

fundamental charges and  $N_{adj}$  of adjoint charges. The unimodular constraint reduces  $N_{fun}$  and  $N_{adj}$  to  $N_{fun} = N_{fun} - 1$  and  $N_{adj} = N_{adj} - 1$ . The  $N_{adj}$  adjoint charges are invariance only over the  $N_{fun}$  fundamental charges. The  $(N_{adj} - N_{fun})$  extra real independent parameters do not appear in the Hamiltonian and they are integrated (washed) out. The unitary ensemble  $U(N)$  has  $N_{fun} = N$  fundamental eigenvalues and  $N_{adj} = N^2$  parameters. The number of extra real independent parameters is  $N_{adj} - N_{fun} = (N^2 - N)$ . This means that the unitary symmetry group has  $N$  fundamental chemical potentials and  $N^2$  adjoint parameters. The  $N^2$  adjoint parameters are transformed (diagonalized) to depend basically only on the  $N$  fundamental chemical potentials. The redundant  $(N^2 - N)$  parameters are, subsequently, integrated out and they disappear from the resultant ensemble. The orthogonal ensemble has  $N_{fun} = N$  fundamental eigenvalues and  $N_{adj} = \frac{1}{2}N(N + 1)$  adjoint parameters. The real symmetric  $N \times N$  matrix has  $N_{adj} - N_{fun} = \frac{1}{2}N(N - 1)$  redundant real parameters and these parameters are washed away by an appropriate transformation. The unitary symmetry group can be broken and decomposed to an orthogonal symmetry group with the same number of conserved charges  $N_{fun} = N$  (or  $N_{fun} = N - 1$  when the unimodular constraint is embedded). The symmetry decomposition (breaking) from  $U(N)$  to orthogonal  $O_{(S)}(N)$  leads to  $N(N - 1)/2$  Goldstone bosons emerge as glueballs (or gluon jets) in the medium. They escape from the Hagedorn bags and enrich the medium with gluonic contents and jets. The  $N(N - 1)/2$  Goldstone bosons are identified as free colorless gluon degrees of freedom while the remaining  $\frac{1}{2}N(N + 1)$  gluons remain as the exchange interacting gluons for the  $O_{(S)}(N)$  symmetry group. The definitions of  $O_{(S)}(N)$  and other symmetry groups such as the symplectic  $Sp(N)$  symmetry group shall be reviewed below in Sec. III. On the other hand, the symplectic ensemble has  $N_{fun} = N$  eigenvalues of  $N \times N$  quaternion-real matrix and  $N_{adj} = N(2N - 1)$ . The number of redundant parameters in the symplectic symmetry group is  $2N(N - 1)$ . The (unimodular) unitary color  $U(N_c)$  symmetry group may merge with other degree of freedom such as flavor symmetry  $U(N_f)$  and the symmetry of the resultant composite transmutes to a symplectic (quaternion) symmetry group through the symmetry modification mechanism that is given by  $U(N_c) \times U(N_f) \rightarrow U(N_c + N_f) \rightarrow O(2N) \rightarrow Sp(N)$ . Hence, under the preceding assumption, there is an indication that the eventual deconfined quark and gluon matter can be reached after a chain of multi-phase processes and this argument is not that simple.

The thermodynamics of quark jets with an internal color structure has been considered