one Rabi parameter, the single-time Rabi frequency, R^{1TD}, determines both the single- and two-time dynamics. The two-fermions spectra are thus obtained by simply substituting the fermionic parameters (Eq. 7) in the expression of the linear model [14].

This simplicity and likeliness to the linear model stems from the fundamental nature of the problem: two identical (indistinguishable) particles coupled linearly, obeying fully their quantum statistics. The two coupled qubits (or the Jaynes-Cummings model [16]), by mixing different types of particles (distinguishable modes) and therefore breaking commutation rules, result in the more complex description and richer dynamics presented in the previous section.

C. four-level system (4LS)

Also in the four-level system (with no cross Lindblad terms), the expressions for the single-time dynamics (populations, coherence, Etc.) are the same than for the linear model, and here also their power spectra are different. The parameters for the 4LS spectra are of a bosonic character, cf. Eq. (8):

$$\tilde{\Gamma}_1 = \gamma_1 - P_1\,, \quad \tilde{\Gamma}_2 = \gamma_2 - P_2\,, \tag{29a}$$

$$\tilde{\Gamma}_\pm = \frac{\tilde{\Gamma}_1 \pm \tilde{\Gamma}_2}{4}\,, \tag{29b}$$

$$\tilde{R} = \sqrt{g^2 - \tilde{\Gamma}_-^2}\,. \tag{29c}$$

The relevant parameters that characterize the coupling simplify to:

$$R = \frac{\tilde{\Gamma}_+}{\Gamma_+}\tilde{R}\,, \tag{30a}$$

$$z_{1,2} = \tilde{R} \pm i\tilde{\Gamma}_+\,, \tag{30b}$$

recovering the conventional strong coupling criterion based on one parameter only, the bosonic (half) Rabi frequency $\tilde{R}$. The resulting spectral structure then consists of two pairs of peaks sitting at $\pm\Re(\tilde{R})$ with:

$$\frac{\gamma_A}{2} + i\omega_A = \frac{3(P_1+P_2)+\gamma_1+\gamma_2}{4} + i\tilde{R}\,, \tag{31a}$$

$$\frac{\gamma_B}{2} + i\omega_B = \frac{3(\gamma_1+\gamma_2)+P_1+P_2}{4} + i\tilde{R}\,, \tag{31b}$$

$$\frac{\gamma_C}{2} + i\omega_C = \frac{3(\gamma_1+\gamma_2)+P_1+P_2}{4} - i\tilde{R}\,, \tag{31c}$$

$$\frac{\gamma_D}{2} + i\omega_D = \frac{3(P_1+P_2)+\gamma_1+\gamma_2}{4} - i\tilde{R}\,. \tag{31d}$$

Note that γ_p are always positive for any combination of the parameters, in contrast with those of two bosonic modes, where the system can diverge. Therefore, the values of pump and decay rates here are not limited, always leading to a physical steady state.

V. STRONG AND WEAK COUPLING REGIMES

The standard criterion for strong coupling (SC) is based on the splitting at resonance of the bare states into dressed states. This manifests in the appearance of τ-oscillations in the two-time correlators and a splitting of the peaks that compose their spectrum.

In a naive approach to the problem of defining strong-coupling in a system other than the linear model, one could think that the condition for SC is $\Re(R^{1TD}) \neq 0$ (at resonance), leading to the familiar inequality, $g > |\Gamma_-|$. However, this is not the case whenever pump and decay are both taken into account. Instead, one must find the condition for a splitting between the new eigenstates, that is, the two pairs of peaks forming the spectrum. The peaks are positioned symmetrically in two pairs about the origin at $\omega_p = \pm\Re(z_{1,2})$ and, therefore,

$$\Re(z_1) \neq 0 \quad \text{or} \quad \Re(z_2) \neq 0 \tag{32}$$

is the mathematical condition for SC in this system. Given that there are two different parameters z_1 and z_2 on which the condition relies, the SC/WC distinction must be extended to cover new possibilities. Thus, instead of only one relevant parameter, Γ_-/g, as was the case in the linear model, SC between two qubits is determined by three parameters:

$$\Gamma_-/g\,, \quad \Gamma_+/g \quad \text{and} \quad D^{\mathrm{s}}\,. \tag{33}$$

This gives rise to the situations listed in Table I, that are discussed in the following sections.

R	$\Re(z_1)$	$\Re(z_2)$	Acronym	Type of coupling
$\lvert R\rvert$	$\neq 0$	$\neq 0$	FSC	First order Strong Coupling
$i\lvert R\rvert$	$\neq 0$	$\neq 0$	SSC	Second order Strong Coupling
$i\lvert R\rvert$	0	$\neq 0$	MC	Mixed Coupling
$i\lvert R\rvert$	0	0	WC	Weak Coupling

TABLE I: Type and nomenclature of coupling for two coupled qubits. Beyond the usual weak coupling (WC) and strong coupling (here denoted FSC) encountered in the linear model, the system exhibits two new regions: Mixed Coupling (coexistence of weak and strong coupling) and Second order Strong Coupling (with two different splittings of two pairs of dressed states).

A. Vanishing pump and spontaneous emission

In the case of vanishing pump, that corresponds as well to spontaneous emission, the standard SC and WC hold. In this limit, we recover the familiar expression for the half Rabi frequency $R, R^{1TD} \to R_0$. The parameters simplify to $z_{1,2} \to \sqrt{(R_0 \pm i\gamma_+)^2} = R_0 \pm i\gamma_+$ [39].

The positions and broadenings of the four peaks are:

$$\frac{\gamma_A}{2} + i\omega_A = \gamma_+ + iR_0\,, \qquad \frac{\gamma_B}{2} + i\omega_B = 3\gamma_+ + iR_0\,, \tag{34a}$$

$$\frac{\gamma_C}{2} + i\omega_C = 3\gamma_+ - iR_0\,, \qquad \frac{\gamma_D}{2} + i\omega_D = \gamma_+ - iR_0\,. \tag{34b}$$