To consider path loss, we now perform a transformation of RVs using (1), $\gamma_k = \bar{K} r_k^{-\beta} \zeta_k$, where, γ_k is the selected user SNR in each ring k. The CDF and PDF of γ_k can then be written as follows:

$$F_{\gamma_k}(\gamma_k) = \prod_{i=1}^{u_k} F_{\zeta_i}(\bar{K}^{-1}\gamma_k r_k^{\beta}) \overset{\text{i.i.d}}{=} \left(F_{\zeta}(\gamma_k r_k^{\beta}\bar{K}^{-1})\right)^{u_k} \tag{7}$$

$$f_{\gamma_k}(\gamma_k) = \frac{1}{r_k^{-\beta}} \sum_{j=1}^{u_k} f_{\zeta_j}(\gamma_k r_k^{\beta}\bar{K}^{-1}) \prod_{i=1, i\neq j}^{u_k} F_{\zeta_i}(\gamma_k r_k^{\beta}\bar{K}^{-1}) \overset{\text{i.i.d}}{=} \frac{u_k}{r_k^{-\beta}} f_{\zeta}(\gamma_k r_k^{\beta}\bar{K}^{-1}) \left(F_{\zeta}(\gamma_k r_k^{\beta}\bar{K}^{-1})\right)^{u_k-1} \tag{8}$$

Step 2 (*Selecting the user with maximum SNR among K rings***)**: In this step, we compute the probability of selecting the k^{th} ring among all other rings. It is important to note that this is equivalent to selecting the ring k which possesses the user with the highest SNR among all rings. Conditioning on γ_k, the PDF of r_{sel} can be written explicitly as follows:

$$P(r_{\text{sel}} = r_k|\gamma_k) = \prod_{i=1, i\neq k}^{K} p(\gamma_i \leq \gamma_k) = \prod_{i=1, i\neq k}^{K} F_{\gamma_i}(\gamma_k) \tag{9}$$

By averaging over the distribution of γ_k, the final expression for the PMF of r_{sel} is

$$P(r_{\text{sel}} = r_k) = \int_0^{\infty} \left(\prod_{i=1, i\neq k}^{K} F_{\gamma_i}(\gamma_k) \right) f_{\gamma_k}(\gamma_k) d\gamma_k \tag{10}$$

Using (7), (10) can be written for i.i.d. case as follows:

$$P(r_{\text{sel}} = r_k) = \int_0^{\infty} \prod_{i=1, i\neq k}^{K} \left(F_{\zeta}(\gamma_k r_i^{\beta})\right)^{u_i} \frac{u_k f_{\zeta}(\gamma_k r_k^{\beta})}{r_k^{-\beta}} \left(F_{\zeta}(\gamma_k r_k^{\beta})\right)^{u_k-1} \tag{11}$$

where $r_{\text{sel}} \in [0, R]$. The results in (11) are generalized for any shadowing and fading statistics. Even though (11) is not a closed form expression, the integration can be solved accurately using standard mathematical software packages such as `MAPLE` and `MATHEMATICA`.

B. Proportional Fair Scheduling Scheme

The proportional fair scheduling scheme allocates the subcarrier to the user with the largest normalized SNR ($\gamma/\bar{\gamma}$) [12], where γ and $\bar{\gamma}$ denote the instantaneous SNR and the short term average SNR of a given user, respectively. In other words, the selection criterion is based on selecting a user who has maximum instantaneous SNR relative to its own average SNR. The distribution of r_{sel} can be derived as follows:

Step 1 (*Selecting the user with maximum normalized SNR in ring k***)**: In this step, the performance of proportional fair scheduling scheme is independent of the path loss factor if users are moving relatively