where

$$\left|\xi'\right|_\mu = \left(1+\mu^2\sum_2^n g_{ij}\xi^i\xi^j\right)^{1/2}.$$

Hence, if $x \in D$, $(\xi^2,...,\xi^n) \in G'$, $t \in [0,d_0]$ are fixed, then applying the mean value theorem (e.g., [13], p.186) to the function $\dot{z}^k\left(x, \frac{1}{|\xi'|_\theta}, \frac{\theta\xi^2}{|\xi'|_\theta}, \cdots, \frac{\theta\xi^n}{|\xi'|_\theta}, t\right)$ with respect to θ on the interval $[0,\mu]$, from the equality $\dot{z}^k\left(x,\nu^0,t\right)=0$ for $2\le k\le n$, we have

$$\dot{z}^k\left(x, \frac{1}{\left|\xi'\right|_\mu}, \frac{\mu\xi^2}{\left|\xi'\right|_\mu}, \cdots, \frac{\mu\xi^n}{\left|\xi'\right|_\mu}, t\right) = \mu\partial_{\mu_0}\dot{z}^k,\ 0<\mu_0<\mu\le 1,\ 2\le k\le n, \tag{A.2}$$

where $\partial_{\mu_0}\dot{z}^k$ is the derivative of the function $\dot{z}^k\left(x, \frac{1}{|\xi'|_\theta}, \frac{\theta\xi^2}{|\xi'|_\theta}, \cdots, \frac{\theta\xi^n}{|\xi'|_\theta}, t\right)$ with respect to θ at a point $\theta=\mu_0$. It is worth to note here that $\dot{z}^1\left(x,\nu^0,t\right)=1$ and equality (A.2) is not valid for $k=1$.

Since $\dot{z}^k(x,\nu,t)\in C^5(\Omega(d_0))$ and $\left(\frac{1}{|\xi'|_\mu}, \frac{\mu\xi^2}{|\xi'|_\mu}, \cdots, \frac{\mu\xi^n}{|\xi'|_\mu}\right)\in S^n(x)$, the function $\partial_\mu\dot{z}^k\left(x, \frac{1}{|\xi'|_\mu}, \frac{\mu\xi^2}{|\xi'|_\mu}, \cdots, \frac{\mu\xi^n}{|\xi'|_\mu}, t\right)$ is bounded on $\Omega(d_0)$. Here we note that $\Omega(d_0)$ is closed and bounded. Therefore, by (A.1) and (A.2), since the vector $\nu=\xi/|\xi|\in S^n(x)$ tends to $\nu^0=(1,0,...,0)\in\mathbb{R}^n$ as $\xi^1\longrightarrow+\infty$, we have

$$\left|\ |\xi|\ \dot{z}^k(x,\nu,t)\right|\le K_1, \tag{A.3}$$

for $2\le k\le n$ in the set Ω, where $K_1>0$ is independent of $(x,\xi)\in D\times G$, but depends on the norm of the vector function $\dot{z}(x,\nu,t)$ in $C^1(\Omega(d_0))$ and the diameter of G'. In the same way as above, we can prove the last inequality for the case $\xi^1\longrightarrow-\infty$.

It is not difficult to verify the following equalities

$$\begin{aligned}\partial_{\xi^1}\left(|\xi|\,\dot{z}^k\left(x,\frac{\xi}{|\xi|},t\right)\right) &= \frac{\xi^1}{|\xi|^2}\left(|\xi|\,\dot{z}^k\left(x,\frac{\xi}{|\xi|},t\right)\right)\\ &\quad+|\xi|\left(-\sum_{j=1}^n\partial_{\nu^j}\dot{z}^k\frac{\xi^j\xi^1}{|\xi|^3}+\frac{1}{|\xi|}\partial_{\nu^1}\dot{z}^k\right),\end{aligned} \tag{A.4}$$