$$\begin{aligned} U(t_0) &= V(t_0) = W(t_0) = 0 \text{ and } f(0) = g(0) = h(0), \\ \theta(s) &= -\int_{s_0}^{s} \tau ds + \theta_0, \qquad (3.6) \\ g^{'}(0) a_{21} m(s) V'(t_0) &= \lambda(s)\sinh\theta,\ h^{'}(0) a_{31} n(s) W'(t_0) = \lambda(s)\cosh\theta, \end{aligned}$$

$\lambda(s) \neq 0$, where $l(s), m(s), n(s), U(t), V(t)$ and $W(t)$ are C^1 functions, $a_{ij} \in \mathbb{R}$ $(k = 1, 2, 3; j = 1, 2, 3, ..., p)$.

Example 4 *Let* $r(s) = \left(a\sinh\left(\frac{s}{c}\right), \frac{bs}{c}, a\cosh\left(\frac{s}{c}\right)\right)$ *be a spacelike curve* ,

$a, b, c \in \mathbb{R},\ a^2 + b^2 = c^2$ and $-2 \leq s \leq 2$. It is easy to show that

$T(s) = \left(\frac{a}{c}\cosh\left(\frac{s}{c}\right), \frac{b}{c}, \frac{a}{c}\sinh\left(\frac{s}{c}\right)\right)$,

$N(s) = \left(\sinh\left(\frac{s}{c}\right), 0, \cosh\left(\frac{s}{c}\right)\right)$,

$B(s) = \left(\frac{b}{c}\cosh\left(\frac{s}{c}\right), -\frac{a}{c}, \frac{b}{c}\sinh\left(\frac{s}{c}\right)\right)$.

By taking $\theta(0) = 0$ we have $\theta(s) = -\frac{bs}{c^2}$. If we choose $\lambda(s) \equiv 1,\ t_0 = 0,\ a_{21} = a_{31} = 1$ and

$u(s,t) = \sum\limits_{k=1}^{3} a_{1k} l(s) U(t) \equiv 0$,

$v(s,t) = \sinh\left(-\frac{bs}{c^2}\right) t + \sum\limits_{k=2}^{3} a_{2k}\sinh^k\left(-\frac{bs}{c^2}\right) t^k$,

$w(s,t) = \cosh\left(-\frac{bs}{c^2}\right) t + \sum\limits_{k=2}^{3} a_{3k}\cosh^k\left(-\frac{bs}{c^2}\right) t^k$

then the Eq. (3.5) is satisfied.

Letting $a = b = 1$, we immediately obtain a member of the surface pencil (Fig. 3.1) as

$P_1(s,t) = (\ \sinh\left(\frac{s}{\sqrt{2}}\right) + \sinh\left(-\frac{s}{2}\right) t \sinh\left(\frac{s}{\sqrt{2}}\right) + \sinh\left(\frac{s}{\sqrt{2}}\right) \sum\limits_{k=2}^{3} a_{2k}\sinh^k\left(-\frac{s}{2}\right) t^k +$
$\frac{\sqrt{2}}{2}\cosh\left(-\frac{s}{2}\right) t \cosh\left(\frac{s}{\sqrt{2}}\right) + \frac{\sqrt{2}}{2}\cosh(\frac{s}{\sqrt{2}}) \sum\limits_{k=2}^{3} a_{3k}\cosh^k\left(-\frac{s}{2}\right) t^k$,

$\frac{s}{\sqrt{2}} - \frac{\sqrt{2}}{2}\cosh\left(-\frac{s}{2}\right) t - \frac{\sqrt{2}}{2}\sum\limits_{k=2}^{3} a_{3k}\cosh^k\left(-\frac{s}{2}\right) t^k, \cosh(\frac{s}{\sqrt{2}}) + \sinh(-\frac{s}{2}) t \cosh(\frac{s}{\sqrt{2}}) +$
$\cosh(\frac{s}{\sqrt{2}}) \sum\limits_{k=2}^{3} a_{2k}\sinh^k\left(-\frac{s}{2}\right) t^k$