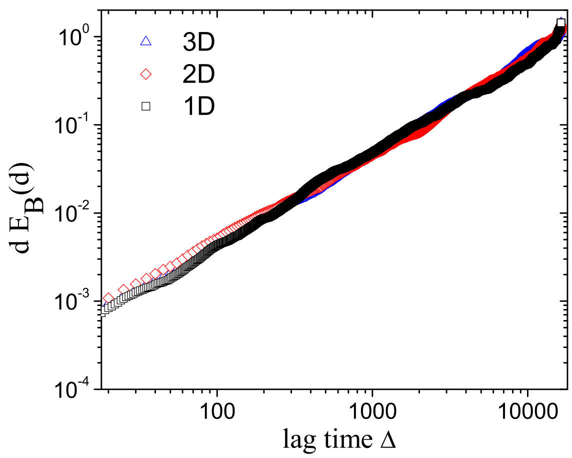

FIG. 7. (Color online) Rescaled ergodicity breaking parameter dE_B versus lag time Δ for interval size $L = 5$ for 1D, 2D, and 3D space (from top to bottom), with Hurst exponent $H = 1/4$ and measurement time $T = 2^{14}$. E_B was evaluated from 200 trajectories with different initial positions.

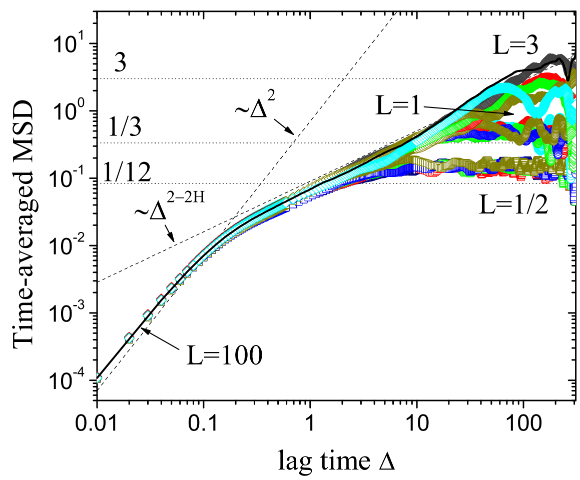

FIG. 8. (Color online) Time-averaged mean squared displacement (MSD) versus lag time Δ. The two dashed lines represent the two asymptotic scaling behaviors $\left\langle\overline{\delta^2(\Delta,T)}\right\rangle \simeq \Delta^2$ and $\left\langle\overline{\delta^2(\Delta,T)}\right\rangle \simeq \Delta^{2-2\overline{H}}$. For each given $L = 1/2$, 1, and 3, five different trajectories are drawn to visualize the scatter. As a reference curve for motion in free space, the curve for $L = 100$ (line) is drawn. In the simulation, we chose the Hurst exponent $\overline{H} = 5/8$ [NB: for the FLE this means subdiffusion], time increment $dh = 0.01$, particle mass $m = 1$, initial velocity $v_0 = 1$, initial position $y_0 = 0$, friction coefficient $\overline{\gamma} = 10$, and $k_BT = 1$.

for FBM big scatter is not caused by higher dimensions in presence of reflecting walls. In fact, from Eqs. (4.2) and (4.6), we can analytically derive the relation

$$E_B(d) = \frac{E_B(d=1)}{d}, \tag{4.7}$$

which still holds in the case of confined motion (see appendix B for the derivation). This relation is numerically confirmed in Fig. 7 where three E_B curves collapse upon rescaling by $dE_B(d)$. According to this relation, we expect that ergodic behavior obtained in one-dimensional confined motion (Figs. 3 and 4) will also be present in multiple dimensions with a factor of $1/d$.

V. FRACTIONAL LANGEVIN EQUATION MOTION IN CONFINED SPACE

In this section we analyze FLE motion under confinement. Due to the different physical basis compared to FBM, in particular, the occurrence of inertia, we observe interesting variations on the properties studied in the previous section.

A. Mean squared displacement

Using the correlation function [55]

$$\langle y(t_1)y(t_2)\rangle = \frac{k_BT}{m}[t_1^2E_{2\overline{H},3}(-\gamma t_1^{2\overline{H}}) + t_2^2E_{2\overline{H},3}(-\gamma t_2^{2\overline{H}}) -(t_2-t_1)^2E_{2\overline{H},3}(-\gamma|t_2-t_1|^{2\overline{H}})], \tag{5.1}$$

one can show analytically that, similarly to the FBM, the ensemble averaged second moment $\langle y^2(t)\rangle$ is identical to its time averaged analog $\left\langle\overline{\delta^2(\Delta)}\right\rangle$, for all Δ in free space, namely

$$\left\langle\overline{\delta^2(\Delta,T)}\right\rangle = 2\frac{k_BT}{m}\Delta^2E_{2\overline{H},3}(-\gamma\Delta^{2\overline{H}}). \tag{5.2}$$

Thus, the time averaged mean squared displacement turns over from a ballistic motion

$$\left\langle\overline{\delta^2(\Delta,T)}\right\rangle \simeq \Delta^2 \tag{5.3}$$

at short lag time to the subdiffusive behavior

$$\left\langle\overline{\delta^2(\Delta,T)}\right\rangle \simeq \Delta^{2-2\overline{H}} \tag{5.4}$$

at long lag times, in free space.

We numerically study how this scaling behavior is affected by the confinement. Figure 8 shows typical curves for the time averaged mean squared displacement, for interval sizes $L = 1/2$, 1, 3, and 100 (regarded as free space motion) with identical initial conditions and Hurst exponent $\overline{H} = 5/8$. The results are summarized as follows: (1) We observe both scaling behaviors, $\left\langle\overline{\delta^2(\Delta,T)}\right\rangle \simeq \Delta^2$