angular momentum commutation relations. The restriction of l to integers is a consequence of restricting m to integers.

Let's go back to equation (6), and write down its famous solutions which are the associated Legendre functions [1, 2](N_l^m is a normalization constant):

$$\Theta(\theta) = N_l^m P_l^m(\cos\theta) \tag{8}$$

where P_l^m are given by

$$P_l^m(x) = (1-x^2)^{\frac{|m|}{2}} \left(\frac{d}{dx}\right)^{|m|} P_l(x) \tag{9}$$

where $P_l(x)$ are the the Legendre polynomials which are given by the Rodrigues formula [1, 2]:

$$P_l(x) = \frac{1}{2^l l!} \left(\frac{d}{dx}\right)^l (x^2-1)^l \tag{10}$$

The solutions $|P_l^m(\cos\theta)|^2$ represent the probability of finding the particle at a certain angle θ, which means that they give the probability distribution of the particle about the z-axis. We will show below that we can view this distribution as being shaped by a polar potential.

III. THE POLAR POTENTIAL

Now we can bring the polar equation (6) to the Schrödinger form, just as we did with the radial equation. Defining:

$$\Theta(\theta) = \sin^{-\frac{1}{2}}(\theta) y(\theta) \tag{11}$$

and multiplying by 1/2, equation (6) becomes

$$-\frac{1}{2}\frac{d^2 y(\theta)}{d\theta^2} + \left\{\frac{m^2-\frac{1}{4}}{2\sin^2(\theta)}\right\} y(\theta) = E y(\theta) \tag{12}$$

where $E = (1/2)(l(l+1)+1/4)$.

This equation can be thought of as a one dimensional Shrödinger equation (with $\hbar = m = 1$) for a particle confined between 0 and π, and satisfying the boundary conditions $y(0) = y(\pi) = 0$. The term in brackets is a one dimensional potential, more precisely it is a family of potentials depending on the choice of the value of m. These potentials