represented in terms of Mellin-Barnes contour integrals. For instance, the following contour integral representation

$$\begin{aligned} &{}_pF_q\left[a_1\,,a_2\,,\ldots a_p\,;b_1\,,b_2\,,\ldots b_q\,;z\right] \\ &\quad=\frac{\Gamma\left(b_1\right)\Gamma\left(b_2\right)\ldots\Gamma\left(b_q\right)}{\Gamma\left(a_1\right)\Gamma\left(a_2\right)\ldots\Gamma\left(a_p\right)}\int_{-i\infty}^{i\infty}\frac{ds}{2\pi i}\frac{\Gamma\left(a_1+s\right)\Gamma\left(a_2+s\right)\ldots\Gamma\left(a_p+s\right)}{\Gamma\left(b_1+s\right)\Gamma\left(b_2+s\right)\ldots\Gamma\left(b_q+s\right)}\Gamma\left(-s\right)\left(-z\right)^s \end{aligned} \tag{B.13}$$

is completely equivalent to the series representation (B.12). The integral can be evaluated using the residue theorem, and the integration along the imaginary axis can be shifted to avoid poles if needed. Note that the Gamma functions of the form $\Gamma\left(c+s\right)$ have poles in the left half plane, while the Gamma function $\Gamma\left(-s\right)$ has poles in the right half plane. In order to reproduce (B.12), the contour should be closed in the right half plane, and the residues can be calculated using (B.2). If the integration contour is closed appropriately, it can be shown that all contributions apart from the integration along the imaginary axis are zero. The proof of this is rather delicate and is not given here (see Refs. [108, 103]). Using (B.13), we can write

$${}_1F_0\left[n\,;\,;z\right]=\frac{1}{\left(1-z\right)^n}=\frac{1}{\Gamma\left(n\right)}\int_{-\infty}^{i\infty}\frac{ds}{2\pi i}\Gamma\left(n+s\right)\Gamma\left(-s\right)\left(-z\right)^s\,. \tag{B.14}$$

This identity permits massive propagators to be expressed as contour integrals of massless propagators and is the foundation of the Mellin-Barnes techniques used in Chapter 2 to calculate loop integrals with two massive propagators. An important identity for contour integrals of the form (B.13) is Barnes' Lemma:

$$\int_{-\infty}^{i\infty}\frac{ds}{2\pi i}\Gamma\left(a+s\right)\Gamma\left(b+s\right)\Gamma\left(c+s\right)\Gamma\left(d+s\right)=\frac{\Gamma\left(a+c\right)\Gamma\left(a+d\right)\Gamma\left(b+c\right)\Gamma\left(b+d\right)}{\Gamma\left(a+b+c+d\right)}\,. \tag{B.15}$$

Most special functions encountered in Mathematical Physics can be expressed in terms of generalized hypergeometric functions (see Ref. [82] for a partial list). The most commonly