The first three of these terms depend solely on the magnitude of the $c_\pm$ coefficients, whereas the remainder of the terms depend upon the phases of the coefficients as well. The latter are clearly contributions to S that rely upon quantum interference between the spin states.

Note, for comparison later below, that $S = 1$ denotes a pure state. If the state is completely decohered in the spin basis then $(\rho_s)_{kl} = \frac{1}{2}\delta_{kl}$ so that the purity, by Eq. (7), is $S = 2(1/2)^2 = 1/2$.

With the $\mathsf{P}, \mathsf{Q}, \mathsf{R}, \mathsf{T}$ matrices known, one can optimize S with respect to c_+ and c_- to determine the state with maximum purity S at a fixed time. In particular, the relative phase of c_+ and c_- is important if either Q_{kl} or R_{kl} is non-zero, and P_{kl} is non-zero . Although not evident from Eq. (8), numerical results below clearly demonstrate a reliance on this relative phase to maximize S.

IV. OVERLAPPING RESONANCES AND SYSTEM PURITY

A. Overlapping Resonances

As noted above, the bare states $|\pm, n\rangle$ are not eigenstates of H, and will evolve in the presence of the system-environment coupling. Insight into the nature of this time evolution is afforded by expanding the zeroth order state in the exact eigenstates $|\gamma\rangle$. For example, a bare state such as $|+, j\rangle$ expands as

$$|+, j\rangle = \sum_\gamma |\gamma\rangle\langle\gamma|+, j\rangle, \tag{9}$$

and evolves as

$$|+, j\rangle_t = \sum_\gamma |\gamma\rangle\langle\gamma|+, j\rangle e^{-iE_\gamma t/\hbar} \tag{10}$$

The E_γ dependence of the square of the expansion coefficient, i.e. $|D_\gamma|^2 \equiv |\langle\gamma|+, j\rangle|^2$ provides the energy width over which the zeroth order state $|+, j\rangle$ is spread due to the system-environment interaction. In the simplest cases the inverse of this width provides a qualitative measure of the time scale for the evolution of $|+, j\rangle$. Hence, the zero order states are indeed resonances with a characteristic width $|D_\gamma|^2$. This is the analog, in a bound state spectrum, of the well known resonance in the continuum.

Similarly, by analogy to the continuum case, we can define *overlapping resonances*, as resonances that overlap in energy space, i.e. resonances that share a common $|\gamma\rangle$ in their