computed from the solenoid geometry. The spheres in Fig. 5 show the computed magnitude of $\mathbf{\Gamma} = -\nabla U$ at the surface of three droplets, radius $a = 5.0\,\mathrm{mm}$, $7.5\,\mathrm{mm}$ and $10.0\,\mathrm{mm}$. Radiating lines indicate the direction and magnitude of $\mathbf{\Gamma}$, which is directed toward the interior of the droplet. The variation in $|\mathbf{\Gamma}|$ over the surface of the droplet is due to the octopole component of the potential trap.

We now consider the effect on the droplet's eigenfrequencies of adding a harmonic component c'_j, $j \geq 2$ to the potential trap. The analysis proceeds as above, however, we must now include higher order harmonics in the eigenfunction of the shape oscillation

$$r = R(\theta, t) = a + \epsilon \sin \omega t \sum_{l \geq 1} b_l P_l(\cos\theta), \tag{14}$$

since l is not, in general, a good eigennumber in a non-spherical potential. In principle, we should decompose the shape into spherical harmonics Y_l^m, since the degeneracy in m is also lifted in a non-spherical potential (see Fig. 6). However, our method of inducing shape oscillations in the droplet tends to excite only the axisymmetric shapes (i.e. with $m = 0$), since the air jet is aligned along the solenoid axis. For this reason, we derive here the frequencies of the $m = 0$ oscillations only (Eqn. 14), which are sufficient to interpret the experimental results. We summarize the treatment of the general case $|m| \leq l$ in the Appendix.

The velocity potential is

$$\phi(r, \theta) = -\epsilon\omega \cos\omega t \sum_{l \geq 1} b_l r^l l^{-1} a^{-l+1} P_l(\cos\theta). \tag{15}$$

The magnetogravitational potential U at the surface of the drop is (see Eqn. 9a)

$$U(R) = U(a) - \epsilon \Gamma_r(a, \theta) \sin(\omega t) \sum_{l \geq 1} b_l P_l(\cos\theta), \tag{16}$$

where $-\Gamma_r(a, \theta) = c'_0(a) + c'_j(a) P_j(\cos\theta)$ in this case (see Eqn. 4). Inserting Eqn. 16, Eqn. 14 and Eqn. 15 into Eqn. 6, and equating the time-varying terms, we obtain

$$a\omega^2 \sum_{l \geq 1} \frac{b_l P_l}{l} = \sum_{l \geq 1} \left[(c'_0(a) + c'_j(a) P_j) + \frac{T}{\rho a^2}(l-1)(l+2) \right] b_l P_l. \tag{17}$$

The product $P_l P_j$ appearing on the RHS of this equation can be expanded as a sum of Legendre polynomials [15], which, for our purposes, is most conveniently written $P_l P_j = \sum_{p=|j-l|}^{j+l} Q(l, j, [j + l - p]/2) P_p$, in which [15]

$$Q(l, j, s) = \frac{A(l-s)A(s)A(j-s)}{A(j+l-s)} \left(\frac{2j + 2l - 4s + 1}{2j + 2l - 2s + 1} \right) \tag{18}$$