as q_L under the $SU(2)_{\rm color}$ symmetry as well as under the Lorentz symmetry. Then, by introducing the field Ψ as

$$\Psi = \begin{pmatrix} q_L^1 \\ q_L^2 \\ \vdots \\ q_L^{N_f} \\ \sigma_2\tau_2 q^*_{R,1} \\ \sigma_2\tau_2 q^*_{R,2} \\ \vdots \\ \sigma_2\tau_2 q^*_{R,N_f} \end{pmatrix}, \tag{A.10}$$

the kinetic term in Eq.(A.8) is rewritten as

$$\int d^4x\, i\bar{\psi}\gamma_\nu D^\nu \psi = \int d^4x\, i\Psi^\dagger \sigma_\nu D^\nu \Psi. \tag{A.11}$$

This is invariant under the $SU(2N_f)$ transformation of Ψ given as

$$\Psi \to g\Psi, \quad (g \in SU(2N_f)). \tag{A.12}$$

Similarly, using the field Ψ, we rewrite $\mathcal{L}_{\text{ext-scalar}}$ and $\mathcal{L}_{\text{ext-vector}}$ in Eqs. (A.4) and (A.5) as

$$\mathcal{L}_{\text{ext-scalar}} = \frac{1}{2}\Psi^T \sigma_2\tau_2 \chi \Psi + (\text{h.c}), \quad \mathcal{L}_{\text{ext-vector}} = \Psi^\dagger \sigma_\mu G^\mu \Psi, \tag{A.13}$$

where χ and G^μ are external fields of $2N_f \times 2N_f$ matrices defined as

$$\chi \equiv \begin{pmatrix} \mathcal{Q} - i\mathcal{R} & -(\mathcal{S} - i\mathcal{P})^T \\ \mathcal{S} - i\mathcal{R} & (\mathcal{Q} + i\mathcal{R})^\dagger \end{pmatrix}, \tag{A.14}$$

$$G^\mu \equiv \begin{pmatrix} \mathcal{V}^\mu + \mathcal{A}^\mu & (\mathcal{B}^\mu + \mathcal{D}^\mu)^\dagger \\ \mathcal{B}^\mu + \mathcal{D}^\mu & -(\mathcal{V}^\mu - \mathcal{A}^\mu)^T \end{pmatrix}. \tag{A.15}$$

Transformation properties of the external fields under $SU(2N_f)$ are given by

$$G_\mu \to gG_\mu g^\dagger + ig(\partial_\mu g^\dagger), \quad \chi \to g^* \chi g^\dagger. \tag{A.16}$$

Appendix B: Explicit realization of the $SU(4)$ generators

In this appendix, we show the explicit representation of the generators of $SU(4)$. We consider the form of the generators following Ref. [8] for convenience. They can be represented