We can derive the asymptotic expression as $x \to -\infty$ ($y \to -\infty$) by using the following formula for the asymptotic behaviour of the hypergeometric function [26]

$$ {}_2F_1(a,b,c;y) = \frac{\Gamma(c)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)}\,(-y)^{-a} + \frac{\Gamma(c)\Gamma(a-b)}{\Gamma(a)\Gamma(c-b)}\,(-y)^{-b}\,, \tag{27} $$

and obtain:

$$ \phi_L(x) \sim G\,e^{-ik(x+L)} + H\,e^{ik(x+L)} \tag{28} $$

where

$$ G = D_1\,A\,e^{i\pi\mu} - D_2\,C\,e^{-i\pi\mu} \tag{29a} $$

$$ H = D_1\,B\,e^{i\pi\mu} - D_2\,D\,e^{-i\pi\mu} \tag{29b} $$

and A, B, C, D are given by:

$$ A = \frac{\Gamma(2\mu)\Gamma(2\nu)}{\Gamma(\mu+\nu-\lambda)\Gamma(\mu+\nu+\lambda)} \tag{30a} $$

$$ B = \frac{\Gamma(2\mu)\Gamma(-2\nu)}{\Gamma(\mu-\nu-\lambda)\Gamma(\mu-\nu+\lambda)} \tag{30b} $$

$$ C = \frac{\Gamma(2-2\mu)\Gamma(2\nu)}{\Gamma(1-\mu+\nu-\lambda)\Gamma(1-\mu+\nu+\lambda)} \tag{30c} $$

$$ D = \frac{\Gamma(2-2\mu)\Gamma(-2\nu)}{\Gamma(1-\mu-\nu-\lambda)\Gamma(1-\mu-\nu+\lambda)} \tag{30d} $$

Similarly we can derive the asymptotic form of lower component $\chi(x)$ from Eq. (5a):

$$ \lim_{x\to-\infty} \chi(x)_L = G\,\frac{(E+k)}{m_0}\,e^{-ik(x+L)} + H\,\frac{(E-k)}{m_0}\,e^{ik(x+L)} \tag{31} $$

Similarly for the solution in the positive region we have from Eq. (24):

$$ \begin{aligned} \phi_R(z) \;&=\; d_1\,z^{-\nu}\,(1-z)^{-\rho}\,{}_2F_1(-\rho-\nu-\lambda, -\rho-\nu+\lambda; 1-2\nu; z) \\ &+\; d_2\,z^{\nu}\,(1-z)^{-\rho}\,{}_2F_1(-\rho+\nu-\lambda, -\rho+\nu+\lambda; 1+2\nu; z)\,. \end{aligned} \tag{32} $$

Now we recall that $z \to 0$ when $x \to \infty$ and imposing the boundary condition of the scattering problem that in the ($x > 0$ region) we only have a wave travelling to the right (only the transmitted wave) we find:

$$ \lim_{x\to+\infty} \phi_R(x) = d_1\,e^{ik(x-L)} \tag{33} $$

and $\chi_R(x)$ is found again through Eq. (5a) in terms of ϕ_R:

$$ \lim_{x\to+\infty} \chi_R(x) = d_1 \frac{(E-k)}{m_0}\,e^{ik(x-L)} \tag{34} $$