follows from the numerical calculation that $\rho_n(r)$ has no maxima for $r > r_{\rm max}(E_n)$ (see Fig. 4). Furthermore, taking into account all directions of the motion in the segment $[0, r_{\rm max}]$ (classical limit does not deal with a single classical orbit but an ansamble of classical orbits [22]) and taking into account that the probability of finding a particle in the spherical layer r, $r+dr$ is inverse proportional to the surface of the sphere with radius r, we find that the classical probability density is given by the formula

$$\rho_{\rm cl}(\boldsymbol{x}) \equiv \rho_{\rm cl}(r) = \frac{\theta(r_{\rm max} - r)}{4\pi r_{\rm max} r^2}, \tag{3.12}$$

where $r_{\rm max} = \frac{\sqrt{2E}}{\kappa}$ and $\theta(x)$ is the Heaviside step function. Clearly, the normalization condition is of the form

$$\int d^3\boldsymbol{x}\rho_{\rm cl}(\boldsymbol{x}) = \int_0^\infty \rho_{\rm cl}(r)d\mu(r) = 1, \tag{3.13}$$

where $d\mu(r) = 4\pi r^2 dr$. The comparison of the quantum probability density $\rho_n(r)$, and the classical one $\rho_{\rm cl}(r)$ for $r_{\rm max}(E_n)$ is shown in Fig. 4. As expected the differences between the quantum and the classical descriptions decrease as the quantum number n increases.

We now discuss the expectation values of both the kinetic and potential energies. Using the identity [23]

$$\frac{1}{[{\rm Ai}'(a_n)]^2}\int_0^\infty x{\rm Ai}^2(x+a_n)dx = -\frac{2}{3}a_n, \tag{3.14}$$

(3.10) and (3.9) we get

$$\langle\psi_n|c\hat{p}\psi_n\rangle = c\int d^3\boldsymbol{k}|\boldsymbol{k}||\psi_n(\boldsymbol{k})|^2 = \frac{2}{3}E_n, \tag{3.15}$$

where $\hat{p} = \sqrt{\hat{\boldsymbol{p}}^2}$. Hence, taking into account the form of the Hamiltonian in Eq. (3.1) we find

$$\left\langle\psi_n\middle|\frac{\kappa^2}{2}\hat{r}^2\psi_n\right\rangle = \frac{1}{3}E_n, \tag{3.16}$$

where $\hat{r} = \sqrt{\hat{\boldsymbol{x}}^2}$. We conclude that the virial theorem takes the nonstandard form in the case of the massless relativistic harmonic oscillator. More precisely, the roles of the kinetic energy and potential energies are exchanged. Interestingly, we have the same formulas on average kinetic and potential energies in the classical case. Indeed, from Eq. (3.12) it follows easily that

$$\left\langle\frac{\kappa^2}{2}r^2\right\rangle_{\rm cl} = \frac{\kappa^2}{2}\int_0^\infty r^2\rho_{\rm cl}(r)d\mu(r) = \frac{1}{3}E. \tag{3.17}$$