$$+\sum_{s^0\le s_k<s} X(s,s_k+)\left[f\left(\psi^{-1}(s_k),x(s_k)\right)-f\left(\psi^{-1}(s_k),\phi_\zeta(s_k)\right)\right]\delta_k$$

implies for $s\ge s^0$ that

$$\|x(s)-\phi_\zeta(s)\|\le Ne^{-\lambda(s-s^0)}\left\|y_0-\phi_\zeta(s^0)\right\|+\int_{s^0}^{s} NL_f e^{-\lambda(s-r)}\|x(r)-\phi_\zeta(r)\|\,dr$$
$$+\sum_{s^0\le s_k<s} NL_f\bar{\delta}e^{-\lambda(s-s_k)}\|x(s_k)-\phi_\zeta(s_k)\|.$$

Applying the Gronwall-Bellman Lemma for piecewise continuous functions [6] to the last inequality, one can obtain that

$$\|x(s)-\phi_\zeta(s)\|\le N(1+NL_f\bar{\delta})^p\left\|y_0-\phi_\zeta(s^0)\right\|e^{[-\lambda+NL_f+p\ln(1+NL_f\bar{\delta})/\psi(\omega)](s-s^0)},\ s\ge s^0.$$

Therefore, we have for $t\ge t^0$, $t\in\mathbb{T}_0$, that

$$\|y(t)-\varphi_\zeta(t)\|\le N(1+NL_f\bar{\delta})^p\left\|y_0-\varphi_\zeta(t^0)\right\|e^{[-\lambda+NL_f+p\ln(1+NL_f\bar{\delta})/\psi(\omega)](\psi(t)-\psi(t^0))}.$$

Consequently, $\|y(t)-\varphi_\zeta(t)\|\to 0$ as $t\to\infty$, $t\in\mathbb{T}_0$. $\square$

In the next section, we will deal with the presence of chaos in system (1.1).

4 The chaotic dynamics

The map (1.2) is called Li-Yorke chaotic on Λ if [11, 13, 29, 35, 36]: (i) For every natural number p_0, there exists a p_0-periodic point of F in Λ; (ii) There is an uncountable set $\mathcal{S}\subset\Lambda$, the scrambled set, containing no periodic points, such that for every $\zeta^1,\zeta^2\in\mathcal{S}$ with $\zeta^1\neq\zeta^2$, we have $\limsup_{k\to\infty}\left\|F^k(\zeta^1)-F^k(\zeta^2)\right\|>0$ and $\liminf_{k\to\infty}\left\|F^k(\zeta^1)-F^k(\zeta^2)\right\|=0$; (iii) For every $\zeta^1\in\mathcal{S}$ and a periodic point $\zeta^2\in\Lambda$, we have $\limsup_{k\to\infty}\left\|F^k(\zeta^1)-F^k(\zeta^2)\right\|>0$.

Let us denote by Θ the set of all sequences $\zeta=\{\zeta_k\}$, $k\in\mathbb{Z}$, obtained by equation (1.2). A pair of sequences $\zeta=\{\zeta_k\}$, $\tilde{\zeta}=\left\{\tilde{\zeta}_k\right\}\in\Theta$ is proximal if $\liminf_{k\to\infty}\left\|\zeta_k-\tilde{\zeta}_k\right\|=0$. Moreover, the pair is frequently separated if $\limsup_{k\to\infty}\left\|\zeta_k-\tilde{\zeta}_k\right\|>0$.

We say that a pair $\varphi_\zeta(t)$, $\varphi_{\tilde{\zeta}}(t)$ of bounded solutions of (1.1) is proximal if for an arbitrary small real number $\epsilon>0$ and arbitrary large natural number E, there exists an integer m such that $\left\|\varphi_\zeta(t)-\varphi_{\tilde{\zeta}}(t)\right\|<\epsilon$ for all $t\in[\theta_{2m-1},\theta_{2(m+E)}]\cap\mathbb{T}_0$. On the other hand, the pair $\varphi_\zeta(t)$, $\varphi_{\tilde{\zeta}}(t)$ is frequently (ϵ_0,Δ)-separated if there exist numbers $\epsilon_0>0$, $\Delta>0$ and infinitely many disjoint intervals $J_q\subset\mathbb{T}_0$, $q\in\mathbb{N}$, each with a length no less than Δ, such that $\left\|\varphi_\zeta(t)-\varphi_{\tilde{\zeta}}(t)\right\|>\epsilon_0$ for each t from these intervals. Furthermore, a pair $\varphi_\zeta(t)$, $\varphi_{\tilde{\zeta}}(t)$ of solutions of (1.1) is called a Li-Yorke pair if it is proximal and frequently (ϵ_0,Δ)-separated for some positive numbers ϵ_0 and Δ.