A. Thermal history

We begin by considering the thermal history of a nearly pure Bino. If one does not allow for co- [40–42] or resonant [40, 43–46] annihilations, then Bino freeze-out is controlled by t-channel sfermion exchange. One can show [9] that in order to produce the observed DM relic density, the sfermion must be $\lesssim 110$ GeV. Since the LEP limits on sfermions are $\mathcal{O}(100\,\text{GeV})$, there is only a small experimentally allowed window for thermal Bino DM.

Either co-annihilations (*e.g.* with the stau or stop) or resonant annihilation through the pseudo-scalar Higgs (A^0) also allow dominantly Bino DM. However, both of these options involve numerical coincidences. In the first case the Boltzmann factor will exponentially suppress the density of the would-be co-annihilator unless $\exp(-\Delta M/T_f)$ is $\mathcal{O}(1)$, where $\Delta M = m_{\text{NLSP}} - m_\chi$, m_{NLSP} is the mass of the next-to-lightest superpartner, and T_f is the DM freeze-out temperature. Since $T_f \approx m_\chi/20$, this requires a mass degeneracy, ΔM, of a few percent. To realize the second case requires a precise relationship between m_χ and m_A. When $m_\chi < m_W$, the Z^0 or h poles may be used to achieve the correct relic density, which requires a similar numerical conspiracy.

Located at the other extreme, far away from the pure Bino, is a pure Wino or a pure Higgsino. In these cases, the requirement of a thermal relic abundance fixes the mass to be $\mathcal{O}(2.5\,\text{TeV})$ and $\mathcal{O}(1\,\text{TeV})$ respectively. Thus, to realize either of these cases implies $\mu \gtrsim \mathcal{O}(100\,\text{GeV})$. Since, in the MSSM, the Z^0 mass is given by

$$\frac{m_Z^2}{2} = -|\mu|^2 + \frac{m_{H_d}^2 - m_{H_u}^2 t_\beta^2}{t_\beta^2 - 1}, \tag{26}$$

where $m_{H_{u,d}}^2$ are the Higgs soft-mass squared parameters, this requires a substantial fine-tuning between μ^2 and $m_{H_{u,d}}^2$ in order to reproduce the measured Z^0 mass of 91 GeV. Therefore, the desire to alleviate fine-tuning in this expression leads to the requirement that $\mu \sim \mathcal{O}(100\,\text{GeV})$. This will also naively lead to well-tempering since the neutralino mixing is proportional to m_Z/μ. Though the accuracy of the current measurement of the DM relic density (see Eq. (1)) requires a precisely determined neutralino composition, one can easily reproduce the DM abundance for any mass of $\mathcal{O}(100$ GeV). The Bino/Higgsino mixed LSP as a good thermal WIMP was pointed out in studies of the focus point region on the MSSM [47, 48].

A Higgs boson mass above the LEP bound requires large radiative corrections from a stop squark. This implies that the scale for these particles, m_{SUSY}, should be around a TeV. These states yield additive corrections to $m_{H_{u,d}}^2$, proportional to m_{SUSY}^2. Hence, even in the case when $\mu \sim \mathcal{O}(100$ GeV), there will naively be fine-tuning between these corrections and the bare value of $m_{H_{u,d}}^2$ in order to reproduce m_Z. Solutions to this "little hierarchy problem"