and then obviously

$$\lim_{q\to 1}[n]_q! = n! \tag{10}$$

The q-binomial can now be defined in an alternative way as

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[k]_q!\,[n-k]_q!} \tag{11}$$

and then it follows that

$$\lim_{q\to 1}\begin{bmatrix} n \\ k \end{bmatrix}_q = \binom{n}{k} \tag{12}$$

Quite analogously it is easy to verify that

$$\lim_{q\to 1}\frac{(q^a;\,q)_n}{(1-q)^n} = (a)_n \tag{13}$$

Finally, note also that $\begin{bmatrix} n \\ k \end{bmatrix}_q$ can be viewed as a formal polynomial in q of degree $k\,(n-k)$ where the coefficient of q^j counts the number of k-subsets of $\{1,\ldots,n\}$ with element sum $j+k\,(k+1)/2$. It is thus a polynomial with positive coefficients.

We have so far only stated what belongs to the standard repertoire on the subject. The q-binomials and the q-Pochhammer function have many interesting properties and we point the interested reader to the books[1],[2],[3] and especially the charming little book[4]. For more on the standard binomial coefficient and Pochhammer function we recommend[5] which contains a wealth of useful information.

A natural extension of the q-binomial coefficient, the p,q-binomial coefficient, was defined in[6] as

$$\begin{bmatrix} n \\ k \end{bmatrix}_{p,q} = \prod_{i=1}^{k}\frac{p^{n-i+1}-q^{n-i+1}}{p^i-q^i}, \quad p\neq q,\; 0\leq k\leq n \tag{14}$$

Clearly, in the case $p=1$ this reduces to a q-binomial coefficient. Also, note that p and q are interchangable so that

$$\begin{bmatrix} n \\ k \end{bmatrix}_{p,q} = \begin{bmatrix} n \\ k \end{bmatrix}_{q,p} \tag{15}$$

Just as the standard binomial coefficients, their p,q-analogues are also symmetric

$$\begin{bmatrix} n \\ k \end{bmatrix}_{p,q} = \begin{bmatrix} n \\ n-k \end{bmatrix}_{p,q} \tag{16}$$

It is an easy exercise to show the following identity and we leave this to the reader.

$$\begin{bmatrix} n \\ k \end{bmatrix}_{p,q} = p^{k\,(n-k)}\begin{bmatrix} n \\ k \end{bmatrix}_{q/p} = q^{k\,(n-k)}\begin{bmatrix} n \\ k \end{bmatrix}_{p/q} \tag{17}$$