TeV scale $m_{N_R^\alpha}$, the contribution of the t-channel diagrams to the signal $e^+e^- \to \xi^+\xi^-$ is much smaller than that from Drell-Yan type diagrams. For most of the possible values of $\hat{h}_\ell^\alpha$ and $m_{N_R^\alpha}$ which satisfy the LFV and the neutrino data, the contribution of the t-channel diagrams is negligible. The production cross section of a charged Higgs pair $\xi^+\xi^-$ is therefore similar to that in the usual THDM: about 92 (10) fb for $m_{\xi^\pm} = 100$ (150) GeV at $\sqrt{s} = 500$ GeV. The produced $\xi^\pm$ decay into $W^{\pm(*)}\xi^0_{r,i}$. [9]

In Fig. 3 (*left*), we show the invariant mass distribution of the di-jet jj of the production cross section of the signal, $e^+e^- \to \xi^+\xi^- \to W^{+*}W^{-*}\xi_r^0\xi_r^0 \to jj\mu\nu\xi_r^0\xi_r^0$ for $m_{\xi^\pm} = 100$ GeV. The main backgrounds come from WW. The $jj\mu\mu$ events from ZZ, $\gamma\gamma$, and $Z\gamma$ can also be the backgrounds. A factor of 0.1 is multiplied to the rate of the $jj\mu\mu$ backgrounds for the miss-identification probability of a muon. The signal is significant around $M(jj) \sim 30$ GeV. The invariant mass cut (such as 15 GeV$< M(jj) <$ 40 GeV) is effective to reduce the backgrounds. For the numerical evaluation, we have used a package CalcHEP 2.5.4 [34].

For $m_{\xi^\pm} > m_W + m_{\xi_r}$, on the other hand, the signal $W^+W^-\xi_r^0\xi_r^0$ can be measured by detecting the events of four jets with a missing energy. The main background comes from $W^+W^-\nu\nu$ and $t\bar{t}$. By the invariant mass cuts of two-jet pairs at the W boson mass, the biggest background from WW can be eliminated. In Fig. 3 (*right*), we show the invariant mass distribution of $jjjj$ of the production cross sections of the signal and the backgrounds without any cut. A factor of 0.1 is multiplied to the rate of tt background, by which the probability of the lepton from a W that escapes from detection is approximately taken into account. The signal is already significant. The invariant mass cut ($M(jjjj) <$ 300 GeV) gives an improvement for the signal/background ratio.

The AKS model

For the AKS model, we take a typical successful scenario for the neutrino data with the the normal mass hierarchy, the LFV data and the DM data as well as the condition for strongly first order phase transition [6, 14];

$$\begin{aligned} & m_\eta = 50 \text{ GeV},\ m_{H^\pm} = 100 \text{ GeV},\ m_{S^\pm} = 400 \text{ GeV},\ m_{N_R^1} = m_{N_R^2} = 3 \text{ TeV}, \\ & h_e^1 = h_e^2 = 2 \gg h_\mu^1, h_\mu^2 \gg h_\tau^1, h_\tau^2,\ \kappa \sim \mathcal{O}(1),\ \sin(\beta-\alpha) = 1,\ \tan\beta = 10\,. \end{aligned} \tag{13}$$

[9] The $\xi_r^0\xi_i^0$ production can also be interesting. The final state should be two jets (or dilepton) plus a missing energy. The cross section for $e^+e^- \to \xi_r^0\xi_i^0 \to \xi_r^0\xi_r^0 jj$ is about 40 fb at $\sqrt{s} = 500$ GeV.