CS 171: Intro to ML and DM

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UC Riverside

Slide Set 3: Linear Regression, I

Slides from CS 171

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  - Professor Christian Shelton
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    - An Introduction to Machine Learning (Kubat)
    - Machine Learning: A Probabilistic Perspective (Murphy)
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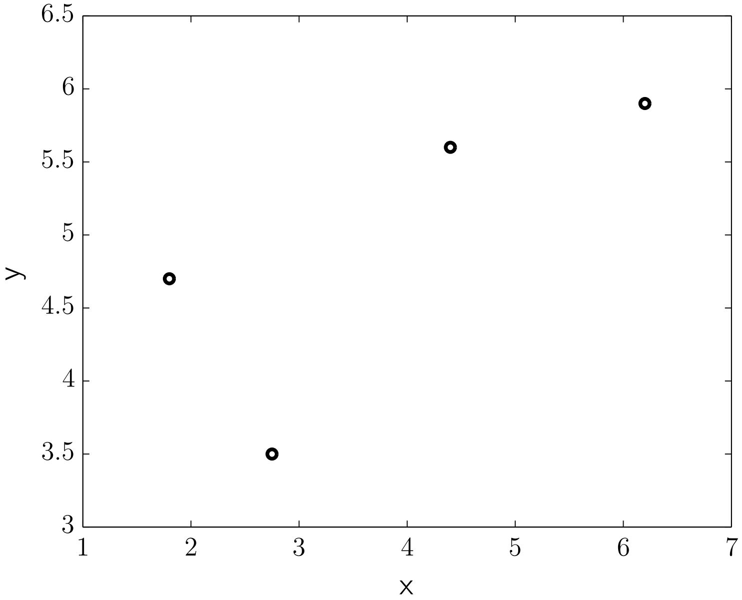
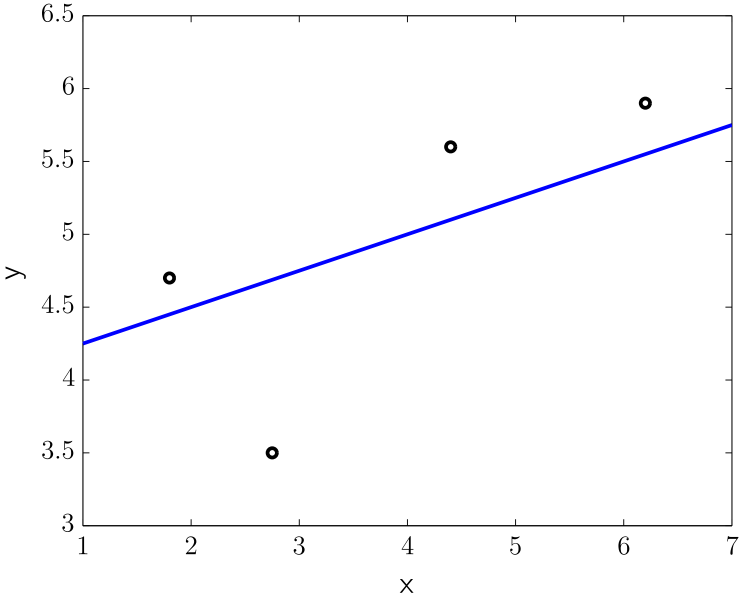
Linear Regression, 1D



Linear Regression, 1D



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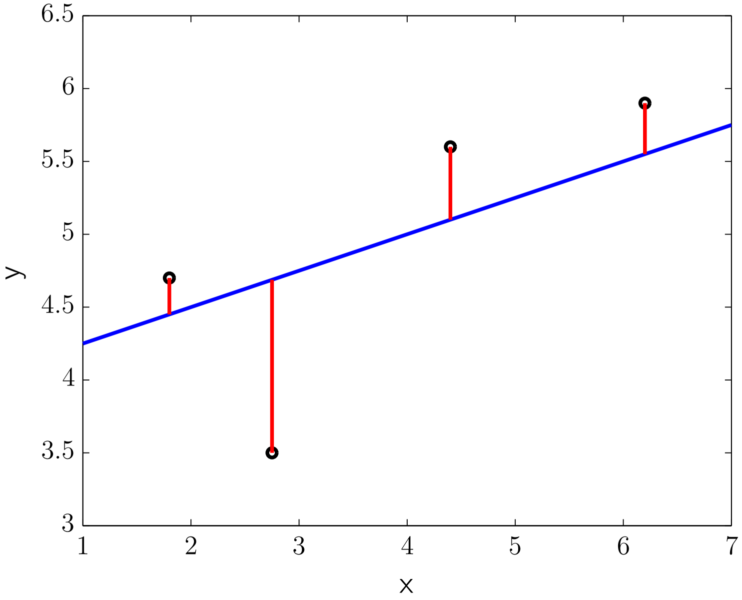

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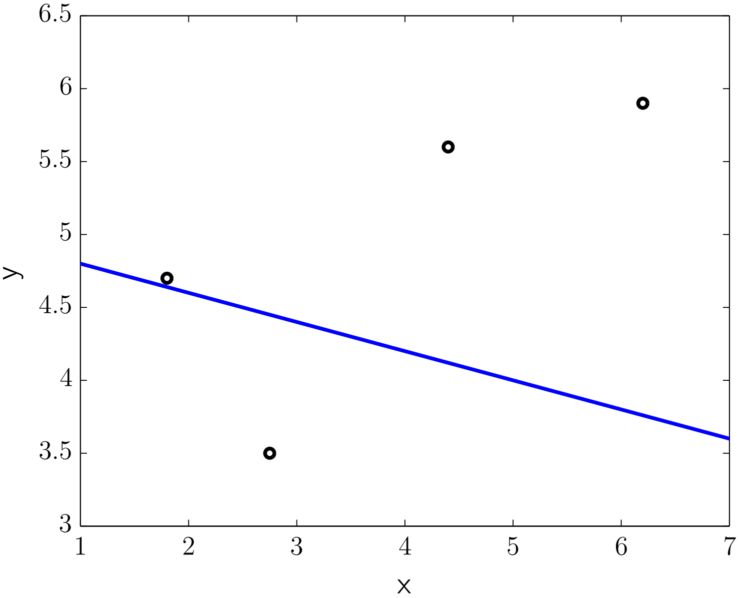

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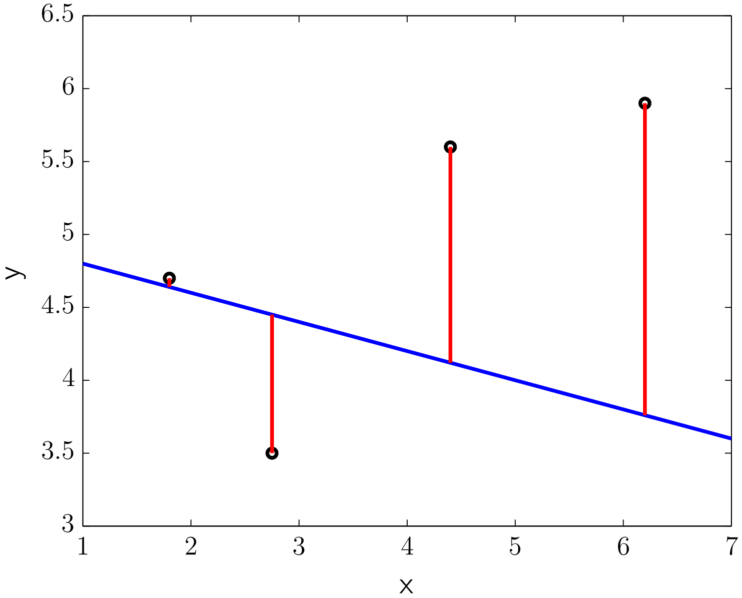

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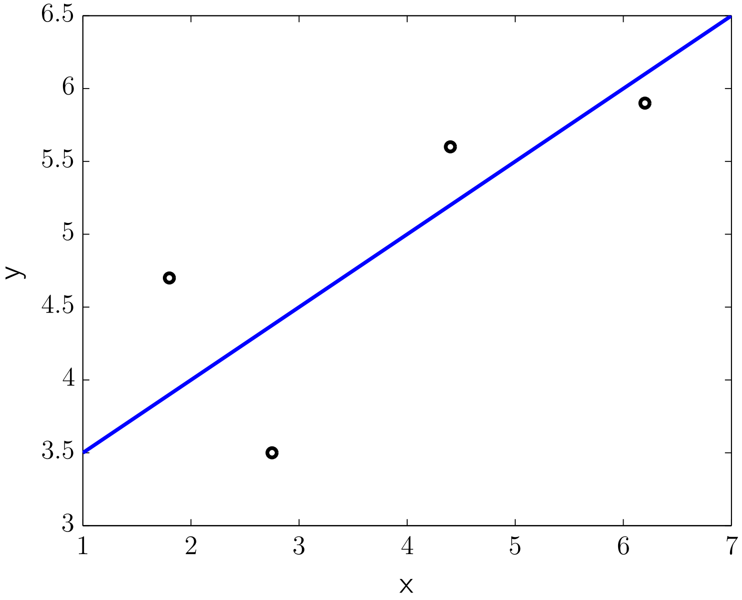

