we have that

$$t\tau(t) = \mathrm{diag}(a_1 a_{\tau'(1)}, \ldots, a_n a_{\tau'(n)}, a_{n+1} a_{\tau'(n+1)}, \ldots, a_{2n} a_{\tau'(2n)}).$$

For each $i = 1, \ldots, n$ such that $\tau'(i) = i$, this gives a_i^2 in the ith diagonal position (and a_i^{-2} in the $(2n+1-i)$th diagonal position). The other positions simply have $a_i a_{\tau'(i)}$.

Now, if this is to be equal to 1, then we have a choice of ± 1 for each position $i = 1, \ldots, n$ such that $\tau'(i) = i$. (The number in position $2n+1-i$ is then determined.) We also have a completely free choice of the other a_i, which then determine the $a_{\tau'(i)}$.

On the other hand, if $t\tau(t)$ were to equal -1, then we would have a choice of $\pm\sqrt{-1}$ for each a_i such that $\tau'(i) = i$. (Again, this choice determines the value of a_{2n+1-i}). We again have a completely free choice of the other a_i, which in turn determine the $a_{\tau'(i)}$.

Thus T_τ can be thought of like this: First, decide whether $t\tau(t)$ is going to be 1 or -1. Having decided that, choose the a_i on the fixed points i to be either ± 1 or $\pm\sqrt{-1}$ (depending on the first choice that was made), and choose arbitrary values in $\mathbb{C}^*$ for the remaining a_i. This shows that

$$T_\tau \cong \mathbb{Z}/2\mathbb{Z} \times ((\mathbb{Z}/2\mathbb{Z})^k \times (\mathbb{C}^*)^{n-k}),$$

where k represents the number of values fixed by τ (or the number of $i = 1, \ldots, n$ fixed by τ') as above.

Now, we turn to $T_0^{-\tau}$. Here, $t\tau(t)$ takes the same form, but now we insist that $t\tau(t) = 1$. Thus $T^{-\tau}$ (by the same analysis of the previous paragraph) is of the form

$$(\mathbb{Z}/2\mathbb{Z})^k \times (\mathbb{C}^*)^{n-k}.$$

However, in considering only the identity component, we get rid of the factors of $\mathbb{Z}/2\mathbb{Z}$. The