Chapter 6

K-orbit Closures as Universal Degeneracy Loci

In this chapter, we describe our main application of the formulas of Chapters 2-5. Namely, in the type A cases, we realize the K-orbit closures as universal degeneracy loci of a certain type determined by K. We describe a translation between our formulas for equivariant fundamental classes of K-orbit closures and Chern class formulas for the fundamental classes of such degeneracy loci. Lastly, we indicate that similar results should hold for the symmetric pairs considered in types BCD, given explicit linear algebraic descriptions of K-orbit closures in those cases.

Before handling the specifics of each case, we first describe the general setup. Denote by E a contractible space with a free action of G. Then E also has a free action of B, and of K, by restriction of the G-action. We shall use the same space $E = EG = EB = EK$ as the total space of a universal principal G, B, or K-bundle, as appropriate. Denote by BG, BB, and BK the quotients of E by the actions of G, B, and K, respectively. These are classifying spaces for the respective groups.

The reason we have worked in S-equivariant cohomology $H^*_S(G/B)$ throughout is to take