3 denote the projections on the set of orts, for example, $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$, for $\vec{m} = m_0\vec{e}_z$ or $\vec{e}_x$, $-\vec{e}_y$, $-\vec{e}_z$ for $\vec{m} = -m_0\vec{e}_z$.

III. FERROMAGNETIC STATE OF ARRAY.

For the FM state of the dot array ($\vec{m} = \vec{e}_z$ for all dots) the same set of orts in Eq. (2), $\vec{e}_1 = \vec{e}_x$, $\vec{e}_2 = \vec{e}_y$ and $\vec{e}_3 = \vec{e}_z$ should be used. In the quadratic approximation over the operators $a^\dagger_{\vec{l}}$ and $a_{\vec{l}}$ the Hamiltonian reads

$$\hat{H} = 2\mu_B \sum_{\vec{l}} \left[\left(H_0 + H_a - \sum_{\vec{\delta}\neq 0} \frac{M}{|\vec{\delta}|^3} \right) a^\dagger_{\vec{l}} a_{\vec{l}} - \frac{1}{2} \sum_{\vec{\delta}\neq 0} \frac{M}{|\vec{\delta}|^3} a^\dagger_{\vec{l}} a_{\vec{l}+a\vec{\delta}} \right] - \frac{3\mu_B M}{2} \sum_{\vec{l}} \left[\sum_{\vec{\delta}\neq 0} \frac{(\delta_x + i\delta_y)^2}{\left|\vec{\delta}\right|^5} a^\dagger_{\vec{l}} a^\dagger_{\vec{l}+a\vec{\delta}} + \text{h.c.} \right] \tag{3}$$

where $\vec{\delta} = l_x\vec{e}_x + l_y\vec{e}_y$ is a dimensionless lattice vector, $H_a = \kappa m_0$ is the anisotropy field, $M = m_0/a^3$ is the characteristic value defining the dipolar interaction intensity and having the same dimension as usual the 3D magnetization. The collective modes are introduced via states a_k and $a^\dagger_k$ of definite quasi-momentum $\vec{k}$

$$a_k = \frac{1}{\sqrt{N}} \sum_{\vec{l}} a_{\vec{l}} e^{i\vec{k}\vec{l}}, \; a^\dagger_k = \frac{1}{\sqrt{N}} \sum_{\vec{l}} a^\dagger_{\vec{l}} e^{-i\vec{k}\vec{l}}, \tag{4}$$

where N is the total number of dots in an array. The collective modes are defined by the quadratic Hamiltonian over a_k, $a^\dagger_k$ which acquires the standard form

$$\hat{H} = 2\mu_B M \sum_k \left[A_k a^\dagger_k a_k + \frac{1}{2} \left(B_k a^\dagger_k a^\dagger_{-k} + B^*_k a_k a_{-k} \right) \right], \tag{5}$$

When the form of coefficients A_k and B_k are established, the collective excitation energy $\varepsilon(\vec{k}) = \hbar\omega(\vec{k})$ may be found by means of $u - v$ Bogolyubov transformations (see, for example,[35]) and universally reads

$$\varepsilon(\vec{k}) = 2\mu_B M \sqrt{A_k^2 - |B_k|^2}, \; \omega(\vec{k}) = \gamma M \sqrt{A_k^2 - |B_k|^2},$$

where $\gamma = 2\mu_B/\hbar$ is the gyromagnetic ratio. The concrete forms of A_k and B_k are defined by the distribution of the magnetic moments within the array. For the case of interest (parallel ordering of dot magnetization) one can find

$$A_k = h + \beta - \frac{3}{2}\sigma(0) + \frac{1}{2} \left[\sigma(0) - \sigma(\vec{k}) \right], \; B_k = 3\sigma_c(\vec{k}), \tag{6}$$