} \land x_1 < 2.1) \!\!\to y = 0[\mathsf{acc} = 2/4] \\ &(x_1 \ge 2.1 \land x_1 < 2.6) \!\!\to y = 1[\mathsf{acc} = 2/2] \\ &(x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) \!\!\to y = 0[\mathsf{acc} = 2/3] \\ &(x_2 \ne \mathsf{blue} \land x_1 < 3.75) \!\!\to y = 1[\mathsf{acc} = 5/5] \\ &(x_1 \ge 3.75 \land x_1 < 5.5) \!\!\to y = 0[\mathsf{acc} = 2/2] \\ &(x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) \!\!\to y = 1[\mathsf{acc} = 1/1] \end{aligned}$$

```
red
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} &(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \!\!\to y = 1[\mathsf{acc} = 1/1] \\ &(x_2 = \mathsf{blue} \land x_1 < 2.1) \!\!\to y = 0[\mathsf{acc} = 2/4] \\ &(x_1 \ge 2.1 \land x_1 < 2.6) \!\!\to y = 1[\mathsf{acc} = 2/2] \\ &(x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) \!\!\to y = 0[\mathsf{acc} = 2/3] \\ &(x_2 \ne \mathsf{blue} \land x_1 < 3.75) \!\!\to y = 1[\mathsf{acc} = 5/5] \\ &(x_1 \ge 3.75 \land x_1 < 5.5) \!\!\to y = 0[\mathsf{acc} = 2/2] \\ &(x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) \!\!\to y = 1[\mathsf{acc} = 1/1] \end{aligned}$$

```
x_2
cold 2
         red
cold 2.3 green 1
hot 1 red
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} &(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \!\!\to y = 1[\mathsf{acc} = 1/1] \\ &(x_2 = \mathsf{blue} \land x_1 < 2.1) \!\!\to y = 0[\mathsf{acc} = 2/4] \\ &(x_1 \ge 2.1 \land x_1 < 2.6) \!\!\to y = 1[\mathsf{acc} = 2/2] \\ &(x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) \!\!\to y = 0[\mathsf{acc} = 2/3] \\ &(x_2 \ne \mathsf{blue} \land x_1 < 3.75) \!\!\to y = 1[\mathsf{acc} = 5/5] \\ &(x_1 \ge 3.75 \land x_1 < 5.5) \!\!\to y = 0[\mathsf{acc} = 2/2] \\ &(x_2 \ne \mathsf{blue} \land x_1 \ge 5.5) \!\!\to y = 1[\mathsf{acc} = 6/8] \end{aligned}$$

```
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} &(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \!\!\to y = 1[\mathsf{acc} = 1/1] \\ &(x_2 = \mathsf{blue} \land x_1 < 2.1) \!\!\to y = 0[\mathsf{acc} = 2/4] \\ &(x_1 \ge 2.1 \land x_1 < 2.6) \!\!\to y = 1[\mathsf{acc} = 2/2] \\ &(x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) \!\!\to y = 0[\mathsf{acc} = 2/3] \\ &(x_2 \ne \mathsf{blue} \land x_1 < 3.75) \!\!\to y = 1[\mathsf{acc} = 5/5] \\ &(x_1 \ge 3.75 \land x_1 < 5.5) \!\!\to y = 0[\mathsf{acc} = 2/2] \\ &(x_1 \ge 5.5) \!\!\to y = 1[\mathsf{acc} = 1/1] \end{aligned}$$

```
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} &(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \!\!\to y = 1[\mathsf{acc} = 1/1] \\ &(x_2 = \mathsf{blue} \land x_1 < 2.1) \!\!\to y = 0[\mathsf{acc} = 2/4] \\ &(x_1 \ge 2.1 \land x_1 < 2.6) \!\!\to y = 1[\mathsf{acc} = 2/2] \\ &(x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) \!\!\to y = 0[\mathsf{acc} = 2/3] \\ &(x_2 \ne \mathsf{blue} \land x_1 < 3.75) \!\!\to y = 1[\mathsf{acc} = 5/5] \\ &(x_1 \ge 3.75 \land x_1 < 5.5) \!\!\to y = 0[\mathsf{acc} = 2/2] \\ &(x_1 \ge 5.5) \!\!\to y = 1[\mathsf{acc} = 1/1] \end{aligned}$$

```
x_2
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \to y = 1[\mathsf{acc} = 1/1]$$

$$(x_2 = \mathsf{blue} \land x_1 < 2.1) \to y = 0[\mathsf{acc} = 2/4]$$

$$(x_1 \ge 2.1 \land x_1 < 2.6) \to y = 1[\mathsf{acc} = 2/2]$$

$$(x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) \to y = 0[\mathsf{acc} = 2/3]$$

$$(x_2 \ne \mathsf{blue} \land x_1 < 3.75) \to y = 1[\mathsf{acc} = 5/5]$$

$$(x_1 \ge 3.75 \land x_1 < 5.5) \to y = 0[\mathsf{acc} = 2/2]$$

$$(x_1 \ge 5.5) \to y = 1[\mathsf{acc} = 11/16]$$

