of the momentum p and the boundary perturbation g by

$$\cos(\beta/2) = \cos(\pi g)\cos p \tag{27}$$

Finally, we can write the partition function in terms of open string variables using the OS/CS duality identity:

$$\frac{1}{\sqrt{2}} \sum_{j=0,\frac{1}{2},1,\dots} \chi_j^{Vir}(q^2) \frac{\sin(2j+1)\beta/2}{\sin\beta/2} = \frac{w^{-\frac{1}{24}}}{f(w)} \sum_{n=-\infty}^{\infty} w^{(n+\beta/4\pi)^2} \tag{28}$$

This gives the partition function

$$Z_{BN}^{R=\infty} = \int_{-\pi}^{\pi} \frac{dp}{2\pi} \frac{w^{-\frac{1}{24}}}{f(w)} \sum_{n=-\infty}^{\infty} w^{(n+\beta(p)/4\pi)^2} \tag{29}$$

The result (29) provides the interpolation between the tight binding limit and the free string limit. Note that, for the critical boundary perturbation $g = \frac{1}{2}$, the rotation angle (27) is $\beta = \pi$, independent of the momentum p. This verifies that the TBA is exact in this case, and the spectrum remains discrete for the noncompact $R = \infty$ theory. For $g < \frac{1}{2}$ each of these states, which are infinitely degenerate in the $R = \infty$ theory, spreads into a band of width $1-2g$ around each integer. In the free string limit $g \to 0$, the gaps disappear and the bands merge to form the continuous dispersion relation of the free string. In the field theory limit, the partition function reduces to that of the lowest band, $n = 0$. For low momentum $p << \pi$ we may expand the exponent in (29),

$$\left(\frac{\beta(p)}{4\pi}\right)^2 - \frac{1}{24} = \left(\frac{g^2}{4} - \frac{1}{24}\right) + \left(\frac{g}{4\pi}\cot\pi g\right)\ p^2 + O(p^4) \tag{30}$$

Thus in the continuum limit, we recover a relativistic dispersion relation with a mass

$$M^2 = \frac{1}{4\alpha'}\left(\frac{g^2}{4} - \frac{1}{24}\right) \tag{31}$$

There is a range of subcritical values of the boundary perturbation, $\frac{1}{\sqrt{6}} < g < \frac{1}{2}$, for which the tachyon has disappeared and the lowest lying mass squared value is positive. [When the confining force of the gauge field is included, the physical states are not charged Z particles, but neutral Z pairs, so M is analogous to the constituent quark mass in QCD, with the physical mass gap being $\approx 2M$. The stringy dynamics of the gauge field is discussed in the next Section.]

The tachyon instability that we have found in the string-cutoff CP^{N-1} model can be identified as the same instability found in the large N approximation of the field theory with momentum space cutoff. A central result of the large N analysis is the spontaneous generation of a mass term for the Z-particles. This results from the fact that the equation obtained from variation of the