a parameter $t_0 \in [T_1, T_2]$ such that $P(s, t_0) = r(s)$ $L_1 \leq s \leq L_2$, that is ,

$$u(s, t_0) = v(s, t_0) = w(s, t_0) \equiv 0 \ , \ L_1 \leq s \leq L_2, T_1 \leq t \leq T_2. \qquad (4.8)$$

Fistly, we derive the condition for n_1 to be parallel to the normal vector $n(s, t)$ of the surface $P(s, t)$:

The normal vector can be expressed as

$n(s,t) = \frac{\partial P(s,t)}{\partial s} \times \frac{\partial P(s,t)}{\partial t}$

$= ((-\tau(s) v(s,t) + \frac{\partial w(s,t)}{\partial s}) \frac{\partial v(s,t)}{\partial t} -$

$(\kappa(s) u(s,t) + \tau(s) w(s,t) + \frac{\partial v(s,t)}{\partial s}) \frac{\partial w(s,t)}{\partial t}) T(s) +$

$((-\tau(s) v(s,t) + \frac{\partial w(s,t)}{\partial s}) \frac{\partial u(s,t)}{\partial t} -$

$(1 + \kappa(s) v(s,t) + \frac{\partial u(s,t)}{\partial s}) \frac{\partial w(s,t)}{\partial t}) N(s) +$

$(-(\kappa(s) u(s,t) + \tau(s) w(s,t) + \frac{\partial v(s,t)}{\partial s}) \frac{\partial u(s,t)}{\partial t} +$

$(1 + \kappa(s) v(s,t) + \frac{\partial u(s,t)}{\partial s})(\frac{\partial v(s,t)}{\partial t})) B(s)$

Thus, we get

$$n(s, t_0) = \phi_1(s, t_0) T(s) + \phi_2(s, t_0) N(s) + \phi_3(s, t_0) B(s),$$

where

$\phi_1(s, t_0) = \frac{\partial w(s,t_0)}{\partial s} \frac{\partial v(s,t_0)}{\partial t} - \frac{\partial v(s,t_0)}{\partial s} \frac{\partial w(s,t_0)}{\partial t},$

$\phi_2(s, t_0) = \frac{\partial w(s,t_0)}{\partial s} \frac{\partial u(s,t_0)}{\partial t} - \left(1 + \frac{\partial u(s,t_0)}{\partial s}\right) \frac{\partial w(s,t_0)}{\partial t},$

$\phi_3(s, t_0) = \left(1 + \frac{\partial u(s,t_0)}{\partial s}\right) \frac{\partial v(s,t_0)}{\partial t} - \frac{\partial v(s,t_0)}{\partial s} \frac{\partial u(s,t_0)}{\partial t}.$

This follows that $n_1(s) // n(s, t_0)$, $L_1 \leq s \leq L_2$, if and only if there exits a function $\lambda(s) \neq 0$ such that

$$\phi_1(s, t_0) = 0, \ \phi_2(s, t_0) = \lambda(s) \cos\theta, \ \phi_3(s, t_0) = \lambda(s) \sin\theta. \qquad (4.9)$$