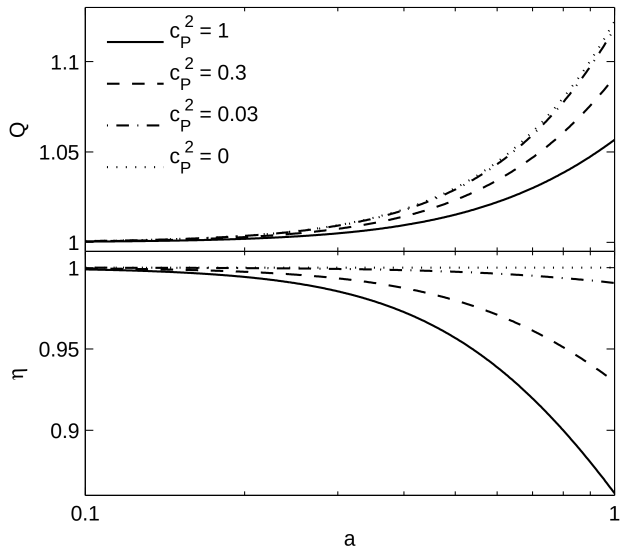

FIG. 1: The evolution of Q and η is presented with varying c_P^2 for the case of scale-invariant growth of dark energy perturbations where the anisotropic stress balances the pressure support ($f_\sigma = 1$). Here we set $w_{de} = -0.8$ as in the sDE reference.

Both, c_P^2 and g_P can be functions of time and space. However, we will consider c_P^2 to be constant [52].

In the absence of anisotropic stress, $\sigma_{de} = 0$, dark energy perturbations are strongly suppressed on comoving scales smaller than $c_P/(aH)$ because of the negative scale-dependent term $\propto k^2 \delta P_{de}$ on the rhs of the evolution equation (27). Therefore, dark energy perturbations become only important on quasi-static scales if $c_P^2 \ll 1$.

The presence of anisotropic stress in the dark energy, however, can mitigate this effect by compensating the pressure support. The extreme case where $\sigma_{de} = \sigma_{de}^0$ with

$$\sigma_{de}^0 \equiv \frac{\delta P_{de}}{(1+w_{de})\rho_{de}}\,, \tag{29}$$

leads to a scale-independent growth of dark energy perturbations if $g_P = 0$. In this case the scale-dependent suppression term on the rhs of the evolution equation for δ_{de} vanishes. More generally we set

$$\sigma_{de} = f_\sigma \sigma_{de}^0 + \frac{g_\sigma}{k^2}\,, \tag{30}$$

such that the constant parameter f_σ re-introduces scale-dependence into the growth of perturbations in case $f_\sigma \neq 1$. This effect becomes more apparent when we substitute the expressions for the pressure and anisotropic stress perturbations into the evolution equation. For the simple case when w_{de} and c_P^2 are constants and $g_P = g_\sigma = 0$ we find from Eq. (27)

$$\begin{aligned}\ddot{\delta}_{de} + (2 + 3c_P^2 - 6w_{de})H\dot{\delta}_{de} = \\ &\frac{3}{2}(1+w_{de})H^2\Big\{\Omega_m\delta_m + (1+3f_\sigma c_P^2)\Omega_{de}\delta_{de} \\ &+2\frac{w_{de}-c_P^2}{1+w_{de}}\Big(2-3w_{de}+\frac{d\ln H}{d\ln a}\Big)\delta_{de}\Big\} \\ &-\frac{k^2}{a^2}(1-f_\sigma)c_P^2\delta_{de}\end{aligned} \tag{31}$$

where we also substituted the source term from Eq. (26). For the matter we find

$$\ddot{\delta}_m + 2H\dot{\delta}_m = \frac{3}{2}H^2\Big\{\Omega_m\delta_m + (1+3f_\sigma c_P^2)\Omega_{de}\delta_{de}\Big\}\,. \tag{32}$$

In the following we will investigate the two cases: scale-independent growth of dark energy perturbations under the conditions $f_\sigma = 1$, $g_P = 0$ and $g_\sigma = 0$, and scale-dependent growth with a deviation from either one of those conditions.

On scales where the quasi-static approximation is applicable, the clustering of dark energy is governed by the competition between the gravitational infall and the pressure support as can be seen from the k^2-dependent term in the full evolution equation (27). However, the anisotropic stress counteracts the pressure. For $\sigma_{de} = \sigma_{de}^0$, i.e. $f_\sigma = 1$, the anisotropic stress exactly cancels the pressure support. The remaining source term is dominated by the matter density. Thus, the dark energy clusters on all scales like dark matter does, and consequently, the growth of perturbations becomes scale-independent.

From the simplified evolution equation (31), with constant w_{de} and c_P^2, we can read off the influence of the sound speed: $c_P^2 > 0$ enhances the Hubble drag, the second term on the lhs of Eq. (31), and also lowers the source term, as the third term in the braces is < -10 for typical values of w_{de} and all $c_P^2 > 0$. In other words, compared to the matter perturbations, dark energy perturbations grow weakly compared with dark matter perturbations but are not completely suppressed. Therefore we expect $Q > 1$ at late times. Dark matter perturbations do not suffer the suppression but are rather enhanced due to the dark energy anisotropic stress adding to the source term, as seen from Eq. (32). The clustering condition of $c_P^2 \ll 1$ is relaxed compared to the case with no anisotropic stress and we find dark energy perturbations cluster on all scales even for $c_P^2 = 1$. In Fig. 1 we show the late-time evolution of Q and η in the case of the scale-independent growth. With the help of a modified version of the CAMB code [47] we plot the mode $k = 0.01\,h^{-1}\,\mathrm{Mpc}$, which is representative for the quasi-static regime. It is important to note that when computing quantities like Q with the help of a full Boltzmann-code like CAMB rather than within the quasi-static approximation it is necessary to use the correct gauge-invariant definition of Q in terms of the comoving density perturbations Δ_i rather than the Newtonian gauge quantities δ_i. Omitting this leads to artificial scale-dependence because it takes time for