where ΔP is the pressure jump at the interface and $\mathcal{K}$ is the interface curvature.

Due to the smallness of the film thickness, h_c, in the lubrication limit only the in-plane components of the velocity field, v_x and v_y, are considered, and velocity gradients are assumed to occur primarily in the perpendicular direction, z. Additional assumptions are that the flow takes place at small velocities and that it is stationary. Hence, the left hand side of equation (2) can be neglected. Following these assumptions, it is possible to average the flow field in the z direction to give

$$\langle \mathbf{v} \rangle = -\frac{h^2}{3\eta_{+1}}(\gamma \boldsymbol{\nabla} \nabla^2 h - \rho \mathbf{g}), \tag{6}$$

where $\langle \mathbf{v} \rangle(x, y)$ is the two dimensional average velocity field, and $h(x, y)$ is the local thickness of the film. In this equation, the major contribution to the local pressure is capillary, $P = -\gamma \mathcal{K} \simeq -\gamma \nabla^2 h$.

In this limit, it follows that the coordinates, velocity, and time in equation (6) can be rescaled as

$$\begin{gathered} x^* = \frac{x}{x_c}, \qquad y^* = \frac{y}{x_c}, \qquad (z^*, h^*) = \left(\frac{z}{h_c}, \frac{h}{h_c} \right), \\ \langle \mathbf{v} \rangle^* = \frac{\langle \mathbf{v} \rangle}{U}, \qquad \text{and} \qquad \frac{t}{t_c}, \end{gathered} \tag{7}$$

using units $x_c = h_c(3Ca)^{-\frac{1}{3}}$, $U = h_c^2 \rho g_x/(3\eta_{+1})$ and $t_c = x_c U^{-1}$. In these expressions, the capillary number, Ca, measures the ratio between viscous and capillary forces, and is defined as $Ca = \eta_{+1} U/\gamma$.

D. Lattice-Boltzmann algorithm

To integrate equations (1), (2) and (3) numerically, we use LUDWIG, a lattice-Boltzmann parallel implementation for binary fluids [28]. Space is represented by a lattice of nodes that are connected by links. The set of links determines a set of velocity vectors $\{\mathbf{c}_i\}$. Here we choose the cubic lattice D3Q19 model for the velocity set $\{\mathbf{c}_i\}$, which consists of nineteen velocity vectors (eighteen pointing towards nearest, next nearest, and next to next nearest neighbors) and one accounting for rest particles. The lattice-Boltzmann method consists of the integration of two linearized discrete Boltzmann equations

$$f_i(\mathbf{r} + \mathbf{c}_i, t + 1) - f_i(\mathbf{r}, t) = \frac{1}{\tau_f} \left(f_i(\mathbf{r}, t) - f_i^{eq}(\mathbf{r}, t) + F_i^f \right), \tag{8}$$