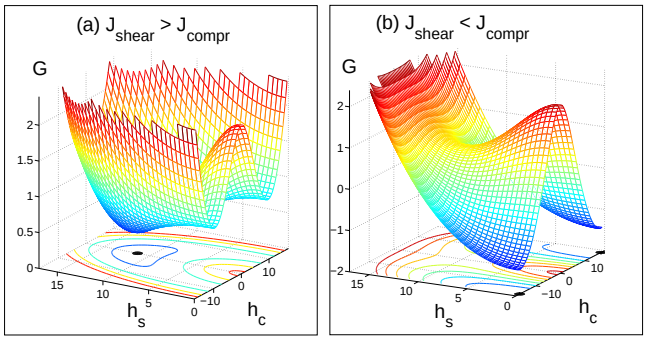

FIG. 3: Free energy from Eq.(24) corresponding to the elastic constants of (a) silica ($J_{\rm compr}/J_{\rm shear} \simeq 0.88$) and (b) salol ($J_{\rm compr}/J_{\rm shear} = 1.5$), corresponding to a high and low values of the shear modulus, relative to the bulk modulus. Cases (a) and (b) exhibit frozen-in shear and uniform compression/dilation respectively. Solid circles at bottom plane denote the locations of minima.

the angular-averaged $\tilde{K}$ tensor:

$$\frac{J_{\rm compr}}{J_{\rm shear}} = \frac{5}{2}\,\frac{K}{K+2\mu} \equiv \frac{5}{2}\,\frac{1+\sigma}{4-5\sigma} \equiv \frac{5}{2}\,\frac{3-4c_t^2/c_l^2}{3+2c_t^2/c_l^2} \quad (22)$$

This ratio varies between 0.88 and 1.6 for non-metallic glasses surveyed by Novikov and Sokolov.[36,37] Here, σ is the Poisson ratio, c_l and c_t are the longitudinal and transverse speeds of sound. Bigger values of the $J_{\rm compr}/J_{\rm shear}$ ratio correspond to a greater Poisson ratio and smaller μ/K ratio, i.e. to a lower shear modulus relative to the bulk modulus. We thus observe that the mean-field Heisenberg model (21) has anisotropic couplings, the anisotropy directly related to the Poisson ratio of the material. Remarkably, the purely isotropic case $J_{\rm compr}/J_{\rm shear} = 1$ (corresponding to $\sigma = 1/5$, $c_t^2/c_l^2 = 3/8$) falls within the experimental range, the implications to be discussed at the end of the article.

The mean-field Hamiltonian (21) can be solved in a standard fashion by a Hubbard-Stratonovich transformation, so that the partition function is given the following six-dimensional integral, up to a multiplicative constant:

$$Z_{\rm MF} = \sum_{\{\vec{s}^{(m)}\}} \int \Pi_{\alpha=1}^{6} dh_\alpha \exp\left\{ -N\frac{h_1^2}{2\beta J_{\rm compr}} - N\sum_{\alpha=2}^{6}\frac{h_\alpha^2}{2\beta J_{\rm shear}} + \sum_{\alpha=2}^{6} h_\alpha \sum_{m}^{N} s_\alpha^{(m)} \right\}, \quad (23)$$

where the sum in front of the integral denotes averaging with respect to spin orientations. (Eq.(23) is a minor variation on the mean field solution of the isotropic Heisenberg model for an arbitrary number of vector-components, which can be found, for instance, in Ref.[28]) Upon the angular averaging (see the derivation of Eq.(14)), the integration can be done by steepest descent. In the leading order, the partition function is equal to $e^{-\beta N G_0}$, where G_0 is the minimum value of the following free energy:

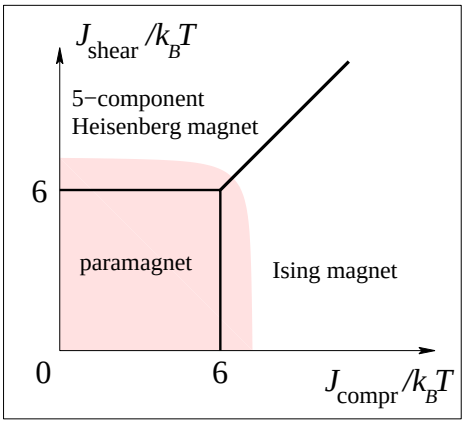

FIG. 4: The phase diagram corresponding to the mean-field Hamiltonian (21). The shaded region schematically denotes the regime where the mapping between liquid dynamics and the spin model does not apply. Labels "Ising magnet" and "Heisenberg magnet" indicate ordering of the compression and shear components in Eq.(21) in the respective regions of the diagram.

$$\frac{G}{N} = \frac{h_{\rm c}^2}{2\beta J_{\rm compr}} + \frac{h_{\rm s}^2}{2\beta J_{\rm shear}} - \ln\frac{I_2(\sqrt{h_{\rm c}^2+h_{\rm s}^2})}{h_{\rm c}^2+h_{\rm s}^2}, \quad (24)$$

with respect to the variables $h_{\rm c}$ and $h_{\rm s}$ (c.f. Eq.(2.3) of Ref.[28]). $h_{\rm c} \equiv h_1$ gives the effective field on each spin in the direction of uniform compression/dilation; it can be of either sign. $h_{\rm s} \equiv (\sum_{\alpha=2}^{6} h_\alpha^2)^{1/2}$ is the *magnitude* of the total field in the direction of pure shear and can be only non-negative, of course. The free energy from Eq.(24) is graphed in Fig.3 for two distinct values of anisotropy, at a temperature below the Curie temperature.

The Curie temperature itself, according to Eq.(24), is determined by the bigger of the two coupling constants:

$$k_B T_{\rm Curie}^{\rm MF} = \frac{1}{6}\max(J_{\rm compr}, J_{\rm shear}). \quad (25)$$

We remind the reader that the activated regime corresponds to the ordered state of the magnet. If $J_{\rm compr} > J_{\rm shear}$, the ordering is along the compression/dilation component of the local displacement, which corresponds to the first sum in Eq.(21), see Fig.3. Note the depths of the two minima are equal, implying there is no net volume change during the ordering transition. From the spin perspective, this state is essentially an ordered Ising ferromagnet. Conversely, if $J_{\rm shear} > J_{\rm compr}$, it is the shear component that becomes ordered; this component corresponds to the second sum in Eq.(21). The frozen-in shear state is an ordered 5-component Heisenberg ferromagnet, from the spin viewpoint. We summarize these notions graphically in the phase diagram shown in Fig.4.

We may further connect the mean-field parameters $J_{\rm shear}$ and $J_{\rm compr}$ to the material constants by enforcing the aforementioned notion that the large g asymptotic of the function $F_0(g)$ be logarithmic in the large g limit, i.e.