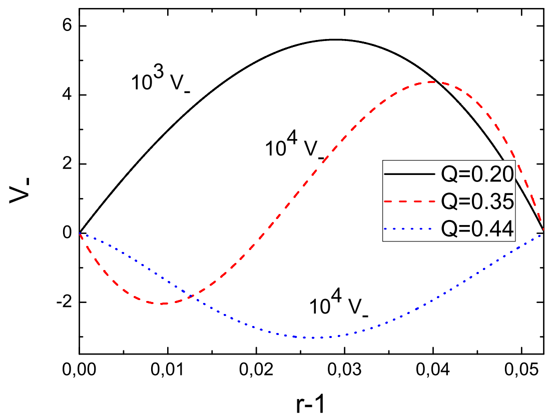

FIG. 1: Behavior of V_- for different parameters, for $D = 8$. Here we fix the event horizon at $r_b = 1$, and the cosmological horizon at $r_c = 1/0.95$. We consider $l = 2$ modes and three different charges, $Q = 0.2, 0.35, 0.44$.

Shadwick [9] and is the following,

$$\int_{r_b}^{r_c} \frac{V}{f} dr < 0 \,. \tag{14}$$

The instability region is depicted in figure 2 for several

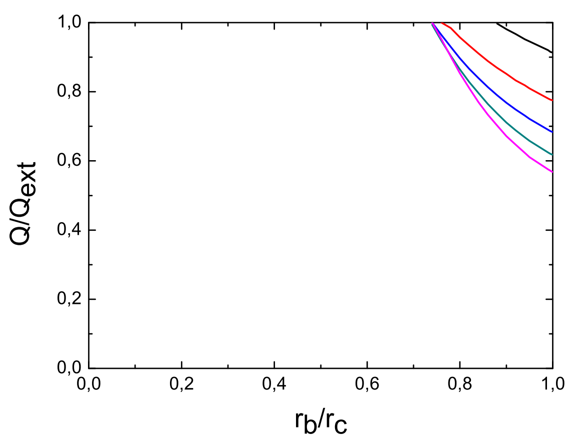

FIG. 2: The parametric region of instability in $Q/Q_{\rm ext} - r_b/r_c$ coordinates, according to criterim (14), for $l = 2$. Top to bottom, $D = 7, 8, 9, 10, 11$.

spacetime-dimension D, which can be compared with the numerical results by KZ, their figure 4. It is apparent that condition (14) very accurately describes the numerical results for $r_b/r_c \sim 1$, a regime we explore below in Section IV. As one moves away from extremality criterium (14) is just too restrictive. An improved analysis and refined criterium would be necessary to describe the whole range of the numerical results. Nevertheless, figure 2 is very clear: higher-dimensional ($D > 6$) RNdS black holes are unstable for a wide range of parameters.

IV. AN EXACT SOLUTION IN THE NEAR EXTREMAL RNDS BLACK HOLE

Let us now specialize to the near extremal RNdS black hole, which we define as the spacetime for which the cosmological horizon r_c is very close (in the r coordinate) to the black hole horizon r_b, i.e. $\frac{r_c - r_b}{r_b} \ll 1$. The wave equation in this spacetime can be solved exactly, in terms of hypergeometric functions [10]. The key point is that the physical region of interest (where the boundary conditions are imposed), lies between r_b and r_c. Thus,

$$f \sim 2\kappa_b \frac{(r - r_b)(r_c - r)}{r_c - r_b} \,, \tag{15}$$

where we have introduced the surface gravity κ_b associated with the event horizon at $r = r_b$, as defined by the relation $\kappa_b = \frac{1}{2} df/dr_{r=r_b}$. For near-extremal black holes, it is approximately

$$\kappa_b \sim \frac{(r_c - r_b)(n-1)}{2r_b^2} \left(1 - nQ^2\right) \,. \tag{16}$$

In this limit, one can invert the relation $r_*(r)$ of (7) to get

$$r = \frac{r_c e^{2\kappa_b r_*} + r_b}{1 + e^{2\kappa_b r_*}} \,. \tag{17}$$

Substituting this on the expression (15) for f we find

$$f = \frac{(r_c - r_b)\kappa_b}{2\cosh\left(\kappa_b r_*\right)^2} \,. \tag{18}$$

As such, and taking into account the functional form of the potentials for wave propagation, we see that for the near extremal RNdS black hole the wave equation (6) is of the form

$$\frac{d^2\Phi(\omega, r)}{dr_*^2} + \left[\omega^2 - \frac{V_0}{\cosh\left(\kappa_b r_*\right)^2}\right]\Phi(\omega, r) = 0 \,, \tag{19}$$

with

$$V_0 = \frac{(r_c - r_b)\kappa_b}{2} \frac{V_{S\pm}(r_b)}{f} \tag{20}$$

The potential in (19) is the well known Pöshl-Teller potential [11]. The solutions to (19) were studied and they are of the hypergeometric type, (for details see Refs. [12, 13]). It should be solved under appropriate boundary conditions:

$$\Phi \sim e^{-i\omega r_*} \,, \quad r_* \to -\infty \tag{21}$$

$$\Phi \sim e^{i\omega r_*} \,, \quad r_* \to \infty. \tag{22}$$