```
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{aligned} &(x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) \!\!\to y = 1[\mathsf{acc} = 1/1] \\ &(x_2 = \mathsf{blue} \land x_1 < 2.1) \!\!\to y = 0[\mathsf{acc} = 2/4] \\ &(x_1 \ge 2.1 \land x_1 < 2.6) \!\!\to y = 1[\mathsf{acc} = 2/2] \\ &(x_0 \ne \mathsf{hot} \land x_1 \ge 2.6) \!\!\to y = 0[\mathsf{acc} = 2/3] \\ &(x_2 \ne \mathsf{blue} \land x_1 < 3.75) \!\!\to y = 1[\mathsf{acc} = 5/5] \\ &(x_1 \ge 3.75 \land x_1 < 5.5) \!\!\to y = 0[\mathsf{acc} = 2/2] \\ &(x_1 \ge 5.5) \!\!\to y = 1[\mathsf{acc} = 1/1] \end{aligned}$$

```
x_2
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_2 \neq \mathsf{blue} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 5/5] \\ (x_1 \geq 2.1 \land x_1 < 2.6) &\to y = 1[\mathsf{acc} = 2/2] \\ (x_1 \geq 3.75 \land x_1 < 5.5) &\to y = 0[\mathsf{acc} = 2/2] \\ (x_2 = \mathsf{blue} \land x_0 = \mathsf{hot}) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_1 \geq 5.5) &\to y = 1[\mathsf{acc} = 1/1] \\ (x_0 \neq \mathsf{hot} \land x_1 \geq 2.6) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_2 = \mathsf{blue} \land x_1 < 2.1) &\to y = 0[\mathsf{acc} = 2/4] \end{split}$$

Rule Set Generation

- Infer a rule set, perhaps by
 - ► Learning a Decision Tree
 - Converting each leaf to a rule
- Prune each rule by removing any preconditions that result not degrading its accuracy on the training set.
- Sort the pruned rules by their estimated accuracy.

When evaluating a rule set:

Consider them in order when classifying a new instance.

Use the first matching rule!

Why Prune Rule Set

Why prune a rule set, instead of the decision tree?

- Converting allows to remove conditions in the "middle" of the tree.
- Converting allows the removal of conditions for some leaves, but not for others.
- Sometimes rule sets are easier for people to understand.

Learn Rule Set

Why not learn the rule set directly?

Learn Rule Set

Why not learn the rule set directly?

Sequential Covering Algorithm:

- Rules $\leftarrow \{\}$
- While not done:
 - ▶ NewRule ← Learn-one-rule(Data)
 - ► Rules ← Rules + NewRule
 - ightharpoonup Data (Examples correctly classified by NewRule)
- Sort Rules by Performance over full Data

Learn Rule Set

Why not learn the rule set directly?

Sequential Covering Algorithm:

- Rules $\leftarrow \{\}$
- While not done:
 - ▶ NewRule ← Learn-one-rule(Data)
 - ▶ Rules ← Rules + NewRule
 - ightharpoonup Data (Examples correctly classified by NewRule)
- Sort Rules by Performance over full Data

Performance could be any of the measures used in decision tree learning:

- mis-classification rate
- Gini index
- Entropy

Learn One Rule

- Rule $\leftarrow \{\}$
- Bestrule \leftarrow Rule
- While rule can be made more specific:
 - Let C be the condition that, when added to Rule, makes Performance (Rule) best
 - ▶ Rule \leftarrow Rule \land C
 - ▶ If **Performance**(Rule) better than **Performance**(Bestrule):
 - ▶ Bestrule ← Rule

Learn One Rule

- Rule $\leftarrow \{\}$
- Bestrule \leftarrow Rule
- While rule can be made more specific:
 - Let C be the condition that, when added to Rule, makes Performance (Rule) best
 - ▶ Rule \leftarrow Rule \land C
 - ▶ If **Performance**(Rule) better than **Performance**(Bestrule):
 - ▶ Bestrule ← Rule

This is a purely greedy algorithm.

Often, we would use a Beam Search instead.

Learning Rule Set

 x_2 cold 2 red cold 2.3 green 1 hot 1 red 1 hot 4.5 red 0 cold 1.5 blue 0 hot 6.5 green 1 cold 3 blue 0 hot 3 blue 1 cold 3.1 blue 1 cold 5.3 green 0 cold 2 blue 0 cold 2.2 blue 1 hot 3 red 1 cold 1.5 red 1 cold 1.8 blue 1 cold 1.8 blue 1

