and the sum is taken over all possible states for the electron-positron pair. Since pair production is symmetric with respect to the electron and the positron, $R'_{jk} = R'_{kj}$; for simplicity we hereafter use R'_{jk} to represent the combined probability of creating the pair in either the state (jk) or (kj) (i.e., $R'^{\rm new}_{jk} = R'^{\rm old}_{jk} + R'^{\rm old}_{kj}$). For a given channel (jk), the threshold condition for pair production is

$$\epsilon' > E_j + E_K, \tag{23}$$

where $\epsilon' = \epsilon \sin\psi$ is the photon energy in the perpendicular frame and $E_n = m_e c^2\sqrt{1+2\beta_Q n}$ is the minimum energy of an electron/positron in Landau level n (the energy of an electron/positron with the momentum along the magnetic field $p_\parallel = 0$). In dimensionless form, the condition [Eq. (23)] can be written as

$$\begin{aligned} x &= \frac{\epsilon'}{2m_e c^2} = \frac{\epsilon}{2m_e c^2}\sin\psi \\ &> x_{jk} \equiv \frac{1}{2}\left[\sqrt{1+2\beta_Q j} + \sqrt{1+2\beta_Q k}\right]. \end{aligned} \tag{24}$$

Note that x_{jk} satisfies

$$x_{00} < x_{01} < x_{02} < \begin{array}{ll} x_{11} < x_{03} < \cdots, & \beta_Q < 4\,; \\ x_{03} < x_{11} < \cdots, & \beta_Q > 4\,. \end{array} \tag{25}$$

The first three attenuation coefficients (corresponding to the three lowest threshold levels x_{00}, x_{01}, x_{02}) for both $\parallel$ and $\perp$ polarizations are given in Appendix B, Eqs. (B6)-(B10); see also Daugherty & Harding (1983). Note that $R'_{\perp,00} = 0$, and thus the first non-zero attenuation coefficient for $\perp$ polarized photons is actually $R'_{\perp,01}$, not $R'_{\perp,00}$.

In our simulation a photon is typically created with x below the first threshold (x_{00} or x_{01}, depending on the photon polarization). As long as x remains below the first threshold, $R' = 0$ and the optical depth to pair production remains zero. As the photon propagates into the magnetosphere and crosses the first threshold, $R' > 0$, $\Delta\tau > 0$, and τ begins to grow. As it continues to travel outward, both τ and the number of Landau levels available for pair production $j_{\max}$ and $k_{\max}$ increase. Depending on the local magnetic field strength [Eq. (11)], the photon may reach a large enough optical depth ($\tau \sim 1$) for pair production after crossing only a few thresholds (so that $j_{\max}$ and $k_{\max}$ are small) or after crossing many thresholds (so that $j_{\max}$ and $k_{\max}$ are very large). For "weak" magnetic fields ($\beta_Q \lesssim 0.1$) the optical depth increases slowly with $s_{\rm ph}$ and it is valid to use the $j_{\max}, k_{\max} \gg 1$ asymptotic attenuation coefficient for pair production (e.g., Erber 1966),

$$R'_{\parallel,\perp} \simeq \frac{0.23}{a_0}\beta_Q \exp\left(-\frac{4}{3x\beta_Q}\right), \tag{26}$$

which applies for both polarizations. For stronger fields, however, pairs are produced in low Landau levels, and the more accurate coefficients of Daugherty & Harding must be used. In Appendix B2 we find that the critical magnetic field strength separating these two regimes is

$$B_{\rm crit} \simeq 3 \times 10^{12}\ {\rm G} \tag{27}$$

[see Eq. (B17)]. We also find that the boundary between the two regimes is very sharp: pairs are either created at the first few Landau levels ($n \le 2$) for $B \gtrsim B_{\rm crit}$ or in very high Landau levels for $B \lesssim B_{\rm crit}$, with very few electrons/positrons

