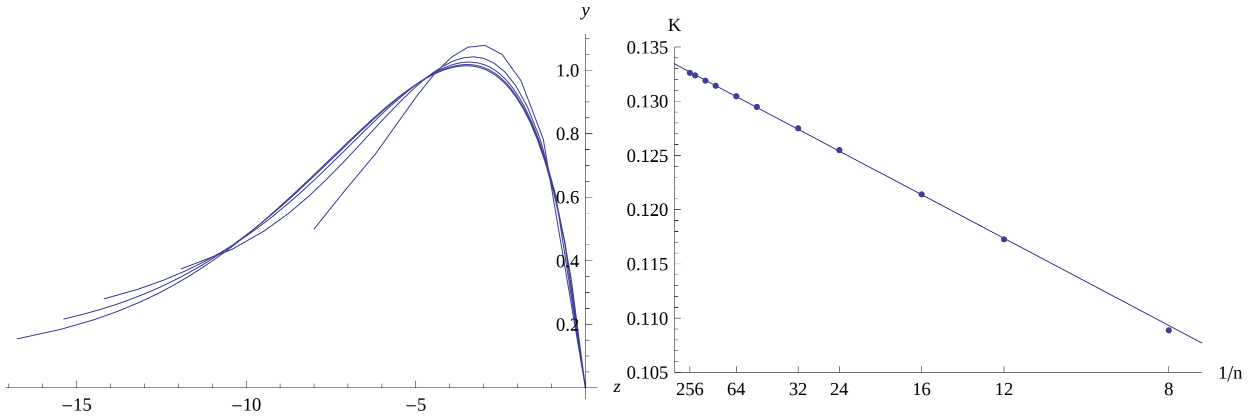

FIG. 8: Left: $y = n\,(p-1)$ versus $z = n\,(q-1)$ for C_n, with $n = 8, 16, 32, 64, 128$ with larger n extending farther to the left. Right: K giving the maximum y vs $1/n$ for C_n, $n = 8, 12$, 16, 24, 32, 48, 64, 96, 128, 192, 256.

Though we can not exactly solve what y and z should be at $K = 2/15$ we can at least see

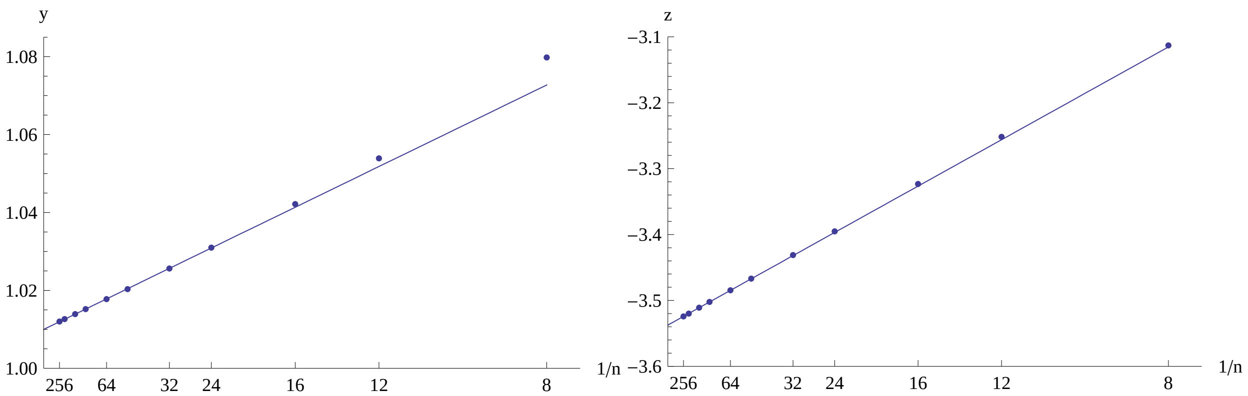

FIG. 9: Left: Maximum value of $y = n\,(p-1)$ versus $1/n$ for C_n. Right: value of $z\,(q-1)$ versus $1/n$ for C_n when y is at its maximum. In both cases $n = 8, 12$, 16, 24, 32, 48, 64, 96, 128, 192, 256.

how y and z relate at this point. For an infinite 1-dimensional lattice we have that $\chi = e^{2\,K}$, see e.g.[13]. The second moment then should behave as

$$\sigma_2 \sim \frac{n\,\chi}{4} = \frac{n\,e^{2\,K}}{4} \tag{144}$$

Let $\ell = k - n/2$ and $\sigma = \sqrt{\sigma_2}$. For high temperatures we expect ℓ/σ to be normally distributed and thus

$$\mathbb{P}\,(\ell) \sim \frac{\exp\left(-(\ell/\sigma)^2/2\right)}{\sigma\,\sqrt{2\,\pi}} \tag{145}$$

The probability ratio is then

$$R\,(n,\,n/2,\,\ell) = \frac{\mathbb{P}\,(\ell)}{\mathbb{P}\,(0)} = \exp\left(-\ell^2/2\,\sigma^2\right) \tag{146}$$