Define $v : [s_1, s_2] \to \mathbb{R}^2$ by

$$v(s) = \begin{cases} \lim_{t\downarrow s_1} \gamma'(s), & s = s_1, \\ \lim_{t\uparrow s} \gamma'(s), & s \in (s_1, s_2]. \end{cases}$$

Note that $v(s)$ is in the tangent cone of $\partial\Omega$ at $\gamma(s)$ so that $\partial\Omega \cap D(\gamma(s), r)$ is graph-like using the orientation given by $v(s)$.

Define $\phi(s) : [s_1, s_2] \to \mathbb{R}$ by $\phi(s) = \langle v(s), p - \gamma(s)\rangle$. Note that from $\gamma(s_1)$ both $v(s_1)$ and $p - \gamma(s_1)$ are directions pointing into the circle so $\phi(s_1) > 0$. Similarly, $v(s_2)$ points out and $p - \gamma(s_2)$ points in so that $\phi(s_2) < 0$.

Observe that v (and therefore ϕ) is piecewise continuous since γ is piecewise C^1. By a piecewise continuous analogue of the intermediate value theorem, there exists $\bar{s} \in [s_1, s_2]$ such that

$$\lim_{t\to\bar{s}^-} \phi(t) \leq 0 \leq \lim_{t\to\bar{s}^+} \phi(t).$$

By continuity of the inner product and γ, we have

$$\lim_{t\to\bar{s}^-} \phi(t) = \langle \lim_{t\to\bar{s}^-} \gamma'(t), p - \gamma(\bar{s})\rangle.$$

Similarly, $\lim_{t\to\bar{s}^+} \phi(t) = \langle \lim_{t\to\bar{s}^+} \gamma'(t), p - \gamma(\bar{s})\rangle$

If γ is differentiable at $\bar{s}$, then $\phi(\bar{s}) = \lim_{t\to\bar{s}} \phi(t) = 0$ and we have our contradiction. Otherwise, let $w_1 = \lim_{t\to\bar{s}^-} \gamma'(t)$ and $w_2 = \lim_{t\to\bar{s}^+} \gamma'(t)$. As both w_1 and w_2 are in the convex tangent cone of $\partial\Omega$ at $\gamma(\bar{s})$, any positive linear combination of them is as well. Letting $\psi(\lambda) = \lambda w_1 + (1 - \lambda)w_2$, we have

$$\langle \psi(0), p - \gamma(\bar{s})\rangle \leq 0 \leq \langle \psi(1), p - \gamma(\bar{s})\rangle.$$

Noting that ψ is continuous in λ, we apply the intermediate value theorem to