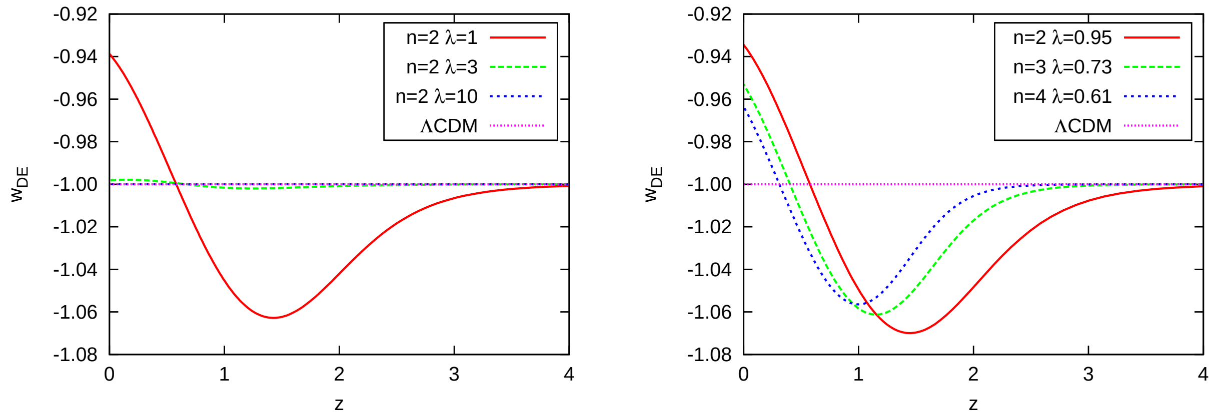

FIG. 1: Evolution of the equation-of-state parameter of effective dark energy.

III. PERTURBATIONS

We now turn to the evolution of density fluctuations. In $f(R)$ gravity, the evolution equation of density fluctuations, δ, deeply in the sub-horizon regime is given by[7]

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\rho\delta = 0, \quad G_{\rm eff} = \frac{G}{F}\frac{1 + 4\frac{k^2}{a^2}\frac{F'}{F}}{1 + 3\frac{k^2}{a^2}\frac{F'}{F}}. \tag{7}$$

In the previous paper[6] we obtained an analytic solution for the high-curvature regime when the scale factor evolves as $a(t) \propto t^{2/3}$ and F takes an asymptotic form $F \simeq 1 - 2n\lambda\left(R/R_s\right)^{-2n-1}$. In the present paper, we numerically integrate (7) up to $z = 0$ without using the approximation $|F - 1| \ll 1$.

The wavenumber of our particular interest is the scale corresponding to σ_8 normalization, for which we find $k_{\rm eff}(r = 8h^{-1}{\rm Mpc}) = 0.174h{\rm Mpc}^{-1}$. Since the standard ΛCDM model normalized by CMB data explains galaxy clustering at small scales well, $\delta_{\rm fRG}$ should not be too much larger than $\delta_{\Lambda\rm CDM}$ on these scales. We may typically require $(\delta_{\rm fRG}/\delta_{\Lambda\rm CDM})^2 \lesssim 1.1$. Although we neglect non-linear effects here, the difference between linear calculation and non-linear N-body simulation remained smaller than 5% at wavenumber $0.174h{\rm Mpc}^{-1}$[8].

The left panel of Fig. 2 represents $(\delta_{\rm fRG}/\delta_{\Lambda\rm CDM})^2$ as a function of λ for $n = 2$ together with two fitting functions. The solid line is from the analytic formula obtained in Ref. [6], and the broken line is numerical fitting using an exponential function $1 + b_n e^{-q_n\lambda}$. From these analysis, we can constrain the parameter space as the right panel of Fig. 2. The region which satisfy $(\delta_{\rm fRG}/\delta_{\Lambda\rm CDM})^2 < 1.1$ lies above the solid line. The region below the dotted line