

# Self-organized chaos through polyhomeostatic optimization

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The goal of polyhomeostatic control is to achieve a certain target distribution of behaviors, in contrast to homeostatic regulation which aims at stabilizing a steady-state dynamical state. We consider polyhomeostasis for individual and networks of firing-rate neurons, adapting to achieve target distributions of firing rates maximizing information entropy. We show that any finite polyhomeostatic adaption rate destroys all attractors in Hopfield-like network setups, leading to intermittently bursting behavior and self-organized chaos. The importance of polyhomeostasis to adapting behavior in general is discussed.

*Introduction.*— Homeostatic regulation plays a central role in all living, as well as in many technical applications. Biological parameters, like the blood sugar level, the heart beating frequency or the average firing rates of neurons need to be maintained within certain ranges in order to guarantee survival. The same holds in the technical regime for the rotation speed of engines and the velocity of airplanes, to give a few examples.

Homeostatic control in the brain goes beyond the regulation of scalar variables like the concentration of proteins and ions, involving the functional stability of neural activity both on the individual as well as on a network level [1–3]. We use here the term ‘polyhomeostasis’ for self-regulating processes aimed at stabilizing a certain target distribution of dynamical behaviors. Polyhomeostasis is an important concept used hitherto mostly implicitly and not yet well studied from the viewpoint of dynamical system theory. Polyhomeostasis is present whenever the goal of the autonomous control is the stabilization of a non-trivial distribution of dynamical states, polyhomeostatic control hence generalizes the concept of homeostasis. The behavior of animals on intermediate time scales, to give an example, may be regarded as polyhomeostatic, aiming at optimizing a distribution of qualitatively different rewards, like food, water and protection; animals are not just trying to maximize a single scalar reward quantity. A concept loosely related to polyhomeostasis is homeokinesis, proposed in the context of closed-loop motion learning [4], having the aim to stabilize non-trivial but steady-state movements of animals and robots.

Here we study generic properties of dynamical systems governed by polyhomeostatic self-regulation using a previously proposed model [5, 6] for regulating the firing-rate distribution of individual neurons based on information-theoretical principles. We show that polyhomeostatic regulation, aiming at stabilizing a specific target distribution of neural activities gives rise to non-trivial dynamical states when recurrent interactions are introduced. We find, in particular, that the introduction of polyhomeostatic control to attractor networks leads to a destruction of all attractors resulting for large networks, as a function of the average firing rate, in either intermittent bursting

behavior or self-organized chaos, with both states being globally attracting in their respective phase spaces.

*Firing-rate distributions.*— We consider a discrete-time, rate encoding artificial neuron with input  $x \in [-\infty, \infty]$ , output  $y \in [0, 1]$  and a transfer function  $g(z)$ ,

$$y(t+1) = g(a(t)x(t)+b(t)), \quad g(z) = \frac{1}{e^{-z} + 1}. \quad (1)$$

The gain  $a(t)$  and the threshold  $-b(t)/a(t)$  in (1) are slow variables, their time evolution being determined by polyhomeostatic considerations.

Information is encoded in the brain through the firing states of neurons and it is therefore plausible to postulate [5], that polyhomeostatic adaption for the internal parameters  $a(t)$  and  $b(t)$  leads to a distribution  $p(y)$  for the firing rate striving to encode as much information as possible given the functional form (1) of the transfer function  $g(z)$ . The normalized exponential distribution

$$p_\lambda(y) = \frac{\lambda e^{-\lambda y}}{1 - e^{-\lambda}}, \quad \mu = \frac{1}{\lambda} \frac{e^\lambda - 1 - \lambda}{e^\lambda - 1}, \quad (2)$$

maximizes the Shannon entropy [7], viz the information content, on the interval  $y \in [0, 1]$ , for a given expectation value  $\mu$ . A measure for the closeness of the two probability distributions  $p(y)$  and  $p_\lambda(y)$  is given by the Kullback-Leibler divergence [7]

$$D_\lambda(a, b) = \int p(y) \log \left( \frac{p(y)}{p_\lambda(y)} \right) dy, \quad (3)$$

which is, through (1), a function of the internal parameters  $a$  and  $b$ . The Kullback-Leibler divergence is strictly positive and vanishes only when the two distributions are identical. By minimizing  $D_\lambda(a, b)$  with respect to  $a$  and  $b$  one obtains [6] the stochastic gradient rules

$$\begin{aligned} \Delta a &= \epsilon_a \left( 1/a + x \tilde{\Delta} b \right) \\ \Delta b &= \epsilon_b \tilde{\Delta} b, \quad \tilde{\Delta} b = 1 - (2 + \lambda)y + \lambda y^2 \end{aligned} \quad (4)$$

which have been called ‘intrinsic plasticities’ [1]. The respective learning rates  $\epsilon_a$  and  $\epsilon_b$  are assumed to be small, viz the time evolution of the internal parameters  $a$  and  $b$  is slow compared to the evolution of both  $x$  and  $y$ . For