$(i_{l+1} - i_l) \times N_g/N$, where $l \in \{0, \ldots, m-1\}$, $i_0 = 0.5$ and $i_m = N + 0.5$. We consider either Pearson's score or the likelihood ratio score for a given cell C,

$$t_C \in \left\{ \sum_{g=1}^{K} \frac{[o_C(g) - e_C(g)]^2}{e_C(g)},\ \sum_{g=1}^{K} o_C(g) \log \frac{o_C(g)}{e_C(g)} \right\}. \tag{2.1}$$

For a given partition $\mathcal{I}$, the score is $T^{\mathcal{I}} = \sum_{C \in \mathcal{C}(\mathcal{I})} t_C$ (where if $t_C = \sum_{g=1}^{K} o_C(g) \log \frac{o_C(g)}{e_C(g)}$ then $T^{\mathcal{I}}$ is the likelihood ratio given the partition). Our test statistics aggregate over all partitions by summation (Cramer–von Mises-type statistics) and by maximization (Kolmogorov–Smirnov-type statistics):

$$S_m = \sum_{\mathcal{I} \in \Pi_m} T^{\mathcal{I}}, \quad M_m = \max_{\mathcal{I} \in \Pi_m} T^{\mathcal{I}}. \tag{2.2}$$

Tables of critical values for given sample sizes $N_1, \ldots, N_K$ can be obtained for (very) small sample sizes by generating all possible $N!/(\Pi_{g=1}^{K} N_g!)$ reassignments of ranks $\{1, \ldots, N\}$ to K groups of sizes $N_1, \ldots, N_K$ and computing the test statistic for each reassignment. The p-value is the fraction of reassignments for which the computed test statistics are at least as large as observed. When the number of possible reassignments is large, the null tables are obtained by large scale Monte Carlo simulations (we used $B = 10^6$ replicates for each given sample size $N_1, \ldots, N_K$). For each of the B reassignment selected at random from all possible reassignments, the test statistic is computed. Clearly, the B computations do not depend on the data, hence the tests based on these statistics are distribution free. Again, the p-value is the fraction of reassignments for which the computed test statistics are at least as large as the one observed, but here the fraction is computed out of the $B + 1$ assignments that include the B reassignments selected at random and the one observed assignment, see Chapter 15 in Lehmann and Romano (2005). The test based on each of these