$$\mathcal{P}(t_1,t_2;z)=\int_{\mathcal{A}_1}d\rho_1\int_{\mathcal{A}_2}d\rho_2\langle\mathcal{Q}|\hat{\psi}^\dagger(\rho_2,t_2)\hat{\psi}^\dagger(\rho_1,t_1)\hat{\psi}(\rho_1,t_1)\hat{\psi}(\rho_2,t_2)|\mathcal{Q}\rangle, \tag{50}$$

where $\mathcal{A}_1$ and $\mathcal{A}_2$ are two areas in the transverse (x,y) plane surrounding waveguides W_1 and S, respectively. The correlation function $\mathcal{P}(t_1,t_2)$ is proportional to the joint probability of detecting one photon in waveguide W_1 at time t_1, and one photon in waveguide S at time t_2, after a propagation distance z from the input plane. In fact, $\langle\mathcal{Q}|\hat{\psi}^\dagger(\rho_2,t_2)\hat{\psi}^\dagger(\rho_1,t_1)\hat{\psi}(\rho_1,t_1)\hat{\psi}(\rho_2,t_2)|\mathcal{Q}\rangle$ is proportional to the joint probability of detecting one photon at point ρ_1 and time t_1, and one photon at point ρ_2 and time t_2 at the same propagation distance z. The integral over the areas $\mathcal{A}_1$ and $\mathcal{A}_2$ thus gives the joint probability of detecting one photon in waveguide W_1 at time t_1, and one photon in waveguide S at time t_2. For photons generated by spontaneous parametric down conversion in type-I nonlinear crystal using a monochromatic pump beam at frequency $\omega_p=2\omega$, the two-photon spectrum $C(\Omega_1,\Omega_2)$ of a pair of signal and idler photons can be expressed as [18, 34]

$$C(\Omega_1,\Omega_2)=\delta(\Omega_1+\Omega_2)G(\Omega_1,\Omega_2)\exp\left[i\frac{\delta(\Omega_1-\Omega_2)}{2}\right] \tag{51}$$

where $G(\Omega_1,\Omega_2)$ is the phase matching function. The last exponential term on the right hand side in Eq.(51) has been introduced to account for a possible time delay δ between signal and idler wave packets introduced by different optical paths from the crystal to waveguides W_1 and W_2. The phase matching function $G(\Omega_1,\Omega_2)$ is assumed to be a real-valued and symmetric function [i.e. $G(\Omega_1,\Omega_2)=G(\Omega_2,\Omega_1)$], peaked at around $\Omega_1=\Omega_2=0$, with e.g. a Gaussian profile [18]. Taking into account that $\hat{\psi}(\rho,t)=(2\pi)^{-1/2}\int d\Omega\hat{\phi}(\rho,\Omega)\exp(-i\Omega t)$, substitution of Eqs.(48) and (51) into Eq.(50), using the commutaion relations $[\hat{\phi}(\rho,\Omega),\hat{a}^\dagger_{1,2}(\Omega')]=u_{1,2}(\rho)\delta(\Omega-\Omega')$, $[\hat{\phi}(\rho,\Omega),\hat{a}^\dagger_3(\Omega')]=\theta(\rho,z)\delta(\Omega-\Omega')$ and the relations $\int_{\mathcal{A}_1}d\rho|u_1(\rho)|^2\simeq 1$, $\int_{\mathcal{A}_2}d\rho|\theta(\rho,z)|^2\simeq 1$, $\int_{\mathcal{A}_{1,2}}d\rho|u_{2,1}(\rho)|^2=\int_{\mathcal{A}_1}d\rho|\theta(\rho,z)|^2=\int_{\mathcal{A}_2}d\rho|u_2(\rho)|^2\simeq 0$, after some lengthy but straightforward calculations one obtains

$$\mathcal{P}(t_1,t_2;z)=|r(\tau+\delta)S_{11}(z)S_{23}(z)+r(\tau-\delta)S_{13}(z)S_{21}(z)|^2 \tag{52}$$

where $\tau=t_2-t_1$ and where we introduced the real-valued correlation function $r(\tau)$ defined by

$$r(\tau)=\frac{1}{2\pi}\int d\Omega G(\Omega,-\Omega)\exp(-i\Omega\tau). \tag{53}$$

In practice, coincidence measurements correspond to an integration of $\mathcal{P}(t_1,t_2;z)$ with respect to the time difference $\tau=t_2-t_1$ over the resolving coincidence time, which is typically much longer than the correlation time τ_c of $g(\tau)$. Integrating Eq.(52) with respect to τ from $-\infty$ to ∞ and taking into account that $S_{21}=S_{12}$ and $|S_{23}|^2=|S_{13}|^2=1-|S_{11}|^2-|S_{12}|^2$, the following expression for the correlation function $\mathcal{P}$ versus time delay δ is finally obtained

$$\mathcal{P}(\delta;z)=\alpha\left[1-|S_{11}(z)|^2-|S_{12}(z)|^2\right]\left[|S_{11}(z)|^2+|S_{21}(z)|^2+\right.$$
$$\left.+2\mathrm{Re}\left[S_{11}(z)S^*_{21}(z)\right]\frac{\int_{-\infty}^{\infty}d\tau r(\tau-\delta)r(\tau+\delta)}{\int_{-\infty}^{\infty}d\tau r^2(\tau)}\right] \tag{54}$$

where $\alpha=\int_{-\infty}^{\infty}d\tau r^2(\tau)$ and the expression of the coefficients $S_{11}(z)$ and $S_{12}(z)$ are given by Eqs.(12) and (13). The behavior of $\mathcal{P}(\delta;z)$ versus δ shows a characteristic dip at $\delta=0$ of width $\sim\tau_c$, which is analogous to the Hong-Ou-Mandel dip observed in two-photon interference from a beam splitter [18]. Far from the dip, $\mathcal{P}(\delta)$