

$$Q = -\kappa \nabla T .$$

It needs to be emphasized that the choice of particular expansion variables, here ∇u and ∇T , affects the solutions of nonlinear problems like shockwave structure. García-Colín and Green emphasized that the description of nonequilibrium continuum mechanics is ambiguous whenever the choice of “equilibrium” variables – energy or longitudinal temperature or transverse temperature in this case – is ambiguous¹⁷. The numerical value of a Taylor’s series in the deviations from equilibrium, truncated after the first nonlinear term, is clearly sensitive to the choice of independent variable.

In the nonequilibrium pressure tensor the superscript t indicates the transposed tensor and I is the unit tensor:

$$I_{11} = I_{22} = 1 ; I_{12} = I_{21} = 0 ,$$

η is the shear viscosity, and $\lambda = \eta_v - \eta$ is defined by the bulk viscosity η_v . In the shockwave problem the pressure-tensor definitions give

$$P_{xx} = P^{\text{eq}} - (\eta_v + \eta) du/dx ; P_{yy} = P^{\text{eq}} - (\eta_v - \eta) du/dx .$$

For a two-temperature continuum model it is necessary to formulate the “equilibrium pressure” P^{eq} as a function of the (nonequilibrium) energy, density, and the two temperatures. The viscosities and conductivity could likewise depend upon these state variables and κ can be a tensor, as we show later, with an example.

When we define T_{xx} and T_{yy} as continuum state variables it becomes necessary for us to formulate constitutive relations for their evolution. The simplest such models begin by separating the energy into two parts: a density-dependent “cold curve” $e^{\text{cold}}(\rho)$ and an additional kinetic or “thermal” part, proportional to temperature:

$$e \equiv e^{\text{cold}}(\rho) + e^{\text{thermal}}(T_{xx}, T_{yy}) = e^{\text{cold}} + (ck)(T_{xx} + T_{yy}) ,$$

where ck is a scalar heat capacity. The functional form of the cold curve produces a corresponding contribution to the pressure:

$$P^{\text{cold}} = -de^{\text{cold}}/d(V/N) = \rho^2 de^{\text{cold}}/d\rho .$$

Grüneisen’s γ defines a corresponding thermal pressure:

$$P^{\text{thermal}} = \gamma \rho e^{\text{thermal}} .$$