where $m = e^M - 1$.

To obtain an expansion in the form of (8), we define two states of the dots as either occupied or vacant. Thus there are $2^3 = 8$ graphs corresponding to the 8 terms in (8). However, up to this point, clusters are defined by the connectivity of Potts sites, *not* by the dotted faces. But the connectivity can be readily translated to that of the dotted faces. A moment's reflections shows that the weight contributing to A, B or C is simply $q^{n_c} m^{n_m}$, where n_c is the number of independent clusters not containing sites 1, 2, or 3, and n_m is the number of occupied dots.

The rules to divide the graphs into five types corresponding to A, B, C are the same as the ones for pure 2-site interactions. For example, the graph in Fig. 7(b) has $n_m = 2$, $n_c = 0$ and corresponds to the term $\sum_{4,5,6} (m\delta_{1,5,6})(m\delta_{2,4,6}) = m^2\delta_{12}$, thus contributing to B_{12} with a term m^2.

The algorithm to obtain expressions of A, B, C is therefore very similar to the one described in the above for 2-site interactions:

1. Generate one term, i.e., a graph, by choosing a set of occupied dots.

2. Count the number of clusters isolated from sites 1, 2 or 3 as n_c.

3. Count the number of occupied dots n_m.

4. Assign $q^{n_c} m^{n_m}$ to A, B or C respectively according to the aforementioned rules.

5. Go to 1 for another graph until all possible graphs are exhausted.

In the Appendix we present expressions of A, B, C for the Potts model on $n \times n$ subnets with 2-site interactions for $n \leq 4$, and for subnets with 3-site interactions for $n \leq 7$.

IV. THE TRANSFER MATRIX AND FINITE-SIZE SCALING

We use the method of transfer matrix to calculate statistical variables for lattice models wrapped on a cylinder with circumference L and length N. For lattices shown in Fig. 1 with hatched triangles, L and N count up- and down-pointing hatched triangles (rather than individual Potts spins within each triangle). Thus, for an $(m \times m) : (n \times n)$ lattice shown in Fig. 8(a), there are actually $(m + n)L$ Potts spins in a length L.