A. Size-related resonances and local-field patterns

Locsitons in a finite 1D array of atoms exhibit size-related resonances, characterized by large increases in their amplitudes at certain frequencies within the locsitonic band, because locsitons are reflected at the boundaries and form standing waves (*strata*) [1, 2]. Essentially, these resonances correspond to locsiton eigenmodes defined by the boundaries. For the *long-wave strata* they are similar to oscillations of a quantum particle in a box, as, e. g., for 1D-confined electrons [10, 11], or a common violin string. It is natural to expect such resonances and eigenmodes to also exist in higher dimensions, in particular, in finite 2D lattices, where we also encounter locsiton reflections at the boundaries. An important distinction of the 2D case is that the wave vector $\mathbf{q}$ of a locsiton may have an arbitrary orientation in the lattice plane with respect to the incident field $\mathbf{E}_{\text{in}}$. Multiple reflections and interference of locsitons with all possible $\mathbf{q}$ quickly make the whole picture very complicated and highly susceptible to minor changes to the size and shape of the lattice patch. We found that at certain geometries only a limited number of locsiton eigenmodes are dominant. Their interference produces various dipole patterns and strata; some of them are reminiscent to "quantum carpets" [12]. An important issue is, therefore, how one can control the locsiton patterns via the geometry of the lattice patch and the frequency and polarization of the laser beam.

One way to engineer a distinct 2D locsiton pattern is to start with a rectangular lattice patch and ensure that size-related resonances are achieved for locsitons with wave vectors parallel to its boundaries. We have to choose the lattice shape, such that the size-related locsiton resonances emerge in both dimensions at the same frequency detuning δ. To simplify our task, we will consider long-wavelength locsitons, which are not too sensitive to the system sizes and thus are easier to control, and, incidentally, also form more pronounced patterns and are described by the simpler formula (18).

In the limit of long-wavelength locsitons ($q \ll 1$, $\delta \approx \delta_{\text{LL}}$), the dispersion relations in the cases (a) and (b) described in Sec. III B coincide with each other:

$$q^2 = 8\frac{(\delta - \delta_{\text{LL}}) + i}{\delta_{\text{LL}}} \tag{29}$$

[cf. Eq. (18) at $\psi = \pi/2$]. In a similar manner, one obtains approximate solutions for the