These equations are given by

$$i\frac{\partial\psi_1}{\partial t}=-\frac{\partial^2\psi_1}{\partial x^2}+v_1(x,t)\psi_1+\sum_{k=1}^{2}(g_{1k}(x,t)|\psi_k|^2)\psi_1, \tag{1}$$

$$i\frac{\partial\psi_2}{\partial t}=-\frac{\partial^2\psi_2}{\partial x^2}+v_2(x,t)\psi_2+\sum_{k=1}^{2}(g_{2k}(x,t)|\psi_k|^2)\psi_2, \tag{2}$$

where $\psi_k=\psi_k(x,t)$, the functions, $v_k(x,t)$, are the trapping potentials, and the $g_{ij},(i,j=1,2)$ describe the strength of the cubic nonlinearities. Note that we are using standard notation, with both the fields and coordinates dimensionless. Note also that we are supposing that our model describe 1D BEC, and then it should engender *cigar-shaped* traps to induce tight confinement in two transverse directions, leaving the condensate almost free along the x axis. In the super-chemistry model of [7, 8], the above equations are amended by a term which goes as $\sqrt{2}\alpha\psi_1\psi_2$ in Eq. (1) and with a term of $\frac{\alpha}{\sqrt{2}}\psi_1^2$ in Eq. (2). These, number-nonconserving, terms arise from treating the Feshbach resonance as a coupling term between atoms and molecules.

It is interesting to comment here that the elimination of, say, ψ_2 in favor of an effective equation for ψ_1, would necessarily end up with a GP equation with depletion effect. This could appear in the form of complex nonlinearity. We leave such study for a future work.

In the following we seek to reduce the above two equations into the following pair of equations

$$\mu_1 A_1=-\frac{\partial^2 A_1}{\partial\zeta^2}+\sum_{k=1}^{2}\left(G_{1k}|A_k|^2\right)A_1, \tag{3}$$

$$\mu_2 A_2=-\frac{\partial^2 A_2}{\partial\zeta^2}+\sum_{k=1}^{2}\left(G_{2k}|A_k|^2\right)A_2, \tag{4}$$

with μ_i and the G_{ij} being constants. We achieve this with the use of the two*Anzatse*:

$$\psi_1(x,t)=\rho(x,t)e^{i\eta(x,t)}A_1(\zeta(x,t)), \tag{5}$$

$$\psi_2(x,t)=\rho(x,t)e^{i\eta(x,t)}A_2(\zeta(x,t)). \tag{6}$$

However, we now have to have

$$\rho\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}\left(\rho^2\frac{\partial\eta}{\partial x}\right)=0, \tag{7}$$

$$\frac{\partial\zeta}{\partial t}+2\frac{\partial\eta}{\partial x}\frac{\partial\zeta}{\partial x}=0, \tag{8}$$

$$\frac{\partial}{\partial x}\left(\rho^2\frac{\partial\zeta}{\partial x}\right)=0. \tag{9}$$