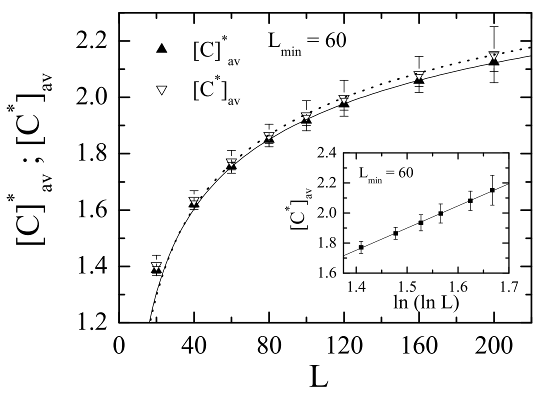

FIG. 7: FSS of the specific heat maxima obtained via the two distinct ways of disorder averaging. The solid and dotted lines are double-logarithmic fittings of the form (9) for lattice sizes in the range $L = 60 - 200$. The inset shows the data for the case $[C^*]_{av}$ as a function of the double logarithm of L. The solid line is an excellent linear fitting.

Tomita and Okabe, using the probability-changing cluster algorithm [48].

As in the pure case, the second alternative estimation of ν is carried out by analyzing the divergency of the logarithmic derivatives of the order parameter. In Fig. 6(a) we illustrate in a double-logarithmic scale the size dependence of the first- (filled squares), second- (filled circles), and fourth-order (filled triangles) logarithmic derivatives (averaged over the individual maxima). The solid lines show linear fittings for the sizes $L \geq 60$. In all cases a value $\nu = 1$ is obtained for the critical exponent ν, providing further evidence to the strong universality scenario emerged from Fig. 5. Figure 6(b) illustrates our method to evaluate and discuss the stability of the estimation for the exponent ν from the scaling behavior of the logarithmic derivatives of panel (a). It shows values of effective exponents (ν_{eff}) determined by imposing a lower cutoff (L_{min}) and applying simultaneous fittings in windows ($L_{min} - L_{max}$), where as for the pure case, $L_{max} = 200$ and $L_{min} = 20, 40, 60, 80,$ and 100 as a function of $1/L_{min}$. The effective estimates show a finite-size effect for small values of the lower cutoff, whereas and for $L \geq 60$ a clear trend towards the value $\nu = 1$ of the Ising universality class is obtained. Let us note here that the same picture emerged from the FSS of the disorder-averaged logarithmic derivatives of the form $[\partial \ln \langle M^n \rangle / \partial K]^*_{av}$ that corresponds to the first way of averaging, but is omitted here for brevity. We should note here that a similar cross-over behavior in the estimates of the critical exponent ν has been observed in the case of the 2d site-diluted SqIM by Ballesteros *et al.* [39] and has been explained as logarithmic corrections. Thus, summarizing our estimates for the critical exponent ν, we feel that it is clear that it maintains the value $\nu = 1$ of the pure case, indicating again the validity of the strong universality scenario.

We continue the presentation of our results by showing in Fig. 7 the FSS of the specific heat maxima averaged over disorder: $[C]^*_{av}$ (up filled triangles) and $[C^*]_{av}$ (down open triangles). Using these data for the larger sizes $L \geq 60$, we tried to observe the quality of the fittings, assuming a double-logarithmic divergence of the form

$$[C]^*_{av}; [C^*]_{av} \sim C_1 + C_2 \ln(\ln L), \tag{9}$$

or a simple power law

$$[C]^*_{av}; [C^*]_{av} \sim C_\infty + C_3 L^{\alpha/\nu}. \tag{10}$$

Although it is rather difficult to numerically distinguish between the above scenarios, our detailed fitting attempts indicated that the double-logarithmic scenario [Eq. (9)] applies better to the numerical data and this is generally true for both $[C]^*_{av}$ and $[C^*]_{av}$ data.

In fact, the double-logarithmic fitting is shown in the main panel, whereas in the corresponding inset of Fig. 7 the data of $[C^*]_{av}$ are plotted as a function of $\ln(\ln L)$. The solid line shown is an excellent linear fit for $L \geq 60$. Let us now give some details on the quality of the applied fittings. We used the following sets of data points ($L_{min} - L_{max}$), with $L_{max} = 200$ and $L_{min} = 20, 40, 60, 80,$ and 100. The quality of the fittings indicated a very good trend for the values of χ^2/DoF for the double logarithmic fittings (9) in the range: $0.2 - 0.7$ and for both sets of data points. However, a strong reliability test in favor of the logarithmic corrections scenario is provided by the stability of the coefficient C_2, for both $[C]^*_{av}$ ($C_2 \approx 1.43(5)$) and $[C^*]_{av}$ ($C_2 \approx 1.48(4)$) data. On the other hand, the estimated values of the exponent α/ν of the power law (10), for both $[C]^*_{av}$ and $[C^*]_{av}$, fluctuate in the range $[-0.12(9), -0.05(6)]$ (as L_{min} increases) with the fitting procedure becoming rather unstable as we move to larger values of L_{min}. The conclusion is that our numerical data are more properly described by the double logarithmic form (9), in agreement with the MC findings of Selke *et al.* [43] and Ballesteros *et al.* [39] for the site-diluted SqIM and also with those of Wang *et al.* [28] for the strong disorder regime ($r = 1/4$ and $r = 1/10$) of the RBSqIM.

In Fig. 8 we provide estimates for the magnetic exponent ratios β/ν and γ/ν of the RBTrIM. In panel (a) we plot the average magnetization at the estimated critical temperature, as a function of the lattice size L in a log-log scale. The solid line is a linear fitting for $L \geq 20$ giving within error bars the value of the pure model, i.e. $\beta/\nu = 0.1253(5) \approx 0.125$. Additional estimate for the ratio β/ν can be obtained from the FSS of the derivative of the absolute order parameter with respect to inverse temperature defined in Eq. (4) which is expected to scale as $L^{(1-\beta)/\nu}$ with the system size [75]. Thus, in panel (b) of Fig. 8 we plot the data for $\partial\langle|M|\rangle/\partial K$ averaged over disorder as a function of L, also in a double-logarithmic scale. The solid line is a linear fitting for the larger lattice sizes $L \geq 60$, which combined with the value $\nu = 1$, gives an estimate of 0.1247(4) for the ratio β/ν. Finally,