In Chapter 1, I obtained sharp bounds on the DTE when marginal distributions are fixed and MTR is imposed. Compared to Figure 1, Figure 4 shows that under MTR the lower bound on the DTE improves by allowing more mass to be added between $Y_1 = Y_0 + \delta$ and $Y_1 = Y_0$. Lemma 5 presents sharp bounds on the DTE under MTR and fixed marginals F_0 an F_1 as follows:

Lemma 5 *(?) Under MTR, sharp bounds on the DTE are given as follows: for fixed marginals F_0 an F_1 and any $\delta \in \mathbb{R}$,*

$$F_\Delta^L(\delta) \le F_\Delta(\delta) \le F_\Delta^U(\delta),$$

where

$$F_\Delta^U(\delta) = \begin{cases} 1 + \inf\limits_{y \in \mathbb{R}} \{\min(F_1(y) - F_0(y-\delta)), 0\}, & \text{for } \delta \ge 0, \\ 0, & \text{for } \delta < 0. \end{cases},$$

$$F_\Delta^L(\delta) = \begin{cases} \sup\limits_{\{a_k\}_{k=-\infty}^{\infty} \in \mathcal{A}_\delta} \sum\limits_{k=-\infty}^{\infty} \max\{F_1(a_{k+1}) - F_0(a_k), 0\}, & \text{for } \delta \ge 0, \\ 0, & \text{for } \delta < 0, \end{cases}$$

where $\mathcal{A}_\delta = \left\{ \{a_k\}_{k=-\infty}^{\infty} ; 0 \le a_{k+1} - a_k \le \delta \text{ for every integer } k \right\}$.

From Lemmas 3, 4, and 5, it is straightforward to derive sharp bounds on the joint distribution and the DTE under $M.1 - M.4$ and MTR.

The specific forms of sharp bounds on marginal distributions of Y_0 and Y_1, their joint distribution, and the DTE under $M.1 - M.4$ and MTR are provided in Theorem 3 in Appendix.

4 Discussion

4.1 Testable Implications

I here show that NSM and MTR yield testable implications.

Note that NSM implies the following: for any $(z', z) \in \mathcal{Z} \times \mathcal{Z}$ such that $p(z') \ge p(z)$, and for