straightforward to verify that U_{AB} is the desired isometry that realizes $\mathcal{E}$ and has q as the heat transfer matrix.

Finally note that any q that satisfies (39) necessarily yields a positive definite Q. However, Theorem 2 allows HTOs that may have negative eigenvalues. This shows that the condition (39) is not necessary. The proof of Theorem 4 is thus completed. $\square$

Note that part (b) of Theorem 4 is proved by finding a closely related Landauer erasure on a hypothetical system. It is possible to extend the scope of part (b) by making use of complete erasures. However, the expressions used in such an extension do not appear to be particularly illuminating and hence they are not given in here. Theorem 4 is a good starting point for discussing the properties of HTOs associated with generic quantum operations. The following result is an example. It is essentially a different way of saying that the set of HTOs associated with a given quantum operation has no upper limit.

Theorem 5 *Let q be a heat transfer matrix for $\mathcal{E}$ which is related to the corresponding HTO by (38). Then, for any q' with $q' > q$, q' is also a possible heat transfer matrix.*

To prove this, first define the matrix $s = q' - q$ and observe that it is strictly positive definite, i.e., all eigenvalues of s are positive. In such a case, it is possible to find a positive number $t \geq 1$ such that the matrix $q + ts$ satisfies (39) and therefore $q + ts$ is a possible heat transfer matrix by part (c) of Theorem 4. Finally, by the convexity of the set of HTOs, the matrix

$$q' = \frac{1}{t}(q + ts) + \frac{t-1}{t} q \tag{46}$$

is a possible heat transfer matrix. This completes the proof. $\square$

Note that this theorem cannot be extended by replacing the matrices q by the associated HTOs mainly due to the restriction in Theorem 4 (a) which must be satisfied by all HTOs. It is plausible that the strict inequality $q' > q$ can be replaced by the weaker requirement of $q' \geq q$, but this conjecture is still awaiting proof.

IV. DIFFERENCES FROM LANDAUER'S BOUND

The restrictions satisfied by the HTOs are closely related to LEP since both follow only from the condition that the transformation on the composite system is an isometry. However, LEP only relates the dumped heat to the drop in the von Neumann entropy of the state of the