by changing coordinates to $U = r/\alpha'$ the α' dependence drops out of S_{D3}. The controlling parameter for Eq. (2.42) then turns out to be $g_s N(\partial U)^2/U^4$ [322]. The lowest order term from the action yields $\mathcal{N} = 4$ $SU(N)$ super-Yang-Mills from the $F_{\alpha\beta}$ term. The quadratic term has no quantum corrections and the quartic term has only a one-loop correction, consistent with $\mathcal{N} = 4$ SYM [326, 327]. This loop correction has been calculated from the gauge theory and string theory, and the two agree [328]. Moreover, it can be argued that all higher order terms in Eq. (2.42) are determined from the fourth-order term [312]. In order for this series to make sense $g_s N(\partial U)^2/U^4 \ll 1$. In particular in the supergravity regime of the AdS/CFT duality $g_s N \gg 1$, and the higher order terms beyond $SU(N)$ may become important.

2.3.3 Motivation for the AdS/CFT Correspondence

In this section we will follow [322] closely for a schematic motivation of the AdS/CFT conjecture (see the same for a very good review of the correspondence as well as an extensive bibliography). Consider from a field theoretic standpoint Type IIB string theory in a 10 dimensional spacetime with a stack of N coincident extremal ($r_+ = r_- \equiv R$) $D3$ branes at the origin. At energies small compared to the string scale $1/l_s = 1/\sqrt{\alpha'}$ only the massless string modes can be excited. The full low energy effective action is then

$$S = S_{\text{bulk}} + S_{\text{branes}} + S_{\text{int}}. \tag{2.44}$$