

In region II, we represent the long-range (in  $R$ , at fixed  $r$  and  $\gamma$ ) potential  $V_{LR}$  for the interaction between  $\text{H}_2$  and  $\text{H}^-$  as [51]:

$$V_{LR}(R; r, \gamma) = D_{as}(r) + \frac{C_3^{\text{th}}}{R^3} + \frac{C_4^{\text{th}}}{R^4} \text{ with} \quad (1)$$

$$C_3^{\text{th}} = -Q(r)P_2(\cos \gamma) \text{ and } C_4^{\text{th}} = -[\alpha_0(r) + \alpha_2(r)P_2(\cos \gamma)]/2,$$

where the first term  $D_{as}(r)$  is the sum of  $\text{H}^-$  and  $\text{H}_2(r)$  energies at a given internuclear distance  $r$  of the  $\text{H}_2$  molecule. The second term is the interaction between the electric charge of  $\text{H}^-$  with the quadrupole moment  $Q(r)$  of  $\text{H}_2$  (taken from Ref. [52]), and the third term is the interaction of the dipole moment of  $\text{H}_2$  induced by  $\text{H}^-$ , involving the second order Legendre polynomial  $P_2(\cos \gamma)$ . The functions  $\alpha_0(r)$  and  $\alpha_2(r)$  are the isotropic and anisotropic polarizabilities of  $\text{H}_2$ , for which we used the analytical functions given in Ref. [11] that were obtained by fitting the numerical values from Ref. [53]. The dispersion energy varying as  $1/R^6$  and other smaller terms are neglected, which is a good approximation because the long-range expansion is only used for  $R > 56.4$  a.u.

In region I we used the following extrapolation formula in  $r$  for fixed  $R$  and  $\gamma$ :

$$V_{SR}(r; R, \gamma) = a(R, \gamma)e^{-b(R, \gamma)r}, \quad (2)$$

where  $a(R, \gamma)$  and  $b(R, \gamma)$  are functions of  $R$  and  $\gamma$  that are obtained considering the two *ab initio* energies  $V(r = 0.8; R, \gamma)$  and  $V(r = 1; R, \gamma)$  calculated at first two values of the coordinate  $r$ . In this way, we obtain the quantities  $a$  and  $b$  given on a two-dimensional grid of points in the  $(R, \gamma)$  space. Then we used the 2D B-spline interpolation to obtain smooth two-dimensional functions  $a(R, \gamma)$  and  $b(R, \gamma)$ .

In region III, we extrapolate the PES in  $r$  and at fixed  $R$  and  $\gamma$  using a dispersion-like expression:

$$V_{LR}(r; R, \gamma) = D_0(R, \gamma) - \frac{C_6(R, \gamma)}{r^6}, \quad (3)$$

where the  $D_0(R, \gamma)$  and  $C_6(R, \gamma)$  (always positive) coefficients are obtained in a way similar to the coefficients  $a$  and  $b$ , considering the two last points  $V(r = 2.2 \text{ a.u.}; R, \gamma)$  and  $V(r = 2.4 \text{ a.u.}; R, \gamma)$ . They are also interpolated using the 2D B-spline method for arbitrary values of  $R$  and  $\gamma$ .

The behavior of the PES in region IV is described by the short-range (in  $R$ ) repulsive expression at given values of  $r$  and  $\gamma$ :

$$V_{SR}(R; r, \gamma) = A(r, \gamma)e^{-B(r, \gamma)R}, \quad (4)$$