

diagrams:

$$\begin{aligned}
& A44 [4], A55 [2], A77 [2], A47 [4], A45 [4], A46 [2], A57 [2], \\
& B44 [4], B55 [4], B47 [4], B45 [4], B46 [2], B57 [2], \\
& C44 [4], C55 [2], C77 [2], C47 [4], C45 [4], C46 [2], C57 [2],
\end{aligned} \tag{3}$$

where the number in the brackets accounts for the symmetry factor for each diagram as well as the doubling due to two directions that a lepton loop takes.

Thus far no one has succeeded in evaluating the diagrams of Set II(e) analytically. We resort to the numerical means utilizing the parametric integral formulation [16, 18, 22, 23]. The evaluation of $g-2$ can be simplified significantly by focusing on the quantity associated with the self-energy diagram G_{ij} , such as the magnetic moment amplitude $M_{G_{ij}}$, using the Ward-Takahashi identity which relates the regularized self-energy function $\Sigma_{G_{ij}}(p)$ of the diagram G_{ij} to the sum $\Lambda_{G_{ij}}(p, q)$ of the contributions from the regularized vertex diagrams obtained by inserting a QED vertex into G_{ij} in all possible ways [24].

The next step is to renormalize the integrals on the computer, which we carry out by subtractive renormalization. Since the bare amplitudes of individual diagrams have different structures of UV singularities, the numerical subtraction of UV singularities must be carried out for each diagram separately. Our aim is to construct subtraction terms that (i) share the same UV singularity as the integrand of the bare amplitude in the common Feynman parameter space, and (ii) do not introduce spurious IR singularities.

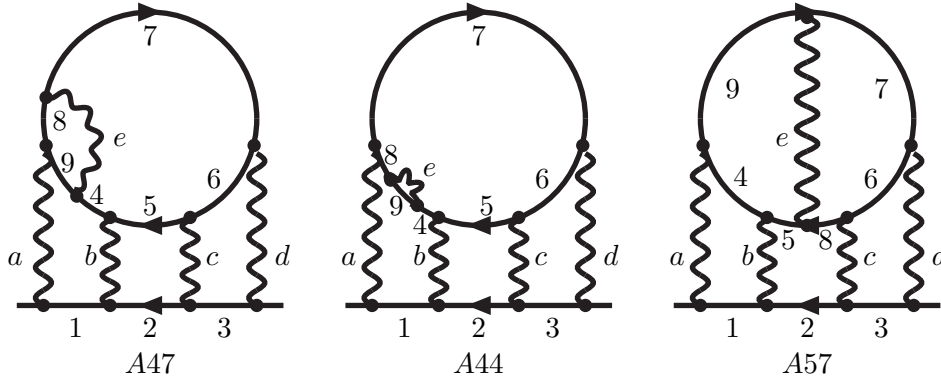


FIG. 3: Representative diagrams of Set II(e). A47 and A44 involve a second-order vertex subdiagram and a second-order self-energy subdiagram, respectively.