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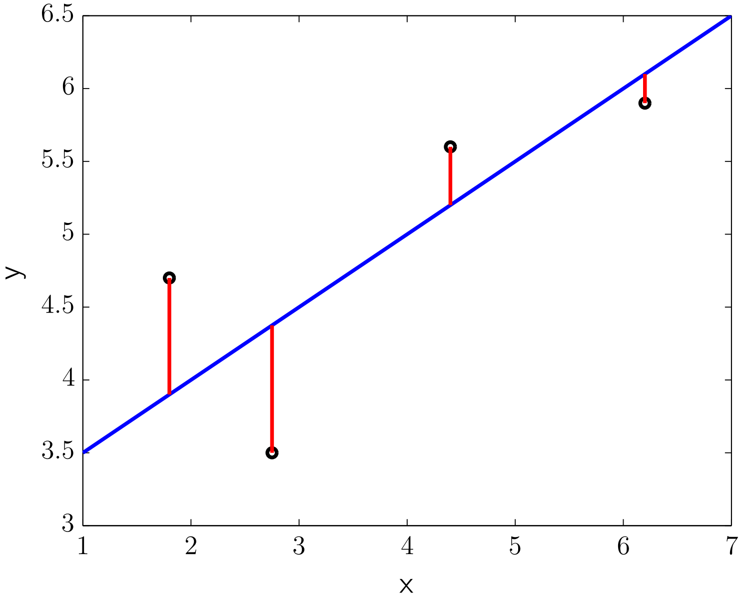

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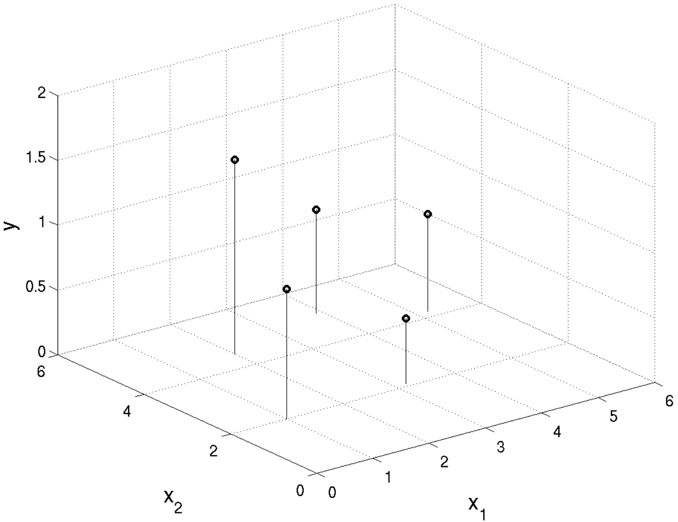
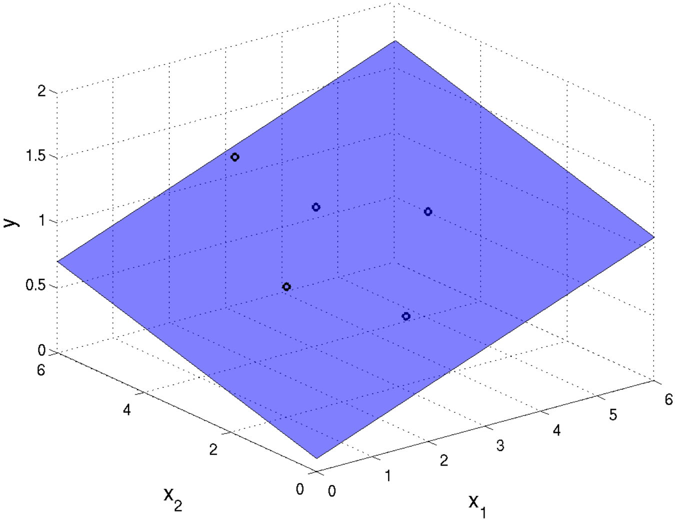


Linear Regression, 2D



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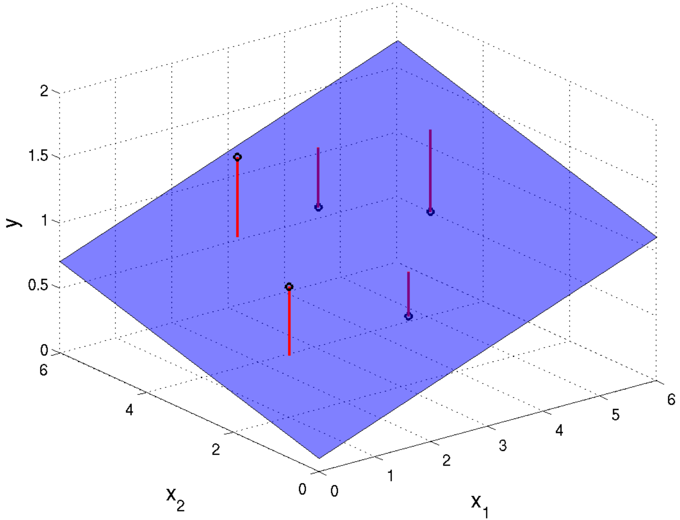

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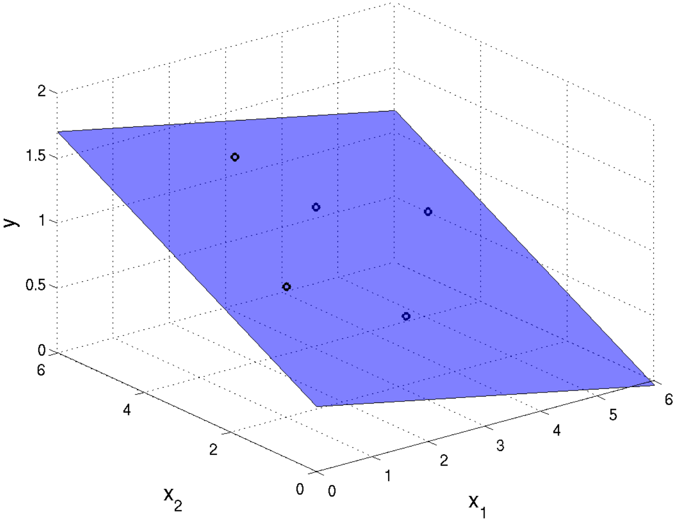

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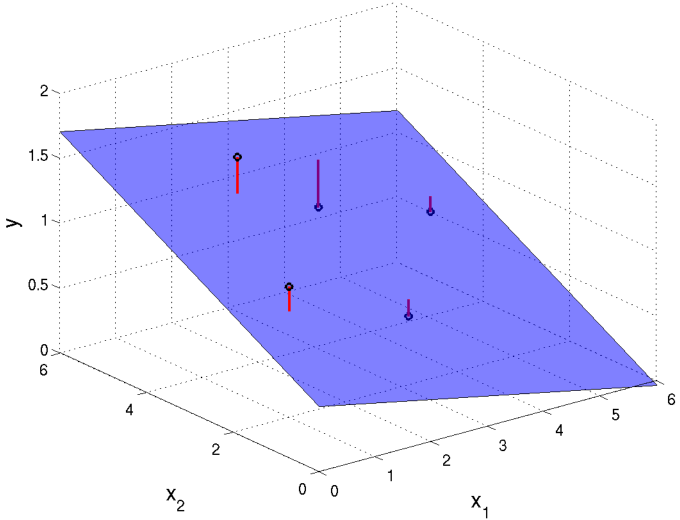

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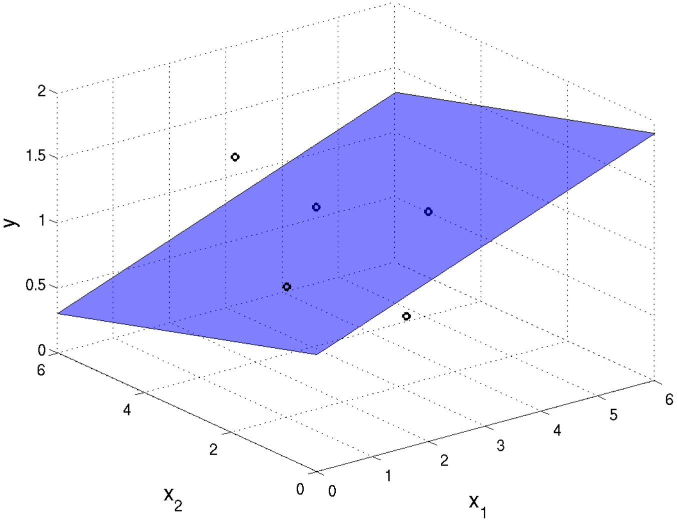

