

tonian spacetime, each frame $\{e_A\}$ is fully characterized by its causal class. The causal class of a frame is the set of all the frames that have same causal signature, which is defined by a set of 14 causal characters:

$$\{c_1 c_2 c_3 c_4, C_{12} C_{13} C_{14} C_{23} C_{24} C_{34}, c_1 c_2 c_3 c_4\}, \quad (1)$$

As notation for the causal characters, we will use lower case roman types (s, t, l) to represent the causal character of vectors (resp. spacelike, timelike, lightlike), and capital types (S, T, L) and lower case italic types (*s, t, l*) to denote the causal character of 2-planes and covectors, respectively.

3.1. Relativistic frames

This new degree of freedom (lightlike) in the causal character, which is proper of Lorentzian relativistic spacetimes but which does not exist in Newtonian spacetimes, allows to obtain (see [5]), as it has been commented in the abstract, the following theorem: *In a relativistic 4-dimensional Lorentzian spacetime, there exists 199, and only 199, causal classes of frames.* These 199 causal classes have been completely classified.

We shall see that among the 199 Lorentzian causal classes, only one is privileged to construct a generic, gravity free and immediate positioning system.

The notion of causal class extends naturally to the set of coordinate lines of the coordinate system and so, to the coordinate system itself. By definition, the causal class of a coordinate system $\{x^\alpha\}_{\alpha=1}^4$ in a domain is the causal class $\{c_\alpha, C_{\alpha\beta}, c_\alpha\}$ of its associated *natural* frame at the events of the domain. In relativity, a specific causal class, among the 199 ones, can be assigned to any of the different coordinate systems used in all the solutions of the Einstein equations. However, for the same coordinate system and the same solution, the causal class can change depending on the region of the spacetime considered and the coordinate system in this case is said to be inhomogeneous.

In fact, see [7], in any spacetime every coordinate x^α plays two extreme roles: that of a hypersurface for every constant value $x^\alpha = \text{const.}$, of gradient dx^α , and that of a coordinate line of tangent vector ∂_α , when the other coordinates remain constant. This simple fact shows that, in spite of the historical custom of associating to a coordinate a causal orientation, saying that *it is* timelike, lightlike or spacelike, *this appellation is not generically coherent.* Causal orientations are generically associated with directions of geometric objects, but not with spacetime coordinates associated to them. In the case of a coordinate x^α , this generic incoherence appears because its two natural variations in the coordinate system, dx^α and ∂_α , have generically different causal orientations. *Only when both causal orientations coincide*, it is possible to extend to the coordinate x^α itself the character of the common causal orientation of its two mentioned variations.

3.2. Newtonian frames

The differences in the geometric description of Lorentzian and Newtonian frames come from the causal structure induced by the different metric descriptions of Lorentzian and Newtonian spacetimes. The main difference comes essentially from the absence of the lightlike character in the Newtonian case. In relativity, the spacetime metric is non-degenerate and defines a one-to-one correspondence between vectors and covectors at the tangent and cotangent space of every event.

In contrast, in a Newtonian space-time no non-degenerate metric structure exists and one have two different metrics, see [8]. This degenerate metric structure is given by a rank one *covariant time metric* $T = dt^2$ and an orthogonal rank three *contravariant space metric* γ . In the time metric appears t which is a absolute time scale and the hypersurfaces $t = \text{const}$ constitute the instantaneous or simultaneity spaces. A vector e is spacelike if it is instantaneous, i.e. if $dt(e) = 0$. Otherwise, it is timelike. So, it is clear that a frame can have at most three spacelike vectors so there only exist four causal types of Newtonian frame bases, namely: $\{tsss\}$, $\{ttss\}$, $\{tsts\}$, $\{tttt\}$.

Correspondingly, a covector $\theta \neq 0$ is timelike if it has no instantaneous part with respect to the contravariant space metric γ , i.e. if $\gamma(\theta) = 0$ and it is necessarily of the form $\theta = N dt$ with $N \neq 0$, being *future* (resp. *past*) oriented if $N > 0$ (resp. $N < 0$). Otherwise, the covector θ is spacelike. Thus, attending to the causal orientation of their covectors, there only exist two causal types of Newtonian coframes bases: $\{tsss\}$, $\{ssss\}$.

In summary, it can be shown (see [7]) that one has the following implications valid only for Newtonian frames: $\{C_A\} \Rightarrow \{C_{AB}, c_A\}$, $\{C_{AB}\} \Rightarrow \{c_A\}$, but $\{C_{AB}\} \not\Rightarrow \{C_A\}$, $\{c_A\} \not\Rightarrow \{C_A, C_{AB}\}$.

The simplicity of the Newtonian causal structure with respect to the Lorentzian one lies in the fact that the causal type of a Newtonian frame determines completely its causal class. This is related to the fact that, in Newtonian space-time, any set of spacelike vectors always generates a spacelike subspace. As a consequence, the number of causally different Newtonian classes of frames is equal to the dimension of the spacetime. Hence, see [7], in the *4-dimensional Newtonian spacetime there exist four, and only four, causal classes of frames.* They are: $\{tsss, TTTSSS, tsss\}$, $\{ttss, TTTTTS, ssss\}$, $\{tsts, TTTTTS, ssss\}$ and $\{tttt, TTTTTS, ssss\}$.

For instance, the standard spatial coordinates ECI and ECEF used in the GPS more the GPS time, i.e. those that are locally realized with three rods and one clock, belong to the same causal class $\{tsss, TTTSSS, tsss\}$, the first one above. The history of the clock is a timelike coordinate line. The other coordinate lines are spacelike straight lines tangent to the rods at every time. Also the reference systems adopted by the I.A.U. for the Earth and the barycenter of the Solar system as, respectively, the 3-