```
red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 4/5]$$

```
cold 2
        red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_0 \neq \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 7/11]$$

```
cold 2
        red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_0 = \operatorname{cold}) \rightarrow y = 1[\operatorname{acc} = 7/11]$$

```
red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_0 \neq \mathsf{cold}) \rightarrow y = 1[\mathsf{acc} = 4/5]$$

```
cold 2
        red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 \ge 1.25) \rightarrow y = 1[acc = 10/15]$$

```
red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 < 1.65) \rightarrow y = 1[acc = 2/3]$$

```
cold 2
        red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 \ge 1.65) \rightarrow y = 1[acc = 9/13]$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 < 1.9) \rightarrow y = 1[acc = 4/5]$$

```
cold 2
        red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 \ge 1.9) \rightarrow y = 1[\mathsf{acc} = 7/11]$$

```
cold 2
        red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 < 2.1) \rightarrow y = 1[acc = 5/7]$$

```
cold 2
       red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 \ge 2.1) \rightarrow y = 1[acc = 6/9]$$

```
cold 2
       red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 < 2.25) \rightarrow y = 1[\mathsf{acc} = 6/8]$$

```
cold 2
       red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 \ge 2.25) \rightarrow y = 1[acc = 5/8]$$

```
cold 2
        red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 < 2.65) \rightarrow y = 1[acc = 7/9]$$

```
cold 2
       red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 \ge 2.65) \rightarrow y = 1[acc = 4/7]$$

```
cold 2
        red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 < 3.05) \rightarrow y = 1[acc = 9/12]$$

```
red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 \ge 3.05) \rightarrow y = 0[acc = 2/4]$$

```
cold 2
        red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 < 3.8) \rightarrow y = 1[acc = 10/13]$$

```
red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 \ge 3.8) \rightarrow y = 0[acc = 2/3]$$

```
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 < 4.9) \rightarrow y = 1[acc = 10/14]$$

```
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_1 < 5.9) \rightarrow y = 1[acc = 10/15]$$

```
cold 2
       red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \operatorname{red}) \to y = 1[\operatorname{acc} = 4/5]$$

```
cold 2
       red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 \neq \mathsf{red}) \rightarrow y = 1[\mathsf{acc} = 7/11]$$

```
cold 2
       red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{green}) \rightarrow y = 1[\mathsf{acc} = 2/3]$$

```
cold 2
       red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 \neq \mathsf{green}) \rightarrow y = 1[\mathsf{acc} = 9/13]$$

```
cold 2 red 1
cold 2.3 green 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 = \mathsf{blue}) \rightarrow y = 1[\mathsf{acc} = 5/8]$$

```
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_2 \neq \mathsf{blue}) \rightarrow y = 1[\mathsf{acc} = 6/8]$$

```
red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_0 = \mathsf{hot}) \rightarrow y = 1[\mathsf{acc} = 4/5]$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_0 = \mathsf{hot} \land x_1 \ge 2) \rightarrow y = 1[\mathsf{acc} = 3/4]$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_0 = \mathsf{hot} \land x_1 < 3.75) \rightarrow y = 1[\mathsf{acc} = 3/3]$$

```
red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_0 = \mathsf{hot} \land x_1 < 5.5) \to y = 1[\mathsf{acc} = 3/4]$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_0 = \mathsf{hot} \land x_2 = \mathsf{red}) \rightarrow y = 1[\mathsf{acc} = 2/3]$$

