

$p^{-2(1-\gamma(g_*)})$. The complex interactions of the quantum field theory have affected the law of rescaling. Finally, since the fixed point is reached at large p , it is known as an ultraviolet fixed point.

An analysis of graph 1.3(b) follows similar lines. Near the fixed point the β function can be approximated by

$$\beta \sim +B(g - g_*) \quad (1.27)$$

which has the solution

$$\bar{g}(p) = g_* + C \left(\frac{p}{M} \right)^B \quad (1.28)$$

This means that for sufficiently small p , the integral in the exponential of (1.8) will be dominated by those values of p where $g(p)$ is close to g_* , giving

$$G(p) \approx G(g_*) \exp \left[-\left(\log \frac{p}{M} \right) 2(1 - \gamma(g_*)) \right] \quad (1.29)$$

$$\approx C \left(\frac{1}{p^2} \right)^{1-\gamma(g_*)} \quad (1.30)$$

Because this occurs at small p , this fixed point is called an infrared fixed point.

Once again, the complex interactions of the field theory have affected the law of rescaling. For this reason, the $\gamma(g)$ function is commonly known as the *anomalous dimension*, even if there is no fixed point in the theory.

1.2.7 Mass gap

Another feature of QCD, and yet to be proved rigorously by anybody, is that of a mass gap. A quantum field theory is said to have a **mass gap** if the energy spectrum has a positive greatest lower bound, but does not