The kinetics of the signal v in model II is represented as follows,

model II

$$\frac{dv(t)}{dt} = \mathrm{g}_2(u_1, \ldots, u_N, v) = c_2 \sum_{i=1}^{N} \frac{u_i^\beta(t)}{K_v^\beta + u_i^\beta(t)} - v(t). \tag{7}$$

We here adopt Hill-type kinetics for the induction of the signal v by u_i, where β is the Hill coefficient, representing the cooperativity in the induction, and K_v denotes the threshold value for the signal induction. The parameter c_2 gives the release rate of v from each cell.

Dependence of the stationary states on the total cell number is shown in Fig. 4. For a small N, all the cells always fall on a single-cluster state of $u_{(1)}$. As N gets larger, the bifurcation to a two-cluster state occurs, where the cells take either $u_{(1)}$ or $u_{(2)}$. Here, the single-cluster state of $u_{(1)}$ ($u_{(2)}$) is realized only at a small (large) number of cells, respectively, so that there is a gap in the total number of cells between the two single-cluster states. The two-cluster state exists within this gap.

To understand the observed dependence of the clustering behavior on the cell number, we first consider the stability of a single-cluster state. From $du_i/dt = 0$, $dv/dt = 0$, and $u_i = u_{(k)}$ $(k = 1,\ \text{or}\ 2)$ for $i = 1, \ldots, N$, we get

$$f(u_{(k)}, v) = 0, \qquad v = c_2 N \frac{u_{(k)}^\beta}{K_v^\beta + u_{(k)}^\beta}. \tag{8}$$

By solving the above equations self-consistently, the solution curve of u is obtained as a function of the total cell number N (Fig. 5). For $N < \tilde{N}_1^*$, a single-cluster state of $u_{(1)}$ is always stable. When the cell number increases beyond $\tilde{N}_1^*$, this single-cluster state becomes unstable, while for much larger N such that $N > \tilde{N}_2^*$, the single-cluster state becomes stable again, where the cell state is $u_{(2)}$ (Fig. 5) . The threshold $\tilde{N}_1^*$ and $\tilde{N}_2^*$ are given by $\tilde{N}_1^* = v_2^*(K_v^\beta + u_{(1)}^\beta(v_2^*))/(c_2 u_{(1)}^\beta(v_2^*))$ and $\tilde{N}_2^* = v_1^*(K_v^\beta + u_{(2)}^\beta(v_1^*))/(c_2 u_{(2)}^\beta(v_1^*))$,