is the least dual D-norm function, corresponding to the case of independent univariate margins, where $\|\cdot\|_D = \|\cdot\|_1$, and

$$\wr \boldsymbol{x} \wr_\infty = \min_{1\le j\le d} |x_j|\,, \qquad \boldsymbol{x}\in\mathbb{R}^d,$$

is the largest dual D-norm function, corresponding to the perfect dependence case, where $\|\cdot\|_D = \|\cdot\|_\infty$. Hence, we have for an arbitrary dual D-norm function the bounds

$$0 = \wr \cdot \wr_1 \le \wr \cdot \wr_D \le \wr \cdot \wr_\infty .$$

For the next examples, the following abbreviation is useful. We define for $\boldsymbol{x}\in\mathbb{R}^d$ and a nonempty subset $T\subset\{1,\dots,d\}$

$$\boldsymbol{x}_T := (x_i,\ i\in T)\in\mathbb{R}^{|T|}.$$

Example 2.5 (Fréchet model). It is well-known that a D-norm is given by the l_λ-norm

$$\|\boldsymbol{x}\|_\lambda := \left(\sum_{i=1}^d |x_i|^\lambda\right)^{1/\lambda}, \qquad \boldsymbol{x}\in\mathbb{R}^d,\ \lambda\in(1,\infty), \tag{9}$$

usually referred to as the logistic model in the literature. Therefore, we obtain by (8)

$$\wr \boldsymbol{x} \wr_\lambda = \sum_{\emptyset\ne T\subset\{1,\dots,d\}} (-1)^{|T|-1}\,\|\boldsymbol{x}_T\|_\lambda\,, \qquad \boldsymbol{x}\in\mathbb{R}^d,\ \lambda\in(1,\infty).$$

A generator $\boldsymbol{Z}=(Z_1,\dots,Z_d)$ of $\|\cdot\|_\lambda$ can easily be found: Put $Z_i := \tilde Z_i/\Gamma\left(1-1/\lambda\right)$, $i=1,\dots,d$, where $\tilde Z_1,\dots,\tilde Z_d$ are iid Fréchet distributed with parameter λ, and Γ denotes the gamma function.

Example 2.6 (Weibull model). We can define a generator $\boldsymbol{Z}=(Z_1,\dots,Z_d)$ by taking independent Weibull distributed random variables $\tilde Z_1,\dots,\tilde Z_d$, i. e. $P(\tilde Z_1 > t) = \exp(-t^\alpha)$, $t>0$, $\alpha>0$, and putting $Z_i := \tilde Z_i/\Gamma(1+1/\alpha)$. It is easy to show that the corresponding dual D-norm function is for $\boldsymbol{x}\in\mathbb{R}^d$, $x_i\ne 0$, $i=1,\dots,d$,