

only one leg take negative helicity. In these cases, the D_2 relations hold trivially. Then we study the tree amplitudes with one and two gravitons in addition to N gluons where $N - 2$ gluons as well as all the gravitons take positive helicity and two gluons take negative helicity. We will show the D_2 relations also hold in this case. The discussions can be extended to tree amplitudes where N gluons minimally coupled to M gravitons with arbitrary helicity configurations. In general, any tree amplitudes for gauge-gravity minimal coupling can be expressed by partial tree amplitudes with $N + 2M$ gluons via D_2 relations. Our conclusions are given in Section V. Some useful properties of spinor helicity formalism are given in Appendix A.

II. KLT RELATIONS VERSUS D_2 RELATIONS

KLT relations[4] in string theory are the relations between amplitudes for closed strings on S_2 and open string tree amplitudes. KLT relations factorize amplitudes for closed strings on S_2 into products of two open string tree amplitudes corresponding to the left- and right-moving sectors except for a phase factor¹

$$\mathcal{M}_{S_2}^{(N)} \sim \kappa^{N-2} \sum_{P, P'} \mathcal{A}^{(N)}(P) \bar{\mathcal{A}}^{(N)}(P') e^{i\pi F(P, P')}, \quad (1)$$

Where $\mathcal{M}_{S_2}^{(N)}$ is N -point amplitude on S_2 while $\mathcal{A}^{(N)}$ and $\bar{\mathcal{A}}^{(N)}$ are the partial tree amplitudes for open strings corresponding to the left- and right-moving sectors. The phase factor only depends on the permutations P and P' of the legs in left- and right-moving sectors. The terms in KLT relations can be reduced by contour deformations. In the reduced form, the phase factors become sine functions. After taking the field theory limit $\alpha' \rightarrow 0$, the KLT

¹ In this paper, we use \sim to omit a proportional factor which does not affect our discussion.