Let us denote by $X(s,r)$ the transition matrix of the linear homogeneous system

$$\begin{aligned} x'(s) &= Ax(s), \ s \neq s_k, \\ \Delta x|_{s=s_k} &= \delta_k A x(s_k). \end{aligned} \tag{2.7}$$

Under the conditions $(C1)$ and $(C2)$ there exist positive numbers N and λ such that $\|X(s,r)\| \leq Ne^{-\lambda(s-r)}$ for $s \geq r$ [6, 42].

The following conditions are also needed.

(C5) $NL_f \left(\dfrac{1}{\lambda} + \dfrac{p\bar{\delta}}{1-e^{-\lambda\psi(\omega)}} \right) < 1$, where $\bar{\delta} = \max\limits_{0\leq k\leq p-1} \delta_k$;

(C6) $-\lambda + NL_f + \dfrac{p}{\psi(\omega)} \ln\left(1 + NL_f\bar{\delta}\right) < 0$;

(C7) $f(t+\omega, y) = f(t,y)$ for all $(t,y) \in \mathbb{T}_0 \times \mathbb{R}^n$.

The next section is devoted to the bounded solutions of system (1.1).

3 Bounded solutions

Under the conditions $(C1) - (C5)$, one can verify by using the results of [6, 42] that for a fixed sequence $\zeta = \{\zeta_k\}$, $k \in \mathbb{Z}$, there exists a unique bounded on $\mathbb{R}$ solution $\phi_\zeta(s)$ of (2.6), which satisfies the relation

$$\begin{aligned} \phi_\zeta(s) = & \int_{-\infty}^{s} X(s,r) \left[f\left(\psi^{-1}(r), \phi_\zeta(r)\right) + g\left(\psi^{-1}(r), \zeta\right) \right] dr \\ & + \sum_{-\infty<s_k<s} X(s, s_k+) \left[f\left(\psi^{-1}(s_k), \phi_\zeta(s_k)\right) + \zeta_k \right] \delta_k. \end{aligned} \tag{3.8}$$

Moreover, $\sup\limits_{s\in\mathbb{R}} \|\phi_\zeta(s)\| \leq K_0$, where $K_0 = N(M_f + M_F)\Big(\dfrac{1}{\lambda} + \dfrac{p\bar{\delta}}{1-e^{-\lambda\psi(\omega)}}\Big)$. Therefore, for a fixed sequence $\zeta = \{\zeta_k\}$, the function $\varphi_\zeta(t) = \phi_\zeta(\psi(t))$ satisfying $\varphi_\zeta(\theta_{2k+1}) = \phi_\zeta(s_k+)$, $k \in \mathbb{Z}$, is the unique solution of (2.5), and hence of (1.1), which is bounded on $\mathbb{T}_0$ such that $\sup\limits_{t\in\mathbb{T}_0} \|\varphi_\zeta(t)\| \leq K_0$.

We say that the bounded solution $\varphi_\zeta(t)$ attracts a solution $y(t)$ of (1.1) if $\|y(t) - \varphi_\zeta(t)\| \to 0$ as $t \to \infty$, $t \in \mathbb{T}_0$. The attractiveness feature of the bounded solutions of (1.1) is mentioned in the next assertion.

Lemma 3.1 *If the conditions $(C1) - (C6)$ are valid, then for a fixed sequence ζ, the bounded solution $\varphi_\zeta(t)$ attracts all other solutions of (1.1).*

Proof. Consider an arbitrary solution $y(t)$, $y(t^0) = y_0$, of (1.1) for some $t^0 \in \mathbb{T}_0$ and $y_0 \in \mathbb{R}^n$. Assume without loss of generality that $t^0 \neq \theta_{2k-1}$ for any $k \in \mathbb{Z}$. Let $s^0 = \psi(t^0)$ and $x(s) = y(\psi^{-1}(s))$. The relation

$$x(s) - \phi_\zeta(s) = X(s, s^0)(y_0 - \phi_\zeta(s^0)) + \int_{s^0}^{s} \left[f\left(\psi^{-1}(r), x(r)\right) - f\left(\psi^{-1}(r), \phi_\zeta(r)\right) \right] dr$$