For values of doping close to the Cooperon band edge ($|\mu - \Delta/2| \gg \Delta$) spectral curvature is important and the action given in Eqn. 2.5 is inadequate. A better description of the Cooperon dynamics is given by the sine-Gordon model

$$\mathcal{L} = \frac{1}{8\pi}\left[v_F^{-1}(\partial_\tau\theta)^2 + v_F(\partial_x\theta - 4\mu)^2\right] - \frac{M}{2}\cos(\theta). \tag{2.6}$$

where $M^2 = \Delta^2 - 4\mu^2$. The mass term here can be thought to arise as follows in a mean field way from the SO(8) Gross-Neveu model. The SO(8) Gross-Neveu model can be written in terms of fundamental fermions (which are non-local with respect to the original fermions in the problem) with an interaction term of the form

$$H_{int}^{SO(8)} = 2g(\sum_a \psi_a^\dagger \tau^y \psi_a)^2 \tag{2.7}$$

Here $a = 1, 4$ and $\psi_a = (\psi_a^R, \psi_a^L)$ and τ^y is a Pauli matrix acting in $R - L$ space. The four fundamental fermions correspond to the different degrees of freedom in SO(8): charge, spin, orbital, parity. The Cooperon (charge) we take to be given by ψ_1. With a finite chemical potential lowering the Cooperon gap, the fluctuations of the Cooperon will be strongest. Invoking mean field theory, we thus replace $\psi_a^\dagger \tau^y \psi_a$ for $a = 2, 3, 4$ by its expectation value. The resulting bosonization of the remaining degree of freedom ψ_a results in the sine-Gordon model.

III. SUPERCONDUCTIVITY OF ARRAYS OF FOUR-LEG LADDERS: TWO SCENARIOS

Having elucidated the properties of individual 4-leg ladders, we now consider an array of such ladders. We assume initially that the electron-electron interaction acts only inside individual ladders and is much smaller than the bandwidth $W \sim 2t_0$. It is also assumed that $W \gg t_\perp$ (the inter-ladder tunneling). We imagine two scenarios. In the first we assume $t_\perp$ is on the same order as Δ_A, the gap on the inner bands of the four leg ladder, but much smaller than Δ_B, the gap on the outer bands. In this case coupling the ladders together lead to small Fermi pockets, very much like in Ref.(10). However in this case the pockets are found near $\pm\pi/2, \pm\pi/2$. The residual coupling between these Fermi pockets and the A-cooperons then leads to superconductivity in the A-bands. And because of a proximity effect, the superconductivity of the A-bands induces superconductivity in the B-bands.