The differential elastic cross-section is given by

$$d\sigma = \frac{1}{|\mathbf{v}_1 - \mathbf{v}_2|}\frac{1}{2E_1}\frac{1}{2p}\left|-ig^2\right|^2 \frac{d^3\mathbf{p}'}{(2\pi)^3 2E_3}\frac{d^3(-\mathbf{p}')}{(2\pi)^3 2p'}(2\pi)^4\delta^4(p_1+p_2-p_3-p_4) \tag{B.5}$$

where $\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocities of the ϕ and χ particles before the collision respectively, where $\mathbf{v}_2 = c$. As we are setting $c = 1$, the first term in Eq. (B.5) ≈ 1. Collecting terms together gives

$$d\sigma \approx -\frac{g^4}{64\pi^2 E_1 E_3 p p'}\delta^4(p_1+p_2-p_3-p_4)d^3\mathbf{p}'\,d^3(-\mathbf{p}') \tag{B.6}$$

Integrating this gives

$$\sigma \approx -\frac{g^4}{64\pi^2 E_1 E_3}\iint \frac{1}{pp'}\delta^4(p_1+p_2-p_3-p_4)d^3\mathbf{p}'\,d^3(-\mathbf{p}') \tag{B.7}$$

$$\approx -\frac{g^4}{64\pi^2 E_1 E_3}\int \frac{1}{pp'}\delta(E_1+p-E_3-p')d^3\mathbf{p}' \tag{B.8}$$

$$\approx -\frac{g^4}{64\pi^2 E_1 E_3}\int \frac{1}{p}\delta(E_1+p-E_3-p')p'dp'\int_\Omega d\Omega \tag{B.9}$$

From the 4-momenta of p_1 and p_3, Eqs. (B.1) and (B.3) respectively, we have

$$E_1 = \sqrt{p^2 + m_{\phi,\text{eff}}^2} \tag{B.10}$$

and

$$E_3 = \sqrt{p'^2 + m_{\phi,\text{eff}}^2} \tag{B.11}$$