The fraction of observed counts in the cell C is a linear combination of empirical cumulative distribution functions

$$\frac{o_C}{N-(m-1)} = \hat{F}_{XY}(x_i, y_i) + \hat{F}_{XY}(x_j, y_j) - \hat{F}_{XY}(x_i, y_j) - \hat{F}_{XY}(x_j, y_i),$$

and the expected fraction under the null is a function of the marginal cumulative distributions

$$\frac{e_C}{N-(m-1)} = \{\hat{F}_X(x_j) - \hat{F}_X(x_i)\}\{\hat{F}_Y(y_j) - \hat{F}_Y(y_i)\}.$$

where $\hat{F}$ denotes the empirical distribution function based on $N-(m-1)$ sample points.

By the Glivenko-Cantelli theorem, uniformly almost surely,

$$\begin{aligned}
&\lim_{N\to\infty}\left(\frac{o_C}{N-(m-1)} - \int_{\{(x,y):x\in(x_i,x_j],y\in(y_i,y_j]\}} h(x,y)dxdy\right) = 0,\\
&\lim_{N\to\infty}\left\{\frac{e_C}{N-(m-1)} - \left(\int_{\{x:x\in(x_i,x_j]\}} f(x)dx\right)\left(\int_{\{y:y\in(y_i,y_j]\}} g(y)dy\right)\right\} = 0.
\end{aligned} \tag{A.1}$$

Therefore, by Slutsky's theorem and the continuous mapping theorem, we have that uniformly almost surely

$$\begin{aligned}
&\lim_{N\to\infty}\frac{1}{N-(m-1)}\frac{(o_C-e_C)^2}{e_C} = \lim_{N\to\infty}\frac{\left(\frac{o_C}{N-(m-1)} - \frac{e_C}{N-(m-1)}\right)^2}{\frac{e_C}{N-(m-1)}}\\
&\geq \lim_{N\to\infty}\left(\frac{o_C}{N-(m-1)} - \frac{e_C}{N-(m-1)}\right)^2\\
&= \lim_{N\to\infty}\left[\int_{\{(x,y):x\in(x_i,x_j],y\in(y_i,y_j]\}}\{h(x,y)-f(x)g(y)\}dxdy\right]^2,
\end{aligned} \tag{A.2}$$