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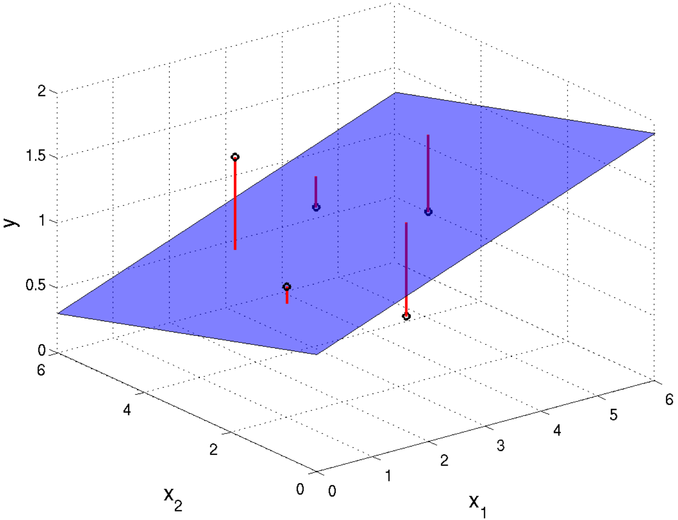
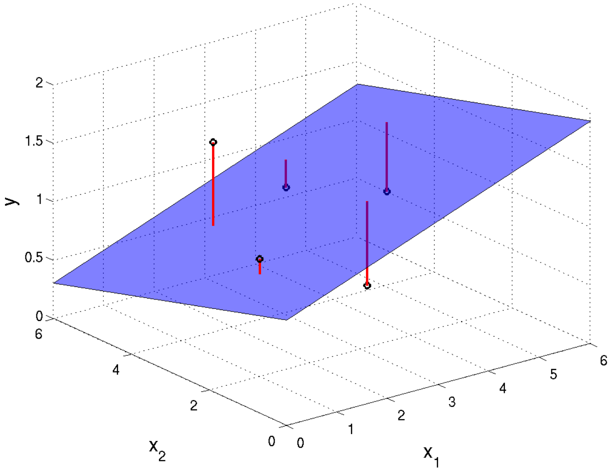
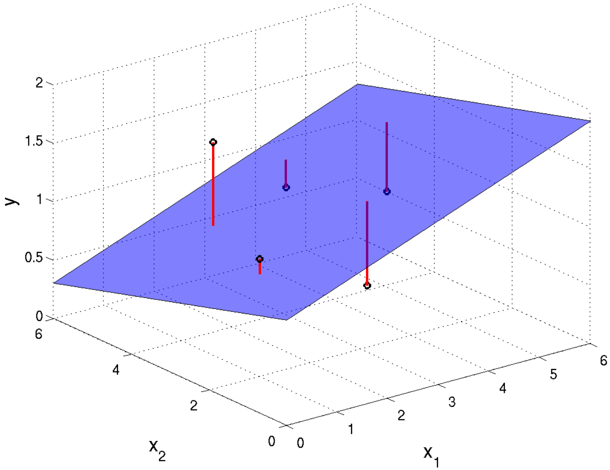


Squared Loss



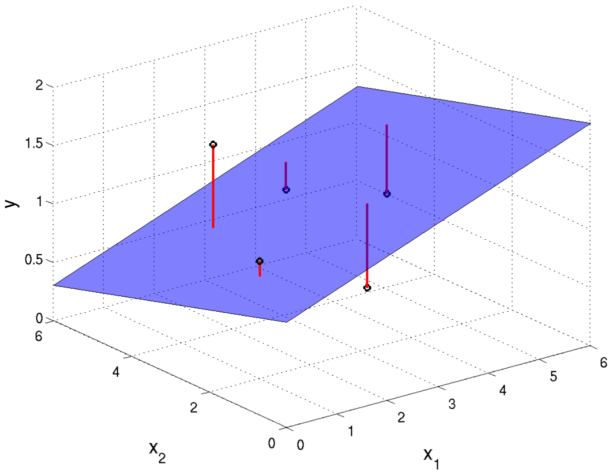
$$f(x) = b + w_1 x_1 + w_2 x_2$$

Squared Loss



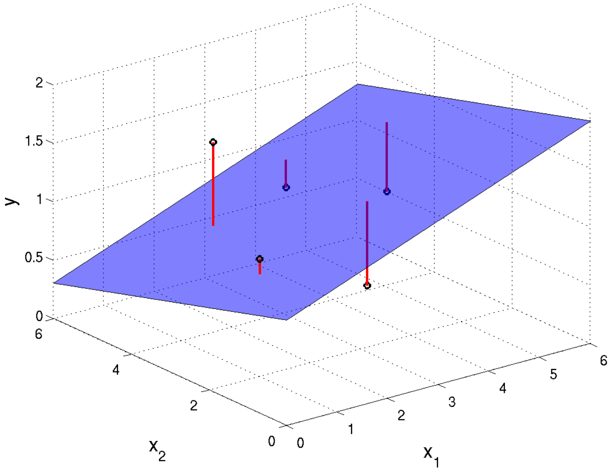
$$f(x) = w_0x_0 + w_1x_1 + w_2x_2$$

Squared Loss



$$f(x) = \sum_{j=0}^{n} w_j x_j$$

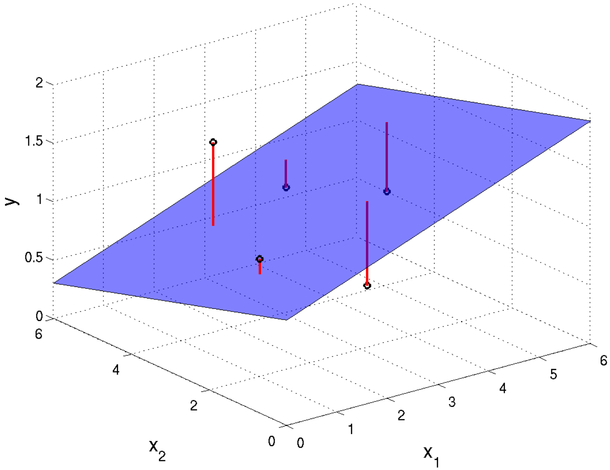
Squared Loss



$$f(x) = \sum_{j=0}^{n} w_j x_j$$

$$(y_i - f(x_i))^2$$

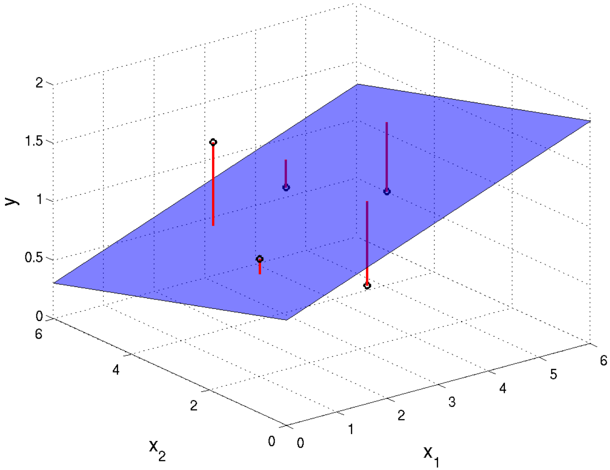
Squared Loss



$$f(x) = \sum_{j=0}^{n} w_j x_j$$

$$L = \sum_{i=1}^{m} (y_i - f(x_i))^2$$

Squared Loss

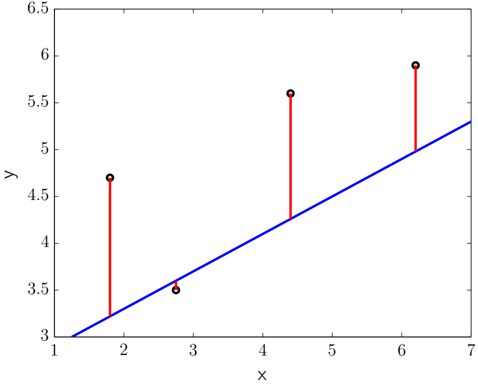


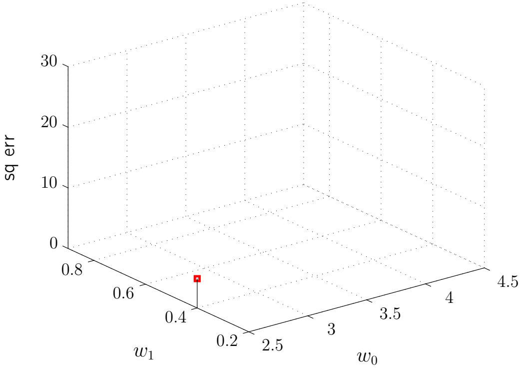
$$f(x) = \sum_{j=0}^{n} w_j x_j$$

$$L = \sum_{i=1}^{m} (y_i - f(x_i))^2$$

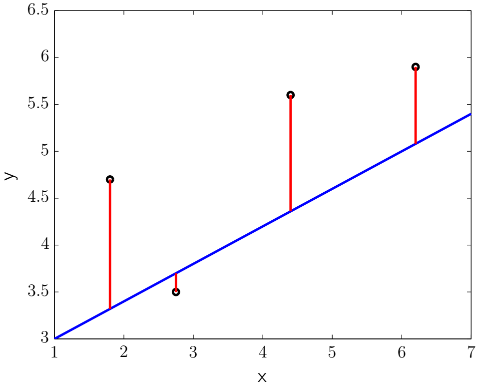
$$= \sum_{i=1}^{m} \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right)^2$$

