

Regardless of different dynamics and different coupling forms, we propose a common design principle for such oscillatory networks.

Design principle: each nonoscillatory node can oscillate if and only if it is driven by one or few oscillatory interactions with *advanced phases*.

The definitions of "advanced phase" for different systems can be different, but they are the same in essence[21]. Let us consider an example of simplest 1D oscillatory networks consisting of nonoscillatory nodes where each node is unidirectionally interacted by only a single other node as shown in Fig. 1. The network consists of  $N$  nodes with  $M$  ( $M = N$ ) unidirectional interactions. As the network is oscillatory, all the nodes are activated. Suppose an arbitrary node  $i_1$  is phase-advancedly driven by a node  $i_2$  via coupling, which is phase-advancedly driven by node  $i_3$  in turn, and this successive unidirectional driving chain goes as  $i_1 \leftarrow i_2 \leftarrow i_3 \leftarrow \dots \leftarrow i_k \leftarrow \dots$ . Since  $N$  is finite we must come to a node  $i_q$ ,  $q \leq N$ , which is driven by one of the previous nodes  $\{i_1, i_2, \dots, i_{q-1}\}$ , said  $i_p$  ( $p < q$ ). Then a successive regulatory loop  $i_p \leftarrow i_{p+1} \leftarrow \dots \leftarrow i_q \leftarrow i_p$  is formed, serving as the oscillation source of all other nodes. From the graph theory, this network has one and only one fundamental loop ( $N - N + 1 = 1$ ), and this fundamental loop is right the dynamic driving loop playing the role of the oscillation source, while all other nodes must in the tree branches radiating from this loop identifying wave propagation paths. The simple structure in Fig. 1 consists of trees from loops, thus is called trees from loops (TFL) pattern.

The TFL structure of Fig. 1 is universal for all self-sustained periodic oscillations in complex networks consisting of nonoscillatory nodes. It illustrates the simplest relationship of these nodes in self-sustained oscillation. Since no nonoscillatory node can oscillate without phase-advanced driving from other nodes, two key rules must be obeyed by any TFL structure:

- (i) There must be some (at least one) successively phase-advanced driving loops.
- (ii) Each node not in the loops must be in a tree branch rooted at a node in a loop.

The simple and instructive structure of Fig. 1 gives an example of simplest 1D network which can self-sustainedly oscillate. However, interaction structures of complex networks are in general much more complex than Fig. 1 which are high dimensional and random (e.g. Fig. 2(a)). In Fig. 1 the only topological loop is right the source loop generating the oscillation. In practical complex connected networks we usually have  $M \gg N$ , and the number of topological loops is large. We therefore propose an operable and physically meaningful method to reduce the original random network graphs (as Fig. 2(a)) to the simple and instructive TFL patterns of Fig. 1. The method consists of the following Complexity Reduction steps:

- (a) Find phase-advanced driving interactions for each node.
- (b) Find the single dominant interaction among these phase-advanced driving interactions.