ranging, in analogy with laser ranging. We send a pulse of light with wavelength λ from a position labeled A to one labeled B, where it is reflected back to A, and measure the travel time with a macroscopic clock visible from A. Since light is a wave we cannot ask that the pulse front can be much more accurately determinable than about λ, so there is an uncertainty of at least about $\Delta l_w \approx \lambda$ in our measurement of the length l.

If nature were actually classical we could use light of arbitrarily short wavelength and arbitrarily low energy so as not to disturb the system, and thereby measure the distance to arbitrarily high accuracy. However in the real world we cannot use arbitrarily short λ since this would put large amounts of energy and effective mass in the measurement region, even if we use only a single photon.[3] According to our comments in section II the energy of the light will distort the spacetime geometry and thus change the length l by a fractional amount $|\phi|/c^2$ according to Eq.(13). We may estimate the Newtonian potential due to the photon, which is somewhere in the interval l, to be about

$$\phi \approx \frac{GM_{ef}}{l} \approx \frac{G(E/c^2)}{l} \approx \frac{Gh\nu}{c^2 l} \approx \frac{G\hbar}{cl\lambda} \tag{18}$$

so the spatial distortion is about

$$\Delta l_{\mathrm{g}} \approx l(\phi/c^2) \approx (G\hbar/c^3)/\lambda = l_p^2/\lambda \tag{19}$$

In Fig.3 the letter B labels a point in space, but we could instead take it to be a small body in free fall, which would move during the measurement (actually only during the return trip of the light pulse), and this would also affect the measurement. We have already estimated just such motion in section III; it is given in Eq.(15) by

$$\Delta x_{\mathrm{g}} \approx l_p^2/\lambda \tag{20}$$

That is the space distortion in Eq.(19) and the motion in Eq.(20) are comparable, and to our desired accuracy we simply write for either effect $\Delta l_{\mathrm{g}} \approx l_P^2/\lambda$.

Since we only know the photon position to be somewhere in l we interpret this as an additional uncertainty due to gravity. We add it to the uncertainty due to the wave nature of the light to obtain

$$\Delta l \approx \Delta l_w + \Delta l_g \approx \lambda + l_p^2/\lambda \tag{21}$$

This expression Eq.(21) for the total uncertainty has a minimum at $\lambda = l_P$, where it is equal to $\Delta l \approx 2l_P$, so we conclude that the best we can do in measuring a distance using