the fact that the effective coordination number increases monotonically as a function of L_z.

The use of periodic boundary conditions in the z direction minimizes finite-size effects, so that for a given L_z, $W(S_{(L_z)_P}, q)$ would be expected to be closer to $W(sc, q)$ than $W(S_{(L_z)_F}, q)$ [30]. Again, to the extent that the lower bounds are close to the actual W functions for these respective slabs, one would expect $W(S_{(L_z)_P}, q)_\ell$ to be closer than $W(S_{(L_z)_F}, q)_\ell$ to $W(sc, q)$. Our results agree with this expectation. In contrast to $W(S_{(L_z)_F}, q)$, $W(S_{(L_z)_P}, q)$ is not, in general, a non-increasing function of L_z, as was discussed in general in [30] (see Fig. 1 therein). Thus, values of $W(S_{(L_z)_P}, q)$, and hence, *a fortiori*, $W(S_{(L_z)_P}, q)_\ell$, may actually lie slightly below those for $W(sc, q)$, as is evident for the $W(S_{3_P}, q)_\ell$ entries in Table I.

VIII. CONCLUSIONS

In this paper we have calculated rigorous lower bounds for the ground state degeneracy per site W, equivalent to the ground state entropy $S_0 = k_B \ln W$, of the q-state Potts antiferromagnet on slabs of the simple cubic lattice that are infinite in two directions and finite in the third. Via comparison with large-q expansions and numerical evaluations, we have shown how the results interpolate between the square (sq) and simple cubic (sc) lattices.

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IX. APPENDIX

We note the following results on $\mathbb{E}^d$ lattices and lattice sections: $W(\Lambda_{bip.}, 2) = 1$ for any bipartite lattice; $W(sq, 3) = (4/3)^{3/2}$ [37]; and $W(\{L\}, q) = W(\{C\}, q) = q - 1$, where L_n and C_n denote the n-vertex line and circuit graphs. For the infinite-length square-lattice strip of width 2, $W(sq[2_F \times \infty], q) = W(sq[2_P \times \infty], q) = \sqrt{q^2 - 3q + 3}$, where, as in the text, the subscripts F and P denote free and periodic boundary conditions in the direction in which the strip is finite. For the infinite-length strip of the square lattice with (transverse) width 3 and free transverse boundary conditions, $sq[3_F \times \infty]$ [31–33]

$$W(sq[3_F \times \infty], q) = (\lambda_{3_F,max})^{1/3} \tag{9.1}$$

where

$$\lambda_{3_F,max} = \frac{1}{2}\left[(q-2)(q^2 - 3q + 5) + \sqrt{R_3}\,\right] \tag{9.2}$$

with

$$R_3 = (q^2 - 5q + 7)(q^4 - 5q^3 + 11q^2 - 12q + 8) \ . \tag{9.3}$$

For the infinite-length strip of the square lattice with width 4 and free transverse boundary conditions, $sq[4_F \times \infty]$ [31, 35]

$$W(sq[4_F \times \infty], q) = (\lambda_{4_F,max})^{1/4} \tag{9.4}$$

where $\lambda_{4_F,max}$ is the largest root of the cubic equation

$$x^3 + b_{4_F,1}x^2 + b_{4_F,2}x + b_{4_F,3} = 0 \tag{9.5}$$

with

$$b_{4_F,1} = -q^4 + 7q^3 - 23q^2 + 41q - 33 \tag{9.6}$$

$$b_{4_F,2} = 2q^6 - 23q^5 + 116q^4 - 329q^3 + 553q^2 - 517q + 207 \tag{9.7}$$

and

$$\begin{aligned} b_{4_F,3} &= -q^8 + 16q^7 - 112q^6 + 449q^5 - 1130q^4 + 1829q^3 \\ &- 1858q^2 + 1084q - 279 \ . \end{aligned} \tag{9.8}$$