degrees of freedom at the more microscopic level 1 according to their relaxation times. By means of the projection-operator technique, one can then eliminate these "fast" degrees of freedom from the time evolution equations for the "slow" variables. The latter are then associated to the macroscopic variables at level 2. The crucial steps in this general procedure are to identify the proper mapping of the variables of one level to another, $\Pi(x) : x \longrightarrow y$, which average in a non-equilibrium ensemble $\rho_y(x)$ is the new coarse-grained variable y

$$y = \Big\langle \Pi\left(x\right) \Big\rangle_y = \int \mathcal{D}x \, \rho_y[x] \, \Pi\left(x\right). \tag{3.1}$$

A. Mappings and ensemble

For coarse graining from Cahn-Hilliard level, the relevant set of variables at the coarse-grained level $y = (\rho, \mathbf{g}, \epsilon, Q, \mathbf{q})$ is motivated by the phenomenological Doi-Ohta model [28, 33]. We assume that the hydrodynamic fields ρ, $\mathbf{g}$, and ϵ are smooth on the more microscopic Cahn-Hilliard length scale. Then the mappings Π_ρ, $\mathbf{\Pi_g}$, and Π_ϵ, simply pick out the hydrodynamic fields, while we need to determine the relationship of mappings Π_Q and $\mathbf{\Pi_q}$ to the underlying configuration. We do this by following the physical meaning rather than the exact definitions (1.2) of the Doi-Ohta variables. This is because, first, the interface orientational distribution function $f(\mathbf{n})$ is not given at the Cahn-Hilliard level, and, second, due to the difference in modeling of the interface. While in the Doi-Ohta model the interface us assumed to be sharp, in the diffuse-interface theories, like Cahn-Hilliard one, the interface is given through continuous variations of the composition c. Therefore, for determining the average interfacial area per unit volume Q and its orientation $\mathbf{q}$ one must use the appropriate combination of the gradients of the composition $\nabla c(\mathbf{r})$, and perform ensemble averaging which incorporates "smoothing" over a certain volume. We introduce a smoothing function $\chi(\mathbf{r} - \mathbf{r}')$, which averages the observable over a certain volume $v(\mathbf{r})$ around position $\mathbf{r}$, and satisfies the normalization condition $\int_V d^3r' \, \chi(\mathbf{r} - \mathbf{r}') = 1$. To obtain the variables which would correspond to the Doi-Ohta averaged interfacial area and its orientation, volume $v(\mathbf{r})$ must comprise many droplets for good statistics. That means that the length scale of the smoothing volume $v(\mathbf{r})$ – smoothing length a – must satisfy

$$\xi \ll L(t) \ll a \ll \Lambda, \tag{3.2}$$