sistent, localized in space, and presumably consist of a few atoms that undergo the irreversible rearrangements responsible for the observed plastic flow. Recently, this plasticity has been further analyzed in simulations with a focus on the quasistatic dynamics at small strain rates, close to the flow arrest.[7–12] With these studies it was possible to trace back the origin of plastic activity to the softening of a vibrational mode and the vanishing of the associated frequency.[7] In real space, this softening is associated with the formation of distinct, localized zones where the plastic failure is nucleated.[8] In turn, this can trigger the failure of nearby zones, such that avalanches of plastic activity form that may "propagate" through the entire system.

It has been argued that the macroscopic extent of these avalanches is a signature of the quasistatic dynamics, which gives the system enough time to propagate the failure throughout the system. Beyond the quasistatic regime, i.e. farther away from the jammed state, the size of these events is expected to be finite. Thus, one naturally finds an increasing length-scale that is connected with the flow arrest upon reducing the stress towards the threshold value.[13–15]

Without external drive, an (un)jamming transition can occur for decreasing particle volume fraction below a critical value, ϕ_c. This special point, which is only present in systems with purely repulsive steric interactions, has been given the name "point J".[16,17] At this point the average number of particle contacts jumps from a finite value z_0 to zero just below the transition. The value of z_0 is given by Maxwell's estimate for the rigidity transition[18,19] and signals the fact that at point J each particle has just enough contacts for a rigid/solid state to exist. This marginally rigid state is called "isostatic". Compressing the system above its isostatic state a number of non-trivial scaling properties emerge.[16,20] As the volume fraction is increased, additional contacts are generated according to $\delta z \sim \delta\phi^{1/2}$. The shear modululs scales as $G \sim p/\delta z$ and vanishes at the transition (unlike the bulk modulus).[16] This scaling is seen to be a consequence of the non-affine deformation response of the system,[21] with particles preferring to rotate around rather than to press into each other.[22] Associated with the breakdown of rigidity at point J is the length-scale, $l^\star \sim \delta z^{-1}$,[23,24] which quantifies the size over which additional contacts stabilize the marginally rigid isostatic state.

In this article we present results from quasistatic and small strain-rate flow simulations of a two-dimensional system in the vicinity of point J. Together with the linear elastic shear modulus, at point J also the yield-stress σ_y vanishes.[25–30] Thus, point J is connected with a transition from plastic-flow behavior ($\phi > \phi_c$, $\sigma_y > 0$) to normal fluid flow ($\phi < \phi_c$, $\sigma_y = 0$), with either Newtonian[26] or Bagnold rheology[27,29] at small strain-rates. In consequence, both (un)jamming mechanisms as described above are present at the same time: the flow arrest, as experienced by lowering the stress towards threshold, is combined with the vanishing of the threshold itself.

In this study we want to address two questions: In how far do the general plastic flow properties carry over to this situation of small or, indeed, vanishing yield-stress? Is the vicinity to point J and its isostatic state at all relevant for the flow properties ? It will be shown that while the stress fluctuations reflect the critical properties at point J, dynamical correlations are typical those observed in the flow of elasto-plastic solids.

We will approach these questions starting with the quasistatic-flow regime. The advantage of quasistatic simulations is to provide a clean way of accessing the transition region between elastic, solid-like behavior and the onset of flow. In the quasistatic regime flow is generated by a succession of (force-)equilibrated solid states. Thus, one can connect a liquid-like flow with the ensemble of solid states that are visited along the trajectory through phase-space.

In Section 3.1, we study the instantaneous statistical properties of the configurations generated by this flow trajectory at zero strain rate, and show that they display large fluctuations in several quantities, that are associated with the proximity to the jamming point.

In Section 3.2, we follow the analysis of recent experiments[31] and use a "four point correlation" tool to define a dynamical correlation length that characterizes the extension of the dynamical heterogeneities observed in the flow process. This dynamical length scale is shown to scale as the system size in the zero strain rate limit, independently of the distance to point J. The heterogeneity in the system is maximal for strains that correspond to the typical duration between the plastic avalanches