and

$$T^{-\tau} := \{t \in T \mid t\tau(t) = 1\}.$$

In the above definitions, the action of $\tau$ on $T$ is twisted by the distinguished involution $\theta$. That is,

$$\tau(t) = \tau.\theta(t),$$

where $\tau.t$ denotes the usual action of $W$ on $T$. In all cases of interest to us save one (the non-equal rank case in type $D$), the distinguished involution is simply the identity, and so the action of $\tau$ is the usual one. At any rate, with these definitions given, we have the following result:

**Proposition A.2.1.** *With notation as above,*

$$|\mathcal{X}_\tau| = |T_\tau / T_0^{-\tau}|.$$

For a proof, see [AdC09, Proposition 11.2 and Remark 11.5] or [dC05, Proposition 2.4].

In our examples, this result allows us to compute the cardinality of $\mathcal{X}$, and then compare it to the total number of clans which correspond to $K_i'$-orbits intersecting $X$, where $K_i = G \cap K_i'$, the groups $K_i$ are the fixed point subgroups of an entire inner class of involutions, and each $K_i'$ is isomorphic to an appropriate $GL(p, \mathbb{C}) \times GL(q, \mathbb{C})$. Since we have already established combinatorial descriptions of such clans in the previous subsection, the latter number is computable.

If these two counts turn out to be equal, then for each $K_i$, it is impossible for the intersection of any $K_i'$-orbit on $X'$ with $X$ to split as a union of multiple $K_i$-orbits — if it did, the cardinality of $\mathcal{X}$ would necessarily be greater than the clan count. Making this counting argument thus establishes in one fell swoop that for *any* $K_i$ in the inner class, each