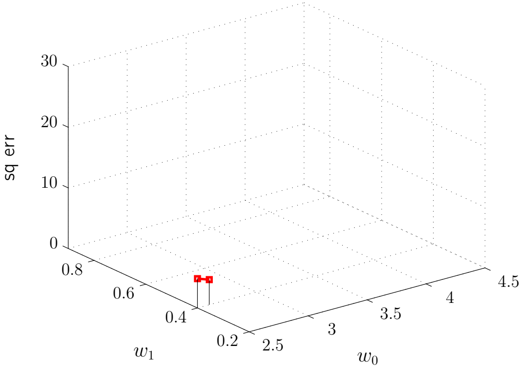
Minimization of Squared Loss





Minimization of Squared Loss





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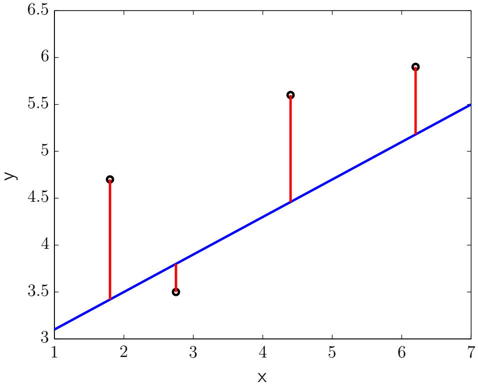

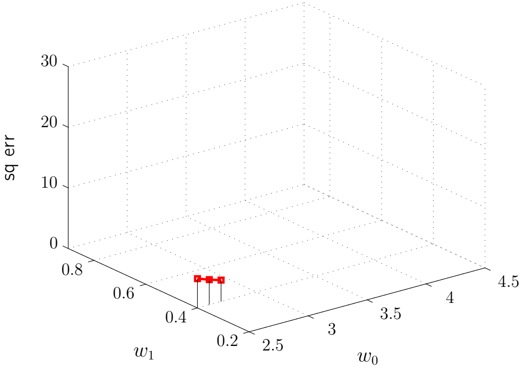

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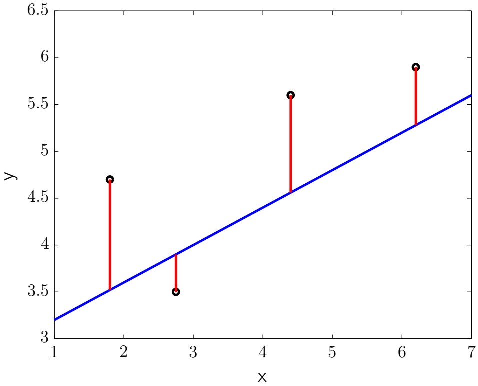

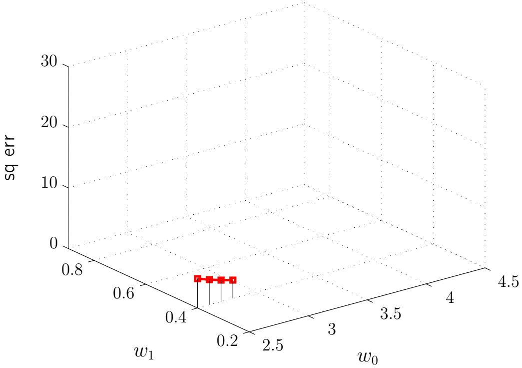

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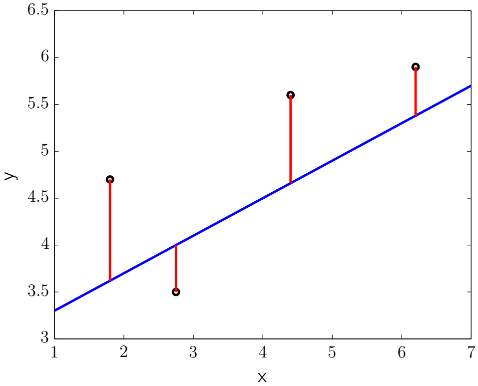

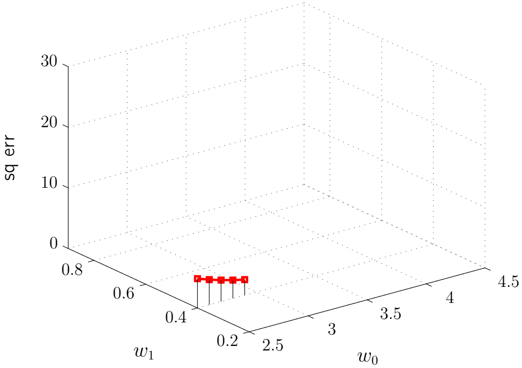

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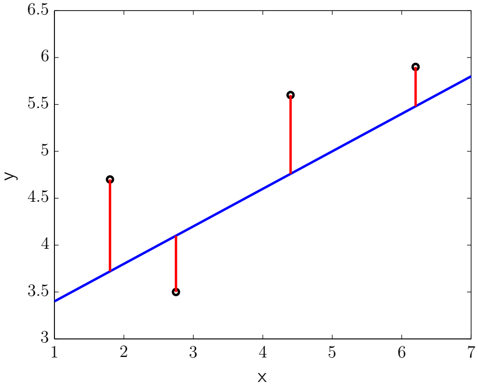

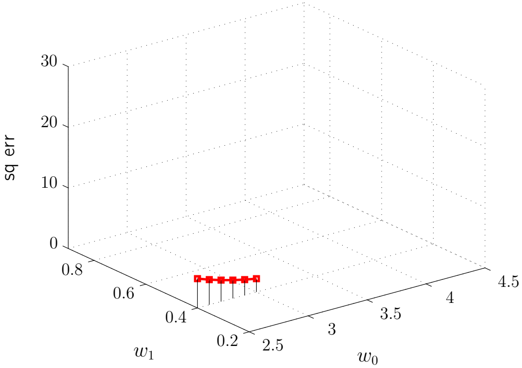

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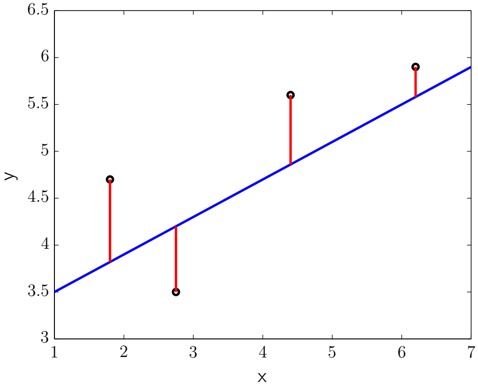

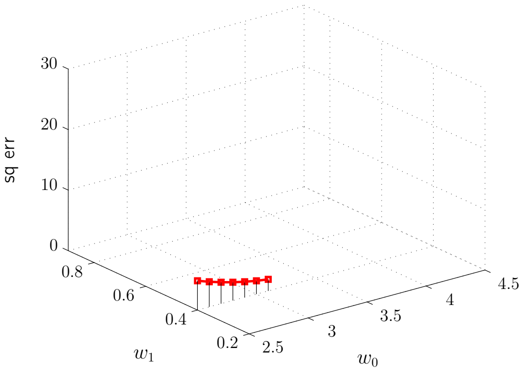

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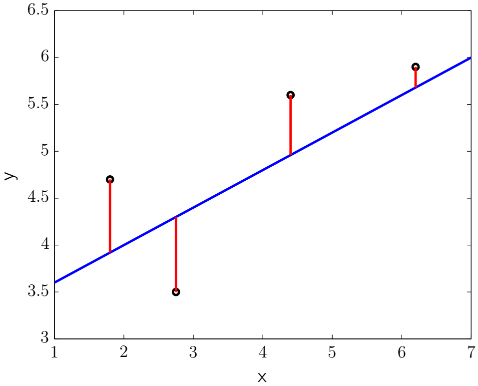

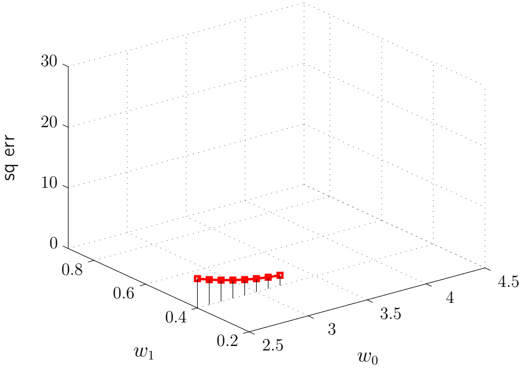

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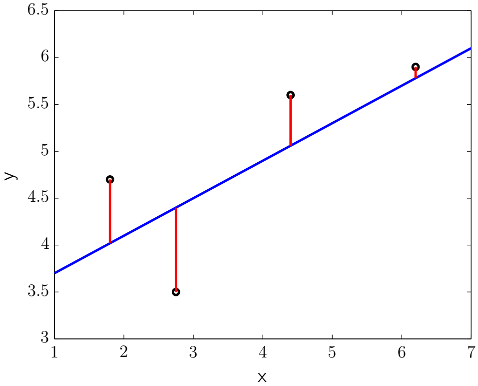

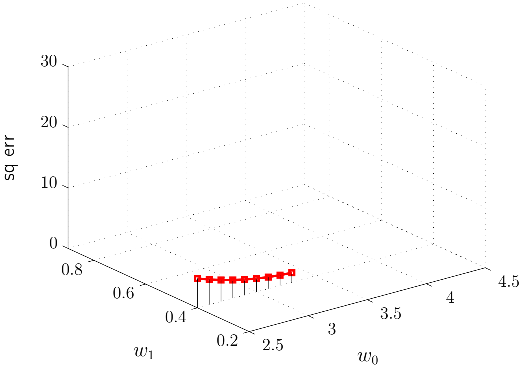

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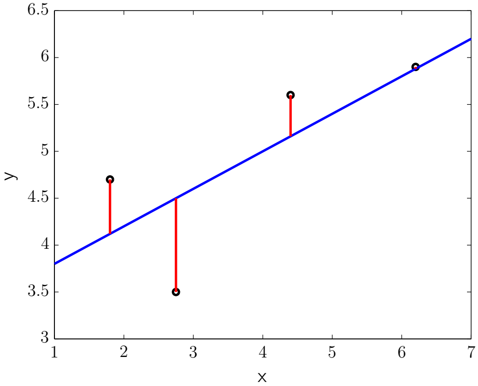

