

$$\sum_{i=1}^q \alpha_i = \bar{\alpha}.$$

When  $\bar{G}(y)$  exists for some  $a$  and some matrix  $S$ , the matrix  $S$  can always be chosen such that  $0 \leq \alpha_1 \leq \dots \leq \alpha_q \leq \bar{\alpha}$ .

**Definition 4.1.** *A  $q \times p$  matrix of polynomials  $G(y)$  satisfies the "continuity of lower degree ranks property" (CLDR) if for some non-singular  $q \times q$  matrix  $S$  and for some  $\alpha = (\alpha_1, \dots, \alpha_q)$  such that  $\sum_{i=1}^q \alpha_i = \bar{\alpha}$ ,  $0 \leq \alpha_1 \leq \dots \leq \alpha_q \leq \bar{\alpha}$ , (15) provides a rank  $q$  matrix of polynomials  $\bar{G}(y)$ .*

Essentially, the CLDR property holds if for some  $S$  the transformed  $SG(y)$  is such that the stabilizing rate  $\bar{a}$  for the determinant is shared between the rows of the matrix  $SG(y)$  according to (15), and the limit matrix is non-singular.

The matrix  $\bar{G}(y)$  depends upon the choice of the matrix  $S$ . Indeed in Example 4.1  $\bar{\alpha} = 4$  but it is clear that for  $S = I$  Lemma 4.1 does not hold. This is a consequence of the fact that there is a linear dependence between the degree one polynomial terms in the rows of the matrix. However setting

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$