to $r = 1.04$. This highest possible rotation frequency seems to coincide with an instability which follows from a Thomas-Fermi solution of the Gross-Pitaevskii equation [9].

As long as we can ignore the presence of two-particle interactions and approximately describe the system with the ideal Bose gas, its many-particle properties in the grand-canonical ensemble are exclusively derivable from one-particle states. When considering the thermodynamic limit, usually the semiclassical approximation is applied, where the one-particle ground state E_0 is retained and treated quantum mechanically, while all one-particles states above E_0 are approximately treated as a continuum [10]. This semiclassical approximation remains reasonable good irrespective of the rotation frequency Ω once the total particle number N is large enough and the trap anharmonicity k small enough. The latter condition implies that the underlying one-particle potential (1) has a small curvature around its minimum. However, in this context the question arises for which system parameters such a semiclassical approximation is not sufficient for a precise description of BEC phenomena, as well as when it finally breaks down, requiring a full quantum mechanical treatment of the system.

In order to analyze this fundamental problem more quantitatively, it is mandatory to determine the one-particle energy eigenvalues and eigenfunctions fully quantum mechanically. To this end we apply a recently developed ultra-fast converging path-integral approach for calculating the imaginary-time evolution amplitude of general non-relativistic many-body systems [11, 12, 13, 14, 15]. In this approach a hierarchy of discretized effective actions is introduced where higher terms, compared to the naive action, substantially reduce errors in calculations of transition amplitudes. In particular, this allows a systematic derivation of discretized effective actions of level p, which lead to a $1/N^p$-convergence of discretized time-sliced transition amplitudes within the continuum limit $N \to \infty$ of infinitely-many time slices. In addition, the improved convergence of transition amplitudes, calculated with the level p effective action, can be used to construct higher-order analytic approximations for short-time propagators. Furthermore, the $N = 1$ time-slice approximation turns out to be valid for short imaginary times and useful for small values of the inverse temperature in applications in quantum statistical physics. The recent Refs. [16, 17] demonstrate that this path-integral approach allows to determine a huge number of both eigenvalues and eigenfunctions of quantum systems with very high accuracy using exact numerical diagonalization.

In this Letter we show how this path-integral approach is applied for studying both global and local properties of fast-rotating Bose-Einstein condensates. To this end we proceed as follows: Sec. 2 briefly reviews the main ingredients of the path-integral approach in the general context of ideal Bose-Einstein condensates. Then we calculate in Sec. 3 a large number of energy eigenvalues and eigenfunctions for the anharmonic one-particle potential (1). Afterwards, Sec. 4 discusses how a finite number of numerically available energy eigenvalues affects the results and how they can be improved by introducing systematic semiclassical corrections. On the basis of this precise numerical one-particle information, Sec. 5 studies global properties of a rotating condensate, for instance the condensation temperature T_c as a function of rotation frequency Ω and the ground-state occupancy N_0/N as a function of temperature T. Finally, Sec. 6 is devoted to the calculation of local properties of the condensate, such as density profiles and time-of-flight absorption pictures. Sec. 7 briefly summarizes the main results presented in this Letter.

2. Many- Versus One-Particle Physics

At first we demonstrate that a precise numerical access to one-particle eigenstates allows the calculation of the condensation temperature and other thermodynamic properties of an ideal Bose gas. Afterwards, we briefly review details of the path-integral effective action approach for a numerical study of a ^{87}Rb BEC in the anharmonic trap (1).

2.1. Ideal Bose Gas

For high temperatures, the grand-canonical partition function of an ideal Bose gas is given by

$$\mathcal{Z} = \sum_{\nu} e^{-\beta(E_\nu - \mu N_\nu)}\,, \tag{2}$$

where ν enumerates all possible configurations of the system, $\beta = 1/k_B T$ represents the inverse temperature, and μ denotes the chemical potential. As the ideal bosons do not interact, the system energy E_ν can be expressed in terms of single-particle energy eigenvalues

$$E_\nu = \sum_{n} N_{\nu(n)}\, E_n\,, \tag{3}$$