The time scale is renormalized because of the critical scaling in the vicinity of the transition.

VI. CONCLUSIONS

In conclusion, we have studied the dynamics of the relative phase of a bilayer of superfluids in 2D, after the hopping between them has been turned off rapidly. We find that on short time scales the dynamics of the correlation function shows a 'light-cone'-like behavior. Depending on the parameters of the system, the light cone dynamics can result in a phase that shows supercritical algebraic scaling, and can therefore be thought of as a superheated superfluid. On long time scales the system relaxes to a disordered state via vortex unbinding, which constitutes a reverse-Kibble-Zurek mechanism. The properties of the dynamical process can be understood with a renormalization group approach. We find that the dynamical evolution of the system resembles the RG flow of the equilibrium system. In particular, using the RG equations we found two possible scenarios of the system reaching the steady state: (i) if initial quantum and thermal fluctuations are weak the vortices are irrelevant and long time long distance behavior is governed by the algebraic fixed point. The only role of vortices is then renormalization of the superfluid stiffness and the sound velocity. (ii) If the initial fluctuations are strong then the vortices become relevant and proliferate resulting in a normal (non-superfluid) steady state. In this case RG gives the time scale of vortex unbinding, which exponentially diverges as the system approaches the non-equilibrium phase transition. The behavior of the relative of phase of two superfluids can be accurately studied by interference experiments of ultra-cold atom systems, and therefore our predictions are of direct relevance to experiment.

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Appendix A

In this Appendix we derive the RG Eqs. 40 and 41, which can also be written as a second order differential equation for θ

$$\frac{1}{\mu}\frac{d^2}{dt^2}\theta \;=\; \lambda\triangle\theta + \frac{g}{a^2}\sin\theta. \tag{A1}$$

To simplify the derivation, here and throughout the Appendix, we formally change notations $\lambda T \to \lambda$, $\mu T \to \mu$, and $gT \to g$. The idea of momentum shell RG is that we treat high momentum components of θ and p (or equivalently $\dot\theta$) perturbatively, while not making any approximations about the low momentum components. Our goal is to find renormalization of the equations of motion governing the low momentum components. So we split

$$\theta(\mathbf{r},t) = \theta^{<}(\mathbf{r},t) + \theta^{>}(\mathbf{r},t), \tag{A2}$$

where the Fourier expansion of $\theta^{>}(\mathbf{r},t)$ only contains momenta in the shell $\Lambda' \equiv \Lambda - \delta\Lambda < |k| < \Lambda$ and $\theta^{<}(\mathbf{r},t)$ contains all other Fourier components:

$$\theta^{<}(\mathbf{r}) \;=\; \frac{1}{\sqrt{V}}\sum_{k<\Lambda'}\exp(i\mathbf{kr})\theta_{\mathbf{k}} \tag{A3}$$

$$\theta^{>}(\mathbf{r}) \;=\; \frac{1}{\sqrt{V}}\sum_{\Lambda'<k<\Lambda}\exp(i\mathbf{kr})\theta_{\mathbf{k}}. \tag{A4}$$

We will treat $\theta^{>}$ (and correspondingly $p^{>}$) perturbatively in g since the nonlinear term should only weakly couple to the high frequency field. We expand the high-momentum field as

$$\theta^{>}(\mathbf{k},t) = \theta^{>}_0(\mathbf{k},t) + \theta^{>}_1(\mathbf{k},t). \tag{A5}$$

Here $\theta^{>}_0(\mathbf{k},t)$ is the solution of the equations of motion, with g set to zero:

$$\theta^{>}_0(\mathbf{k},t) = \frac{\mu}{\omega_\Lambda}p^{>}_{0,\mathbf{k}}\sin(\omega_\Lambda t) + \theta^{>}_{0,\mathbf{k}}\cos(\omega_\Lambda t), \tag{A6}$$

where $\omega_k = v|k|$, and the velocity v is $v=\sqrt{\lambda\mu}$. In the next leading order we have

$$\theta^{>}_1(\mathbf{k},t) = \frac{g\omega_\Lambda}{\lambda}\int_0^t d\tau F_1(\mathbf{k},\tau)\sin(\omega_\Lambda(t-\tau)), \tag{A7}$$

where

$$F_1(\mathbf{k},\tau) = \int d^2r\exp[-i\mathbf{kr}]\sin(\theta^{<}_0(\mathbf{r},\tau)), \tag{A8}$$

and we used $\Lambda a = 1$. Note that in the last equation in the argument of the sinus we changed θ_0 to $\theta_0^{<}$ because the contribution from $\theta_0^{>}$ is smaller by the factor $\delta\Lambda/\Lambda$. So we see that in the leading order in g the high momentum component of θ oscillates with time at very high frequency ω_Λ. In the next order in g the high momentum component also acquires a low frequency component (as we will discuss below).

Next we consider the equation of motion (A1) expanding it up to the second order in $\theta^{>}$:

$$\frac{1}{\mu}\frac{d^2}{dt^2}\theta(\mathbf{r},t) \;\approx\; \lambda\Delta\theta(\mathbf{r},t) + \frac{g}{a^2}\cos(\theta^{<}(\mathbf{r},t))\theta^{>}(\mathbf{r},t) + \frac{g}{a^2}\sin\theta^{<}(\mathbf{r},t)\left(1-\frac{(\theta^{>}(\mathbf{r},t))^2}{2}\right). \tag{A9}$$

Because of the nonlinearity high-momentum modes couple to the low momentum modes leading to the renormalization of the couplings governing the dynamics of