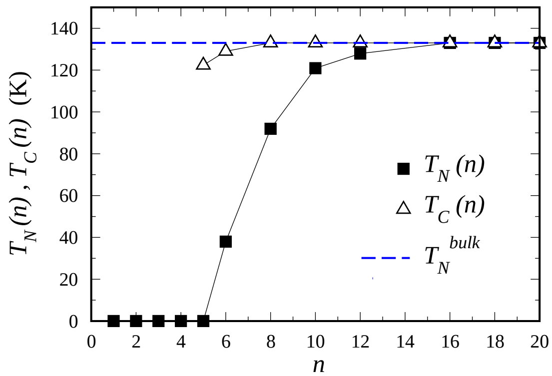

FIG. 5: Transition temperatures $T_N(n)$ and $T_C(n)$ vs. film thickness n.

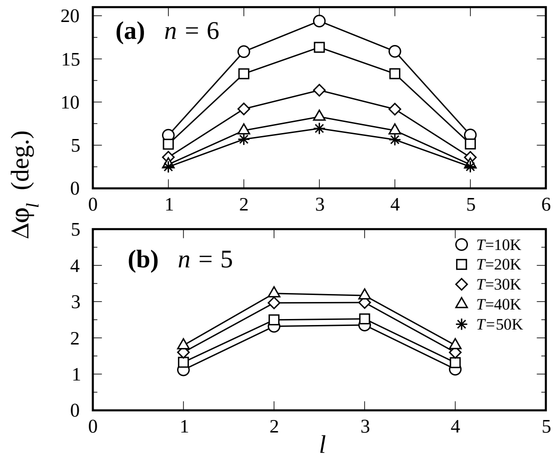

FIG. 6: Rotation angle $\Delta\varphi_l$ between magnetic moments on NN layers $(l+1, l)$ at some low temperatures, for thickness $n = 5$ and $n = 6$, and lateral dimension $L = 64$.

the same is true for the crossing point of the Binder cumulant of the average magnetization M (not reported in figure), which is located at $T_C(8) = 133.3(3)\,\mathrm{K}$. These data give a first rough indication that also for $n = 8$ all the planes of the sample are still ordering almost at the same temperature; such property has been observed for all the investigated thicknesses n below 16, so that $T_C(n)$ results quite n-independent (see also Fig. 5) .

Although the layer subtraction does not seem to modify $T_C(n)$, the onset of helical arrangement is observed to shift at lower temperatures as n decreases. The chirality κ defined in Eq. (4) is reported in Fig 4b for $n = 8$. As the temperature decreases, around $T \sim 80\,\mathrm{K}$ we can identify a finite-size behaviour of κ which, at variance with the previous one, can be easily recognized as typical of an effective phase transition. Such conclusion is confirmed by the analysis of the chiral susceptibility χ_κ (Fig. 4c), which for the largest L has a maximum at $T = 85\,\mathrm{K}$. Assuming that the order parameter (4) is the relevant one to single out the onset of the fan arrangement, we can get a more accurate estimate of $T_N(8)$ by looking at the Binder cumulant $u_4(\kappa)$, reported in Fig. 4d. By making use of the MH technique, we locate the crossing point at $T_N(8) = 92(2)\,\mathrm{K}$. Finally, it is worthwhile to observe as the specific heat does not show any anomaly at $T_N(8)$, being the entropy substantially removed at $T_C(8)$.

The scenario just outlined for $n = 8$ results to be correct in the thickness range $6 \le n \lesssim 15$, where a clear separation between $T_N(n)$ and $T_C(n)$ can be easily figured out. In such temperature window, the strong surface effects produce a *quasi*-FM set-up of the magnetic film structure along the z-direction. While leaving to the next Section a more detailed discussion of this regime, we report in Fig. 5 a plot of $T_N(n)$ and $T_C(n)$ vs. n for all the simulated thicknesses. The separation between the two critical temperatures is maximum for $n = 6$, where $T_N(6) = 38(4)$, that is $T_N(6) \sim \frac{1}{3}T_C(6)$. For films with less than six layers no fan order is observed, i.e. for $n = 5$ and below the chirality does not display any typical feature of fan ordering at any temperature below $T_C(n)$. As a representative quantity we finally look at the rotation angle of the magnetization between nearest planes:

$$\Delta\varphi_l = \varphi_{l+1} - \varphi_l = \arccos\left[M_l^x M_{l+1}^x + M_l^y M_{l+1}^y\right] \quad (10)$$

where (M_l^x, M_l^y) is the magnetic vector profile for each plane l. $\Delta\varphi_l$ is displayed in Fig. 6a and Fig. 6b, for $n = 6$ and $n = 5$, respectively. In Fig. 6a, a quite clear fan stabilization is observed when the temperature decreases, while in Fig. 6b, i.e. for $n = 5$, $\Delta\varphi_l$ keeps an almost temperature independent very small value; what's more, $\Delta\varphi_l$ seems to loose any temperature dependence as $T = 0$ is approached. We attribute the absence of fan arrangement for $n \le 5$ as simply due to the lack of "bulk planes" inside the film, so that we are left with only a 2d trend at $T_C(n)$, i.e. at the temperature where the order parameters defined in Eqs. (2) and (3) show a critical behaviour.

IV. DISCUSSION AND CONCLUSION

A possible framework to analyze the results presented in the previous Section is suggested by Fig. 5, where we can easily distinguish three significant regions: *i*) high thickness, $n \geqslant 16$, where the films substantially display a bulk behaviour, with the single planes ordering temperature coinciding with the helical phase transition one; *ii*) intermediate thickness, $6 \le n \lesssim 15$, where the temperature corresponding to the onset of in-plane order, $T_C(n)$, is still $\simeq T_N^{Ho}$, but where the helical/fan arrangement stabilizes only below a finite temperature $T_N(n) < T_C(n)$; *iii*) low thickness,$1 \le n \le 5$, where $T_C(n) \lesssim T_N^{Ho}$ but no fan phase is present at any temperature.

The observed behaviour in region *iii*) can be reasonably attributed to the decreasing relevance of the contribution to the total energy of the system coming from the competitive interactions among NNN planes as the film thickness decreases; moreover, the thinness of the