for $\psi \in G$ is

$$(e_{nhj}, \psi) = \int \frac{1}{\Pi h_i} K(\frac{x - x_j}{h}) \psi(x) dx - (f, \psi)$$

and consider $e_{nh} = \frac{1}{n} \sum_{j=1}^{n} e_{nhj}$; this generalized function provides $\widehat{f} - f$.

The expectation functional Ee_{hn} gives the generalized bias of the estimator $\hat{f}$, $Bias\left(\hat{f}\right)$, see (8).

Next to derive the variance functional consider $T_{lj} = E(e_{nhl}, \psi_1)(e_{hnj}, \psi_2))$.

For $l \neq j$ by independence

$$\begin{aligned} T_{lj} &= E(e_{nhl}, \psi_1)(e_{nhj}, \psi_2) = E(e_{nhl}, \psi_1) E(e_{nhj}, \psi_2) \\ &= \left(Bias\left(\hat{f}\right), \psi_1\right) \left(Bias\left(\hat{f}\right), \psi_2\right). \end{aligned}$$

For $l = j$

$$\begin{aligned} T_{jj} &= E(e_{nhj}(x), \psi_1)(e_{nhj}(x), \psi_2) \\ &= \int \left[\int \frac{1}{\Pi h_i} K(\frac{x_j - x}{h}) \psi_1(x) dx - (f, \psi_1)\right] \times \\ &\quad \left[\int \frac{1}{\Pi h_i} K(\frac{x_j - x}{h}) \psi_2(x) dx - (f, \psi_2)\right] dF(x_j) \\ &= T_{jj}^1 + T_{jj}^2, \end{aligned}$$

where

$$T_{jj}^1 = \int \left(\int \frac{1}{\Pi h_i} K(\frac{x_j - x}{h}) \psi_1(x) dx\right) \left(\int \frac{1}{\Pi h_i} K(\frac{x_j - x}{h}) \psi_2(x) dx\right) dF(x_j)$$