(2.1) [13, 21].

We say that the response (2.3) replicates the period-doubling cascade of (2.1) if for each periodic $x(t)$, system (2.3) possesses a periodic solution with the same period. To illustrate the replication of period-doubling cascade, let us use $\overline{\sigma} = 10$, $\overline{r} = 28$, $\overline{b} = 8/3$ in (2.3) such that the corresponding non-perturbed Lorenz system (2.2) is chaotic [3, 21]. Moreover, we set $g_1(x_1, x_2, x_3) = 6.5x_1$, $g_2(x_1, x_2, x_3) = 5.2x_2$, $g_3(x_1, x_2, x_3) = 7.1x_3$. One can numerically verify that the solutions of (2.3) are ultimately bounded by a bound common for each $x(t)$. Therefore, according to Theorem 15.8 [22], the response (2.3) replicates the period-doubling cascade of the drive (2.1). It is worth noting that the coupled system (2.1)+(2.3) possesses a period-doubling cascade as well. For the value of the parameter $r < r_\infty$, the instability of the infinite number of periodic solutions is ensured by Theorem 3.1.

Figure 4 shows the stable periodic orbits of system (2.3). The period-1 and period-2 orbits of (2.3) corresponding to the y^2x and y^2xy^2x periodic orbits of the drive system (2.1) are depicted in Figure 4 (a) and (b), respectively. The value $r = 100.36$ is used in Figure 4 (a), whereas $r = 99.75$ is used in Figure 4 (b). The figure reveals the presence of periodic motions in the dynamics of (2.3). Figure 5, on the other hand, represents the projection of the chaotic trajectory of the coupled system (2.1) + (2.3) with $r = 99.51$ on the $y_1 - y_3$ plane. The initial data $x_1(0) = -1.15$, $x_2(0) = 3.52$, $x_3(0) = 77.01$, $y_1(0) = 0.27$, $y_2(0) = 2.17$, $y_3(0) = 254.09$ are used in the simulation. Figures 5 manifests that (2.3) replicates the period-doubling cascade of (2.1).

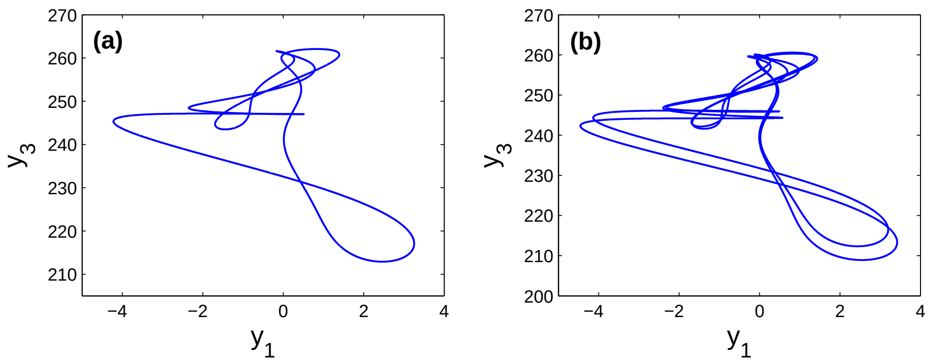

Figure 4: Stable periodic orbits of the response system (2.3). (a) Period-1 orbit corresponding to $r = 100.36$; (b) Period-2 orbit corresponding to $r = 99.75$. The pictures in (a) and (b) demonstrate the presence of periodic motions in the dynamics of (2.3).

6 Conclusions

In the present study, we demonstrate the persistence of chaos in unidirectionally coupled Lorenz systems by checking for the existence of sensitivity and infinitely many unstable periodic motions. This is the first time in the literature that the presence of sensitivity in the dynamics of the response is theoretically proved regardless of GS. The obtained results certify that the applied perturbation does not suppress