

lattice and look at the pictures.

II. DEFINITIONS, NOTATIONS, THE VERY BASICS

The q -binomial coefficient

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \prod_{i=1}^k \frac{1 - q^{n-i+1}}{1 - q^i}, \quad q \neq 1, \quad 0 \leq k \leq n \quad (1)$$

is a natural extension of the standard binomial coefficient

$$\binom{n}{k} = \frac{n!}{k! (n-k)!}, \quad 0 \leq k \leq n \quad (2)$$

The Pochhammer symbol (or shifted factorial) is defined as

$$(a)_n = \prod_{i=0}^{n-1} (a + i) \quad (3)$$

so that $(1)_n = n!$. Its q -deformed relative, the q -Pochhammer symbol, is defined as

$$(a; q)_n = \prod_{i=0}^{n-1} (1 - a q^i), \quad n \geq 0 \quad (4)$$

The q -binomial coefficient can then be expressed as

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{(q; q)_n}{(q; q)_k (q; q)_{n-k}} = \frac{(q^{n-k+1}; q)_k}{(q; q)_k} \quad (5)$$

The q -numbers are defined for any real number a as

$$[a]_q = \frac{1 - q^a}{1 - q}, \quad q \neq 1 \quad (6)$$

and it is easy to show that

$$\lim_{q \rightarrow 1} [a]_q = a \quad (7)$$

Note that for integers $n \geq 1$ we have

$$[n]_q = \frac{1 - q^n}{1 - q} = 1 + q + \cdots + q^{n-1}, \quad q \neq 1 \quad (8)$$

so that $[n]_q \rightarrow n$ when $q \rightarrow 1$. To continue, the q -number factorials are defined as

$$[n]_q! = \prod_{k=1}^n [k]_q \quad (9)$$