

computing the  $B$ -meson decay constants and  $\Delta B = 2$  mixing matrix elements. We therefore use the small-volume ( $16^3$ ) ensembles with only one lattice spacing and have relatively heavy light-quark masses and limited statistics. A novel feature of this work is the use of  $SU(2)$  heavy-light meson chiral perturbation theory (HM $\chi$ PT) to extrapolate  $N_f = 2 + 1$  lattice QCD results for  $B$ -meson quantities to the physical quark masses. This follows the approach taken by the RBC and UKQCD Collaborations in the light pseudoscalar meson sector in Ref. [34], and differs from the calculations of HPQCD and Fermilab/MILC, both of whom use  $SU(3)$  HM $\chi$ PT for their chiral and continuum extrapolations [14, 15]. The use of  $SU(2)$   $\chi$ PT is based on the fact that the strange quark is much heavier than the up and down quarks, and can therefore be integrated out of the chiral effective theory. Because lattice QCD simulations at the physical strange-quark mass are possible via tuning, interpolation, or reweighting,  $SU(3)$   $\chi$ PT is generally not needed to extrapolate the strange quark to its physical value. When the masses of the light valence and sea quarks are sufficiently small such that  $SU(2)$  chiral perturbation theory is applicable,  $SU(2)$   $\chi$ PT for light pseudoscalar meson masses and decay constants converges more rapidly than  $SU(3)$   $\chi$ PT [34–37]. Although we do not have enough data or sufficiently light quark masses to perform a thorough comparison of  $SU(2)$  and  $SU(3)$  HM $\chi$ PT in this work, we believe that the use of  $SU(2)$  HM $\chi$ PT provides a promising alternative to  $SU(3)$  and warrants further study when better data is available.

In this work we compute both the ratio of decay constants,  $f_{B_s}/f_{B_d}$ , and the ratio of  $\Delta B = 2$  matrix elements,  $\xi$ . We focus on the  $SU(3)$ -breaking ratios because both the statistical and systematic errors are smaller and under better control than for the individual decay constants and mixing matrix elements. Our results have large total uncertainties compared to those of HPQCD and Fermilab/MILC. Within errors, however, our results for the  $SU(3)$ -breaking ratios are consistent with the values presented in the literature and we expect to improve upon them and present values for the individual decay constants and matrix elements in a future work.

This paper is organized as follows. First, in Section II, we present the actions and parameters used in our lattice simulations. Next, in Sec. III, we briefly discuss the perturbative matching of the heavy-light current and the four-fermion operators; the details of the lattice perturbation theory calculation will be presented in another publication [38]. We compute