we then have

$$\begin{aligned} \vec{\mathcal{E}}(\vec{k}) &= ik^0\vec{\mathcal{A}}(\vec{k}) - i\vec{k}\mathcal{A}^0(\vec{k}) && \text{(F.45)} \\ \vec{\mathcal{B}}(\vec{k}) &= i\vec{k}\times\vec{\mathcal{A}}(\vec{k}). && \text{(F.46)} \end{aligned}$$

Let's now prove some nice relations regarding $\mathcal{E}$ and $\mathcal{B}$ that will be useful later on: transversality and orthogonality. To prove that $\vec{\mathcal{E}}$ is orthogonal to $\vec{k}$ recall that $\mathcal{A}^\mu \propto (p'^\mu/k\cdot p') - (p^\mu/k\cdot p)$. Then

$$\begin{aligned} \vec{k}\cdot\vec{\mathcal{E}}(\vec{k}) &\propto -i|\vec{k}|^2(\hat{k}\cdot\vec{\mathcal{A}} - \mathcal{A}^0) \\ &\propto -i|\vec{k}|\left[\left(\frac{\vec{k}\cdot\vec{p}'}{k\cdot p'} - \frac{\vec{k}\cdot\vec{p}}{k\cdot p}\right)\right. \\ &\quad \left. - \left(\frac{k^0p'^0}{k\cdot p'} - \frac{k^0p^0}{k\cdot p}\right)\right] \\ &\propto -i|\vec{k}|\left[\frac{k\cdot p'}{k\cdot p'} - \frac{k\cdot p}{k\cdot p}\right] = 0. && \text{(F.47)} \end{aligned}$$

Also

$$\begin{aligned} \vec{k}\times\vec{\mathcal{E}} &= \vec{k}\times\left[ik^0\vec{\mathcal{A}} - i\vec{k}\mathcal{A}^0\right] \\ &= ik^0\vec{k}\times\vec{\mathcal{A}}; && \text{(F.48)} \end{aligned}$$

since $k^0 = \pm|\vec{k}|$, and we won't be interested in the sign, we have that

$$\vec{\mathcal{B}} = \hat{k}\times\vec{\mathcal{E}}. \quad \text{(F.49)}$$