$\chi_i(t<0)=0$. The Fourier transform of Eq. 3 for a purely ac field $E(t)=E\cos(\omega t)$ gives:

$$
\begin{aligned}
\frac{P(\omega')}{\epsilon_0} &= \frac{E}{2}\left[\chi_1(\omega)+\frac{3E^2}{4}\chi_3(-\omega,\omega,\omega)+...\right]\delta(\omega'-\omega)\\
&+ \frac{E}{2}\left[\chi_1(-\omega)+\frac{3E^2}{4}\chi_3(\omega,-\omega,-\omega)+...\right]\delta(\omega'+\omega)\\
&+ \frac{E^3}{8}\chi_3(\omega,\omega,\omega)\delta(\omega'-3\omega)\\
&+ \frac{E^3}{8}\chi_3(-\omega,-\omega,-\omega)\delta(\omega'+3\omega)+...,
\end{aligned}
\tag{4}
$$

where the polarization P and the susceptibilities χ_i are now taken in the frequency domain and the dots indicate again infinite sums involving higher order terms. The response $P(t)$ to $E(t)=E\cos(\omega t)$ can thus be written

$$
P(t)/\epsilon_0 = Re\left[(E\chi_1(\omega)+3/4E^3\chi_{\bar{3}}(\omega)+...)e^{-i\omega t}\right]+Re\left[1/4E^3\chi_3(\omega)e^{-i3\omega t}+...\right]+.... \tag{5}
$$

To obtain Eq. (5), we have used the fact that because χ_1 and χ_3 are real in the time domain, their Fourier transform verify $\chi_1^*(\omega)=\chi_1(-\omega)$ and $\chi_3^*(\omega_1,\omega_2,\omega_3)=\chi_3(-\omega_1,-\omega_2,-\omega_3)$ (the star denotes the complex conjugate), and the invariance of χ_3 by permutation of its arguments. For simplicity, we write $\chi_3(\omega)=\chi_3(\omega,\omega,\omega)$ and $\chi_{\bar{3}}(\omega)=\chi_3(-\omega,\omega,\omega)$. Eq. (5) can be written

$$
\begin{aligned}
P(t)/\epsilon_0 = E(\chi_1'\cos\omega t+\chi_1''\sin\omega t)+3/4E^3(\chi_{\bar{3}}'\cos\omega t+\chi_{\bar{3}}''\sin\omega t)+...\\
+1/4E^3(\chi_3'\cos 3\omega t+\chi_3''\sin 3\omega t)+...,
\end{aligned}
\tag{6}
$$

where the susceptibilities χ_i are given as a function of their real and imaginary parts χ_i' and χ_i''. For practical applications, the modulii and arguments $|\chi_i|$ and δ_i are rather used:

$$
\begin{aligned}
P(t)/\epsilon_0 = E\left|\chi_1\right|\cos(\omega t-\delta_1)+3/4E^3\left|\chi_{\bar{3}}\right|\cos(\omega t-\delta_{\bar{3}})+\\
+...+1/4E^3\left|\chi_3\right|\cos(3\omega t-\delta_3)+...
\end{aligned}
\tag{7}
$$

We see in the rhs in Eqs 5-7 that the nonlinear susceptibility of interest, namely χ_3