the two regimes coïncide in the standard CQED picture). On the contrary, the good cavity regime does not necessarily imply a coherent energy exchange between the atom and the cavity (which on resonance requires $2g > \kappa+\gamma+\gamma^*$), as it would be the case in usual CQED experiments performed with atoms. On the contrary, pure dephasing opens a new regime where the quantum of energy can stay in the cavity mode without being reabsorbed by the atom. As it will be studied in section V, the good cavity regime is a necessary condition to implement single emitter lasers.

III. GENERALIZED PURCELL FACTOR

In this section, we focus on the bad cavity regime and show that the effective coupling R allows to define a generalized Purcell factor. By definition, in this regime the cavity behaves like a source of losses, and the atom-cavity coupling is incoherent. As a consequence, the parameter R has the dynamical meaning of an effective spontaneous emission rate. If $R \ll \kappa$ (which corresponds to the so called Purcell regime), one can easily extract the atomic relaxation rate from the set of Eqs. (6), which is $\gamma + R$. As expected, "switching on" the cavity mode corresponds to creating an additional relaxation channel for the atom, whose typical rate is R. One can thus define a generalized Purcell factor $F^* = R/\gamma$, quantifying the enhancement of spontaneous emission rate that simultaneously takes into account the influence of pure dephasing. This factor can be expressed as

$$F^* = \frac{4g^2}{\gamma(\kappa+\gamma+\gamma^*)} \frac{1}{1+\left(\frac{2\delta}{\kappa+\gamma+\gamma^*}\right)^2} \,. \tag{9}$$

We can notice that one recovers the usual expression for the Purcell factor, $F = 4g^2/\kappa\gamma$ [21–23], for $\gamma^* = 0$. With respect to the standard expression, F^* is obtained by replacing the cavity mode linewidth, κ, with the sum of κ and the total emitter's linewidth, $\gamma + \gamma^*$. This essentially reduces to replacing the bare cavity mode Q-factor, $Q_{cav} = \omega_{cav}/\kappa$, which usually appears in the standard Purcell expression, with an effective quality factor, Q_{eff}, depending also on the emitter's quality factor, $Q_{em} = \omega_0/(\gamma+\gamma^*)$, as

$$\frac{1}{Q_{eff}} = \frac{1}{Q_{cav}} + \frac{1}{Q_{em}} \,. \tag{10}$$

The existence of a generalized Purcell factor had already been heuristically derived in [21, 24] and finds here a demonstration in the case of a single emitter homogeneously broadened.