Recursion relations for generalized Fresnel coefficients: Casimir force in a planar cavity

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We emphasize and demonstrate that, besides using the usual recursion relations involving successive layers, generalized Fresnel coefficients of a multilayer can equivalently be calculated using the recursion relations involving stacks of layers, as introduced some time ago [M. S. Tomaš, Phys. Rev. A **51**, 2545 (1995)]. Moreover, since the definition of the generalized Fresnel coefficients employed does not imply properties of the stacks, these nonstandard recursion relations can be used to calculate Fresnel coefficients not only for a local but also for a general multilayer consisting of various types (local, nonlocal, inhomogeneous, etc.) of layers. Their utility is illustrated by deriving a few simple algorithms for calculating the reflectivity of a Bragg mirror and extending the formula for the Casimir force in a planar cavity to arbitrary media.

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Generalized Fresnel coefficients are basic ingredients in the theory of electromagnetic processes and effects in layered systems such as light propagation in stratified media [1, 2], molecular fluorescence and energy-transfer near interfaces [3], (dipole) radiation from multilayers [4–6], spontaneous emission and light scattering at surfaces [7] and in planar cavities[8, 9], the Casimir effect between multilayered stacks [10–12] etc. Correspondingly, the problem of calculating Fresnel coefficients arises in many area of physics: optics, surface physics, spectroscopy of multilayers, cavity QED, theory of the Casimir effect etc. For systems consisting of few layers, these coefficients are standardly calculated using (ordinary) recursion relations involving successive layers. With increasing number of layers, however, this method soon leads to cumbersome formulas and thus becomes impractical. Therefore, despite the possibility of a polynomial representation of the generalized Fresnel coefficients [13], for more complex systems these coefficients are conveniently calculated using the transfer matrix method [1, 2, 5, 12].

Based on the definition of the generalized Fresnel coefficients given in Ref. [6], in our consideration of the Green function for a (local) multilayered system [9] we have used the recursion relations for the reflection (r) and transmission (t) coefficients involving *stacks* of layers, which enabled us to write the Green function in a simple compact form. For a stack of layers between, say, layers j and m (denoted shortly as $j/m \equiv j \dots m$) these recursion relations read [9] (unless necessary, we omit the polarization index $q = p, s$)

$$r_{j/m} \equiv r_{j/k/m} = r_{j/k} + \frac{t_{j/k}t_{k/j}r_{k/m}e^{2i\beta_k d_k}}{1 - r_{k/j}r_{k/m}e^{2i\beta_k d_k}}, \qquad (1)$$

$$t_{j/m} \equiv t_{j/k/m} = \frac{t_{j/k}t_{k/m}e^{i\beta_k d_k}}{1 - r_{k/j}r_{k/m}e^{2i\beta_k d_k}}. \qquad (2)$$

where k is an intermediate layer, as depicted in Fig. 1, and where by using the notation $r_{j/k/m}$ we simply stress to which intermediate layer we address. As seen, the above recurrence relations look the same as the standard ones [1, 2] (to which they reduce in case of a system $jk/m \equiv jk \dots m$), however, this time they generally involve Fresnel coefficients for stacks of layers.

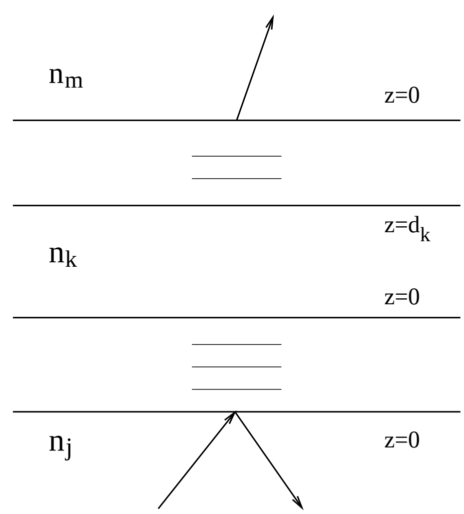

FIG. 1: Stack considered when deriving recursion relations for generalized Fresnel coefficients shown schematically. Layers j, k and m are local and described by (complex) refraction indexes $n_a(\omega) = \sqrt{\varepsilon_a(\omega)\mu_a(\omega)}$, $a = j, k, m$, whereas stacks between them are unspecified. Arrows indicate propagation of a wave incident on the stack. A shifted-z coordinate system is adopted, as explained in the text.

Although Eqs. (1) and (2) have also been known for some time in optics of multilayers [14], it seems that the possibility of grouping the layers in stacks when calculating Fresnel coefficients is not widely recognized. Since this method is particularly convenient in some cases, e.g., when calculating Fresnel coefficients of periodic media, in this work we explicitly derive the above recurrence relations starting from the definition of the generalized Fres-