



FIG. 12: (Color online) The sudden death of (a) the atomic squeezing  $\xi_3^2$  and (b) the concurrence  $C$  under DPC. The parameters are chosen as  $N = 20$ ,  $n = 4$  and  $K = 0$ . The atomic squeezing  $\xi_3^2$  always disappear later than the concurrence  $C$ .

#### D. Depolarizing channel

Based on the map in Eq. (45) for DPC, one can find the following relations for single quasi-spin operators [14]

$$\langle \sigma_z(p) \rangle = s \langle \sigma_z \rangle_0, \quad (58a)$$

$$\langle \sigma_\alpha^1 \sigma_\beta^2(p) \rangle = s^2 \langle \sigma_\alpha^1 \sigma_\beta^2 \rangle_0 \quad (\alpha, \beta = x, y, z). \quad (58b)$$

Then the corresponding relations for the collective operators are  $\langle J_z(p) \rangle = s \langle J_z \rangle_0$  and

$$\langle J_z^2(p) \rangle = s^2 \langle J_z^2 \rangle_0 + (1 - s^2) \frac{N}{4}, \quad (59a)$$

$$\langle \mathbf{J}^2(p) \rangle = s^2 \langle \mathbf{J}^2 \rangle_0 + (1 - s^2) \frac{3N}{4}. \quad (59b)$$

Substituting the relevant expectation values and the correlation function into Eqs. (25,26) leads to the explicit expression of the spin-squeezing parameters

$$\xi_3^2(p)^D = \frac{s^2 \xi_0^2 + 1 - s^2}{\frac{4}{N^2} \langle \mathbf{J}^2(p) \rangle - \frac{2}{N}}. \quad (60)$$

In the same sense to investigate the evolution of the concurrence, the two-qubit density matrix is present as

$$\rho_{12}^D = \begin{bmatrix} v_+^D & 0 & 0 & u^{D,*} \\ 0 & w^D & y^D & 0 \\ 0 & y^D & w^D & 0 \\ u^D & 0 & 0 & v_-^D \end{bmatrix} \quad (61)$$

with relations  $y^D = s^2 y$ ,  $u^D = s^2 u$  and

$$v_\pm^D = \frac{s^2 + s}{2} v_\pm + \frac{s^2 - s}{2} v_\mp + \frac{1 - s^2}{4}, \quad (62a)$$

$$w^D = s^2 w_0 + \frac{1 - s^2}{4}. \quad (62b)$$

Thus we obtain the evolution of the concurrence as

$$C^D = \max \left\{ 0, 2 \left( s^2 w |\cos \phi| - \sqrt{v_+^D v_-^D} \right) \right\}. \quad (63)$$

The subindex D in the above Eqs. (60,61,63,63) represents that the evolutions of the spin-squeezing and the concurrence are taken under DPC. The sudden death of the spin-squeezing and the concurrence implied in Eq. (60,63) are shown in Fig. 12. In contrast of the ADC case, the atomic squeezing  $\xi_3^2$  always disappear later than the concurrence  $C$ .

#### V. OPTIMAL TIME FOR GENERATING ATOMIC SQUEEZING

Now, two competitive processes dominate generating the spin-squeezing. One is adiabatically manipulating the parameter  $\theta$  from 0 to  $\pi/2$ , which increases the spin-squeezing and the concurrence for storing the information of phonons into the atomic ensemble. However, the other process resulting from the various decoherence channels decreases the spin-squeezing and the concurrence, which means that the system loses information continuously. Therefore, the competition between this two processes leads to the existence of an optimal time for storing information, after which the information stored in the atomic ensemble is always losing. We can define two time scales: if only considering the adiabatic manipulation, one time scale is  $t_1$  representing the time when the spin squeezing increases to the half maximum value; if only considering the decoherence processes, the other time scale is  $t_2$  representing the time when the spin squeezing decreases to the half maximum value. Only when  $t_1 > t_2$  the optimal time exists.

We would like to demonstrate such competition in a typical quantum memory based on the cold  $^{87}\text{Rb}$  atomic ensemble, which is released from a magneto-optical trap at a temperature of about  $100\mu\text{K}$  [7]. In such quantum memory, the ground state  $|g\rangle$ , the metastable state  $|m\rangle$  and the excited state  $|e\rangle$  are chosen as  $|5S_{1/2}, F=1, m_F=1\rangle$ ,  $|5S_{1/2}, F=2, m_F=-1\rangle$  and  $|5S_{1/2}, F=2, m_F=0\rangle$ , respectively. An off-resonant  $\sigma^-$ -polarized write pulse contributes to the transition from  $|g\rangle$  to  $|e\rangle$  and the Stokes photon with  $\sigma^-$  polarization associates with the transition from  $|e\rangle$  to  $|m\rangle$ . The dark state for storing the photonic information is described in Eq. (7).

To realize the adiabatic creation of the non-classical correlation, we adopt the hyperbolic tangent type pulse sequence for both Rabi frequencies [26] as

$$g(t) = \Omega_m \left[ 1 - \tanh\left(\frac{a}{t} + \frac{a}{t-\tau}\right) \right], \quad (64a)$$

$$\Omega(t) = \Omega_m \left[ 1 + \tanh\left(\frac{a}{t} + \frac{a}{t-\tau}\right) \right] \quad (64b)$$

instead of the usual gaussian type ones [27] or solitary type ones [28]. Here,  $\Omega_m$  is the maximum Rabi frequency,  $\tau$  is the pulse length and  $a$  corresponds to the half width of the pulse. The pulse sequences of both Rabi frequencies are depicted in Fig. 13. Only at the time interval