complete review see Ref. [18]). The perturbed metric has ten degrees of freedom, but only six of them are physical. The others, are unphysical because of gauge (coordinate) transformations. Therefore, introducing gauge invariant quantities, helps us to have results that are independent of the chosen coordinate system. Among different gauge invariant quantities, the comoving curvature perturbation R would be an appropriate quantity to analyze the power spectrum of the CMB. Symmetries of perturbation equations under translations make it easier to work with Fourier modes, R_q.

The evolution of R_q in the non–improved theory follows the MSE [18]:

$$\frac{d^2R_q}{d\tau^2} + \frac{2}{z}\frac{dz}{d\tau}\frac{dR_q}{d\tau} + q^2R_q = 0 \tag{8}$$

where $\tau = \int_{t_0}^{t} \frac{dt'}{a(t')}$ is the conformal time and z which is related to the time derivative of the scalar field as $z = a\dot{\varphi}/H$ can be considered as the redshift. Since the Fourier transformation of 2-point function of the inflation field, i.e. the power spectrum, needs the solution of this differential equation, introducing the initial conditions is unavoidable.

The wave number q is proportional to the inverse of the comoving wavelength of the perturbations, and the conformal time is the comoving horizon, hence the quantity $-q\tau$ will be the ratio of causal horizon length to the comoving wavelength of the perturbations. For $-q\tau \ll 1$, the wavelength of the perturbations is larger than the length of causal horizon and the perturbations exit the horizon before the end of inflation. At the end of inflation, with growing the comoving Hubble radius, $\frac{1}{aH}$, the wavelength of perturbations becomes smaller than the comoving Hubble radius and thus the inhomogeneities and their effect on CMB gradually become observable.

Here we are interested in investigating the effects of quantum improvements of G_0 and Λ_0 on these observables. Therefore, at the first step, we have to study briefly the effect of improvement on the perturbation equations. Then IMSE in the ΛCDM universe will be obtained.