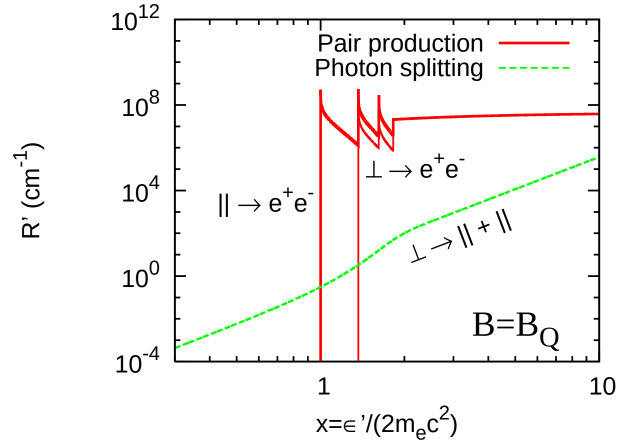

Figure 5. Attenuation coefficients in the perpendicular frame (the frame where the photon is traveling perpendicular to the magnetic field), for both photon splitting, labeled by $\perp\to\parallel + \parallel$, and pair production, labeled by $\parallel\to e^+e^-$ and $\perp\to e^+e^-$. The local magnetic field strength is $B = B_Q \equiv 4.414 \times 10^{13}$ G.

created in intermediate Landau levels. Therefore, in our simulation we only consider the first three attenuation coefficients for $\parallel$-polarized photons ($R'_{\parallel,00}, R'_{\parallel,01}, R'_{\parallel,02}$) and the first two non-zero attenuation coefficients for $\perp$-polarized photons ($R'_{\perp,01}, R'_{\perp,02}$). If the photon reaches the threshold for the (03) or (11) channel [whichever is reached first; see Eq. (25)], we use the asymptotic formula, Eq. (26). The total attenuation coefficient for pair production (as given by this approximation) is plotted in Fig. 5 for both $\parallel$ and $\perp$ polarizations at $\beta_Q = 1$.

We include photon splitting in our simulations. Based on the kinetic selection rule (Adler 1971; Usov 2002, but see Baring & Harding 2001), only the process $\perp\to\parallel\parallel$ is allowed. Therefore, for $\parallel$-polarized photons, the attenuation coefficient for photon splitting is zero ($R'^{\rm sp}_{\parallel} = 0$). For $\perp$-polarized photons we use the following formula, adapted from the numerical calculation of Baring & Harding (1997):

$$R'^{\rm sp}_{\perp\to\parallel\parallel} \simeq \frac{\frac{\alpha_f^2}{60\pi^2 a_0}\left(\frac{26}{315}\right)^2 (2x)^5 \beta_Q^6}{[g(\beta_Q, x) + 0.05]\,[0.25 g(\beta_Q, x) + 20]}, \tag{28}$$

where $g(\beta_Q, x) = \beta_Q^3 \exp(-0.6x^3)$. For $x \le 1$, this expression reproduces the results of Baring & Harding to better than 10% at both $\beta_Q \le 0.5$ and $\beta_Q \gg 1$, while underestimating the results at $\beta_Q = 1$ by less than 30%. The $\perp\to\parallel\parallel$ attenuation coefficient for photon splitting is plotted in Fig. 5 at $B = B_Q$. Because the attenuation coefficient $R'^{\rm sp}$ drops rapidly with field strength for $\beta_Q < 1$, photon splitting is unimportant for $\beta_Q \lesssim 0.5$ (e.g., Baring & Harding 2001). However, for $\perp$-polarized photons propagating in superstrong fields $\beta_Q \gtrsim 0.5$, photon splitting is the dominant attenuation process: even though above the first threshold ($x \ge x_{01}$ for $\perp$ photons) the attenuation coefficient for photon splitting is much smaller than that for pair production, in superstrong fields the photon splits before reaching the first threshold (see Fig. 5).

In the simulation, whenever $\tau \ge 1$ or $\tau_{\rm sp} \ge 1$ the photon is destroyed (i.e., turned into a pair or two photons). More precisely, the photon should only be destroyed with probability $1 - \exp(-\tau)$. But in practice we find that such