



Figure 1: The power envelopes $\beta_\lambda(h)$ (upper panel) and $\beta_\mu(h)$ (lower panel) for $\alpha = 0.05$, as functions of $h/\sqrt{c} = (h_1, h_2)/\sqrt{c}$.

Proposition 4 *Let Φ denote the standard normal distribution function. Then,*

$$\beta_\lambda(h) = 1 - \Phi \left[\Phi^{-1}(1 - \alpha) - \sqrt{-\frac{1}{2} \sum_{i,j=1}^r \ln \left(1 - \frac{h_i h_j}{c} \right)} \right] \quad \text{and} \quad (14)$$

$$\beta_\mu(h) = 1 - \Phi \left[\Phi^{-1}(1 - \alpha) - \sqrt{-\frac{1}{2} \sum_{i,j=1}^r \left(\ln \left(1 - \frac{h_i h_j}{c} \right) + \frac{h_i h_j}{c} \right)} \right]. \quad (15)$$

Figure 1 shows the asymptotic power envelopes $\beta_\lambda(h)$ and $\beta_\mu(h)$ as functions of h_1/\sqrt{c} and h_2/\sqrt{c} when $h = (h_1, h_2)$ is two-dimensional.

It is important to realize that the asymptotic power envelopes derived in Proposition 4 are valid not only for λ - and μ -based tests but also for any test invariant under left orthogonal transformations of the observations ($X \mapsto QX$, where Q is a $p \times p$ orthogonal matrix), and for any test invariant under multiplication by any non-zero constant and left orthogonal transformations of the observations ($X \mapsto aQX$, where $a \in \mathbb{R}_0^+$ and Q is a $p \times p$ orthogonal matrix), respectively.