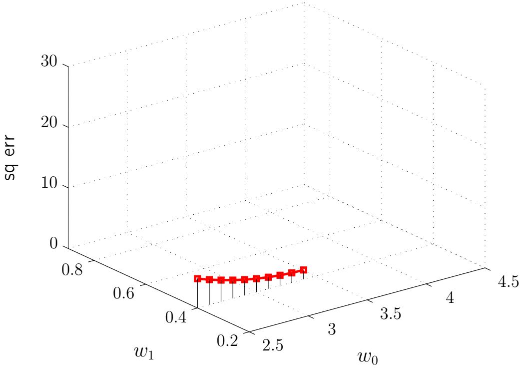

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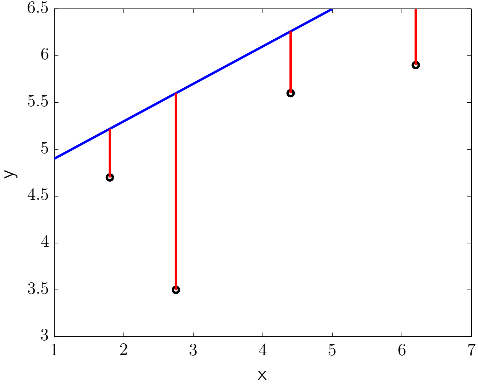

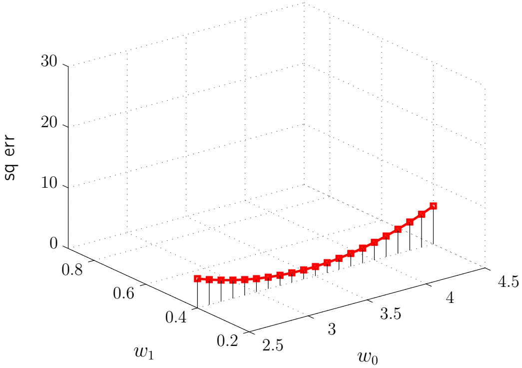

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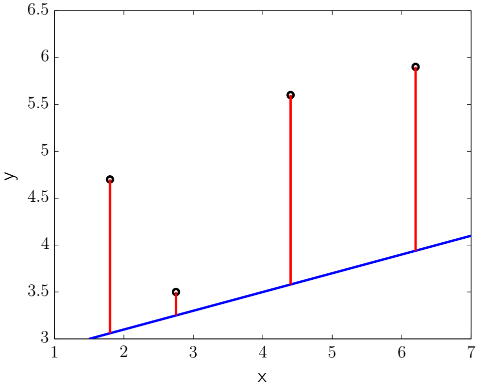

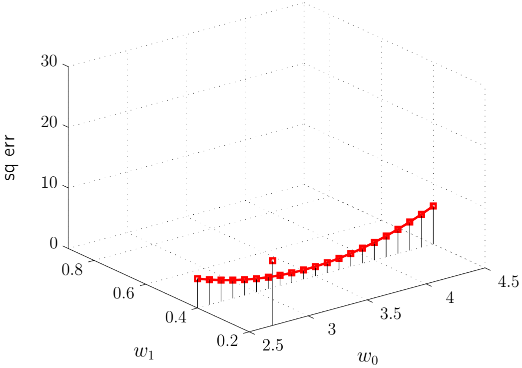

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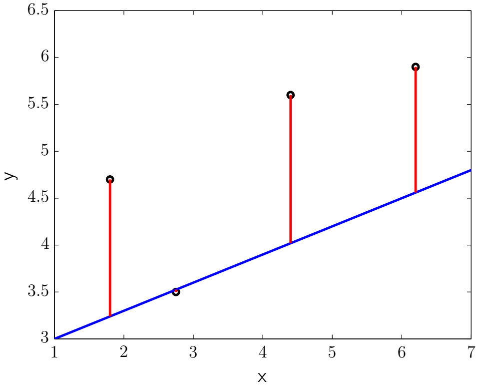

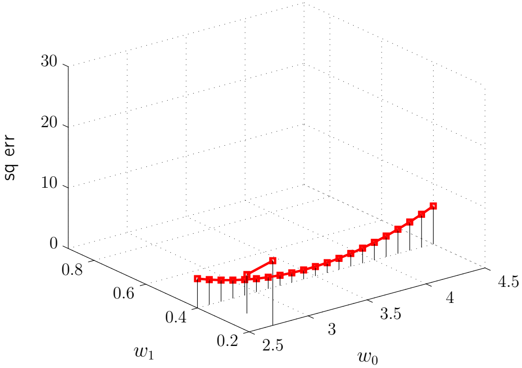

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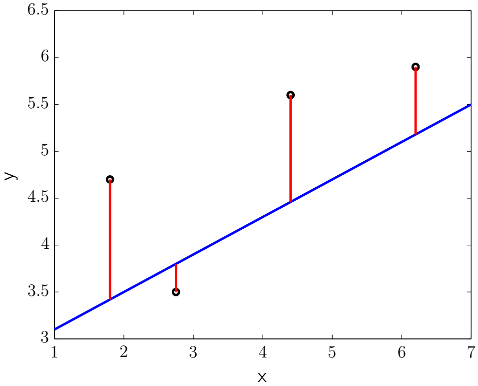

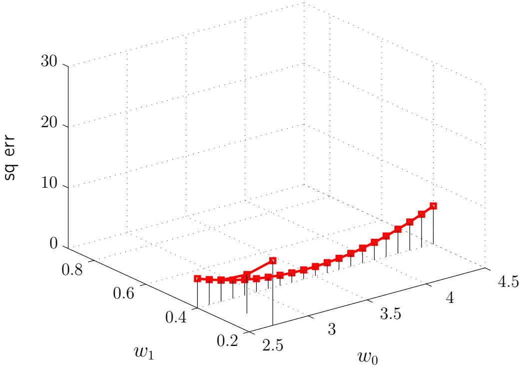

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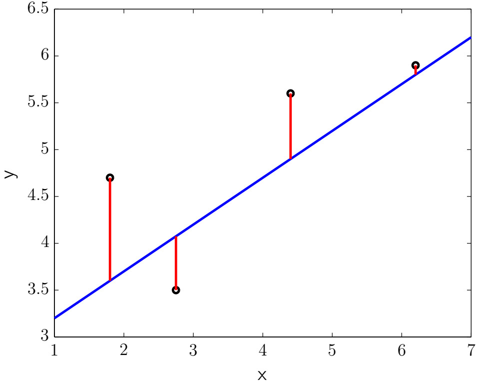

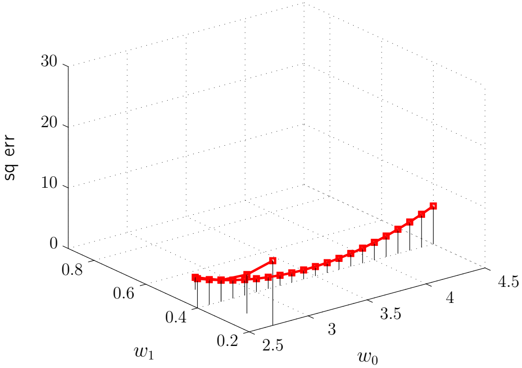

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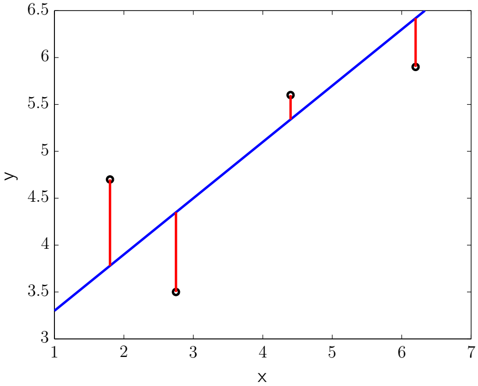

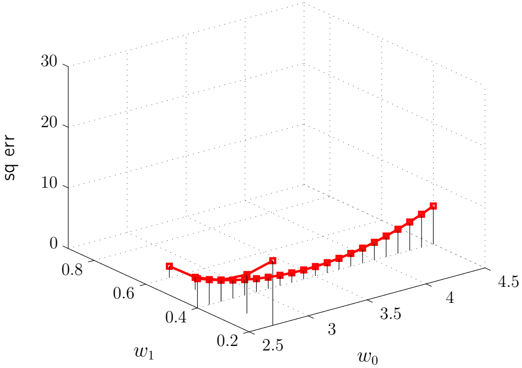

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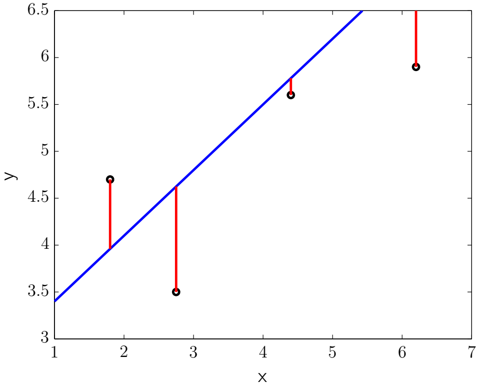

