where ξ is a rv with $P(\xi = 1) = \gamma = 1 - P(\xi = 0)$, and ξ is independent of $\boldsymbol{Z}^*$ which is a random permutation of the vector $(d, 0, \dots, 0)$ with equal probability $1/d$. Obviously, $P(\boldsymbol{Z} > 0, \xi = 0) = 0$. On the other hand, $\xi = 1$ implies $\boldsymbol{Z} = 1$. Thus, we obtain by (13) for all $\boldsymbol{x} \leq \boldsymbol{0}$

$$\begin{aligned}\bar{H}_\gamma(\boldsymbol{x}) &= 1 - \frac{E\left(\left(\|1/\boldsymbol{Z}\|_{M_\gamma}\right)^{-1} \exp\left(\|1/\boldsymbol{Z}\|_{M_\gamma} \max_{i=1,\dots,d}(x_i Z_i)\right) \cdot 1_{\{\boldsymbol{Z}>0,\xi=1\}}\right)}{E\left(\left(\|1/\boldsymbol{Z}\|_{M_\gamma}\right)^{-1} \cdot 1_{\{\boldsymbol{Z}>0,\xi=1\}}\right)} \\ &= 1 - \exp\left(\|\boldsymbol{1}\|_{M_\gamma} \max_{i=1,\dots,d} x_i\right).\end{aligned}$$

□

In order to generalize Proposition 3.7 to stochastic processes in $C(S)$ with arbitrary margins, the following lemma is needed.

Lemma 3.9. *Let f_n, $n \in \mathbb{N}$, be a sequence of functions in $\bar{E}^-(S)$ converging uniformly to $f \in \bar{E}^-(S)$. Then, under the conditions and notation of Proposition 3.7,*

$$\bar{H}_n(f_n) = \frac{\Pi_n(f_n)}{\pi_n} \to_{n\to\infty} \bar{H}_D(f).$$

Proof. Let $\varepsilon > 0$. Due to the uniform convergence of f_n, there exists $N \in \mathbb{N}$ such that $f - \varepsilon \leq f_n \leq f + \varepsilon$ for $n \geq N$. Assume without loss of generality $f + \varepsilon < 0$, otherwise consider $\min(f + \varepsilon, 0)$. Clearly, for such n,

$$\Pi_n(f + \varepsilon) \leq \Pi_n(f_n) \leq \Pi_n(f - \varepsilon).$$

Now with $n \to \infty$, Proposition 3.7 shows

$$E\left(\inf_{s\in S} |\max(\eta_s, f(s) - \varepsilon)| Z_s\right) \leq \lim_{n\to\infty} \Pi_n(f_n) \leq E\left(\inf_{s\in S} |\max(\eta_s, f(s) + \varepsilon)| Z_s\right).$$

Now check

$$\inf_{s\in S} |\max(\eta_s, f(s) \pm \varepsilon)| Z_s \leq -\eta_{s_0} Z_{s_0}, \qquad s_0 \in S,$$

and let $\varepsilon \downarrow 0$. The assertion now follows from the dominated convergence theorem.

□