We observe that the Dirac commutator given in equation (21) has exactly the same structure as equation (2) obtained in chapter 2.

The explicit form of the Dirac commutator for the electronic density operator $\vartheta_e(x)$ and the conjugate field $P_e(x')$, $[\vartheta_e(x), P_e(x')]_{Dirac}$ will be given by:

$$[\vartheta_e(x), P_e(x')]_{Dirac} = [\vartheta_e(x), P_e(x')](1 - \frac{\mathbf{h}(x,d)(sin(2K_F x + \sqrt{\pi}\vartheta_e(x)))^2}{(sin(\sqrt{2\pi}\vartheta_s(x)))^2 + (sin(2K_F x + \sqrt{\pi}\vartheta_e(x)))^2}) \tag{23}$$

The constraint operator $Q_1(x)|F >= 0$ causes any operator $R(x)$ to obey the equation: $R(x)Q_1(x)|F >= 0$. We choose $R(x) = cos[2K_F x + \sqrt{\pi}\vartheta_e(x)] - cos[\sqrt{2\pi}\vartheta_s(x)]$ and construct the product $R(x)Q_1(x) = (cos[2K_F x + \sqrt{\pi}\vartheta_e(x)])^2 - (cos[\sqrt{2\pi}\vartheta_s(x)])^2$. Adding and subtracting one from $R(x)Q_1(x)$ gives us a new equation $R_2(x)$ which obeys:

$$R_2(x)|F >\equiv [(sin(\sqrt{2\pi}\vartheta_s(x)))^2 - (sin(2K_F x + \sqrt{\pi}\vartheta_e(x)))^2]|F >= 0 \tag{24}$$

This gives as the result:

$$\begin{aligned}&\frac{(sin(2K_F x + \sqrt{\pi}\vartheta_e(x)))^2}{(sin(\sqrt{2\pi}\vartheta_s(x)))^2 + (sin(2K_F x + \sqrt{\pi}\vartheta_e(x)))^2})|F >= \\ &\frac{(sin(2K_F x + \sqrt{\pi}\vartheta_e(x)))^2}{2(sin(2K_F x + \sqrt{\pi}\vartheta_e(x)))^2[1 + \frac{((sin(\sqrt{2\pi}\vartheta_s(x)))^2 - (sin(2K_F x + \sqrt{\pi}\vartheta_e(x)))^2}{(sin(2K_F x + \sqrt{\pi}\vartheta_e(x)))^2}]}|F >= \frac{1}{2}\end{aligned} \tag{25}$$

The result in equation (25) was obtained after expanding the last expression in powers of $(\frac{R_2(x)}{(sin(2K_F x + \sqrt{\pi}\vartheta_e(x)))^2}])^n$ and using the condition $(R_2(x))^n|F >= 0$ The commutator $[\vartheta_e(x), P_e(x')]$ is finite only for an infinite number of particles and vanishes otherwise. The anomalous commutator will be given by the expectation value $< F|[\vartheta_e(x), P_e(x')]|F >$ [31, 33]. This allows us to define the **Dirac commutator** $[\vartheta_e(x), P_e(x')]_{Dirac}$.

$$\begin{aligned}&[\vartheta_e(x), P_e(x')]_{Dirac} \equiv < F|[\vartheta_e(x), P_e(x')]_{Dirac}|F > \\ &= i\delta(x - x')[1 - \mathbf{h}(x,d) < F|\frac{(sin(2K_F x + \sqrt{\pi}\vartheta_e(x)))^2}{(sin(\sqrt{2\pi}\vartheta_s(x)))^2 + (sin(2K_F x + \sqrt{\pi}\vartheta_e(x)))^2})|F >] \\ &= i\delta(x - x')[1 - \frac{1}{2}\mathbf{h}(x,d)]\end{aligned} \tag{26}$$

This result shows that the anomalous commutator (Dirac commutator) for the wire has been modified to $\frac{i}{2}\hbar$. This is consistent with fact that the electronic density for the wire is half the density of the leads (see figure 1).