

2. LOCATION SYSTEMS

Location systems are physical realizations of 4D coordinate systems. Hence there is a differentiation of a coordinate system as a mathematical object from its realization through physical objects and protocols. A location system is thus a precise protocol on a set of physical fields allowing to materialize a coordinate system. However, different physical protocols, involving different physical fields, may be given for a unique mathematical coordinate system.

A location system must include the protocols for the physical construction of the coordinate lines, coordinate surfaces or coordinate hypersurfaces of specific causal orientations of the coordinate system that it realizes. Thus, for instance, these coordinate elements may be realized by means of clocks for timelike lines, laser pulses for null lines, synchronized inextensible threads for spacelike lines, laser beams or inextensible threads for time like surfaces and light-front surfaces for null hypersurfaces. The different protocols involved in the construction of location systems give rise to coordinate elements (lines, surfaces and hypersurfaces) of different causal orientations, i.e., they realize coordinate systems of different causal nature.

2.1. Reference systems

Location systems are of two different types: reference systems and positioning systems. The first ones are 4D reference systems which allow one observer, considered at the origin, to assign four coordinates to the events of its neighborhood by means of electromagnetic signals. In relativity due to the finite speed of the transmission of information, this assignment is retarded with a time delay.

A paradigmatic reference system in relativity is the *radar system* which is based in the Poincaré protocol of synchronization which uses two-way light signals from the observer to the events to be coordinated. Unfortunately, the radar system suffers from the bad property of being constructed from a retarded protocol due to the finite speed of the transmission of information.

2.2. Positioning systems

The second kind of location systems are 4D positioning systems, which allow to every event of a given domain to know its proper coordinates in an immediate or instantaneous way without delay. In addition to be immediate, the positioning systems must verify other two conditions, they must be generic and free of gravity. A positioning system is generic, if it can be constructed in any spacetime and, it is free of gravity, if the knowledge of the gravitational field is not necessary to construct it. Reference systems privilege one specific observer among all others, whereas in positioning systems no observers

are necessary at all and hence there is no necessity of any synchronization procedure between different observers.

In relativity, a (retarded) reference system can be constructed starting from an (immediate) positioning system, it is sufficient that each event sends its coordinates to the observer at the origin of the reference system, but not the other way around. In contrast, in Newtonian theory, 3D reference and positioning systems are interchangeable and as the velocity of transmission of information is infinite, the Newtonian reference systems are not retarded but immediate. The reference and positioning systems defined here are 4-dimensional objects, including time location.

3. CAUSAL CLASSIFICATION OF FRAMES

In the Lorentzian spacetime of general relativity, directions and planes or hyperplanes of directions at any event are said to be spacelike, lightlike (or null or isotropic) or timelike oriented if they are respectively exterior, tangent or secant to the light-cone of this event. These causal orientations can be extended in a natural way to vectors, covectors and volume forms on these sets of directions. Thus, every one of the vectors e_A of a frame of the tangent space $\{e_A\}$ ($A = 1, \dots, 4$) has a particular causal orientation c_A .

However, the causal orientations C_{AB} ($A < B$) of the six different associated or adjoint planes $\Pi(e_A, e_B)$ of the frame $\{e_A\}$ are *not* determined by the specific causal orientations c_A of the vectors of the frame. For instance, the plane associated to two spacelike vectors may have any causal orientation. So, in general, the causal characters c_A and C_{AB} are independent. Moreover, in order to give a complete description of the causal properties of the frames, one needs also to specify the causal orientations c_A of the four covectors or 1-forms θ^A giving the dual coframe $\{\theta^A\}$, i.e. $\theta^A(e_B) = \delta_B^A$. Following [5], the best way to visualize and characterize a spacetime coordinate system is to start from four families of coordinate 3-surfaces, then, their mutual intersections give six families of coordinate 2-surfaces and four congruences of coordinate lines.

Alternatively, one can use the related covectors or 1-forms θ^A , instead of the 3-surfaces, and the vectors of a coordinate tangent frame $\{e_A\}$, instead of four congruences of coordinate lines which are their integral curves. The covector θ^A is timelike (resp. spacelike) iff the 3-plane $\Pi(e_B, e_C, e_D)$ generated by the three vectors $\{e_B\}_{B \neq A}$ is spacelike (resp. timelike). This applies for both Newtonian and Lorentzian spacetimes. In addition, for the latter, the covector θ^A is lightlike (or null) iff the 3-plane generated by $\{e_B\}_{B \neq A}$ is lightlike (or null). Thus, to specify the causal orientations of hyperplanes is not necessary because is redundant with the causal orientation of the covectors.

In this way, for a specific domain of a Lorentzian or New-