mean-field, linear stability, perturbative and numerical analyses, and found that if an angular perturbation is large in magnitude and highly localized in space, it will be amplified and thus serves as an indication of the onset of collective motion. Our calculations also demonstrate the importance of particle speed for collection motion transition. As a result, it is indicative that the critical point for CM may depend on the speed u, the perturbation magnitude b and the perturbation wavelength σ. This is in contrast to the mean-field and linear stability analyses where only the hydrodynamic, or infinite-wavelength, mode dictates the onset of CM. Our results therefore highlights the importance of incorporating the nonlinear term into the analysis.

The main limitation of this work is on the approximation adopted – the omissions of higher order modes. While we believe that such an approximation is appropriate at the onset of CM, it would be highly desirable to have a systematic method to incorporate the higher order modes into the dynamics. Besides the consideration of the higher order modes, singular perturbation method would also be needed to investigate the long-time behaviour of the system [22]. We believe that these aspects would constitute two promising directions for future investigation.

Appendix

We are unable to solve the set of differential equations shown in Eqs (41) to (43) analytically. But since only the leading orders in x and t are of interests, we will replace the $U^{\pm}$ in $\alpha_0, \beta_0, \gamma_0$ (c.f. Eqs (37) to (39)) by

$$\tilde{U}^{\pm} \equiv 1 - \frac{(x \pm t)^2}{2\sigma^2} + \left[\frac{(x \pm t)^2}{2\sigma^2}\right]^2 \ . \qquad \text{(A.1)}$$

With this simplification, the differential equations can be solved by the Laplace-Fourier Transform method and the relevant results are:

$$\alpha_1(t, y=0) = \mathcal{O}(t^3) \quad , \quad \gamma_1(t, y=0) = \frac{g\xi^2}{\pi\sigma^2} t + \mathcal{O}(t^3) \ . \qquad \text{(A.2)}$$

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