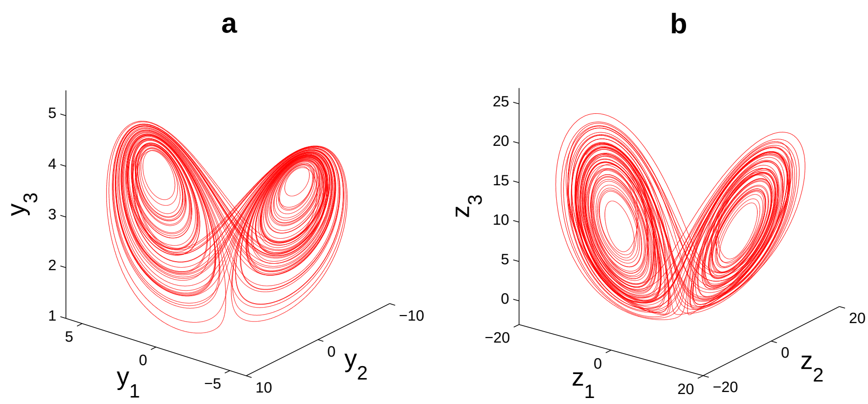

Figure 3: The 3−dimensional projections of the chaotic trajectory of system (1.1)+(3.5)+(3.6). (a) The projection on the $y_1 - y_2 - y_3$ space, (b) The projection on the $z_1 - z_2 - z_3$ space. The picture represented in (a) illustrates the chaotic trajectory of the perturbed Lorenz system (3.5), while the picture in (b) corresponds to the perturbed Lorenz system (3.6). The pictures represented in (a) and (b) confirm both the extension of chaos and the existence of a chaotic attractor in the 9−dimensional phase space.

More precisely, "x" is written every time when the orbit spirals round in $x_1 > 0$, while "y" is written every time when it spirals round in $x_1 < 0$. As r decreases towards 99.98 a period-doubling bifurcation occurs in the system such that two new symmetric stable periodic orbits (x^2yx^2y and y^2xy^2x) appear and the previous periodic orbits lose their stability [24, 65]. According to Franceschini [24], system (1.1) undergoes infinitely many period-doubling bifurcations at the parameter values 99.547, 99.529, 99.5255 and so on. The sequence of bifurcation parameter values accumulates at $r_\infty = 99.524$. For values of r smaller than r_∞ infinitely many unstable periodic orbits take place in the dynamics of (1.1) [24, 65].

To extend the period-doubling cascade of (1.1), we take into account the system

$$\begin{aligned} \frac{dy_1}{dt} &= -10y_1 + 10y_2 + 1.8x_1(t) \\ \frac{dy_2}{dt} &= -y_1y_3 + 0.27y_1 - y_2 + x_2(t) \\ \frac{dy_3}{dt} &= y_1y_2 - (8/3)y_3 + 0.3x_3(t), \end{aligned} \tag{4.7}$$

where $x(t) = (x_1(t), x_2(t), x_3(t))$ is a solution of (1.1). Note that in the absence of driving, system (4.7) admits a stable equilibrium point, i.e., system (2.3) with $\overline{\sigma} = 10$, $\overline{r} = 0.27$ and $\overline{b} = 8/3$ does not admit chaos.

By using Theorem 15.8 [70], one can verify that for each periodic $x(t)$, there exists a periodic solution of (4.7) with the same period.

In Figure 4, we illustrate the stable periodic orbits of systems (1.1) and (4.7). Figure 4, (a) shows the y^2x periodic orbit of (1.1) for $r = 100.36$, while Figure 4, (b) depicts the corresponding periodic orbit of system (4.7). Similarly, Figure 4, (c) and (d) represent the y^2xy^2x periodic orbit of (1.1) with $r = 99.74$ and the corresponding periodic orbit of (4.7), respectively. Figure 4 confirms that if (1.1) has a periodic