of the slope is negative by construction, in order to predict a red spectrum, [519]

$$V(\phi) = M^4 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right] \ . \tag{172}$$

This model emerges naturally when studying natural inflation [504], modular inflation [517] in the small field regime or from generic SUGRA theories, see [10, 520, 521]. Again the model differs from the chaotic limit if $\phi \ll \mu$, which is therefore the limit of interest. Another motivation to assume that the form of the potential is only valid at small VEVs is the fact the potential is not bounded from below; at large VEV, it must therefore be compensated by other terms for field theory to be well defined. In this limit, η is negative and so is $n_s - 1$, since $\epsilon \ll |\eta|$ if $p > 1$. However, n_s is very close to unity for $p \neq 2$ or well below for $p = 2$ and therefore $p = 2$ is incompatible with data [10]. Note that in the case $p = 2$, the limit of small fields could be abandoned and super-Planckian VEVs would then be necessary to obtain CMB predictions in agreement with the WMAP5 data. But in addition to the problem of super-Planckian VEV, inflation is then realized in a sector where the potential is not trustable because not bounded from below. As an example of the completion of an inverted hybrid model, in Ref. [522] authors have analyzed the potential:

$$V = V_0 - \frac{1}{2}m^2\phi^2 + \lambda\phi^4 \ , \tag{173}$$

where the amplitude of the CMB predictions can be matched, with almost a flat spectral tilt. In this model ϵ remains negligible and $n_s - 1 \approx 2|\eta|$.

The running of one (or more) parameters of the general scalar potential can also be the origin of the function $f(\phi)$ in Eq. (167). The most common one, the "hybrid running mass" model is driven by a potential of the form [523–526]

$$V(\phi) = M^4 \left[1 + \frac{\eta_0\phi^2}{2M_{\rm P}^2} \ln\left(\frac{\phi}{\phi_*} - \frac{1}{2}\right)\right] \ . \tag{174}$$

A reasonable fit to the CMB data has been found in Ref. [527], with a significant running of the spectral tilt for $\eta_0 < 0$, and ϕ_* the scale of the RG flow. The current WMAP data actually disfavors running of the spectral tilt and therefore these models are now very well constrained. The validity of the running of a quartic coupling constant has also been proposed in Ref. [10].

We will see below that another form of inflationary potential can also emerge of type [528]:

$$V(\phi) = M^4 \left[1 - \left(\frac{\phi_*}{\phi}\right)^n\right] \ , \tag{175}$$