nal results, we will take V and N to infinity while holding the photon density N/V constant.

The primary quantity investigated will be the quantum mutual information $\mathcal{I}_{\mathcal{S}:\mathcal{F}} = H_{\mathcal{S}} + H_{\mathcal{F}} - H_{\mathcal{S},\mathcal{F}}$ between the system $\mathcal{S}$ and fragment $\mathcal{F}$, where H denotes the von Neumann entropy. From this we will calculate the *redundancy* R_δ, which is the number of distinct fragments in the environment that supply, up to an *information deficit* δ, the classical information about the state of the system. More precisely, $R_\delta = 1/f_\delta$, where f_δ is the smallest fragment such that $\mathcal{I}_{\mathcal{S}:\mathcal{F}_{f_\delta}} = (1-\delta)H_{\mathcal{S}}$. (Only very large fragments $f \geq 0.5$ will have complete classical information about the object [4].)

The sphere and the photons in the environment are assumed to be initially unentangled: $\rho^0 = \rho^0_{\mathcal{S}} \otimes \rho^0_e \otimes \cdots \otimes \rho^0_e$, where $\rho_{\mathcal{S}}$ and ρ_e are the density matrices of the system and of a single photon, respectively, and a superscript "0" denotes prescattering states. The photon momenta are distributed according to $\rho^0_e = \int_0^\infty \mathrm{d}k\, p(k) k^2 |\vec{k}\rangle\langle\vec{k}|$ for $p(k) \propto k^2/[\exp(kc/k_B T) - 1]$ and $\hat{n} = \hat{k_i}$ a unique direction.

The decoherence of the superposition is governed by

$$|\langle\vec{x_1}|\rho_{\mathcal{S}}|\vec{x_2}\rangle|^2 = \gamma^N |\langle\vec{x_1}|\rho^0_{\mathcal{S}}|\vec{x_2}\rangle|^2, \tag{1}$$

where

$$\gamma \equiv \left|\mathrm{Tr}\left[S_{\vec{x}_1}\rho^0_e S^\dagger_{\vec{x}_2}\right]\right|^2 \tag{2}$$

and $S_{\vec{x}_p}$ is the scattering matrix acting on the single photon state when the particle is located at $\vec{x}_p$. Because γ controls the suppression of the off-diagonal terms of the object's density matrix in the position basis, γ and $\Gamma \equiv \gamma^N$ are the *decoherence factors* attributable to a single scattering photon and the environment as a whole, respectively. The two-dimensional $\rho_{\mathcal{S}}$ can be diagonalized and its entropy is

$$\begin{aligned} H_{\mathcal{S}} &= \ln 2 - \sum_{n=1}^{\infty} \frac{\Gamma^n}{2n(2n-1)} &(3)\\ &= \ln 2 - \sqrt{\Gamma}\,\mathrm{arctanh}\sqrt{\Gamma} - \ln\sqrt{1-\Gamma}, &(4)\end{aligned}$$

We use the classical cross section of a dielectric sphere [15] in the dipole approximation ($\lambda \gg a$) and assume the photons are not sufficiently energetic to resolve the superposition individually ($\lambda \gg \Delta x$). We further assume that the object is heavy enough to have negligible recoil and that photon energy is conserved. Under these conditions, the key matrix element (which coincides with γ in the case of *monochromatic* radiation) is

$$\begin{aligned}&|\langle\vec{k}(\lambda)|S_{\vec{x}_1}{}^\dagger S_{\vec{x}_2}|\vec{k}(\lambda)\rangle|^2 = \\ &\qquad 1 - \frac{1}{V}\frac{256\,\pi^7}{15}(3 + 11\cos^2\theta)\frac{\tilde{a}^6\Delta x^2 tc}{\lambda^6}\end{aligned} \tag{5}$$

to leading order in $1/V$. Above, $\tilde{a} \equiv a[(\epsilon-1)/(\epsilon-2)]^{1/3}$ is the effective radius of the object and t is the elapsed time. The states $|\vec{k}(\lambda)\rangle$ are photon momentum eigenstates with wavelength λ making an angle θ with the separation vector $\vec{\Delta x}$.

