

# Uniform cross phase modulation for nonclassical radiation pulses

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We propose a scheme to achieve a uniform cross phase modulation (XPM) for two nonclassical light pulses and study its application for quantum non-demolition measurements of the photon number in a pulse and for controlled phase gates in quantum information. We analyze the scheme by quantizing a common phenomenological model for classical XPM. Our analysis first treats the ideal case of equal cross-phase modulation and pure unitary dynamics. This establishes the groundwork for more complicated studies of non-unitary dynamics and difference in phase shifts between the two pulses where decohering effects severely affect the performance of the scheme.

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## I. INTRODUCTION

Optical cross phase modulation (XPM) is a specific variant of nonlinear optical phenomena of Kerr type in which the refractive index  $n_1$  of light in pulse 1 varies linearly with the intensity  $I_2$  of another light field, so that  $\epsilon_1 = 1 + \chi_1 + \chi_3 I_2$ , with  $\epsilon_1 = n_1^2$  the permittivity of the medium,  $\chi_1$  the linear, and  $\chi_3$  the nonlinear XPM susceptibility, respectively. In ordinary optical media this effect is small and requires large intensities, but the proposal by Schmidt and Imamoglu [1] to generate giant nonlinearities using electromagnetically induced transparency (EIT) [2–4] has made very large Kerr coefficients possible [1, 5–8] and may even lead to nonlinear effects at the single-photon level [9–15].

XPM is a strong candidate for the design of optical quantum controlled-phase gates (CPG) for photonic quantum information processing [16–18], in which two single-photon pulses would become entangled. For sufficiently high fidelity, XPM would allow the construction of deterministic gates, as opposed to the non-deterministic optical CPG that are based on linear optics [19] and establish entanglement through measurements [20]. It has been suggested that double EIT – EIT for both pulses with matched group velocities – would generate the maximal XPM phase shift because matched group velocities for both photons would maximize their interaction time [9–15]. However, XPM based on double EIT still faces some challenges: (i) to achieve sufficiently high intensities at the two-photon level, the photon pulses must be tightly confined in the transversal direction; (ii) if the nonlinear medium has a finite response time, the matter-light interaction unavoidably induces noise. It was shown by Shapiro [21] that for a large class of XPM models this would limit the fidelity of a CPG to only about 65%. Further theoretical studies on this topic confirmed these results [22–25]. (iii) For matched co-propagating pulses, each point of the pulse will experience a different XPM phase shift because the intensity, and hence the refractive index  $n_1$ , varies over the shape of the pulse. This would severely affect the entanglement between the pulses.

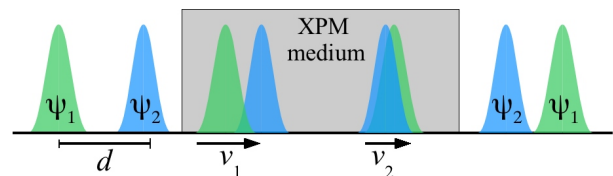


FIG. 1: Scheme to achieve uniform XPM: two light pulses  $\psi_1, \psi_2$  travel at different group velocities  $v_1 > v_2$  through an XPM medium such that  $\psi_1$  can overtake  $\psi_2$ .  $d$  denotes the initial distance between the two pulses.

In this paper we will address the third problem and do not deal with problem (i) and (ii), i.e., we will assume that transverse confinement of the photon pulses can be achieved by some means, such as hollow core fibers [26–29] or nano wires [30], and that the medium’s response time is so short that the polarizability of the medium reacts instantaneously to a photon’s electric field. The omission of the noise associated with a finite response time is also made for clarity because, despite that we are working in the instantaneous regime, we will show that very similar effects may appear if the two light pulses have different group velocities. To solve problem (iii) we extend an idea of Rothenberg [31] who demonstrated a uniform XPM phase shift for classical pulses in birefringent fibers: two light pulses travel through a XPM medium with different group velocities  $v_1 > v_2$ , see Fig. 1. Pulse 2 (blue) reaches the Kerr medium first but pulse 1 (green) is faster and leaves it first. While the pulses overlap they interact via the Kerr effect, which is proportional to the pulse intensity. Because pulse 1 overtakes pulse 2 the acquired phase shift will be averaged over the pulse shape and thus be nearly uniform.

Here we generalize this idea to characterize the propagation of quantized light pulses and show that a uniform phase shift can also be achieved for quantum interference effects. We will show that quantizing the classical XPM equations is generally a subtle problem which may make the introduction of noise terms similar to those in problem (i) necessary. We exploit our results to suggest exper-