and wavelet series LS estimators attain the optimal sup-norm rates, allowing for weakly dependent data and heavy tailed error terms. It also presents general sharp L^2-norm convergence rates of series LS estimators with an arbitrary basis under very mild conditions. Section 3 provides the asymptotic normality of sieve t statistics for possibly nonlinear functionals of h_0. Section 4 provides new exponential inequalities for sums of weakly dependent random matrices, and a reinterpretation of equivalence of the theoretical and empirical L^2 norms as a criterion regarding convergence of a certain random matrix. Section 5 shows the sup-norm stability of the empirical L^2 projections onto compactly supported wavelet bases, which provides a tight upper bound on the sup-norm bias term for the wavelet series LS estimator. The results in Sections 4 and 5 are of independent interest. Section 6 contains a brief review of spline and wavelet sieve bases. Proofs and ancillary results are presented in Section 7.

Notation: Let $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the smallest and largest eigenvalues, respectively, of a matrix. The exponent $^-$ denotes the Moore-Penrose generalized inverse. $\|\cdot\|$ denotes the Euclidean norm when applied to vectors and the matrix spectral norm (i.e., largest singular value) when applied to matrices, and $\|\cdot\|_{\ell^p}$ denotes the ℓ^p norm when applied to vectors and its induced operator norm when applied to matrices (thus $\|\cdot\| = \|\cdot\|_{\ell^2}$). If $\{a_n : n \geq 1\}$ and $\{b_n : n \geq 1\}$ are two sequences of non-negative numbers, $a_n \lesssim b_n$ means there exists a finite positive C such that $a_n \leq C b_n$ for all n sufficiently large, and $a_n \asymp b_n$ means $a_n \lesssim b_n$ and $b_n \lesssim a_n$. $\#\mathcal{S}$ denotes the cardinality of a set $\mathcal{S}$ of finitely many elements. Given a strictly stationary process $\{X_i\}$ and $1 \leq p < \infty$, we let $L^p(X)$ denote the function space consisting of all (equivalence classes) of measurable functions f for which the $L^p(X)$ norm $\|f\|_{L^p(X)} \equiv E[|f(X_i)|^p]^{1/p}$ is finite, and we let $L^\infty(X)$ denote the space of bounded functions under the sup norm $\|\cdot\|_\infty$, i.e., if $f : \mathcal{X} \to \mathbb{R}$ then $\|f\|_\infty \equiv \sup_{x\in\mathcal{X}} |f(x)|$.

2 Uniform Convergence Rates

In this section we present some general results on uniform convergence properties of nonparametric series LS estimators with weakly dependent data.

2.1 Estimator and basic assumptions

In nonparametric series LS estimation, the conditional mean function h_0 is estimated by least squares regression of $Y_1, \ldots, Y_n$ on a vector of sieve basis functions evaluated at $X_1, \ldots, X_n$. The standard