although it has been widely adopted in nonextensive classical statistics [37]. The situation is the same also in the FA for nonextensive quantum systems as recently pointed out in Refs. [38, 39].

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Appendix: A. Evaluations of averages by the exact approach

1. The q-average

We first discuss an evaluation of the q-average given by

$$Z_q^{(N)}(\alpha) \equiv \int [1-(1-q)\alpha\, \Phi(\boldsymbol{x})]^{\frac{1}{1-q}}\; d\boldsymbol{x}, \tag{A1}$$

$$Q_q^{(N)}(\alpha) = [Q(\boldsymbol{x})]_q \equiv \frac{1}{\nu_q^{(N)} Z_q^{(N)}} \int Q(\boldsymbol{x})\; [1-(1-q)\alpha\, \Phi(\boldsymbol{x})]^{\frac{q}{1-q}}\; d\boldsymbol{x}, \tag{A2}$$

by using the exact expressions for the gamma function [38, 40, 41]:

$$y^{-s} = \frac{1}{\Gamma(s)} \int_0^\infty u^{s-1} e^{-yu}\, du \qquad \text{for } s>0, \tag{A3}$$

$$y^{s} = \frac{i}{2\pi}\Gamma(s+1) \int_C (-t^{-s-1}) e^{-yt}\, dt \qquad \text{for } s>0. \tag{A4}$$

Here $\alpha = 1/(2\nu_q^{(N)}\sigma^2)$, $\Phi(\boldsymbol{x})$ is given by Eq. (19), $Q(\boldsymbol{x})$ denotes an arbitrary function of $\boldsymbol{x}$, C the Hankel path in the complex plane, and Eq. (32) is employed. We obtain [38, 40, 41]

$$Z_q^{(N)}(\alpha) = \begin{cases} \frac{1}{\Gamma\left(\frac{1}{q-1}\right)} \int_0^\infty u^{\frac{1}{q-1}-1} e^{-u} Z_1^{(N)}[(q-1)\alpha u]\, du & \text{for } q>1.0, \\ \frac{i}{2\pi}\Gamma\left(\frac{1}{1-q}+1\right) \int_C (-t)^{-\frac{1}{1-q}-1} e^{-t} Z_1^{(N)}[-(1-q)\alpha t]\, dt & \text{for } q<1.0, \end{cases} \tag{A5}$$