On making this choice, we can then either work directly with (6.2) and (6.3), whilst remembering that ϕ, π and σ are constants under the SO(3) isometry, or we can explicitly recast the entire ensemble in five dimensions. This is how it was presented in [142], and we reproduce the five dimensional theory here.

The five dimensional metric is

$$ds^2 = H^{-1/2} f^{1/8} \sum_{i=0}^{3} \eta_{\mu\nu} dx^\mu dx^\nu + H^{1/2} f^{3/8} h dr^2 \tag{6.5}$$

with

$$f = \frac{(\sigma(r)^2 + r^2)^2 + 1}{(\sigma(r)^2 + r^2)^2 - 1}, \qquad h = \frac{(\sigma(r)^2 + r^2)^2 - 1}{(\sigma(r)^2 + r^2)^2}, \qquad H = f^{1/2} - 1 \tag{6.6}$$

The explicit values of ϕ, g_5 and $\mathcal{L}_\sigma$ are

$$e^\phi = H^{5/4} f^{15/16 - \sqrt{39}/4} h^{5/2} r^3 (1 + \dot{\sigma}^2)^{-1/2} \tag{6.7}$$

$$g_5^2 = 4\pi^2 H^{1/2} f^{3/8 + \sqrt{39}/4} h \tag{6.8}$$

$$\mathcal{L}_\sigma = \sqrt{-g} f^{\sqrt{39}/4} g_{rr}^{3/2} \sqrt{1 + \dot{\sigma}^2} \tag{6.9}$$

So on substituting (6.9) into (6.1) we get the five dimensional action. Note that as $r \to \infty$, the entire model reverts back to the simple AdS model considered in section 5.

We now go on to calculate the masses and decay constant of the vector and axial meson sectors.