where V is the volume of the system and $Q_\sigma(t)$ is the space integral of $q_\sigma(x,t)$. The quantity $\chi_4(t)$ is a measure of the differences that are observed during the evolution from region to region [8–13]. In a similar fashion we can define the time dependent correlation function

$$G_4(x-y,t) = \overline{q_\sigma(x,t)q_\sigma(y,t)} - C(t)^2 \ . \tag{3}$$

The explicit time dependence is quite a complex problem that we do not address in this letter; different dynamical exponents are involved and they are known to be not universal (at least in the mean field approximation). We will consider here only relations where the time is not explicitly presents, as the dependence of χ_4 on C, that can be obtained by eliminating the time parametrically, e.g. by plotting $\chi_4(t)$ versus $C(t)$ exactly in the same way as for the fluctuation dissipation relations. A particular example of a time independent quantity is χ_4^*, that is defined as the maximum of $\chi_4(t)$ (that happens at time t^*) (the corresponding correlation will be denoted by $G_4^*(x)$). Analogously we define a C-dependent correlation function $G_4(x-y|C)$ as follows

$$G_4(x-y|C(t)) = G_4(x-y,t) \ . \tag{4}$$

We are interested in the universal properties of the previous quantities when we approach the dynamical transition. We denote by ϵ the distance in temperature from the dynamical transition and we indicate from here on the dependence on ϵ. We are interested in the behaviour in the double scaling limit $\epsilon \to 0$ and $C \to C_P$. Within mean field theory, the susceptibility $\chi_4(C_P,\epsilon)$ is divergent when $\epsilon \to 0$ from above [32]. Given the obvious relations $\chi_4^*(\epsilon) = \int G_4^*(x,\epsilon)\,dx$ the divergence of $\chi_4^*(\epsilon)$ implies the existence of a divergent correlation length $\xi(\epsilon)$. In the same way as in standard phase transitions, we expect the following scaling laws

$$\chi_4^*(\epsilon) \propto \epsilon^{-\gamma} \ , \quad \widetilde{G}_4^*(k,\epsilon) = \chi_4^*(\epsilon)\tilde{g}_4(k\xi(\epsilon)) \ , \quad \xi(\epsilon) \propto \epsilon^{-\nu} \ , \tag{5}$$

where $\widetilde{G}_4^*(k,\epsilon)$ is the Fourier transform of $G_4^*(x,\epsilon)$.

What is the replica counterpart of this behaviour? In the replica approach [5] we consider two replicas σ and τ. We denote by $H(\sigma)$ the original Hamiltonian, while the Hamiltonian of the τ system is

$$H(\tau) - hQ_{\sigma,\tau} \ . \tag{6}$$

The thermal averages are taken first with respect to the τ variables and later with respect to σ variables. The dynamical phase transition is defined by the behaviour of $q_0(\epsilon) \equiv \lim_{h\to 0^+} q(\epsilon,h)$. For $\epsilon > 0$ we should have $q_0 = q_{bulk}$, i.e. a small value usually temperature independent; at $\epsilon = 0$ we should have $q_0(0) = C_P$ and for $\epsilon < 0$ we should have $q_0(\epsilon) > C_P$. This behaviour is present in mean field model where metastable states do exist. It survives in the real world in the approximation where metastable states can be observed. In other words the two replicas system (with the replica σ quenched respect to the replica τ) becomes critical at the same point where the dynamics display the mode-coupling singularity.

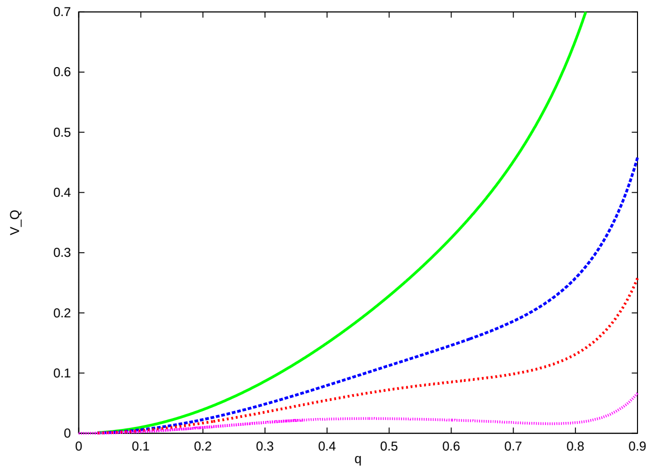

FIG. 1: A schematic view of the potential $W(q)$ computed at various temperatures, decreasing from above to below.

One can sharpen the physical picture by introducing a potential $W(q)$ defined as follows [2, 5, 18]. We consider an equilibrium configuration σ. We call $P_\sigma(q)$ the probability that another configuration τ has an overlap $q_{\sigma,\tau} = q$. We define

$$W(q) = -\lim_{V\to\infty} \frac{\ln(P_\sigma(q))}{V} \ . \tag{7}$$

With probability one when the volume V goes to infinity, the potential $W(q)$ does not depend on the reference configuration σ. In other words $P_\sigma(q) \approx \exp(-VW(q))$. By construction $W(q_{bulk}) = 0$ and the vanishing of the potential $W(q)$ for more than one q value is the distinctive characteristic of replica symmetry breaking (this should happen below an eventual thermodynamical glass transition). In other words we are considering an equilibrium configuration σ and we define by $W(q)$ the increase in the free energy density if we constrain an other equilibrium configuration τ to stay at overlap q.

The behaviour of the potential in the mean field approximation is described in Fig. 1. The dynamical transition is characterized by the presence of an horizontal flex for the potential $W(q)$. Beyond the mean field approximation the Maxwell construction should hold and the non convex part of the potential disappear, but we will not consider this effect.

A standard assumption is that the behaviour in the dynamics mirrors the behaviour in the equilibrium properties of two replicas of the system. This assumption is usually accepted and there are partial proofs of its validity in perturbation theory (we will come later to this point by showing how to complete these proofs). Let us try to formulate this assumption in a sharp way.

In the same way as the time may be eliminated parametrically, also the forcing field h may be eliminated in