Properties of the perturbative expansion around the mode-coupling dynamical transition in glasses

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In this letter we show how to perform a systematic perturbative approach for the mode-coupling theory. The results coincide with those obtained via the replica approach. The upper critical dimension turns out to be always 8 and the correlations have a double pole in momentum space in perturbations theory. Non-perturbative effects are found to be very important. We suggest a possible framework to compute these effects.

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In the mean field theory of glasses there are two transitions, the dynamical-replica transition, that corresponds to the mode-coupling transition in the dynamics and happens at a temperature T_c, and a thermodynamical transition (the Kauzmann transition) that happens at a lower temperature T_K [1, 2]. The dynamical-replica transition can be identified by looking at equilibrium properties of the system, i.e. its landscape; it corresponds to the formation of local minima in the free energy and it is usually studied using replicas [3–5]. The mode-coupling transition is defined by the dynamical properties of the system. The two transitions are related as far as in the mean field approximation the time needed to escape from a local minimum of the free energy is infinite.

This picture is exact in many solvable models. However it should be modified in the real world, where the mean field approximation is no more exact. In this letter we do not address the fate of the thermodynamical transition and we concentrate our attention on the dynamical mode-coupling transition.

It is quite evident that in short range systems there is no dynamical transition, exactly for the same reasons for which there are no infinite lifetime metastable states. However if we neglect the so-called "activated" process, the dynamical transition is present. Moreover there is a very large amount of experimental and numerical data that are well fitted by the predictions of the mode-coupling theory, so that it is certain interesting to try to understand which is the critical behaviour associated to the mode-coupling transition.

In this letter we present a computation of the upper critical dimensions and of the critical properties of the dynamical mode-coupling transition: in the dynamics we consider only the mutual dependence of quantities that do not depend explicitly on the time, i.e. time has been eliminated parametrically as it happens in the generalized fluctuation dissipation relations [6, 7]. We will firstly present the results in the framework of the equilibrium replica approach and we will later show how the same results hold for the mode-coupling transition.

The critical behaviour at the dynamical transition stems from the presence of dynamical heterogeneities [8–13]. In the dynamics these heterogeneities are related to the presence of correlated movements of cooperatively rearranging regions [14] that have been observed both above and below the critical region around T_c. We are interested in getting precise predictions on the properties of dynamical heterogeneities.

Let us start with the basic definitions. Given two configurations of the coordinates (that we label with σ and τ), we indicate with $q_{\sigma,\tau}(x)$ the similarity (overlap) of the two configurations in the region of space around the point x (many different definitions are possible). Usually q is equal to one for identical configurations and it takes a small value for uncorrelated configurations [10, 15–17]. For example we can take $q_{\sigma,\tau}(x)$ to be one if a region around x of size a [31] has the same particle content in the configurations σ and τ; otherwise $q_{\sigma,\tau}(x) = 0$, if the particle content is different.

Let us consider the case where σ is an equilibrium configuration of the system and $\tau(t)$ is a configuration obtained using some dynamics at time t starting from the σ configuration (i.e. $\tau(0) = \sigma$). If the dynamics is non-deterministic, the configuration $\tau(t)$ will depend also on some extra random variables η. For simplicity of notation we will not indicate the dependence of $\tau(t)$ on η, unless we need it in an explicit way. We can define

$$C(t) = \overline{q_\sigma(x,t)} \text{ where } q_\sigma(x,t) \equiv \langle q_{\sigma,\tau(t)}(x)\rangle \ . \quad (1)$$

Here the overline denotes the average over the Boltzmann distribution of the initial configuration (σ) and the angular brackets the average over η. $C(t)$ is the usual equal point (smeared over a region of size a) density-density correlation. Approaching the dynamical transition, $C(t)$ will decay slower and slower, and will also develop a plateaux (as function of $\ln(t)$) at the value C_P. This plateaux becomes infinitely long at the mode-coupling temperature; below the mode-coupling temperature, neglecting *activated processes*, the correlation does not decay any more, i.e. $\lim_{t\to\infty} C(t) \equiv C_\infty > C_P > 0$.

For the study of dynamical heterogeneities it is usual to consider a dynamical susceptibility $\chi_4(t)$ defined as

$$V\chi_4(t) = \overline{Q_\sigma(t)^2} - \overline{Q_\sigma(t)}^2 = \overline{(Q_\sigma(t) - C(t))^2} \ , \quad (2)$$