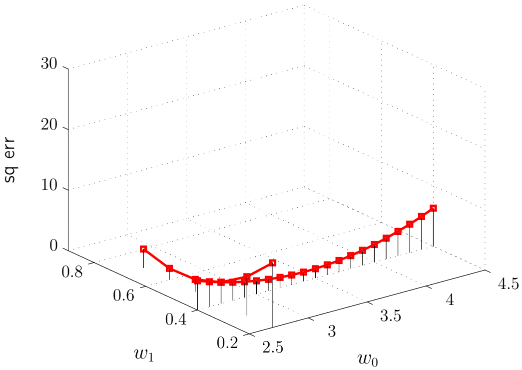

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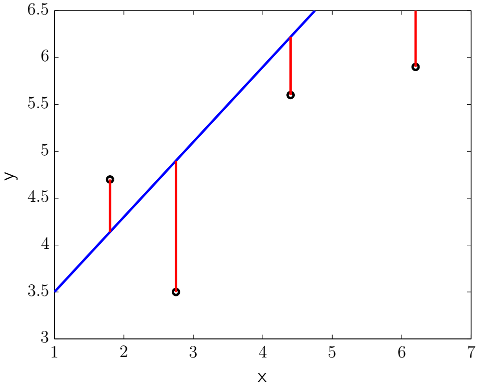

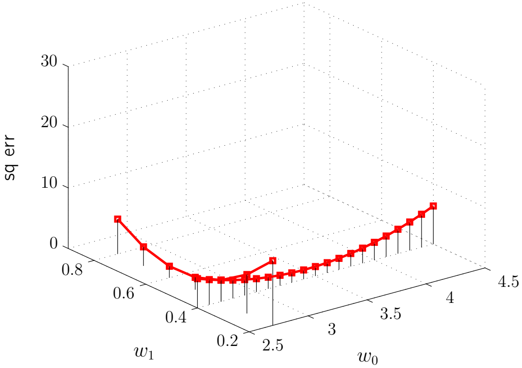

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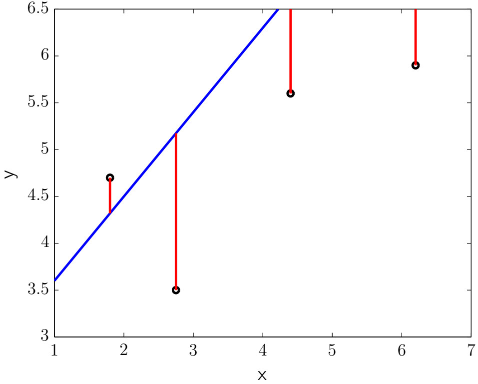

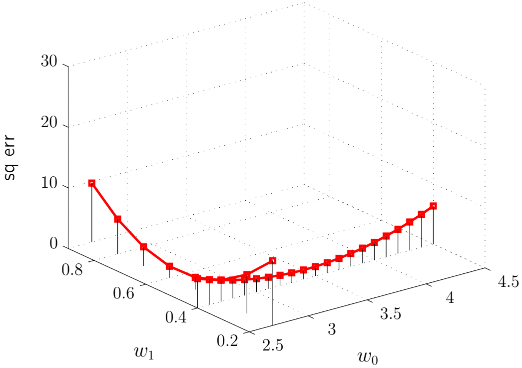

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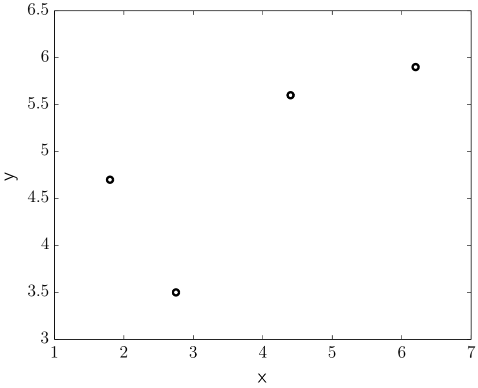

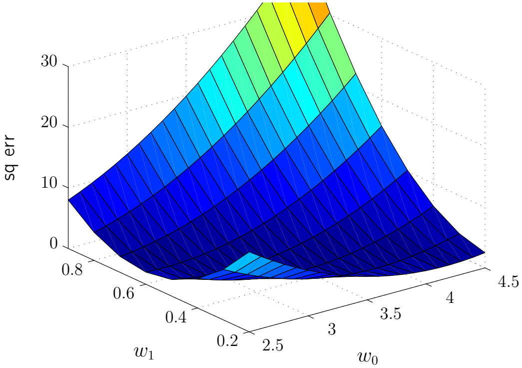

Minimization of Squared Loss

$$L = \sum_{i=1}^{m} \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right)^2$$

Minimization of Squared Loss

$$L = \sum_{i=1}^{m} \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right)^2$$

$$\frac{\partial L}{\partial w_0} = \sum_{i=1}^{m} -2 \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right) x_{i,0}$$

Minimization of Squared Loss

$$L = \sum_{i=1}^{m} \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right)^2$$

$$\frac{\partial L}{\partial w_0} = \sum_{i=1}^{m} -2 \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right) x_{i,0}$$

$$\frac{\partial L}{\partial w_1} = \sum_{i=1}^{m} -2 \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right) x_{i,1}$$

Minimization of Squared Loss

$$L = \sum_{i=1}^{m} \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right)^2$$

$$\frac{\partial L}{\partial w_0} = \sum_{i=1}^{m} -2 \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right) x_{i,0}$$

$$\frac{\partial L}{\partial w_1} = \sum_{i=1}^{m} -2 \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right) x_{i,1}$$

$$\vdots$$

$$\frac{\partial L}{\partial w_n} = \sum_{i=1}^{m} -2 \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right) x_{i,n}$$

Minimization of Squared Loss

$$\frac{\partial L}{\partial w_0} = \sum_{i=1}^{m} -2 \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right) x_{i,0}$$

$$\frac{\partial L}{\partial w_1} = \sum_{i=1}^{m} -2 \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right) x_{i,1}$$

$$\vdots$$

$$\frac{\partial L}{\partial w_n} = \sum_{i=1}^{m} -2 \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right) x_{i,n}$$

Minimization of Squared Loss

$$0 = \sum_{i=1}^{m} -2\left(y_i - \sum_{j=0}^{n} w_j x_{i,j}\right) x_{i,0}$$

$$0 = \sum_{i=1}^{m} -2\left(y_i - \sum_{j=0}^{n} w_j x_{i,j}\right) x_{i,1}$$

$$\vdots$$

$$0 = \sum_{i=1}^{m} -2\left(y_i - \sum_{j=0}^{n} w_j x_{i,j}\right) x_{i,n}$$

Minimization of Squared Loss

$$0 = \sum_{i=1}^{m} \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right) x_{i,0}$$

$$0 = \sum_{i=1}^{m} \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right) x_{i,1}$$

$$\vdots$$

$$0 = \sum_{i=1}^{m} \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right) x_{i,n}$$

Minimization of Squared Loss

$$0 = \sum_{i=1}^{m} \left(y_i x_{i,0} - \sum_{j=0}^{n} w_j x_{i,j} x_{i,0} \right)$$

$$0 = \sum_{i=1}^{m} \left(y_i x_{i,1} - \sum_{j=0}^{n} w_j x_{i,j} x_{i,1} \right)$$

$$\vdots$$

$$0 = \sum_{i=1}^{m} \left(y_i x_{i,n} - \sum_{j=0}^{n} w_j x_{i,j} x_{i,n} \right)$$

Minimization of Squared Loss

$$0 = \sum_{i=1}^{m} y_i x_{i,0} - \sum_{i=1}^{m} \sum_{j=0}^{n} w_j x_{i,j} x_{i,0}$$

$$0 = \sum_{i=1}^{m} y_i x_{i,1} - \sum_{i=1}^{m} \sum_{j=0}^{n} w_j x_{i,j} x_{i,1}$$

$$\vdots$$

$$0 = \sum_{i=1}^{m} y_i x_{i,n} - \sum_{i=1}^{m} \sum_{j=0}^{n} w_j x_{i,j} x_{i,n}$$

Minimization of Squared Loss

$$0 = \sum_{i=1}^{m} y_i x_{i,0} - \sum_{j=0}^{n} \sum_{i=1}^{m} w_j x_{i,j} x_{i,0}$$
$$0 = \sum_{i=1}^{m} y_i x_{i,1} - \sum_{j=0}^{n} \sum_{i=1}^{m} w_j x_{i,j} x_{i,1}$$
$$\vdots$$
$$0 = \sum_{i=1}^{m} y_i x_{i,n} - \sum_{j=0}^{n} \sum_{i=1}^{m} w_j x_{i,j} x_{i,n}$$

