respectively, for $\delta = 0$. First we focus on the case where pure dephasing is negligible (blue solid line). With the set of parameters used, this corresponds to the good cavity regime. Note again that the chosen parameters are within reach of current technology regarding quantum dots coupled to optical semi-conducting microcavities. Typical coupling strengths of $g = 50 - 100$ μeV can be reached [15, 35], whereas quality factors exceeding $Q_{cav} = 10^5$ (i.e. cavity linewidths smaller than $\kappa = 10$ μeV) have separately been demonstrated [36]. However, the physics of single two-level atom lasers and single quantum dot lasers are drastically different from one another. We will come back to discuss this point in Sec D.

As it can be seen from the plotted quantities, the device perfectly converts the pump energy into cavity photons, whatever the pump power (log-log scale), which was expected since the device shows a high β [32, 33]. Very intuitively, the critical value $n_a = 1$ is reached as soon as the pump rate is of the same order of magnitude as the cavity damping rate. At this point, the atomic population remains clamped at a value nearly equal to $n_x \sim 0.5$, which is already a signature of lasing. This is confirmed studying the statistics of the emitted field, that clearly shows a transition from antibunched ($g^2(0) < 1$) to Poissonian ($g^2(0) = 1$), for nearly the same value of the pump power. Qualitatively similar results were recently shown in [13, 37]. Indeed, in the spontaneous emission regime the device produces streams of single photons, and the emitted field is antibunched [38]. When stimulated emission is reached, more than one photon can be stored in the cavity mode before the intra-cavity field is dissipated, leading to the buildup of a Poissonian field that reflects the statistics of the single atom excitation events during a typical cavity lifetime. Thus, in the single atom device, in addition to be an efficient relaxation channel (just as in the conventional laser case), the cavity plays the role of a photon delayer, which keeps the photons emitted by a single atom for a sufficiently long time so that a Poissonian field can build up in the mode. This crossover in the statistics of the emitted field is a signature of the transition from the "single photon source" to the "single two-level emitter laser" operating regime. We mention here that this behavior is quite different from what happens for "conventional" high-β lasers involving several emitters. In the latter case indeed, the statistics of the field maps the statistics of the pump, whatever its power is [32]. The single photon source regime has been observed, e.g., for a Caesium atom strongly coupled to an optical cavity [39]. A crossover to Poissonian statistics has been observed - to a certain extent - for a single quantum dot coupled to a micropillar cavity [40], and a photonic crystal cavity [35, 41]. The