

Remark 2.1 For the application of the results, it is possible to take the initial moment as $t_0 = 0$, without being the discontinuity moment since of the group property. Then $x_0 \notin \Gamma \cup \tilde{\Gamma}$.

Denote by $\widehat{[a, b]}$, $a, b \in \mathbb{R}$, the interval $[a, b]$, whenever $a \leq b$ and $[b, a]$, otherwise. Let $x_1(t) \in PC(\mathbb{R}_+, \theta^1)$, $\theta^1 = \{\theta_i^1\}$, and $x_2(t) \in PC(\mathbb{R}_+, \theta^2)$, $\theta^2 = \{\theta_i^2\}$, be two different solutions of (2.1).

Definition 2.3 The solution $x_2(t)$ is in the ϵ -neighborhood of $x_1(t)$ on the interval \mathcal{J} if

- the sets θ^1 and θ^2 have same number of elements in \mathcal{J} ;
- $|\theta_i^1 - \theta_i^2| < \epsilon$ for all $\theta_i^1 \in \mathcal{J}$;
- the inequality $\|x_1(t) - x_2(t)\| < \epsilon$ is valid for all t , which satisfy $t \in \mathcal{J} \setminus \bigcup_{\theta_i^1 \in \mathcal{J}} (\theta_i^1 - \epsilon, \theta_i^1 + \epsilon)$.

The topology defined with the help of ϵ -neighborhoods is called the B-topology. It can be apparently seen that it is Hausdorff and it can be considered also if two solutions $x_1(t)$ and $x_2(t)$ are defined on a semi-axis or on the entire real axis.

Definition 2.4 The solution $x_0(t) = x(t, 0, x_0)$, $t \in \mathbb{R}$, $x_0 \in D$, of (2.1) B-continuously depends on x_0 for increasing t if there corresponds a positive number δ to any positive ϵ and a finite interval $[0, b]$, $b > 0$ such that any other solution $x(t) = x(t, 0, \tilde{x})$ of (2.1) lies in ϵ -neighborhood of $x_0(t)$ on $[0, b]$ whenever $\tilde{x} \in B(x_0, \delta)$. Similarly, the solution $x_0(t)$ of (2.1) B-continuously depends on x_0 for decreasing t if there corresponds a positive number δ to any positive ϵ and a finite interval $[a, 0]$, $a < 0$ such that any other solution $x(t) = x(t, 0, \tilde{x})$ of (2.1) lies in ϵ -neighborhood of $x_0(t)$ on $[a, 0]$ whenever $\tilde{x} \in B(x_0, \delta)$. The solution $x_0(t)$ of (2.1) B-continuously depends on x_0 if it continuously depends on the initial value, x_0 , for both increasing and decreasing t .

If conditions (C1)-(C7) hold, then each solution $x_0(t) : \mathbb{R} \rightarrow \mathbb{R}^n$, $x_0(t) = x(t, 0, x_0)$, of (2.1) continuously depends on x_0 [1].

2.1 B-equivalence to a system with fixed moments of impulses

In order to facilitate the analysis of the system with variable moments of impulses (2.1), a B-equivalent system [1] to the system with variable moments of impulses will be utilized in our study. Below, we will construct the B-equivalent system.

Let $x(t) = x(t, 0, x_0 + \Delta x)$ be a solution of system (2.1) neighbor to $x_0(t)$ with small $\|\Delta x\|$. If the point $x_0(\theta_i)$ is a (β) - or (γ) - type point, then it is a boundary point. For this reason, there exist two different possibilities for the near solution $x(t)$ with respect to the surface of discontinuity. They are:

- (N1) The solution $x(t)$ intersects the surface of discontinuity, Γ , at a moment near to θ_i ,
- (N2) The solution $x(t)$ does not intersect Γ , in a small time interval centered at θ_i .