```
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_0 = \mathsf{hot} \land x_2 \neq \mathsf{green}) \rightarrow y = 1[\mathsf{acc} = 3/4]$$

```
red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_0 = \mathsf{hot} \land x_2 \neq \mathsf{blue}) \rightarrow y = 1[\mathsf{acc} = 3/4]$$

```
cold 2 red 1
cold 2.3 green 1
hot 1 red 1
hot 4.5 red 0
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red 1
cold 1.5 red 1
cold 1.8 blue 1
cold 1.8 blue 1
```

$$(x_0 = \mathsf{hot} \land x_1 < 3.75) \rightarrow y = 1[\mathsf{acc} = 3/3]$$

```
x_0 x_1 x_2 y cold 2 red 1 cold 2.3 green 1
```

hot 4.5 red 0 cold 1.5 blue 0 hot 6.5 green 1 cold 3 blue 0

cold 3.1 blue 1 cold 5.3 green 0 cold 2 blue 0 cold 2.2 blue 1

cold 1.5 red 1 cold 1.8 blue 1 cold 1.8 blue 1

$$(x_0 = \mathsf{hot} \land x_1 < 3.75) \rightarrow y = 1[\mathsf{acc} = 3/3]$$

```
x_0 x_1 x_2 y cold 2 red 1 cold 2.3 green 1
```

hot 4.5 red 0 cold 1.5 blue 0 hot 6.5 green 1 cold 3 blue 0

cold 3.1 blue 1 cold 5.3 green 0 cold 2 blue 0 cold 2.2 blue 1

cold 1.5 red 1 cold 1.8 blue 1 cold 1.8 blue 1

$$(x_0 = \text{hot } \land x_1 < 3.75) \rightarrow y = 1[\text{acc} = 3/3]$$

 $(x_1 < 2.65) \rightarrow y = 1[\text{acc} = 6/8]$

