VII. SCHRÖDINGER'S CAT

It is instructive to use the formalism of subsystems developed here to analyze the classic example of macroscopic entanglement: Schrödinger's Cat. The Schrödinger's Cat thought experiment can be divided into five subsystems:

- $\mathcal{R}$, a radioactive atom, with projection operators $\hat{P}_{\mathcal{R}}^{\text{yes}}$ and $\hat{P}_{\mathcal{R}}^{\text{no}}$ indicating that it has or has not decayed.
- $\mathcal{D}$, a detector/poison gas apparatus, with projection operators $\hat{P}_{\mathcal{D}}^{\text{yes}}$ and $\hat{P}_{\mathcal{D}}^{\text{no}}$ indicating that a decay product has been detected, with a consequent release of poison gas, or not.
- $\mathcal{C}$, the cat, with projection operators $\hat{P}_{\mathcal{C}}^{\text{alive}}$ and $\hat{P}_{\mathcal{C}}^{\text{dead}}$ indicating that the cat is alive or dead.
- $\mathcal{B}$, the box (consisting of just the bounding container but not its interior), with projection operators $\hat{P}_{\mathcal{B}}^{\text{open}}$ and $\hat{P}_{\mathcal{B}}^{\text{closed}}$ indicating that the box is open or closed.
- $\mathcal{E}$, the environment with projection operators $\hat{P}_{\mathcal{E}}^{\text{closed}}$, $\hat{P}_{\mathcal{E}}^{\text{alive}}$ and $\hat{P}_{\mathcal{E}}^{\text{dead}}$ indicating that either that the box is closed or it is open and the cat is alive or dead.

The experiment is presumed to have a finite duration, so that, as before, the complete hypervolume $\mathcal{V} = \mathcal{E} \cup \mathcal{B} \cup \mathcal{C} \cup \mathcal{D} \cup \mathcal{R}$ has both upper and lower time bounds.

Let $|\Psi_I\rangle_0$ be an initial state in which the box already exists, with the cat, atom and detector sealed inside it. That is,

$$\hat{P}_{\mathcal{R}}^{\text{no}}|\Psi_I\rangle_0 = \hat{P}_{\mathcal{D}}^{\text{no}}|\Psi_I\rangle_0 = \hat{P}_{\mathcal{C}}^{\text{alive}}|\Psi_I\rangle_0 = \hat{P}_{\mathcal{B}}^{\text{closed}}|\Psi_I\rangle_0 = |\Psi_I\rangle_0 \,.$$

Consider first the interior of the box, consisting of $\mathcal{I} = \mathcal{R} \cup \mathcal{D} \cup \mathcal{C}$. Clearly,

$$\begin{aligned} \hat{G}_{\mathcal{I}} &= \hat{P}_{\mathcal{C}}^{\text{alive}}\hat{G}_{\mathcal{C}}\hat{P}_{\mathcal{D}}^{\text{no}}\hat{G}_{\mathcal{D}}\hat{P}_{\mathcal{R}}^{\text{no}}\hat{G}_{\mathcal{R}}|\Psi_I\rangle_0 + \hat{P}_{\mathcal{C}}^{\text{dead}}\hat{G}_{\mathcal{C}}\hat{P}_{\mathcal{D}}^{\text{yes}}\hat{G}_{\mathcal{D}}\hat{P}_{\mathcal{R}}^{\text{yes}}\hat{G}_{\mathcal{R}}|\Psi_I\rangle_0 \\ &= \psi_{\mathcal{R}}^{\text{no}}|r^{\text{no}}, d^{\text{no}}, c^{\text{alive}}\rangle_0 + \psi_{\mathcal{R}}^{\text{yes}}|r^{\text{yes}}, d^{\text{yes}}, c^{\text{dead}}\rangle_0 \,. \end{aligned} \tag{41}$$

The key issue in the Schrödinger's Cat scenario is, of course, whether opening the box has any relevance to the state of the interior of the box (i.e., by "collapsing the wave function"). Initially, the box is closed. However, at some time during the course of the experiment, the box may be opened, presumably by an experimenter who is part of the environment. But,