with curve features.

3 The warped ANOVA model

Let us go back now to the original problem of a one-factor design, where the sample of n individuals can be separated into I groups, with group i containing J_i individuals. For subject j in group i we observe certain variable (e.g. mass) at time points $t_{ij1}, \ldots, t_{ij\nu_{ij}}$, obtaining observations $y_{ij1}, \ldots, y_{ij\nu_{ij}}$. The number of observations ν_{ij} as well as the time points may change from individual to individual. We assume

$$y_{ijk} = x_{ij}(t_{ijk}) + \varepsilon_{ijk}, \tag{5}$$

where $\{x_{ij}(t)\}$ are underlying smooth curves, no directly observable, and $\{\varepsilon_{ijk}\}$ are i.i.d. $N(0, \sigma^2)$ random errors independent of the underlying $x_{ij}(t)$s. Observational model (5), which treats the smooth curves $\{x_{ij}(t)\}$ as latent variables, is the usual way to bridge functional data analysis and longitudinal data analysis (Müller, 2008). As discussed in Section 2, we can write $x_{ij}(t) = z_{ij}\{w_{ij}^{-1}(t)\}$ for a warped process $z_{ij}(t)$ and a warping function $w_{ij}(t)$. These will inherit the dependence structure of the x_{ij}s, so we can assume

$$z_{ij}(t) = \mu(t) + \alpha_i(t) + \beta_{ij}(t), \quad j = 1, \ldots, J_i, \quad i = 1, \ldots, I, \tag{6}$$

with $\{\alpha_i(t)\}$ and $\{\beta_{ij}(t)\}$ zero-mean random factors independent of each other and among themselves. For the main factor $\alpha(t)$ and the residual term $\beta(t)$ we assume expansions analogous to (4):

$$\alpha(t) = \sum_{k=1}^{p} U_k \phi_k(t), \tag{7}$$