with the parameters $\omega_0 = \gamma(H_a^{\text{eff}} + H_0)$, $\omega_{\text{int}} = \omega_M LR^2/8a^3$, where ω_0 can be thought of as the mode gap frequency, ω_{int} determines the magnitude of the mode dispersion caused by interaction, $\omega_M = 4\pi\gamma M_s$ is a characteristic frequency of the material, which is ω_M= 30 GHz for permalloy. It is worth noting, the same structure (11) appears for the collective mode of vortex precession for the array of vortex state dots.[33] To define the parameters in (11) we used $\omega_{\text{int}} = \gamma M/2 = \gamma m_0/2a^3$ and the value of the magnetic moment $m_0 = \pi M_s LR^2$ for the single dot of cylindrical shape of thickness L and radius R.

Let us discuss the characteristic features of the dispersion relation represented by Eq. (11) in more detail. For small values of $|\vec{k}|$ both $\sigma(0) - \sigma(\vec{k})$ and $\left|\sigma_c(\vec{k})\right|$ are linearly increasing functions of $|\vec{k}|$ for all directions of $\vec{k}$, $\sigma(0)-\sigma(\vec{k}) \to 2\pi a|\vec{k}|$, $\left|\sigma_c(\vec{k})\right| \to 2\pi a|\vec{k}|/3$, as explained in the Appendix. Because of this the first bracket in Eq. (11) in the long-wave limit acquires the form $H_{tot}^{\text{eff}} + 4\pi akM$, $k =|\vec{k}|$. Note that the multiplier before k, in contrast with that for static demagnetizing field, is exactly the same as for continuum films. On the other hand the linear components in k compensate each other in the second bracket as $\vec{k} \to 0$ and only the quadratic terms in k remain (see Fig. 1). Thus the magnon spectra have a peculiarity as $\vec{k} \to 0$, $\omega(\vec{k}) \cong \omega_0 + 2\pi\gamma Ma|\vec{k}|$ (see Fig. 2), while the value of coefficient of $|\vec{k}|$ is exactly the same as for a thin magnetic film of saturation magnetization, M and thickness, a, see Ref. 37,38. Note the interesting feature that the lattice constant of the array, related to an in-plane space scale, plays the role of film thickness.

Since the spectrum's behavior depends on the value of the external magnetic field by a simple additive way, the sole parameter, determining the form of the spectrum, is the ratio $\lambda = \omega_0/\omega_{\text{int}}$. First, this dispersion relation is strongly anisotropic. For all values of $|\vec{k}|$ inside the Brillouin zone the frequency $\omega(\vec{k})$ is increasing monotonically for $\vec{k}$ parallel to the (1,1) direction, but for $\vec{k}$ along the (1,0) axis the dependence can be non-monotonic, as seen in Fig. 2. The oscillations have $\omega^2 > 0$ and the ferromagnetic state is stable for weak enough interaction, $\omega_0 > 1.172\omega_{\text{int}}$. Near the point of instability the dependence $\omega(\vec{k})$ for $\vec{k} \quad || \ (1{,}0)$ has a minimum near the boundary of the Brillouin zone, which is present until $\lambda = 3.6$ which is also seen in Fig. 2. In all this region of parameter space $1.172 < \omega_0/\omega_{\text{int}} < 3.6$ some more extrema and saddle points are present inside the Brillouin zone. For weaker interactions, $\omega_{\text{int}} < \omega_0/3.6$, the dependence of $\omega(\vec{k})$ becomes monotonic inside all of the Brillouin zone.

Here it is remarked that the stability condition for the dot array against small perturbations is nothing but the condition of positive definiteness of the function $\omega^2(\vec{k})$. The values