QFT operator tends to generate highly entangled states when it is applied on product states [7]. Furthermore, the superior efficiency of Shor's algorithm is attributed to the QFT, and since entanglement is considered a necessary resource for quantum computational speedup, one would expect that the QFT will induce it. It seems as though for the purpose of quantum speedup it suffices for the QFT to simply operate on a highly entangled register rather than generate entanglement by itself.

The states generated by the preprocessing stage of Shor's algorithm are called periodic states. These states consist of an equal superposition of basis states whose indices take the form $i = jr + l$, where $j = 0, 1, 2, \ldots,$ r is the period and l is referred to as a shift. It was shown by numerical simulations that these states have the property of not being further entangled by the QFT [7]. In this article we explain this surprising property using an approximated formula for the Groverian entanglement measure of periodic states.

The article is organized as follows. In Sec. II we present the Groverian measure. The periodic states generated by the preprocessing stage of Shor's algorithm are described in Sec. III and their entanglement is analyzed in Sec. IV. The effect of the QFT on their entanglement is considered in Sec. V. The results are discussed in Sec. VI and summarized in Sec. VII.

II. THE GROVERIAN ENTANGLEMENT MEASURE

The Groverian measure of a quantum state $|\psi\rangle$ of q qubits is based on the maximal overlap that $|\psi\rangle$ may have with any product state $|\varphi\rangle$, with the same number of qubits. The smaller this overlap gets, the more entangled the quantum state becomes. We define the square of this overlap as

$$P_{\max}(\psi) = \max_{|\varphi\rangle = |\varphi_1\rangle \otimes \cdots \otimes |\varphi_q\rangle} |\langle \varphi | \psi \rangle|^2 , \tag{1}$$

where $|\varphi_m\rangle$, $m = 1, \ldots, q$ are single qubit states. This quantity cannot be decreased by local operations and classical communication between the parties holding the different qubits. Therefore, any nonincreasing function of $P_{\max}$ that vanishes for product states (where $P_{\max} = 1$) is a valid entanglement measure. Among all these possible measures, we have found it useful to use the *logarithmic Groverian entanglement measure* [24]