

[Note that to get the correct equations of motion, the action should also include a metric factor $e^{-1}(\tau)$ to incorporate invariance under reparametrizations of the path $x(\tau)$. For our discussion, we have fixed the gauge by setting $e(\tau) = 1$.]

By performing the path integral over the λ and A_μ fields (with the effective gauge action generated by closed Z -loops), we obtain the full two-point function for the model. So far we have not explicitly introduced an ultraviolet cutoff. In a continuum formulation of the large N approximation an ordinary momentum space cutoff can be used. A lattice formulation of the CP^{N-1} model provides a more general nonperturbative cutoff procedure, but, as discussed in the Introduction, Monte Carlo results raise subtle issues regarding the scaling behavior of the topological susceptibility χ_t in the continuum limit of the lattice cutoff [3, 4, 9, 10]. The dominance of topological charge structures with transverse size of order the lattice spacing (pointlike instantons for small N and line-like excitations for $N > 4$ [4]) clearly indicates that the structure of the nonperturbative vacuum is crucially affected by gauge field dynamics taking place at or near the cutoff scale. In this paper we explore this dynamics by introducing a cutoff at short proper time, $\tau \approx 0$ in Eq. (7) [18]. As a nonperturbative regulator, the proper time cutoff procedure provides a viable alternative to the lattice formulation of the model. Moreover, it leads rather naturally to a string-theoretic ultraviolet completion in which the CP^{N-1} field theory is obtained as the $\alpha' \rightarrow 0$ limit of an open string theory. In the string theory completion of this model, the parameter $\sqrt{\alpha'}$ plays a role analogous to lattice spacing in the lattice cutoff. In the field theory limit the strings become very short, and the world sheet parametrization of the string coordinates $X^\mu(\tau, \sigma)$ reduces to the proper time parametrization of the particle path $x^\mu(\tau)$. The open string propagator reduces to the propagator for the CP^{N-1} field $Z(x)$. For the full string theory, the contribution of massive string excitations to the propagator at a given proper time is determined by an elliptic modulus parameter $q = e^{-\tau/4\alpha'}$. For $\tau \gg 4\alpha'$, only the lowest mass states contribute significantly (including the tachyon, which is associated with the vacuum instability as we discuss below). Thus, in the proper time decomposition of the propagator, the stringy corrections affect the theory only at proper time separations comparable to or smaller than α' , and we can regard the “string length” $l = \sqrt{\alpha'}$ as a short distance cutoff. It is important for the following analysis that both the Lagrange multiplier field λ and the gauge field A_μ are introduced as *boundary terms* (rather than bulk terms) on the open string world sheet. In this way, the spacetime conformal symmetry is broken but conformal symmetry on the world sheet may be preserved by certain boundary fields. As is familiar in string theory, the boundary perturbations that preserve worldsheet conformal symmetry correspond to background fields which satisfy the spacetime equations of motion. We