unpredictable point. More precisely, we say that the dynamics on the quasi-minimal set Ω_p is chaotic if the dynamics on it is sensitive, transitive and there exists a continuum of Poisson stable trajectories dense in the quasi-minimal set. Nevertheless, in the framework of chaos there may be infinitely many periodic motions. For instance, chaos in the sense of Devaney [12] and Li-Yorke [4] admit a basis consisting of periodic motions. However, our definition does not contradict to this, and this possibility is exemplified in the next section.

4 Applications

In this section, we will mainly investigate symbolic dynamics [12, 13] and show the presence of unpredictable points as well as chaos on a quasi-minimal set in the sense mentioned in Section 3. Moreover, we will reveal by means of the topological conjugacy that the same is true for the logistic, Hénon and horseshoe maps.

Let us take into account the following space of bi-infinite sequences [13],

$$\Sigma^2 = \{s = (\ldots s_{-2}s_{-1}.s_0 s_1 s_2 \ldots) : s_j = 0 \,\text{or}\, 1 \text{ for each } j\}$$

with the metric

$$d[s, \overline{s}] = \sum_{k=-\infty}^{\infty} \frac{|s_k - \overline{s}_k|}{2^{|k|}},$$

where $s = (\ldots s_{-2}s_{-1}.s_0 s_1 s_2 \ldots)$, $\overline{s} = (\ldots \overline{s}_{-2}\overline{s}_{-1}.\overline{s}_0 \overline{s}_1 \overline{s}_2 \ldots) \in \Sigma^2$. The shift map $\sigma : \Sigma^2 \to \Sigma^2$ is defined as

$$\sigma(\ldots s_{-2}s_{-1}.s_0 s_1 s_2 \ldots) = (\ldots s_{-2}s_{-1}s_0.s_1 s_2 \ldots).$$

The map σ is continuous and the metric space Σ^2 is compact [13].

In order to show that the map σ possesses an unpredictable point in Σ^2, we need an ordering on the collection of finite sequences of 0's and 1's as follows [13]. Suppose that two finite sequences $s = \{s_1 s_2 \ldots s_k\}$ and $\bar{s} = \{\bar{s}_1 \bar{s}_2 \ldots \bar{s}_{k'}\}$ are given. If $k < k'$, then we say that $s < \bar{s}$. Moreover, if $k = k'$, then $s < \bar{s}$ provided that $s_i < \bar{s}_i$, where i is the first integer such that $s_i \neq \bar{s}_i$. Note that there are 2^m distinct sequences of 0's and 1's with length m. Thus, one can denote the sequences having length m as $s_1^m < \ldots < s_{2^m}^m$, where the superscript represents the length of the sequence and the subscript refers to a particular sequence of length m which is uniquely specified by the above ordering scheme.

Now, consider the following sequence,

$$s^* = (\ldots s_8^3 s_6^3 s_4^3 s_2^3 s_4^2 s_2^2 . s_1^1 s_1^2 s_3^2 s_1^3 s_3^3 s_5^3 s_7^3 \ldots).$$

It was demonstrated in [13] that the trajectory of s^* is dense in Σ^2. We will show that s^* is an unpre-