

bottom (2) superconductors. The kinetic inductance fraction,  $\alpha = \frac{L_k}{L_T}$ , is defined as the ratio of the kinetic inductance  $L_k$  to the total inductance,  $L_T = L_k + L_m$ , where  $L_m$  is the magnetic inductance of the transmission line. Using Equation 1 and the phase velocity of a normal metal transmission line  $v_{pN} = c/\sqrt{\epsilon_r}$  we can calculate  $\alpha$  for a uniform distribution of quasiparticles in both the top and bottom microstrip wiring as:

$$\alpha = 1 - \left( \frac{v_p}{v_{pN}} \right)^2 \quad (2)$$

$$= 1 - \left( 1 + \frac{\lambda_1}{d} \coth \left( \frac{t_1}{\lambda_1} \right) + \frac{\lambda_2}{d} \coth \left( \frac{t_2}{\lambda_2} \right) \right)^{-1} \quad (3)$$

The microstrip MKID is read out by sending a microwave probe signal past the resonator, and a homodyne mixing scheme is used to recover the phase and dissipation changes imprinted onto the carrier by the MKID [1]. Using Equation 3 and Equation 4 [14], we can express the expected responsivity, normalized so the microwave transmission past the resonator  $S_{21}$  far off resonance is unity, of a microstrip MKID in both dissipation and phase as:

$$\frac{\partial S_{21}}{\partial N_{qp}} = \frac{\alpha |\gamma| \kappa Q_m^2}{V Q_c} \quad (4)$$

with

$$\begin{aligned} \kappa \approx & \frac{1}{\pi N_0} \sqrt{\frac{2}{\pi k T \Delta_0}} \sinh(\xi) K_0(\xi) \\ & + j \frac{1}{2 N_0 \Delta_0} \left[ 1 + \sqrt{\frac{2 \Delta_0}{\pi k T}} e^{-\xi} I_0(\xi) \right] \end{aligned}$$

where  $N_{qp}$  is the number of quasiparticles in the resonator,  $Q_m$  is the measured quality factor,  $Q_c$  is the coupling quality factor,  $V$  is twice the volume of the top microstrip wiring layer since this is where the current flows and where quasiparticles effectively contribute to the surface impedance,  $N_0$  is the single spin density of states,  $\Delta_0$  is the effective gap at  $T \approx 0$ ,  $\xi = \hbar\omega/2kT$ , and  $\gamma$  is constant that varies from -1/3 in the extreme anomalous limit to -1 in the thin film local limit. The predicted phase responsivity in radians per quasiparticle,  $\partial\theta/\partial N_{qp}$ , can be found by taking the imaginary part of Equation 4, while the dissipation response  $\partial D/\partial N_{qp}$  is found by taking the real part.  $Q_m$  is related to  $Q_c$  and the quality factor resulting from any source of dissipation in the system,  $Q_i$ , by the relation  $Q_m^{-1} = Q_c^{-1} + Q_i^{-1}$ . We operate the devices in this paper at  $T < T_c/8$ , so there are