where the formula is exact in the limit $n \to \infty$. The approximation in (3.19) is very good. The maximal relative error $|(\langle\psi_n|\hat{r}\psi_n\rangle - r_{\rm max}(E_n)/2)/\langle\psi_n|\hat{r}\psi_n\rangle|$ arising in the case with $n = 1$ is about 5% and is lesser than 1% for $n = 2$. The fact that the limit $n \to \infty$ when we have the exact equality in (3.19), is the classical limit is confirmed by the classical formula

$$\langle r\rangle_{\rm cl} = \int_0^\infty r\rho_{\rm cl}(r)d\mu(r) = \frac{r_{\rm max}}{2} = \frac{\sqrt{2E}}{2\kappa}, \tag{3.20}$$

following directly from Eq. (3.12).

IV. CONCLUSION

In this work we study the relativistic massless harmonic oscillator in both classical and quantum cases. It seems that the obtained results concerning the classical oscillator are of importance not only from the physical point of view. Indeed, Eq. (2.2) is one of the simplest examples of a nonlinear Hamiltonian system with constant length of velocity. As far as we are aware such an interesting class of nonlinear dynamical systems was not discussed in the literature. Referring to the observations of this work related to the quantum mechanics of the relativistic massless harmonic oscillator we wish to point out that Eq. (3.11) is, to our best knowledge, the first example of the nontrivial exact solution to the Salpeter equation. We also stress the good behavior of the corresponding probability density and expectation values of observables which confirms the correctness of the quantization based on the massless spinless Salpeter equation. Furthermore, we obtain the exact formula (3.9) on the spectrum of the Hamiltonian. It should be noted that for the Salpeter equation only energy bounds were analyzed in the literature so far (for the massive relativistic harmonic oscillator see Ref. [10]). Finally, we have obtained the interesting form of the virial theorem for the massless relativistic harmonic oscillator with the exchanged roles of the kinetic and potential energies.

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