ions, due to the distance dependence of the Coulomb interaction within the chain of ions, for more details see e.g. [26]. The atomic distances are scaled by an externally controlled length parameter γ, which can be properly adjusted to the number N of trapped ions.

One also cannot ignore fluctuations in the positions, as the individual ions are not really fixed due to the uncertainty principle. Assuming a harmonic potential around each of the equilibrium positions of the ions, the frequency of these harmonic oscillators are nearly equal to the frequency of the trap oscillation. This simple assumption overestimates the variances of the atomic positions, since the potential around a given ion rises quickly due to anharmonic effects. Hence the true position uncertainty can be much smaller than that of an ion in the full trap potential. Nevertheless, we choose this larger value, to effectively account for other possible experimental imperfections. Using elementary quantum mechanics and statistics, we can approximate the physical limit of the variance of the individual ions as

$$\Delta z \approx \sqrt[4]{\frac{4\pi\epsilon_0\hbar^2\gamma^3}{MQ^2}}, \tag{13}$$

where M is the mass of one of the ions and Q its charge. Assuming for example $Q = e$ and $M = 3.3309 \times 10^{-25}$ kg and an optical transition wavelength $\lambda = 194.2$ nm, we obtain $\Delta z \approx 1.014 \times 10^{-9}\mathrm{m}\left(\frac{\gamma}{\lambda}\right)^{3/4}$. This is the situation to be realized with mercury ions, cf. the experiments in [9]. As the variance should be small compared to the transition wavelength to keep statistical averaging effects in the minor μ given by Eq. (9) sufficiently small, we assume a limit of $\Delta z \leq 0.1\lambda$. This leads to a maximal value for γ of about 50λ. Thus we conclude that linear traps would be suited to provide sufficient positioning accuracy of the atoms to realize a continuously radiating entangled-light source with regularly arranged atoms for average distances large compared to the wavelength.

To verify these theoretical predictions by simulations, we choose the parameter γ in such a way, that for each N the minor μ becomes minimal. The resulting Δz from Eq. (13) is implemented as an upper bound for a random deviation from the ideal position along the z-axis, variations perpendicular to this axis are assumed to be negligible due to properly chosen potentials. The minors have been calculated for the same number of ions and the same parameters as in the ideal case shown in Fig. 1. The calculations for the realistic chain are in reasonable agreement with the idealized results given above. For $N = 2, 3$ the uncertainties of the ion's positions modify the idealized result by less than one percent. For four ions in the linear trap, we could optimize the negativities to about 98.8% of the ideal case. Thus it is possible to obtain almost optimal entanglement in the atomic resonance fluorescence with ions in a linear trap, as long as the number N of ions is not too large. By applying alternative technologies for realizing regular atomic systems, e.g. by trap arrays [25], one may also overcome this limitation.

In conclusion, we have studied the realization of a continuous radiation source emitting two multi-mode light beams in different spatial directions, which show bipartite entanglement. The desired radiation source consists of the atomic resonance fluorescence of a regular system of several non-interacting atoms. This represents a non-Gaussian source of entangled radiation. For a single atom, entanglement is obtained under the same conditions as required for the realization of squeezing in resonance fluorescence. For a system of N atoms, the situation can be significantly improved, as by increasing the number of atoms the entanglement of the emitted light beams is getting more robust against atomic dephasing. As an example, we have estimated the practical limitations of the achievable entanglement for a system of ions in a linear trap. Altogether, the resonance fluorescence of regular atomic systems is a continuous and non-Gaussian source of entangled light, which may open interesting perspectives for various applications in quantum information technology.

This work was supported by the Deutsche Forschungsgemeinschaft, SFB 652.

[1] H. J. Carmichael and D. F. Walls, J. Phys. B **9**, L43 (1976); H. J. Kimble and L. Mandel, Phys. Rev. A **13**, 2123 (1976).
[2] H. J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. **39**, 691 (1977).
[3] F. Diedrich and H. Walther, Phys. Rev. Lett. **58**, 203 (1987); M. Schubert et al., Phys. Rev. Lett. **68**, 3016 (1992).
[4] R. Short and L. Mandel, Phys. Rev. Lett. **51**, 384 (1983).
[5] D. F. Walls and P. Zoller, Phys. Rev. Lett. **47**, 709 (1981).
[6] W. Vogel and D.-G. Welsch, Phys. Rev. Lett. **54**, 1802 (1985).
[7] A. Heidman and S. Reynaud, J. Physique **45**, 1937 (1985).
[8] L. L. Jin et al., Opt. Comm. **283**, 790 (2010).
[9] W. M. Itano et al., Phys. Rev. A, **57**, 4176 (1998).
[10] Z. H. Lu, S. Bali, and J. E. Thomas, Phys. Rev. Lett. **81**, 3635 (1998)
[11] W. Vogel, Phys. Rev. Lett. **67**, 2450 (1991).
[12] E. Shchukin and W. Vogel, Phys. Rev. Lett. **96**, 200403 (2006).
[13] W. Vogel, Phys. Rev. Lett. **100**, 013605 (2008).
[14] S. Gerber, D. Rotter, L. Slodicka, J. Eschner, H. J. Carmichael, and R. Blatt, Phys. Rev. Lett. **102**, 183601 (2009).
[15] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[16] O. Gühne and G. Toth, Phys. Rep. **474**, 1 (2009).
[17] Z. Ficek and R. Tanaś, Phys. Rep. **327**, 369 (2002).
[18] D. Vitali, G. Morigi, and J. Eschner, Phys. Rev. A **74**, 053814 (2006).
[19] W. Vogel and D.-G. Welsch, *Quantum Optics* (Wiley-VCH, Berlin, 2006).
[20] A. Peres, Phys. Rev. Lett. **77**, 1413 (1996).
[21] E. Shchukin and W. Vogel, Phys. Rev. Lett. **95**, 230502 (2005).
[22] E. Shchukin and W. Vogel, Phys. Rev. A **74**, 030302(R) (2006).
[23] R. M. Gomes et al., Proc. Nat. Acad. Sci. USA **106**, 21517 (2009).
[24] A. A. Ourjoumtsev et al., Science **312**, 83 (2006); J. Heersink et al., Phys. Rev. Lett. **96** 253601 (2006); A. Franzen et al., Phys. Rev. Lett. **97** 150505 (2006).
[25] R. Blatt and D. Wineland, Nature **453**, 1008 (2008).
[26] M. Šašura and V. Bužek, J. Mod. Opt. **49**, 1593 (2002).