find

$$I = \int_{t_0}^{\infty} \frac{dt}{2\pi i} \frac{\Pi\left(te^{-i\pi}\right) - \Pi\left(te^{i\pi}\right)}{t^2\left(t+Q^2\right)}\,, \tag{1.75}$$

where $\Pi\left(te^{-i\pi}\right)$ and $\Pi\left(te^{i\pi}\right)$ are the values of the correlation function at points below and above the branch cut, respectively. Therefore Eq. (1.75) effectively requires the discontinuity of the correlation function across the branch cut. However, the correlation function satisfies Schwarz reflection [101], which implies that

$$\left[\Pi\left(z^*\right)\right]^* = \Pi\left(z\right) \quad \rightarrow \quad \Pi\left(te^{-i\pi}\right) - \Pi\left(te^{i\pi}\right) = 2i\,\mathrm{Im}\Pi\left(te^{-i\pi}\right)\,, \tag{1.76}$$

where z^* denotes the complex conjugate of z and $\mathrm{Im}\Pi\left(te^{-i\pi}\right)$ is the imaginary part of the correlation function evaluated at a point below the branch cut. The imaginary part of the correlation function is equivalent to the hadronic spectral function $\rho^{\mathrm{had}}\left(t\right)$ (see Ref. [89] for a proof of this). Using this and substituting Eq. (1.76) into Eq. (1.75) yields

$$I = \frac{1}{\pi}\int_{t_0}^{\infty} dt\, \frac{\rho^{\mathrm{had}}\left(t\right)}{t^2\left(t+Q^2\right)}\,. \tag{1.77}$$

The contour integral (1.72) can also be evaluated using the residue theorem, with the result

$$I = \frac{1}{Q^4}\left[\Pi\left(Q^2\right) - \Pi\left(0\right) - Q^2\Pi'\left(0\right)\right]\,, \quad \Pi'\left(0\right) = \left.\frac{d}{dQ^2}\Pi\left(Q^2\right)\right|_{Q^2=0}\,. \tag{1.78}$$

Equating the results for the contour integral given in Eq. (1.77) and Eq. (1.78), the following dispersion relation results:

$$\Pi\left(Q^2\right) = \Pi\left(0\right) + Q^2\Pi'\left(0\right) + \frac{Q^4}{\pi}\int_{t_0}^{\infty} dt\, \frac{\rho^{\mathrm{had}}\left(t\right)}{t^2\left(t+Q^2\right)}\,. \tag{1.79}$$

1.4.2 Borel Transform

The dispersion relation (1.79) relates the correlation function $\Pi\left(Q^2\right)$ that can be calculated in QCD to the hadronic spectral function $\rho^{\mathrm{had}}\left(t\right)$ which can be parametrized in terms of the hadronic parameters. In principle this can be used to calculate hadronic parameters, such