The form of the corresponding terms is common to that for FM state, see Eq. (3), and we do not present this long expression here.

The translation symmetry within any sublattice allows us to introduce magnon creation and annihilation operators for magnetic oscillations through the Bose operators for each of the sublattices

$$a_{\vec{\delta}} = \frac{1}{\sqrt{N}} \sum_{\vec{q}} a_{\vec{q}} e^{ia\vec{q}\vec{\delta}}, \quad b_{\vec{\mu}} = \frac{1}{\sqrt{N}} \sum_{\vec{q}} b_{\vec{q}} e^{ia\vec{q}\vec{\mu}}, \tag{16}$$

where, naturally, the quasi-momentum $\vec{q}$ takes values within the first Brillouin zone of the sublattice (see Fig. 4). In the basis of the sublattices the vector $\vec{q} = q_1\vec{\varepsilon}_1 + q_2\vec{\varepsilon}_2$, where $|q_{1,2}| \leq \pi/a\sqrt{2}$, and in the basis of the basic lattice the vector $\vec{q} = q_x\vec{e}_x + q_y\vec{e}_y$, where $q_x = (q_1{+}q_2)/\sqrt{2}$, $q_y = q_x = (q_2{-}q_1)/\sqrt{2}$, $|q_{x,y}| \leq \pi/a$. After this transform, the Hamiltonian acquires the standard form $\hat{H} = \sum_{\vec{q}} \hat{H}_{\vec{q}}$, where

$$\begin{aligned} \frac{\hat{H}_{\vec{q}}}{2\mu_B M} = (A_{\vec{q}} + h)a^{\dagger}_{\vec{q}}a_{\vec{q}} + (A_{\vec{q}} - h)b^{\dagger}_{\vec{q}}b_{\vec{q}} - \\ - \left[\frac{C_{\vec{q}}}{2}\left(a^{\dagger}_{\vec{q}}a^{\dagger}_{-\vec{q}} + b_{\vec{q}}b_{-\vec{q}}\right) + D_{\vec{q}}a^{\dagger}_{\vec{q}}b_{\vec{q}} + F_{\vec{q}}a_{\vec{q}}b_{-\vec{q}} + h.c.\right], \end{aligned} \tag{17}$$

where the following notation are used

$$\begin{aligned} A_{\vec{q}} = \sum_{\vec{\mu}} \frac{1}{|\vec{\mu}|^3} - \sum_{\vec{\delta}\neq 0} \frac{1}{|\vec{\delta}|^3} - \frac{1}{2}\sum_{\vec{\delta}\neq 0} \frac{e^{ia\vec{q}\vec{\delta}}}{|\vec{\delta}|^3} + \beta\,, \\ C_{\vec{q}} = \frac{3}{2}\sum_{\vec{\delta}\neq 0} \frac{\left(\delta_x + i\delta_y\right)^2 e^{ia\vec{q}\vec{\delta}}}{|\vec{\delta}|^5}\,, \\ D_{\vec{q}} = \frac{3}{2}\sum_{\vec{\mu}} \frac{\left(\mu_x + i\mu_y\right)^2 e^{ia\vec{q}\vec{\mu}}}{|\vec{\mu}|^5}\,,\ F_{\vec{q}} = \frac{1}{2}\sum_{\vec{\mu}} \frac{e^{ia\vec{q}\vec{\mu}}}{|\vec{\mu}|^3}. \end{aligned} \tag{18}$$

There is a simple connection between the sums over the sublattices and the previously introduced sums over the whole lattice $\sigma(\vec{k})$ and $\sigma_{\rm c}(\vec{k})$, see Eq. (8). For example, a simple geometrical transformation gives

$$\sum_{\vec{\delta}\neq 0} \frac{e^{ia\vec{q}\vec{\delta}}}{|\vec{\delta}|^3} = \frac{1}{2^{3/2}}\sigma\left(\tilde{\vec{q}}\right),$$

where we introduced the vector $\tilde{\vec{q}} = \sqrt{2}(q_1\vec{e}_x + q_2\vec{e}_y)$, which is derived from the vector $\vec{q}$ by rotation by the angle $\pi/4$ and by stretching by the value $\sqrt{2}$ (see Fig. 4). It is easy to see that for the $\vec{q}$ values within the first Brillouin zone of the sublattice, the corresponding