TBRE and, therefore, analytically accessible. In the model parity is the only quantum number, and m spinless fermions occupy ℓ_1 (ℓ_2) degenerate single-particle states with positive (negative) parity, respectively. The fermions interact via a parity–conserving two–body interaction with random Gaussian–distributed uncorrelated two–body matrix elements. The nth moments $\mathcal{M}_n(\pm)$ of the Hamiltonian H of the model (defined as normalized traces of H^n with positive integer n) can be worked out analytically for the many–body states of both positive (+) and negative (−) parity. The case of large matrix dimension is attained in the "dilute limit" defined by $\ell_1, \ell_2, m \to \infty$, $m/\ell_1 \to 0$, $m/\ell_2 \to 0$. It is shown that $\mathcal{M}_n(+) = \mathcal{M}_n(-)$ for all n up to a maximum value that is bounded from above by m but tends to ∞ in the dilute limit, and that the two moments differ ever more strongly when n grows beyond that bound. That result shows that the spectra of states with positive and negative parity are strongly correlated. In particular, the shapes of the two average spectra are extremely similar. At the same time, the result suggests that the local spectral fluctuations of the two spectra are uncorrelated. Indeed, in their work on the use of moments for nuclear spectroscopy, French and collaborators concluded that such fluctuations are determined by the very highest moments of the Hamiltonian (Brody *et al.*, 1981).

The results just stated apply in the dilute limit only. They do not contradict earlier findings (Papenbrock and Weidenmüller, 2007) for the TBRE on correlations between spectra carrying different quantum numbers. These calculations involved Hilbert spaces of small dimension only.

Assuming that a result similar to the one just stated holds in the limit of large matrix dimension for the TBRE and observing that the fluctuation properties of the S–matrix are caused by the local spectral fluctuation properties of the underlying Hamiltonian (see Eqs. (28,29)), we conclude that within the framework of the nuclear shell model, S–matrix elements carrying different quantum numbers are likely to be uncorrelated, in agreement with Eq. (5).

VI. TESTS AND APPLICATIONS OF THE STATISTICAL THEORY

The statistical theory reviewed in Sections IV and V has been tested thoroughly. Moreover, it has found numerous applications, both within the realm of nuclear physics and beyond. In this Section we review some recent such tests and applications.

A. Isolated and Weakly Overlapping CN resonances

In the regime of isolated resonances, thorough tests of the statistical theory of nuclear reactions ($\Gamma \ll d$) were undertaken already many years ago. Especially for neutron resonances there exists a comprehensive review (Lynn, 1968). In the regime of weakly overlapping resonances, tests have so far not been performed in nuclei. Here we do not summarize the early works but rather focus attention on recent data and tests of the theory. These have become possible in microwave billiards (Dietz *et al.*, 2008). Such devices simulate the CN and its resonances or, for that matter, any other chaotic quantum scattering system. Indeed, in sufficiently flat microwave resonators and for sufficiently low values of the radio frequency (rf) — so–called quantum billiards — only one vertical mode of the electric field is excited, and the Helmholtz equation is mathematically equivalent to the Schrödinger equation for a two–dimensional quantum billiard (Gräf *et al.*, 1991; Richter, 1999; So *et al.*, 1995; Sridhar, 1991; Stöckmann and Stein, 1990; Stöckmann, 2000). If the classical dynamics (free motion within the microwave resonator and elastic scattering by its boundary) is chaotic, the statistical properties of the eigenvalues and eigenfunctions of the closed resonator in the quantum case follow RMT predictions (Bohigas *et al.*, 1984), and the scattering of rf amplitudes by the resonator corresponds to quantum chaotic scattering.

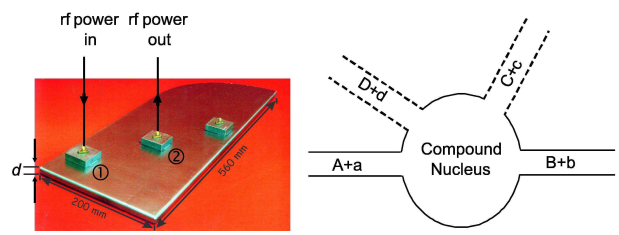

FIG. 6 Flat microwave resonator (left–hand side) as a model for the compound nucleus (right–hand side). The height $d = 0.84$ cm of the flat resonator makes it a chaotic quantum billiard up to a frequency of 18.75 GHz.

The left–hand side of Fig. 6 shows a typical quantum billiard realized in the form of a flat microwave resonator. For the measurement of the spectrum, rf power is coupled via an antenna labeled 1 into the resonator, thereby exciting an electric field mode within the resonator, and the reflected output signal at the same antenna (or the transmitted one at the antenna labeled 2) is determined in magnitude and phase in relation to the input signal. Hence, the resonator is an open scattering system where the antennas act as single scattering channels. The scattering process is analogous to that of a CN reaction as indicated schematically on the right–hand side of Fig. 6. The incident channel $A + a$ consists of a target nucleus A bombarded by a projectile a leading to a compound nucleus which eventually decays after some time into the channel with the residual nucleus B and the outgoing particle b. (We disregard angular momentum and spin). Attaching more antennas to the resonator (or dissipating microwave power in its walls) corresponds to more open channels $C + c$, $D + d$, ... of the compound nucleus. The resonator in Fig. 6 has the shape of a so–called Bunimovich stadium billiard which is known to be fully