Graph theoretical concepts allow us to unravel the scale dependence of the modular structure of the NHCD. Graph theory [12] defines the distance between two nodes (also called the chemical distance) as the number of links along the shortest path between the nodes in the network. We use this notion to propose a community detection algorithm that identifies modules of size ℓ composed of highly connected cell types. The algorithm finds the optimal tiling of the network with the smallest possible number of modules, N_B, of size ℓ [13] (each node is assigned to a module or box and all nodes in a module are at distance smaller than ℓ). This process results in an optimization problem which can be solved using the box-covering algorithm explained in Fig.2a, Materials and Methods Section III and reported in [29] as the Maximum Excluded Mass Burning algorithm (MEMB, the algorithm can be downloaded from `http://lev.ccny.cuny.edu/~hmakse/soft_data.html`). The requirement of minimal number of modules to cover the network (N_B) guarantees that the partition of the network is such that each module contains the largest possible number of nodes and links inside the module with the constraint that the modules cannot exceed size ℓ. This optimized tiling process gives rise to modules with the fewest number of links connecting to other modules implying that the degree of modularity, defined by [10–12, 30]

$$\mathcal{M}(\ell) \equiv \frac{1}{N_B} \sum_{i=1}^{N_B} \frac{L_i^{\text{in}}}{L_i^{\text{out}}}, \tag{1}$$

is maximized. Here L_i^{in} and L_i^{out} represent the number of links that start in a given module i and end either within or outside i, respectively. Large values of $\mathcal{M}$ ($L_i^{\text{out}} \to 0$) correspond to a higher degree of modularity. The value of the modularity of the network $\mathcal{M}$ varies with ℓ, so that we can detect the dependence of modularity on different length scales, or equivalently how the modules themselves are organized into larger modules that enhance the degree of modularity.

For a given ℓ, we obtain the optimal coverage of the network with N_B modules (we use the MEMB algorithm [29] explained in Fig. 2a and Materials and Methods). Analysis of the modularity Eq. (1) in Fig. 3a reveals a monotonic increase of $\mathcal{M}(\ell)$ with a lack of a characteristic value of ℓ. Indeed, the data can be approximately fitted with a power-law functional form:

$$\mathcal{M}(\ell) \sim \ell^{d_M}, \tag{2}$$

which is detected through the modularity exponent d_M. We characterize the network using different snapshots in time and we find that $d_M \simeq 2.0$ is approximately constant over the