we obtain the "radial" function $\psi_{\ell mk}$ obeys the equation

$$\frac{d^2\psi_{\ell mk}}{d\tau^{*2}} + \left[k^2 - V(\tau^*)\right]\psi = 0 \qquad (10)$$

where the potential is given by

$$V(\tau^*) = \frac{g_{\rho\rho}}{\tau}\left[\frac{\ell(\ell+1)}{\tau} + 2\frac{g_{\rho\rho}}{\tau} + \partial_\tau g_{\rho\rho}\right]. \qquad (11)$$

For our purposes we are interested in the asymptotic solutions of (10) as $\tau^* \to -\infty$. The potential $V(\tau^*)$ decays exponentially in time as

$$V(\tau^*) \approx e^{\mp\alpha_\pm\tau^*}, \qquad \tau^* \to \pm\infty. \qquad (12)$$

where $2\alpha_\pm \equiv \pm(dg_{00}/dr)_{r=r_\pm}$. Finally, near the Cauchy horizon the asymptotic solutions of (10) are

$$e^{-ik\rho}\psi_{\ell mk}(\tau^*) \approx\ e^{\pm ikx_\pm}\ \left[1 + O(e^{\alpha_-\tau^*})\right]. \qquad (13)$$

Up to exponentially vanishing corrections, the solution are plane waves approaching the left and right branch of the horizon r_-, respectively. As in [7] the energy density in the scalar field ϕ as measured by a freely falling observer near a horizon with four velocity U^μ will be proportional to

$$\mathcal{E} = (\phi,_{\ \alpha}U^\alpha)(\phi,_{\ \beta}U^\beta) + \frac{1}{2}\phi,_{\ \alpha}\phi^{*,\ \alpha}. \qquad (14)$$

Since $-\tau^*\pm\rho = const$ are null surfaces and taking into account for the form of ϕ nearby the horizon, we have that the energy density is dominated by the term $|\phi,_{\ \alpha}U^\alpha|^2$. Therefore we can restrict our analysis to the term $\phi,_{\ \alpha}U^\alpha$ only. To this purpose, we need the form of the velocity vector field associated to the FFO which can be written as

$$U = U^{x_-}\frac{\partial}{\partial x_-} + U^{x_+}\frac{\partial}{\partial x_+} \qquad (15)$$

where

$$U^{x_\pm} = -\frac{\tau}{2m(\tau)-\tau}\left(h \mp \sqrt{h^2 + \frac{2m(\tau)-\tau}{\tau}}\right), \qquad (16)$$

with h a dimensionless parameter. If $h > 0$ the FFO worldline enters region III from region I and exits region III through the left-hand branch ($x_- = \infty$) of the inner horizon. If $h < 0$ the worldline enters region III from region II and exits through the right-hand ($x_+ = \infty$) branch of r_-. If $h = 0$ the worldline will move through the region III passing through the bifurcation points of the horizon r_-. In the vicinity of the Cauchy horizon, we have for $h > 0$

$$U^{x_+} \approx \frac{1}{2}, \quad U^{x_-} \approx e^{-\alpha_-\tau^*} = e^{\alpha_-(x_-+x_+)/2} \qquad (17)$$

whereas for $h < 0$

$$U^{x_+} \approx e^{\alpha_-(x_-+x_+)/2}, \quad U^{x_-} \approx \frac{1}{2}. \qquad (18)$$

Hence, for $h > 0$ and asymptotically for $x_- \to \infty$ we have

$$\phi,_{\ \alpha}U^\alpha \approx e^{\alpha_-(x_-+x_+)/2}\frac{\partial\phi}{\partial x_-} + \frac{1}{2}\frac{\partial\phi}{\partial x_+} \qquad (19)$$

whereas for $h < 0$ and $x_+ \to \infty$

$$\phi,_{\ \alpha}U^\alpha \approx \frac{1}{2}\frac{\partial\phi}{\partial x_-} + e^{\alpha_-(x_-+x_+)/2}\frac{\partial\phi}{\partial x_+}. \qquad (20)$$

In order that the FFO can measure a nondivergent amount of field energy density near the r_- horizon, the appropriate derivative of the field times the exponential blue-shift factor must be finite. In the classical picture [5, 7] from the last two relations above one concludes that the e^{-ikx_+} waves are singular along the left branch of r^- and the e^{ikx_-} waves become singular along the right branch of r_-. We shall see that due to the noncommutativity of the field, (19) and (20) stay bounded at the Cauchy horizon. Indeed, we find that for the left-going component

$$\phi,_{\ \alpha}U^\alpha \sim \frac{x_-}{\theta^{3/2}}\ e^{\alpha_-(x_-+x_+)/2}e^{-x_-^2/\theta} \qquad (21)$$

which vanishes as $x_- \to \infty$, keeping x_+ constant. Analogously, we find for the right-going component

$$\phi,_{\ \alpha}U^\alpha \sim \frac{x_+}{\theta^{3/2}}\ e^{\alpha_-(x_-+x_+)/2}\ e^{-x_+^2/\theta} \qquad (22)$$

which vanishes as $x_+ \to \infty$, for x_- fixed. The above result confirms that, probing higher momenta the field basically triggers the noncommutative nature of the manifold, which shows graininess and prevents any spacetime resolution beyond the value $\sqrt{\theta}$. This let us also conclude that in this framework no mass inflation can occur. To this purpose, we define

$$T_{ab} = \mathcal{E}_{in}l_al_b + \mathcal{E}_{out}n_an_b \qquad (23)$$

as the two-dimensional section of the stress tensor, which describes the cross flowing stream of infalling and outgoing of light like particles following null geodesics. Here l_a is the radial null vector pointing inwards, n_a is the radial null vector pointing outwards, while $\mathcal{E}_{in}$ and $\mathcal{E}_{out}$ represent the energy density of the fluxes. The mass inflation is a huge boom of the black hole internal mass parameter, which becomes classically unbounded at the Cauchy horizon. Contrary to the intuition, the inflation is due to both the outflux and the blueshifted influx of a collapsing star as shown in [9]. On the other hand, in the present framework, energy densities cannot diverge even at the Cauchy horizon. Therefore, the mass inflation which is in general proportional to the product