Let us stress that the vector field $B_\mu$ *is not obtained by a gauge transformation from the gauge field* ${b_\mu}^3$.

Firstly, this is obvious from the fact the $U$ fields are some of the dynamical variables of the theory: a gauge transformation would require a *fixed* element in $\underline{SU(2)}$, but, as dynamical variables, the function $U$ is not such a fixed element. Secondly, a gauge transformation should be applied at the same time on gauge fields and to any scalar and spinor fields coupled to them. In the present situation, the scalar fields $\phi$ are not subject to a transformation accompanying (3). $\phi$ is written in terms of $U$ but it will not be transformed any further. Finally, the following actions of the gauge groups $\underline{U(1)}$ and $\underline{SU(2)}$ on $B_\mu$ prove that $B_\mu$ *is no more a gauge potential for* $\underline{SU(2)}$. This rules out the fact that (3) can be a gauge transformation.

Indeed, for the action of $s \in \underline{U(1)}$, one gets

$$\begin{aligned} B^s_\mu &= (U^s)^{-1} b^s_\mu U^s + \tfrac{2i}{g}(U^s)^{-1}(\partial_\mu(U^s)) \\ &= \widehat{s}^{-1} U^{-1} b_\mu U \widehat{s} + \tfrac{2i}{g}\widehat{s}^{-1} U^{-1}[(\partial_\mu U)\widehat{s} + U(\partial_\mu \widehat{s})] \\ &= \widehat{s}^{-1} B_\mu \widehat{s} + \tfrac{2i}{g}\widehat{s}^{-1}\partial_\mu \widehat{s}\,, \end{aligned}$$

whereas for the action of $u \in \underline{SU(2)}$, one gets

$$\begin{aligned} B^u_\mu &= (U^u)^{-1} b^u_\mu U^u + \tfrac{2i}{g}(U^u)^{-1}(\partial_\mu(U^u)) \\ &= U^{-1} u [u^{-1} b_\mu u + \tfrac{2i}{g} u^{-1}\partial_\mu u] u^{-1} U \\ &\qquad + \tfrac{2i}{g} U^{-1} u [(\partial_\mu u^{-1}) U + u^{-1}(\partial_\mu U)] \\ &= B_\mu\,. \end{aligned}$$

Developing $B_\mu$ as $B_\mu = B^a_\mu \sigma_a$ and defining $W^\pm_\mu = \frac{1}{\sqrt{2}}(B^1_\mu \mp i B^2_\mu)$, such that

$$B_\mu = \begin{pmatrix} B^3_\mu & \sqrt{2}W^+_\mu \\ \sqrt{2}W^-_\mu & -B^3_\mu \end{pmatrix},$$

one gets

$$B^s_\mu = \begin{pmatrix} B^3_\mu + \frac{2i}{g} s^{-1}\partial_\mu s & s^{-2}\sqrt{2}W^+_\mu \\ s^2\sqrt{2}W^-_\mu & -(B^3_\mu + \frac{2i}{g}s^{-1}\partial_\mu s) \end{pmatrix},$$

from which we deduce the transformations of these new fields under the action of the two