1 Introduction

In many scientific problems, it is of interest to determine whether two particular functions are equal to each other. In many settings these functions are unknown and may be viewed as features of a data-generating mechanism from which observations can be collected. As such, these functions can be learned from available data, and estimates of these respective functions can then be compared. To reduce the risk of deriving misleading conclusions due to model misspecification, it is appealing to employ flexible statistical learning tools to estimate the unknown functions. Unfortunately, inference is usually extremely difficult when such techniques are used, because the resulting estimators tend to be highly irregular. In such cases, conventional techniques for constructing confidence intervals or computing p-values are generally invalid, and a more careful construction, as exemplified by the work presented in this article, is required.

To formulate the problem statistically, suppose that n independent observations $O_1, \ldots, O_n$ are drawn from a distribution P_0 known only to lie in the nonparametric statistical model, denoted by $\mathcal{M}$. Let $\mathcal{O}$ denote the support of P_0, and suppose that $P \mapsto R_P$ and $P \mapsto S_P$ are parameters mapping from $\mathcal{M}$ onto the space of univariate bounded real-valued measurable functions defined on $\mathcal{O}$, i.e. R_P and S_P are elements of the space of univariate bounded real-valued measurable functions defined on $\mathcal{O}$. For brevity, we will write $R_0 \triangleq R_{P_0}$ and $S_0 \triangleq S_{P_0}$. Our objective is to test the null hypothesis

$$\mathcal{H}_0 : R_0(O) \stackrel{d}{=} S_0(O)$$

versus the complementary alternative $\mathcal{H}_1$: not $\mathcal{H}_0$, where O follows the distribution P_0 and the symbol $\stackrel{d}{=}$ denotes equality in distribution. We note that $R_0(O) \stackrel{d}{=} S_0(O)$ if $R_0 \equiv S_0$, i.e. $R_0(O) = S_0(O)$ almost surely, but not conversely. The case where $S_0 \equiv 0$ is of particular interest since then the null simplifies to $\mathcal{H}_0 : R_0 \equiv 0$. Because P_0 is unknown, R_0 and S_0 are not readily available. Nevertheless, the observed data can be used to estimate P_0 and hence