where $\mathcal{D}$ is a set of separable states and $S(\rho||\sigma)$ is a quantum relative entropy-i.e., $S(\rho||\sigma) = \mathrm{tr}(\rho \ln \rho - \rho \ln \sigma)$. It was shown in Ref.[12] that $E_R(\rho)$ is a upper bound of the distillable entanglement. Subsequently, Rains[13, 14] has shown that

$$\tilde{E}_R(\rho) = \min_{\sigma \in \mathcal{D}_{PPT}} S(\rho||\sigma), \tag{1.3}$$

where $\mathcal{D}_{PPT}$ is a set of positive partial transposition (PPT) states, is more tight upper bound when ρ is an higher-dimensional bipartite state. Using the facts that the REE is an upper bound of the distillable entanglement and Smolin state[15] is a bound entangled state, the distillable entanglement for the various Bell-state mixtures has been analytically computed[16–18]. In order to understand the distillable entanglement more deeply, therefore, it is important to develop the various techniques for the explicit computation of the REE. Of course, regardless of the distillable entanglement, the development of the calculation technique for the REE itself is important to understand the characterization of entanglement more profoundly. For last few years many properties of the REE were investigated[19]. Furthermore, recently relation between the REE and other distance measures has been studied[20, 21].

In this paper we confine ourselves to the REE when ρ is two-qubit states-i.e., $\rho \in \mathcal{H}^2 \otimes \mathcal{H}^2$. Since there is no bound-entangled state in this case, $E_R(\rho)$ and $\tilde{E}_R(\rho)$ defined in Eq.(1.2) and Eq.(1.3) are same. Let σ^* be the closest separable state (CSS) of ρ. Then, $E_R(\rho)$ is given by

$$E_R(\rho) = \min_{\sigma \in \mathcal{D}} S(\rho||\sigma) = S(\rho||\sigma^*). \tag{1.4}$$

When the CSS σ^* is explicitly given and it is full rank, Ref.[22] has presented how to construct the set of the entangled states, whose CSS are σ^*. Let $|i\rangle$ and λ_i be eigenvectors and corresponding eigenvalues of σ^*. If σ^* is the CSS (hence, edge) state, then its partial transposition σ^Γ is rank deficient. Let $|\phi\rangle$ be the kernel of σ^Γ-i.e.,

$$\sigma^\Gamma |\phi\rangle = 0. \tag{1.5}$$

Then, the set of the entangled states $\rho(x)$, whose CSS are σ^*, is given by the following one-parameter family expression:

$$\begin{aligned} \rho(x) &= \sigma^* - x G(\sigma^*) \qquad (1.6) \\ G(\sigma^*) &= \sum_{i,j} G_{ij} |i\rangle\langle i| (|\phi\rangle\langle\phi|)^\Gamma |j\rangle\langle j|, \end{aligned}$$