



FIG. 2: The skeleton diagrams for correlation function  $\Lambda_\lambda$ . Double dashed lines depict the tunneling Green's functions  $T_\lambda$  and the rectangles depict the irreducible Green's functions.

### III. THERMODYNAMIC POTENTIAL DIAGRAMS

The thermodynamic potential of the system is determined by the connected part of the mean value of the evolution operator <sup>[11,12]</sup>

$$F = F_0 - \frac{1}{\beta} \langle U(\beta) \rangle_0^c. \quad (14)$$

Let us consider a more general quantity first

$$F(\lambda) = F_0 - \frac{1}{\beta} \langle U_\lambda(\beta) \rangle_0^c, \quad (15)$$

and put then  $\lambda = 1$ .

By using the perturbation theory we have obtained the first orders of diagrams for  $\langle U_\lambda(\beta) \rangle_0^c$ , depicted in Fig. 3.

In order to obtain the better understanding of these diagrammatic contributions we examine the expression

$$\sum_{xx'} G_\lambda(x|x') \lambda t(\vec{x}' - \vec{x}) \delta(\tau - \tau' - 0^+) \delta_{\sigma\sigma'}, \quad (16)$$

where double repeated indices suppose summation and integration. Consequently (16) is equal to

$$\begin{aligned} & - \beta \sum_{\vec{x}\vec{x}'} \sum_{\sigma} G_{\lambda\sigma}(\vec{x} - \vec{x}' | -0^+) \lambda t(\vec{x}' - \vec{x}) \\ & = -\lambda \sum_{\vec{k}} \sum_{\sigma} \epsilon(\vec{k}) G_{\lambda\sigma}(\vec{k} | i\omega_n) \exp(i\omega_n 0^+). \end{aligned} \quad (17)$$

Here we have carried out the integration by time.

From diagrammatic point of view equation (16) implies the procedure of locking of the external lines of propagators  $G_\lambda$  diagrams depicted on the Fig. 1 with the tunneling matrix element  $t(\vec{x}' - \vec{x})$  and obtaining in such a way the diagrams without external lines similar with ones for  $\langle U_\lambda(\beta) \rangle_0^c$  depicted on Fig. 3. These two series of diagrams differ by coefficients in front of them.

In expression (17) the coefficients  $\frac{1}{n}$  before each diagram are absent, where  $n$  is the order of perturbation theory. These coefficients are present in Fig. 3. In order to restore these  $\frac{1}{n}$  coefficients in (17) and obtain the coincidence with  $\langle U_\lambda(\beta) \rangle_0^c$  series it is enough to integrate by  $\lambda$  the expression (17) and obtain:

$$- \sum_{\vec{x}\vec{x}'} \sum_{\sigma} \beta \int d\lambda t(\vec{x}' - \vec{x}) G_{\lambda\sigma}(\vec{x} - \vec{x}' | -0^+). \quad (18)$$

The expression (18) displayed in a diagrammatic representation coincides exactly with mean value of the evolution operator:

$$\begin{aligned} \langle U_\lambda(\beta) \rangle_0^c &= - \sum_{\vec{x}\vec{x}'} \beta t(\vec{x}' - \vec{x}) \\ &\times \int_0^\lambda d\lambda' G_{\lambda'\sigma}(\vec{x} - \vec{x}' | -0^+). \end{aligned} \quad (19)$$

In Fourier representation we have

$$\langle U_\lambda(\beta) \rangle_0^c = - \int_0^\lambda d\lambda' \sum_{\vec{k}\sigma\omega_n} \epsilon(\vec{k}) G_{\lambda'\sigma}(\vec{k} | i\omega_n) \exp(i\omega_n 0^+). \quad (20)$$

From (15) and (20) we obtain

$$\begin{aligned} F(\lambda) &= F_0 + \int_0^\lambda d\lambda' \sum_{\vec{k}\sigma} \frac{1}{\beta} \sum_{\omega_n} \epsilon(\vec{k}) \\ &\times G_{\lambda'\sigma}(\vec{k} | i\omega_n) \exp(i\omega_n 0^+). \end{aligned} \quad (21)$$

Using the definition (14), the equation (21) can be written in the form:

$$\begin{aligned} F(\lambda) &= F_0 + \int_0^\lambda \frac{d\lambda'}{\lambda'} \sum_{\vec{k}\sigma} \frac{1}{\beta} \sum_{\omega_n} T_{\lambda'}(k) \\ &\times \Sigma_{\lambda'}(k) \exp(i\omega_n 0^+). \end{aligned} \quad (22)$$