momentum, q_d, which gives the current density

$$j = n_{2D} q_d = \frac{E_F}{\pi} q_d. \tag{26}$$

Since the samples we consider are typically longer than the electron's coherence length, we have used the ShF occupations within the main text. For completeness we give also the results for the 2FR case. The spin polarization per unit length of the sample to third order in α_E is

$$m_z = -\frac{\alpha_E}{\pi^2}\sqrt{2E_F}\Delta\mu - \frac{2\alpha_E^3 \Delta V}{3\pi^2} E_F^{3/2}\Delta\mu - \frac{4\alpha_E^3}{45\pi^2} E_F^{5/2}\Delta\mu \tag{27}$$

which after expressing the applied bias in terms of the current density results in

$$m_z = -\alpha_E\, j - \frac{2\alpha_E^3 \Delta V}{3} E_F\, j + \frac{4\alpha_E^3}{45} E_F^2\, j. \tag{28}$$

Comparing the last equation with Eq. 14 we see that while the first order term is identical for both occupations, the higher orders differ by a numerical prefactor. Both of these are larger in the case of partially equilibrated shifted Fermi-like occupations.

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[33] We will use effective atomic units; the distances are measured in multiplies of the effective Bohr radius, $a_B^* = \frac{\varepsilon_r}{m_{ef}} = 9.79$nm, energy in the effective Hartrees, $Ha^* = \frac{m_{ef}}{\varepsilon_r^2} Ha = 11.9$meV, both numerical values are given for GaAs where $m_e^* = 0.067 m_e$ and $\varepsilon_r = 12.4$ are the effective mass of the electron and the relative permittivity.