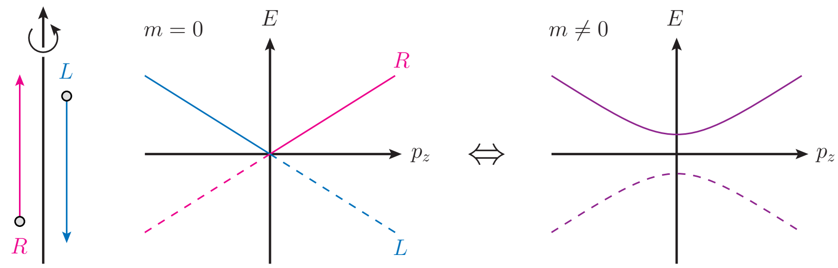

FIG. 1: Low-energy spectrum of fermions localized around the vortex line in the chiral limit (left panel) and with the nonzero fermion mass for the even parity pairing (right panel). The $E < 0$ part of the spectrum is redundant in the superconductor and thus shown by the dashed line.

B. Spectrum of fermions localized around a vortex line

The spectrum of fermions in the presence of a singly quantized vortex line is obtained by solving the Bogoliubov–de Genne equation:

$$\begin{pmatrix} -i\boldsymbol{\alpha}\cdot\boldsymbol{\partial}+\beta m-\mu & e^{i\theta}|\Delta(r)| \\ e^{-i\theta}|\Delta(r)| & i\boldsymbol{\alpha}\cdot\boldsymbol{\partial}-\beta m+\mu \end{pmatrix}\Phi(\boldsymbol{x}) = E\Phi(\boldsymbol{x}). \tag{18}$$

Here we assumed that the vortex line extends in the z direction and $\Delta(\boldsymbol{x})$ does not depend on z; $\Delta(\boldsymbol{x}) = e^{i\theta}|\Delta(r)|$ where (r,θ,z) are cylindrical polar coordinates. Note that we do not make any assumptions on the form of $|\Delta(r)|$ except that it has a nonvanishing asymptotic value; $|\Delta(r\to\infty)| > 0$. Therefore the existence of localized fermions that we will find below is independent of the vortex profile and thus in this sense they are *universal*. This would be because these solutions have topological origins and, in particular, the zero energy solutions are guaranteed by the index theorem [18, 31]. In contrast, there will be other Caroli–de Gennes–Matricon-type bound fermions on the vortex line which typically have the energy gap $\sim |\Delta(\infty)|^2/\mu$ [32]. Because their spectrum depends on the vortex profile, we shall not investigate such nonuniversal solutions in this paper.

Because of the translational invariance in the z direction, we look for solutions of the form

$$\Phi(r,\theta,z) = e^{ip_z z}\phi_{p_z}(r,\theta). \tag{19}$$

We rewrite the Hamiltonian in Eq. (18) as

$$\begin{aligned} e^{-ip_z z}\,\mathcal{H}\,e^{ip_z z} &= \mathcal{H}|_{m=p_z=0} + \begin{pmatrix} \alpha_z p_z+\beta m & 0 \\ 0 & -\alpha_z p_z-\beta m \end{pmatrix} \\ &\equiv \mathcal{H}_0 + \delta\mathcal{H}. \end{aligned} \tag{20}$$

We first construct zero energy solutions for $\mathcal{H}_0$ at $m = p_z = 0$ and then determine their dispersion relations with treating $\delta\mathcal{H}$ $(m, p_z \neq 0)$ as a perturbation.

1. *Zero energy solutions at $m = p_z = 0$*

Consider the zero energy Bogoliubov–de Genne equation at $m = p_z = 0$; $\mathcal{H}_0\phi_0 = 0$. We can find two exponentially localized solutions (see Appendix A):

$$\phi_R \equiv \frac{e^{i\frac{\pi}{4}}}{\sqrt{\lambda}} \begin{pmatrix} J_0(\mu r) \\ ie^{i\theta}J_1(\mu r) \\ 0 \\ 0 \\ e^{-i\theta}J_1(\mu r) \\ -iJ_0(\mu r) \\ 0 \\ 0 \end{pmatrix} e^{-\int_0^r |\Delta(r')|dr'} \tag{21}$$

and

$$\phi_L \equiv \frac{e^{-i\frac{\pi}{4}}}{\sqrt{\lambda}} \begin{pmatrix} 0 \\ 0 \\ J_0(\mu r) \\ -ie^{i\theta}J_1(\mu r) \\ 0 \\ 0 \\ e^{-i\theta}J_1(\mu r) \\ iJ_0(\mu r) \end{pmatrix} e^{-\int_0^r |\Delta(r')|dr'}, \tag{22}$$

where λ is a normalization constant:

$$\lambda = 2\pi\int_0^\infty dr\, r\left[2J_0^2(\mu r) + 2J_1^2(\mu r)\right] e^{-2\int_0^r |\Delta(r')|dr'}. \tag{23}$$

These two solutions have definite chirality; $\gamma^5\phi_{R/L} = \pm\phi_{R/L}$, and hence their index.

2. *Perturbations in terms of m and p_z*

We now evaluate matrix elements of $\delta\mathcal{H}$ with respect to ϕ_R and ϕ_L. It is easy to find

$$\int_0^{2\pi} d\theta \int_0^\infty dr\, r \begin{pmatrix} \phi_R^\dagger\,\delta\mathcal{H}\,\phi_R & \phi_R^\dagger\,\delta\mathcal{H}\,\phi_L \\ \phi_L^\dagger\,\delta\mathcal{H}\,\phi_R & \phi_L^\dagger\,\delta\mathcal{H}\,\phi_L \end{pmatrix} = v\begin{pmatrix} p_z & -im \\ im & -p_z \end{pmatrix}, \tag{24}$$