of S_{2n}.

These fixed points correspond to elements of S_{2n} which constitute 2 (left) W_K-cosets — namely, $W_K \cdot 1$ and $W_K \cdot s_n$. Thus there are two closed orbits, as follows from the discussion immediately following the statement of Proposition 1.3.7. $\square$

With the closed orbits determined, we now give a formula for the S-equivariant class of each:

Proposition 2.3.2. *With Q_1 and Q_2 as in the previous proposition, $[Q_1]$ is represented by the polynomial $P_1(x,y)$, and $[Q_2]$ by the polynomial $P_2(x,y)$, where*

$$P_1(x,y) = 2^{n-1}(x_1 \dots x_n + y_1 \dots y_n) \prod_{1 \leq i < j \leq n} (y_i + y_j)(y_i + y_{2n+1-j});$$

and

$$P_2(x,y) = -2^{n-1}(x_1 \dots x_n - y_1 \dots y_n) \prod_{1 \leq i < j \leq n} (y_i + y_j)(y_i + y_{2n+1-j}).$$

Proof. We demonstrate the correctness of the formula for $[Q_1]$. The argument is similar to that given in the previous case for the lone closed orbit of the odd orthogonal group.

As stated, Q_1 consists of those S-fixed points corresponding to elements of W_K — that is, signed permutations with an even number of sign changes. Take $w \in Q_1$ to be such a fixed point. We use Proposition 1.3.3 to compute the restriction $[Q_1]|_w$. As in the previous example, we first determine the restriction of the positive roots Φ^+ to $\mathfrak{s}$, then apply the signed permutation w to that set of weights.

Restricting the positive roots $\{Y_i - Y_j \mid 1 \leq i < j \leq 2n\}$ to $\mathfrak{s}$, we get the following set of weights:

1. $X_i - X_j$, $1 \leq i < j \leq n$, each with multiplicity 2 (one is the restriction of $Y_i - Y_j$, the other the restriction of $Y_{2n+1-j} - Y_{2n+1-i}$)