in the ratios. We therefore average the APE and HYP determinations to obtain our final results. After adding the statistical and systematic errors for each link smearing in quadrature, we compute the average assuming that the two determinations are 100% correlated using the method of Ref. [79]:

$$\frac{f_{B_s}}{f_{B_d}} = 1.15(12) \tag{86}$$

$$\xi = 1.13(12), \tag{87}$$

where the errors reflect the combined statistical and systematic uncertainties. Although we computed these quantities in the static b-quark limit, the inclusion of the neglected $1/m_b$ corrections (which we estimate to be about 2%) produces a negligible change in the total errors in Eqs. (86) and (87) given the size of our other uncertaintites; thus our results can be directly compared to phenomenological determinations and other lattice QCD results using relativistic b-quarks. As shown in Fig. 9, our results agree with the published results of the HPQCD Collaboration ($\xi = 1.258 \pm 0.025_{\mathrm{stat.}} \pm 0.021_{\mathrm{sys.}}$) [14] and the preliminary results of the Fermilab Lattice and MILC Collaborations ($\xi = 1.205 \pm 0.037_{\mathrm{stat.}} \pm 0.034_{\mathrm{sys}}$) [15].

Although our results have significantly larger errors than the other $N_f = 2+1$ flavor determinations, in this work we have demonstrated the viability of our lattice computation method. In particular, we have introduced the new approach of using $SU(2)$ heavy-light meson chiral perturbation theory to extrapolate $N_f = 2+1$ lattice QCD results for B-meson quantities to the physical quark masses. The largest sources of error in our calculation are from statistics and the chiral extrapolation, and we expect to reduce the sizes of both in a future work that analyzes the 24^3 domain-wall ensembles with the same lattice spacing [34]. Some of the 24^3 ensembles contain almost three times as many configurations as we have analyzed in this work. Furthermore, the use of a larger spatial volume will allow us to simulate at lighter valence and sea quark masses. Once we have made these improvements, our results will provide a valuable cross-check of these important inputs to the CKM unitarity triangle analysis and determination of the ratio of CKM matrix elements $|V_{td}|/|V_{ts}|$.

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