the inflaton evolves according to the formula Eq. (381). Thus, there exists a final time after which $|\phi| < m_\psi/h$, and the total mass no longer vanishes, then the resonant production of fermion ends.

The Dirac equation (in conformal time η) for a fermionic field is given by:

$$\left(\frac{i}{a}\,\gamma^\mu\,\partial_\mu + i\,\frac{3}{2}H\gamma^0 - m(\eta)\right)\psi = 0\,, \tag{390}$$

where a is the scale factor of the universe, $H = a'/a^2$ the Hubble rate and $'$ denotes derivative w.r.t. η, and $m(\eta) = m_\psi + h\phi(\eta)$, where m_ψ is the bare mass of the fermion. The particle density per physical volume $V = a^3$ at time η is given by:

$$n(\eta) \equiv \langle 0|\frac{N}{V}|0\rangle = \frac{1}{\pi^2\,a^3}\int dk\,k^2\,|\beta_k|^2\,, \tag{391}$$

where α_k, β_k are the Bugolyubov's coefficients satisfying: $|\alpha_k|^2 + |\beta_k|^2 = 1$. The occupation number of created fermions is thus given by $n_k = |\beta_k|^2$, and the above condition ensures that the Pauli limit $n_k < 1$ is respected. One important physical quantity is the scaling of the total energy

$$\rho_\psi \propto m_\psi N_\psi \propto q m_\psi^{1/2} \tag{392}$$

which is linear in $q = h^2\hat{\phi}^2/m_\phi^2$, as generally expected [338, 774–776], but also note that $m_\psi(t) \propto q^{1/2}$.

In a realistic case, since the SM fermions are chiral, if the inflaton is a SM gauge singlet, then it can only couple via dimension-5 operators, i.e.

$$\frac{\lambda}{M_{\rm P}}\phi(H\bar{q}_l)q_R\,, \tag{393}$$

where $\lambda \sim \mathcal{O}(1)$, H is the SM Higgs doublet and q_l, q_R are the $SU(2)_l$ doublet and the right handed SM fermions, respectively. As a result preheating of SM fermions from a gauge singlet inflaton becomes less important due to weak coupling.

In Ref. [338], it was argued that an inflaton coupling to right handed neutrino, $h\phi\bar{N}N$, where N is right handed neutrino, will induce non-thermal leptogenesis, where the right handed neutrinos were treated gauge singlets. Anyway, if we embed the right handed neutrinos in a gauge sector, where they get their masses via some Higgs mechanism, then one requires non-renormalizable couplings like Eq. (393).

Similar argument holds for coupling to the SM gauge bosons, where the inflaton can only couple via non-renormalizable operator, i.e.

$$\frac{\lambda}{M_{\rm P}}\phi F_{\mu\nu}F^{\mu\nu}\,, \tag{394}$$