XII. CONCLUSIONS

The magnetisation distribution for the complete graph is exactly described by the p, q-binomial distribution, corresponding to the special (or limit) case of $p = q$. For balanced complete bipartite graphs this is most likely also true in some limit sense, yet to be made precise. Actually, it appears that for most graphs, at least those which are more or less regular, the magnetisations are well-fitted by a p, q-binomial distribution for some choice of p and q. The exact extent to which the p, q-binomial approximation is *good* we do not yet know (e.g. convergence in moment) nor the exact class of graphs that would satisfy this. We have investigated the matter more closely for lattices of dimension one through five. In general they are always well-fitted by p, q-binomial distributions for high- and low-temperatures but the problems arise near K_c, or rather K^* where the distribution changes from unimodal to bimodal.

For the 1-dimensional lattices (having no such bounded K^*) the situation is basically always that of high temperatures. It seems possible to give expressions for p and q in terms of K in this case though we have not done so. For 2-dimensional lattices the distributions near K^* are least well-fitted by the p, q-binomials but slightly better fitted in the 3-dimensional case. We made theory-based predictions of how z should scale with n near K^*. Unfortunately, scaling is probably very slow, involving logarithms and double logarithms, making it near impossible to test the prediction. For 4-dimensional lattices the distributions are clearly much better fitted by p, q-binomials, though some discrepancy still remains just above K^*. For 5-dimensional lattices even this small discrepancy is gone, leaving us perfectly fitted (that is, to the human eye) p, q-binomial distributions. In this case the values of z at K^* should approach a limit value. We estimated this limit and, using this limit value, compared the first four normalised cumulants for finite lattices with the (possible) limit curves.

We described and used a rather simple method to determine p and q given a distribution. Possibly this method is not optimal since it simply forces the distribution to be correct at a single point rather than providing a good overall-fit. It is also sensitive to noise when the distributions are unimodal, thus making it difficult to determine p and q. On the other hand it works extremely well for bimodal distributions where the noise sensitivity problem vanishes.

The p, q-binomial coefficients are just a tweaked form of q-binomials, i.e. they are multi-