τ_b is given by

$$\mu_{\tau_b}(p_{F\tau_b}) = m_{\tau_b} + \frac{p^2_{F\tau_b}}{2m_{\tau_b}} + U_{\tau_b}(p_{F\tau_b}), \qquad (12)$$

where $p_{F\tau_b}$ is their Fermi momentum, and U_{τ_b} and m_{τ_b} are, respectively, the single-particle potential and mass of the baryon. For leptons ($l = \mu,\ e$) their chemical potential is given by $\mu_l = (m_l^2 + p_{Fl}^2)^{1/2}$ with $p_{Fl} = (3\pi^2\rho_l)^{1/3}$ being their Fermi momentum. The relative abundances of various hadrons and leptons for a given total baryon density are then obtained by solving above equations.

In terms of the densities of various particles, the total energy density of the hadron phase can be written as

$$\epsilon^H = V_H + V_L, \qquad (13)$$

where V_H and V_L are the contributions from baryons and leptons, respectively. The former can be written as

$$V_H = V_{HP} + V_{HK} + V_{HM}, \qquad (14)$$

where $V_{HP} = (1/2)\sum_{b,b'} V_{bb'}$ is the potential energy density of baryons with $V_{bb'}$ calculated from Eq. (2), and V_{HK} and V_{HM} are, respectively, the kinetic energy and mass contributions given by

$$V_{HK} = \sum_b \sum_{\tau_b} \frac{p^5_{F\tau_b}}{10\pi^2 m_{\tau_b}}, \qquad (15)$$

$$V_{HM} = \sum_b \sum_{\tau_b} \rho_{\tau_b} m_{\tau_b}. \qquad (16)$$

The contribution V_L from leptons is calculated by treating them as a free Fermi gas, i.e.,

$$\begin{aligned} V_L &= V_e + V_\mu, \\ V_e &= \frac{p^4_{Fe}}{4\pi^2}, \\ V_\mu &= \frac{1}{4\pi^2}\left[p_{F\mu}\mu_\mu^3 - \frac{1}{2}m_\mu^2 p_{F\mu}\mu_\mu \right. \\ &- \left. \frac{1}{2}m_\mu^4 \ln\left(\frac{p_{F\mu}+\mu_\mu}{m_\mu}\right)\right]. \end{aligned} \qquad (17)$$

The pressure of the hadron phase is obtained from the thermodynamical relation

$$P^H = P_H + P_L = (\sum_b \sum_{\tau_b} \mu_{\tau_b}\rho_{\tau_b} - V_H) + (\sum_l \mu_l \rho_l - V_L), \qquad (18)$$

where b and l run over all species of baryons and leptons, respectively. We note that the thermodynamical consistency condition

$$P^H = \rho^2 \frac{d(\epsilon^H/\rho)}{d\rho} \qquad (19)$$

is satisfied.

B. The quark phase

As the nuclear matter density increases, such as in the core of neutron stars, not only hyperons appear but also a quark matter could exist [15]. To take into consideration possible transition between the hadronic matter and the quark matter, we follow many previous studies to use in the present study the MIT bag model [16, 29] to describe the cold quark matter that might exist in the dense core of neutron stars.

For the quark phase consisting of quarks and leptons, the baryon number conservation and charge neutrality conditions are given by expressions similar to Eqs. (9) and (10) with i denoting now quarks and leptons. For the β-equilibrium condition in the quark phase, it is given by $\mu_i = \mu_b^Q b_i - \mu_c^Q q_i$ with μ_b^Q and μ_c^Q being the baryon and charge chemical potentials of the quark phase, respectively.

The total energy density and pressure of the quark phase can be calculated from

$$\epsilon^Q = V_Q + V_L, \qquad (20)$$

$$P^Q = P_Q + P_L, \qquad (21)$$

where V_Q and P_Q are the energy density and pressure of quarks, which can be calculated from the MIT bag model as described in the following, and V_L and P_L are the energy density and pressure of leptons given by same expressions as those in the hadron phase.

At zero temperature, the density ρ_f, chemical potential μ_f, and energy density V_f of quarks of flavor $f = u, d, s$ in the quark matter are given, respectively, by

$$\begin{aligned} \rho_f &= \frac{p^3_{Ff}}{\pi^2}, \\ \mu_f &= \sqrt{m_f^2 + p^2_{Ff}}, \\ V_f &= \frac{3}{4\pi^2}\left[p_{Ff}\mu_f^3 - \frac{1}{2}m_f^2 p_{Ff}\mu_f \right. \\ &- \left. \frac{1}{2}m_f^4 \ln\left(\frac{p_{Ff}+\mu_f}{m_f}\right)\right], \end{aligned} \qquad (22)$$

where p_{Ff} is the Fermi momentum of quarks of flavor f. For the quark masses, they are taken to be $m_u = m_d = 0$ and $m_s = 150$ MeV. In the bag model, the energy density is modified by a bag constant B, resulting in an energy density given by

$$V_Q = \sum_f V_f + B. \qquad (23)$$

This leads to the following pressure for the quark matter:

$$P_Q = \sum_f \mu_f \rho_f - V_Q = \rho^2 \frac{d(V_Q/\rho)}{d\rho}, \qquad (24)$$

where ρ is the total baryon density of the quark phase

$$\rho = \frac{1}{3}\sum_f \rho_f. \qquad (25)$$