$n = 1$. In fact, the fermionic field behavior can be classified according to the value of the exponent n. In particular, for $n < 1/2$ the pressure of the fermion field is negative and it could represent (in a universe ruled by Einstein gravity) either the inflaton or a dark energy constituent[7]. The fermionic energy density is given by $\rho_D = \left[(\overline{\psi}\psi)^2\right]^n$. On the other hand the second constituent, the matter field, is described, as mentioned in the precedent section, following a barotropic equation of state[7]. After some algebraic manipulation we can put the model dynamics in the following form:

$$2\frac{\ddot{a}}{a} + H^2 + \alpha^2\ddot{\varphi} + 2\alpha H\dot{\varphi} + \frac{\alpha^2\omega}{2}\dot{\varphi}^2 = e^{-\alpha\varphi}\Big[(2n-1)(\bar{\psi}\psi)^{2n} + p_m\Big], \tag{20}$$

$$\alpha^2\ddot{\varphi} + 3\alpha H\dot{\varphi} = \frac{e^{-\alpha\varphi}}{3+2\omega}\Big[V + \rho_m - 3(2n-1)(\bar{\psi}\psi)^{2n} - 3p_m\Big], \tag{21}$$

$$\dot{\rho_m} + 3H(\rho_m + p_m) = 0, \tag{22}$$

$$\dot{\psi} + \frac{3}{2}H\psi + \imath\gamma^0\frac{dV}{d\overline{\psi}} = 0, \quad \dot{\overline{\psi}} + \frac{3}{2}H\overline{\psi} - \imath\frac{dV}{d\psi}\gamma^0 = 0 \tag{23}$$

We consider a pressureless matter field ($p_m = 0$), so that one can obtain from equation (22) that $\rho_m(t) = \rho_m(0)/a(t)^3$. Furthermore, the equations (23) can also be integrated for the potential (19) resulting that the fermionic bilinear evolution is governed by $[\overline{\psi}\psi](t) = [\overline{\psi}\psi](0)/a(t)^3$. The remaining equations (20) and (21) constitute a highly non-linear system of differential equations and we proceed to solve it numerically. We analyze the time evolution of our model choosing first the conditions for $t = 0$:

$$a(0) = 1, \quad \dot{a}(0) = 1, \quad [\bar{\psi}\psi](0) = 0.001, \quad \rho_m(0) = 1,$$
$$\varphi(0) = 1, \quad \dot{\varphi}(0) = 0.001.$$

These conditions characterize qualitatively an initial proportion between the constituents; an era when matter predominates over the fermionic density. Besides that, we have to specify the magnitude of the remaining parameters: we suppose initially that $\alpha = 1.0$, $\omega = 4 \times 10^4$ [11] and a value for the potential power n, $n = 0.2$ [7].

These choices are reference values that permit final adjustments to follow several cosmological constraints, like the spectrum of the Brans-Dicke coupling ω[11] and the present value of the Brans-Dicke scalar field φ. These parameters can be in fact adjusted due to invariance properties of the Brans-Dicke gravitational field equations, by using the following change of variables: $\alpha \to \bar{\alpha}$, $\varphi \to \bar{\varphi}$, $\bar{\alpha}\bar{\varphi} = \gamma/\varphi + \alpha$ where $\exp(-\gamma) = x_0$ is associated to the asymptotic value of $\bar{\varphi}_0 = (2\omega+4)/(2\omega+3)$. These transformations show how a value of the Brans-Dicke parameter ω is linked to the definition of a new Brans-Dicke scalar field $\bar{\varphi}$. In fact, after numerical integration, it is possible to verify that these features are included in the φ evolution (see figure 1) that imply into the evolution of the gravitational "constant" as $G(t) = (2\omega+4)/[(2\omega+3)\bar{\varphi}(t)]$.

In figure 2 it is plotted the acceleration field $\ddot{a}$ as function of time t for two different values of n. The results show that initially the universe is expanding with negative acceleration, a period where the matter constituent predominates over the fermionic field. With the evolution of time we have increasing values in energy transference to the fermionic field (this is not happening directly but via the gravitational field $a(t)$ and the scalar field $\varphi(t)$, as the equations of motion (20-23) show). The negative pressure of $\psi(t)$ help in fact to promote a final accelerated period (also shown in figure 1) indicating that in this model the dark energy role would be played by a *combination of the fermionic constituent with the scalar* $\varphi(t)$. Other interesting results appear when we choose the power n to be in the neighborhood of values $n \approx 0.33$. In fact, for a fixed value of $\dot{\varphi}(0)$ and increasing values of n what emerges is a universe that fails to show a final accelerated period, even when the fermionic field still exhibits a negative pressure ($p_D = (2n-1)\left[(\overline{\psi}\psi)^2\right]^n$). On the other hand, for values $n \leq 0.33$ we will find a universe that is permanently in accelerated expansion. Another important remark here is that this general qualitative behavior depends strongly on the initial value of the time derivative of the scalar field $\varphi(t)$: what we verify is that increasing values of $\dot{\varphi}(0)$ promote an earlier entrance on the accelerated period.

In figure 3 it is shown the behavior of the energy densities of the fermionic field ρ_D and matter ρ_m as functions of time t. The numerical results show that eventually the energy density of the femionic field overcomes the energy density of the matter field, although this does not coincide with the instant when the universe goes into an accelerated period (opposed to what occurs in some Einstein gravity based models[12]). Another feature showed by the numerical results is that for larger values of n it follows that: (a) the energy density of fermionic field grows more slowly causing a larger decelerated period and (b) the energy density of the matter field has a less significant decay and also promotes a larger decelerated period. On the other hand, as figure 3 shows, both densities have decreasing values in time due to the permanent expansion of the universe (this was verified with plots of the scale factor against time). Again, all cases are strongly dependent on the exponent n of the self-interacting potential and one can obtain different behaviors, that are in tune with the acceleration patterns presented above. Finally we verify that the scalar field φ approaches a final constant value that validates the accord between Brans-Dicke and Einstein gravitation for large t.

As concluding remarks we stress that we have investigated the role of a fermionic field – with a self-interacting potential – in an old Brans-Dicke universe. We have shown that the fermionic field, in combination with the