

the large uncertainty in D ($\sigma_D = 0.13$) can potentially hide, within the $2\text{--}3\sigma$, the wavelength dependence of $D \simeq 0.3$ observed in optical studies. Another possibility is that the mid-IR wavelength dependence is much weaker than that observed in optical.

Table 7 shows the variability amplitudes at the observed time lag of 4 years for various classes of objects from Figure 4 assuming a fixed slope of $\tau^{1/2}$. Objects with $z > 1$ are AGNs, and show uniform and consistent variability amplitudes of ~ 0.09 mag as a function of magnitude in both channels for fixed $\gamma = 0.5$. The X-ray selected sources inside the modified AGN wedge are expected to be AGNs, and not surprisingly, they also show similar variability amplitudes of ~ 0.10 mag to the $z > 1$ objects. The mid-IR counterparts of the $24\text{ }\mu\text{m}$ (MIPS) selected sources inside the modified AGN wedge, again, have almost identical variability amplitudes. The radio selected objects, however, have a drastically different distribution in their mid-IR colors (top right panel in Figure 4) from that of AGNs, suggesting that a large fraction of them may not be AGNs. The structure functions based on the radio-selected objects inside the modified AGN wedge may be driven by non-AGN objects, and we find somewhat smaller variability amplitudes of $S_0 = 0.05 \pm 0.01$ mag and $S_0 = 0.06 \pm 0.02$ mag in $[3.6]$ and $[4.5]$, respectively.

6. Increasing Completeness with Future Surveys

Given the structure function, we can now estimate the expected statistical properties of the AGNs in the current survey or any extension by adding new epochs. The structure function $S(\tau)$ is related to the correlation function by $C(\tau) = V_\infty^2 - S(\tau)$, where V_∞ is the variance as $\tau \rightarrow \infty$. This makes it straightforward to compute the mean χ^2 of an AGN $\langle \chi^2 \rangle = \langle (m_i - \langle m \rangle)^2 / \sigma^2 \rangle$ since $\langle m_i m_j \rangle = \sigma^2 \delta_{ij} + C(|t_i - t_j|)$, where σ is the measurement uncertainty. Because we fit for the mean, $\langle m \rangle$, the exact value of V_∞ is irrelevant. For our observed frame power-law structure function, the result for the SDWFS data and cadence is $\langle \chi^2 \rangle = 3 + 1.6 S_0^2 / \sigma^2$, where $S = S_0 (\tau / \tau_0)^{0.52}$, $S_0 \simeq 0.1$ mag, and σ is the typical photometric uncertainty. The constant 3 is simply the number of degrees of freedom after fitting the mean, so a non-variable source would have $\langle \chi^2 \rangle = 3$. This expression makes it clear why our AGN completeness is so low and drops rapidly with magnitude (increasing σ , see Figure 2).

We can, in fact, simulate the completeness. We can decompose the correlation matrix $C_{ij} = C(|t_i - t_j|)$ into eigenvectors \mathbf{v}_i and eigenvalues λ_i^2 . If $G(x)$ is a Gaussian deviate of dispersion x , then random realizations of light curves consistent with the structure function are

$$\mathbf{m} = \sum_i \mathbf{v}_i G(\lambda_i) + G(\sigma) \quad (9)$$