$$A_2(x,t) = \mathrm{sech}(\zeta(\mathrm{x},\mathrm{t})). \tag{29}$$

We take

$$g_{jk} = G_{jk}\chi^{-1}(1+\lambda\exp(-\xi^2))^3, \tag{30}$$

with $\chi(t) > 0$. We use Eq. (11) to get

$$\rho(x,t) = \frac{1}{\left(\chi\left(1+\lambda\exp(-\xi^2)\right)\right)^{1/2}} \tag{31}$$

Now, the potentials are given by

$$v_j(x,t) = s_1(t)x^2 + s_2(x,t), \tag{32}$$

with

$$s_1 = -\frac{1}{4\chi}\frac{d^2\chi}{dt^2}, \tag{33}$$

$$s_2 = \frac{\lambda\exp(-\xi^2)}{\chi^2[1+\lambda\exp(-\xi^2)]}\left(1+\frac{[\lambda\exp(-\xi^2)-2]}{\chi^2[1+\lambda\exp(-\xi^2)]}x^2\right). \tag{34}$$

This choice of nonlinearity gives $\zeta(x,t) = \xi + \sqrt{\pi}\lambda\mathrm{erf}(\xi)/2$. With this, the solutions are given by

$$\psi_1(x,t) = \frac{e^{i\eta}}{\sqrt{2}}\left[\chi\left(1+\lambda e^{-\xi^2}\right)\right]^{-1/2}\tanh(\zeta), \tag{35}$$

$$\psi_2(x,t) = e^{i\eta}\left[\chi\left(1+\lambda e^{-\xi^2}\right)\right]^{-1/2}\mathrm{sech}(\zeta), \tag{36}$$

with η real, given by Eq. (13), and $\zeta(x,t)$ as shown above. Due to the several kinds of modulations of the nonlinearities, we can choose $\chi(t)$ in several distinct ways. For instance, we may choose $\chi(t) = 1 + \alpha\sin(t) + \beta\sin(\sqrt{2}t)$, with α and β being constants related to the periodic or quasiperiodic choice of χ.

In Fig. (5), we plot the potential (32) for the (a) periodic and (b) quasiperiodic cases. Since these potentials have some structure at small distances, in Fig. (6) we show the same potentials, but now for small x. The parameters were chosen to be $\alpha = 0.1$ and $\beta = 0$ for the periodic solution, and $\alpha = \beta = 0.1$ for the quasiperiodic solution, with $\lambda = 0.5$. Despite the presence of structures at small distances, the main importance of the potentials are their long distance behavior, which allows the presence of localized excitations, even though the potential changes from attractive to repulsive behavior periodically. We get to this result after changing $\lambda \to -\lambda$, which significantly alters the small distance behavior of the potential, but preserving its long distance behavior; even