

where  $\mathbb{A}$ ,  $\mathbb{B}$  and  $\mathbb{C}$  are

$$\begin{aligned}\mathbb{A} &= ig^{N-2} \left(-\frac{\kappa}{2}\right)^2 \frac{\langle 1i \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle N1 \rangle} \\ &\times \sum_{l \in \{g^+\}} s_{N+1, N+2} s_{l, N+1} \frac{\langle 1l \rangle}{\langle 1, N+1 \rangle \langle N+1, l \rangle} \frac{\langle li \rangle}{\langle l, N+1 \rangle \langle N+1, i \rangle} \\ &\times \frac{\langle 1, N+1 \rangle}{\langle 1, N+2 \rangle \langle N+2, N+1 \rangle} \frac{\langle N+1, i \rangle}{\langle N+1, N+2 \rangle \langle N+2, i \rangle},\end{aligned}\tag{23}$$

$$\begin{aligned}\mathbb{B} &= ig^{N-2} \left(-\frac{\kappa}{2}\right)^2 \frac{\langle 1i \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle N1 \rangle} \\ &\times \sum_{l \in \{g^+\}} s_{N+2, N+1} s_{l, N+2} \frac{\langle 1l \rangle}{\langle 1, N+2 \rangle \langle N+2, l \rangle} \frac{\langle li \rangle}{\langle l, N+2 \rangle \langle N+2, i \rangle} \\ &\times \frac{\langle 1, N+2 \rangle}{\langle 1, N+1 \rangle \langle N+1, N+2 \rangle} \frac{\langle N+2, i \rangle}{\langle N+2, N+1 \rangle \langle N+1, i \rangle},\end{aligned}\tag{24}$$

$$\begin{aligned}\mathbb{C} &= ig^{N-2} \left(-\frac{\kappa}{2}\right)^2 \frac{\langle 1i \rangle^4}{\langle 12 \rangle \langle 23 \rangle \dots \langle N1 \rangle} \\ &\times \sum_{l \in \{g^+\}} s_{l, N+2} \frac{\langle l1 \rangle}{\langle N+2, 1 \rangle \langle l, N+2 \rangle} \frac{\langle li \rangle}{\langle l, N+2 \rangle \langle N+2, i \rangle} \\ &\times \sum_{k \in \{g^+\}} s_{k, N+1} \frac{\langle k1 \rangle}{\langle N+1, 1 \rangle \langle l, N+1 \rangle} \frac{\langle ki \rangle}{\langle k, N+1 \rangle \langle N+1, i \rangle}.\end{aligned}\tag{25}$$

We first look at  $\mathbb{A}$  part.  $\frac{\langle 1l \rangle}{\langle 1, N+1 \rangle \langle N+1, l \rangle}$  and  $\frac{\langle li \rangle}{\langle l, N+1 \rangle \langle N+1, i \rangle}$  can split into sums of terms as in (18) and (19). For a given  $l$ , as in  $M = 1$  case, they insert the two gluons corresponding to the graviton  $(N+1)_h$  into positions between  $1, l$  and  $l, i$  respectively. After this insertion, we consider the insertion of gluons corresponding to  $(N+2)_h$ . With the eikonal identity(A9), for  $1 < l < i$ ,  $\frac{\langle 1, N+1 \rangle}{\langle 1, N+2 \rangle \langle N+2, N+1 \rangle}$  and  $\frac{\langle N+1, i \rangle}{\langle N+1, N+2 \rangle \langle N+2, i \rangle}$  in (23) can be given as

$$\begin{aligned}\frac{\langle 1, N+1 \rangle}{\langle 1, N+2 \rangle \langle N+2, N+1 \rangle} &= \left( \frac{\langle 1, r \rangle}{\langle 1, N+2 \rangle \langle N+2, r \rangle} + \frac{\langle r, N+1 \rangle}{\langle r, N+2 \rangle \langle N+2, N+1 \rangle} \right), \\ \frac{\langle N+1, i \rangle}{\langle N+1, N+2 \rangle \langle N+2, i \rangle} &= \left( \frac{\langle t+1, i \rangle}{\langle t+1, N+2 \rangle \langle N+2, i \rangle} + \frac{\langle N+1, t+1 \rangle}{\langle N+1, N+2 \rangle \langle N+2, t+1 \rangle} \right),\end{aligned}\tag{26}$$