small. This will occur when the energy required for reorienting $\langle\vec{\mathbf{M}}\rangle$ is nonzero, but not large compared to T. If this were to occur, then there would be no local order parameter, and thus no terms in the Landau-Ginsburg free-energy functional which depended on gradients of $\langle\vec{\mathbf{M}}\rangle$.

Monte Carlo calculations for Eqn. (1) on simple cubic lattices were performed some time ago by Gingras and Huse[2] and Fisch[3]. More recently, additional calculations were carried out.[12] The results of these studies, which used several different choices for the probability distribution $P(h_i)$, indicate that for typical samples the order parameter $\langle M^2\rangle$ becomes positive at some positive temperature T_c, as long as the random fields are not too strong. However, the precise nature of what happens near T_c remained unclear. In this work we report additional studies using new choices of $P(h_i)$, which display phenomena which were not seen clearly in the earlier studies.

The computer program which was used was an enhanced version of the one used before.[12] The data were obtained from $L\times L\times L$ simple cubic lattices with $L = 64$ using periodic boundary conditions. Some preliminary studies for smaller values of L were also done. The program approximates $O(2)$ with Z_{12}, a 12-state clock model. It was modified to enable the study of random-field probability distributions of the form

$$P(h_i) \;=\; (1-p)\delta(h_i - h_r) \;+\; p\delta(h_i)\,. \qquad (5)$$

This modification makes the program run a few percent slower than before. In this work we study two cases. Type A samples have $h_r = 1.5$ with $p = 0$, and Type B samples have $h_r = 2$ with $p = 0.5$. For both of these cases we find T_c/J near 1.5, which was suggested by Gingras and Huse[2] to be optimal for this sort of calculation.

On a simple cubic lattice of size $L\times L\times L$, the magnetic structure factor, $S(\vec{\mathbf{k}})$, for $n = 2$ spins is

$$S(\vec{\mathbf{k}}) \;=\; L^{-3}\sum_{i,j}\cos(\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}_{ij})\langle\cos(\phi_i-\phi_j)\rangle\,, \qquad (6)$$

where $\vec{\mathbf{r}}_{ij}$ is the vector on the lattice which starts at site i and ends at site j. Thus, setting $\vec{\mathbf{k}}$ to zero yields

$$S(0)/L^3 \;=\; \langle M^2(L)\rangle\,. \qquad (7)$$

As we will see, the extrapolation of $\langle M^2(L)\rangle$ to $L = \infty$ is not trivial.

In this work, we present results for the average over angles of $S(\vec{\mathbf{k}})$, which we write as $S(k)$. Eight different $L = 64$ samples of the random fields θ_i were used for each $P(h_i)$. The same samples of random fields were used for all values of T.

The Monte Carlo calculations were performed using both hot start (*i.e.* random) initial conditions followed by slow cooling, and also cold start (*i.e.* ferromagnetic) initial conditions followed by slow warming. For the Type A samples three different cold start initial conditions were used, equally spaced around the circle. For the Type B samples, where the average random field is somewhat weaker, only two cold start initial conditions were used.

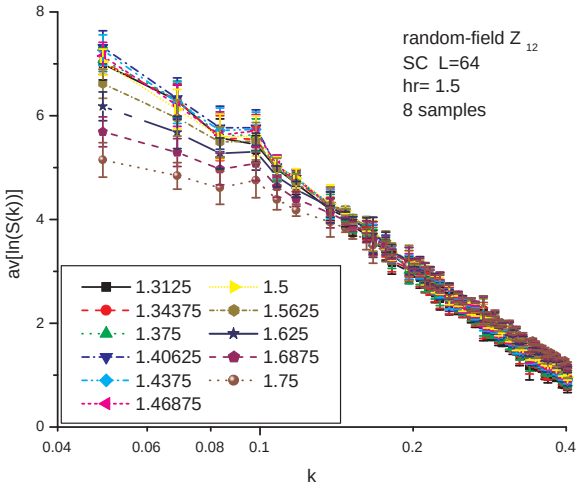

FIG. 1: (color online) Average over 8 samples of $\ln(S(k))$ for $64\times64\times64$ lattices with $h_r = 1.5$ and $p = 0$ at various temperatures. The error bars indicate approximately three standard deviation statistical errors, and the x-axis is scaled logarithmically.

In Fig. 1 we show data for the average over our eight Type A samples of $\ln(S(k))$ for T/J between 1.3125 and 1.75. The data for $T/J \geq 1.4375$ are taken from the hot start runs, and the data for lower values of T/J are taken from the cold start run which gave the lowest average value for the energy, $\langle E\rangle$. When $T = 1.4375$ or more, all runs for a given sample converge in a rather short time to give similar values of $\langle E\rangle$ and $\langle\vec{\mathbf{M}}\rangle$. However, for lower values of T/J the hot start runs remain stuck in metastable states.

An attempt was made to drive the hot start runs into equilibrium by deep undercooling, followed by slow warming. This thermal cycling technique had proven successful earlier,[12] for the weak random field case of $h_r = 1.0$ with $p = 0$. For the cases studied here, however, a single undercooling was only partially successful in driving the samples to equilibrium. The author believes that repeating cycling would have worked successfully. However, since a detailed study of the nonequilibrium behavior was not the purpose of the work described here, this was not attempted.

Somewhat surprisingly, the cold start runs were much more successful in finding a single free energy minimum at T/J between 1.3125 and 1.40625. In the majority of samples, all of the cold start initial conditions were converging to the same free energy minimum. This was