solving the Schrödinger equation. We can observe that for $n \geq 4$ and $r_{sc} > 30$ fm the second order estimate coincides with the exact results. In this example the integral relations derived from the Kohn Variational Principle have been used to extract a phase-shift in the presence of the Coulomb potential using wave functions with free asymptotic conditions.

IV. USE OF THE INTEGRAL RELATIONS IN THE THREE-BODY CASE

The integral relations derived from the KVP are general and their validity is not limited to two-body systems. The two-body system is the simplest system in which different applications can be studied and compared to the exact solution of the Schrödinger equation and, therefore, a detailed analysis of the variational character of the relations can be performed. In this section the study of the integral relations is extended to describe a $2+1$ collision in the three-body system, below the breakup threshold into three particles. The description of the breakup channel remains outside the scope of the present work. In the following we will consider the two examples already discussed in the previous section: the computation of phase-shifts using bound state like wave functions and the calculation of phase-shifts in presence of the Coulomb potential using wave functions having free asymptotic conditions. To this end we will use the s-wave MT I-III nucleon-nucleon interaction [24], active in the singlet and triplet spin states, respectively:

$$\begin{aligned} V_{MT\,I}(r) =& \frac{1438.72}{r}\mathrm{e}^{-3.11r} - \frac{513.968}{r}\mathrm{e}^{-1.55r} \\ V_{MT\,III}(r) =& \frac{1438.72}{r}\mathrm{e}^{-3.11r} - \frac{626.885}{r}\mathrm{e}^{-1.55r} \end{aligned} \tag{34}$$

with distances in fm and energies in MeV. This interaction has been used many times in the literature to study the three-nucleon system at low energies. It is considered a semi-realistic interaction since it describes reasonably well the deuteron binding energy and the singlet and triplet $n-p$ scattering lengths. Its predictions for these quantities are $E_d = -2.23069$ MeV, ${}^1a_{n-d} = -23.582$ fm, and ${}^3a_{n-d} = 5.5132$ fm. To be noticed that this potential has a strong repulsion at short distances.

To compute bound and scattering wave functions we make use of the pair hyperspherical harmonic (PHH) method which has proven to be extremely accurate [18, 25]. In the following a brief illustration of the method is given. For bound states, the three-nucleon wave function