the n transverse dimensions [4, 6],

$$\Delta_n(r) = \frac{1}{(2\pi)^{\frac{n}{2}}} \left(\frac{m_\chi}{M_*}\right)^{n-2} (m_\chi r)^{-\frac{n-2}{2}} K_{\frac{n-2}{2}}(m_\chi r) \,. \quad (5)$$

With the natural choice $r = R$ and $\langle\sigma\rangle \lesssim M_*$, it is easy to read

$$\langle\chi\rangle \sim M_* \Delta_n(R) \,. \quad (6)$$

We will clarify later a heavy mass m_χ is necessary for a successful leptogenesis. However, the modified Bessel function $K_{\frac{n-2}{2}}(m_\chi r)$ will exponentially suppress $\langle\chi\rangle$ if $m_\chi \gg 1/r$. This $\langle\chi\rangle$ is too small to generate the desired neutrino masses. Therefore, we consider the brane-lattice crystallization scenario [18] where the bulk is populated with large numbers of branes. One finds the lepton number violation in our brane would be [2]

$$\begin{aligned} \langle\chi\rangle &\sim M_* \int d^n r n_{brane} \Delta_n(r) \\ &= M_* n_{brane} \left(\frac{m_\chi}{M_*}\right)^{n-2} \frac{1}{m_\chi^n} \\ &= M_* \left(\frac{M_*}{m_\chi}\right)^2 \left(\frac{M_*}{M_{\rm Pl}}\right)^{\frac{4}{n}} \end{aligned} \quad (7)$$

with the brane density [18]

$$n_{brane} \sim M_*^n \left(\frac{M_*}{M_{\rm Pl}}\right)^{\frac{4}{n}} \,. \quad (8)$$

By taking the natural assumption $m_\chi \lesssim M_*$, the VEV $\langle\chi\rangle$ is power suppressed by the ratio of M_* over $M_{\rm Pl}$. In the following we will consider

$$\langle\chi\rangle \sim 260\,\mathrm{eV} \quad \text{for } n = 6\,, M_* = 3\,\mathrm{TeV} \text{ and } m_\chi \lesssim M_* \,. \quad (9)$$

Due to the VEV $\langle\chi\rangle$, the masses of the right-handed neutrinos $N_{R_{1,2}}$ would be

$$\begin{aligned} \mathcal{L} \supset & -M_N \bar{N}_{R_1}^c N_{R_2} - \frac{1}{2} m_1 \bar{N}_{R_1}^c N_{R_1} - \frac{1}{2} m_2 \bar{N}_{R_2}^c N_{R_2} \\ & + \mathrm{H.c.} \end{aligned} \quad (10)$$

with

$$m_{1,2} = h_{1,2} \langle\chi\rangle \ll M_N \,. \quad (11)$$

We can diagonalize the above mass terms to be

$$\mathcal{L} \supset -\frac{1}{2} M_\pm \bar{X}_R^{\pm c} X_R^\pm + \mathrm{H.c.} \quad (12)$$

by taking the rotations as below,

$$N_{R_1} = c X_R^+ - i s X_R^- \,, \quad (13a)$$
$$N_{R_2} = s X_R^+ + i c X_R^- \,. \quad (13b)$$

Here we have take the following notations,

$$c \equiv \cos\vartheta \,, \quad s \equiv \sin\vartheta \quad \text{for} \quad \vartheta = \frac{1}{2} \arctan \frac{2M_N}{m_2 - m_1} \quad (14)$$

and

$$M_+ = 2scM_N + c^2 m_1 + s^2 m_2 \,, \quad (15a)$$
$$M_- = 2scM_N - s^2 m_1 - c^2 m_2 \,. \quad (15b)$$

Without loss of generality we will assume $m_1 < m_2$ so that

$$M_+ > M_- > 0 \,. \quad (16)$$

Actually, the small and large masses (11), i.e. $m_{1,2} \ll M_N$ will induce

$$\vartheta \simeq \tfrac{\pi}{4} \,, \quad (17a)$$
$$M_\pm \simeq M_N \pm \tfrac{1}{2}(m_1 + m_2) \gg M_+ - M_- \,. \quad (17b)$$

It is convenient to define the Majorana fermions

$$X^+ = X_R^+ + X_R^{+c} \,, \quad (18a)$$
$$X^- = X_R^- + X_R^{-c} \,, \quad (18b)$$

as the mass eigenstates,

$$\mathcal{L} \supset -\frac{1}{2} M_\pm \bar{X}^\pm X^\pm \,. \quad (19)$$

We further assume the doublet scalar η much heavier than the Majorana fermions $X^\pm$, say $m_\eta^2 \gg M_\pm^2$. In this case, we can integrate out η and then simplify the Yukawa couplings given by the first line of Eq. (2) in a new form,

$$\begin{aligned} \mathcal{L} \supset & -(c y_{\alpha 1} - s y'_{\alpha 2}) \bar{\psi}_{L_\alpha} \phi X^+ \\ & - i (s y_{\alpha 1} + c y'_{\alpha 2}) \bar{\psi}_{L_\alpha} \phi X^- + \mathrm{H.c.} \end{aligned} \quad (20)$$

with

$$y'_{\alpha 2} = -y_{\alpha 2} \frac{\rho \langle\chi\rangle}{m_\eta^2} \,. \quad (21)$$

For $M_\pm \gg M_+ - M_-$, we can follow the standard method [12] of the resonant leptogenesis [1] to compute

[1] The right-handed neutrinos for the type-I seesaw will induce the vertex loop besides the self-energy loop in their decays. At the same time, the Higgs triplet for the type-II seesaw, which will be introduced later, will mediate another vertex correction in the decays of the right-handed neutrinos. The right-handed neutrinos and the Higgs triplet are at the TeV scale so that the CP asymmetry constrained by the neutrino masses would be too small unless we take the resonant enhancement into account. However, the resonant effect only exists in the self-energy loop. So, the vertex corrections induced by the Higgs triplet or the right-handed neutrinos would have no significant contributions to the leptogenesis.