profiles may admit the same response distribution. In other words, there exist $\boldsymbol{\alpha}_1 \neq \boldsymbol{\alpha}_2$ such that $f_j(y|\theta, \boldsymbol{\alpha}_1) = f_j(y|\theta, \boldsymbol{\alpha}_2)$ for all y. In this case, responses to this item of subjects in latent classes $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ admit the same probability law. Thus, each item usually provides partial information of the entire attribute profile. This information structure will be formulated mathematically in the sequel. Each Q-matrix along with a specific model parameterization (such as the DINA and DINO models, etc) maps to a unique item-specific partial information structure.

3 On the identifiability of diagnostic classification models

We present the main identifiability results in this section. In the current formulation, the identifiability of diagnostic classification models consists of two components: 1. the item parameters and the attribute population, 2. the partial information structure of each item.

3.1 Identifiability of item parameters and the attribute distribution

We first present four theorems that are applicable to different situations. We start with the simplest case that the responses are binary and each item only has two possible response distributions. The binary response settings will be relaxed in subsequent theorems. The result is applicable to stylized models such as the DINA and the DINO models.

Theorem 1 *We consider the general setting of a latent class model with $M > 2$ latent classes. The responses are binary and take values in $\{0, 1\}$. For each item j, let $p_{j\boldsymbol{\alpha}} = P(Y_j = 1|\boldsymbol{\alpha})$. Let $\pi_{\boldsymbol{\alpha}}$ be the probability of each latent class. Suppose that the following assumptions are satisfied.*

A1 There exist three non-overlap subsets of items denoted by I_1, I_2, and I_3 such that for each $\boldsymbol{\alpha}_1 \neq \boldsymbol{\alpha}_2$ and $l = 1, 2$, and 3, the conditional distributions of $(Y_j : j \in I_l)$ on classes $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ are distinct.

A2 For each $j \in I_1 \cup I_2 \cup I_3$, the response probabilities $(p_{j\boldsymbol{\alpha}} : \boldsymbol{\alpha} = 1, ..., M)$ take only two possible values.