by exact full diagonalization, are not representative for the thermodynamic limit, see also Refs. 16 and 17. Note that for the 1D case, the ED data for $N = 20$ are in good agreement with RGM data for $N \to \infty$.[28] Consistent to that observation, in two dimensions the system of $N = 400$ sites with the same linear extension is already close to the thermodynamic limit, see Fig. 5.

Next we use the RGM for $N \to \infty$ to investigate the critical behavior of χ and ξ for $T \to 0$ in more detail. We start with a brief discussion of the low-temperature behavior of the unfrustrated ferromagnet. Contrary to the 1D case, where for $J_2 = 0$ exact Bethe results are available,[47] we have only approximate results for the 2D model. From low-temperature expansions of the susceptibility and the correlation length for the model with $J_2 = 0$ using renormalization group approaches,[30,31] the modified spin-wave threory,[29] and the RGM[37] it is known that for $T \to 0$ the susceptibility behaves as $\chi \propto T^s \exp(b/T)$ and the correlation length as $\xi \propto T^\sigma \exp(\beta/T)$. While different leading exponents s and σ for the (less important) preexponential factor were obtained by different methods, the exponential divergence is obtained by all authors.[29–31,37] However, different values for the coefficients b and β were reported, namely, $b(J_2 = 0) = \pi/2$ using RGM,[37] $b(J_2 = 0) = \pi$ using modified spin-wave theory[29] or renormalization group approach,[30] and $b(J_2 = 0) = 0.1327\pi$ using a different version of the renormalization-group approach.[31] In all these papers[29–31,37] it was found that the coefficients in the exponents fulfill the relation $\beta = b/2$. We mention further, that early numerical studies based on quantum Monte-Carlo calculations (using, however, χ and ξ data only for quite large temperatures) give $b(J_2 = 0) \approx 4.5 \approx 1.43\pi$ (Ref. 33) and $\beta(J_2 = 0) = 0.254\pi$ (Ref. 32). Let us finally argue, that the results for the coefficient b obtained by modified spin-wave theory[29] as well as renormalization group approach[30] and by the RGM[37] seem to be most reliable, since these methods are well-tested. Moreover, for the 1D spin-1/2 Heisenberg ferromagnet it was shown that the RGM[28,37] as well as the modified spin-wave theory[29] reproduce the exact Bethe-ansatz results for the low-temperature behavior of χ and ξ. Hence, it is to some extent surprising, that there is a difference by a factor of 2 in the coefficient b between the value obtained by modified spin-wave theory[29] (as well as renormalization group approach[30]) and the RGM for the 2D unfrustrated ferromagnet. This discrepancy is a known but unresolved problem.[35,37] However, we believe that our general conclusions concerning the exponential divergence of the frustrated model, see the discussion below, are not affected by this problem.

Next we use the RGM results to determine the coefficients b and β as functions of J_2. We assume that the general low-T behavior of these quantities is preserved for $0 < J_2 < J_2^c$, cf. Ref. 28, and fit our numerical RGM data for χ and ξ at low temperatures using the ansatzes $\chi = \left(a_0 T^{-1} + a_1 + a_2 T\right) \exp\left(\frac{b}{T}\right)$ and $\xi = \sqrt{(\alpha_0 T^{-1} + \alpha_1 + \alpha_2 T)} \exp\left(\frac{\beta}{T}\right)$. Note that the leading power T^{-1} in the preexponential functions was derived for the unfrustated case with the RGM in Ref. 37. We consider b, a_0, a_1, and a_2 as well as β, α_0, α_1, and α_2 as independent fit parameters. Recall that we can calculate RGM data only for temperatures down to a certain T_0, where T_0 (e.g., $T_0 = 0.161$ for $J_2 = 0$ and $T_0 = 0.052$ for $J_2 = 0.4$) is the lowest temperature where the system of RGM equations converges (see Sec. II). Thus, in addition to the leading power T^{-1} in the preexponential function it is reasonable to consider higher order terms to achieve an optimal fit of the RGM data. For the fit we use 500 equidistant data points in the interval $T_0 \ldots T_0 + T_{cut}$, where T_{cut} is set to 0.05. The fit of the numerical RGM data reproduces the analytical results of Ref. 37 for $J_2 = 0$, i.e., $b(J_2 = 0) = 2\beta(J_2 = 0) = \pi/2$, with a precision of four digits.

After having tested our fitting procedure by comparison with the analytical predictions for $J_2 = 0$, we now consider the frustrated model, where (to the best of our knowledge) no other results are available. From our numerical data for χ and ξ we determine the J_2 dependence of the coefficients b and β. We find that the numerical data for b and β obtained by the fitting procedure described above are very well described by a linear decrease in both parameters with increasing frustration,

$$b = 2\beta = -\frac{\pi}{2}\left(J_1 + 2J_2\right). \tag{11}$$

Obviously, both parameters would be zero at the classical transition point $J_2 = 0.5$, but they are still finite at the transition point $J_2^c \approx 0.44$ of the quantum model. Hence, the exponential divergence is present in the full parameter range $J_2 \leq J_2^c$ where the ground state is ferromagnetic. We emphasize that this result is contrary to the behavior observed for the 1D frustrated spin-1/2 ferromagnet, where the critical properties change at the zero-temperature transition point.[28] We mention further that the leading coefficients a_0 and α_0 of the preexponential functions, see the expressions for χ and ξ given above, vanish at the transition point J_2^c, whereas the next coefficients a_1 and α_1 remain finite at J_2^c. Hence, the preexponential temperature dependence is changed approaching the zero-temperature transition.

Let us mention, that the linear decrease in the coefficients b and β, see Eq. (11), found by fitting the low-temperature behavior of χ and ξ is the same as that obtained analytically for the zero-temperature spin-stiffness ρ_s, see Sec. II. This relation between ρ_s and the divergence of the correlation length and the susceptibility is in accordance with general arguments[48,49] concerning the low-temperature physics of low-dimensional Heisenberg ferromagnets.

Another interesting quantity is the specific heat C_V shown in Fig. 6. For $J_2 = 0$ the specific heat exhibits a typical broad maximum at about $T = 0.562$. Increasing the frustration the height of this maximum becomes smaller and it is shifted to lower temperatures.