Note that dispersion is represented in these equations only through the terms $\hat{v}\psi_X$ and $\hat{v}a_X$, representing advection of the pattern envelope with the group velocity $\hat{v}$. Note also that the group velocity of the large scale mode f is zero, and hence no corresponding term appears in the second of these equations.

The three amplitude equations may be reduced to the single (nonlinear) phase equation

$$\left(\frac{\partial}{\partial T} - 4\frac{\partial^2}{\partial X^2} + \hat{v}\frac{\partial}{\partial X}\right)^2 \left(\frac{\partial}{\partial T} - \frac{\partial^2}{\partial X^2}\right)\psi = -16a_0^2\left(\frac{\partial\psi}{\partial X} + q\right)\frac{\partial^2\psi}{\partial X^2}. \tag{21}$$

Then linearising this equation and setting $\psi = e^{\mathrm{i}LX+\sigma T}$ yields the dispersion relation

$$\sigma^3 + 9\sigma^2L^2 + 24\sigma L^4 - \hat{v}^2\sigma L^2 + 16L^6 - \hat{v}^2L^4 - 16a_0^2qL^2 + \mathrm{i}\hat{v}(2\sigma^2L + 10\sigma L^3 + 8L^5) = 0. \tag{22}$$

Before considering this dispersion relation for general L, it is helpful to consider the two limiting cases, of small and large L. First, if L is small, then $\sigma^3 \sim 16a_0^2qL^2$. Thus, to leading order in L, $\sigma = \sigma_{2/3}L^{2/3}$, where $\sigma_{2/3}^3 = 16a_0^2q$; hence all traveling waves are unstable if L is small. On the other hand, if L is large, then we have $\sigma^3 + 9\sigma^2L^2 + 24\sigma L^4 + 16L^6 \approx 0$, and so $\sigma \approx -L^2$ or $-4L^2$ (twice); hence traveling waves are stable to large-L disturbances. In summary, all traveling waves are unstable at onset (provided $a_0^2q \neq 0$; in fact we shall see later that when a_0^2q is suitably small, we shall need to reconsider this conclusion). The rest of the section provides more details of the instability, for general values of L.

In order to find the secondary stability boundary for the traveling waves, we set $\sigma = \mathrm{i}\Omega$ in the dispersion relation (22), where Ω is real. From the real and the imaginary parts, we obtain

$$\begin{aligned}
\Omega^2 - \frac{16}{9}L^4 + \frac{16}{9}a_0^2q + \frac{\hat{v}^2}{9}L^2 + \frac{10}{9}\hat{v}L\Omega &= 0,\\
\Omega^3 - 24\Omega L^4 + \hat{v}^2\Omega L^2 + 2\hat{v}L\Omega^2 - 8\hat{v}L^5 &= 0,
\end{aligned}$$

and then after eliminating Ω between these two equations we find that this stability boundary is given by

$$16a_0^6q^3 - 2500L^{12} + 2100L^8a_0^2q + 384L^4a_0^4q^2 - 200\hat{v}^2L^{10} - 4\hat{v}^4L^8 - 44\hat{v}^2L^6a_0^2q + \hat{v}^2L^2a_0^4q^2 = 0. \tag{23}$$

We note that in this equation L and $\hat{v}$ appear only as even powers and thus we can restrict our attention to positive L and $\hat{v}$ with no loss of generality. However, both even and odd powers of q occur, so no such economy is possible in considering q (indeed, in the light of [3], we should expect different behaviors for $q > 0$ and $q < 0$).