is Galilean invariant, i.e., it is invariant under the transformation indicated in Eqs. (44). Hence, the nonlinear Hopf-Cole transformation is responsible for the loss of Galilean invariance. Note that these results are independent of whether we consider this discretization scheme or a more accurate one.

Galilean invariance has been always associated with the exactness of the 1D KPZ exponents, and with a relation that connects the critical exponents in higher dimensions. If the numerical solution obtained from a finite differences scheme as Eq. (45), which is not Galilean invariant, *yields the well known critical exponents*, that would strongly suggest that Galilean invariance is not a fundamental symmetry as usually considered.

In Sec. VII we present some numerical results for the critical exponents using the consistent discretization schemes indicated in Eqs. (8) and (31), and compare with those found with the standard one. All the cases exhibit the same critical exponents. Moreover, let us note that the discretization used in Refs. [10, 11], which also violates Galilean invariance, yields the same critical exponents too.

When we compare the classical discretization given by Eq. (43), that explicitly reads

$$\dot{h}_j = \nu \frac{h_{j+1} + h_{j-1} - 2h_j}{a^2} + \frac{\lambda}{2}\left(\frac{h_{j+1} - h_{j-1}}{2a}\right)^2 + F + \xi_j(t). \tag{47}$$

with the alternative one in Eq. (45), that reads

$$\dot{h}_j = \nu \frac{h_{j+1} + h_{j-1} - 2h_j}{a^2} + \frac{\lambda}{4}\left[\left(\frac{h_{j+1} - h_j}{a}\right)^2 + \left(\frac{h_j - h_{j-1}}{a}\right)^2\right] + F + \xi_j(t), \tag{48}$$

we find that this second one presents excess fluctuations with respect to the first. This can be easily seen by means of the inequality

$$\left(h_{j+1} - h_{j-1}\right)^2 = \left(h_{j+1} - h_j + h_j - h_{j-1}\right)^2 \leq 2\left(h_{j+1} - h_j\right)^2 + 2\left(h_j - h_{j-1}\right)^2, \tag{49}$$

which immediately translates into

$$\frac{\lambda}{2}\left(\frac{h_{j+1} - h_{j-1}}{2a}\right)^2 \leq \frac{\lambda}{4}\left[\left(\frac{h_{j+1} - h_j}{a}\right)^2 + \left(\frac{h_j - h_{j-1}}{a}\right)^2\right], \tag{50}$$

where the inequality is strict unless $h_j = (h_{j+1} + h_{j-1})/2$, an event which happens with zero probability (note that in 1D and for long times, the KPZ interface has independent Gaussian distributed increments, as Brownian motion). This implies that the excess fluctuations are genuinely present in the interface dynamics.