also known as a Schwinger string, defined as

$$S_{\alpha\omega}\left[x\,,0\right]=P\exp\left[ig\frac{\lambda^{a}_{\alpha\omega}}{2}\int_{0}^{x}dz^{\mu}\;A^{a}_{\mu}\left(z\right)\right]\,, \tag{4.5}$$

where P denotes path-ordering and g is the strong coupling. In Ref. [42] the correlation function (4.2) was calculated for diquarks composed of light quarks, and it was demonstrated that the correlation function is gauge invariant to next-to-leading order in the strong coupling. Because physical observables are gauge invariant, only gauge invariant correlation functions can be used in QSR analyses. Therefore, in order to extract physically meaningful heavy-light diquark masses, it is crucial to verify the gauge invariance of the heavy-light diquark correlation function. In this chapter we perform an explicit calculation that confirms that Eq. (4.2) is gauge invariant to next-to-leading order in the strong coupling. Using a straight line geometry, the Schwinger string is given by

$$S_{\alpha\omega}\left[x\,,0\right]=\delta_{\alpha\omega}+ig\frac{\lambda^{a}_{\alpha\omega}}{2}\int_{0}^{1}d\xi A^{a}_{\mu}\left(\xi x\right)x^{\mu}+\mathcal{O}\left(g^{2}\right)\,. \tag{4.6}$$

The first term in Eq. (4.6) simply generates a trace over the colour indices in the correlation function (4.2). However, the second term in Eq. (4.6) leads to a non-trivial contribution. Note that this term is not calculated in the leading-order analysis performed in Ref. [125]. In order to verify that the heavy-light correlation function is gauge invariant, all calculations must be performed in a general covariant gauge. That is, the gauge parameter a in the gluon propagator (1.39) must be retained in all calculations.

The next-to-leading order perturbative contributions to the heavy-light diquark correlation function also introduce gauge dependent terms. In this chapter it is shown that the gauge dependent contributions of the Schwinger string (4.6) exactly cancel the gauge dependence in the next-to-leading order perturbative contribution. Therefore the heavy-light diquark correlation function (4.2) is gauge invariant to next-to-leading order in the strong coupling and can be utilized in QSR to determine the heavy-light diquark mass.

Once the gauge invariance of the heavy-light diquark correlation has been established the bare correlation function must be renormalized. This can be achieved by renormalizing the heavy quark mass and the diquark current, whose renormalization factor is calculated