equations

$$
\begin{aligned}
ik_y E_x - \frac{\partial E_y}{\partial x} &= -i\omega H_z, \\
ik_y H_x - \frac{\partial H_y}{\partial x} &= i\omega\varepsilon E_z
\end{aligned}
\tag{51}
$$

we get

$$
\begin{aligned}
E_z &= \frac{1}{i\omega\varepsilon}\left(ik_y H_x - \frac{\partial H_y}{\partial x}\right), \\
H_z &= -\frac{1}{i\omega}\left(ik_y E_x - \frac{\partial E_y}{\partial x}\right).
\end{aligned}
\tag{52}
$$

Substituting into Eq. (52) the expansion (50) and analogous expansions for all other components of $\boldsymbol{E}$ and $\boldsymbol{H}$, we arrive at

$$
\begin{aligned}
E_z^n &= \sum_m \left(\frac{1}{i\omega\varepsilon}\right)_{n-m}(ik_y H_x^m - i\alpha_m H_y^m), \\
H_z^n &= -\frac{1}{i\omega}(ik_y E_x^n - i\alpha_n E_y^n),
\end{aligned}
\tag{53}
$$

where $(\Theta)_{n-m}$ is the Toeplitz matrix.

The following equations follow from the remaining four Maxwell equations:

$$
\begin{aligned}
\frac{dE_y^n}{dz} &= ik_y E_z^n - i\omega H_x^n, \\
\frac{dE_x^n}{dz} &= i\alpha_n E_z^n + i\omega H_y^n, \\
\frac{dH_y^n}{dz} &= ik_y H_z^n + \sum_m (i\omega\varepsilon)_{n-m} E_x^m, \\
\frac{dH_x^n}{dz} &= i\alpha_n H_z^n - \sum_m (i\omega\varepsilon)_{n-m} E_y^m.
\end{aligned}
\tag{54}
$$

One can substitute Eq. (53) into Eq. (54) and obtain a system of the first order differential equations for the Fourier components of the electromagnetic field $E_y^n, E_x^n, H_y^n, H_x^n$ in the region $0 \le z \le h$.

Now we have to determine the Rayleigh coefficients $R_{np}^{(e)}, R_{np}^{(h)}$ for the specific periodic geometry profile. One can determine these coefficients by matching the solution of equations inside the corrugation region $0 \le z \le h$ with Rayleigh expansions (44) at $z = h$ and expansions (48) at $z = 0$. This can be done by imposing the continuity conditions on each Fourier component of the fields E_y, E_x, H_y, H_x at $z = 0$ and $z = h$.