

that it restricts correctly at all of the  $T$ -fixed points. The observation is that had a formula for this class not already been discovered by Fulton using other methods, it might have been determined simply by identifying how it should restrict at each fixed point and attempting to guess a class which restricts as required.

With this in mind, we return now to the setting of symmetric subgroups. Let  $S = K \cap T$ , a maximal torus of  $K$  contained in  $T$ . (When  $\text{rank}(K) = \text{rank}(G)$ , we have  $S = T$ , but in general,  $S$  may be strictly smaller than  $T$ .) In the present paper, we apply equivariant localization and divided difference operators as described above to discover previously unknown formulas for the  $S$ -equivariant fundamental classes of  $K$ -orbit closures on  $G/B$ . We do so for all symmetric pairs  $(G, K)$  when  $G = SL(n, \mathbb{C})$ ,  $SO(n, \mathbb{C})$ , or  $Sp(2n, \mathbb{C})$ . As a means to this end, along the way we also partially handle some pairs  $(G, K)$  with  $G = Spin(n, \mathbb{C})$ .

In each case, this is done in two steps. First, we identify the closed orbits and their restrictions at the various  $S$ -fixed points. Using this information, we produce polynomials in the generators of  $H_S^*(G/B)$  which restrict at the  $S$ -fixed points as required. We then conclude by the localization theorem that these polynomials represent the equivariant fundamental classes of the closed  $K$ -orbits. (As an interesting aside, we remark that the aforementioned work of Fulton turns out to be vital to this step in two cases, namely the cases  $(Sp(2n, \mathbb{C}), GL(n, \mathbb{C}))$  and  $(SO(2n, \mathbb{C}), GL(n, \mathbb{C}))$ . Indeed, our formulas for the closed orbits in those cases are “determinantal” in nature, and are very similar to corresponding formulas of Fulton for the smallest Schubert locus in the type  $C$  and type  $D$  flag bundles. Moreover, some algebraic properties of these determinants established in [Ful96b] turn out to amount precisely to a proof of the correctness of our formulas.)

Second, we outline how divided difference operators can be used to deduce formulas for the fundamental classes of the remaining orbit closures. This is analogous to what is done for Schubert varieties. However, combinatorial parametrizations of  $K \backslash G/B$ , as well as descriptions of its weak closure order in terms of such parametrizations, are typically more