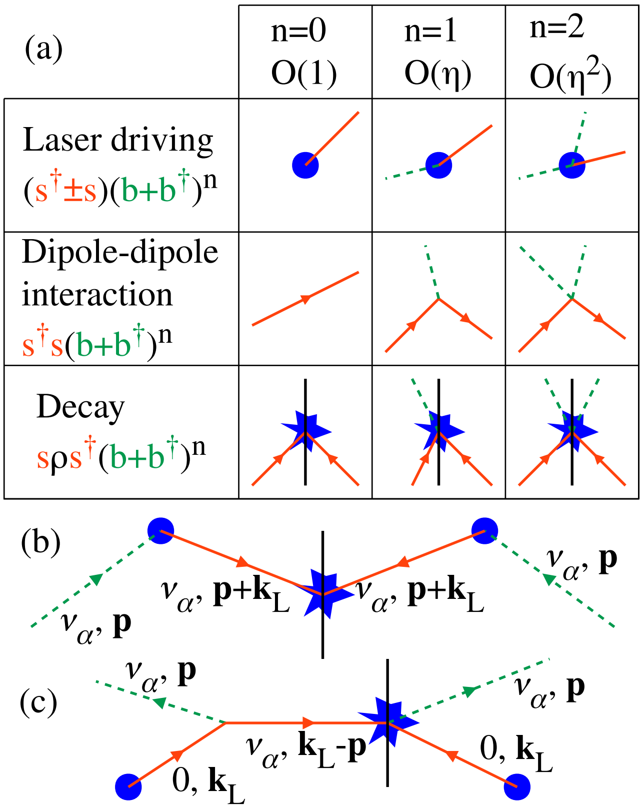

FIG. 4: (Color online) Feynman diagrams. Solid orange line: electronic excitation, dotted green line: phonon, blue circle: laser (de)excitation, blue star: spontaneous decay. The black vertical line is used to keep track of which operators act from which side of ρ. (a) Table of allowed vertices, by number of phonon lines (order in η_α) n and type of interaction. Vertices exist with arbitrarily high n, but for our purposes we need only consider up to 2nd order, i.e. $n \leq 2$. In the case of decay, the operator ordering shown is not the only one allowed: there is always one atomic excitation on each side of ρ, but the phonon(s) can be on either or both sides. Where no arrowhead is shown either propagation direction is allowed. (b) Example of a rate term (diagram symmetrical about the ρ line), the ordinary sideband cooling process. (c) Example of an interference term (diagram asymmetrical). Lines labelled with frequency $E/\hbar$ and wavevector $\mathbf{p}$.

are necessary to fully identify an operator, here $\mathbf{p}$ for an electronic excitation and $\mathbf{p}\alpha$ for a phonon, and also with an energy E, which is generally *not* on that particle's dispersion. Each operator in a vertex must be connected to a matching propagation line, which may go to another vertex or leave the diagram. The energy E is conserved at vertices (total in=total out), and a particular master equation may have additional conserved quantities, e.g. wavevector if it is translationally invariant (such as ours). Only particles appearing as operators in the master equation appear as lines in the diagram; particles eliminated during the master equation derivation (here, decay photons) do not. Classical entities such as the laser field also do not appear as lines, but may contribute momentum, here $\mathbf{k}_\mathrm{L}$ at each laser vertex.

A master equation contains operators acting on both sides of the density matrix ρ, and we hence need to keep track of which operators act on the left of ρ and which on the right, which we do with the black vertical line in the diagrams. Propagation lines and vertices from $H_\mathrm{cond}\rho$ are on the left of this ρ line, while those from $\rho H_\mathrm{cond}^\dagger$ are on the right; a line or vertex on the right hence has the complex conjugate coefficient of an identical one on the left. Reset operator vertices contain operators acting on both sides of ρ, and hence appear only on the ρ line, with propagation lines attached to them from both sides. Propagation lines cannot cross the ρ line without going through a reset vertex.

We choose to make the arrow on a propagation line point forward in time, i.e. from the vertex where that excitation is created to that where it is destroyed. Since $s_\mathbf{p}^\dagger |0\rangle = |1_\mathbf{p}\rangle$ but $\langle 1_\mathbf{p}| s_\mathbf{p}^\dagger = \langle 0|$, this is the usual creation operator (of the vertex) to annihilation operator on the left side of the diagram, but annihilation operator to creation operator on the right side. With this convention, energy/momentum conservation at a reset vertex means equal energy/momentum on each side of ρ. (The alternative of creation operator to annihilation operator on both sides would also be self-consistent, and would make the conservation laws in=out everywhere, but would make the arrows point backwards in time on the right.)

To evaluate such a diagram, we replace each vertex by its coefficient (in the master equation, i.e. including the $\pm\mathrm{i}/\hbar$ where applicable) and each internal line by its propagator $1/(-\text{coefficient} \mp \frac{\mathrm{i}}{\hbar}E)$, where the $-$ applies to the left side of ρ and the $+$ to the right. Lines entering or leaving the diagram have on-resonance energy. As in conventional Feynman diagrams, for tree diagrams the external lines' energies (and momenta, if conserved) determine all the internal ones by conservation, while for diagrams containing loops it is necessary to integrate over the undetermined energies and momenta. For example, ordinary sideband cooling is represented by the diagram Fig. 4(b), which has the value

$$\begin{aligned}
&\left(\frac{\eta_\alpha \Omega \hat{k}_{\mathrm{L}\alpha}}{2}\right)\left(\frac{1}{\frac{1}{2}c(\mathbf{p}+\mathbf{k}_\mathrm{L}) - \mathrm{i}\nu_\alpha}\right)\mathrm{Re}\, c(\mathbf{p}+\mathbf{k}_\mathrm{L}) \\
&\times\left(\frac{1}{\frac{1}{2}c^*(\mathbf{p}+\mathbf{k}_\mathrm{L}) + \mathrm{i}\nu_\alpha}\right)\left(\frac{\eta_\alpha \Omega \hat{k}_{\mathrm{L}\alpha}}{2}\right) \\
&= \frac{\left(\eta_\alpha \Omega \hat{k}_{\mathrm{L}\alpha}\right)^2 \mathrm{Re}\, c(\mathbf{p}+\mathbf{k}_\mathrm{L})}{|c(\mathbf{p}+\mathbf{k}_\mathrm{L}) - 2\mathrm{i}\nu_\alpha|^2}\,, \qquad (28)
\end{aligned}$$

where $\mathbf{p}$ is the phonon momentum (which is the same on both sides by conservation).

The resulting value is that diagram's contribution to the transition rate (not amplitude) between its start and end states, which are given by the particles entering and leaving it. Symmetrical diagrams such as Fig. 4(b) give the rate of the process appearing on both sides, while