mensions, and spacetime momenta q^μ in d Euclidean dimensions, respectively. Therefore these propagators have the form shown in (4.18). In a free theory

$$D(q) = \begin{cases} 1/(2q) & \text{Coulomb gauge} \\ 1/q^2 & \text{Landau gauge} \end{cases} \tag{5.1}$$

where the propagator in Coulomb gauge is at equal times, with q the space (rather than spacetime) momentum. The behavior (5.1) is expected at high momenta, but it is certainly not correct at low momenta, as seen from lattice Monte Carlo simulations. In Landau gauge, the current evidence is that $D(0)$ is finite and non-zero at $q=0$ in three and four dimensions [7, 8], while $D(q)\rightarrow 0$ in two dimensions [9]. In Coulomb gauge it appears that $D(q)\rightarrow 0$ in four dimensions [10].

In order to allow for non-singular power behavior in the transverse gluon propagator as $p\rightarrow 0$, I will adopt the ansatz that

$$D(q) = \frac{1}{2\sqrt{q^2+m^{2+\alpha}/q^\alpha}} \tag{5.2}$$

in Coulomb gauge, and

$$D(q) = \frac{1}{q^2+m^{2+\alpha}/q^\alpha} \tag{5.3}$$

in Landau gauge. Gribov's proposal for the gluon propagator in these cases corresponds to $\alpha=2$. The propagators go over to free-field behavior as $q\rightarrow\infty$.

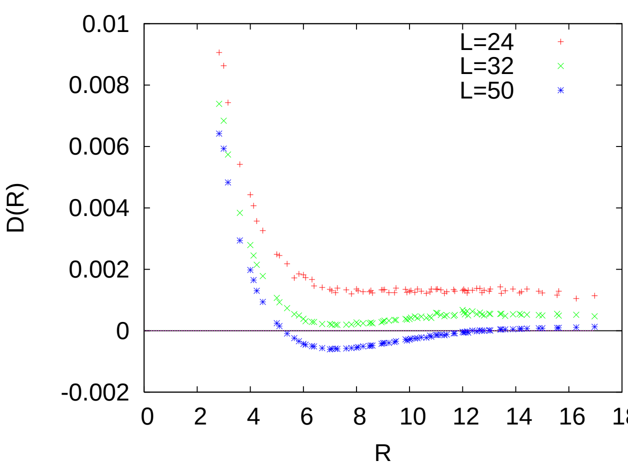

FIG. 2: Equal-times Coulomb-gauge gluon propagator in 2+1 dimensions, at $\beta=6$ and L^3 lattice volume, for $L=24,32,50$.

I am not aware of any lattice Monte Carlo computation of the transverse gluon propagator in Coulomb gauge in $d=3$ dimensions, in position space. In Fig. 2 I show data for $D(R)$ obtained from the equal times correlator

$$\langle \mathrm{Tr}[\mathtt{A}_j(\mathbf{x},t)\mathtt{A}_j(\mathbf{y},t)] \tag{5.4}$$

of gluon fields

$$\mathtt{A}_j(\mathbf{x},t) = \frac{1}{2i}(U_j(\mathbf{x},t)-U_j^\dagger(\mathbf{x},t)) \tag{5.5}$$

on the lattice. The correlator is calculated via lattice Monte Carlo with an SU(2) Wilson action, on an L^3 lattice volume at coupling $\beta=6$ and $L=24,32,50$, with the equal-times correlator computed after transforming the gauge fields to Coulomb gauge. Note that as the lattice volume increases, the gluon propagator develops a "dip" and actually becomes negative at the larger R values. This behavior appears to rule out $\alpha=0$, in which the propagator should be everywhere positive. A reliable computation of $D(q)$ as $q\rightarrow 0$ will probably require a large-scale lattice calculation, as has been done for the Landau gauge.

VI. RESULTS FOR F-P SPECTRA

In section III I introduced a parameter d_H to control the approach to the first Gribov horizon, and speculated on the low-p behavior of λ_p as the horizon is approached. In the perturbative calculation, the mass parameter m in the gluon propagator plays essentially the same role as d_H. Note that in dimensions lower than 3+1, where $I[p,m,\alpha]$ is convergent, the coupling g^2 is dimensionful, and we may as well choose units such that $g^2=1$. Then

$$\langle\lambda_p\rangle = p^2\Big((1-R_d I[p,m,\alpha]\Big) \tag{6.1}$$

Expanding $I[p,m,\alpha]$ in leading powers of p near $p=0$, we have

$$\begin{aligned} R_d I[p,m,\alpha] &= a[m,\alpha]-b[m,\alpha]p^s \\ &\qquad +\text{higher powers of } p \end{aligned} \tag{6.2}$$

in which case

$$\begin{aligned} \langle\lambda_p\rangle &= (1-a[m,\alpha])p^2+b[m,\alpha]p^{2+s} \\ &\qquad +\text{higher powers of } p \end{aligned} \tag{6.3}$$

Suppose, for a given α, it is possible to find a critical value $m=m_c$ such that $a[m_c,\alpha]=1$ and $b[m,\alpha]>0$. In that case we have the Type I scenario conjectured in Fig. 1(a) above; i.e.

1. $m<m_c$ and $a[m,\alpha]>1$: The low-lying F-P eigenvalue spectrum has a range of negative eigenvalues, starting at $p=0$. We interpret this to mean that the transverse gluon propagator, which determines the spectrum at second order, is determined by configurations outside the Gribov region.

2. $m=m_c$ and $a[m_c,\alpha]=1$: The region of negative eigenvalues just disappears, and $\lambda_p\sim p^{2+s}$. This is the case of particular interest, where the gluon propagator is derived from configurations which mainly lie right on the Gribov horizon.