



FIG. 10: Superfluid and normal fluid velocity fields associated with first and second sound in a unitary Fermi superfluid. The solid (black) line shows the ratio $v_s^{(1)}/v_n^{(2)}$ of the superfluid and normal fluid velocity fields involved with first sound, (B8). The dashed (red) line shows the ratio $\rho_{s0}v_s^{(2)}/\rho_{n0}v_n^{(2)}$ of the superfluid and normal fluid currents associated with second sound, (B9). If we set $\epsilon_{\text{LP}} = 0$, these ratios would be 1 and -1 , respectively, describing the velocity fields for pure density and temperature oscillations.

and second sound are significantly different from those given in (B2). This is a reflection of the fact that first and second sound are not pure uncoupled density and temperature waves, respectively, which would be described by (B2) and correspond to $\epsilon_{\text{LP}} = 0$ (and $\rho_{n0} \neq 0$). As we have shown in [8], such in-phase ($\mathbf{v}_s = \mathbf{v}_n$) and out-of-phase zero current ($\mathbf{j} = 0$) solutions correspond to first and second sound velocities given by

$$u_1^2 = v_s^2, \quad u_2^2 = T \frac{\bar{s}_0^2}{\bar{c}_v} \frac{\rho_{s0}}{\rho_{n0}} \equiv v^2. \quad (\text{B12})$$

As we show in figure 2 of [8], these sound velocities are in good agreement with a full calculation including the coupling associated with ϵ_{LP} . In this Appendix and section IV, we have carried out a careful analysis based on expanding to first order in the parameter $\epsilon_{\text{LP}}x$, which is small ($\ll 1$) at all temperatures [14, 15]. The results in figure 10 show that the