where in our case each term in the sum is the same, yielding

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H}{2\pi}\right)^2 \frac{1}{m} \frac{H^2}{\dot{\phi}_1^2} \,. \tag{194}$$

Note that this last expression only contains one of the scalar fields, chosen arbitrarily to be ϕ_1. This estimation for the spectral tilt is given by [213]:

$$n - 1 = 2\frac{\dot{H}}{H^2} - 2\frac{\frac{\partial N}{\partial \phi_i}\left(\frac{\dot{\phi}_i\dot{\phi}_j}{M_{\rm P}^2 H^2} - \frac{M_{\rm P}^2 V_{,i,j}}{V}\right)\frac{\partial N}{\partial \phi_j}}{\delta_{ij}\frac{\partial N}{\partial \phi_i}\frac{\partial N}{\partial \phi_j}} \,, \tag{195}$$

where there is a summation over repeated indices and the commas indicate derivatives with respect to the corresponding field component. Under our assumptions, the complicated second term on the right-hand side of the above equation cancels out, and Eq. (195) reduces to the simple form

$$1 - n = -2\frac{\dot{H}}{H^2} = \frac{m_{\rm Pl}^2}{8\pi}\left[\frac{\frac{\partial V(\phi_1)}{\partial \phi_1}}{V(\phi_1)}\right]^2 = \frac{2}{mp} \,. \tag{196}$$

This result shows that the spectral index also matches that produced by a single scalar field with $\tilde{p} = mp$. The more scalar fields there are, the closer to scale-invariance is the spectrum that they produce. Note however that if the fields have such steep potentials as to be individually non-inflationary, $p < 1$, then many fields are needed before the spectrum is flat enough. The above calculation can be repeated for arbitrary slopes, p_i in Eq. (188). In which case the spectral tilt would have been given by $n = 1 - 2/\tilde{p}$, where $\tilde{p} = \sum p_i$. The above scenario has been generalized to study arbitrary exponential potentials with couplings, $V = \sum^n z_s \exp(\sum^m \alpha_{sj}\phi_j)$ in Ref. [548], see also [551]. Such potentials are expected to arise in dimensionally reduced SUGRA models [552].

One particular nice observation for m scalar potentials of chaotic type, $V \sim \sum_i f(\phi_i^n/M_{\rm P}^{n-4})$ (for $n \geq 4$), is that inflation can now be driven at VEVs below the Planck scale [285, 286, 549, 550] [52]. The *effective* slow-roll parameters are given by: $\epsilon_{eff} = \epsilon/m \ll 1$ and $|\eta_{eff}| = |\eta|/m \ll 1$, where ϵ, η are the slow-roll parameters for the individual fields. Inflation can now occur for field VEVs [286]:

$$\frac{\Delta\phi}{M_{\rm P}} \sim \left(\frac{600}{m}\right)\left(\frac{N_Q}{60}\right)\left(\frac{\epsilon_{eff}}{2}\right)^{1/2} \ll 1 \,, \tag{197}$$

[52] The double inflation model has been studied extensively with two such fields, $V = m_1^2\phi_1^2 + m_2\phi_2^2$, in Refs. [214, 217, 553–556]. In general one could expect: $V \sim \sum_i m_i^2\phi_i^2$ [285, 549, 550], or $V \sim \lambda_i(\phi_i^n/M_{\rm P}^{n-4})$, where $n \geq 4$ [286], where $\phi_i \ll M_{\rm P}$.