```
\begin{bmatrix} x_0 & x_1 & x_2 & y \\ \text{cold 2} & \text{red 1} \\ \text{cold 2.3 green 1} \end{bmatrix}
```

hot 4.5 red 0 cold 1.5 blue 0 hot 6.5 green 1 cold 3 blue 0

cold 3.1 blue 1 cold 5.3 green 0 cold 2 blue 0 cold 2.2 blue 1

cold 1.5 red 1 cold 1.8 blue 1 cold 1.8 blue 1

$$(x_0 = \mathsf{hot} \land x_1 < 3.75) \rightarrow y = 1[\mathsf{acc} = 3/3]$$

 $(x_1 < 2.65 \land x_2 \neq \mathsf{blue}) \rightarrow y = 1[\mathsf{acc} = 3/3]$

$$x_0 \quad x_1 \quad x_2 \quad y$$

hot 4.5 red 0 cold 1.5 blue 0 hot 6.5 green 1 cold 3 blue 0

cold 3.1 blue 1 cold 5.3 green 0 cold 2 blue 0 cold 2.2 blue 1

cold 1.8 blue 1 cold 1.8 blue 1

$$(x_0 = \mathsf{hot} \land x_1 < 3.75) \rightarrow y = 1[\mathsf{acc} = 3/3]$$

$$(x_1 < 2.65 \land x_2 \neq \mathsf{blue}) \rightarrow y = 1[\mathsf{acc} = 3/3]$$

 $x_0 \quad x_1 \quad x_2 \quad y$

hot 4.5 red 0 cold 1.5 blue 0 hot 6.5 green 1 cold 3 blue 0

cold 3.1 blue 1 cold 5.3 green 0 cold 2 blue 0 cold 2.2 blue 1

cold 1.8 blue 1 cold 1.8 blue 1

$$(x_0 = \text{hot} \land x_1 < 3.75) \rightarrow y = 1[\text{acc} = 3/3]$$

 $(x_1 < 2.65 \land x_2 \neq \text{blue}) \rightarrow y = 1[\text{acc} = 3/3]$
 $(x_1 < 1.9) \rightarrow y = 1[\text{acc} = 2/3]$

 $x_0 \quad x_1 \quad x_2 \quad y$

hot 4.5 red 0 cold 1.5 blue 0 hot 6.5 green 1 cold 3 blue 0

cold 3.1 blue 1 cold 5.3 green 0 cold 2 blue 0 cold 2.2 blue 1

cold 1.8 blue 1 cold 1.8 blue 1

$$(x_0 = \text{hot} \land x_1 < 3.75) \rightarrow y = 1[\text{acc} = 3/3]$$

 $(x_1 < 2.65 \land x_2 \neq \text{blue}) \rightarrow y = 1[\text{acc} = 3/3]$
 $(x_1 < 1.9) \rightarrow y = 1[\text{acc} = 2/3]$

$$x_0 \quad x_1 \quad x_2 \quad y$$

hot 4.5 red 0

hot 6.5 green 1 cold 3 blue 0

$$\begin{split} (x_0 &= \mathsf{hot} \land x_1 < 3.75) {\to} \ y = 1 [\mathsf{acc} = 3/3] \\ (x_1 < 2.65 \land x_2 \neq \mathsf{blue}) {\to} \ y = 1 [\mathsf{acc} = 3/3] \\ (x_1 < 1.9) {\to} \ y = 1 [\mathsf{acc} = 2/3] \end{split}$$

 $x_0 \quad x_1 \quad x_2 \quad y$

hot 4.5 red 0

hot 6.5 green 1 cold 3 blue 0

$$\begin{split} (x_0 &= \mathsf{hot} \land x_1 < 3.75) {\to} \ y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 2.65 \land x_2 \neq \mathsf{blue}) {\to} \ y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 1.9) {\to} \ y = 1[\mathsf{acc} = 2/3] \\ (x_1 < 5.9) {\to} \ y = 0[\mathsf{acc} = 4/6] \end{split}$$

 $x_0 \quad x_1 \quad x_2 \quad y$

hot 4.5 red 0

hot 6.5 green 1 cold 3 blue 0

$$(x_0 = \text{hot} \land x_1 < 3.75) \rightarrow y = 1[\text{acc} = 3/3]$$

 $(x_1 < 2.65 \land x_2 \neq \text{blue}) \rightarrow y = 1[\text{acc} = 3/3]$
 $(x_1 < 1.9) \rightarrow y = 1[\text{acc} = 2/3]$
 $(x_1 < 5.9 \land x_1 \ge 2.6) \rightarrow y = 0[\text{acc} = 3/4]$

$$x_0 \quad x_1 \quad x_2 \quad y$$

hot 4.5 red 0

hot 6.5 green 1 cold 3 blue 0

$$\begin{split} (x_0 = \mathsf{hot} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 2.65 \land x_2 \neq \mathsf{blue}) &\to y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 1.9) &\to y = 1[\mathsf{acc} = 2/3] \\ (x_1 < 5.9 \land x_1 \geq 2.6 \land x_0 \neq \mathsf{hot}) &\to y = 0[\mathsf{acc} = 2/3] \end{split}$$

$$x_0 \quad x_1 \quad x_2 \quad y$$

hot 4.5 red 0

hot 6.5 green 1 cold 3 blue 0

$$\begin{split} (x_0 = \mathsf{hot} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 2.65 \land x_2 \neq \mathsf{blue}) &\to y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 1.9) &\to y = 1[\mathsf{acc} = 2/3] \\ (x_1 < 5.9 \land x_1 \geq 2.6 \land x_0 \neq \mathsf{hot}) &\to y = 0[\mathsf{acc} = 2/3] \end{split}$$

$$x_0 \quad x_1 \quad x_2 \quad y$$

hot 4.5 red (

hot 6.5 green 1

cold 2 blue 0 cold 2.2 blue 1

$$\begin{split} (x_0 = \mathsf{hot} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 2.65 \land x_2 \neq \mathsf{blue}) &\to y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 1.9) &\to y = 1[\mathsf{acc} = 2/3] \\ (x_1 < 5.9 \land x_1 \geq 2.6 \land x_0 \neq \mathsf{hot}) &\to y = 0[\mathsf{acc} = 2/3] \end{split}$$

 $x_0 \quad x_1 \quad x_2 \quad y$

hot 4.5 red (

hot 6.5 green 1

cold 2 blue 0 cold 2.2 blue 1

$$\begin{split} (x_0 &= \mathsf{hot} \land x_1 < 3.75) {\to} \ y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 2.65 \land x_2 \neq \mathsf{blue}) {\to} \ y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 1.9) {\to} \ y = 1[\mathsf{acc} = 2/3] \\ (x_1 < 5.9 \land x_1 \geq 2.6 \land x_0 \neq \mathsf{hot}) {\to} \ y = 0[\mathsf{acc} = 2/3] \\ (x_1 \geq 2.1) {\to} \ y = 1[\mathsf{acc} = 2/3] \end{split}$$

 $x_0 \quad x_1 \quad x_2 \quad y$

hot 4.5 red (

hot 6.5 green 1

cold 2 blue 0 cold 2.2 blue 1

$$\begin{split} (x_0 &= \mathsf{hot} \land x_1 < 3.75) {\to} \ y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 2.65 \land x_2 \neq \mathsf{blue}) {\to} \ y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 1.9) {\to} \ y = 1[\mathsf{acc} = 2/3] \\ (x_1 < 5.9 \land x_1 \geq 2.6 \land x_0 \neq \mathsf{hot}) {\to} \ y = 0[\mathsf{acc} = 2/3] \\ (x_1 \geq 2.1) {\to} \ y = 1[\mathsf{acc} = 2/3] \end{split}$$

$$x_0 \quad x_1 \quad x_2 \quad y$$

$$\begin{split} (x_0 = \mathsf{hot} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 2.65 \land x_2 \neq \mathsf{blue}) &\to y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 1.9) &\to y = 1[\mathsf{acc} = 2/3] \\ (x_1 < 5.9 \land x_1 \geq 2.6 \land x_0 \neq \mathsf{hot}) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_1 \geq 2.1) &\to y = 1[\mathsf{acc} = 2/3] \end{split}$$

cold 2 blue 0

$$\begin{split} (x_0 &= \mathsf{hot} \land x_1 < 3.75) {\to} \ y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 2.65 \land x_2 \neq \mathsf{blue}) {\to} \ y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 1.9) {\to} \ y = 1[\mathsf{acc} = 2/3] \\ (x_1 < 5.9 \land x_1 \geq 2.6 \land x_0 \neq \mathsf{hot}) {\to} \ y = 0[\mathsf{acc} = 2/3] \\ (x_1 \geq 2.1) {\to} \ y = 1[\mathsf{acc} = 2/3] \\ () {\to} \ y = 0[\mathsf{acc} = 1/1] \end{split}$$

