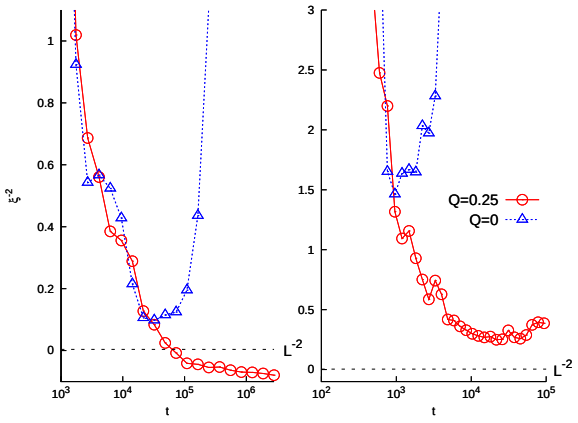

FIG. 3: Left: $A/B = \xi^{-2}(t)$ (see Eq. 2) at $T = T_{MC}$ in the constrained (circles) and unconstrained (triangle) cases. Right: the same at $T = 1.55T_{MC}$. $L = 16$.

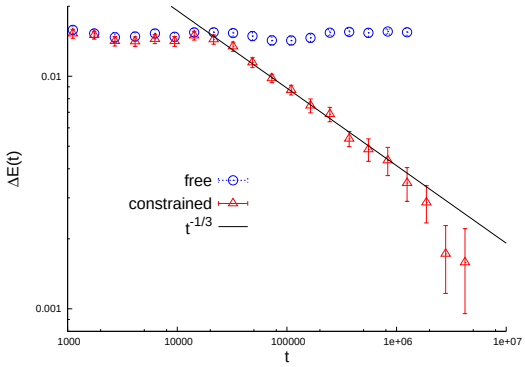

FIG. 4: Energy difference $\Delta E(t) = E(t) - E_0$ vs t at $T = T_{MC}$ with constrained dynamics, $\hat{Q} = 0.25$. E_0 is a parameter of the fit $E(t) = E_0 + \gamma t^{-1/3}$. The line is $1/t^{1/3}$, corresponding to the surface tension exponent $\theta = 2$.

unconstrained case ξ^{-2} keeps well clear of L^{-2}, while in the constrained case $\xi^{-2}(t)$ unmistakably goes below L^{-2}. This is exactly what we expect in a system with nonzero surface tension undergoing phase separation. We studied two other sizes, $L = 8$ and $L = 25$, and in both cases $\xi^{-2}(t)$ drops below L^{-2}, indicating phase separation. At higher temperatures, however, though the correlation is enhanced by the constraint, the latter is ineffective to make $\xi^{-2}(t)$ drop below L^{-2} (Fig. 3, right). These results are consistent with the idea that the surface tension decays at high temperature, thus preventing phase separation [16].

In systems with conserved order parameter undergoing phase separation the domains size $\xi(t)$ grows as $t^{1/3}$ and the dynamics proceeds by reducing the total amount of interfaces, and therefore of energy, in the system [15]. The interface energy per domain scales like ξ^{θ}, where θ is the surface tension exponent. The total number of domains is L^d/ξ^d, so that the total interface energy density is $\Delta E(t) \sim 1/\xi(t)^{d-\theta} \sim 1/t^{(d-\theta)/3}$. In the standard case $\theta = d - 1$, so that $\Delta E(t) \sim 1/t^{1/3}$ [15]. Fig.4 shows that something remarkably similar happens in our case. After the constraint kicks in, $\Delta E(t)$ decays compatibly with an exponent 1/3. Hence, even though fitting coarsening exponents is notoriously difficult, and one must be careful in drawing any conclusion, our data seems to be compatible with the 'naive' exponent $\theta = 2$ [16, 17].

In general, phase-separation is the landmark of first order phase transitions and metastability. At the mean-field level one can normally define a thermodynamic potential as a function of the order parameter that, below some spinodal point, exhibits a stable and a metastable minimum, corresponding to the two phases. In finite dimension Maxwell's construction makes the potential convex, so that the derivative of the potential is constant (zero second derivative) in a finite interval (Fig. 5, inset). Maxwell's construction implies that when the order parameter is conserved and constrained to take a value in the non-convex interval, phase separation occurs. We have clearly observed phase-separation. Can we define a thermodynamic potential displaying Maxwell's construction?

Let us proceed minimalistically. Our phase-separating order parameter is the overlap Q, so it is a potential $W(Q)$ we are after. Besides, the potential must determine the observed probability distribution of Q through the relation, $P(Q) = \exp\left[-NW(Q)\right] \theta(Q - \hat{Q})$, where the θ-function enforces the constraint [18]. If we compute the average linear fluctuation of Q and expand the exponential, we obtain,

$$W'(\hat{Q}) \sim N^{-1} \langle Q - \hat{Q} \rangle^{-1}. \qquad (3)$$

This quantity is easy to compute: we let the system evolve until the constraint is hit, and then we measure the (very small) average fluctuation of the overlap Q over $\hat{Q}$ (see [19] for a different definition of the potential). We report $W'(\hat{Q})$ in Fig.(5). The second derivative of the potential is clearly nonzero at high T, whereas around the Mode Coupling temperature a finite region with $W''(\hat{Q}) \sim 0$ develops. This is evidence of Maxwell's construction and it supports the link between phase separation and metastability in our system.

$W(Q)$ is a finite-dimensional variant of the two-replica potential originally introduced in mean-field spin-glasses [20], and later generalized to structural glasses [21]. This potential is the free energy cost to keep a configuration (the running one in the present work) at fixed overlap Q with a generic equilibrium configuration (the initial reference one). Below a dynamic transition (roughly, the Mode Coupling temperature), the mean-field potential develops a metastable minimum at a finite value of Q. In this framework relaxation at low temperatures can be interpreted as a barrier crossing process, bringing the system from the metastable minimum (short times, finite Q) to the stable minimum (long times, zero Q) [22]. The