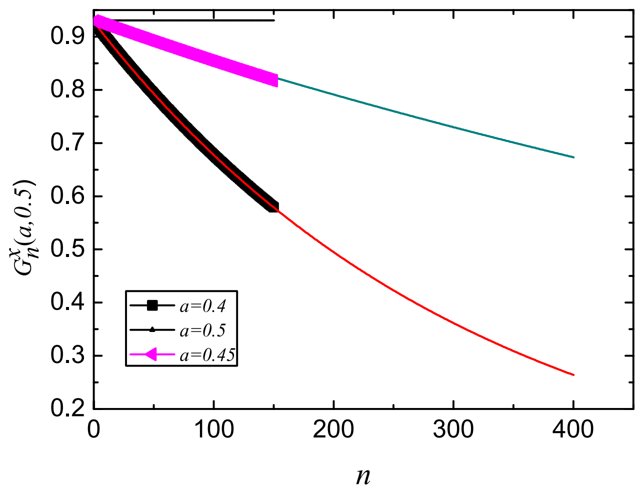

FIG. 5. Numerical fitting of $G_n^x(a,b)$ at $b = 0.5$ up to $n = 150$. Through numerical fit of the two typical data $G_n^x(0.4, 0.5)$ and $G_n^x(0.45, 0.5)$, we find $G_n^x(a,b) \sim n^{-c_1} e^{-c_2 n}$, with $G_n^x(0.4, 0.5) = n^{-0.0000404236} e^{(-0.074916n-0.0031414)}$ and $G_n^x(0.45, 0.5) = n^{-0.0000460715} e^{(-0.07259538n-0.00080686)}$. We extrapolate those two curves to $n = 400$.

it after a long enough time evolution (Fig.4(d)). So, only when static case $a = b < 1$, the long-range correlation in x direction is preserved. In other cases we can have when $n \to \infty$, $\langle \sigma_0^x(a,b)\sigma_n^x(a,b)\rangle \to 0$, leading to $\langle \sigma_x(a,b)\rangle = 0$ straightforwardly. A physical intuition tells us that because the magnetic field is in the z direction, the long-range correlation in the z direction can survive after the quench but the long-range correlation in the x direction is destroyed by the quench.

We note that the long range correlation is also researched by Sengupta *et al.* in Ref.[3]. They give some analytic results of the two point correlation function perpendicular to the magnetic field direction in quantum Ising model. However, they only consider two limit cases, namely the initial magnetic field is fixed at $a = 0$ and $a = \infty$. In both cases, $G_n^x(a,b)$ tends to zero when $n \to \infty$ but in different ways which depend on the value of the final magnetic field b. In Fig.4, one can see that $G_n^x(0, 0.5)$ tends to zero much slower than $G_n^x(0, 2)$ and there is a clear spatial oscillation of $G_n^x(2, 0.5)$. These phenomena are consistent with Ref.[3]. Therefore, our numerical results confirm their analytic results about long range correlation and give an extension to the general situation beyond the two limit cases.

V. A FURTHER DISCUSSION: FIXED INITIAL MAGNETIC FIELD

Up to now we only consider the case in which the final magnetic field b is fixed and all quench quantities are regarded as functions of initial magnetic field a. For completeness, we discuss the case in which a is fixed and all quench quantities are regarded as functions of b. As one can expect, the quench quantities will have different behaviors. If we make a similar calculation with that in Sec.II using perturbation theory, we can obtain

$$\begin{aligned}\partial_b\langle A(a,b)\rangle &= 2\sum_n\sum_{m\neq n}\frac{\langle\phi_n(b)|A|\phi_n(b)\rangle}{\omega_n(b)-\omega_m(b)}\times\Re[(H_I)_{nm}\langle\psi_0(a)|\phi_n(b)\rangle\langle\phi_m(b)|\psi_0(a)\rangle]\\&+2\sum_n\sum_{m\neq n}\frac{\langle\psi_0(a)|\phi_n(b)\rangle\langle\phi_n(b)|\psi_0(a)\rangle}{\omega_n(b)-\omega_m(b)}\times\Re[(H_I)_{nm}\langle\phi_m(b)|A|\phi_n(b)\rangle].\end{aligned} \tag{3}$$

It is difficult to study the behavior of Eq.(3) near $b = \lambda_c$ for a general a. However, the case in which $a = \lambda_c$ is easy to analyze. When $a = \lambda_c = b$, $\phi = \psi$. Therefore in the first term of the right hand side of Eq.(3), $\langle\psi_0(a)|\phi_n(b)\rangle\langle\phi_m(b)|\psi_0(a)\rangle = \delta_{m,0}\delta_{n,0}$, meaning that this term is 0 because $m \neq n$ in the sum. Similarly, we have $\langle\psi_0(a)|\phi_n(b)\rangle\langle\phi_n(b)|\psi_0(a)\rangle = \delta_{n,0}$ in the second term of Eq.(3). So we have

$$\partial_b\langle A(\lambda_c,b)\rangle|_{b=\lambda_c} = 2\sum_{m\neq 0}\frac{\Re[(H_I)_{0m}\langle\phi_m(\lambda_c)|A|\phi_0(\lambda_c)\rangle]}{\omega_0(\lambda_c)-\omega_m(\lambda_c)}.$$

This equation is the same with Eq.(1) and means that $\partial_b\langle A(\lambda_c,b)\rangle$ will diverge at $b = \lambda_c$ because the vanish of the energy gap.

In the following we use $\partial_b G_1^z(a,b)$ as an example to analyze (see Fig.6). As we expect above, $\partial_b G_1^z(a,b)$ is divergent at $b = 1$ when $a = 1$. While $a \neq 1$, it is discontinuous at $b = 1$. How to explain the discontinuity is still an open question to us.

VI. SUMMARY

In summary, starting from a general Hamiltonian which may undergo a QPT with the change of a controllable parameter, we obtain a general conclusion that in a sudden quench system, when the final Hamiltonian is fixed, the behavior of the time-averaged expectation of any observable has close relationship with the gapless excitation of the initial Hamiltonian. This general conclusion, clarified by our example of XY model, to a large extent explains the similarity between the critical phe-