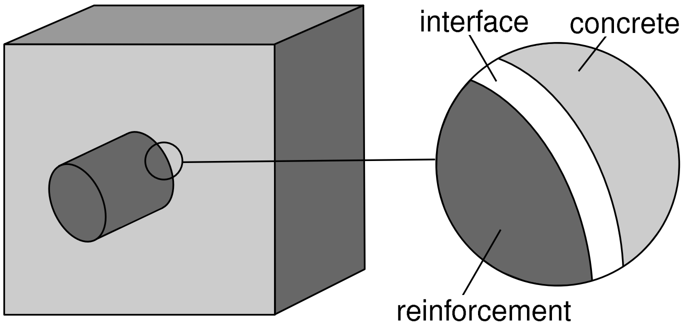

Figure 1: Three-dimensional modelling of reinforced concrete: Concrete, reinforcement and bond between concrete and reinforcement are considered as individual phases. Left: Concrete cube (light grey) with reinforcement bar (dark grey); right: detail of interface between reinforcement and concerte.

plain the small number of models which are available based on this three-dimensional approach. Discrete approaches, such as lattice and particle models, might be a favourable alternative to describe this discontinuous problem. Their potential to model corrosion induced cracking and its influence on bond is assessed in the present study.

Lattice approaches have been used successfully in the past to model the failure of concrete, as reported by Schlangen and van Mier (1992) and Bolander and Saito (1998). The model by Bolander has been shown to accurately reproduce analytical solutions for elasticity and potential flow problems (Yip et al. 2005; Bolander and Berton 2004). Furthermore, it allows for the use of constitutive models, which are formulated by means of tractions and displacement jumps, as commonly used in interface approaches for concrete fracture (Caballero et al. 2006). These have been shown to result in an element-size independent description of crack-openings. This lattice approach is used in the present study to describe the mechanical response of three phases, namely reinforcement, concrete and bond between reinforcement and concrete. In this approach, the lattice elements do not represent the meso-structure of the materials (Zubelewicz and Bažant 1987). Instead, they are used to discretise the continuum. Thus, constitutive models are required for all three phases, which are described in more detail in the following sections.

2 MODELLING APPROACH

The present lattice approach for the modelling of corrosion induced cracking follows the framework developed by Bolander and his co-workers. The nodes of the lattice are randomly located in the domain to be analysed, subject to the constraint of a minimum distance (Zubelewicz and Bažant 1987). The arrangement of the lattice elements is determined from the edges of the tetrahedra of the Delaunay tessellation based on the randomly placed nodes. The cross-sectional properties of these elements are obtained from the dual Voronoi tessellation of the same set of random nodes. For the interface between reinforcement and concrete, the lattice nodes are not placed randomly but at special locations, such that the middle cross-sections of the lattice elements form the boundaries between the two phases (Bolander and Berton 2004). The nodes of the lattice elements have six degrees of freedom, namely three translations and three rotations. These degrees of freedom are related to three displacement and three rotation discontinuities at the centroid of the middle cross-section of the elements. The three rotation discontinuities are related to moments by elastic relationships. The three displacement discontinuities are used in interface constitutive models to determine the corresponding tractions. In the present study, an elastic interface model is used for the reinforcement. One limitation of the present lattice approach is that it only predicts Poisson's ratios of less than $1/4$ for 3D and less than $1/3$ for 2D. This is restrictive for the 3D modelling of the steel reinforcement, which has a Poisson's ratio greater than $1/4$. This limitation could be overcome by combining the lattice model with continuum tetrahedra as discussed for the 2D case by Grassl et al. (2006). However, this approach is beyond the scope of the present study. The interface model for concrete is based on a combination of plasticity and damage, which describes the softening and reduction of the unloading stiffness in tension as well as the nonlinear hardening response in high confined compression (Grassl 2009). For the interface between concrete and reinforcement, a new cap-plasticity model is proposed which is described in more detail in the next section.

2.1 *Elasto-plastic cap model for the bond between concrete and reinforcement*

In the lattice model, the nodal degrees of freedom are related to displacement jumps at the middle cross-section of the lattice element. This three-dimensional displacement jump $\mathbf{u}_\mathrm{c} = (u_\mathrm{n}, u_\mathrm{s}, u_\mathrm{t})^T$ is transformed into strains $\boldsymbol{\varepsilon} = (\varepsilon_\mathrm{n}, \varepsilon_\mathrm{s}, \varepsilon_\mathrm{t})^T$ by means of the interface thickness h as

$$\boldsymbol{\varepsilon} = \frac{\mathbf{u}_\mathrm{c}}{h} \tag{1}$$

The three subscripts n, s and t denote the normal and two tangential directions in the local coordinate system of the interface (Figure 2a). The thickness of the interface h (Figure 2b) is chosen to be equal to the lattice elements crossing the interface between reinforcement steel and concrete. The strains are related to the nominal stress $\boldsymbol{\sigma} = (\sigma_\mathrm{n}, \sigma_\mathrm{s}, \sigma_\mathrm{t})^T$ by the elasto-plastic stress-strain relationship

$$\boldsymbol{\sigma} = \mathbf{D}_\mathrm{e} \left(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_\mathrm{c} - \boldsymbol{\varepsilon}_\mathrm{p} \right) \tag{2}$$