sented a detailed discussion of this scenario in the $CP^{N-1}$ model, which provides a two-dimensional model with a topological charge structure very similar to QCD. For our discussion of the $CP^{N-1}$ model, the full superstructure of a gauge/gravity dual was not necessary to arrive at the BCFT which describes the tachyonic decay of the vacuum. Instead we showed that the $CP^{N-1}$ model could be formulated as the field theoretic limit of an open bosonic string in background fields $\lambda$ and $A_\mu$. In a proper time formulation, the propagator for the charged field $Z$ is given by the $\alpha' \to 0$ limit of the open string propagator. The string length parameter $\sqrt{\alpha'}$ plays a role analogous to lattice spacing in a lattice-regulated gauge theory. This string-theoretic cutoff naturally incorporates one of the most striking features of the observed topological charge sheets on the lattice, the fact that the thickness of each sheet and the separation between sheets is determined by the cutoff scale and approaches zero in the continuum limit.

An important issue which has not been adequately addressed in this paper is the question of scaling and universality in a theory whose properties depend so crucially on dynamics that occurs near the cutoff scale. In particular, the proper scaling behavior of the topological susceptibility is not a foregone conclusion. As an example of what can go wrong, the simplest lattice formulation of the $CP^1$ and $CP^2$ models do not exhibit proper scaling for $\chi_t$ due to small instantons[4, 9], and it is not clear whether the topological susceptibility in these models can be defined as the continuum limit of a lattice calculation. Fortunately, for both $CP^{N-1}$ with $N > 4$ [3] and for 4-dimensional $SU(3)$ gauge theory [21], small instantons do not contribute, and $\chi_t$ is found to scale properly. So at least the numerical evidence indicates that the topological sandwich structure leads to a finite topological susceptibility in the continuum limit and does not cause the same scaling difficulties as small instantons. It is interesting to note that as the scaling limit is approached the scaling of $\chi_t$ takes place by virtue of an increasingly precise cancellation in the integrated correlator between the positive contribution at the origin and the negative tail at $|x| \geq 2$ lattice spacings, with the positive and negative contributions separately diverging in the continuum limit [3, 24]. Since the positive contribution comes from each individual brane, while the negative contribution comes from the juxtaposition of oppositely charged branes, this cancellation is strong evidence that the positions of the layered membranes are highly correlated and arranged in a regular periodic array, as would be expected in the tachyon crystal scenario. We can conclude from this that the presence of thin membranes of topological charge in the vacuum does indeed pose a threat to proper scaling, just as small instantons do, in the sense that a random distribution of such membranes would lead to a divergent $\chi_t$.. It is only through the regular "antiferromagnetic" ordering of the membranes that proper scaling is achieved.