Then the only properties of interest at t_1 will be those about $\mathcal{B}$, and those of interest at t_2 will be about $\mathcal{A}$. That is, the chain operators will have the form

$$C^{(\alpha\beta)} = \hat{P}^{\alpha}_{\mathcal{A}} T(t_2, t_1) \hat{P}^{\beta}_{\mathcal{B}} T(t_1, t_0) \,, \tag{7}$$

where the projection operators $\hat{P}^{\alpha}_{\mathcal{A}}$ act only on the component of the system state for subsystem $\mathcal{A}$ while the $\hat{P}^{\beta}_{\mathcal{B}}$ act only on the component for $\mathcal{B}$.

Now, for an ideal measurement process, the *dynamics* correlates the pointer states of the apparatus with the states of the measured subsystem. That is, if α in Eq. (7) enumerates pointer states of $\mathcal{A}$ corresponding to similarly enumerated measured states β of $\mathcal{B}$, then the time evolution $T(t_2, t_1)$ ensures that $C^{(\alpha\beta)}$ is zero unless $\alpha = \beta$. Note that this does not change that fact that the $\hat{P}^{\alpha}_{\mathcal{A}}$ act *only* on $\mathcal{A}$ states and the $\hat{P}^{\beta}_{\mathcal{B}}$ act *only* on $\mathcal{B}$ states.

The above simple analysis motivates the following approach for moving from non-relativistic to relativistic consistent histories.

Consider the subsystems $\mathcal{A}$ and $\mathcal{B}$ to occupy physical three-dimensional volumes within the overall combined system (e.g., the physical space occupied by the apparatus $\mathcal{A}$, etc.). Then take the time interval $[t_1, t_2]$ along with the 3-volume for $\mathcal{A}$, forming a *four-dimensional* hypervolume within which all interesting dynamics happens for $\mathcal{A}$. Similarly, take the time interval $[t_0, t_1]$ with the 3-volume for $\mathcal{B}$ to form a hypervolume of interest for $\mathcal{B}$.

Heuristically, what is desired is to recast chain operators of the form of Eq. (7) into a form something like

$$C^{(\alpha\beta)} = \hat{P}^{\alpha}_{\mathcal{A}} \hat{G}_{\mathcal{A}} \hat{P}^{\beta}_{\mathcal{B}} \hat{G}_{\mathcal{B}} \,, \tag{8}$$

where the operators $\hat{P}^{\alpha}_{\mathcal{A}}$ represent properties of interest about the hypervolume associated with $\mathcal{A}$ and $\hat{G}_{\mathcal{A}}$ represents the dynamical interactions that occur in that hypervolume, and similarly for $\hat{P}^{\beta}_{\mathcal{B}}$, $\hat{G}_{\mathcal{B}}$ and $\mathcal{B}$. The point of this is to develop a spacetime formulation for the chain operator that is manifestly Lorentz invariant.

The use of spacetime hypervolumes here has some similarity to previous analyses of the probabilities for a particle to enter a specific spacetime region in timeless quantum theories [18, 19, 35]. However, in the present case there will generally be *many* particles in each hypervolume (i.e., the particles that physically make up the subsystem in that hypervolume) and these particles will be interacting within and across the hypervolumes. Thus, to handle multiple, interacting particles in spacetime, we turn to the formalisms of quantum field theory.