But there is a limit to how much probe energy E or effective mass $M_{ef} \approx E/c^2 = \hbar/lc$ can be packed into a region of size l, as we discussed in section II. According to Eq.(13) the fractional distance uncertainty in the volume containing such an effective mass is about

$$\frac{\Delta l}{l} \approx \frac{|\phi|}{c^2} \approx \frac{1}{c^2}\Big(\frac{GM_{ef}}{l}\Big) = \frac{G}{c^2 l}\Big(\frac{\hbar}{lc}\Big) = \frac{G\hbar}{c^3 l^2} = \frac{l_p^2}{l^2} \tag{26}$$

If l is made so small that this approaches 1 then the geometry becomes greatly distorted and the measurement fails, which happens at $l \approx l_P$.

Another way to see the limit effect is to note that the effective mass $M_{ef} = h/cl$ injected into the region by the probe can induce gravitational collapse to form a black hole (as already noted in section V) when the region size approaches the Schwarzschild radius of about GM_{ef}/c^2; this happens for

$$l \approx \frac{GM_{ef}}{c^2} \approx \frac{G\hbar}{c^3 l},\ l \approx \sqrt{G\hbar/c^3} \approx l_p \tag{27}$$

We thus conclude that any attempt to measure physical properties in a region of about the Planck size involves so much energy that large fluctuations in the geometry must occur, including the formation of black holes and probably more exotic objects such as wormholes.[35] Such wild variations in geometry were first dubbed spacetime foam by J. A. Wheeler; the phrase has become quite popular to express vividly the supposed chaotic nature of geometry at the Planck scale.[36]

VII. ENERGY DENSITY OF GRAVITATIONAL FIELD

This argument is based on the uncertainty in the energy density of the gravitational field, and field fluctuations that correspond to the uncertainty. Algebraically it resembles somewhat the argument of section VI, but has a different conceptual basis.

We first obtain an expression for the energy density of the gravitational field in Newtonian theory. Consider assembling a spherical shell of radius R and mass M by moving small masses from infinity to the surface, as shown in Fig.6. From Newtonian theory the energy done moving a small mass dM to the surface is

$$dE = -GMdM/R \tag{28}$$

and the total energy for the assembly is the integral of this energy over the mass, which is

$$E = -GM^2/2R \tag{29}$$