longer diagonalize the polarization operator already because the vectors $(\tilde{F}k)_\mu$ and $(Fk)_\mu$ stop being mutually orthogonal, since their scalar product $-k\tilde{F}Fk = \mathfrak{G}k^2$ is now nonzero.

When $\mathfrak{G} \neq 0$, the first eigenvector is expressed in terms of the fields by the same formula as in (7):

$$\flat^{(1)}_\mu = (F^2k)_\mu k^2 - k_\mu(kF^2k), \qquad (\flat^{(1)}\tilde{F}k) = (\flat^{(1)}Fk) = 0,$$
$$(\flat^{(1)})^2 = k^2(k^2\mathcal{E}^2 - kF^2k)(k^2\mathcal{B}^2 + kF^2k) \Leftrightarrow k^2(B^2+E^2)^2k_\perp^2(k_3^2 - k_0^2) \tag{8}$$

and the first eigenvalue is given by the formula

$$\varkappa_1 = \frac{k^2(\mathcal{B}^2+\mathcal{E}^2)}{k^2\mathcal{B}^2 + kF^2k}\Lambda_1 \Leftrightarrow \frac{k^2}{k_3^2 - k_0^2}\Lambda_1, \tag{9}$$

where the scalar function of the fields and momentum Λ_1 here, as well as other Λ's below, is a *linear* superposition of the polarization tensor components $\Pi_{\mu\nu}$. The other two eigenvectors are the linear combinations

$$\flat^{(2,3)}_\mu = -2\Lambda_3 c^-_\mu + \left[\Lambda_2 - \Lambda_4 \pm \sqrt{(\Lambda_2-\Lambda_4)^2 + 4\Lambda_3^2}\right] c^+_\mu \tag{10}$$

(where the square root is understood algebraically: $\sqrt{Z^2} = Z$, and not $|Z|$) of two orthonormalized vectors :

$$c^-_\mu = \frac{\mathcal{B}(Fk)_\mu + \mathcal{E}(\tilde{F}k)_\mu}{(\mathcal{B}^2+\mathcal{E}^2)^{1/2}(k^2\mathcal{E}^2 - kF^2k)^{1/2}} \Leftrightarrow \frac{B(Fk)_\mu + E(\tilde{F}k)_\mu}{(B^2+E^2)|\mathbf{k}_\perp|},$$
$$c^+_\mu = \mathrm{i}\frac{\mathcal{E}(Fk)_\mu - \mathcal{B}(\tilde{F}k)_\mu}{(\mathcal{B}^2+\mathcal{E}^2)^{1/2}(k^2\mathcal{B}^2 + kF^2k)^{1/2}} \Leftrightarrow \frac{E(Fk)_\mu - B(\tilde{F}k)_\mu}{(B^2+E^2)(k_0^2-k_3^2)^{1/2}},$$

$$(c^+c^-) = (c^\pm\flat^{(1)}) = (c^\pm k) = 0, \quad (c^\pm)^2 = 1, \tag{11}$$

thereby of the former basic vectors $(\tilde{F}k)_\mu$ and $(Fk)_\mu$, too. The corresponding two eigenvalues are

$$\varkappa_{2,3} = \frac{1}{2}\left[-(\Lambda_2+\Lambda_4) \pm \sqrt{(\Lambda_2-\Lambda_4)^2 + 4\Lambda_3^2}\right]. \tag{12}$$

The scalar coefficients in the linear combination (10) cannot be expressed in a universal way in terms of the field and momentum, but are irrational functions of the polarization tensor components. The reason is that the polarization operator is a linear combination of four independent matrices with four scalar coefficients, whereas there may be only three eigenvalues in accordance with three polarization degrees of freedom of a vector field. (When