n, $m =, -1, 0, 1$. It is further seen that the ground state is not the $u_{1,0,0}(x, y)$ Bloch state, as for a regular square lattice lacking SO couplings. This fact is at the heart of the *Jahn-Teller effect* [49], where the Dirac cone (or *conical intersection* as it is usually referred to when it appears in position space rather than in momentum space) induces a symmetry breaking that lowers the ground state energy. Furthermore, we note that the two lowest bands are not gapped, while there is a gap between the second and third band. Indeed, due to the SO coupled two-level structure of the Hamiltonian, the bands come in pairs where each pair possesses Dirac cones for quasi-momenta $(k_x, k_y) = (n, m)$. The effective number of Dirac cones is four (easily seen by shifting the Brillouin zone by $\frac{1}{2}$ in both directions), compared to two Dirac cones encountered in honeycomb lattices, like in graphene. We notice that recently it was shown how more complex lattice potentials may render single Dirac cones within the first Brillouin zone [50].

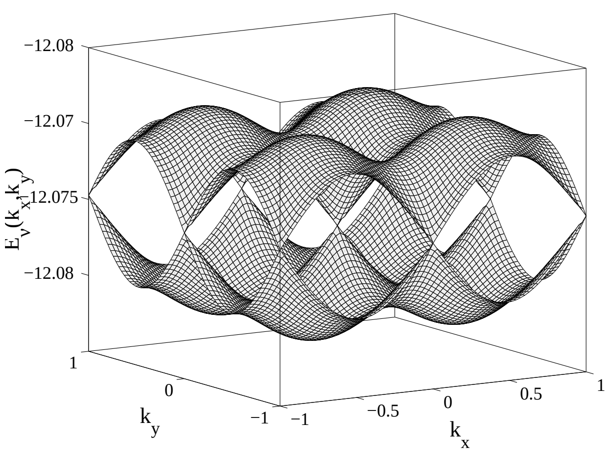

FIG. 4: The two lowest Bloch bands $E_\nu(k_x, k_y)$ for a square lattice with Rashba spin-orbit coupling. In this example, $v_{so} = 1$ and $V = 5$.

In the adiabatic regime, a non-zero extrinsic force $F_{ex}\mathbf{e}_x$ is treated by using the same approximation as above, yielding

$$u_{\nu,k_x,k_y}(x, y) \to u_{\nu,k_x-F_{ex}t,k_y}(x, y). \tag{17}$$

In the presence of a periodic potential, this adiabatic assumption has become known as the *acceleration theorem* [51]. It implies that no population is transferred between the individual bands ν. As a direct consequence, an initial state with well localized quasi-momentum within one single band will show an oscillatory motion rather than a constantly accelerating one, once the extrinsic force $F_{ex}\mathbf{e}_x$ has been turned on. These are the well studied Bloch oscillations [52], which have been experimentally verified in a variety of systems, such as cold atomic gases [33], BECs [25], semiconductor superlattices [53], waveguide arrays [54], and photonic crystals [55].

BOs are clearly an outcome of the periodic potential and its characteristic energy spectrum possessing gaps of forbidden energies at the center and edges of the Brillouin zone. As the amplitude of the potential V vanishes, the gaps close and the acceleration theorem breaks down. In particular, for weak lattice amplitudes, the gap between the first two energy bands scales as $\sim V$. By linearizing the dispersions around the band gap and replacing k_x by $k_x - F_{ex}t$, one obtains a realization of the celebrated *Landau-Zener model* [56]. Here, k_x is the initial quasi-momentum in the x-direction. This model is analytically solvable, giving the probability of population transfer between the two states of the corresponding bands. It has been shown that the decay of BOs can be correctly estimated using the Landau-Zener theory [25, 57].

In the present model, the situation becomes qualitatively different. At the Dirac points, the gap vanishes and the acceleration theorem cannot be imposed. However, from Fig. 4 it is seen that the Dirac cones appear between the first two bands, and not between the second and third band. Hence, assuming a state initially prepared on the lowest band one may expect predominant population transfer between the $\nu = 1$ and $\nu = 2$ bands and not to higher bands. Thereby, further acceleration is hindered by the gap between the second and third band and spin-dependent BOs may still be encountered. In other words, a modified acceleration theorem involving pair of bands could result. In fact, as already mentioned, the acceleration theorem is a specific example of the more general adiabatic theorem, and in molecular and chemical physics it has long been known that adiabaticity breaks down in the vicinity of a conical intersection [58]. In this context, it should be kept in mind that adiabaticity is a widespread concept in physics, and the general definition concerns systems whose dynamics is governed by an explicitly time-dependent Hamiltonian. Adiabaticity, as it appears in the acceleration theorem, is in this sense more reminiscent of the *Born-Oppenheimer approximation* [58], applicable to time-independent Hamiltonians describing light and heavy quantal degrees of freedom corresponding here to the spin and the spatial motion, respectively, of the particles.

III. NUMERICAL RESULTS

Our numerical analysis will be restricted to a system consisting of weakly interacting atoms, for which a mean-field description is expected to capture the phenomena we are interested in. In the s-wave scattering regime, we thereby consider dynamics rendered by a spinor Gross-Pitaevskii equation [14, 21]