

After carrying out the angular integrals we obtain

$$I_0 = \nu(p_F)^{-1} \mu \int \frac{dpp^2}{2\pi^2} \frac{\xi_p}{\epsilon_p^3} \left[\frac{4\epsilon_p^2}{\omega^2} - 1 + \frac{2\mu\xi_p}{\epsilon_p^2} \right] L_0, \quad (\text{B4})$$

$$I_1 = \frac{2}{3} \nu(p_F)^{-1} \mu \int \frac{dpp^2}{2\pi^2} \frac{\xi_p}{\epsilon_p^3} \left[\frac{4\epsilon_p^2}{\omega^2} - 1 \right] L_1, \quad (\text{B5})$$

$$I_2 = 2\nu(p_F)^{-1} \int \frac{dpp^2}{2\pi^2} \frac{1}{\epsilon_p} \left[\frac{4\epsilon_p^2}{\omega^2} - 1 \right] \langle L_2(x) \rangle, \quad (\text{B6})$$

where $\langle L_2 \rangle$ is the angle integrated loop L_2 , which is obtained from (A12) via the substitution $x^2 = 1/3$. Using the transformation $dpp^2 = m^* p_F d(p^2/2m^*) = m^* p_F d\xi_p$, and the relation $\epsilon_p d\epsilon_p = \xi_p d\xi_p$ we obtain

$$I_0 = \mu \int_{\Delta}^{\infty} d\epsilon_p \frac{1}{\epsilon_p^2} \left[\frac{4\epsilon_p^2}{\omega^2} - 1 + \frac{2\mu\xi_p}{\epsilon_p^2} \right] L_0, \quad (\text{B7})$$

$$I_1 = \frac{2\mu}{3} \int_{\Delta}^{\infty} d\epsilon_p \frac{1}{\epsilon_p^2} \left[\frac{4\epsilon_p^2}{\omega^2} - 1 \right] L_1, \quad (\text{B8})$$

$$I_2 = 2 \int_{\Delta}^{\infty} d\epsilon_p \frac{1}{\xi_p} \left[\frac{4\epsilon_p^2}{\omega^2} - 1 \right] \langle L_2(x) \rangle. \quad (\text{B9})$$

To make further progress we need to separate the real and imaginary parts of the integrals. We shall first compute the imaginary parts. They are extracted with the help of the identity

$$\frac{1}{(D + i\delta)^{n+1}} = \frac{P}{D^{n+1}} - i\pi \frac{(-1)^n}{n!} \delta^{(n)}(D), \quad (\text{B10})$$

where P denotes the principal value and $\delta^{(n)}(D)$ is the n -th derivative of the delta function.

For positive ω the integral (B1) gives [21]

$$\begin{aligned} \text{Im} I_0 &= -\pi\mu \int_{\Delta}^{\infty} d\epsilon_p \frac{1}{\epsilon_p^2} \left[\frac{4\epsilon_p^2}{\omega^2} - 1 + \frac{2\mu\xi_p}{\epsilon_p^2} \right] \tanh\left(\frac{\epsilon_p}{2T}\right) \delta(\omega^2 - 4\epsilon_p^2) \\ &= -8\pi\mu^2 \frac{\sqrt{\omega^2 - 4\Delta^2} \text{sgn}(\omega)}{\omega^5} \tanh\left(\frac{\omega}{4T}\right) \theta(\omega - 2\Delta). \end{aligned} \quad (\text{B11})$$

Consider the integral (B2). First, note that the term $\propto D^{-1}$ vanishes, since the delta function enforces $\omega^2 = 4\epsilon_p^2$. After dropping this term we are left with the integral

$$\begin{aligned} \text{Im} I_1 &= \frac{16\pi\mu^2}{3} \int_{\Delta}^{\infty} d\epsilon_p \frac{1}{\epsilon_p^2} \left(\frac{4\epsilon_p^2}{\omega^2} - 1 \right) \sqrt{\epsilon_p^2 - \Delta^2} \\ &\quad \tanh\left(\frac{\epsilon_p}{2T}\right) \delta^{(1)}(\omega^2 - 4\epsilon_p^2). \end{aligned} \quad (\text{B12})$$

This and similar integrals, which contain derivatives of the delta function, are computed via the formula

$$\int f(x) \delta^n(x - a) dx = (-)^n f^{(n)}(a). \quad (\text{B13})$$

The result of integration is

$$\text{Im} I_1 = -\frac{8\pi\mu^2}{3} \frac{\sqrt{\omega^2 - 4\Delta^2} \text{sgn}(\omega)}{\omega^5} \tanh\left(\frac{\omega}{4T}\right) \theta(\omega - 2\Delta). \quad (\text{B14})$$

In the integral (B3) we again omit terms $\propto D^{-1}$, since their prefactors are zero after integration. After inserting $x^2 = 1/3$ in the remainder we obtain

$$\begin{aligned} \text{Im} I_2 &= \frac{8\pi}{3} \int_{\Delta}^{\infty} d\epsilon_p \left[\frac{4\epsilon_p^2}{\omega^2} - 1 \right] \left\{ \frac{\mu^2 \sqrt{\epsilon_p^2 - \Delta^2}}{\epsilon_p T} \text{sech}^2\left(\frac{\epsilon_p}{2T}\right) + \frac{2}{\sqrt{\epsilon_p^2 - \Delta^2}} \mu^2 \tanh\left(\frac{\epsilon_p}{2T}\right) \right. \\ &\quad \left. + 3\mu \tanh\left(\frac{\epsilon_p}{2T}\right) - \frac{\mu^2 \sqrt{\epsilon_p^2 - \Delta^2}}{\epsilon_p^2} \tanh\left(\frac{\epsilon_p}{2T}\right) \right\} \delta^{(1)}(\omega^2 - 4\epsilon_p^2) \\ &\quad - \frac{64\pi\mu^2}{3} \int_{\Delta}^{\infty} d\epsilon_p \left[\frac{4\epsilon_p^2}{\omega^2} - 1 \right] \sqrt{\epsilon_p^2 - \Delta^2} \tanh\left(\frac{\epsilon_p}{2T}\right) \delta^{(2)}(\omega^2 - 4\epsilon_p^2). \end{aligned} \quad (\text{B15})$$

Applying Eq. (B13) we obtain

$$\text{Im} I_2 = -\frac{\pi\mu \text{sgn}(\omega)}{3T\omega^5 \sqrt{\omega^2 - 4\Delta^2}} \text{sech}^2\left(\frac{\omega}{4T}\right) \left\{ T [24\Delta^2\mu + \omega^2 (2\mu + 3\sqrt{\omega^2 - 4\Delta^2})] \sinh\left(\frac{\omega}{2T}\right) + 4\mu\omega (\omega^2 - 4\Delta^2) \right\} \theta(\omega - 2\Delta). \quad (\text{B16})$$

Finally, adding the three integrals (B11), (B14), and (B16) we obtain Eq. (42) of the main text.