parton $a$ in a proton with momentum fraction between $x_a$ and $x_a + dx$, where $x_a = p_a/P_p$ and something of an abuse of notation occurred by which $p_a$ stands for the momentum of parton $a$ and $P_p$ stands for the momentum of the proton. The FF gives the probability of parton $c$ hadronizing into $h$ with fractional momentum between $z_c$ and $z_c+dz$, where $P_h = z_c p_c$. All allowed combinations of $a$, $b$, and $c$ are summed over where $a+b \to c+X$ such that $c$ may hadronize into $h$. We note that we have slightly extended the factorization theorem to include the fragmentation process, which introduces the new scale $\mu_F'$. Additionally a renormalization scale $\mu_R$ associated with the running coupling $\alpha_s$ is included in the perturbative cross section.

Parameterizations of PDFs may be found from, e.g., the CTEQ collaboration [161], GRV [162], MRST [163], or Alekhin [164]. Hadronic fragmentation functions have similarly been parameterized by, e.g., DSS [165], AKK [166], HKNS [167], and KKP [168]; [169] provides a photon FF. We note that the posted code for KKP has a bug in its leading order (LO) kaon output that has still not been corrected; ported *Mathematica* codes for the DSS and KKP fragmentation functions, as well as a corrected FORTRAN code for KKP can be found online [170]. Although PDFs and FFs at a specific scale must be found from experiment, their form is constrained by QCD and general sum rule considerations and their evolution is governed by pQCD and DGLAP dynamics [171–173].

Derivations of perturbative cross sections vary depending on the specific reaction considered. For those involving gluons and light guarks NLO calcu-