in Fig. F.1.

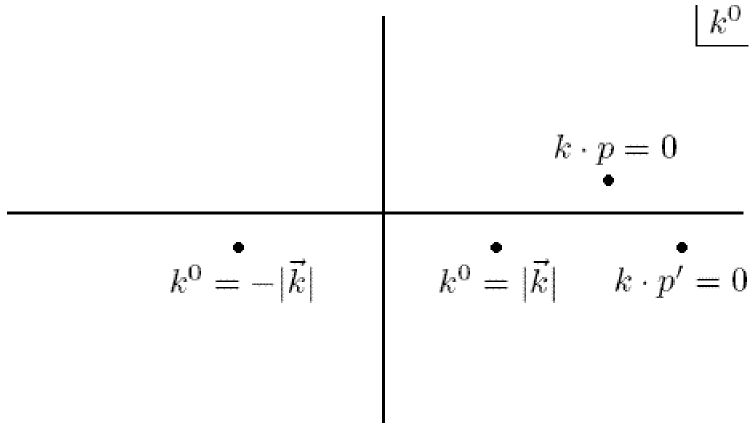

Figure F.1: *Pole structure of Eq. (F.34).*

For $t < 0$ we close the contour in the upper half of the plane, picking up the $k^0 = \vec{k} \cdot \vec{p}/p^0$ pole; the result is

$$A^\mu(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} e^{-i\left(\frac{\vec{k}\cdot\vec{p}}{p^0}\right)t} \frac{(2\pi i)}{2\pi} \frac{-ie}{k^2} \frac{-p^\mu}{p^0}, \tag{F.35}$$

where the k^0 in the k^2 term is given by its value at the pole. Note that the $1/p^0$ comes from correctly evaluating the contour integral using

$$\begin{aligned} \frac{p^\mu}{k \cdot p - i\epsilon} &= \frac{p^\mu}{k^0 p^0 - \vec{k} \cdot \vec{p} - i\epsilon} \\ &= \frac{p^\mu}{p^0} \left[\frac{1}{k^0 - \frac{\vec{k}\cdot\vec{p}}{p^0} - i\epsilon} \right]. \end{aligned} \tag{F.36}$$

We can see that the $k \cdot p = 0$ pole for $t < 0$ corresponds to the Coulomb field of the particle by boosting to its rest frame; we have that $k^2 = -|\vec{k}|^2$