friendly faces are in the majority, the triad relation is in one state, or the gauge invariance state. While the hostile faces are in the majority, the triad relation is in the other state, or the gauge variance state. The explicit expressions of the gauge invariance and the gauge variance states [19–22] are as follows.

For each triad relation, we propose if

$$\sigma_1+\sigma_2+\sigma_3\geq 0 \ or \ \sigma_1=\sigma_2=\sigma_3=0, \qquad (20)$$

it satisfies the gauge invariance. If

$$\sigma_1+\sigma_2+\sigma_3< 0 \ or \ \sigma_1\neq\sigma_2\neq\sigma_3=0, \qquad (21)$$

the triad relation does not satisfy the gauge invariance. Here σ_1, σ_2 and σ_3 are the values of the three nodes of the triangle. So $a_1, a_2, a_5, a_8, a_{10}$ are gauge invariance states, while a_3, a_4, a_6, a_7, a_9 are gauge variance ones, as shown by Fig. 1 and Fig. 3.

We study the ternary network in association with the Potts spin systems. Since each node has three values, the way of spin changing is the spin transition [18]. Besides, we propose the spin transition depends both on itself and the surroundings [18, 19]. Considering the SOC is determined by the structure of the system, we set the geometric temperature as $\beta=\frac{1}{T}>0$, where $T=|\sum_{<i,j>}\sigma_j/n_i-\sigma_i'|$, with σ_i' the final value of i, and $\sum_{<i,j>}\sigma_j/n_i$ the average value of all neighboring spins of i. Based on these considerations, we give the spin transition probability of each node as follows,

$$W(\sigma_i\rightarrow\sigma_i')=\frac{1}{Z}e^{-\beta H(\sigma_i',\Sigma_{<i,j>}\sigma_j)}. \qquad (22)$$

In Eq. (22), $H(\sigma_i',\sum_{<i,j>}\sigma_j)=-\sigma_i'\sum_{<i,j>}\sigma_j$, where $\sum_{<i,j>}\sigma_j$ is the sum of values of all neighboring spins of i, and $Z=\sum_{\sigma_i'=-1,0,+1}e^{\beta\sigma_i'\Sigma_{<i,j>}\sigma_j}$ is the normalizing factor. In order to avoid the denominator of β being zero, we propose if the final value of the spin results in $T=0$, it happens with probability zero, while the other values happen with probability one. In this way, whatever the initial condition is, the network will evolve to the sensitive state.

The evolving rules are as follows. We set all triad relations in the network to satisfy gauge invariance initially. The simplest way is to set all spins to be zero, which stands for all persons in the society being neutral with each other initially. After a long enough time of evolution, the network will reach the sensitive state. Then any small disturbance may result in an avalanche.

For any small disturbance:

(1) Choose a triangle at random. If it is gauge variance, nothing happens. If it is gauge invariance, we choose one of the three nodes at random, and let the node do spin transition according to Eq. (22). If after the spin transition, the triangle is still gauge invariance, nothing happens. Otherwise, we store all the gauge invariance triangles attached to the selected triangle, and goto step (2).

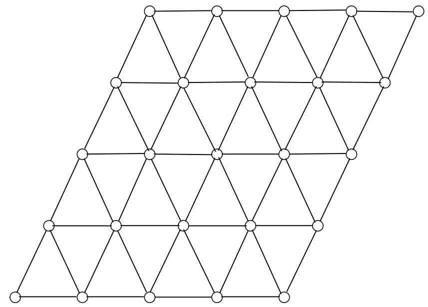

FIG. 6: There are 9×10^4 nodes in the two-dimensional lattice with equal length and width. It is one of the simplest two-dimensional regular networks satisfying the triad relations.

(2) Choose all the stored triangles successively at random. Because of the interaction, first we judge whether the triangle is gauge invariance. If it is gauge variance, nothing happens. Otherwise, let one of the three nodes do spin transition. If the final triangle is gauge invariance, nothing happens, otherwise we find out all the gauge invariance triangles attached to the selected triangle.

(3) Find out all the gauge invariance triangles by the combined action of the stored triangles in step (2). If the number of gauge invariance triangles to store is nonzero, goto step (2), otherwise goto step (1).

For each avalanche, we define the size as the number of triangles stored in step (2) changed from the gauge invariance state to the other state, and plus the one changed in step (1). Besides, we define the time of avalanche as follows. If the number of stored triangles in step (2) changed from the gauge invariance state to the other state is nonzero, the time is increased by one, and plus the one changed in step (1).

At first, we study the SOC on the regular network. The structure of the regular network is given in Fig. 6. The size distribution and the time distribution are given in Fig. 7 (a) and 7 (b) respectively. In the following content, we keep the number of nodes in both the regular network and the small-world network constant as 9×10^4. Besides, the numerical results are obtained by taking the average of 100 simulations. For each simulation, we execute 9×10^6 time of disturbance.

In Fig. 7 (a) and Fig. 7 (b), the diagrams are curved when s and t are small. Because we set all spins to be zero initially. It takes a long time for the network to reach the sensitive state. While the diagrams curved at the ends is caused by the finite size effect.

Next, we study the SOC on the small-world network [15, 16]. The structure of the small-world network is given in Fig. 8.

When a triangle changes from gauge invariance to gauge variance, the value of the selected node is de-