$$
\begin{aligned}
G^4\langle\chi_1^4\rangle = \langle C_1^4\rangle &- 12G^2\langle V^2\rangle\langle C_1C_2\rangle - 12G^2\langle V^2\rangle\langle C_1^2\rangle \\
&+ 6\langle C_1C_2\rangle\langle C_1^2\rangle + 5\langle C_1C_2\rangle^2 + 12G^4\langle V^2\rangle^2 \\
&- 4\langle C_1\rangle\langle C_1^3\rangle - 4\langle C_1\rangle\langle C_1^2C_2\rangle + 8\langle C_1\rangle^2\langle C_1^2\rangle \\
&+ 8\langle C_1\rangle^2\langle C_1C_2\rangle - \langle C_1^2C_2^2\rangle + \langle C_1^2\rangle\langle C_2^2\rangle
\end{aligned}
$$

Similar formulas can be derived for χ_2. Furthermore, we assume the first moment of the detector noise to vanish for both chains throughout the experimental part of this work. Practically, this implies an offset correction (c.f. appendix). Finally, from Eqs. (1)–(3), the central moments can be retrieved with the binomial transformation

$$\langle(S-\langle S\rangle)^n\rangle = \sum_{k=0}^{n}\binom{n}{k}(-1)^{n-k}\langle S^k\rangle\langle S\rangle^{n-k}\,. \tag{4}$$

We emphasize the practical relevance of the above theory by conducting proof-of-principle experiments with weak classical microwaves. The setup is shown in Fig. 1. As signals, we use pulsed coherent microwaves with a frequency of 5.85 GHz generated by a source at room temperature. A series of cold attenuators ensures that the thermal noise at the signal port of the hybrid ring is restricted to that of an effective 50 Ω-termination at the base temperature of 300 mK. The source power at the input of the hybrid ring, P_{in}, is related to an equivalent number of signal photons on average (poa) as described in the appendix. Figure 2a shows the ensemble average of a typical signal used in our experiments. The pulse duration of 1 µs mimics standard cavity decay times in circuit QED experiments [10, 22]. We first demonstrate the suppression of the amplifier noise via cross-correlations. The auto-variance $\langle C_1^2\rangle - \langle C_1\rangle^2$ of the ensemble is depicted in Fig. 2b, where one immediately notices the large offset of $35.7\times10^{-3}\,\mathrm{V}^2$ due to the amplifier noise. In the cross-variance $\langle C_1C_2\rangle - \langle C_1\rangle\langle C_2\rangle$, this offset is efficiently suppressed by two orders of magnitude. As expected for a coherent signal, the variances are flat and do not allow to distinguish between the "on"- and "off"-regions of the pulses. The fluctuations of the variance signals are smaller for the cross-correlation than for the auto-correlation by a factor of 1.6, see Fig. 2c. Next, we prove that our method works efficiently at the quantum level, i.e., for signals of few photons on average. To this end, we investigate the resolution limits of the constituents of the variance, mean value and cross-product. In Fig. 2d, the root mean square power inside the pulse region is plotted against the signal power at the input of the hybrid ring. We find a large dynamic range of the mean value extending over six decades down to 0.001 poa. This means that