Thermodynamic properties of a dipolar Fermi gas

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Based on the semiclassical theory, we investigate the thermodynamic properties of a dipolar Fermi gas. Through a self-consistent procedure, we numerically obtain the phase-space distribution function at finite temperature. We show that the deformations in both momentum and real space become smaller and smaller as the temperature is increased. For the homogeneous case, we also calculate pressure, entropy, and heat capacity. In particular, at the low-temperature limit and in the weak interaction regime, we obtain an analytic expression for the entropy which agrees qualitatively with our numerical result. The stability of a trapped gas at finite temperature is also explored.

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I. INTRODUCTION

The experimental success in creating ultracold ^{40}K^{87}Rb molecular gas near quantum degeneracy [1–4] has drawn considerable attention to in studying the fundamental properties of degenerate dipolar Fermi gases. Within the framework of the semi-classical theory, the ground state properties, the collective excitation, and the free expansion dynamics of a normal state dipolar Fermi gas were studied theoretically [5–7]. A recent theoretical work based on a variational approach reveals that, due to the Fock exchange interaction, the momentum distribution is stretched along the direction of dipole moment such that the Fermi surface becomes an ellipsoid [8]. This result was confirmed numerically for both homogeneous [9] and trapped [10] systems. Taking into account the effect of the exchange interaction, further theoretical work regarding the normal state of the zero temperature dipolar Fermi gas includes studying the free expansion [7, 11], collective excitation [6, 11], zero sound [9], and the Fermi liquid properties [12].

Another interesting feature of the dipole-dipole interaction is that the partially attractive dipolar force is responsible for the formation of anisotropic BCS pairing [13–16]. With the recent experimental development on control the hyperfine states of ^{40}K^{87}Rb molecules [3, 4], the BCS pairings in a mixture of fermionic polar molecules with two different hyperfine states are also studied theoretically [17–19]. In particular, the effects of the Fock exchange interaction to pairing was considered in Ref. [16, 18].

In this paper, we extend our previous work on the ground state properties of the dipolar Fermi gases to finite temperature case. Employing the semi-classical theory, we numerically obtain the phase-space distribution function through a self-consistent procedure. We show that the deformations of the distribution function in both momentum and real space become smaller and smaller as the temperature is increased. For homogeneous system, we also calculate the thermodynamic quantities such as pressure, entropy, and heat capacity. In particular, at low temperature limit and in weak interaction regime, we derive an analytic expression for the entropy, which agrees qualitatively with our numerical result. For the trapped gases, we also explore the temperature dependence of the stability.

The remainder of this paper is organized as follows. In Sec. II, we introduce our model and briefly outline the semi-classical theory for a dipolar Fermi gas at finite temperature. The numerical and analytical results are presented in Sec. III. Finally, we conclude in Sec. IV.

II. THEORY

Here we consider a system of N spin polarized dipolar fermions interacting via dipole-dipole interaction

$$V_d(\mathbf{r}) = \frac{c_d}{r^3}\left(1 - \frac{3z^2}{r^2}\right), \tag{1}$$

where $c_d = d^2/(4\pi\varepsilon_0)$ with d being the electric dipole moment. For simplicity, we have assumed that the dipole moments of all fermions are orientated along z-axis. Additionally, we shall consider both homogeneous and trapped systems. In the former case, $U_{\rm ext}$ can be regarded as a box of volume $\mathcal{V}$; while for the latter, the trap is assumed to be a harmonic potential with axial symmetry, i.e.,

$$U_{\rm ext}(\mathbf{r}) = \frac{1}{2}m\overline{\omega}^2\lambda^{-2/3}(x^2 + y^2 + \lambda^2 z^2), \tag{2}$$

where $\overline{\omega}$ is the geometric average of the trap frequencies and λ is trap aspect ratio.

Within the framework of the semiclassical theory, the thermodynamic properties are completely characterized by the phase-space distribution function $f(\mathbf{r}, \mathbf{k})$, which, at finite temperature T, satisfies the Fermi-Dirac statistics

$$f(\mathbf{r}, \mathbf{k}) = \frac{1}{e^{(\varepsilon(\mathbf{r},\mathbf{k})-\mu)/k_BT} + 1}, \tag{3}$$

where μ is the chemical potential introduced to fix the