Slater's condition holds in the context of our problem as long as Ω is a strictly positive number (i.e., $\mathbf{p}$ such that $p_i = q_i$ for $\forall i$ is a strictly feasible solution). Therefore, strong duality holds for our inner minimization problem as long as Ω is strictly positive.

Using strong duality, our max-min problem can be transformed into (53).

$$\begin{array}{ll}
\multicolumn{2}{c}{\max_{\boldsymbol{\xi},\mathbf{x},\mathbf{d}} \max_{\mu,\nu} -\mu \sum_{i=1}^{N} q_i e^{-1-\frac{d_i+\nu}{\mu}} - \mu\Omega - \nu} \\
\multicolumn{2}{c}{\text{such that}} \\
\text{(A)} & d_i = z_i \exp\left[-\alpha \sum_{j=1}^{J} x_j \left(\left(\mathbf{A}^T\boldsymbol{\xi}\right)_j - \mathbf{A}_{ij}\right)\right] \text{ for } \forall i \in \{1,..,N\} \\
\text{(B)} & \mu \geq 0 \\
\text{(C)} & \boldsymbol{\xi} \geq \mathbf{0} \\
\text{(D)} & \sum_{i=1}^{N} \xi_i = 1 \\
\text{(E)} & (\mathbf{x},\boldsymbol{\xi}) \in \mathbf{F}
\end{array} \tag{53}$$

Two maximization operators can be collapsed into a single operator.

$$\begin{array}{ll}
\multicolumn{2}{c}{\max_{\boldsymbol{\xi},\mathbf{x},\mu,\nu,\mathbf{d}} -\mu \sum_{i=1}^{N} q_i e^{-1-\frac{d_i+\nu}{\mu}} - \mu\Omega - \nu} \\
\multicolumn{2}{c}{\text{such that}} \\
\text{(A)} & d_i = z_i \exp\left[-\alpha \sum_{j=1}^{J} x_j \left(\left(\mathbf{A}^T\boldsymbol{\xi}\right)_j - \mathbf{A}_{ij}\right)\right] \text{ for } \forall i \in \{1,..,N\} \\
\text{(B)} & \mu \geq 0 \\
\text{(C)} & \boldsymbol{\xi} \geq \mathbf{0} \\
\text{(D)} & \sum_{i=1}^{N} \xi_i = 1 \\
\text{(E)} & (\mathbf{x},\boldsymbol{\xi}) \in \mathbf{F}
\end{array} \tag{54}$$

8.4.3 Further Simplification through Algebraic Manipulation

Because the objective function of (54) is a strictly concave function of ν, we can find the global optimizing value of ν from the first-order condition.

$$-\mu \sum_{i=1}^{N} q_i \left(-\frac{1}{\mu}\right) e^{-1-\frac{d_i+\nu}{\mu}} - 1 = 0$$

$$\sum_{i=1}^{N} q_i e^{-1-\frac{d_i+\nu}{\mu}} = 1$$