

$\mathfrak{G} = 0$, the number of independent matrices reduces to three). The orthogonality $(b^{(2)}b^{(3)}) = 0$ is explicit in (10). The Lorentz-invariant coefficients $\Lambda_{1,2,3,4}$ are functions of the background fields and momenta. Expressions for them as simple linear superpositions of the components $\Pi_{\mu\nu}$,

$$\Lambda_1 = \frac{(kF^2)_\mu \Pi_{\mu\nu} (F^2 k)_\nu}{(\mathcal{B}^2 + \mathcal{E}^2)(k^2 \mathcal{E}^2 - kF^2 k)}, \quad \Lambda_2 = -c_\mu^- \Pi_{\mu\nu} c_\nu^-, \quad \Lambda_3 = -c_\mu^- \Pi_{\mu\nu} c_\nu^+, \quad \Lambda_4 = -c_\mu^+ \Pi_{\mu\nu} c_\nu^+ \quad (13)$$

are obtained in Appendix 1 from a less transparent representation to be found in [4, 5]; their calculations in one-loop approximation of QED are given in [4, 5].

The transparency domain of momenta is such a region where absorption is absent. The electron-positron pair production by a photon is an example of absorption. The region, where it is kinematically allowed, is not the transparency domain. The absence of absorption of small perturbation of the background field is reflected in the property of Hermiticity [13] of the matrix $\Pi_{\mu\nu}$. It is symmetric when the charge conjugation invariance holds [5, 13] (no charge-asymmetric plasma background, no spontaneous pair creation). Hence, in the transparency region all the components of $\Pi_{\mu\nu}$ are real in the case under consideration, once the charge conjugation invariance is assumed. Then, all Λ 's defined by (13) are also real there, except the region $k_3^2 - k_0^2 > 0$ (or, in invariant terms, $k^2 \mathcal{B}^2 + kF^2 k > 0$) wherein Λ_3 becomes imaginary due to (11). (We shall see later that dispersion curves cannot get into this region without violating the stability). In this exceptional region the quantity under the square root in (10), (12) stops being manifestly positive. Nevertheless, it should remain nonnegative, since eigenvalues of a Hermitian matrix should be real.

The dispersion equations that define the mass shells of the three eigenmodes are

$$\varkappa_a(k\tilde{F}^2 k, kF^2 k, \mathfrak{F}, \mathfrak{G}^2) = k^2, \quad a = 1, 2, 3. \quad (14)$$

We have explicitly indicated here that the eigenvalues should be even functions of the pseudoscalar \mathfrak{G} .

When, due to a certain reason, Λ_3 is small as compared to $|\Lambda_2 - \Lambda_4|$, the small mixing of eigenmodes is obtained by expanding (10) in powers of $\Lambda_3/|\Lambda_2 - \Lambda_4|$. In this way we get, with the linear accuracy in Λ_3 , after normalizing out the common factors 2 and $2\Lambda_3/(\Lambda_2 - \Lambda_4)$

$$b_\mu^{(2)} = -\Lambda_3 c_\mu^- + (\Lambda_2 - \Lambda_4) c_\mu^+, \quad b_\mu^{(3)} = (\Lambda_2 - \Lambda_4) c_\mu^- + \Lambda_3 c_\mu^+. \quad (15)$$

Such situation occurs, first of all, when the electric field is small as compared to the magnetic