B. Optical analogue of population trapping

If one of the two channel waveguides, say e.g. W_2, were removed, light initially injected into waveguide W_1 would decay into the slab waveguide, a phenomenon fully analogous to the quantum mechanical decay of a bound state coupled to a continuum. In the markovian approximation, valid for weak coupling and for a nearly unstructured continuum, the decay is well described by an exponential law. The presence of waveguide W_2 generally modifies the decay behavior and, under certain conditions, the decay can be suppressed owing to a destructive Fano-like interference of different decay channels. Full or fractional suppression of the decay is related to the appearance of a trapped (or dark) state in the continuum. To derive the decay laws of light waves in the channel waveguides W_1 and W_2, we follow a standard procedure [13], detailed for instance in Ref.[9], and eliminate the amplitudes c_k of continuous modes by a formal integration of Eq.(7) with the initial condition $c_k(0,\Omega) = 0$. This yields a set of two coupled integro-differential equations for the amplitudes c_1 and c_2 of discrete modes. In the weak coupling limit and assuming a nearly unstructured continuum, the markovian approximation can be made and the following reduced equations for amplitudes c_1 and c_2 are derived

$$\frac{\partial c_1}{\partial z} = -i\Delta\beta_0 c_1 - \sigma(c_1+c_2) \ , \ \ \frac{\partial c_2}{\partial z} = -i\Delta\beta_0 c_2 - \sigma(c_1+c_2), \tag{8}$$

where

$$\sigma = \int_0^\infty d\tau \int dk |g(k)|^2 \exp\{-i[\Delta\beta(k) - \Delta\beta_0]\tau\} \tag{9}$$

is the decay rate of the single channel waveguide into the continuum. The solution to Eqs.(8) reads explicitly

$$c_1(z,\Omega) = S_{11}(z)c_1(0,\Omega) + S_{12}(z)c_2(0,\Omega) \tag{10}$$
$$c_2(z,\Omega) = S_{21}(z)c_1(0,\Omega) + S_{22}(z)c_2(0,\Omega) \tag{11}$$

where

$$S_{11}(z) = \frac{1}{2}\exp(-i\Delta\beta_0 z)\,[1+\exp(-2\sigma z)] \tag{12}$$
$$S_{12}(z) = -\frac{1}{2}\exp(-i\Delta\beta_0 z)\,[1-\exp(-2\sigma z)] \tag{13}$$
$$S_{22}(z) = S_{11}(z) \ , \ \ S_{21}(z) = S_{12}(z). \tag{14}$$

Note that, in the quasi-monochromatic approximation assumed in this work, the matrix coefficients $S_{n,l}$ are independent of frequency Ω. To understand the appearance of the optical analogue of population trapping, let us consider the monochromatic case, with the only nonvanishing spectral component at $\Omega = 0$, and two different input excitations, corresponding the former to single waveguide excitation and the latter to simultaneous excitation of the two channel waveguides. In the former case, assuming for instance $c_1(0,\Omega) = \sqrt{2\pi}\delta(\Omega)$ and $c_2(0,\Omega) = 0$, one obtains

$$\psi(\rho,z) = S_{11}(z)u_1(\rho) + S_{12}(z)u_2(\rho) + S_{13}(z)\theta(\rho,z) \tag{15}$$

where the last term on the right hand side of Eq.(15) accounts for the light field tunnelled into the slab waveguide and $\theta(\rho,z)$ ia a suitable superposition of continuous modes $u_k(\rho)$ normalized such that $\int d\rho|\theta(\rho,z)|^2 = 1$. For power conservation, the relation $|S_{11}|^2+|S_{12}|^2+|S_{13}|^2 = 1$ then holds. Note that, after a propagation distance z a few times the decay length $l_d \equiv (1/\sigma)$, one has $|S_{11}|^2 = |S_{12}|^2 = 1/4$ and $|S_{13}|^2 \simeq 1/2$, i.e. half of the injected light power has decayed into the slab, whereas the other half of light power is equally distributed into the two channel waveguides. The fact that the decay is not complete (fractional decay) indicates that a bound state embedded in the continuum does exist. When both waveguides W_1 and W_2 are initially excited with coherent fields of equal amplitudes but opposite sign, i.e. $c_1(0,\Omega) = -c_2(0,\Omega) = \sqrt{2\pi}\delta(\Omega)$, one obtains $\psi(\rho,z) = [u_1(\rho) - u_2(\rho)]\exp(-i\Delta\beta_0 z)$, i.e. the decay into the slab waveguide is fully suppressed. This is due to a destructive interference effect between different decay channels when $c_2 = -c_1$ [see Eqs.(7) and (8)] and to the existence of a trapped state embedded in the continuum. Numerical examples of fractional light decay for single waveguide excitation, and of full decay suppression for simultaneous waveguide excitation in the trapped state, as obtained by a direct numerical analysis of Eq.(1) in the monochromatic regime, are shown in Fig.2. In the simulations, we assumed circular channel waveguides with a Gaussian index core profile of radius r_c (at $1/e$), and a step-index slab waveguide of thickness a. Equation (1) has been integrated by a standard split-step pseudospectral method with absorbing boundary conditions [9].

Generalization of light propagation in the non-monochromatic case simply follows from the superposition principle. For instance, if waveguides W_1 and W_2 are excited at the input plane by two pulses with envelopes $r_1(t)$ and $r_2(t)$, from Eqs.(4), (10) and (11) it follows that the field envelope $\psi(\rho,z,t)$ can be cast in the form