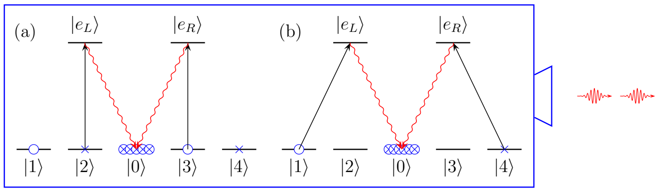

FIG. 2: (Color online) Generation of the photonic Bell state $(|R\rangle_1|L\rangle_2 - |L\rangle_1|R\rangle_2)/\sqrt{2}$ from the collective atomic state $(|1010\rangle - |0101\rangle)/\sqrt{2}$ (the first term is indicated with circles, and the second with crosses). (a) A light pulse transfers the populations in $|2\rangle$ and $|3\rangle$ to the excited states $|e_L\rangle$ and $|e_R\rangle$, respectively, and the subsequent atomic decay to the state $|0\rangle$ leads to emission of the first photon, which is L-polarized if $|e_L\rangle$ is populated and R-polarized if $|e_R\rangle$ is populated. (b) The second photon is similarly obtained via spontaneous emission after transfer of the populations in the states $|1\rangle$ and $|4\rangle$ to the excited states.

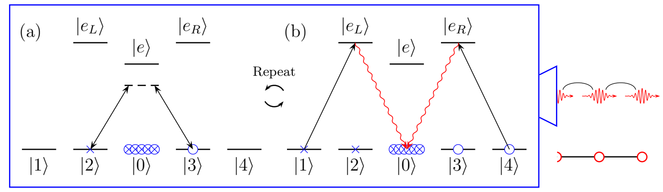

FIG. 3: (Color online) Generation of a one-dimensional cluster state. Initially, one atom is moved to $|3\rangle$. (a) In each iteration step, a $\pi/2$-pulse is applied to the (Raman) transition between $|2\rangle$ and $|3\rangle$. An atom is then moved from $|0\rangle$ to $|1\rangle$ if and only if $|3\rangle$ is unoccupied, and an atom is moved from $|0\rangle$ to $|4\rangle$ if and only if $|2\rangle$ is unoccupied. (Note that the sum of the populations in $|2\rangle$ and $|3\rangle$ is always one atom.) (b) The populations in $|1\rangle$ and $|4\rangle$ are transferred to $|e_L\rangle$ and $|e_R\rangle$, respectively, which leads to emission of a photon, and the process is repeated. The bonds between the emitted photons illustrate the cluster state entanglement, and the graph representing the state is shown below the photons.

atom in $|j'\rangle$. A train of entangled pulses, all containing either zero or one photon, can then be emitted by conditional feeding of the state $|j\rangle$, as shown in Fig. 1(c). After $M-1$ emissions, we apply a π-pulse between $|j'\rangle$ and $|r'\rangle$, a π-pulse between $|r'\rangle$ and $|0\rangle$, and a π-pulse between $|j'\rangle$ and $|r'\rangle$, which coherently changes the population in $|j'\rangle$ between zero and one. The last mode is then released by transferring the population in $|j'\rangle$ to an excited state.

The maximally entangled two-mode Bell state $|\Psi_-\rangle = (|R\rangle_1|L\rangle_2 - |L\rangle_1|R\rangle_2)/\sqrt{2}$, where R and L denote right and left polarized photons, respectively, can be prepared, if we first prepare the atoms in the collective state $|\Psi_-^{\rm at}\rangle = (|1010\rangle - |0101\rangle)/\sqrt{2}$, making use of the five hyperfine states in Fig. 2. This is done as follows: *i)* Transfer one atom from $|0\rangle$ to $|2\rangle$ via a Rydberg level. *ii)* Use a Raman transition between $|2\rangle$ and $|3\rangle$ to obtain the transformation $|0100\rangle \to (|0010\rangle - |0100\rangle)/\sqrt{2}$. *iii)* Move an atom from $|0\rangle$ to $|4\rangle$ if and only if $|3\rangle$ is unoccupied, and an atom from $|0\rangle$ to $|1\rangle$ if and only if $|2\rangle$ is unoccupied using the conditional method in Fig. 1(c). The optical release of the state $|\Psi_-^{\rm at}\rangle$ is then achieved as explained in Fig. 2. If the light pulses are released by spontaneous Raman transitions in the atoms, the temporal shape of the emitted photon pulse can be controlled by the coupling laser amplitude. Conversely, incident light pulses with a known temporal shape can be transferred via Raman processes to collective atomic population in different states. By subsequent application of Rydberg quantum gates, the state can be processed and analyzed, and the ensemble can thus, for example, be used to distinguish between all four polarization or photon number encoded Bell states.

As a specific application of the two-mode optical Bell state, we implement the double trine scheme for quantum key distribution, proposed recently by Tabia and Englert [20]. This scheme has the appealing features that the users of the communication channel, Alice and Bob, do not need to share a common reference frame and it is possible to extract 0.573 key bits per trine state which is close the the theoretical maximum. The scheme applies a sequence of groups of three photons with orthogonal polarizations R and L. The groups are chosen at random among three mixed quantum states ρ_i, $i = 1, 2, 3$, each being a direct product of a completely mixed state $(|R\rangle\langle R| + |L\rangle\langle L|)/2$ of the ith photon and a pure Bell state $(|R\rangle|L\rangle - |L\rangle|R\rangle)/\sqrt{2}$ of the other two photons. To produce such a state, we prepare the atomic state $|\Psi_-^{\rm at}\rangle$ and interweave its conversion to light with the emission of a mixed polarization state of a single photon. The mixed state $(|R\rangle\langle R| + |L\rangle\langle L|)/2$ can be generated by using a classical random decision to determine whether an atom is moved from $|0\rangle$ to $|e_L\rangle$ or from $|0\rangle$ to $|e_R\rangle$ via a Rydberg level. The subsequent decay back to $|0\rangle$ then produces the photon with the apparent random polarization. Note that the emission of the randomly polarized photon may occur independently of the state of the levels $|1\rangle$, $|2\rangle$, $|3\rangle$, and $|4\rangle$, and the three photons can thus be emitted in any order, which allows Alice to prepare any of the states ρ_i. The detection by Bob consists simply in measuring the polarization of the individual photons in any basis and does not require any further complicated processing of the states.

IV. CLUSTER STATES

Recently, Lindner and Rudolph proposed to use a quantum dot to prepare one-dimensional cluster states of photons [21, 22]. Following their approach, such cluster states can be produced as shown in Fig. 3. Starting with all atoms in $|0\rangle$, the initialization consists in moving one atom to the state $|3\rangle$ via a Rydberg level. We then use a Raman transition to apply a $\pi/2$ pulse between $|2\rangle$ and $|3\rangle$. Using the controlled feeding of atomic states shown in Fig. 1(c), we can apply the transformations $|0100\rangle \to |1100\rangle$ and $|0010\rangle \to |0011\rangle$ and subse-