```
x_2
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_0 = \mathsf{hot} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 2.65 \land x_2 \neq \mathsf{blue}) &\to y = 1[\mathsf{acc} = 4/4] \\ (x_1 < 1.9) &\to y = 1[\mathsf{acc} = 4/5] \\ (x_1 < 5.9 \land x_1 \ge 2.6 \land x_0 \neq \mathsf{hot}) &\to y = 0[\mathsf{acc} = 2/3] \\ (x_1 \ge 2.1) &\to y = 1[\mathsf{acc} = 6/9] \\ () &\to y = 0[\mathsf{acc} = 5/16] \end{split}$$

```
x_2
cold 2
        red
cold 2.3 green 1
hot 1 red 1
hot 4.5 red
cold 1.5 blue 0
hot 6.5 green 1
cold 3 blue 0
hot 3 blue 1
cold 3.1 blue 1
cold 5.3 green 0
cold 2 blue 0
cold 2.2 blue 1
hot 3 red
cold 1.5 red
cold 1.8 blue 1
cold 1.8 blue 1
```

$$\begin{split} (x_1 < 2.65 \land x_2 \neq \mathsf{blue}) &\to y = 1[\mathsf{acc} = 4/4] \\ (x_0 = \mathsf{hot} \land x_1 < 3.75) &\to y = 1[\mathsf{acc} = 3/3] \\ (x_1 < 1.9) &\to y = 1[\mathsf{acc} = 4/5] \\ (x_1 \ge 2.1) &\to y = 1[\mathsf{acc} = 6/9] \\ (x_1 < 5.9 \land x_1 \ge 2.6 \land x_0 \ne \mathsf{hot}) &\to y = 0[\mathsf{acc} = 2/3] \\ () &\to y = 0[\mathsf{acc} = 5/16] \end{split}$$