mesons ($\sigma^*$ and $\phi$). It is well known that the meson-hyperon couplings play an important role in determining the properties of neutron star matter. We have used the couplings constrained by reasonable hyperon potentials that include the updated information from recent developments in hypernuclear physics. To examine the $^1S_0$ pairing of $\Lambda$ hyperons, we have adopted several $\Lambda\Lambda$ potentials. Most are based on the Nijmegen models and have been used in double-$\Lambda$ hypernuclei studies. NFs, NSC97s, and Urbana potentials have simulated the experimental value of $B_{\Lambda\Lambda}({}^6_{\Lambda\Lambda}\text{He})$ from the Nagara event.

We have calculated the $^1S_0$ pairing gap of $\Lambda$ hyperons at the Fermi surface, $\Delta_F$, using the $\Lambda\Lambda$ potentials adopted in this article. It is found that $\Delta_F$ depends both on the $\Lambda\Lambda$ interaction and on the treatment of neutron star matter. The maximal $\Delta_F$ obtained in the present calculation is about 0.8 MeV with the ESC00 potential in the TM1 case. This is because the ESC00 potential has the strongest attraction among the $\Lambda\Lambda$ interactions used in this article. The ND1 and ND2 potentials yield somewhat smaller $\Delta_F$ of the order of 0.1–0.2 MeV. For the NSC97e, NFs, NSC97s, and Urbana potentials, the values of $\Delta_F$ are of the order of $10^{-4}$ MeV (TM1) or absent (NL3). The $\Lambda$ pairing does not appear for the NSC97b and NSC97f potentials. The difference in these results reflects the dependence of $\Delta_F$ on the $\Lambda\Lambda$ interaction. On the other hand, the magnitude and the threshold density of $\Delta_F$ are also dependent on properties of neutron star matter, especially on the effective mass and particle fraction of $\Lambda$ hyperons. In the case of TM1 (NL3) with the ESC00 potential, the threshold density of $\Delta_F$ is around 0.31 fm$^{-3}$ (0.28 fm$^{-3}$), reaches a maximum value at $\rho_B \sim 0.34$ fm$^{-3}$ ($\rho_B \sim 0.30$ fm$^{-3}$), and finally vanishes at $\rho_B < 0.46$ fm$^{-3}$ ($\rho_B < 0.38$ fm$^{-3}$). By solving the TOV equation, we have calculated neutron star properties and found that whether the $^1S_0$ superfluidity of $\Lambda$ hyperons exists in the core of neutron stars mainly depends on the $\Lambda\Lambda$ interaction used. With stronger $\Lambda\Lambda$ interactions, such as ESC00, ND1, and ND2, the $\Lambda$ superfluidity may exist in massive neutron stars. It is unlikely that $\Lambda$ superfluidity can exist in neutron stars with the NFs, NSC97s, and Urbana interactions, which have simulated the experimental value of $B_{\Lambda\Lambda}({}^6_{\Lambda\Lambda}\text{He})$ from the Nagara event.

In this article, we have considered the updated information from recent developments in hypernuclear physics and used the weak attractive $\Lambda\Lambda$ interactions suggested by the Nagara event. However, there are still large uncertainties in the hyperon-hyperon interaction and the EOS of neutron star matter. A more precise study of the $\Lambda$ pairing in neutron stars requires further development in hypernuclear physics.