as $H_0 \le H_c$ the unstable mode has the maximal increment at $\vec{k} = (\pi/a)\vec{e}_x$ or $\vec{k} = (\pi/a)\vec{e}_y$ corresponding to the transition from the FM state with parallel magnetic moments for all the dots to the chessboard AFM state. For continuous films the instability occurs only for long wave excitations and leads to appearance of long-period domain structures.[39]

B. Density of states.

The complicated behavior of the dispersion curves play an important role in the formation of Van Hove singularities. The Van-Hove singularities are connected with the extrema of the dispersion law of quasiparticles; namely, with minima, maxima and saddle points. Any branch of collective excitations has at least one of these extrema within the Brillouin zone. For the interaction of a finite number of neighboring spins, the dispersion law is described by an analytical function, and the function $\omega(\vec{k})$ can be approximated by parabolic functions in the vicinity of an extremum. In this case the van Hove singularities have a standard form. In particular, for a two-dimensional case the points of minima and maxima of $\omega(\vec{k})$, where $\omega = \omega_{\min}$ or $\omega = \omega_{\max}$, result in a finite jump of the density of states, $D(\omega) = C \cdot \Theta(\omega - \omega_{\min})$ or $D(\omega) = C \cdot \Theta(\omega_{\max} - \omega)$, respectively, where $\Theta(x)$ is the Heaviside step function, $\Theta(x) = 1$ for $x > 0$ and $\Theta(x) = 0$ for $x < 0$. Saddle points with $\omega = \omega_{\mathrm{c}}$ leads to logarithmic singularities of the form $\Delta D(\omega) = C \cdot \ln[\omega_{\mathrm{c}}/|\omega - \omega_{\mathrm{c}}|]$, and the appearance or disappearance of one more singularity of such form at some value of frequency has to be clearly seen.

In our case the structure can be richer due to the long-range character of the interaction, as extrema can correspond to an non-standard behavior of $D(\omega)$. First, note, the non-standard (linear in k) dispersion near the gap frequency, ω_0 produce much weaker singularities in the density of states; for $\omega > \omega_0$ simple calculations give

$$D(\omega) = C(\omega - \omega_0) \cdot \Theta(\omega - \omega_0) \tag{13}$$

instead of finite jump. In addition, the number of extrema could be more than three. In our case all these possibilities can be realized at various values of the parameter $\lambda = \omega_0/\omega_{\mathrm{int}}$.

The analysis of the spectrum shows that for the FM state the dependence $\omega = \omega(\vec{k})$ always has a minimum in the point Γ ($\vec{k} = 0$); moreover, in the vicinity of this point $(\omega - \omega_0) \propto |\vec{k}|$. As well $\omega(\vec{k})$ always has the standard parabolic maximum at the point M