For a conserved current, $\partial_\beta j^\beta = 0$, we have a consistency equation

$$\begin{aligned}
\partial_\beta\left(\partial_\alpha\partial^\alpha A^\beta - \partial_\alpha\partial^\beta A^\alpha + \mu^2 A^\beta\right) &= \partial_\beta j^\beta \\
\Rightarrow \partial_\alpha\partial^\alpha\partial_\beta A^\beta - \partial_\beta\partial^\beta\partial_\alpha A^\alpha + \mu^2\partial_\beta A^\beta &= 0 \\
\Rightarrow \partial_\beta A^\beta &= 0.
\end{aligned} \tag{F.62}$$

Our EOM, Eq. (F.61), become

$$\partial^2 A^\beta + \mu^2 A^\beta = j^\beta. \tag{F.63}$$

Fourier analysis immediately leads one to

$$A^\mu(x) = \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x}\frac{-ie}{k^2-\mu^2}\left(\frac{p'^\mu}{k\cdot p' + i\epsilon} - \frac{p^\mu}{k\cdot p - i\epsilon}\right). \tag{F.64}$$

The poles in this occur at $k\cdot p' = 0$, $k\cdot p = 0$, and $k^2 - \mu^2 = 0$; the pole structure is exactly the same as for the massless case, Fig. F.1, but with the radiation poles giving a dispersion relation of

$$k^0_\pm = \pm\sqrt{\vec{k}^2 + \mu^2}. \tag{F.65}$$

Let's examine the $t < 0$ case and make sure we recover the correct Yukawa