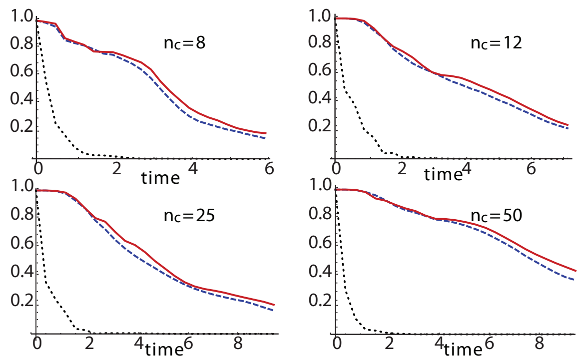

FIG. 3: **Simulations.** Normalized signal decay for the proposed (red, solid lines), spin-echo sequence (blue, dashed lines) and no control (black, dotted lines). We assumed perfect delta function pulses. A leading order cluster expansion was used [23]. 20 dark spins were randomly placed in a cube of side-length $\sqrt[3]{20}$ with the sensor spin at the center. We set $g\mu_B \equiv 1\,[m]^{3/2}\,[s]^{-1/2}$ to use dimensionless quantities. WAHUHA sequences [17] with n_c=8, 12, 25 and 50 cycles per echo interval were simulated. Each curve is an average over 10 Monte Carlo simulations. For simplicity we set $P=0$.

quence is applied (for the same environment the decay is now described by the dephasing time T_2^*). We simulated a spin bath composed of spin 1/2 paramagnetic impurities, undergoing a WAHUHA sequence (which is designed to refocus the dipole-dipole coupling of the environment spins, but does not cancel out the coupling to the external field [17, 26]).

A different limitation on the sensing time τ is set by the fact that the orientation of the dark spin will not be static as assumed. Dipole-dipole couplings among dark spins during each $\tau/4$ period of free evolution rotate each spin away from the initial direction. This rotation means that the spins cease to build up a phase difference between the two arms of the interferometer for time scales comparable to the correlation time of the dark spin bath τ_c^d. The optimum sensing time is thus $T \sim \min\left\{T_2^s,\,\tau_c^d\right\}$. Since in most systems $\tau_c^d \geq T_2^s$ [17], the optimum sensing time of this pulse sequence is comparable to that of spin-echo based magnetometry, thus the sensitivity enhancement is roughly the same as the signal strength enhancement.

In conclusion, we proposed a scheme to enhance precision measurement by exploiting the possibility to coherently control the ancillary qubits. In solid state implementations we are able to exploit dark spins in the bath while preserving roughly the same coherence times as in spin-echo based magnetometry. Thus signal enhancement leads directly to sensitivity enhancement. For trapped ion implementations we can use imperfect phase gates and still achieve Heisenberg-limited sensitivity. Our method has the potential to be applied more generally, using different systems and more sophisticated pulse sequences [26]. It opens the possibility to use a broad class of partially entangled states to achieve Heisenberg limited metrology, even in the presence of disordered couplings, partial control and decoherence.

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[34] Although our pulse sequence is more sensitive to dark spin dephasing (since it creates superposition states of the dark spins), this noise source is not relevant in current experiments, where noise is dominated by single species dipole-dipole interactions [28–30].
[35] For $P \approx 1$ spin-echo achieves longer coherence times than the proposed sequence, since flip-flops are quenched in a fully polarized bath, they would still be allowed in the proposed pulse sequence. However the improvement in coherence times for spin echo scales poorly as $\sim \sqrt[4]{1-P^2}$ and homonuclear decoupling sequences such as WAHUHA can be used to reduce the flip-flop effects.