$$
\begin{array}{ll}
\multicolumn{2}{c}{\max_{\boldsymbol{\xi},\mathbf{x}} \min_{\mathbf{p}\in\Psi} -\sum_{i=1}^{N} p_i \exp\left[-\alpha w_i - \alpha \sum_{j=1}^{J} x_j \left(\left(\mathbf{A}^T\boldsymbol{\xi}\right)_j - A_{ij}\right)\right]} \\
\multicolumn{2}{c}{\text{such that}} \\
\text{(A)} & \Psi = \left\{\mathbf{p} \in \mathbb{R}^{N\times 1} | \mathbf{p} \geq \mathbf{0}, \sum_{i=1}^{N} p_i = 1, \sum_{i=1}^{N} p_i \ln\left(\frac{p_i}{q_i}\right) \leq \Omega\right\} \\
\text{(B)} & \boldsymbol{\xi} \geq \mathbf{0} \\
\text{(C)} & \sum_{i=1}^{N} \xi_i = 1 \\
\text{(E1)} & \forall j \in \{1,2,..,J\},\ \ x_j = 0 \quad \text{if} \quad \left(\mathbf{A}^T\boldsymbol{\xi}\right)_j > B_j b_j \\
\text{(E2)} & \forall j \in \{1,2,..,J\},\ \ x_j \in [0, Q_j] \quad \text{if} \quad \left(\mathbf{A}^T\boldsymbol{\xi}\right)_j = B_j b_j \\
\text{(E3)} & \forall j \in \{1,2,..,J\},\ \ x_j = Q_j \quad \text{if} \quad \left(\mathbf{A}^T\boldsymbol{\xi}\right)_j < B_j b_j
\end{array}
\tag{8}
$$

Corollary 1 *Suppose that the market maker holds zero inventory:* $w_i = 0$ *for* $\forall i$. *As the value of* Ω *increases to infinity, the KPM becomes completely pari-mutuel. The market maker incurs no loss regardless of the outcome.*

Proof. *See the Appendix.* ■

The KPM may not be completely pari-mutuel in the sense that the market maker can lose money with positive probability. However, Corollary 1 shows that the KPM subsumes a completely pari-mutuel market. By adjusting the value of Ω, the market designer can fine-tune the extent to which the market is close to being completely pari-mutuel. The larger the value of Ω, the more completely pari-mutuel the market becomes.

For example, consider increasing the value of Ω. Problem (8) then models the auctioneer with a large level of ambiguity aversion. The ambiguity-averse DM is very sensitive to the worst-case scenario. Thus, the auctioneer clears the market such that he/she performs moderately even in the worst-case scenario. In other words, the auctioneer does not want to lose too much money even in the worst-case scenario.[5] In the extreme case in which Ω diverges to infinity, the auctioneer becomes so conservative that he/she does not want to lose any money under any circumstances. The market should become completely pari-mutuel.

4.3 The Market-Clearing Algorithm

Before further discussion, we introduce new notations: $z_i = -e^{-\alpha w_i}$ and $\theta_i = q_i e^{\Omega}$ for each $i \in \{1, 2, ..., N\}$. In addition, let $\mathbf{F}$ be the set of pairs $(\boldsymbol{\xi}, \mathbf{x})$ that satisfy the limit order logic constraints (E1), (E2), and (E3).

[5] The cost of this strategy is that the market maker may not be able to make a great deal of money on the upside.