Universality of vertex corrections to the electrical conductivity in models with elastically scattered electrons

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We study quantum coherence of elastically scattered lattice fermions. We calculate vertex corrections to the electrical conductivity of electrons scattered either on thermally equilibrated or statically distributed random impurities. We demonstrate that the sign of the vertex corrections to the Drude conductivity is in both cases negative. Quantum coherence due to elastic back-scatterings always leads to diminution of diffusion.

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I. INTRODUCTION

It is very difficult to describe full electron correlations due to a number of complex phenomena related to the quantum character of the electron. It is hence appropriate to approach the full description of electron correlations iteratively in several stages. The first one is the static mean-field approximation of the Hartree type. Such a mean-field theory completely neglects both charge and spin fluctuations and reduces the interacting system to a Fermi gas with renormalized, self-consistently determined, densities. Such a simplification may deliver reliable results only for macroscopic static quantities in the weak-coupling limit. Electron correlations in this approximation have no impact on dynamical and transport properties.

The next level in a comprehensive modeling of strong electron correlations are models and approximations allowing for charge fluctuations. In these models the spin of electrons does not play a significant role and electrons are subjected only to a potential scattering. It means that electrons are only scattered on fluctuations of the atomic potential in the lattice. The potential fluctuations are caused by impurities that may be distributed in the crystal either regularly or randomly. The paradigm for the former situation is the Falicov-Kimball model[1] (FKM) and for the latter the Anderson model of disordered electrons (DAM).[2] Unlike the static mean-field approximations, the models with a potential scattering lead to quantum dynamical effects and are applicable to the entire range of the interaction strength (variance of the potential fluctuations). The potential scattering does consequently affect spectral and transport properties of the system.

A common feature of the models with potential scatterings only is that energy is conserved during scattering events and need not be treated as a dynamical variable. Each energy, however, is renormalized in a different manner and hence the energy (frequency) is used as an external label. Conservation of energy in scattering events is a significant simplification in the description of electron correlations. It allows for an exact solution in the limit of infinite spatial dimensions (dynamical mean-field theory), where the effect of strong potential fluctuations may be studied without uncontrolled approximations.[3–6]

The two models, FKM and DAM, are standardly used for different purposes. The former one is aimed at a description of quantum fluctuations caused by electron correlations in thermally equilibrated states, while the latter one was introduced so that a response of a disordered electron gas to weak electromagnetic non-equilibrium perturbations can be estimated in a controlled way. Both the models have served well their original purposes. The Falicov-Kimball model has been successfully applied to a simplified description of correlation-induced metal-insulator[7,8] and valence-change[9] transitions in rare-earth compounds, or atoms in optical lattices.[10,11] The disordered Anderson model has been used to describe the spectral and transport properties of metallic alloys[12] and vanishing of diffusion, called Anderson localization.[13] There have been efforts to describe the combined effect of electron correlations and randomness in the disordered Falicov-Kimball model[6] or Anderson localization in FKM.[14] Only a few attempts have, however, been made in the calculation of the response of FKM to non-equilibrium perturbations beyond the mean-field approach.[15] In particular, it is little known about the electrical conductivity of FKM beyond the mean-field, Drude contribution.[16]

It is the aim of this paper to fill up this gap and to propose a systematic way how to calculate vertex corrections to the Drude (mean-field) electrical conductivity in models with elastic scatterings only, that is FKM, DAM or a disordered FKM. The method we use is an expansion around the mean-field solution obtained from the asymptotic limit to high spatial dimensions. We calculate the leading-order vertex correction in high spatial dimensions being of order $1/d^2$, while the Drude conductivity is of order $1/d$. We demonstrate that the vertex corrections have a universal behavior for all models of elastically scattered electrons and are always negative, independent of whether they are caused by an external random potential (quenched randomness) or by static, thermally equilibrated electron correlations (annealed randomness).