For increasing V, photon momentum eigenstates become diffuse so individual photons decohere the state less and less (i.e. $\gamma \to 1$). This is balanced, of course, by an increasing number of photons in the box, which will lead to a finite decoherence factor for the whole environment, $\Gamma = \gamma^N$. In the $V \to \infty$ limit we use $e = \lim_{q\to\infty}(1+1/q)^q$ to get $\Gamma = \exp(-t/\tau_D)$, where [16]

$$\frac{1}{\tau_D} = C_\Gamma(3 + 11\cos^2\theta)\frac{I\tilde{a}^6\Delta x^2 k_B^5 T^5}{c^6\hbar^6}. \tag{6}$$

and $C_\Gamma = 161280\,\zeta(9)/\pi^3 \approx 5210$ is a numerical constant. We have replaced the photon density N/V with the more physical *irradiance* I (radiative power per unit area). Given Eq. (1), we identify τ_D as the *decoherence time*. Although the rate of decoherence (and, as we shall see, the redundancy) depends on the angle of illumination θ, decoherence is usually so rapid that it hardly matters.

To get the mutual information, we can avoid calculating $H_{\mathcal{SF}}$ by using the identity [Eq. (8) of [6]]

$$\mathcal{I}_{\mathcal{S}:\mathcal{F}} = \left[H_{\mathcal{F}} - H^0_{\mathcal{F}}\right] + \left[H_{\mathcal{S}d\mathcal{E}} - H_{\mathcal{S}d\mathcal{E}/\mathcal{F}}\right] \tag{7}$$

where $H_{\mathcal{S}d\mathcal{E}} = H_{\mathcal{S}}$ is the entropy of the system as decohered by the entire environment $\mathcal{E}$ and $H_{\mathcal{S}d\mathcal{E}/\mathcal{F}}$ is the entropy of the system if it were decohered by only $\mathcal{E}/\mathcal{F}$. We obtain $H_{\mathcal{S}d\mathcal{E}/\mathcal{F}}$ from $H_{\mathcal{S}}$, Eq. (3), by making the replacement $\Gamma \to \Gamma^{1-f}$. Despite the mixedness of the environment, it is possible to diagonalize the post-scattering state $\rho_{\mathcal{F}}$ to get $H_{\mathcal{F}}$ because of the special form of our model; the photons are of mixed energy but are in directional eigenstates, while the elastic scattering conserves energy but mixes photon direction. This allows us to write

$$\rho_{\mathcal{F}} = \int \mathrm{d}\chi_{\mathcal{F}}\, p(\chi_{\mathcal{F}})\, |\chi_{\mathcal{F}}\rangle\langle\chi_{\mathcal{F}}| \otimes \rho^{\chi}_{\hat{\mathcal{F}}}\,, \tag{8}$$

$$\rho^{\chi}_{\hat{\mathcal{F}}} = \frac{1}{2}\left[\bigotimes_{i=1}^{fN} S^{k_i}_{\vec{x}_1}|\hat{n}\rangle\langle\hat{n}|S^{k_i}_{\vec{x}_1}{}^\dagger + \bigotimes_{i=1}^{fN} S^{k_i}_{\vec{x}_2}|\hat{n}\rangle\langle\hat{n}|S^{k_i}_{\vec{x}_2}{}^\dagger\right], \tag{9}$$

where we have broken the momentum eigenstates into a tensor product $|\vec{k_i}\rangle = |k_i\rangle|\hat{n}\rangle/k_i$ of magnitude and directional eigenstates. Above, $\chi_{\mathcal{F}} = (k_1, \ldots, k_{fN})$ is the vector of the magnitudes of the photon momenta of $\mathcal{F}$, $p(\chi_{\mathcal{F}}) = \prod_{i=1}^{fN} p(k_i)$ is the momentum probability distribution, and $|\chi_{\mathcal{F}}\rangle\langle\chi_{\mathcal{F}}| = \bigotimes_{i=1}^{fN} |k_i\rangle\langle k_i|$. $S^{k_i}_{\vec{x}_p}$ is defined by

$$S_{\vec{x}_p}|\vec{k_i}\rangle = S_{\vec{x}_p}|k_i\rangle|\hat{n}\rangle/k = |k_i\rangle S^{k_i}_{\vec{x}_p}|\hat{n}\rangle/k\,. \tag{10}$$

We then have

$$H_{\mathcal{F}} = fNH^0_e + \int \mathrm{d}\chi_{\mathcal{F}}\, p(\chi_{\mathcal{F}})\, H^{\chi}_{\hat{\mathcal{F}}}\,, \tag{11}$$