Minimization of Squared Loss

$$0 = \sum_{i=1}^{m} y_i x_{i,0} - \sum_{j=0}^{n} \left(\sum_{i=1}^{m} x_{i,0} x_{i,j} \right) w_j$$

$$0 = \sum_{i=1}^{m} y_i x_{i,1} - \sum_{j=0}^{n} \left(\sum_{i=1}^{m} x_{i,1} x_{i,j} \right) w_j$$

$$\vdots$$

$$0 = \sum_{i=1}^{m} y_i x_{i,n} - \sum_{j=0}^{n} \left(\sum_{i=1}^{m} x_{i,n} x_{i,j} \right) w_j$$

Minimization of Squared Loss

$$
\begin{aligned}
0 &= \sum_{i=1}^{m} y_i x_{i,0} - \sum_{j=0}^{n} \left(\sum_{i=1}^{m} x_{i,0} x_{i,j} \right) w_j \\
0 &= \sum_{i=1}^{m} y_i x_{i,1} - \sum_{j=0}^{n} \left(\sum_{i=1}^{m} x_{i,1} x_{i,j} \right) w_j \\
&\vdots \\
0 &= \sum_{i=1}^{m} y_i x_{i,n} - \sum_{j=0}^{n} \left(\sum_{i=1}^{m} x_{i,n} x_{i,j} \right) w_j
\end{aligned}
$$

where

$$
\begin{aligned}
A_{k,l} &= \sum_{i=1}^{m} x_{i,k} x_{i,l} \\
c_k &= \sum_{i=1}^{m} y_i x_{i,k}
\end{aligned}
$$

Minimization of Squared Loss

$$
\begin{aligned}
0 &= c_0 - \sum_{j=0}^{n} A_{0,j} w_j \\
0 &= c_1 - \sum_{j=0}^{n} A_{1,j} w_j \\
&\vdots \\
0 &= c_n - \sum_{j=0}^{n} A_{n,j} w_j
\end{aligned}
$$

where

$$
\begin{aligned}
A_{k,l} &= \sum_{i=1}^{m} x_{i,k} x_{i,l} \\
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Minimization of Squared Loss

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&\vdots \\
0 &= c_n - \sum_{j=0}^{n} A_{n,j} w_j
\end{aligned}
$$

where

$$
A_{k,l} = \sum_{i=1}^{m} x_{i,k} x_{i,l} \qquad\qquad A = X^\top X
$$

$$
c_k = \sum_{i=1}^{m} y_i x_{i,k} \qquad\qquad c = X^\top Y
$$

Minimization of Squared Loss

$$
\begin{aligned}
0 &= c_0 - \sum_{j=0}^{n} A_{0,j} w_j \\
0 &= c_1 - \sum_{j=0}^{n} A_{1,j} w_j \\
&\vdots \\
0 &= c_n - \sum_{j=0}^{n} A_{n,j} w_j
\end{aligned}
\qquad 0 = c - Aw
$$

where

$$
A_{k,l} = \sum_{i=1}^{m} x_{i,k} x_{i,l} \qquad A = X^\top X
$$

$$
c_k = \sum_{i=1}^{m} y_i x_{i,k} \qquad c = X^\top Y
$$

Minimization of Squared Loss

$$
\begin{aligned}
0 &= c_0 - \sum_{j=0}^{n} A_{0,j} w_j \\
0 &= c_1 - \sum_{j=0}^{n} A_{1,j} w_j \\
&\vdots \\
0 &= c_n - \sum_{j=0}^{n} A_{n,j} w_j
\end{aligned}
\qquad\qquad c = Aw
$$

where

$$
A_{k,l} = \sum_{i=1}^{m} x_{i,k} x_{i,l} \qquad\qquad A = X^\top X
$$

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Minimization of Squared Loss

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\begin{aligned}
0 &= c_0 - \sum_{j=0}^{n} A_{0,j} w_j \\
0 &= c_1 - \sum_{j=0}^{n} A_{1,j} w_j \\
&\vdots \\
0 &= c_n - \sum_{j=0}^{n} A_{n,j} w_j
\end{aligned}
$$

$$\boxed{w = A^{-1}c}$$

where

$$A_{k,l} = \sum_{i=1}^{m} x_{i,k} x_{i,l} \qquad A = X^\top X$$

$$c_k = \sum_{i=1}^{m} y_i x_{i,k} \qquad c = X^\top Y$$

Minimization of Squared Loss, reprise

$$L = \sum_{i=1}^{m} \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right)^2$$

Minimization of Squared Loss, reprise

$$L = \sum_{i=1}^{m} \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right)^2$$

$$\begin{bmatrix} \sum_{j=0}^{n} x_{0,j} w_j \\ \sum_{j=0}^{n} x_{1,j} w_j \\ \vdots \\ \sum_{j=0}^{n} x_{m,j} w_j \end{bmatrix} = \begin{bmatrix} \text{———} x_{1,\cdot} \text{———} \\ \text{———} x_{2,\cdot} \text{———} \\ \vdots \\ \text{———} x_{m,\cdot} \text{———} \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \\ w_m \end{bmatrix} = Xw$$

Minimization of Squared Loss, reprise

$$L = \sum_{i=1}^{m} \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right)^2$$

$$\begin{bmatrix} y_1 - \sum_{j=0}^{n} x_{1,j} w_j \\ y_2 - \sum_{j=0}^{n} x_{2,j} w_j \\ \vdots \\ y_m - \sum_{j=0}^{n} x_{m,j} w_j \end{bmatrix} = Y - \begin{bmatrix} \text{———} x_{1,\cdot} \text{———} \\ \text{———} x_{2,\cdot} \text{———} \\ \vdots \\ \text{———} x_{m,\cdot} \text{———} \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \\ w_m \end{bmatrix} = Y - Xw$$

Minimization of Squared Loss, reprise

$$L = \sum_{i=1}^{m} \left(y_i - \sum_{j=0}^{n} w_j x_{i,j} \right)^2$$

$$L = (Y - Xw)^\top (Y - Xw)$$

$$\begin{bmatrix} y_1 - \sum_{j=0}^{n} x_{1,j} w_j \\ y_2 - \sum_{j=0}^{n} x_{2,j} w_j \\ \vdots \\ y_m - \sum_{j=0}^{n} x_{m,j} w_j \end{bmatrix} = Y - \begin{bmatrix} \text{———} x_{1,\cdot} \text{———} \\ \text{———} x_{2,\cdot} \text{———} \\ \vdots \\ \text{———} x_{m,\cdot} \text{———} \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \\ w_m \end{bmatrix} = Y - Xw$$

Minimization of Squared Loss, reprise

$$L = (Y - Xw)^\top (Y - Xw)$$

Minimization of Squared Loss, reprise

$$
\begin{aligned}
L &= (Y - Xw)^\top (Y - Xw) \\
\frac{\partial L}{\partial w} &=
\end{aligned}
$$

Minimization of Squared Loss, reprise

$$L = (Y - Xw)^\top (Y - Xw)$$

$$\frac{\partial L}{\partial w} = \nabla_w L$$

Minimization of Squared Loss, reprise

$$
\begin{aligned}
L &= (Y - Xw)^\top (Y - Xw) \\
\frac{\partial L}{\partial w} &= \nabla_w L = -2(Y - Xw)^\top X
\end{aligned}
$$

Minimization of Squared Loss, reprise

$$
\begin{aligned}
L &= (Y - Xw)^\top (Y - Xw) \\
\frac{\partial L}{\partial w} &= \nabla_w L = -2(Y - Xw)^\top X \\
&= -2(Y^\top X - w^\top X^\top X)
\end{aligned}
$$

Minimization of Squared Loss, reprise

$$
\begin{aligned}
L &= (Y - Xw)^\top (Y - Xw) \\
\frac{\partial L}{\partial w} &= \nabla_w L = -2(Y - Xw)^\top X \\
&= -2(Y^\top X - w^\top X^\top X) \\
0^\top &= -2(Y^\top X - w^\top X^\top X)
\end{aligned}
$$

Minimization of Squared Loss, reprise

$$
\begin{aligned}
L &= (Y - Xw)^\top (Y - Xw) \\
\frac{\partial L}{\partial w} &= \nabla_w L = -2(Y - Xw)^\top X \\
&= -2(Y^\top X - w^\top X^\top X) \\
0^\top &= -2(Y^\top X - w^\top X^\top X) \\
0 &= X^\top Y - X^\top X w
\end{aligned}
$$

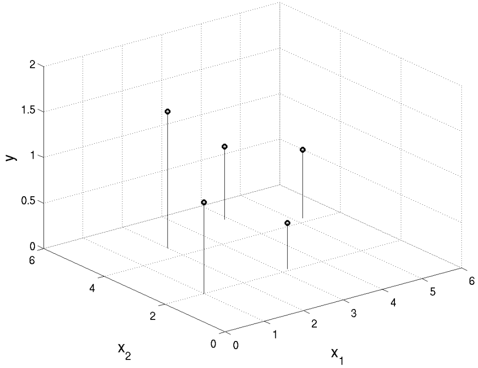
Minimization of Squared Loss, reprise

$$
\begin{aligned}
L &= (Y - Xw)^\top (Y - Xw) \\
\frac{\partial L}{\partial w} &= \nabla_w L = -2(Y - Xw)^\top X \\
&= -2(Y^\top X - w^\top X^\top X) \\
0^\top &= -2(Y^\top X - w^\top X^\top X) \\
0 &= X^\top Y - X^\top X w \\
X^\top X w &= X^\top Y
\end{aligned}
$$

