The match of the two solutions $\phi_L(x)$ and $\phi_R(x)$ is done by imposing the continuity of the wave function and of its derivative at $x = 0$ which gives:

$$\begin{aligned} D_1\, e^{i\pi\mu}\, e^{-\mu a L} - D_2\, e^{-i\pi\mu}\, e^{-(1-\mu)aL} &= d_1 \left[M\, e^{\mu a L} + N\, e^{-(\mu+1)aL} \right] \\ -\mu a\, D_1\, e^{i\pi\mu}\, e^{-\mu a L} + (1-\mu)a\, D_2\, e^{-i\pi\mu}\, e^{-(1-\mu)aL} &= d_1 \left[-\mu a\, M\, e^{\mu a L} + N\,(\mu+1)a\, e^{-(\mu+1)aL} \right] \end{aligned}$$

and solving:

$$D_1 = \frac{d_1\, e^{-i\pi\mu}}{1-2\mu} \left[M\,(1-2\mu)\, e^{2\mu a L} + 2N e^{-aL} \right] \tag{39a}$$

$$D_2 = \frac{d_1\, e^{i\pi\mu}}{1-2\mu} \left[N(2\mu+1) e^{-2\mu a L} \right] \tag{39b}$$

We would like to remark that when solving the Dirac equation the continuity condition at a given boundary ($x = 0$ in our case) should be imposed by requiring the match of both the upper and lower spinor components ($u_1(x)$ and $u_2(x)$). In our second order approach based on the introduction of the auxiliary components $\phi(x)$ and $\chi(x)$, the derivative of one of the two (ϕ') is connected to the other (χ) because of Eq. 5a. In turns both the initial upper u_1 and lower u_2 components can be expressed in terms of ϕ and ϕ'. Indeed solving Eq. 4a and Eq. 4b and using Eq. 5a one finds for u_1 and u_2:

$$\begin{aligned} u_1(x) &= \frac{1}{2}\left[\left(1 + \frac{E - V(x)}{m} \right) \phi(x) + \frac{i}{m} \phi'(x) \right] \\ u_2(x) &= -\frac{i}{2}\left[\left(1 - \frac{E - V(x)}{m} \right) \phi(x) - \frac{i}{m} \phi'(x) \right] \end{aligned}$$

which shows how the matching of the wave function $\phi(x)$ and of its derivative $\phi'(x)$ is totally equivalent to requiring the continuity of $u_1(x)$ and $u_2(x)$. We also remind the reader that this method of matching the solution in $x = 0$ has been used also in ref. [24]. The above consideration applies as well to subsection C of section IV.

F. Probability current density, reflection and transmission coefficients

The reader might wonder whether the Dirac equation with a position dependent mass still has a conserved current. It is well know that a continuity equation for the current is related to the conservation of probability, or unitarity. It is quite straightforward to show that a coordinate dependence of the mass does not bring in any change in the derivation of the conserved current. This is related to the fact the mass multiplies the spinor wave function and in deriving the conserved