

FIG. 1: (Color online) Charge density profiles in the semi-infinite system. The dashed line is the solution of the linearized equation (55), $\phi^{(1)}(z)$, the dash-dotted line is the correction term (74), $\phi^{(2)}(z)$, and the solid line is the sum of the two functions, $\phi^{(1)}(z) + \phi^{(2)}(z)$, for $\kappa = 0.01$, $\sigma = 0.5$, $\xi_b = 100$, and $n_1 = 0.5$. The charge density is in e/v_0 units, where e is the elementary charge, and z is in $v_0^{1/3}$ units, where v_0 is the volume per molecule in the close-packed system and we assume that all molecules are of similar size.

For $T \to T_c$ we have $\xi_b^{-1} \to 0$, and using $\xi_b^{-1} \ll \kappa$ we obtain the approximation

$$\phi^{(2)}(z)/\phi^{(1)}(z) \simeq \frac{n_1}{2} \left[1 - \kappa \frac{(1 - e^{-z/\xi_b})}{\xi_b^{-1}} \right] \qquad \kappa \gg \xi_b^{-1}$$
 (78)

For $z \ll \xi_b$ the above takes the simple form

$$\phi^{(2)}(z)/\phi^{(1)}(z) \simeq \frac{n_1}{2} \Big[1 - \kappa z \Big]$$
 (79)

The correction term changes sign for $z_0 \approx 1/\kappa$.

Equations for the first corrections to the nonelectrostatic part read

$$\left(\mathbf{v}^{(2)}\right)^{"} = \mathbf{M}\mathbf{v}^{(2)} + \mathbf{D},\tag{80}$$

where $\mathbf{v}^{(2)} = (\vartheta^{(2)}, \eta^{(2)})$, and the components of the vector $\mathbf{D}^T = (D_s, D_\rho)$ are

$$D_s = \frac{D_{\vartheta\vartheta}^a \vartheta^2 + D_{\vartheta\eta}^a \vartheta \eta + D_{\eta\eta}^a \eta^2 + D_{\phi\phi}^a \phi^2}{J}$$
(81)

$$D_{\rho} = \frac{D_{\vartheta\vartheta}^b \vartheta^2 + D_{\eta\vartheta}^b \vartheta \eta + D_{\eta\eta}^b \eta^2 + D_{\phi\phi}^b \phi^2}{I}$$
(82)