Resonance induced by repulsive interactions in a model of globally-coupled bistable systems

T. Vaz Martins[1,2], V. N. Livina[3], A. P. Majtey[4], R. Toral[1]
(1)IFISC, Instituto de Física Interdisciplinar y Sistemas Complejos, CSIC-UIB, Campus UIB, E-07122 Palma de Mallorca, Spain
(2)Centro de Física do Porto, DF, FCUP, 4169-007 Porto, Portugal
(3)School of Environmental Sciences, University of East Anglia, Norwich, NR4 7TJ, UK
(4) Departament de Física, Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain
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We show the existence of a competition-induced resonance effect for a generic globally coupled bistable system. In particular, we demonstrate that the response of the macroscopic variable to an external signal is optimal for a particular proportion of repulsive links. Furthermore, we show that a resonance also occurs for other system parameters, like the coupling strength and the number of elements. We relate this resonance to the appearance of a multistable region, and we predict the location of the resonance peaks, by a simple spectral analysis of the Laplacian matrix.

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I. INTRODUCTION

The amplification of an external forcing acting upon a dynamical system under the presence of the right amount of disorder has attracted much attention in the last decades. The phenomenon of *stochastic resonance*, proposed in 1981 to explain the periodicity of ice ages, [1, 2] is a somehow counterintuitive effect arising from the cooperation between deterministic dynamics and dynamical disorder or noise. By this effect, a system's coherent response to a weak signal can be optimally amplified by an intermediate level of noise. The prototypical example is that of a continuous variable whose deterministic dynamics is relaxational in a double-well potential. Noise induces jumps between the wells with a rate given by Kramers' expression [3]. The system becomes optimally synchronized with the signal when the signal half-period matches Kramers' rate, as reflected by a maximum value in a suitably defined response. Applications of stochastic resonance wer e addressed in many areas, and the theory evolved in several directions (see [4] and [5] for reviews). We would like to stress here the focus on many-component, or extended, systems [6, 7] for which it was shown that it is possible to tune some of the parameters, like the number of elements [8–11], or coupling strength [12–20], to further enhance the resonance effect.

Another, more recent, related line of research considers the role that other types of disorder, such as quenched noise (identified with heterogeneity or disorder), can play in producing a resonance effect in many-component systems. Tessone *et al.* [21, 22] have shown that in generic bistable or excitable systems, an intermediate level of diversity in the individual units can enhance the global response to a weak signal. In bistable systems, a mean field analysis interprets the phenomenon macroscopically as a result of the two equilibrium states getting closer to each other and the lowering of the height of the potential barrier, thereby turning the signal suprathreshold. The resonance appears close to an order-disorder transition, and it was suggested that any source of disorder would lead to the same result. This *diversity-induced resonance* effect has already found extensions in systems concerned with cellular signaling [23], complex networks [24], intercellular Ca^{2+} wave propagation induced by cellular diversity [25], linear oscillators [26] and others. Focusing on the double-well model, Perc *et al.* [27] studied the combined effect of dynamic and static disorder, where static disorder was either diversity, the presence of competitive interactions, or a random field. Namely, they showed that the random presence of repulsive bonds decreases the level of noise warranting the optimal response.

It is the purpose of this work to show that competitive interactions can actually replace -not merely enhance- noise in its constructive effect. In a recent study [28] we focused on a network of Ising-like units, and found that the addition of an intermediate fraction of repulsive links can increase the sensitivity to an external forcing. In this work, we expand the result by looking at a generic double-well model, and clarifying some of the previous conclusions. We resort to a spectral analysis of the Laplacian [29] matrix, to locate the amplification region, and unveil the mechanism of resonance.

The outline of this paper is as follows: in section II we will introduce the model; we show that there is an amplification and discuss how the amplification mechanism is related to a break of stability in section III; and how we can predict the resonance peaks in section IV; Conclusions are drawn in section V.

II. THE MODEL

We consider a system of N globally-coupled bistable units described by real variables $s_i(t), i = 1, \ldots, N$ under the influence of a periodic forcing.

$$\frac{ds_i}{dt} = s_i - s_i^3 + \frac{C}{N}\sum_{j=1}^{N} J_{ij}(s_j - s_i) + A\sin(2\pi t/T), \quad (1)$$