

complicated than the weak Bruhat order on Schubert varieties. We refer to known results on these combinatorics ([MØ90, MT09, RS90, Yam97]). Although these parametrizations are likely familiar to experts on the representation theory of real groups, proofs of their correctness have not appeared in the literature in all cases. As such, we have written down the details here for lack of a suitable reference.

One application of our formulas is that they allow one to deduce Chern class formulas for varieties analogous to the degeneracy loci considered by Fulton. In general, such loci involve a vector bundle  $V$  on a scheme  $X$  equipped with a complete flag of subbundles and a further structure determined by  $K$ . Given such a setup, degeneracy loci can be defined by conditions on the “relative position” of the flag and the extra structure over various points of  $X$ . In the type  $A$  cases, this extra structure is either a splitting as a direct sum of subbundles of ranks  $p$  and  $q$  (corresponding to  $K = S(GL(p, \mathbb{C}) \times GL(q, \mathbb{C}))$ ), or a non-degenerate bilinear form taking values in the trivial bundle. The form is symmetric in the case of  $K = SO(n, \mathbb{C})$ , and skew-symmetric in the case of  $K = Sp(2n, \mathbb{C})$ .  $K$ -orbit closures are universal cases of such loci, in exactly the same way that Schubert varieties are universal cases of the degeneracy loci studied by Fulton. We describe the dictionary between these two viewpoints explicitly in the type  $A$  cases, and indicate our thoughts on how this should extend to cases where  $G$  is of type  $BCD$ .

After giving preliminary background in Chapter 1, we treat the various examples in types  $ABCD$  in Chapters 2, 3, 4, and 5, respectively. Each of these chapters is organized as follows: For each symmetric pair  $(G, K)$ , we realize  $K$  explicitly as a subgroup of  $G$ , and describe the corresponding embeddings of Weyl groups and root systems. We then identify the closed orbits explicitly — their number, and the fixed points contained in each. Using this information, we determine formulas for each closed  $K$ -orbit using equivariant localization as described above. The identification of the closed orbits is straightforward in the cases where  $K$  is connected, but we do deal with some cases where  $K$  is disconnected. In those