Exact calculation of the number of degrees of freedom of a rigid body constituted by n particles

Jorge Bernal,* Roberto Flowers-Cano,† and Adrian Carbajal-Dominguez‡

Universidad Juárez Autónoma de Tabasco,

km. 1 Carretera Cunduacán-Jalpa,

Cunduacán, Tabasco, México, C.P. 86690

Abstract

In this work we correct a calculation made by Albert Einstein that appears in his book titled "The Meaning of Relativity" (Princeton, 1953), and by means of which he tries to obtain the number of degrees of freedom of a system constituted by n particles with fixed relative distances and which are immerse in a three-dimensional space. As a result of our analysis, we develop expressions which yield the number of degrees of freedom of an analogous system, not only in three, but in any arbitrary number D of dimensions.

The number of independent coordinate variables needed to simultaneously determine the position of every particle in a dynamical system is called the number of degrees of freedom of that system. So a system of n free particles in a three-dimensional space has $3n$ degrees of freedom, because three coordinates are needed to specify the location of the center of mass of each particle. However, if the particles are no longer all free, but there are restrictions imposed on the system, the number of degrees of freedom will be less than $3n$; $3n$ coordinates are still needed to locate the centers of mass, but less than $3n$ values are assignable at will to the coordinate variables [1]. Specifically, we are interested in the system made up of n particles in three-dimensional space, which hold fixed distances between them. In the sake of clarity, this system will be referred to from now on as S_3, and the number of its degrees of freedom will be referred to as N_3.

Usually, N_3 is calculated by giving S_3 the treatment of a rigid body. Mechanics recognizes two types of rigid bodies: the ones made up by a continuous distribution of mass; and those formed by n mass points joined by rigid links [2]. Thus, S_3 is equivalent to a rigid body of the second type.

It is not difficult to calculate the number of degrees of freedom of a rigid body of con-

*Electronic address: jorge.bernal@dacb.ujat.mx

†Electronic address: flowerscano@hotmail.com

‡Electronic address: adrian.carbajal@dacb.ujat.mx