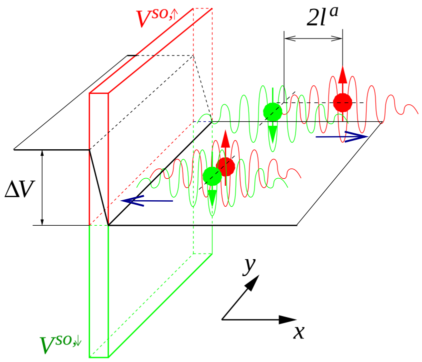

FIG. 1: (Color online) The edge's confinement potential of depth ΔV leads, via the SO coupling (Eq. 3) to attractive potential ($V^{SO,\downarrow}$, in green) for electrons with spin down in the vicinity of the edge. The initially compensated spins of electrons approaching the edge becomes locally un-compensated due to the difference in the scattering shift l^a.

wavepacket will give rise to nonzero contributions (the reflected part will be extremely small due to the quickly oscillating factors) and the wavepacket will be approaching the edge (Fig. 1, moving to the left). On the other hand, for large positive times only the reflected wave will give nonzero contribution. The position and form of the reflected wavepacket will depend on the spin of the incoming wavepacket since the reflection amplitude, $r^{q,\sigma}(k)$ will be different for the two possible spin orientations. Specifically, it is shown in the Appendix A that the reflection amplitude has the form $r^{q,\sigma}(k) = e^{i\theta_k^{q,\sigma}}$ where the phase shift up to 3rd order in α_E is

$$\begin{aligned}
\theta_k^{q,\uparrow/\downarrow} &= \pi + 2\mathrm{atan}\frac{k}{\kappa} \mp 2k\alpha_E q + 2k\kappa\alpha_E^2 q^2 \\
&\mp 4\alpha_E^3 q^3 k(\Delta V - \frac{2}{3}k^2), \qquad (5)
\end{aligned}$$

where $\kappa = \sqrt{2\Delta V - k^2}$. Assuming the width Δk of the wavepacket $\psi_{k,\Delta k}^{q,\sigma}(t)$ small, one can easily find that the probability density of the reflected wavepackets with the phase-shift given by Eq. 5 for large times will take a form

$$|\psi_{k,\Delta k}^{q,\sigma}(t)|^2 \approx \frac{1}{\pi}\frac{\sin^2(x(t)\Delta k/2)}{x(t)^2\Delta k/2} \qquad (6)$$

where $x(t) = x + d\theta^{q,\sigma}/dk - kt$, i.e. the maximum of the probability is at $x = kt - d\theta^{q,\sigma}/dk$. From this it is evident that the electron with spin down will be lagging behind that with spin up by a distance $2l^a = d\theta^{q,\uparrow}/dk - d\theta^{q,\downarrow}/dk$ (indicated for the reflected right-going wavepackets in Fig. 1). This behavior of the scattering of single electron in the wavepacket can be directly extended to many-electrons in view of the stroboscopic wavepacket basis[22]. Namely, the above wavepackets for times $t_m = t + 2\pi m/(k\Delta k)$, $m = 0, \pm 1, \pm 2, ...$, where t is arbitrary moment of physical time, form orthogonal set of states which are all occupied with electrons. Hence the product of the length $2l^a$ times the number of electrons per area must give an estimate of the spin-polarization per unit length of the edge, as given in our previous work. Here we will show that this physically motivated estimate is in fact a rigorous result that we formally prove in the following, directly using the eigenstates of the Hamiltonian of the studied system.

The spin polarization *per unit length of the edge* is given in terms of the spin-resolved density as

$$m_z = \int_{-\infty}^{+\infty} dx \int \frac{dq}{2\pi}\int\frac{dk}{2\pi}(n_\uparrow^{k,q}(x) - n_\downarrow^{k,q}(x)), \qquad (7)$$

where the integration in q and k goes over all the occupied eigenstates and $n_\sigma^{k,q}(x)$ is the contribution to the density at position x, y from an occupied eigenstate

$$n_\sigma^{k,q}(x) = \left|e^{iqy}\left(e^{-ikx} + r^{q,\sigma}(k)e^{ikx}\right)\right|^2 = 2 + 2\Re\left\{e^{i(2kx+kl^{q,\sigma})}\right\}. \qquad (8)$$

Since the spatial shift $l^{q,\sigma}$ is very small (given by the strength of the SO interaction), we can approximate the expression for the spin density to the first order

$$\begin{aligned}
n_\sigma^{k,q}(x) &= n_\sigma^{k,q}(x)\Big|_{l=0} + \frac{d}{dl}n_\sigma^{k,q}(x)\Big|_{l=0} l^{q,\sigma}, \qquad (9)\\
&= n_\sigma^{k,q}(x)\Big|_{l=0} + \frac{1}{2}\frac{d}{dx}n_\sigma^{k,q}(x)\Big|_{l=0} l^{q,\sigma} \qquad (10)
\end{aligned}$$

where we have interchanged the differentiation in view of the functional dependence of the density, Eq. 8 on x and $l^{q,\sigma}$. Hence, using Eq. 7 and Eq. 10 we obtain for the spin polarization

$$m_z = \int\frac{dq}{2\pi}\int\frac{dk}{2\pi}\, n_\uparrow^{k,q}(x)\Big|_{x=-\infty}^{x=+\infty} l_a^q, \qquad (11)$$

where we have introduced the antisymmetric part of the spatial shift

$$l_a^q = \frac{\theta^{q,\uparrow} - \theta^{q,\downarrow}}{2k} = -2\alpha_E q - 4\alpha_E^3 q^3(\Delta V - \frac{2}{3}k^2). \qquad (12)$$

The density at $x = -\infty$ is exponentially small, where as the contribution in the bulk of the 2DEG is according to Eq. 8 equal to 2.

Including higher order terms in Eq. 10 would result in appearance of contributions with higher spatial derivatives of the density at $x = \pm\infty$. There, however, is the density constant (either zero for $x = -\infty$ or the homogeneous 2DEG value for $x = +\infty$) so that the expression for the spin polarization, Eq. 11 is correct also for large values of $l^{q,\sigma}$ in principle.

To complete the derivation we need to integrate over the occupied states. The simplest way is to assume that the non-equilibrium distribution is well described by a Fermi sphere with the radius (Fermi momentum) k_F and the corresponding Fermi energy $E_F = k_F^2/2$ in 2D, shifted by a drift momentum q_d in the $+y$ direction. Such a model corresponds to the distribution function maximizing the information entropy for a fixed average energy, density and current[28,29], and is also supported by Boltzmann-equation based analysis[30]. In the Appendix B we discuss the results obtained using the non-equilibrium model with two Fermi energies, conventional