Proposition 1.5.2 ([RS90, Proposition 7.9, Part (i)]). *Let $Q \in K\backslash G/B$ be given, with $a = \phi(Q)$. Suppose that*

$$id <_{i_1} a_1 <_{i_2} a_2 <_{i_3} \ldots <_{i_n} a_n = a.$$

Then there exists some closed K-orbit Q' such that

$$Q = s_{i_n} \cdot (s_{i_{n-1}} \cdot \ldots (s_{i_2} \cdot (s_{i_1} \cdot Q')) \ldots).$$

In particular, when ϕ is injective, we have the following:

Corollary 1.5.3. *Suppose $\phi : K\backslash G/B \to \mathcal{I}$ is injective. Let Q_1, Q_2 be any two orbits, with $a_1 = \phi(Q_1)$ and $a_2 = \phi(Q_2)$. Then $Q_1 < Q_2$ if and only if $a_1 < a_2$. Thus the weak order on $K\backslash G/B$ corresponds precisely to the order on $\phi(K\backslash G/B)$ induced by the weak order on $\mathcal{I}$.*

Proof. That $Q_1 < Q_2 \Rightarrow a_1 < a_2$ follows from Proposition 1.5.1, part (1). So we prove only the direction $\Leftarrow$ here.

We first note that for any i, it is impossible to have three distinct twisted involutions a_1, a_2, b with $a_1 <_i b$ and $a_2 <_i b$. Suppose by contradiction that we have such a situation. Let $s = s_i$. By definition of the $M(W)$-action on $\mathcal{I}$, the possible ways this could happen are

1. $s * a_1 = a_1$, $s * a_2 = a_2$: In this case, we would have that

   $$m(s) * a_1 = sa_1 = sa_2 = m(s) * a_2,$$

   and this contradicts $a_1 \neq a_2$.

2. $s * a_1 \neq a_1$, $s * a_2 \neq a_2$: In this case, we have

   $$m(s) * a_1 = sa_1\theta(s)^{-1} = sa_2\theta(s)^{-1} = m(s) * a_2,$$

   again contradicting $a_1 \neq a_2$.