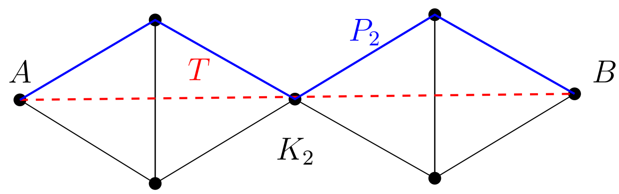

Figure 3.3: A sequence of simplicial chains that converges in the flat norm (i.e., $P_n \to T$) need not have convergent simplicial flat norm values (i.e., $\mathbb{F}_{K_n}(P_n) \to \mathbb{F}(T)$ need not hold). The current T is the segment from A to B, the complex K_n is the arrangement of $2n$ equilateral triangles of appropriate size stretching from A to B and P_n is the top chain from A to B on K_n. Clearly, $\mathbb{F}(T - P_n) \to 0$ but $\mathbb{F}_{K_n}(P_n) = \frac{2}{\sqrt{3}}\mathbb{F}(T) \not\to \mathbb{F}(T)$.

$$\begin{tikzcd} T \arrow[rr, "\text{Polyhedral approximation}"] \arrow[d, equal, "\text{Optimal flat norm decomposition}"'] & & P_\delta \\ X + \partial S \arrow[rr, "{X \to X_\delta,\ S \to S_\delta}", "\text{Polyhedral approximation}"'] & & X_\delta + \partial S_\delta \end{tikzcd}$$

Figure 3.4: Various approximations and decompositions used in our results.

flat norm of an integral chain in codimension 1 has an optimal integral current decomposition; by the compactness theorem from geometric measure theory, the limit of these decompositions is also integral.

In order to show that an integral current T has integral flat norm decomposition, we therefore find suitable simplicial approximations to T and take the limit of their simplicial flat norm decompositions to obtain an integral decomposition for T.

We must also show that this decomposition achieves the flat norm value for T (that is, express T using integral currents in such a way that it remains an optimal flat norm decomposition). This is immediate if our simplicial approximations to T have simplicial flat norm values that converge to the flat norm of T but this is not necessary (see