true also for each couple of sequences inside $\left(\mathscr{C}_y^\beta \times \mathscr{G}_y^\beta\right)$ for the set $\mathscr{G}_y^\beta$ of all almost periodic functions in $\mathscr{C}_y^\beta$. This verifies the point (iii) in the definition of Li-York chaotic set for $\mathscr{A}_y^\beta$. Consequently, the set of functions $\mathscr{A}_y^\beta$ is a Li-Yorke chaotic set. The proof is finalized. $\square$

Remark 3.1 *When we consider "$i\bar{l}$-periodic for some $i \in \mathbb{N}$" in the definitions of scrambled set $\mathscr{C}_x$ and Li-Yorke chaotic set $\mathscr{A}_x$ instead of "almost periodic", then by using Lemma 2.2, we see that Theorem 3.1 is valid also for this alternative periodic case by considering any periodic $\nu_\beta(t)$. Roughly writing, now scrambled set $\mathscr{C}_x$ contains functions nonresonant with the unperturbed, i.e., without $h(x)$, part of (2.9) for any periodic $\nu_\beta(t)$. On the other hand, the set in the part (i) of definition of Li-York chaotic set contains resonant functions.*

Remark 3.2 *System (2.4) may possess bounded solutions other than $\nu_\beta(t)$, $\beta \in S_m$. Then there may exist a replicated chaos corresponding for each of such solution, but verification of that is a difficult task in general, which would need additional assumptions for the system. This is why we are satisfied with the proof of the chaos around an almost periodic (or just periodic) solution $\nu_\beta(t)$ for $\beta \in S_m$.*

4 An Example

This part of the paper is devoted to an illustrative example. First of all, we will take into account a forced Duffing equation, which is known to be chaotic in the sense of Li-Yorke, as the source of chaotic perturbations. The forcing term in this equation will be in the form of a relay function to ensure the presence of Li-Yorke chaos. Detailed theoretical as well as numerical results concerning relay systems can be found in the papers [2, 3, 4, 5, 7]. In order to provide the replication of chaos, we will perturb another Duffing equation, which admits a homoclinic orbit, by the solutions of the former.

Another issue that we will focus on is the stabilization of unstable quasi-periodic motions. In the literature, control of chaos is understood as the stabilization of unstable periodic orbits embedded in a chaotic attractor [27, 46]. However, in this section, we will demonstrate the stabilization quasi-periodic motions instead of periodic ones, and this is one of the distinguishing features of our results. The existence of unstable quasi-periodic motions embedded in the chaotic attractor will be revealed by means of an appropriate chaos control technique based on the Ott-Grebogi-Yorke (OGY) [39] and Pyragas [43] control methods.

Let us consider the following forced Duffing equation,

$$x'' + 0.82x' + 1.4x + 0.01x^3 = v(t, \zeta, \lambda), \tag{4.20}$$