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Appendix A: Measure for integrating over reparametrization

Introducing

$$v_i = s_i - s_{i-1} \qquad s_N = s_f \,, \tag{A1}$$

we rewrite the measure of [6] for the integration over reparametrizations as

$$\begin{aligned} \int_{s_0}^{s_f} \mathcal{D}_{\rm diff} s \;&\equiv\; \lim_{N\to\infty} \prod_{i=2}^{N-1} \int_{s_0}^{s_{i+1}} \frac{{\rm d}s_i}{(s_{i+1}-s_i)} \int_{s_0}^{s_2} \frac{{\rm d}s_1}{(s_2-s_1)(s_1-s_0)} \\ &= \lim_{N\to\infty} \prod_{i=1}^{N} \int_0^\infty \frac{{\rm d}v_i}{v_i}\, \delta^{(1)}\Big(s_f - s_0 - \sum_{j=1}^N v_j\Big). \end{aligned} \tag{A2}$$

The integration over v_i's in Eq. (A2) can be represented through the integration over a scalar field as follows. Writing

$$v_i = \,{\rm e}^{\,\psi_i/2} \,, \tag{A3}$$

we have

$$\int_0^\infty \frac{{\rm d}v_i}{v_i} \cdots = \frac{1}{2} \int_{-\infty}^{+\infty} {\rm d}\psi_i \tag{A4}$$

and

$$\int_{s_0}^{s_f} \mathcal{D}_{\rm diff} s = \lim_{N\to\infty} \frac{1}{2^N} \prod_{i=1}^{N} \int_{-\infty}^{+\infty} {\rm d}\psi_i \, \delta^{(1)}\Big(s_f - s_0 - \sum_{j=1}^N \,{\rm e}^{\,-\psi_j/2}\Big). \tag{A5}$$

This represents the continuous measure as

$$\int_{s_0}^{s_f} \mathcal{D}_{\rm diff} s = \int \mathcal{D}\psi \, \delta^{(1)}\Big(s_f - s_0 - \int_{s_0}^{s_f} {\rm d}t \,{\rm e}^{\,-\psi(t)/2}\Big), \tag{A6}$$

where t is a certain parametrization of the contour (e.g. through the proper time) and $\mathcal{D}\psi$ is the usual measure

$$\int \mathcal{D}\psi = \prod_{s=s_0}^{s_f} \int_{-\infty}^{+\infty} {\rm d}\psi(s)\,. \tag{A7}$$

The scalar field ψ, that appears in Eqs. (A1), (A3), is in fact a discretization of the boundary value of the Liouville field

$$\varphi(\tau,s)\Big|_{\rm boundary} = \psi(s)\,, \tag{A8}$$