second differential of $g(B) = r\log\left|BB^T\right|$ is

$$\mathrm{d}^2 g = \mathrm{d}\,\mathrm{tr}\left\{2rB^{-1}\mathrm{d}B\right\} = \mathrm{tr}\left\{-2r\left[B^{-1}(\mathrm{d}B)\right]^2\right\}\ . \tag{49}$$

By Theorem 10.6.1 of (Magnus and Neudecker 2007), the Hessian of g is $-2rK_n\left(B^{-\top}\otimes B^{-1}\right)$, where K_n is the order-n commutation matrix and $\otimes$ denotes the Kronecker product. We now show that $K_n\left(B^{-\top}\otimes B^{-1}\right)$ is (matrix) positive-definite.

$$(\mathrm{d}\,\mathrm{vec}\,X)^\top K_n\left(B^{-\top}\otimes B^{-1}\right)(\mathrm{d}\,\mathrm{vec}\,X) = (\mathrm{d}\,\mathrm{vec}\,X)^\top K_n\,\mathrm{vec}\left\{B^{-1}(\mathrm{d}X)B^{-1}\right\} \tag{50}$$

$$= (\mathrm{vec}\,\mathrm{d}X)^\top\,\mathrm{vec}\left\{B^{-\top}(\mathrm{d}X)^\top B^{-\top}\right\} \tag{51}$$

$$= \mathrm{tr}\left\{\left(B^{-1}\mathrm{d}X\right)^2\right\} \geq 0\ . \tag{52}$$

Equation (50) follows from the well-known fact that $\mathrm{vec}\,ABC = (C^\top\otimes A)\,\mathrm{vec}\,B$. Thus, the Hessian of g is negative definite, and $r\log\left|BB^T\right|$ is concave.

Concavity of the middle term in (48) follows in the usual way from the univariate convexity of the function

$$g(t) := \mathrm{tr}\left(Q(M+tP)(M+tP)^\top\right) = \sum_{i=1}^{n}(m_i+tp_i)^\top Q(m_i+tp_i) \tag{53}$$

for fixed matrices M and P, with columns m_i and p_i. To see that the rightmost term in (48) is concave, define

$$g_j(t) := a_j + c_j^\top(M+tQ)(M+tQ)^\top c_j$$

for $j = 1,\ldots,d$ and fixed matrices M and Q. Each g_j is convex in t, and the rightmost term in (48) is (minus) the log-sum-exp function composed with the g_j's. Concavity of this term in t, and hence in B, follows from (Boyd and Vandenberghe 2004; p. 86).