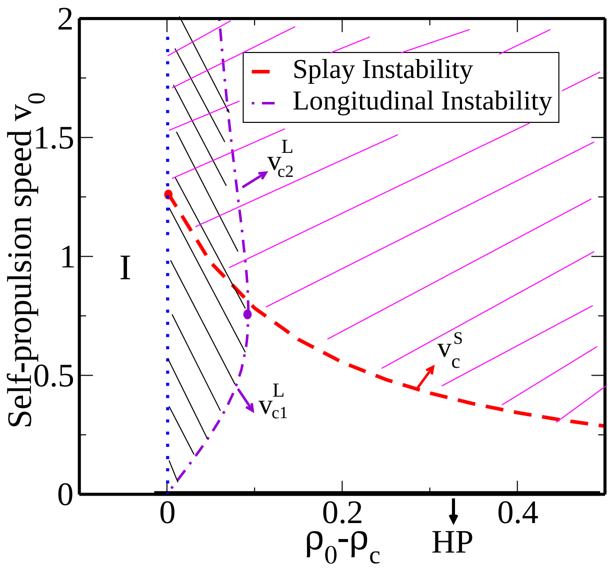

FIG. 2: (color online) The figure displays the linear stability boundaries in the (v_0, ρ_0) plane. All lines have been calculated using the microscopic parameter values of Table I. The vertical dotted line (blue online) is the mean field continuous transition from the isotropic (I) to the homogeneous polarized (HP) state. The dashed-dotted lines (purple online) are the boundaries (calculated by numerical solution of $D_{eff}(v_0, \rho_0) = 0$, with D_{eff} given by Eq. (20)) that define the region $v^L_{c1} \leq v_0 \leq v^L_{c2}$ where the homogeneous polarized state is unstable due to the growth of coupled density and polarization fluctuation associated with spatial gradients along the direction of mean order (longitudinal instability). The linear theory predicts that the homogeneous polar state is unstable in the ruled region to the right of the vertical mean-field transition and bounded by these two lines. The dashed line (red online) is the splay instability boundary given in Eq. (A11). It terminates at a finite value at $\rho_0 = \rho_c$. The linear theory predicts that splay fluctuations destabilize the polar state in the entire ruled region above the dashed (red) line. The region where the system exhibits both the longitudinal and splay instabilities is cross-hatched. The longitudinal instability boundary $v^L_{c1}(\rho_0)$ vanishes for $v_0 \to 0$ and $\rho_0 \to \rho_c^+$, in agreement with the numerics.

the other hand, the instability line obtained from the linear theory agrees remarkably well with the numerical onset of stripes in this high density region (Fig. (1)). More work is needed to understand stripe formation at high density and the origin of the associated length scale.

IV. NONLINEAR REGIME

To go beyond the linear stability analysis and investigate the nature of the flocking state above v_c, we have solved numerically the full nonlinear hydrodynamic equations. The numerical analysis has been carried out using the specific parameter values obtained for the self-propelled hard rod model of Ref. [23], summarized in Table I. All diffusion coefficients are enhanced by self propulsion of an additive contribution proportional to v_0^2 that arises from the persistent nature of the random walk performed by Brownian, self-propelled rods. In the following we discuss the properties of the system in terms of two dimensionless parameters, the self-propulsion speed, v_0, and the density of particles, ρ_0. In the numerics the coefficients a_2 and a_4 that control the continuous mean field phase transition from an isotropic to a polar state have been taken to be of the simple form given in Eqs. (10b) with $\rho_c = 0.5$ in units of the rod length. This form yields $P_0 \sim (\rho - \rho_c)^{1/2}$ for $\rho \to \rho_c$ and $P_0 \to 1$ for $\rho >> \rho_c$.

D/D_0	D_b/D_0	D_s/D_0	λ_1	λ_2	λ_3
$(3+2v_0^2)/2$	$(7+6v_0^2)/8$	$(9+10v_0^2)/8$	$3\pi v_0^2$	πv_0^2	πv_0^2

TABLE I: Diffusion constants and convective parameters for the model of self-propelled hard rods with excluded volume interactions discussed in Ref. [23]. All diffusion constants are in units of $\ell^2 D_r$ and all convective parameters are in units of $\ell^3 D_r$. The diffusion coefficients have been expressed in terms of the longitudinal diffusion constant D_0 of a long, thin rod. Below we use the low density value, $D_0 = 1/4$.

For generality we include fluctuations beyond the mean field level in the numerical analysis by adding Gaussian white noise terms in both the density and polarization equations of the forms $\nabla \cdot \mathbf{f}_\rho(\mathbf{r}, t)$ and $\mathbf{f}_P(\mathbf{r}, t)$, respectively. The random forces are chosen to have zero mean and correlations

$$< f_{i\rho}(\mathbf{r}, t) f_{j\rho}(\mathbf{r}', t') >= \delta_{ij} \Delta_\rho \rho(\mathbf{r}, t) \delta(\mathbf{r} - \mathbf{r}') \delta(t - t'), \tag{21}$$

$$< f_{iP}(\mathbf{r}, t) f_{jP}(\mathbf{r}', t') >= \delta_{ij} \frac{\Delta_P}{\rho(\mathbf{r}, t)} \delta(\mathbf{r} - \mathbf{r}') \delta(t - t'), \tag{22}$$

where Δ_ρ and Δ_P are dimensionless noise strengths. The noise in the density equation scales as $[\rho(\mathbf{r}, t)]^{1/2}$, while the polarization noise scales as $[\rho(\mathbf{r}, t)]^{-1/2}$ [22, 36]. This difference arises because the fields ρ and $\mathbf{P}$ are extensive and intensive quantities, respectively. The numerical results described below are all for fixed values of the noise amplitudes, $\Delta_p = \Delta_\rho = 0.3$. We have solved the nonlinear equations using the Euler method for numerical differentiation on a grid with $\Delta x = 1.0$ and $\Delta t = 0.1$ (we have verified that the numerical scheme is convergent and stable for $\Delta t / (\Delta x)^2 < 0.5$) We consider a square system of size $L \times L$ with both periodic and shifted boundary conditions and a range of system sizes.

The behavior of the system as a function of the self-propulsion velocity v_0 and the density of particles ρ is summarized in the phase diagram shown in Fig 1 discussed in the Introduction. The isotropic state is stable