A Proofs

In order to prove the STP of the rejection threshold, we first need to understand the the *scaled inverse rejection process* $L(t) = t(\max\{R(t), 1\})^{-1}$ with $t \in [0,1]$, with which the regularity of $\tilde{\pi}_0(\lambda)$, the property of the stopped backward filtration $\mathcal{G}$ and a contrapositive argument will yield Theorem 1. In what follows, no assumption will be made about the independence between the p-values, the continuity of the p-value distributions, or their stochastic orders wrt the standard uniform distribution.

A.1 Downward jumps of the scaled inverse rejection process

Order the p-values into $p_{(1)} < p_{(2)} < ... < p_{(n)}$ distinctly, where the multiplicity of $p_{(i)}$ is n_i for $i = 1, ..., n$. Let $p_{(n+1)} = \max\{p_{(n)}, 1\}$ and $p_{(0)} = 0$. Define $T_j = \sum_{l=1}^{j} n_l$ for $j = 1, ..., n$.

Lemma A.1. *The process $\{L(t), t \in [0,1]\}$ is such that*

$$L(t) = \begin{cases} t & \text{if} \quad t \in [0, p_{(1)}), \\ tT_j^{-1} & \text{if} \quad t \in [p_{(j)}, p_{(j+1)}) \text{ for } j = 1, .., n-1, \\ tm^{-1} & \text{if} \quad t \in [p_{(n)}, p_{(n+1)}]. \end{cases} \tag{A.1}$$

Moreover, it can only be discontinuous at $p_{(i)}$, $1 \le i \le n$, where it can only have a downward jump with size

$$L(p_{(i)}-) - L(p_{(i)}) = \frac{p_{(i)} n_i}{R(p_{(i)})\left[R(p_{(i)}) - n_i\right]} > 0.$$

Proof. Clearly

$$R(t) = \begin{cases} 0 & \text{if} \quad 0 \le t < p_{(1)}, \\ T_j & \text{if} \quad p_{(j)} \le t < p_{(j+1)}, j = 1, .., n-1, \\ m & \text{if} \quad p_{(n)} \le t \le p_{(n+1)}, \end{cases}$$

and (A.1) holds. Therefore, the points of discontinuities of $L(\cdot)$ are the original distinct p-values. This justifies the first part of the assertion.