where the inequality follows from the fact that $\frac{e_C}{N-(m-1)} \leq 1$.

We shall show that this limit can be bounded from below by a positive constant that depends on (x_0, y_0) and R but not on $\mathcal{I}$. Since

$$\begin{aligned}
&\{(x,y) : x \in (x_0 + R/4, x_0 + 3R/4], y \in (y_0 + R/4, y_0 + 3R/4]\} \\
&\subseteq \{(x,y) : x \in (x_i, x_j], y \in (y_i, y_j]\},
\end{aligned}$$

a positive lower bound on expression (A.2) can be obtained:

$$\begin{aligned}
&\lim_{N\to\infty} \left[\int_{\{(x,y):x\in(x_i,x_j],y\in(y_i,y_j]\}} \{h(x,y) - f(x)g(y)\} dxdy \right]^2 \\
&\geq \left[\int_{\{(x,y):x\in(x_0+R/4,x_0+3R/4],y\in(y_0+R/4,y_0+3R/4]\}} \{h(x,y) - f(x)g(y)\} dxdy \right]^2 \\
&\geq c^2 \int_{\{(x,y):x\in(x_0+R/4,x_0+3R/4],y\in(y_0+R/4,y_0+3R/4]\}} dxdy = c^2 R^2/4, \qquad \text{(A.3)}
\end{aligned}$$

where the first inequality follows since $h(x,y) - f(x)g(y) > 0$ in $\mathcal{A}$, and the second inequality follows since the minimum value is $c > 0$. Therefore, it follows that $\frac{1}{N-m+1} \frac{(o_C - e_C)^2}{e_C}$ converges uniformly almost surely to a positive constant greater than $c' = c^2 R^2/4$,

$$Pr\left(\lim_{N\to\infty} \frac{1}{N-m+1} \frac{(o_C - e_C)^2}{e_C} \geq c' \right) = 1. \qquad \text{(A.4)}$$

The partition $\mathcal{I}$ either contains the cell C, or a group of cells that divide C. By Jensen's inequality, it follows that if the partition $\mathcal{I}$ contains a group of cells that divide C, the score is made larger, since for any partition of the cell C, $C = \cup_l C_l$,

$$\left(\frac{o_C - e_C}{e_C} \right)^2 = \left(\sum_l \frac{e_{C_l} \left(\frac{o_{C_l}}{e_{C_l}} - 1 \right)}{\sum_h e_{C_h}} \right)^2 \leq \frac{\sum_l e_{C_l} \left(\frac{o_{C_l}}{e_{C_l}} - 1 \right)^2}{\sum_l e_{C_l}} = \frac{\sum_l \frac{(o_{C_l} - e_{C_l})^2}{e_{C_l}}}{e_C},$$