# Homoclinic and Heteroclinic Motions in Hybrid Systems with Impacts

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**Abstract**

In this paper, we present a method to generate homoclinic and heteroclinic motions in impulsive systems. We rigorously prove the presence of such motions in the case that the systems are under the influence of a discrete map that possesses homoclinic and heteroclinic orbits. Simulations that support the theoretical results are represented by means of a Duffing equation with impacts.

**Keywords:** Impulsive systems; Stable and unstable sets; Homoclinic motion; Heteroclinic motion; Duffing equation with impacts

## 1 Introduction

Impulsive differential equations describe the dynamics of real world processes in which abrupt changes occur. Such equations play an increasingly important role in various fields such as mechanics, electronics, biology, neural networks, communication systems, chaos theory and population dynamics [1, 2, 4, 5, 15, 17, 19, 24, 25, 28]. In this paper, we investigate the existence of homoclinic and heteroclinic motions in systems with impulsive effects.

The main object of the present study is the following impulsive system,

$$\begin{aligned} & x' = A(t)x + f(t,x) + g(t,\zeta), \ t \neq \theta_k, \\ & \Delta x|_{t=\theta_k} = B_k x + J_k(x) + \zeta_k, \end{aligned} \tag{1.1}$$

where $\{\theta_k\}$, $k \in \mathbb{Z}$, is a strictly increasing sequence of real numbers such that $|\theta_k| \to \infty$ as $|k| \to \infty$, $A(t)$ is an $n \times n$ continuous matrix function, $B_k$ are constant $n \times n$ real valued matrices, $\Delta x|_{t=\theta_k} = x(\theta_k+) - x(\theta_k)$, $x(\theta_k+) = \lim\limits_{t \to \theta_k^+} x(t)$, the functions $f : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ and $J_k : \mathbb{R}^n \to \mathbb{R}^n$ are continuous in all their arguments, the function $g(t,\zeta)$ is defined by the equation $g(t,\zeta) = \zeta_k$, $t \in (\theta_{k-1}, \theta_k]$, and the sequence $\zeta = \{\zeta_k\}$, $k \in \mathbb{Z}$, is a solution of the map

$$\zeta_{k+1} = F(\zeta_k), \tag{1.2}$$

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