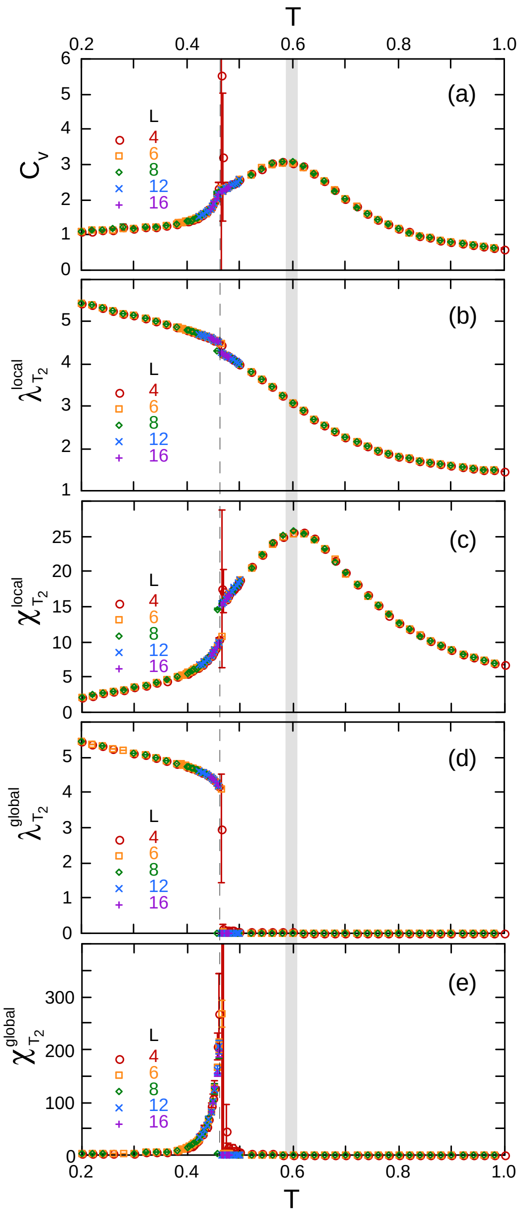

FIG. 20: (Color online) The same plots as Fig. 19 but for $J_3 = -0.02$, showing both a crossover in to the plateau liquid for $T^* \approx 0.6$ (bold grey line) and a transition into the ordered phase at $T_N = 0.46(1)$ (dashed line).

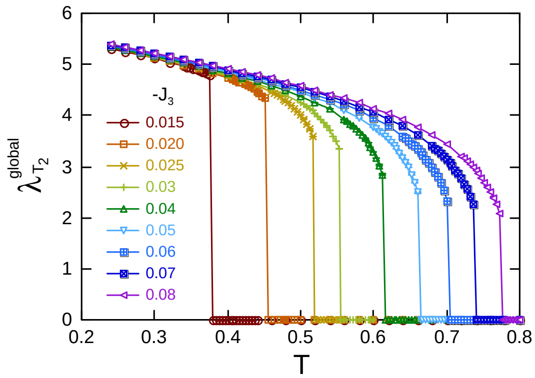

FIG. 21: (Color online) Temperature dependence of the order parameter $\lambda^{\text{global}}_{\text{T}_2}$ for four-sublattice $uuud$ order [as defined by Eq. (8)], for a range of values of $J_3 < 0$. The transition temperature becomes smaller as $|J_3| \to 0$. At the same time the transition becomes more strongly first-order. All results are for $h = 4$ and $b = 0.6$, in a cluster with $L = 8$. The lines are guides for the eye.

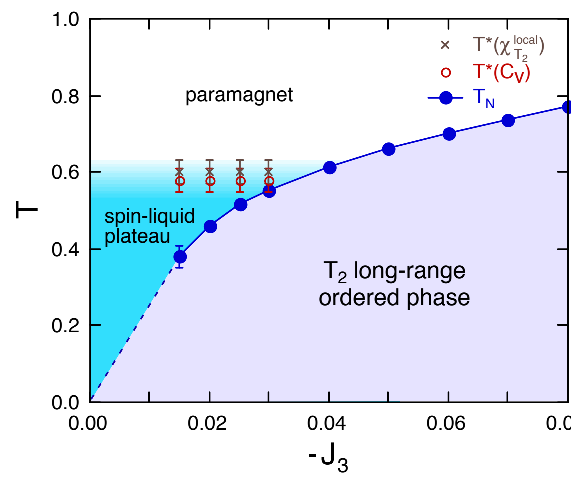

FIG. 22: (Color online) Phase diagram for the classical Heisenberg antiferromagnet on a pyrochlore lattice in applied magnetic field $h = 4$, with additional biquadratic interactions $b = 0.6$ [Eq. (1)]. The transition temperature T_N associated with the gapped, ordered, half–magnetization plateau state vanishes as the strength of ferromagnetic third–neighbor interactions $J_3 \to 0$, as determined by Monte Carlo simulation. A state exhibiting a half–magnetization plateau but no long–range magnetic order exists above T_N up to a crossover temperature T^*. Estimates of the crossover temperature are taken from peaks in the local susceptibility and heat capacity.

ature [Fig. 20(a)]. And, at $T = T_N \approx 0.46 < T^*$, there is a small jump in C_v, accompanied by a clear singularity in global order parameter susceptibility $\chi^{\text{global}}_{\text{T}_2}$ [Fig. 20(e)]. While there is no true phase transition at T^*, it is clear that the bulk of the entropy of the paramagnet is lost in the smooth crossover into the plateau liquid, and not in the first order transition into the ordered phase.

So what happens for $J_3 \to 0$? Unfortunately this question is hard to answer by Monte Carlo simulation, as the massive degeneracy of the uud states translates into many competing local minima in the free energy. However T_N is strictly zero for $J_3 = 0$, and there are two obvious scenarios for how this can be achieved.

The first is that the first–order transition into the ordered phase becomes weaker as $T_N \to 0$, terminating in a critical end point for $J_3 = 0$, $T_N = 0$. This end point would in fact be *multicritical*, since many different ordered $uuud$ states can be formed out of the dimer manifold for different choice of long range interactions. Within this scenario, the power–law correlations between spins in the plateau liquid for $T \to 0$ could be viewed as evidence of critical fluctuations. The second scenario is that the first order transition into the ordered phase persists down to $T_N = 0$. Since an infinite number of