(because $k \cdot p = 0 \;\Rightarrow\; k^0 = 0$ for $p^\mu = (m, \vec{0})^\mu$), and Eq. (F.35) becomes

$$\begin{aligned} A^\mu(x) &= \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \frac{e}{|\vec{k}|^2}(1,\vec{0})^\mu \\ &= \frac{2\pi}{(2\pi)^3} e \underbrace{\int_0^\infty dk \int_0^\pi \sin\theta d\theta e^{ikr\cos\theta}}_{\pi/r}(1,\vec{0})^\mu \\ &= \frac{1}{4\pi r}(1,\vec{0})^\mu. \end{aligned} \tag{F.37}$$

For $t > 0$ we will ignore the $k \cdot p' = 0$ pole because that merely gives the Coulomb field; radiation is produced by the $k^0 = |\vec{k}|$ and $k^0 = -|\vec{k}|$ poles. Since $k^2 = (k^0 - |\vec{k}|)(k^0 + |\vec{k}|)$ and $k \cdot p = k^0 p^0 - \vec{k} \cdot \vec{p}$ we have that

$$\begin{aligned} A^\mu(x) &= \int \frac{d^3k}{(2\pi)^3} \frac{(-2\pi i)}{2\pi}(-ie)\Bigg[\\ &\quad e^{-i|\vec{k}|t} e^{i\vec{k}\cdot\vec{x}} \frac{1}{2|\vec{k}|}\left(\frac{p'^\mu}{|\vec{k}|p'^0 - \vec{k}\cdot\vec{p}'} - \frac{p^\mu}{|\vec{k}|p^0 - \vec{k}\cdot\vec{p}}\right) \\ &\quad + e^{i|\vec{k}|t} e^{i\vec{k}\cdot\vec{x}} \frac{1}{-2|\vec{k}|}\left(\frac{p'^\mu}{-|\vec{k}|p'^0 - \vec{k}\cdot\vec{p}'} - \frac{p^\mu}{-|\vec{k}|p^0 - \vec{k}\cdot\vec{p}}\right)\Bigg] \\ &= -e\int \frac{d^3k}{(2\pi)^3}\frac{1}{2|\vec{k}|}\Bigg[e^{-i|\vec{k}|t} e^{i\vec{k}\cdot\vec{x}}\left(\frac{p'^\mu}{|\vec{k}|p'^0 - \vec{k}\cdot\vec{p}'} - \frac{p^\mu}{|\vec{k}|p^0 - \vec{k}\cdot\vec{p}}\right) \\ &\quad + e^{i|\vec{k}|t} e^{-i\vec{k}\cdot\vec{x}}\left(\frac{p'^\mu}{|\vec{k}|p'^0 - \vec{k}\cdot\vec{p}'} - \frac{p^\mu}{|\vec{k}|p^0 - \vec{k}\cdot\vec{p}}\right)\Bigg], \end{aligned} \tag{F.38}$$

where, to get to the last line, we were able to freely change $\vec{k} \to -\vec{k}$ because