Minimization of Squared Loss, reprise

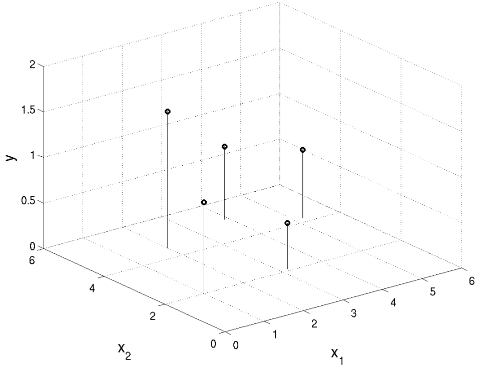
$$
\begin{aligned}
L &= (Y - Xw)^\top (Y - Xw) \\
\frac{\partial L}{\partial w} &= \nabla_w L = -2(Y - Xw)^\top X \\
&= -2(Y^\top X - w^\top X^\top X) \\
0^\top &= -2(Y^\top X - w^\top X^\top X) \\
0 &= X^\top Y - X^\top X w \\
X^\top X w &= X^\top Y \\
w &= (X^\top X)^{-1} X^\top Y
\end{aligned}
$$

Least-Squares, Example



$$X = \begin{bmatrix} 1.0 & 2.0 \\ 2.0 & 4.5 \\ 3.2 & 2.1 \\ 3.9 & 5.1 \\ 5.2 & 4.2 \end{bmatrix} \qquad Y = \begin{bmatrix} 1.0 \\ 1.5 \\ 0.5 \\ 0.8 \\ 0.75 \end{bmatrix}$$

Least-Squares, Example



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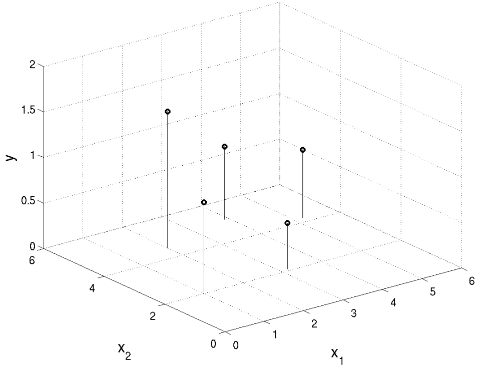
Least-Squares, Example

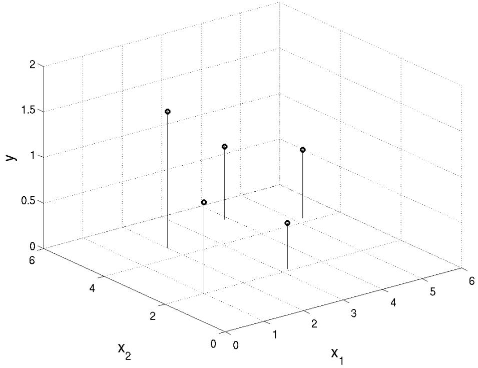


$$X = \begin{bmatrix} 1.0 & 1.0 & 2.0 \\ 1.0 & 2.0 & 4.5 \\ 1.0 & 3.2 & 2.1 \\ 1.0 & 3.9 & 5.1 \\ 1.0 & 5.2 & 4.2 \end{bmatrix} \qquad Y = \begin{bmatrix} 1.0 \\ 1.5 \\ 0.5 \\ 0.8 \\ 0.75 \end{bmatrix}$$

$$A = X^\top X = \begin{bmatrix} 5.0 & 15.3 & 17.9 \\ 15.3 & 57.49 & 59.45 \\ 17.9 & 59.45 & 72.31 \end{bmatrix}$$

$$c = X^\top Y = \begin{bmatrix} 4.55 \\ 12.62 \\ 17.03 \end{bmatrix}$$

Least-Squares, Example



$$X = \begin{bmatrix} 1.0 & 1.0 & 2.0 \\ 1.0 & 2.0 & 4.5 \\ 1.0 & 3.2 & 2.1 \\ 1.0 & 3.9 & 5.1 \\ 1.0 & 5.2 & 4.2 \end{bmatrix} \qquad Y = \begin{bmatrix} 1.0 \\ 1.5 \\ 0.5 \\ 0.8 \\ 0.75 \end{bmatrix}$$

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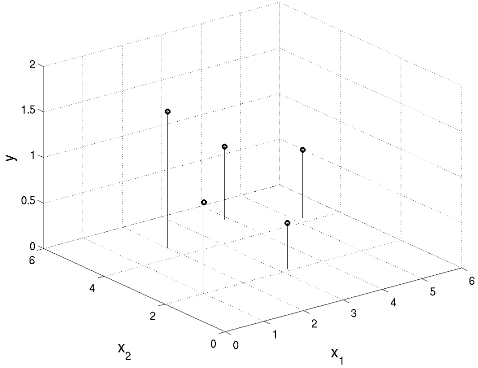
$$\begin{bmatrix} 4.55 \\ 12.62 \\ 17.03 \end{bmatrix} = \begin{bmatrix} 5.0 & 15.3 & 17.9 \\ 15.3 & 57.49 & 59.45 \\ 17.9 & 59.45 & 72.31 \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix}$$

Least-Squares, Example



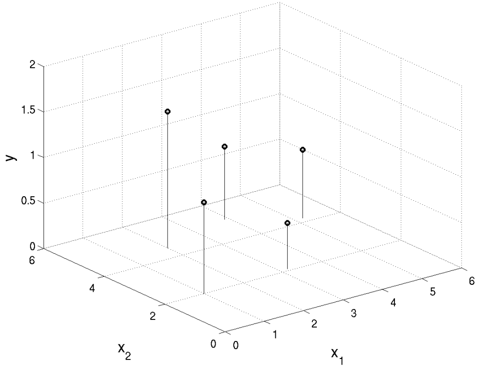
$$X = \begin{bmatrix} 1.0 & 1.0 & 2.0 \\ 1.0 & 2.0 & 4.5 \\ 1.0 & 3.2 & 2.1 \\ 1.0 & 3.9 & 5.1 \\ 1.0 & 5.2 & 4.2 \end{bmatrix} \qquad Y = \begin{bmatrix} 1.0 \\ 1.5 \\ 0.5 \\ 0.8 \\ 0.75 \end{bmatrix}$$

$$A = X^\top X = \begin{bmatrix} 5.0 & 15.3 & 17.9 \\ 15.3 & 57.49 & 59.45 \\ 17.9 & 59.45 & 72.31 \end{bmatrix}$$

$$c = X^\top Y = \begin{bmatrix} 4.55 \\ 12.62 \\ 17.03 \end{bmatrix}$$

$$w = \begin{bmatrix} 5.0 & 15.3 & 17.9 \\ 15.3 & 57.49 & 59.45 \\ 17.9 & 59.45 & 72.31 \end{bmatrix}^{-1} \begin{bmatrix} 4.55 \\ 12.62 \\ 17.03 \end{bmatrix}$$

Least-Squares, Example



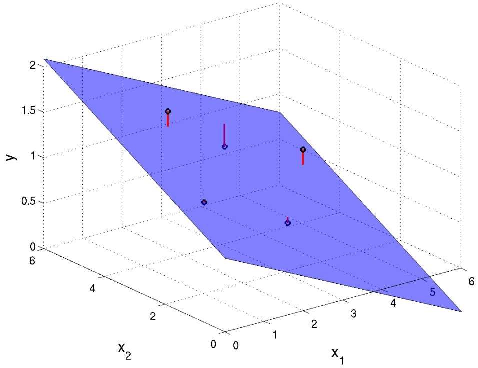
$$X = \begin{bmatrix} 1.0 & 1.0 & 2.0 \\ 1.0 & 2.0 & 4.5 \\ 1.0 & 3.2 & 2.1 \\ 1.0 & 3.9 & 5.1 \\ 1.0 & 5.2 & 4.2 \end{bmatrix} \qquad Y = \begin{bmatrix} 1.0 \\ 1.5 \\ 0.5 \\ 0.8 \\ 0.75 \end{bmatrix}$$

$$A = X^\top X = \begin{bmatrix} 5.0 & 15.3 & 17.9 \\ 15.3 & 57.49 & 59.45 \\ 17.9 & 59.45 & 72.31 \end{bmatrix}$$

$$c = X^\top Y = \begin{bmatrix} 4.55 \\ 12.62 \\ 17.03 \end{bmatrix}$$

$$w = \begin{bmatrix} 0.808 \\ -0.215 \\ 0.212 \end{bmatrix}$$

Least-Squares, Example



$$X = \begin{bmatrix} 1.0 & 1.0 & 2.0 \\ 1.0 & 2.0 & 4.5 \\ 1.0 & 3.2 & 2.1 \\ 1.0 & 3.9 & 5.1 \\ 1.0 & 5.2 & 4.2 \end{bmatrix} \qquad Y = \begin{bmatrix} 1.0 \\ 1.5 \\ 0.5 \\ 0.8 \\ 0.75 \end{bmatrix}$$

$$A = X^\top X = \begin{bmatrix} 5.0 & 15.3 & 17.9 \\ 15.3 & 57.49 & 59.45 \\ 17.9 & 59.45 & 72.31 \end{bmatrix}$$

$$c = X^\top Y = \begin{bmatrix} 4.55 \\ 12.62 \\ 17.03 \end{bmatrix}$$

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