According to (2.4), the bounded solutions $\phi_\eta(t)$ and $\phi_\zeta(t)$ satisfy the following relation

$$\phi_\eta^{ij}(t) - \phi_\zeta^{ij}(t) = -\int_{-\infty}^{t} e^{-a_{ij}(t-s)} \Big[\sum_{C_{hl}\in N_r(i,j)} C_{ij}^{hl} f\big(\phi_\eta^{hl}(s)\big)\phi_\eta^{ij}(s) - P_{ij}(s,\eta)$$
$$- \sum_{C_{hl}\in N_r(i,j)} C_{ij}^{hl} f\big(\phi_\zeta^{hl}(s)\big)\phi_\zeta^{ij}(s) + P_{ij}(s,\zeta)\Big] ds.$$

Using the inequality $|P_{ij}(t,\eta) - P_{ij}(t,\zeta)| < \dfrac{\epsilon}{\alpha}$, $t \in (\theta_{k_0}, \infty)$, one can obtain on the same interval that

$$\big|\phi_\eta^{ij}(t) - \phi_\zeta^{ij}(t)\big| \le 2(M_f K_0 \sum_{C_{hl}\in N_r(i,j)} C_{ij}^{hl} + M_F) \int_{-\infty}^{\theta_{k_0}} e^{-a_{ij}(t-s)} ds$$
$$+ \int_{\theta_{k_0}}^{t} e^{-a_{ij}(t-s)} M_f \sum_{C_{hl}\in N_r(i,j)} C_{ij}^{hl} \big|\phi_\eta^{ij}(s) - \phi_\zeta^{ij}(s)\big| ds$$
$$+ \int_{\theta_{k_0}}^{t} e^{-a_{ij}(t-s)} L_f K_0 \sum_{C_{hl}\in N_r(i,j)} C_{ij}^{hl} \big|\phi_\eta^{hl}(s) - \phi_\zeta^{hl}(s)\big| ds + \int_{\theta_{k_0}}^{t} e^{-a_{ij}(t-s)} \frac{\epsilon}{\alpha} ds.$$

Therefore, we have for $t > \theta_{k_0}$ that

$$\begin{aligned} \big\|\phi_\eta(t) - \phi_\zeta(t)\big\| &\le 2 \max_{(i,j)} \frac{1}{a_{ij}} \left(M_f K_0 \sum_{C_{hl}\in N_r(i,j)} C_{ij}^{hl} + M_F \right) e^{-\gamma(t-\theta_{k_0})} + \frac{\epsilon}{\gamma\alpha}\left(1 - e^{-\gamma(t-\theta_{k_0})}\right) \\ &+ (M_f + L_f K_0) \max_{(i,j)} \sum_{C_{hl}\in N_r(i,j)} C_{ij}^{hl} \int_{\theta_{k_0}}^{t} e^{-\gamma(t-s)} \big\|\phi_\eta(s) - \phi_\zeta(s)\big\| ds. \end{aligned} \tag{3.5}$$

Define the function $v(t) = e^{\gamma t}\,\|\phi_\eta(t) - \phi_\zeta(t)\|$ and let

$$R_0 = 2 \max_{(i,j)} \frac{1}{a_{ij}} \left(M_f K_0 \sum_{C_{hl}\in N_r(i,j)} C_{ij}^{hl} + M_F \right) - \frac{\epsilon}{\gamma\alpha}.$$

The inequality (3.5) yields

$$v(t) \le R_0 e^{\gamma\theta_{k_0}} + \frac{\epsilon}{\gamma\alpha} e^{\gamma t} + (M_f + L_f K_0)\delta \int_{\theta_{k_0}}^{t} v(s) ds, \ \ t > \theta_{k_0}.$$

Applying the Gronwall's Lemma [45] to the last inequality one can verify that

$$\begin{aligned} v(t) \ &\le \frac{\epsilon}{\gamma\alpha} e^{\gamma t} + R_0 e^{(M_f + L_f K_0)\delta(t-\theta_{k_0})} e^{\gamma\theta_{k_0}} \\ &+ \frac{\epsilon(M_f + L_f K_0)\delta}{\gamma\alpha\left[\gamma - (M_f + L_f K_0)\delta\right]} e^{\gamma t}\left(1 - e^{-[\gamma - (M_f + L_f K_0)\delta](t-\theta_{k_0})}\right). \end{aligned}$$

Multiplying both sides of the last inequality by $e^{-\gamma t}$ we obtain that

$$\|\phi_\eta(t) - \phi_\zeta(t)\| < \frac{\epsilon}{\alpha\left[\gamma - (M_f + L_f K_0)\delta\right]} + R_0 e^{-[\gamma - (M_f + L_f K_0)\delta](t-\theta_{k_0})}, \ \ t > \theta_{k_0}.$$