corrections to fermionic and bosonic masses, has been determined in the real-time formulation in [22]. The inevitable breaking of supersymmetry at finite temperature has sometimes been called *spontaneous collapse* of supersymmetry [23].

In Sect. IV we derive the RG flow equations at finite temperature. In addition to the momentum integrals we are confronted with sums over Matsubara frequencies. For the three-dimensional Wess-Zumino model and for a particular regulator the thermal sums can be calculated analytically. Related sums have been discussed in earlier works on finite-temperature renormalization group flow equations, for example in [24–27, 29–32]. We observe that the Wess-Zumino model in three dimensions at finite temperature in the $\mathbb{Z}_2$ symmetric phase behaves similarly to a gas of massless bosons. In particular we show in Sect. IV A that it obeys the Stefan-Boltzmann law in three dimensions. For high temperatures the fermions do not contribute to the flow equations since they do not have a thermal zero-mode. On the other hand we observe dimensional reduction in the bosonic part of the model due to the presence of a thermal zero-mode. We show in Sect. IV B how this is manifested in our RG framework. In a similar way dimensional reduction has been observed in $O(N)$-models at finite temperature in [33, 34]. Finally we compute the phase diagram for the restoration of the global $\mathbb{Z}_2$ symmetry at finite temperature in Sect. IV C.

II. THE $\mathcal{N}=1$ WESS-ZUMINO MODEL IN THREE DIMENSIONS AT $T=0$

There are many works on the supersymmetric Wess-Zumino models in both four and two space-time dimensions. Actually the two-dimensional model with $\mathcal{N}=2$ supersymmetries is just the toroidal compactification of the four-dimensional $\mathcal{N}=1$ model. The three-dimensional model with $\mathcal{N}=1$ supersymmetry, on the other hand, cannot be obtained by dimensional reduction of a local field theory in four dimensions. Thus it may be useful to recall the construction of the three-dimensional model starting from the real superfield

$$\Phi(x,\alpha)=\phi(x)+\bar\alpha\psi(x)+\frac{1}{2}\bar\alpha\alpha F(x) \tag{1}$$

with real (pseudo)scalar fields ϕ, F and Majorana spinor-field ψ. The supersymmetry variations are generated by the supercharge

$$\delta_\beta\Phi=i\bar\beta\mathcal{Q}\Phi,\quad \mathcal{Q}=-i\frac{\partial}{\partial\bar\alpha}-(\gamma^\mu\alpha)\partial_\mu\,. \tag{2}$$

We use the metric tensor $(\eta_{\mu\nu})=\mathrm{diag}(1,-1-1)$ to lower Lorentz indices. With the aid of the symmetry relations for Majorana spinors $\bar\psi\chi=\bar\chi\psi,\ \bar\psi\gamma^\mu\chi=-\bar\chi\gamma^\mu\psi$ and the particular Fierz identity $\alpha\bar\alpha=-\bar\alpha\alpha\,\mathbb{1}/2$ the transformation laws for the component fields follow from Eq. (2):

$$\delta\phi=\bar\beta\psi,\quad \delta\psi=(F+i\partial\!\!\!/\phi)\beta,\quad \delta F=i\bar\beta\partial\!\!\!/\psi\,. \tag{3}$$

The anticommutator of two supercharges yields $\{\mathcal{Q}_\alpha,\bar{\mathcal{Q}}^\beta\}=2(\gamma^\mu)_\alpha{}^\beta\partial_\mu$. The supercovariant derivatives are

$$\mathcal{D}=\frac{\partial}{\partial\bar\alpha}+i(\gamma^\mu\alpha)\partial_\mu,\quad\text{and}\quad \bar{\mathcal{D}}=-\frac{\partial}{\partial\alpha}-i(\bar\alpha\gamma^\mu)\partial_\mu. \tag{4}$$

Up to a sign they obey the same anticommutation relation as the supercharges

$$\{\mathcal{D}_\alpha,\bar{\mathcal{D}}^\beta\}=-2(\gamma)_\alpha{}^\beta\partial_\mu\,. \tag{5}$$

As kinetic term we choose the D term of $\bar{\mathcal{D}}\Phi\mathcal{D}\Phi=2\bar\alpha\alpha\mathcal{L}_{\mathrm{kin}}+\dots$ which reads

$$\mathcal{L}_{\mathrm{kin}}=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{i}{2}\bar\psi\partial\!\!\!/\psi+\frac{1}{2}F^2. \tag{6}$$

The interaction term is the D term of $2W(\Phi)=\bar\alpha\alpha\mathcal{L}_{\mathrm{int}}+\dots$ and contains a Yukawa term,

$$\mathcal{L}_{\mathrm{int}}=FW'(\phi)-\frac{1}{2}W''(\phi)\bar\psi\psi. \tag{7}$$

The complete off-shell Lagrange density $\mathcal{L}_{\mathrm{off}}=\mathcal{L}_{\mathrm{kin}}+\mathcal{L}_{\mathrm{int}}$ takes then the simple form

$$\mathcal{L}_{\mathrm{off}}=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{i}{2}\bar\psi\partial\!\!\!/\psi+\frac{1}{2}F^2+FW'(\phi)-\frac{1}{2}W''(\phi)\bar\psi\psi. \tag{8}$$

Eliminating the auxiliary field via its equation of motion $F=-W'(\phi)$, we end up with the on-shell density

$$\mathcal{L}_{\mathrm{on}}=\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-\frac{i}{2}\bar\psi\partial\!\!\!/\psi-\frac{1}{2}W'^2(\phi)-\frac{1}{2}W''(\phi)\bar\psi\psi. \tag{9}$$

From this expression we read off that $W'^2(\phi)$ acts as a self-interaction potential for the scalar fields. For a polynomial superpotential $W(\phi)$ in which the power of the leading term is even, $W(\phi)=c\phi^{2n}+\mathcal{O}(\phi^{2n})$, we do not observe supersymmetry breaking in our present non-perturbative renormalization group study[1]. On the other hand spontaneous supersymmetry breaking is definitely possible for a superpotential in which the power of the leading term is odd. In the explicit calculations we shall use a Majorana representation for the γ-matrices, $\gamma^0=\sigma_2$, $\gamma^1=i\sigma_3$ and $\gamma^2=i\sigma_1$.

III. FLOW EQUATION AT ZERO TEMPERATURE

To find a manifestly *supersymmetric flow equation* in the off-shell formulation we extend our earlier results on

[1] In a two-loop calculation a ground state with broken supersymmetry has been found in Ref. [36]. Since we neglect higher F-terms in our non-perturbative study it is not possible to check whether the findings of this perturbative analysis of the Wess-Zumino model hold when higher-order corrections are taken into account.