

of no replicability null hypotheses. In the alternative procedure, H_j is rejected if $p_{1j} \leq \alpha/m$ and $p_{2j} \leq \alpha/m$. The two procedures differ in the thresholds used in each of the studies. The cut-off for p_{1j} is larger in the alternative procedure, since $\alpha_1 < \alpha$. However, the cut-off for p_{2j} may be substantially smaller in the alternative procedure, since $(\alpha - \alpha_1) / \sum_{i=1}^m \mathbf{I}[p_{1j} \leq \alpha_1/m]$ may be significantly larger than α/m . This is so in the common setting where signal is sparse in the primary study, i.e. $\sum_{j=1}^m h_{1j} \ll m$.

Example E.1. Suppose we have m independent normal outcomes in each of the two studies $T_{1j}, T_{2j}, j = 1 \dots, m$. In this example, $E(T_{11}) = \mu_{11}, E(T_{21}) = \mu_{21}, \text{Var}(T_{11}) = \text{Var}(T_{21}) = 1$, and outcomes $j = 2, \dots, m$ have expectation 0 and variance 1. Consider first the power of the alternative procedure that applies Bonferroni on the maximum of the two study p -values for FWER control at level $\alpha = 0.05$:

$$\pi_1 = \tilde{\Phi}(z_{1-\alpha/m} - \mu_{11}) \times \tilde{\Phi}(z_{1-\alpha/m} - \mu_{21}),$$

where $\tilde{\Phi}(\cdot)$ is the right tail of the standard normal distribution. Next, we compute the power of Procedure 3.1 with Bonferroni as the FWER controlling procedure. The probability of correctly selecting (PCS) the non-null hypothesis in the first study as well as $k - 1$ null hypotheses along with it is

$$PCS(k) = \tilde{\Phi}(z_{1-\alpha_1/m} - \mu_{11}) \binom{m-1}{k-1} (\alpha_1/m)^{k-1} (1 - \alpha_1/m)^{m-k},$$

so the power is

$$\pi_2 = \sum_{k=1}^m PCS(k) \times \tilde{\Phi}(z_{1-(\alpha-\alpha_1)/k} - \mu_{21}).$$

Figure 5 shows the power of the Bonferroni on maximum p -values procedure (left panel) and the power of Procedure 3.1 (right panel) for different configurations of (μ_{11}, μ_{21}) , where $(\alpha_1, \alpha) = (0.025, 0.05)$. In most configurations of μ_{11} and μ_{21} , Procedure 3.1 is more powerful than the Bonferroni on maximum p -values proce-