15]. Here μ_l and $\mathbf{R}_l$ are magnetic moment, and position of the lth nucleus and sum goes over all nucleus in the lattice. In second quantization it can be written as

$$H_{en} = -\sum_{\alpha\beta}\sum_{l}\frac{A_{\alpha\beta}(\mathbf{R}_l)}{2}[I_{zl}(c^{\dagger}_{\alpha\uparrow}c_{\beta\uparrow}-c^{\dagger}_{\alpha\downarrow}c_{\beta\downarrow}) + I^{+}_{l}c^{\dagger}_{\alpha\downarrow}c_{\beta\uparrow}+I^{-}_{l}c^{\dagger}_{\alpha\uparrow}c_{\beta\downarrow}], \quad (8)$$

where $A_{\alpha\beta}(\mathbf{R}_l)=\tilde{A}_l\varphi^{*}_{\alpha}(\mathbf{R}_l)\varphi_{\beta}(\mathbf{R}_l)$. Finally

$$H_{\rm int}=-\frac{1}{2}\sum_{\alpha\alpha'}\sum_{\sigma\sigma'}\vec{Q}_{\alpha\alpha'}\cdot\tau_{\sigma\sigma'}c^{\dagger}_{\alpha\sigma}c_{\alpha'\sigma'}, \quad (9)$$

where $\vec{Q}_{\alpha\alpha'}=\sum_I J_{\alpha\alpha'}(\vec{R}_j)\vec{M}_j-\sum_n A_{\alpha\alpha'}(\vec{R}_l)\vec{I}_l$.

From the total Hamiltonian H an effective Hamiltonian is obtained by tracing over the degrees of freedom of the electron wavefunction and by taking the interaction term $H_{\rm in}$ into account to second order of perturbation theory, which yields

$$H_{\rm eff}=H_{\rm m}+H_{\rm n}+\sum_{\rm x}\frac{|\langle\Psi_{\rm x}|H_{\rm in}|\Psi_{\rm g}\rangle|^2}{E_{\rm g}-E_{\rm x}}. \quad (10)$$

Here we limit our calculation to a two electron and two MI system in a DQD. The two-electron wavefunction is confined to the Hilbert sub-space constructed from the bonding and anti-bonding (HOMO, LUMO) one-electron orbitals of the DQD, $\varphi_{\pm}$. Below the magnetic field corresponding to spin singlet-triplet transition, this results in six basis functions of two-electron, a spin singlet (S_0) ground state $\Psi_{\rm g}$ that can be expressed as superposition of $\varphi_{+}(\vec{r}_1)\varphi_{+}(\vec{r}_2)|S_0\rangle$ and $\varphi_{-}(\vec{r}_1)\varphi_{-}(\vec{r}_2)|S_0\rangle$ with binding energy $E_{\rm g}$, and five excited states $\Psi_{\rm x}$ with energy $E_{\rm x}$, consisting of three degenerate first excited triplet states, and two higher excited singlet states [11].

Calculating the matrix elements in Eq. 10 yields the effective Hamiltonian

$$H_{\rm eff}=H_{\rm m}+H_{\rm n}+H_{\rm mm}+H_{\rm mn}, \quad (11)$$

where $H_{\rm mm}=\sum_{j,j'}\Delta_{jj'}\vec{M}_j\cdot\vec{M}_{j'}$ is the electron mediated (RKKY-type [14]) interaction between the MIs, and $H_{mn}=\sum_{j,l}\Delta_{jl}\vec{I}_l\cdot\vec{M}_j$ is the electron mediated interaction between MIs and nuclear spin bath; the electron mediated interaction between host nuclear spins $H_{\rm nn}$ is neglected. Here, $\Delta_{jj'}=-\gamma^2J^2_{em}U(\vec{R}_j,\vec{R}_{j'})/(2\Delta_e)$ and $\Delta_{j,l}=\gamma^2\tilde{A}J_{em}U(\vec{R}_j,\vec{R}_l)/\Delta_e$. Further, $\gamma=\alpha_+-\alpha_-$, where α_+, and α_- are the coefficients of the two-electrons ground state that is expressed as linear combination of bonding-antibonding in two level model $\Psi_g(\vec{r}_1,\vec{r}_2)=[\alpha_+\varphi_+(\vec{r}_1)\varphi_+(\vec{r}_2)+\alpha_-\varphi_-(\vec{r}_1)\varphi_-(\vec{r}_2)]\,|S_0\rangle$, Δ_e is the two electron singlet-triplet splitting, $\tilde{A}=1/L\sum_{l=1}^{L}\tilde{A}_l$, and $U(\vec{R}_j,\vec{R}_\lambda)=\varphi_+(\vec{R}_j)\varphi_-(\vec{R}_j)\varphi_+(\vec{R}_\lambda)\varphi_-(\vec{R}_\lambda)$ with $\lambda=j',l$.

