B. Positive region

We consider the same substitution of scattering states for the variable x in the positive region. So we use the variable $1/z = 1 + e^{a(x-L)}$, and the transformation $\phi = z^{-\tau}(1-z)^{-\eta}g(z)$

$$
\begin{aligned}
& z(1-z)\frac{d^2g(z)}{dz^2} + [1-2\tau - z(1-2\eta-2\tau)]\frac{d\,g(z)}{dz} + \\
& + \frac{1}{a^2 z(1-z)}[(E+Wz)^2 - m_0^2(1-z)^2 + iaWz(1-z) + iaz(E+Wz)]g(z) \\
& + \frac{1}{z(1-z)}[\tau(\tau+1)(1-z)^2 + \eta(\eta+1)z^2 - \tau(1-z) + \eta z - 2\eta z^2 - \tau z(1-z) + \eta z^2]g(z) \\
& + (2\tau - 2\tau\eta)g(z) = 0
\end{aligned} \tag{49}
$$

Following a line of thought similar to that outlined in the previous subsection ($x < 0$) we obtain $\tau^2 = -(E^2 - m_0^2)/a^2 = \sigma^2 = \nu^2$ and $\eta = -i(E+W)/a = \epsilon$ so that Eq. 49 reduces to:

$$
z(1-z)\frac{d^2 f(z)}{dz^2} + [1 - 2\nu - (1-2\epsilon-2\nu)\,z]\frac{d\,f(z)}{dz^2} - (-\epsilon-\nu+\lambda)(-\epsilon-\nu-\lambda)f(z) = 0. \tag{50}
$$

C. Bound state wave function and match at $x = 0$

We note that the wave function in the $x > 0$ region can be obtained from that of the $x < 0$ region simply letting $\nu \to -\nu$ and $\epsilon \to -\epsilon$. The general solutions to Eqs. (48,50) are:

$$
\begin{aligned}
h(y) &= A'\ {}_2F_1(\epsilon+\nu+\lambda, \epsilon+\nu-\lambda; 1+2\nu; y) + B'\ y^{-2\nu}\ {}_2F_1(\epsilon-\nu+\lambda, \epsilon-\nu-\lambda; 1-2\nu; y)\,, \\
g(z) &= C'\ {}_2F_1(-\epsilon-\nu+\lambda, -\epsilon-\nu-\lambda; 1-2\nu; z) + D'\ z^{2\nu}\ {}_2F_1(-\epsilon-\nu+\lambda, -\epsilon-\nu-\lambda; 1+2\nu; z)\,.
\end{aligned}
$$

Recall the parametric transformation for $\phi_{L,R}$: $\phi_R = z^{-\nu}(1-z)^{-\epsilon}g(z)$ and $\phi_L = y^{\nu}(1-y)^{\epsilon}h(y)$ and that in the limit of $x \to \pm\infty$ the variable $y \to 0$ as well as $z \to 0$. Therefore imposing the boundary condition of a bound state (vanishing wave function at infinity) we obtain $B' = C' = 0$ and we are left with:

$$
\begin{aligned}
\phi_L(y) &= A'\ y^{\nu}(1-y)^{\epsilon}\ {}_2F_1(\epsilon+\nu+\lambda, \epsilon+\nu-\lambda, 1+2\nu; y) \\
\phi_R(z) &= D'\ z^{\nu}(1-z)^{-\epsilon}\ {}_2F_1(-\epsilon+\nu+\lambda, -\epsilon+\nu-\lambda, 1+2\nu; z)
\end{aligned}
$$

With the help of the continuation formula of the Hypergeometric function [26] we can extract the behavior of the solution in the vicinity of $x = 0$ (recall that for $x \to 0$, $y, z \to 1$ and $1-y \approx e^{-a(x-L)}$