we have confirmed that the renormalized λ^* approaches the strong coupling limit value 0.312. In this sense, the size dependence of λ^* might be related with the scaling function of the Wilson NRG(Eqs. VIII.45, 46 or IX.58 in Ref.[5]). For the detailed analysis of λ^*, the wavefunction of much larger system size is needed beyond the exact diagonalization.

VI. SUMMARY AND DISCUSSION

We have clarified that the key mechanism of Wilson NRG is the scale free property of the Wilson Hamiltonian (1); The cutoff Λ results in the wavepacket basis described by Eq. (8) and its lattice translation enables for us to control the energy scale of the system with no reference to rescaling of the length unit. In addition, we have shown that the Hamiltonian (1) has the edge states at the lowest-energy scale, which has not been mentioned so far. On the basis of wavepacket, we have performed the exact diagonalization including the Kondo coupling. We then revealed that the nontrivial effect of the Kondo coupling can be detectable as a renormalization of the decay rate of the groundstate spin density; The value of λ^* is close to the canonical value λ in the strong coupling limit, while the renormalization effect becomes large in the crossover regime. Since the effective length scale of the ground state wavefunction is given by $\xi^* = 1/\lambda^*$, it may be efficient to increase the number of the basis in the crossover regime.

In the context of the uniform 1D critical system, the critical phenomena are usually characterized by the power-low decay of physical quantities. However, the exponential modulation introduces the cutoff of the infrared divergence and then the criticality of the uniform system would be converted to the nontrivial renormalization of λ^*. In particular, the connection of the renormalized scale factor in Wilson NRG to the DMRG analysis of the Kondo screening cloud for the uniform chain with a boundary[23] is an interesting problem. We also remark that, as in Ref.[17], the regulator for the infrared divergence due to such scale-free modulation of the interaction is a generally applicable idea to the 1D quantum many body system. For further research, it may be essential to clarify the direct relation between the entanglement and the renormalized scale factor. Recently, the connection of Wilosn NRG to the matrix product state is pointed out in Refs.[24,25]. Moreover, very recently, the boundary critical phenomena can be correctly captured by the entanglement renormalization.[26,27] We believe that the present result stimulates a new frontier in the quantum RG.

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