In the context of Corollary A.6, let $\{(z_m, r_m)\}_{m\geq 1}$ be a sequence with values in $\ell^\infty(\mathcal{F}) \times [0, \infty)$ and such that $\sup_{f\in\mathcal{F}} |z_m(f) - z(f)| + |r_m| \to 0$. Lemma A.5 implies that $h(z_m, r_m) \to h(z, 0) \equiv 0$. Since the sequence was arbitrarily chosen, h is indeed continuous at $(z, 0)$ and the proof of Corollary A.6 is complete. $\square$

Lemma A.7. *If* Π *is Donsker, then* $\mathcal{F}$ *is also Donsker.*

Proof. Similar to (A.1), the key is the following remark: for every policy $\pi : \mathcal{X} \to \{-1, 1\}$,

$$\begin{aligned} 2f_\pi(O) &= |A + \pi(X)| \frac{Y - Q(A, X)}{P(A|X)} \\ &\quad + (1 + \pi(X))Q(1, X) + (1 - \pi(X))Q(-1, X) - 2\mathcal{V}(\pi). \end{aligned} \tag{A.10}$$

For future use, we first note that (A.1) and (A.2) imply, for any $\pi_1, \pi_2 \in \Pi$,

$$|f_{\pi_1}(O) - f_{\pi_2}(O)| \lesssim |\pi_1(X) - \pi_2(X)| + \|\pi_1 - \pi_2\|$$

hence

$$\|f_{\pi_1} - f_{\pi_2}\| \lesssim \|\pi_1 - \pi_2\|. \tag{A.11}$$

Introduce $\phi : \mathbb{R}^5 \to \mathbb{R}$ given by $2\phi(u) = u_1|u_2 + u_3| + (1 + u_3)u_4 + (1 + u_3)u_5$ and f_1, f_2, f_4, f_5 be the function given by $f_1(o) \equiv (y - Q(a, x))/P(A = a|X = x)$, $f_2(o) \equiv x$, $f_4(o) \equiv Q(1, x)$ and $f_5(o) \equiv Q(-1, x)$. Let $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{F}_4$, $\mathcal{F}_5$ be the singletons $\{f_1\}$, $\{f_2\}$, $\{f_4\}$ and $\{f_5\}$, each of them a Donsker class. Let $\tilde{\mathcal{F}} \equiv \mathcal{F}_1 \times \mathcal{F}_2 \times \Pi \times \mathcal{F}_4 \times \mathcal{F}_5$ and note that $\phi \circ \mathbf{f} = \tilde{f}_\pi$ if $\mathbf{f} \in \tilde{\mathcal{F}}$ writes as $\mathbf{f} = (f_1, f_2, \pi, f_4, f_5)$. In light of (A.10), observe now that, for every $\mathbf{f}_1 = (f_1, f_2, \pi_1, f_4, f_5), \mathbf{f}_2 = (f_1, f_2, \pi_2, f_4, f_5) \in \tilde{\mathcal{F}}$, it holds that

$$|\phi \circ \mathbf{f}_1(o) - \phi \circ \mathbf{f}_2(o)| \lesssim |\pi_1(x) - \pi_2(x)|$$

(the bound on Y implies that $\|\gamma\|_\infty$ is finite). By [13, Theorem 2.10.6], whose conditions are obviously met, $\phi \circ \tilde{\mathcal{F}} = \{\tilde{f}_\pi : \pi \in \Pi\}$ is Donsker. Because $\Lambda \equiv \{\mathcal{V}(\pi) : \pi \in \Pi\}$ (viewed