[19]. These Hamiltonians may be regarded as the sum of $\mathcal{H}_\uparrow = (\psi_\uparrow^\dagger, \psi_\downarrow) H_\uparrow (\psi_\uparrow, \psi_\downarrow^\dagger)^T$ and $\mathcal{H}_\downarrow = (\psi_\downarrow^\dagger, \psi_\uparrow) H_\downarrow (\psi_\downarrow, \psi_\uparrow^\dagger)^T$[19], where $H_\uparrow$ and $H_\downarrow$ have the form of Eq. (6) with $M_{11} = h, M_{12} = \Delta, M_{22} = -h^T$ and $M_{11} = h, M_{12} = -\Delta, M_{22} = -h^T$ respectively.

TABLE I: Conditions for superconductors in the various symmetry classes to support protected zero modes in vortex cores, expressed as conditions on $\mathcal{H}_I$, the insulator associated with the superconductor. The last column indicates whether there is a single protected Majorana mode(M) or a protected pair of modes(D). No protected modes exist for the class CI.

Class	Time-rev	Spin-rot	Condn. on $\mathcal{H}_I$	Mode
D	No	No	$\sigma_{xy} 2\pi\hbar/e^2 = 2k-1$	M
C	No	Yes	$\sigma_{xy} 2\pi\hbar/e^2 = 2(2k-1)$	D
DIII	Yes	No	non-trivial Z_2	D
CI	Yes	Yes	-	-

The spectra of $H_\uparrow$ and $H_\downarrow$ are identical. If $(u, v)^T$ written in the particle hole basis, is a zero mode of $H_\uparrow$, then $(u, -v)^T$ is a zero mode of $H_\downarrow$. The superconductor may be mapped onto an insulator, $\mathcal{H}_I$, which is the sum of two single particle Hamiltonians, $\mathcal{H}_{I,\uparrow}$ and $\mathcal{H}_{I,\downarrow}$, and which may be regarded as separate systems. The condition that in the presence of a vortex, the matrix $H_\uparrow$ has an odd number of zero modes is, as deduced in the study of Hamiltonians of class D, that the Hall conductance of the corresponding insulator, $\sigma_{xy}(\mathcal{H}_{I,\uparrow})2\pi\hbar/e^2$ is an odd integer. The net Hall conductance, $\sigma_{xy}(H_I)$ is twice that of $H_{I,\uparrow}$ and is thus always an even integer. Thus, when the Hall conductance of $\mathcal{H}_I$ has the form $2pe^2/2\pi\hbar$ where p is an odd integer, the system has an odd number of pairs of zero modes in its vortices, while when p is even, the system has an even (or zero) number of pairs of zero modes.

Superconductors with both time reversal and spin rotational symmetry, which belong to the class CI, may be regarded as belonging to the trivial Z_2 class of superconductors with time reversal symmetry. These superconductors thus have no topologically protected zero modes in their vortex cores.

Our results are summarized in Table I. In the notation used in Ref. [22], superconductors in the class D have protected Majorana modes in their vortex cores when the integer invariant for this class is odd, superconductors in class C and DIII have protected Dirac modes when the invariants for these classes are odd and non-trivial respectively. Most continuum models can be simulated to an arbitrary degree of accuracy by a series of lattice models. Since the results derived above are not limited to a particular tightbinding model, one expects that the analysis presented above extends also to continuum models [24].

Our analysis thus far has been restricted to a single vortex in an infinite superconductor. A finite superconductor with a vortex is topologically equivalent to a system with two vortices and periodic boundary conditions. One may then start with the associated insulator with no flux inserted and gradually insert flux into two tubes in such a way that the flux entering one tube is equal to the flux leaving the other leaving the total flux through the system zero. At $2\pi\hbar/2e$ flux, the states localized at the two flux tubes will in general hybridize, giving rise to a finite splitting. The magnitude of the splitting when the two vortices are a certain distance d is proportional to the overlap between two zero-energy eigenstates of the infinite system placed at the same distance and therefore falls exponentially as the distance between the vortices.

Finally, we note that the arguments made above would are also applicable in the case when there is a mobility gap in the absence of flux rather than a gap to all states since the states within the mobility gap have zero Chern number and their energies return to their initial values when the flux inserted varies from 0 to $2\pi\hbar/e$.

In summary, we have provided a simple and general argument which shows that certain topological classes of superconductors have topologically protected, robust zero modes, which can either be unpaired Majorana modes, or come in pairs. We applied this analysis to the various symmetry classes of superconducting Hamiltonians.

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