where we assume a Maxwell-Boltzmann distribution of relative velocities given by

$$f(v_z, v_r) \propto \exp\left(-\frac{\mu v_r^2}{2k_B T_r}\right) \exp\left(-\frac{\mu v_z^2}{2k_B T_z}\right) \tag{9}$$

with $v_r^2 = v_x^2 + v_y^2$. A factor of v_z^2 in both the numerator and denominator of Eqn. (7) accounts for the fact that the $m_L = 0$ p-wave inelastic collision rate scales as collision energy $\propto v_z^2$. Similarly, for $m_L = \pm 1$ collisions, the collision rate scales as v_r^2 and we find the average relative energy for two colliding particles using

$$\begin{aligned} E^1_{z,rel} &= \frac{1}{2}\mu \frac{\int_{-\infty}^{\infty} f(v_z, v_r) v_z^2 dv_z}{\int_{-\infty}^{\infty} f(v_z, v_r) dv_z} \\ &= \frac{1}{2} k_B T_z \qquad (10) \\ E^1_{x,rel} &= \frac{1}{4}\mu \frac{\int_0^{\infty} f(v_z, v_r) v_r^4 (2\pi v_r) dv_r}{\int_0^{\infty} f(v_z, v_r) v_r^2 (2\pi v_r) dv_r} \\ &= k_B T_x. \qquad (11) \end{aligned}$$

Adding this to the $k_B T_i$ from the center-of-mass energies, we get

$$\begin{aligned} \Delta E_z^0 &= \frac{5}{2} k_B T_z \qquad (12) \\ \Delta E_x^0 &= \frac{3}{2} k_B T_x \qquad (13) \\ \Delta E_z^1 &= \frac{3}{2} k_B T_z \qquad (14) \\ \Delta E_x^1 &= 2 k_B T_x. \qquad (15) \end{aligned}$$

Putting this into the expression for the heating rate (Eqn. (6)), we find that

$$\begin{aligned} \frac{dT_z}{dt} &= -\frac{1}{2} T_z \Gamma^0_{coll} + \frac{1}{2} T_z \Gamma^1_{coll} \qquad (16) \\ \frac{dT_x}{dt} &= +\frac{1}{2} T_x \Gamma^0_{coll}. \qquad (17) \end{aligned}$$

In the main text, we defined the loss rate coefficient as

$$\begin{aligned} \beta &= K_0 T_z + 2 K_1 T_x \qquad (18) \\ &= \frac{2}{n}\left(\Gamma^0_{coll} + \Gamma^1_{coll}\right), \end{aligned}$$

where K_0 and K_1 depend on the induced dipole moment. The inelastic collision rate per particle is related to β through

$$\Gamma^0_{coll} = \frac{K_0 T_z n}{2} \tag{19}$$

$$\Gamma^1_{coll} = \frac{2 K_1 T_x n}{2}. \tag{20}$$