

Recall that the charged lepton sector is already diagonal in this basis and therefore, U_{HPS} gives the PMNS mixing matrix with the predicted TBM form. The resulting neutrino mass spectrum reads

$$U_{HPS}^T \tilde{\mathcal{M}}^\nu U_{HPS} = -\frac{\tilde{m}}{l} \begin{pmatrix} \frac{1}{\epsilon_s + \epsilon_t} & 0 & 0 \\ 0 & \frac{1}{\epsilon_s} & 0 \\ 0 & 0 & \frac{1}{\epsilon_t - \epsilon_s} \end{pmatrix}, \quad (44)$$

implying the neutrino mass-squared differences

$$\Delta m_{21}^2 \equiv |m_2|^2 - |m_1|^2 = \left| \frac{\tilde{m}}{\nu \epsilon_s} \right|^2 \left[1 - \frac{1}{(1+r)^2} \right], \quad (45)$$

$$\Delta m_{31}^2 \equiv |m_3|^2 - |m_1|^2 = \left| \frac{\tilde{m}}{\nu \epsilon_s} \right|^2 \left[\frac{4r}{(1-r^2)^2} \right], \quad (46)$$

where $r \equiv \epsilon_t/\epsilon_s$. From Eq. (45) we see that Δm_{21}^2 is positive, as conventionally assumed, for $r < -2$ or $r > 0$. (For $-2 < r < 0$ we would have to exchange the ordering of the first two neutrinos, thus ruining the TBM prediction.) Hence, the neutrino spectrum is normal ($\Delta m_{31}^2 > 0$) for $r > 0$ and inverted ($\Delta m_{31}^2 < 0$) for $r < -2$ (see Eq. (46)). There are three solutions to Eqs. (45) and (46) reproducing the observed mass-squared differences, $\Delta m_{21}^2 \approx 7.67 \times 10^{-5} \text{ eV}^2$ and $\Delta m_{31}^2 \approx 2.46 \text{ } (-2.37) \times 10^{-3} \text{ eV}^2$ for normal (inverted) hierarchy [27], in the allowed r range,

$$r \approx -2.01, 0.79, 1.20. \quad (47)$$

The other solution $r \approx -1.99$ does not give the correct mixing pattern and is therefore ignored. However, both, the normal ($r = 0.79, 1.20$) and the inverted ($r = -2.01$) mass hierarchy, can be realized in these models, with similar phenomenology in either case. On the other hand, the correct scale of neutrino masses is easily obtained varying the localization parameter c_2 , which lies in the interval $-0.4 \lesssim c_2 \lesssim -0.2$ for $c_{1,3}$ values giving the τ mass and $|\epsilon_{t,s}| \sim \mathcal{O}(10^{-2} - 10^{-1})$.

These results receive three types of corrections. First, there are bulk lepton KK modes with masses $\sim \text{TeV}$ which mix with the zero modes. This mixing is small for leptons localized near the UV brane, and therefore the modifications they induce on the fermion masses and mixings are small too. However since the inter-generational mixing is large in the lepton sector, it is important to check that no large LFV is introduced. The second source of corrections is related to the perturbative treatment of the Higgs effects. This is justified