likely to follow the baryon peculiar velocities. Dark matter over-densities are subject to bias and we would not be able to measure them directly. The above expression for the induced Q in case II is also applicable to other kinds of interaction models, e.g. the one proposed by Amendola [35] where the interaction is described by a quintessence field coupled to dark matter. In Fig. 3 we plot the parameter $Q(a,k)$ from Eq. (47) for different values of the interaction rate Γ_{de}.

VI. TRAJECTORIES ON THE Σ AND μ PLANE

In the previous three sections we examined predictions for Q and η in MG models and GR models with exotic dark energy. As discussed in section III, MG models characterised by the BD theory in the quasi-static approximation have a distinct path on the plane of (Σ, μ). In this section, we summarise our predictions in this plane. For simplicity we only consider the case where Σ and μ are scale independent.

Let us consider the simplest massless case in the MG models described by the BD theory. In this case, as is seen from Eq. (14), Q and η is expressed in terms of a single parameter w_{BD}, i.e. both are not independent quantities. If we project this constraint on the Σ and μ plane, we get

$$\Sigma(k,a) = 1, \quad \mu(k,a) = \frac{2(2+\omega_{\rm BD})}{3+2\omega_{\rm BD}}\,, \qquad (48)$$

and interestingly, $\Sigma(k,a) = 1$ is the same as GR with varying μ only. Note that μ and Σ are scale independent, as the massless case is considered. As is shown in Fig. 4, the MG models described by the BD theory trace a path along the direction of μ at fixed $\Sigma = 1$.

In clustering dark energy (cDE) models, it is difficult to give physical models for dark energy perturbations. We have considered a simple toy model where the pressure perturbation is modeled as in Eq. (28). If we impose scale-independence of μ and Σ, we find $g_p = 0$ and that the anisotropic stress should be related to the pressure perturbation via $\sigma_{de} = \delta P_{de}/(1+w_{de})\rho_{de}$. Then we are left with only one free parameter, either the pressure perturbation δP_{de} or the anisotropic stress σ_{de}, thus again there is a unique path on the (Σ, μ) plane. Of course, it is possible to vary δP_{de} and σ_{de} fully independently, but then the scale independence of Σ and μ is lost, or absolute fine-tuning.

In interacting dark energy (IDE) models, no anisotropic stress is introduced with the assumption that dark energy is smooth and, thus, only Q is non-trivial. Therefore, the condition $\eta = 1$ reduces the degrees of freedom again to only one such that we have the relation

$$\Sigma = \mu\,. \qquad (49)$$

This turns out to be quite distinct from the trajectory of the MG models as is shown in Fig. 4.

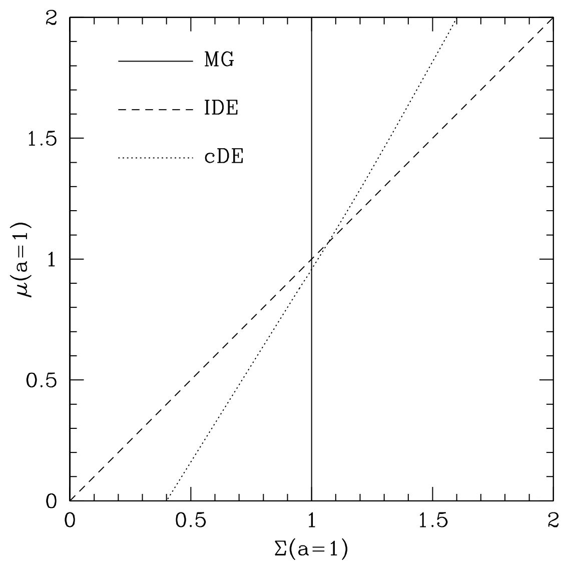

FIG. 4: Trajectories on Σ and μ plane of BD type MG models (solid curve), clumping dark energy (dotted curve) and interacting dark energy (dash curve).

These examples suggest that if we can measure Σ and μ from observations, the path on the (Σ, μ) plane enables us to identify the underlying physics of the cosmic acceleration.

VII. CONCLUSIONS

In this paper, we proposed to parameterise the relation between the lensing potential and the matter over-densities, Σ, and the dynamic relation between the Newtonian potential and the matter over-densities, μ, which enable us to characterise theoretical models and constrain them with observations. If dark energy is described by a perfect fluid that is homogeneous (smooth) on sub-horizon scales these parameters are trivial, i.e. $\Sigma = 1$ and $\mu = 1$. We showed that Σ and μ can depart from unity in some theoretical models; such as modified gravity models, interacting dark energy models and clustering dark energy models. Interestingly, both parameters are related to each other in an unique way depending on the underlying theory:

- With the assumption of the scale-independent evolution of perturbations, Σ and μ in Brans-Dicke type MG models are described by a single variable, $\omega_{\rm BD}$, which leads to a specific trajectory with $\Sigma = 1$. This comes from the fact that there is no coupling between photons and the BD scalar field.

- In clustering dark energy the scale-independence of Σ and μ and the simple assumption that $\delta P_{de} \propto \delta\rho_{de}$ lead to a constraint equation between the pressure perturbation and anisotropy stress. Therefore,