## III. HIGHER-ORDER ELECTRON-CORRELATION CORRECTIONS TO THE TRANSITION ENERGIES

Electron-electron interaction within the basic principles of QED is described by exchange of virtual photons. The one-photon exchange leads to the operator

$$I(\omega) = e^2 \alpha_1^\mu \alpha_2^\nu D_{\mu\nu}(\omega, \boldsymbol{r}_{12}), \tag{8}$$

where $D_{\mu\nu}$ is the photon propagator, which in the Coulomb gauge is written as

$$\begin{aligned} D_{00}(\omega, \boldsymbol{r}_{12}) &= \frac{1}{4\pi r_{12}}, \quad D_{i0} = D_{0i} = 0 \quad (i = 1, 2, 3)\,, \\ D_{il}(\omega, \boldsymbol{r}_{12}) &= \int \frac{d\boldsymbol{k}}{(2\pi)^3} \frac{\exp(i\boldsymbol{k}\cdot\boldsymbol{r}_{12})}{\omega^2 - \boldsymbol{k}^2 + i0} \Big(\delta_{il} - \frac{k_i k_l}{\boldsymbol{k}^2}\Big) \quad (i, l = 1, 2, 3)\,, \end{aligned} \tag{9}$$

$r_{12} = |\boldsymbol{r}_{12}| = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$, $\boldsymbol{r}_i$ is the position vector of the $i$th electron, and $\alpha^\mu = (1, \boldsymbol{\alpha})$ are the Dirac matrices.

Expanding expression (9) in powers of the photon frequency one can derive a simplified form of the interaction. The low-frequency limit of this interaction consists of two parts, referred to as the Coulomb and the Breit interaction,

$$V(i,j) = V_{\rm C}(i,j) + V_{\rm B}(i,j) = \frac{\alpha}{r_{ij}} - \alpha \left[ \frac{\boldsymbol{\alpha}_i \cdot \boldsymbol{\alpha}_j}{2r_{ij}} + \frac{(\boldsymbol{\alpha}_i \cdot \mathbf{r}_{ij})(\boldsymbol{\alpha}_j \cdot \mathbf{r}_{ij})}{2r_{ij}^3} \right]. \tag{10}$$

The most traditional approach for the treatment of the electron-electron interaction in relativistic many-electron atoms consists in using so-called Breit approximation. In this approximation the total Hamiltonian can be represented as the sum of the one-electron Dirac Hamiltonians and the Coulomb and Breit electron-electron interactions, projected on the positive-energy Dirac's states. In this way one gets the Dirac-Coulomb-Breit equation. Traditional methods for solving the Dirac-Coulomb-Breit equation are the many-body perturbation theory (MBPT) [48, 49], the multi-configuration Dirac-Fock method [50], and the configuration-interaction (CI) method [3, 34]. All these methods treat the one-photon exchange exactly and the higher-order electron correlation is accounted for within the Breit approximation only.

The current level of experimental accuracy demands rigorous QED calculations of two-photon exchange contributions, which for $n = 2$ states of Li-like ions were performed in Refs. [6, 8–11, 13, 51]. Meanwhile rigorous QED calculations of three- and more photon exchange contributions have not been performed up to now. For high-$Z$ few-electron ions