spinon phase $\vartheta_s(x)$ must be pinned to a constant value $\vartheta_s(x) = (\sqrt{2\pi})^{-1}[\frac{\pi}{2} + n\pi]$, $n = 0, 1, 2...$ Therefore, the system has no spinon excitations, and the only excitations are spinless fermions. It is this constraint which gives strong exponential decays of the spinon correlation function. As a result, the system is described effectively by a spinless system. Therefore, due to the exclusion of double occupancy, the electronic density in the wire will be half in comparison to the leads, where the constraint is absent. This changes will modify the commutation equation for the electron number from $[\vartheta_e(x), P_e(x')] = i\delta(x - x')$ to a new commutator $[\vartheta_e(x), P_e(x')]_D = \frac{i}{2}\delta(x - x')$.

For the remaining part, we will work with the full constraint operator given in equation (16). The constraint $Q_1(x,t)$ must be satisfied at any time. Therefore, we must have $\frac{dQ_1(x,t)}{dt}|F >= 0$. Using the Heisenberg equations of motion with the non-interacting anomalous commutator given in equation (6), we compute : $i\hbar\frac{dQ_1(x,t)}{dt} = [H_{wire}, Q_1(x,t)]$. The condition $\frac{dQ_1(x,t)}{dt}|F >= 0$ can be satisfied only if one introduces a new constraint $Q_2(x)$ given by $[Q_1(x,t), H_{wire}]$:

$$Q_2(x) = \sqrt{2}sin[2K_F x + \sqrt{\pi}\vartheta_e(x)]P_e(x) + sin[\sqrt{2\pi}\vartheta_s(x)]P_s(x) \tag{17}$$

The constraint must annihilate the ground state, $Q_2(x)|F >= 0$, and we must also have $\frac{dQ_2(x,t)}{dt}|F >= 0$. In order for this to happen, we need to include a third constraint $Q_3(x) \propto [[Q_1(x,t), H_{wire}], H_{wire}]$ which is given by: $Q_3(x) = sin[2K_F x + \sqrt{\pi}\vartheta_e(x)](\sqrt{2}P_e(x))^2 + sin[\sqrt{2\pi}\vartheta_s(x)]P_s^2(x)$. Continuing this process, we identify additional constraints $Q_4(x)$, $Q_5(x)$..... The third order constraint $Q_3(x,t) \propto [[Q_1(x,t), H_{wire}], H_{wire}]$ can be written as a second order time derivative $Q_3(x,t) \propto \frac{d^2Q_1(x,t)}{dt^2}$. The higher order constraints $Q_i(x,t)$, $i = 3, 4, 5, ..$ can be neglected since they are represented by higher order time derivatives : $Q_i(x,t) \propto \frac{d^{i-1}Q_1(x,t)}{dt^{i-1}}$, $i \geq 3$ [35].

Following [1, 30], we find that the operators $Q_i(x)$ i=1,2 form a set of *Second Class* constraints, $[Q_1(x), Q_2(x)] \approx < F|[Q_1(x), Q_2(x)]|F > \neq 0$. In order to find the new commutation rules for our system, we will use the Lagrange multiplier fields $\lambda_i(x)$ which will be multiplied by the step function $\mathbf{h}(x,d)$ (which is one for $|x| \leq \frac{d}{2}$ and zero otherwise). We are interested to find the commutation rules for the full wire-leads system. In order to do this, we replace the Hamiltonian $H_{wire} + H_{leads}$ by the the Hamiltonian H_T which is a function of the *Lagrange* fields $\lambda_i(x)$ that enforce the constraints $Q_i(x)$ for the wire [1]. Since we have no constraints for the leads, we will introduce the step function $\mathbf{h}(x,d)$ which is zero for the