$R_0 > 1$ a specified distance in excess of the diameter of the disks. None of the currently available pair correlation function theories are able to predict this effect, including the crucial dependence of R_0 on $R_{min}(N)$, because the underlying approximations of the currently available theories cannot account for the basic many-body geometrical features involved.

F. Imperfect symmetry

Not all DLP optimal packings exhibit perfect symmetry; for many N, a subset of disks in an optimal configuration appear to mimic a symmetric packing, but the packing as a whole exhibits only imperfect symmetry. One situation in which this occurs frequently is when the number of disks N in the optimal packing is close to a different number for which the optimal packing is relatively unusually dense. For example, the optimal $N = 59$ packing shown in the top panel of of Fig. 15 lacks any of the symmetry elements described above but nonetheless closely resembles the particularly dense $N = 60$ curved hexagonal packing (top panel of Fig. 7).

Other packings in which imperfect symmetry is present include the $N = 80$ packing shown in the center panel of Fig. 15, the disks closer to the center of which are ordered with centers on the sites of a slightly distorted triangular lattice, and the $N = 46$ packing shown in the bottom panel of Fig. 15, which has imperfect five-fold symmetry. The $N = 46$ packing, along with the $N = 45$ packing (center panel of Fig. 12), together illustrate another finding: that the structure of optimal packings even for consecutive numbers of disks can vary substantially.

G. Surface effects

DLP optimal packings with N in the higher range of the packings studied appear, as N increases, to more and more resemble the triangular lattice in the bulk of the packing. Nonetheless, the surface of the packing always deviates significantly from the bulk crystal. In general, the optimal packings at higher N consist of a "bulk zone" with disk centers arranged in the triangular lattice surrounded by a "surface zone" with disk centers arranged in circular rings. This effect can be seen in all of the optimal packings with N in the higher range of N studied, including in those shown in Figs. 8, 9, 11, in the center panel of Fig.