Write $\widetilde{r}_{bi}$ for the response of unit i in block b in the uniformity trial. There is only one such $\widetilde{r}_{bi}$, not one for each $\mathbf{z} \in \Omega$, because the realized treatment assignment $\mathbf{Z}$ that was recorded in an office has no way to affect the biological response of unit i in block b. Because the uniformity trial was not actually performed, none of the $\widetilde{r}_{bi}$ are observed. Generally, $\widetilde{r}_{bi}$ need not equal any of $r_{bi\mathbf{z}}$, $\mathbf{z} \in \Omega$. If there were no interference between units, then $\widetilde{r}_{bi}$ would equal $r_{bi\mathbf{z}}$ for every $\mathbf{z} \in \Omega$ with $z_{bi} = 0$, because without interference the response of unit bi depends only on the treatment z_{bi} assigned to bi; however, with interference, it can happen that $\widetilde{r}_{bi} \neq r_{bi\mathbf{z}}$ for every $\mathbf{z} \in \Omega$. Write $\widetilde{\mathbf{r}} = (\widetilde{r}_{11}, \ldots, \widetilde{r}_{B,N_B})^T$.

In the presence of interference between units, the magnitude of the treatment effect is understood not as a comparison of treated and control groups both of which are affected by the treatment, but as a comparison of the actual experiment and the uniformity trial.

3 Inference with Interference

3.1 Preliminaries: a nonlinear rank statistic; testing no treatment effect

Fix an integer $k \geq 2$, with $k \leq \min_{b \in \{1,\ldots,B\}} m_b + 1$. As will be seen, the familiar choice is $k = 2$, and it yields the Mann-Whitney U-statistic, but there are reasons to prefer a larger value of k when only some treated units respond to treatment. Ties among responses are not an issue in the fMRI experiment of §1.3, where blood oxygenation is recorded to many digits. We assume no ties in the discussion that follows.

The technical material that follows is not difficult but does require a certain amount of notation. To simplify, the reader may consider the special case of a single block ($B = 1$) with the parameter k set to $k = 2$; then, one is considering a single-subject completely randomized trial, like the lady tasting tea, using the Mann-Whitney-Wilcoxon statistic, which happens to be the only linear placement statistic that is also a linear rank statistic (Orban and Wolfe 1982).