evaluation of leading order contributions to the Wilson coefficients involves the calculation of multiple two-loop momentum integrals. These loop integrals are often quite difficult to evaluate and constitute a significant technical barrier to extending QSR calculations to higher orders. Chapter 2 discusses techniques for evaluating loop integrals.

1.4.4 Hadronic Spectral Function

As mentioned previously, the hadronic spectral function can be measured experimentally. For instance, the spectral function for hadronic states with $J^{PC} = 1^{--}$ is related to the ratio of the cross sections

$$R(s) = \frac{\sigma\left(e^- e^+ \to \text{ hadrons }\right)}{\sigma\left(e^- e^+ \to \mu^- \mu^+\right)} \,. \tag{1.110}$$

This spectral function is shown in Fig. 1.9.

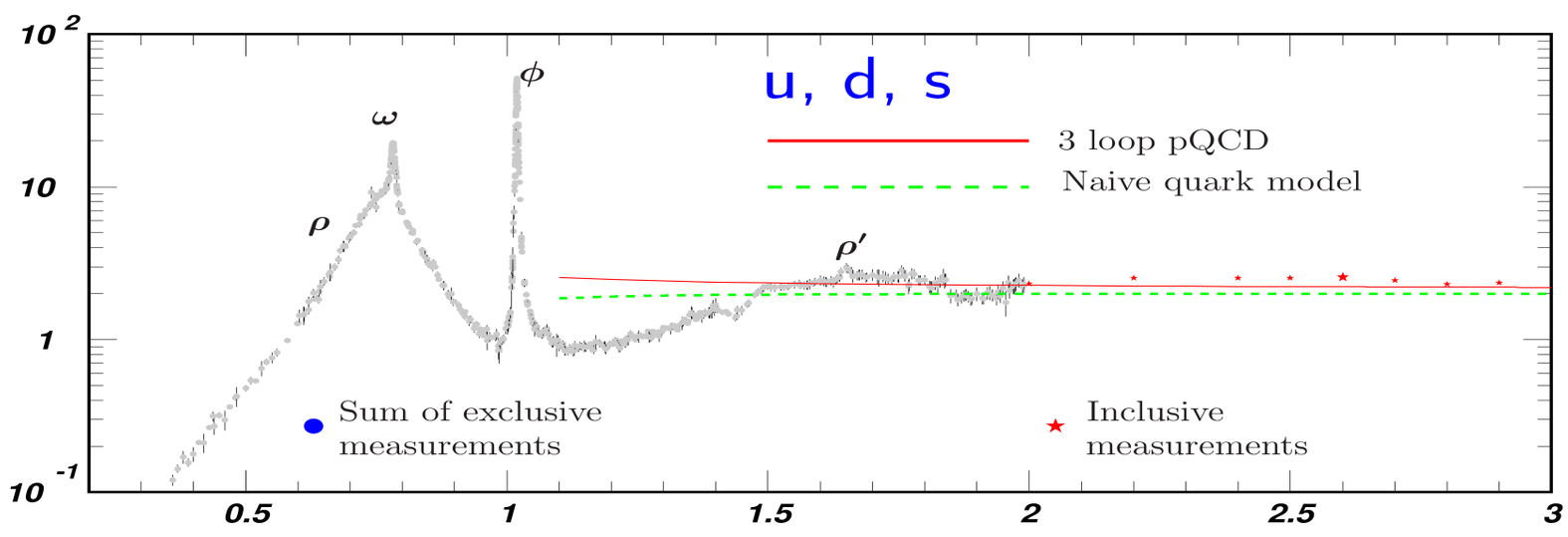

Figure 1.9: The hadronic spectral function $R(s)$. The horizontal axis is the center of mass frame collision energy of the electron and positron in units of GeV and the vertical axis is the dimensionless number $R(s)$. The resonances labeled ρ, ω and ϕ correspond to distinct hadrons. The electron and positron annihilate through a virtual photon or Z boson, both of which have the quantum numbers $J^{PC} = 1^{--}$. Therefore all of these hadrons must have these quantum numbers. The horizontal location of each resonance peak is the mass of the hadron corresponding to the resonance. The region between 1.5 GeV and 3.0 GeV is the continuum which is described well by the three-loop perturbative QCD calculation. Note that in the region below 1.5 GeV the QCD prediction and resonance features agree in the sense of a global average. This is an example of the concept of quark-hadron duality which is crucial to QSR. Figure taken from Ref. [20].