Lemma 13. *Consider the same assumptions of Theorem 4 and suppose that* $\mathbf{\Sigma}_0 = \mathbf{I}$. *Then, if* H_0 *is the distribution of* $(\mathbf{y}', \mathbf{x}')'$, *we have that*

$$(i) \qquad E_{H_0}\left(\frac{\partial W(d(\mathbf{B}_0, \mathbf{\Sigma}_0)/S)vec(\widehat{\mathbf{u}}(\mathbf{B}_0)\mathbf{x}')}{\partial vec(\mathbf{\Sigma})\prime}\right) = 0 \text{ for all } S,$$

$$(ii) \qquad E_{H_0}\left(W(d(\mathbf{B}_0, \mathbf{\Sigma}_0)/S)vec(\widehat{\mathbf{u}}(\mathbf{B}_0)\mathbf{x}')\right) = 0 \text{ for all } S.$$

Proof: (i) By (A.15) we have

$$E_{H_0}\left(\frac{\partial W(d(\mathbf{B}_0, \mathbf{\Sigma}_0)/S)\mathrm{vec}(\widehat{\mathbf{u}}(\mathbf{B}_0)\mathbf{x}')}{\partial \mathrm{vec}(\mathbf{\Sigma})\prime}\right) = -E_{H_0}\left(\frac{W'(\|\mathbf{u}\|/S)}{2S\|\mathbf{u}\|}(\mathbf{x}\mathbf{u}' \otimes \mathbf{I})(\mathbf{I} \otimes \mathbf{u}\mathbf{u}')\right)$$

Since the distribution of $\mathbf{u}$ is assumed elliptical with $\mathbf{\Sigma}_0 = \mathbf{I}$, for any function h we have, $E_{H_0}(h(\|\mathbf{u}\|)u_i u_j u_l x_k) = 0$. Then, since all the elements of the right side of the above equation have this form, part (i) of the lema is proved. (ii) follows from $E_{H_0}(h(\|\mathbf{u}\|)u_i x_j) = 0$ for all i and j. ■

Proof Theorem 4: Assume $\mathbf{z} = (\mathbf{y}', \mathbf{x}')'$ satisfying the MLM (1.1). Consider first the case with $\mathbf{\Sigma}_0 = \mathbf{I}$. Using Lemma 11 with $\boldsymbol{\theta}_1 = \mathrm{vec}(\mathbf{B}_0')$, $\boldsymbol{\theta}_2 = \mathrm{vec}(\mathbf{\Sigma}_0)$, $S(\mathbf{T}_0(H), H) = S(H)$ and lemmas 12 and 13 we obtained

$$\begin{aligned} IF(\mathbf{z}_0, \mathbf{T}_1, (\mathbf{B}_0, \mathbf{I})) &= -\left(\frac{\partial E_{H_0} W\left(d(\mathbf{B}_0, \mathbf{\Sigma}_0)/\sigma_0\right) \mathrm{vec}((\mathbf{y} - \mathbf{B}_0'\mathbf{x})\mathbf{x}')}{\partial \mathrm{vec}(\mathbf{B}')'}\right)^{-1} \\ &\quad \times W\left(\|\mathbf{y}_0 - \mathbf{B}_0'\mathbf{x}_0\|/\sigma_0\right) \mathrm{vec}((\mathbf{y}_0 - \mathbf{B}_0'\mathbf{x}_0)\mathbf{x}_0'). \qquad \text{(A.27)} \end{aligned}$$