Substituting $\langle\phi\rangle$, ψ_* and Γ, Eqs. (4.7), (4.92) and (4.195) respectively, into Eq. (4.209) gives the constraint

$$h \ll \left(g^2 m_0 M_P^{\alpha-1}\right)^{2\alpha-1} \left(\frac{\sqrt{(2n+4)\,\lambda}}{m_0 M_P^n}\right)^{\frac{2\alpha-1}{n+1}} \left(\frac{100\sqrt{H_*}}{m_0 M_P^{\alpha-1}}\right)^{2\alpha-2} \tag{4.210}$$

4.5.2.3 Energy Density of the Spectator Field

We require the energy density of ψ to be subdominant after thermal inflation up until it decays, in order that it does not cause any inflation by itself. The energy density of ψ after thermal inflation is

$$\rho_\psi = h^2 \frac{\psi_*^{2\alpha}}{M_P^{2\alpha-2}} \overline{\phi}^2 + \frac{1}{2} m_\psi^2 \psi_*^2 \tag{4.211}$$

$$\sim h^2 \frac{\psi_*^{2\alpha}}{M_P^{2\alpha-2}} \langle\phi\rangle^2 + \frac{1}{2} m_\psi^2 \psi_*^2 \tag{4.212}$$

For simplicity, we assume that ψ decays around the same time as ϕ, i.e. that H does not change much between the time when ϕ decays and the time when ψ decays. Therefore, the energy density of the Universe at the time when ψ decays is $\sim M_P^2 \Gamma^2$. We therefore require

$$\rho_\psi \ll M_P^2 \Gamma^2 \tag{4.213}$$

$$h^2 \frac{\psi_*^{2\alpha}}{M_P^{2\alpha-2}} \langle\phi\rangle^2 + \frac{1}{2} m_\psi^2 \psi_*^2 \ll M_P^2 \Gamma^2 \tag{4.214}$$