

spinors: the only flavour transformation we still can make is one where we simultaneously make the same transformation on both Ψ_L and Ψ_R . We say that the $SU(2)_L \times SU(2)_R$ flavour symmetry has been spontaneously broken to $SU(2)_V$.

Goldstone's theorem [25] states that whenever a continuous symmetry of a quantum field theory is spontaneously broken, a massless particle will appear. In breaking $SU(2)_L \times SU(2)_R$ to $SU(2)_V$, we started with six group generators, and ended up with three: according to Goldstone's theorem we would expect three massless particles to appear in the spectrum of QCD.

However, the Lagrangian of (1.31) is not exactly that of real QCD. The proper Lagrangian of QCD has massive quarks and is given in (1.1). Hence QCD does not have an exact $SU(2)_L \times SU(2)_R$ symmetry. But because the up and down quarks are *almost* massless, there is an approximate symmetry, so a perturbative approach about $m_q \approx 0$ may still be useful.

Looking at the hadron spectrum there are three suspiciously light hadrons, namely the pions, whose masses are about one fifth of the next lightest hadron. Furthermore they have the correct parity to be created by the axial isospin current, $j^{\mu 5a}$:

$$j^{\mu 5a} = \bar{Q} \gamma^\mu \gamma^5 \tau^a Q \quad (1.34)$$

The coupling between the pions and the vector axial current is defined as

$$\langle \pi^b(p) | \bar{Q} \gamma_\mu \gamma^5 T^a Q(0) | 0 \rangle = -i f_\pi p_\mu \delta^{ab} \quad (1.35)$$

where T^a is a generator of the broken symmetry group and f_π is a number with dimensions of mass. It is called the pion decay constant. For an $SU(2)$ isospin symmetry, meaning two flavours of quark, we expect three types of pion.