$$\mathrm{Tr_C}\left[t^a t^b\right] g^2 n_1^\mu n_1^\nu \left\{ \left(\frac{1}{-l_1^+ + i\epsilon}\right)\left(\frac{1}{-l_2^+ + i\epsilon}\right) + \left(\frac{1}{l_1^+ + i\epsilon}\right)\left(\frac{1}{-l_2^+ + i\epsilon}\right) + \right.$$
$$\left. + \left(\frac{1}{-l_1^+ + i\epsilon}\right)\left(\frac{1}{l_2^+ + i\epsilon}\right) + \left(\frac{1}{l_1^+ + i\epsilon}\right)\left(\frac{1}{l_2^+ + i\epsilon}\right)\right\}$$
$$= \mathrm{Tr_C}\left[t^a t^b\right] g^2 n_1^\mu n_1^\nu \left(\frac{1}{l_1^+ + i\epsilon} + \frac{1}{-l_1^+ + i\epsilon}\right)\left(\frac{1}{l_2^+ + i\epsilon} + \frac{1}{-l_2^+ + i\epsilon}\right) = -4\pi^2 g^2 n_1^\mu n_1^\nu \mathrm{Tr_C}\left[t^a t^b\right]\delta(l_1^+)\delta(l_2^+). \quad (26)$$

The anomalous eikonal factor here is the same as in Eq. (25), apart from an overall minus-sign. However, the propagator denominators for graphs like Fig. 7(a) (both gluons on opposite sides of the cut) are different from the propagator denominators for graphs like Fig. 7(b) (gluons on the same side). For graphs with the extra gluons on opposite sides of the cut, one finds the following contribution to the TMD PDF of hadron H_1:

$$I_1(k_{1T}) =$$
$$\frac{g^2\lambda_1^2 \mathrm{Tr_C}\left[t^a t^b\right]\mathrm{Tr_C}\left[t^b t^a\right]}{(2\pi)^{12}} x_1 p_1^+ \int dk^- d^4 l_1 d^4 l_2 \frac{\left[2(p_1^+ - k_1^+) + l_1^+\right]\left[2(p_1^+ - k_1^+) + l_2^+\right]}{(l_1^2 + i\epsilon)(l_2^2 - i\epsilon)\left[(k_1 - l_1)^2 - m_q^2 + i\epsilon\right]\left[(k_1 - l_2)^2 - m_q^2 - i\epsilon\right]} \times$$
$$\times \frac{(2\pi)^3 \delta(l_1^+)\delta(l_2^+)\delta((p_1 - k_1)^2 - m_q^2)}{\left[(p_1 - k_1 + l_1)^2 - m_q^2 + i\epsilon\right]\left[(p - k + l_2)^2 - m_q^2 - i\epsilon\right]}$$
$$= \frac{g^2\lambda_1^2 T_F^2 (N_c^2 - 1) x_1 (1 - x_1)}{256\pi^7} \int d^2\mathbf{l}_{1T}\, d^2\mathbf{l}_{2T} \prod_{j=1,2} \frac{1}{l_{jT}^2\left[(\mathbf{k}_{1T} - \mathbf{l}_{jT})^2 + m_q^2\right]}. \quad (27)$$

This is the same result as in Ref. [18], except that the gluon is massless and there is a non-Abelian color factor multiplying the integral. Equation (26) allows for a similar calculation of the remaining contribution to the TMD PDF for hadron H_1 from the graphs with the extra gluons on the same side of the cut:

$$I_2(k_{1T}) = \frac{-g^2\lambda_1^2 T_F^2 (N_c^2 - 1) x_1 (1 - x_1)}{256\pi^7} \int d^2\mathbf{l}_{1T}\, d^2\mathbf{l}_{2T} \frac{1}{l_{1T}^2 l_{2T}^2 \left[(\mathbf{k}_{1T} - \mathbf{l}_{1T} - \mathbf{l}_{2T})^2 + m_q^2\right]\left[k_{1T}^2 + m_q^2\right]}. \quad (28)$$

The mismatch in denominators between Eq. (27) and Eq. (28) means that the full contribution $I_1(k_{1T}) + I_2(k_{1T})$ does not generally vanish point-by-point in k_{1T}. The sum of graphs like Fig. 7 therefore results in uncanceled terms that are not accounted for by the standard Wilson lines. Hence, the standard TMD-factorization formula Eq. (24) fails for unpolarized scattering.

Exactly analogous observations apply to the TMD PDF of the other hadron if the two extra gluons are radiated from the spectator in H_2, collinear to the minus direction. In that case, the eikonal factors analogous to Eqs. (25, 26) will instead use a vector n_2 and the delta functions from the eikonal factors will be $\delta(l_1^-)$ and $\delta(l_2^-)$ [33]. So, both TMD PDFs yield factorization anomalies at the two-gluon level in the unpolarized cross section.

V. GENERALIZED TMD-FACTORIZATION

From the factorization anomaly terms Eqs. (25, 26) one may determine what the modified gauge link structure must be for the TMD PDF of hadron H_1 in a generalized factorization formula [9, 12, 15]. In our example, the sequence of eikonal factors in the factorization anomaly terms of Eqs. (25,26), including the trace around a color loop, require a color-traced Wilson loop operator to be inserted into the definition, Eq. (19), of the TMD PDF for H_1. Each of the eikonal factors in Eqs. (25,26) corresponds to an attachment to a leg of the Wilson loop. Therefore, the standard TMD PDF in Eq. (24) for H_1 should be replaced with,

$$\Phi_{H_1}^{[n_1,(\Box)]}(x_1, k_{1T}) =$$
$$x_1 p_1^+ \int \frac{dw^- d^2\mathbf{w}_t}{(2\pi)^3} e^{-ix_1 p_1^+ w^- + i\mathbf{k}_t\cdot\mathbf{w}_t} \times$$
$$\times \langle H_1, s_1 | \phi_{1,r}^\dagger(0, w^-, \mathbf{w}_t)\, U_{rs}^{n_1}[0,w] U_{(\Box)}^{n_1}\, \phi_{1,s}(0) | H_1, s_1\rangle. \quad (29)$$

This is the same as the standard TMD PDF definition in Eq. (19) apart from the insertion of the following color-traced Wilson loop operator:

$$U_{(\Box)}^{n_1} = U_{ij}^{n_1}[0,w]\,(U^{n_1\,\dagger}[0,w])_{ji} =$$
$$\mathrm{Tr_C}\left[V_0(n_1) I(n_1) V_w^\dagger(n_1) V_w(n_1) I^\dagger(n_1) V_0^\dagger(n_1)\right]. \quad (30)$$