Physics With Two Time Dimensions

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We explore the properties of physical theories in space-times with two time dimensions. We show that the common arguments used to rule such theories out do not apply if the dynamics associated with the additional time dimension is thermal or chaotic and does not permit long-lived time-like excitations. We discuss several possible realizations of such theories, including holographic representations and the possibility that quantum dynamics emerges as a consequence of a second time dimension.

I. INTRODUCTION

Modern unified theories of the fundamental interactions generally posit that we live, at microscopic scales, in a universe with more than three spatial dimensions. Following Kaluza's [1] and Klein's [2] original idea, the extraneous dimensions are assumed to be compactified so that excited modes of the elementary particle fields involving nontrivial dynamics in these dimensions are exceedingly massive and therefore not directly observable. The sole exception to this rule may be gravity [3].

The Kaluza-Klein concept is naturally realized within superstring theory, which requires at least six additional dimensions for internal consistency [4]. It has been noted in recent years that the additional spatial dimensions do not have to be aspects of a genuine space-time continuum, but could be artefacts of the algebraic structure of some underlying quantum mechanical system [5].

One common feature of most theoretical speculations of this kind is that the additional dimensions are assumed to be spatial. The reason for this restriction is that physics in the presence of more than one *temporal* dimension is generally considered to violate several fundamental and well established properties of nature. The two most important of these principles are causality and unitarity, but there are also concerns that additional time dimensions imply tachyonic modes and ghost fields, i. e. field components with negative norm.

We will discuss the case against more than a single time dimension in Sect. II, followed by an overview of the arguments that have led us to reconsider this case. In Sect. III, we first review the mathematical difficulties associated with the initial-value problem for wave equations in spaces with multiple time dimensions. After explaining why a recently proposed resolution of the ill-posed nature of the initial-value problem does not work in general for interacting fields, we will argue that interacting fields without propagating time-like modes (with respect to the additional time dimension) can evade this difficulty. In particular, we show how the difficulty can be circumnavigated if the field is thermalized in the additional time dimension. We propose a specific model of such a quantum field theory, constructed as the boundary field theory of a six-dimensional Anti-deSitter space-time with two time dimensions containing a Schwarzschild black-hole.

In Sect. IV we point out that, once the notion of field theories in space-times with two time dimensions is accepted as physically viable, the *quantum* dynamics of the reduced field theory in Minkowski space can be generated by the mechanism of micro-canonical quantization from the underlying *classical* dynamics of a field in a higher-dimensional space-time that contains two time dimensions. Finally, we summarize our results and present some thoughts about the possible cosmological context of a second time dimension.

II. MULTIPLE TIME DIMENSIONS

A. The case against multiple time dimensions

Let us briefly review why the basic principles of causality, unitarity, and vacuum stability are thought to be violated in the presence of additional time dimensions.

1. *Causality:* In space-times with at least two time dimensions, it is always possible to construct closed time-like curves [6]. This is especially worrisome in the context of classical physics, where observers are not part of the physical system and can "act" in unpredictable ways. The existence of closed time-like curves implies that an observer can revisit the past and, if we accept the tenet of "free will", change it in a manner that is incompatible with the already experienced future. The classic example of a resulting inconsistency is the trip back in time, where the hero unwittingly kills his own ancestor (the grandfather paradox). Obviously, anything resembling the common notion of causality cannot be maintained under such circumstances. The question of the well-posed nature of the Cauchy (initial-value) problem in space-times with closed time-like paths was investigated in detail by Aref'eva *et al.* [9].

2. *Unitarity:* While violations of causality can arise both in classical and quantum dynamics, violations