A new approach for doing theoretical and numeric work with Lemaître–Tolman–Bondi dust models.

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We introduce quasi–local integral scalar variables for the study of spherically symmetric Lemaître–Tolman–Bondi (LTB) dust models. Besides providing a covariant, and theoretically appealing, interpretation for the parameters of these models, these variables allow us to study their dynamics (in their full generality) by means of fluid flow evolution equations that can be handled with simple numerical techniques and has a significant potential for astrophysical and cosmological applications. These evolution equations can also be understood in the framework of a gauge invariant and covariant formalism of spherical non–linear perturbations on a FLRW background. The covariant time splitting associated the new variables leads, in a natural way, to rephrase the known analytic solutions within an initial value framework in which covariant scalars are given by simple scaling laws. By using this re–parametrization of the analytic solutions, we re–examine and provide an alternative outlook to various theoretical issues already treated in the literature: regularity conditions, an Omega parameter, as well as the fitting of a given LTB model to radial profiles of density or velocity at different cosmic times. Other theoretical issues and numeric applications will be examined in separate articles.

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I. INTRODUCTION.

Spherically symmetric LTB dust models are among the best known exact solutions of Einstein's equations [1]. Since they provide access to non–linear effects associated with inhomogeneous sources by means of analytic and/or tractable numeric solutions, they have received widespread attention in the literature (see [2, 3] for comprehensive reviews). These exact solutions are used mostly to study cosmological inhomogeneities, as for example the series of articles in the references [7–10], but have also been employed to examine a variety of issues, for example as useful toy models to describe gravitational collapse under a classical [4, 5] and quantum [6] approach. LTB models have also been widely used [11–18] in the context of the ongoing widespread theoretical and empiric effort to explore the possibility that cosmic observations could be explained without resorting to dark energy, but by taking into consideration the fully non–linear effects of inhomogeneities in the context of General Relativity (see [19] for a review). These models are also helpful as theoretical tools to probe various averaging techniques applied to cosmological inhomogeneities [14, 15, 20–22], in particular, the scalar averaging formalism developed by Buchert and coworkers [23]. One finds in the extensive literature on these models a preferred set of free functions and analytic solutions (implicit and parametric) that has become a sort of standard [2, 3, 24] (even in numeric work on these models [14–18]).

Given the success of these models and the fact that their standard parametrization is adequate and works in practice, then it is legitimate to ask for the justification of an article proposing and discussing alternative variables. The answer to this question is simple: alternative variables may provide or motivate either new theoretical and empiric developments or allow us to grasp known results under a different perspective, thus illuminating possible unsuspected connections with other models or theoretical issues. In this context, we propose an alternative description of LTB dust models that is based on a set of covariant scalar variables defined by applying to arbitrary scalars the same type of integral construct that yields the Misner–Sharp quasi–local mass function [26–28]. Hence, we find it natural to call these variables quasi–local scalars.

The quasi–local scalars mentioned above have already been used for a dynamical system study of dust LTB models [29] and in the application of Buchert's averaging formalism to LTB models [21, 22], as well as in a numeric study of inhomogeneous dark energy sources associated with an LTB metric, but with an energy–momentum tensor that is more general than dust [30, 31]. The pure dust case merits the separate study that we provide here, not only because dust is an adequate source to model cold dark matter in cosmological scales (and a much less artificial source for an LTB metric than the fluid tensor in [30, 31]), but because dust sources allow for a qualitative analytic framework that complements numeric work and yields very useful theoretical and practical information.

The potential of the quasi–local variables for analytic and qualitative work on LTB models follows readily from parametrizing their known analytic expressions in terms of these variables and their fluctuations. The new variables lead in a natural way to an initial value parametrization of LTB models, which in turn leads to the introduction of a FLRW–like metric and scaling laws for density, Hubble expansion, spatial curvature, shear and all other covariant quantities of physical or geomet-