where q_U is given by Eq. (38). Equation (50) yields specific heat given by

$$\tilde{C}_q = \frac{k_B DN}{2}, \tag{53}$$

which is the same as that of the BG statistics.

The Tsallis entropy is expressed by

$$\tilde{S}_q = k_B \left(\frac{\tilde{c}_q - 1}{1-q} \right), \tag{54}$$

with

$$\tilde{c}_q = (\tilde{X}_q)^{1-q} = \frac{(\tilde{Z}_q)^{1-q}}{\tilde{\nu}_q}, \tag{55}$$

$$\tilde{\nu}_q = \frac{DN}{2}\left(1 - \frac{1}{q}\right) + 1 = \nu_{1/q}, \tag{56}$$

where ν_q is given by Eq. (42).

For $|q-1| \ll 1.0$, we obtain $\tilde{S}_q$ given by

$$\tilde{S}_q = N S_1^{(1)} - (q-1)\left[\frac{N^2}{2}\left(S_1^{(1)}\right)^2 + \frac{DN}{4} \right] + \cdot\cdot, \tag{57}$$

where $S_1^{(1)}$ is given by Eq. (44). Equation (57) is the same as Eq. (46) except for the $O(DN)$ term in the bracket.

For large $N(q-1)$, $\tilde{S}_q$ is given by

$$\tilde{S}_q \simeq \frac{k_B}{q-1}\left[1 - \frac{2q\, e^{-(q-1)NS_1^{(1)}/k_B}}{N(q-1)} \right] \quad \text{for } N(q-1) \gg 1, \tag{58}$$

which is quite different from S_q in Eq. (47) obtained by the q-average.

C. Model calculations

We will present some numerical calculations of the Tsallis entropy of ideal gases with $D=1$. Equations (38) and (52) show that the conceivable q values for q- and normal averages are $0 < q < q_U$ and $q > q_L$, respectively, where $q_U = 1 + 2/DN$ and $q_L = 1 - 2/(DN+2)$ (see the inset of Fig. 1). q_L and q_U approach unity for $N \to \infty$. Although the N dependence of the Tsallis entropy in the q-average is rather different from that in the normal average, as Eqs. (47) and (58) show, both results are in fairly good agreement for $q_L < q < q_U$ where both the q- and normal averages are valid. Dashed and solid curves in Fig. 1 show