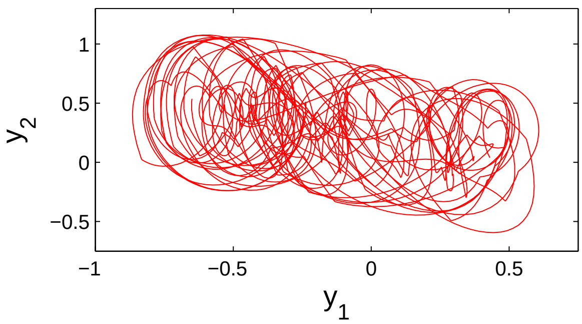

Figure 4: The chaotic trajectory of system (5.17). The simulation supports the result of Theorem 4.1 such that the system remains to be chaotic even if the system is perturbed with the solutions of (5.12).

6 Control of chaos

In this part of the paper, we will present a numerical technique to control the chaos of system (5.17). For that purpose, we will make use of the OGY control method [25] applied to the logistic map (1.5), which is the main source of chaos in the unidirectionally coupled system (5.12)+(5.17).

Let us briefly explain the OGY algorithm for the logistic map (1.5) [25, 37]. Denote by $\kappa^{(j)}$, $j = 1, 2, \ldots, p$, the target $p-$periodic orbit of (1.5) with $\mu = 3.9$ to be stabilized. Suppose that the parameter μ in (1.5) is allowed to vary in the range $[3.9 - \varepsilon, 3.9 + \varepsilon]$, where ε is a given small positive number. In the OGY control method [37], after the control mechanism is switched on, we consider (1.5) with the parameter value $\mu = \overline{\mu}_i$, where

$$\overline{\mu}_i = 3.9 \left(1 + \frac{(2\kappa^{(j)} - 1)(\overline{\kappa}_i - \kappa^{(j)})}{\kappa^{(j)}(1 - \kappa^{(j)})} \right), \tag{6.18}$$

provided that the number on the right-hand side of the formula (6.18) belongs to the interval $[3.9 - \varepsilon, 3.9 + \varepsilon]$. Here, the sequence $\{\overline{\kappa}_i\}$, $i \geq 0$, satisfying $\overline{\kappa}_0 \in [0, 1]$ is an arbitrary solution of the map

$$\overline{\kappa}_{i+1} = G_{\overline{\mu}_i}(\overline{\kappa}_i). \tag{6.19}$$

Formula (6.18) is valid only if the trajectory $\{\overline{\kappa}_i\}$ is sufficiently close to the target periodic orbit $\kappa^{(j)}$, $j = 1, 2, \ldots, p$. Otherwise, we take $\overline{\mu}_i = 3.9$ in (6.19) so that the system evolves at its original parameter value, and wait until the trajectory $\{\overline{\kappa}_i\}$ enters in a sufficiently small neighborhood of the periodic orbit such that the inequality $-\varepsilon \leq 3.9 \dfrac{(2\kappa^{(j)} - 1)(\overline{\kappa}_i - \kappa^{(j)})}{\kappa^{(j)}(1 - \kappa^{(j)})} \leq \varepsilon$ holds. If this is the case, the control of chaos is not achieved immediately after switching on the control mechanism. Instead, there is a transition time before the desired periodic orbit is stabilized. The transition time increases if the number ε decreases [9].

Consider the sequence $\overline{\zeta} = \{\overline{\zeta}_i\}$, $i \geq 0$, generated through the equation $\overline{\zeta}_i = 1.05i + \overline{\kappa}_i$, where $\{\overline{\kappa}_i\}$, $\overline{\kappa}_0 \in [0, 1]$, is a solution of (6.19). To control the chaos of (5.17), we replace the sequence $\zeta = \{\zeta_i\}$