in which the self-energy has been expressed as $i\Sigma_C(q) = q^2 b(q^2)$.

Thus, with the expressions (31) and (46), we can rewrite the identity (42) as follows

$$q_\mu \Gamma^\mu(k,p,q)[1+b(q^2)] = ie[S_F^{-1}(k)H_1(k,p,q) - H_2(k,p,q)S_F^{-1}(p)]. \tag{47}$$

By considering that energy-momentum conservation holds at the vertices $\Gamma^\mu(k,p,q)$ and $H(k,p,q)$, we can write

$$\Gamma^\mu(k,p,q) = ie(2\pi)^4\delta^4(k-p-q)\tilde{\Gamma}^\mu(p,p+q) \tag{48}$$

and

$$H(k,p,q) = (2\pi)^4\delta^4(k-p-q)\tilde{H}(p,p+q). \tag{49}$$

With this representation we may obtain from Eq. (47) that

$$q_\mu \tilde{\Gamma}^\mu(p,p+q)[1+b(q^2)] = S_F^{-1}(p+q)\tilde{H}_1(p,p+q) - \tilde{H}_2(p,p+q)S_F^{-1}(p). \tag{50}$$

C. The triple photon vertex

To obtain ST identity for the triple photon vertex, the proper part of $\langle 0|T(A_\mu A_\nu A_\lambda)|0\rangle$, we differentiate the functional equation (15) with respect to $\zeta(x)$, $J^\nu(y)$ and $J^\lambda(z)$ and turn off all the sources. The result is

$$\frac{1}{\alpha e}\partial_x^\mu \frac{\delta^3 W}{\delta J^\mu(x)\delta J^\nu(y)\delta J^\lambda(z)}\Bigg| = \frac{\delta^3 W}{\delta\zeta(x)\delta K^\nu(y)\delta J^\lambda(z)}\Bigg| + \frac{\delta^3 W}{\delta\zeta(x)\delta J^\nu(y)\delta K^\lambda(z)}\Bigg|, \tag{51}$$

or in terms of the Green functions,

$$\begin{aligned} -\frac{1}{\alpha}\partial_x^\mu\langle 0|T(A_\mu(x)A_\nu(y)A_\lambda(z))|0\rangle \;&=\; \langle 0|T(\bar{C}(x)D_\nu^{AD}(y)C(y)A_\lambda(z))|0\rangle \\ &\quad+\langle 0|T(\bar{C}(x)A_\nu(y)D_\lambda^{AD}(z)C(z))|0\rangle, \end{aligned} \tag{52}$$

where $D_\nu^{AD}(y)$ denotes the covariant derivative in the adjoint representation, $D_\nu^{AD}(y)C(y) = \partial_{y\nu}C(y) - ie[A_\nu(y), C(y)]_\star$. Thus, we can rewrite the above expression as

$$\begin{aligned} &-\frac{1}{\alpha}\partial_x^\mu\langle 0|T(A_\mu(x)A_\nu(y)A_\lambda(z))|0\rangle = \partial_{y\nu}\langle 0|T(\bar{C}(x)C(y)A_\lambda(z))|0\rangle \\ &+2e\sin(\partial_y\wedge\partial_{\hat{y}})\langle 0|T(\bar{C}(x)A_\nu(y)C(\hat{y})A_\lambda(z))|0\rangle + \partial_{z\lambda}\langle 0|T(\bar{C}(x)A_\nu(y)C(z))|0\rangle \\ &+2e\sin(\partial_z\wedge\partial_{\hat{z}})\langle 0|T(\bar{C}(x)A_\nu(y)A_\lambda(z)C(\hat{z}))|0\rangle, \end{aligned} \tag{53}$$

where, after the application of the differential operators, we must identify $\hat{y}$ and $\hat{z}$ respectively with y and z.