U_i^δ, W_j^δ and V_2^δ be d-, $(d+1)$- and $(d+2)$-dimensional currents such that:

$$P_\delta - T = U_0^\delta + \partial W_0^\delta, \qquad \mathbb{F}(P_\delta - T) = \mathrm{M}(U_0^\delta) + \mathrm{M}(W_0^\delta), \tag{3.18a}$$

$$X - X_\delta = U_1^\delta + \partial W_1^\delta, \qquad \mathbb{F}(X - X_\delta) = \mathrm{M}(U_1^\delta) + \mathrm{M}(W_1^\delta), \tag{3.18b}$$

$$S - S_\delta = W_2^\delta + \partial V_2^\delta, \qquad \mathbb{F}(S - S_\delta) = \mathrm{M}(W_2^\delta) + \mathrm{M}(V_2^\delta). \tag{3.18c}$$

To clarify the notation, we adopt the convention that variables with a δ subscript are chains on the simplicial complex K_δ whereas a δ superscript merely indicates dependence on δ.

Let K_δ be any simplicial complex that triangulates P_δ, X_δ and S_δ separately as well as the convex hull of their union. We may assume (applying the subdivision algorithm of Edelsbrunner and Grayson [18] and Theorem 3.3.6 if necessary) that the currents U_0, U_1, W_0, W_1, and W_2 can be pushed to K_δ with expansion bound at most L and the maximum diameter Δ of a simplex of K_δ satisfies

$$\Delta \leq \frac{\delta}{\max\{1, \mathrm{M}(\partial U_0^\delta), \mathrm{M}(\partial U_1^\delta), \mathrm{M}(\partial W_0^\delta), \mathrm{M}(\partial W_1^\delta), \mathrm{M}(\partial W_2^\delta)\}}. \tag{3.19}$$

Claim 3.3.7.1. $\mathbb{F}(T) \leq \lim_{\delta\downarrow 0} \mathbb{F}_{K_\delta}(P_\delta)$

Proof of claim. By the triangle inequality and since any simplicial flat norm decomposition is a candidate decomposition for the flat norm, we have

$$\begin{aligned} \mathbb{F}(T) &\leq \mathbb{F}(T - P_\delta) + \mathbb{F}(P_\delta) \\ &\leq \mathbb{F}(T - P_\delta) + \mathbb{F}_{K_\delta}(P_\delta). \end{aligned}$$

The claim follows from letting $\delta \downarrow 0$ and noting that $\mathbb{F}(T - P_\delta) \to 0$.

Claim 3.3.7.2. $\mathbb{F}(T) = \lim_{\delta\downarrow 0} \mathbb{F}_{K_\delta}(P_\delta)$

Proof of claim. In light of Claim 3.3.7.1, we must show that $\mathbb{F}(T) \geq \lim_{\delta\downarrow 0} \mathbb{F}_{K_\delta}(P_\delta)$.