Note that the integrating measure in CvM_n is a random measure, but Corollary 1 shows that the asymptotic distribution is not affected by this fact. Further details can be found in the Appendix A.

3.2 Asymptotic power

Now, we investigate the power properties of tests based on continuous functionals $\Gamma(\hat{R}_n^p)$, like CvM_n and KS_n in (2.8) and (2.9), respectively. We consider fixed alternatives, and a sequence of local alternatives H_{1n} that converges to H_0 at the parametric rate $n^{-1/2}$.

3.2.1 Power against fixed alternatives

Next theorem analyzes the asymptotic properties of our tests under fixed alternatives of the type

$$H_1 : \mathbb{E}[(D - q(X,\theta))1\{q(X,\theta) \leq u\}] \neq 0 \text{ for all } \theta \in \Theta \text{ and for some } u \in \Pi, \tag{3.2}$$

where $\Pi = [0,1]$ is the unit interval. Note that H_1 is simply the negation of H_0 in (2.3).

Theorem 2 *Suppose Assumptions 3.1-3.3 hold. Then, under the fixed alternative hypothesis H_1 in (3.2), we have that*

$$\sup_{u\in\Pi}\left|\frac{1}{\sqrt{n}}\hat{R}_n^p(u) - \mathbb{E}\left[\left(p\left(X\right) - q\left(X,\theta_0\right)\right)\mathcal{P}1\left\{q\left(X,\theta_0\right) \leq u\right\}\right]\right| = o_p\left(1\right).$$

From Theorem 2, we see that test statistics of the form of $\Gamma(\hat{R}_n^p)$ are not consistent against all fixed alternative hypotheses in (3.2), but only those not collinear to the score function $g(X,\theta_0)$. To see this, note that

$$\begin{aligned}\mathbb{E}\left[\left(p\left(X\right) - q\left(X,\theta_0\right)\right)\mathcal{P}1\left\{q\left(X,\theta_0\right) \leq u\right\}\right] = &\\ \mathbb{E}\left[\left(p\left(X\right) - q\left(X,\theta_0\right)\right)1\left\{q\left(X,\theta_0\right) \leq u\right\}\right] - &\\ \mathbb{E}\left[\left(p\left(X\right) - q\left(X,\theta_0\right)\right)g'\left(X,\theta_0\right)\right]\Delta^{-1}\left(\theta_0\right)G\left(u,\theta_0\right)&\end{aligned}$$

is equal to zero under (3.2) if $p\left(X\right) - q\left(X,\theta_0\right)$ and $g(X,\theta_0)$ are collinear almost surely. We do not see this as a limitation. First, when one estimates θ_0 using the NLS method, the population first order condition for θ_0 sets $\mathbb{E}\left[\left(D - q\left(X,\theta_0\right)\right)g'\left(X,\theta_0\right)\right] = 0$, implying that, for some $u \in \Pi$,

$$\mathbb{E}\left[\left(p\left(X\right) - q\left(X,\theta_0\right)\right)\mathcal{P}1\left\{q\left(X,\theta_0\right) \leq u\right\}\right] = \mathbb{E}\left[\left(p\left(X\right) - q\left(X,\theta_0\right)\right)1\left\{q\left(X,\theta_0\right) \leq u\right\}\right] \neq 0.$$