the LSR holds if we consider the sign changes the Green's function undergoes not at $\omega = 0$ but at $\omega = \mu$.

2. *Superconducting Instability*

The residual interactions between the Fermi pockets and the Cooperons will lead to instabilities in the RPA solutions as temperature goes to zero. Provided a finite chemical potential is present the leading instability will be to a superconducting state. While gapless quasi-particles only exist in the A-bands, both A and B bands will go superconducting simultaneously. The general form of the Cooperon-quasiparticle interaction is

$$\begin{aligned} H_{\phi QP} &= \sum_{i=A,B;\mathbf{k},\mathbf{q}} \frac{\Gamma_i(\mathbf{k},\mathbf{q})}{(NLa)^{1/2}} \left[\Phi_i(\mathbf{q})\Delta^\dagger_{QPA}(\mathbf{k},\mathbf{q}) + h.c.\right] \\ &+ \frac{1}{2}\sum_{\mathbf{q},\mathbf{k},\mathbf{k}'} \frac{g(\mathbf{q},\mathbf{k},\mathbf{k}')}{NLa} \Delta^\dagger_{QPA}(\mathbf{k},\mathbf{q})\Delta_{QPA}(\mathbf{k}',\mathbf{q}) \\ \Delta^\dagger_{QPA}(\mathbf{k},\mathbf{q}) &= \epsilon_{\sigma\sigma'}[A^\dagger_{1\sigma}(\mathbf{k}+\mathbf{q})A^\dagger_{1\sigma'}(-\mathbf{k}) - A^\dagger_{2\sigma}(\mathbf{k}+\mathbf{q})A^\dagger_{2\sigma'}(-\mathbf{k})]. \end{aligned} \tag{3.11}$$

Here L is the length of the ladders, a is the interladder spacing, and N is the number of ladders in the array. $\Phi_{A,B}$ are the Cooperon fields whose bare propagators are defined as

$$D^0_i(\omega_n, k) = \langle T\Phi_i(\mathbf{k},\omega_n)\Phi^\dagger_i(\mathbf{k},\omega_n)\rangle_0 = \frac{v_{Fi}}{-(i\omega_n - 2\mu)^2 + \Delta_i^2 + (v_{Fi}k_x)^2}. \tag{3.12}$$

We see that g has the dimensionality of energy$\times$length^2 and Γ_i has the dimensionality of energy$\times$length$^{1/2}$.

The different terms in Eqn. (3.11) have different origins. The strongest interactions are presumably Γ_i as this term already exists for uncoupled ladders. Inter-ladder interactions, such as interladder Coulomb repulsion, also contribute to Γ_i. However interladder hoping does not – this contribution is suppressed due to a mismatch between the Fermi momenta of the A_i and B_i bands. The coupling g is smaller than Γ_i: it arises only in second order perturbation theory from intraladder interactions and from presumed weak inter-ladder Coulomb interactions.

The pair susceptibility for the quasiparticles Δ_{QPA} in an RPA approximation is given by

$$\chi^{RPA}_{QPA}(\omega_n,\mathbf{q}) = \frac{1}{LNa}\sum_{\mathbf{k_1},\mathbf{k_2}}\int_0^\beta d\tau e^{i\omega_n\tau}\langle T\Delta_{QPA}(\mathbf{k_1},\mathbf{q},\tau)\Delta^\dagger_{QPA}(\mathbf{k_2},\mathbf{q},0)\rangle$$