<u>**Proof of Theorem 6:**</u> We denote $\boldsymbol{\theta}_n = (\widehat{\mathbf{B}}_n, \widehat{\boldsymbol{\Sigma}}_n)$ and $\boldsymbol{\theta}_0 = (\mathbf{B}_0, \sigma_0^2 \boldsymbol{\Sigma}_0)$.

Since we assumed that the distribution of errors $\mathbf{u}$ is elliptical with density of the form (4.1), for any function h we have $E_{H_0}\left(x_j u_i h(\mathbf{u}'\boldsymbol{\Sigma}_0^{-1}\mathbf{u})\right) = 0$. This implies that

$$E_{H_0} \frac{1}{\sigma_0^2} W\left(d(\mathbf{B}, \sigma_0^2 \boldsymbol{\Sigma}_0)\right)(\mathbf{y} - \mathbf{B}'\mathbf{x})\mathbf{x}' \tag{A.53}$$

vanishes at $\mathbf{B} = \mathbf{B}_0$. Then $\boldsymbol{\theta}_0$ is a zero of the function $\Phi(\boldsymbol{\theta}) = E_{H_0}\phi(\mathbf{z}; \boldsymbol{\theta})$.

By Lemma 19 there exists a bounded subset $\mathcal{C}$ and a function $\boldsymbol{\theta}(\xi)$ such that $\boldsymbol{\theta}_0$ is an interior point of $\boldsymbol{\theta}(\mathcal{C})$ and since $\boldsymbol{\theta}_n \to \boldsymbol{\theta}_0$ a.s., $\boldsymbol{\theta}_n \in \boldsymbol{\theta}(\mathcal{C})$ for n large enough, i.e., $\phi_{kj}(\mathbf{z}; \boldsymbol{\theta}_n)$ and $\phi_{kj}(\mathbf{z}; \boldsymbol{\theta}_0)$ belong to the Euclidean class $\mathfrak{F}_{kj}$ for n sufficiently large. By (A5), the functions $\phi_{kj}(\mathbf{z}; \boldsymbol{\theta}_n)$ and $\phi_{kj}(\mathbf{z}; \boldsymbol{\theta}_0)$ are in the class $[\delta]$ of Lemma 20 for each $\delta > 0$ and n sufficiently large. Hence,

$$|\sqrt{n}\{\nu_n(\phi_{kj}(\cdot; \boldsymbol{\theta}_n)) - \nu_n(\phi_{kj}(\cdot; \boldsymbol{\theta}_0))\}| \longrightarrow 0 \tag{A.54}$$

in probability. Then since $\nu_n(\phi_{kj}(\cdot; \boldsymbol{\theta}_n)) - \nu_n(\phi_{kj}(\cdot; \boldsymbol{\theta}_0)) = \mathrm{o}_P(1/\sqrt{n})$ for all $k = 1, \ldots, q$ and $j = 1, \ldots, p$ and $\phi_{kj}(\mathbf{z}; \boldsymbol{\theta})$ corresponds to the element $h = (j-1)q + k$ of the function ϕ, we conclude that

$$\nu_n(\phi(\cdot; \boldsymbol{\theta}_n)) - \nu_n(\phi(\cdot; \boldsymbol{\theta}_0)) = \mathrm{o}_P(1/\sqrt{n}). \tag{A.55}$$

Since $\partial\Phi/\partial\mathrm{vec}(\mathbf{B}')'$ is continuous in $\boldsymbol{\theta}_0$, we have that

$$\Phi(\mathbf{B}, \boldsymbol{\Sigma}) = \Phi(\mathbf{B}_0, \boldsymbol{\Sigma}) + \left(\frac{\partial\Phi(\mathbf{B}, \boldsymbol{\Sigma})}{\partial\mathrm{vec}(\mathbf{B}')'}(\mathbf{B}_0, \boldsymbol{\Sigma})\right)\mathrm{vec}(\mathbf{B}' - \mathbf{B}_0') + r(\boldsymbol{\theta})\mathrm{vec}(\mathbf{B}' - \mathbf{B}_0') \tag{A.56}$$

where $r(\boldsymbol{\theta}) \to \mathbf{0}$ when $\boldsymbol{\theta} \to \boldsymbol{\theta}_0$.