

geometric function ${}_3F_2$. The result is

$$B(d; n_1, m; n_2, m) = \frac{i}{(4\pi)^2} (-m^2)^{2-n_1-n_2} \left[\frac{m^2}{4\pi\mu^2} \right]^{\frac{d}{2}-2} \frac{\Gamma(n_1 + n_2 - \frac{d}{2})}{\Gamma(n_1 + n_2)} {}_3F_2 \left[\begin{matrix} n_1, n_2, n_1 + n_2 - \frac{d}{2} \\ \frac{1}{2}(n_1 + n_2), \frac{1}{2}(n_1 + n_2 + 1) \end{matrix} \middle| \frac{q^2}{4m^2} \right]. \quad (2.33)$$

which agrees with an expression given for this integral in Ref. [27]. Note that this technique can be used to calculate integrals such as (2.27) where the masses in each propagator are distinct. In Ref. [27] this is done, and the results are given in terms of multivariable generalized hypergeometric functions. However, these functions are somewhat unwieldy and are not widely implemented in computer algebra systems. Fortunately, all of the integrals in Chapters 3, 4, 5 and 6 can be evaluated in terms of only two scales, the external momentum q and the heavy quark mass m .

Consider now a two-loop integral with two massive propagators

$$J(d; n_1, m; n_2, m; n_3, 0) = \frac{1}{\mu^{2(d-4)}} \int \frac{d^d k_1}{(2\pi)^d} \int \frac{d^d k_2}{(2\pi)^d} \frac{1}{(k_1^2 - m^2)^{n_1} [(k_2 - q)^2 - m^2]^{n_2} (k_1 - k_2)^{2n_3}}. \quad (2.34)$$

This integral occurs in Chapter 3, and is represented by the Feynman diagram in Fig. 2.5.

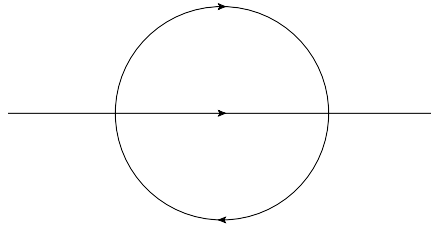


Figure 2.5: Feynman diagram representing the J -type two-loop sunset integral.