In the following we use the effective Hamiltonian 11 to calculate the decoherence time of the DQD with two MIs and two electrons, where each MI represents a spin qubit. The result is compared to a two qubit system realized by two electrons in a DQD. The calculation is performed by using the quasi-static bath approximation [15–17], where the host nuclear spins are approximated by a random magnetic field $\vec{B}_{\rm n}$ with a Gaussian distribution. In this limit the two-electron and two-MI nuclear bath Hamiltonian are given by

$$H_{\rm kn}=\sum_{i=1}^{2}g_{\rm k}\mu_{\rm b}\vec{B}_{\rm n}\cdot\vec{K}_i, \quad (12)$$

where k = e, m and $\vec{K}=\vec{S},\vec{M}$ for electrons and MI, respectively. The coherence time is obtained by solving the equation of motion for $\vec{K}_1$ and $\vec{K}_2$ with initial state $|\uparrow\downarrow\rangle$ and by averaging over the Gaussian magnetic field distribution. From that we obtain $\langle B_{\rm e}\rangle=\langle B_{\rm m}\rangle=0$, $\langle B^2_{\rm e}\rangle=1/(g_{\rm e}\mu_{\rm b})^2\sum_l I_l(I_l+1)\tilde{A}^2_l|\varphi_+(\mathbf{R}_l)|^4$, and $\langle B^2_{\rm m}\rangle=1/(g_{\rm m}\mu_{\rm b})^2\sum_l I_l(I_l+1)\tilde{A}^2_l|\varphi_+(\mathbf{R}_l)\varphi_-(\mathbf{R}_l)|^2$. From there an effective Zeeman splitting $\tilde{\Delta}_{\rm k}=(2\langle B^2_{\rm k}\rangle/3)^{1/2}$ is calculated, hence $\tau_{\rm k}=\hbar/(g_{\rm k}\mu_{\rm b}\tilde{\Delta}_{\rm k})$. Assuming $I_l=1/2$ we find the spin relaxation time $\tau_{\rm e}=\hbar/(2A_e)$ and $\tau_{\rm m}=C\hbar/(2A_m)$. Here $A_e=(\sum_l\tilde{A}^2_l|\varphi_+(\mathbf{R}_l)|^4)^{1/2}$, $A_m=(\sum_l\tilde{A}^2_l|\varphi_+(\mathbf{R}_l)\varphi_-(\mathbf{R}_l)|^2)^{1/2}$, and $C=\Delta_{\rm e}/[\gamma^2J_{\rm em}\Lambda(\vec{R}_1,\vec{R}_2)]$ is the RKKY correction to the MI coherence time, stems from the MI-nuclear-spin in-

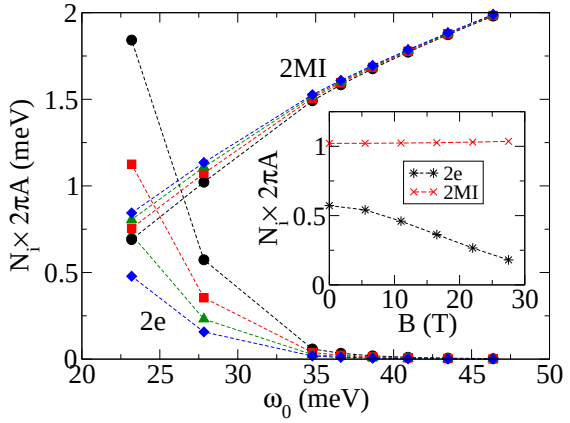

FIG. 2: $N_{\rm i}$, the maximum number of elementary gate operations within the coherence-time normalized by factor $2\pi A$ ($A\equiv A_e\approx A_m$) in meV in a Fe:ZnSe DQD (2MI), compared with ZnSe DQD filled with two-electrons (2e) versus parabolic strength of confining potential of each dot, ω_0, for various gate voltages, $V_g=110$ (circles), $V_g=154$ (squares), $V_g=198$ (triangles), and $V_g=242$ (diamonds) in meV. $N_{\rm i}$ as a function of external magnetic field for $\omega_0=27.5$ meV and $V_g=110$ meV is shown in the inset.