I. INTRODUCTION

The most dramatic effects related to superfluidity in liquid ^{4}He arise [1] when the dynamics of the two components are described by the two-fluid hydrodynamics first discussed by Landau [2]. These equations only describe the dynamics when the non-equilibrium states are in local hydrodynamic equilibrium [3], which requires short collision times between the excitations forming the normal fluid. (This requirement is usually summarized as $\omega\tau \ll 1$, where ω is the frequency of a collective mode and τ is the appropriate relaxation rate.) The study of ultracold gases when they are in local equilibrium has been difficult because the density and the s-wave scattering length are typically not large enough. However, recent experimental work on trapped Bose-condensed gases has reported some success, with evidence for a second sound mode in highly elongated (cigar-shaped) traps [4].

Another approach to achieving conditions where the Landau two-fluid description is correct has been to consider a Fermi superfluid gas close to unitarity [5], where the s-wave scattering length between Fermi atoms in two different hyperfine states is infinite. Developing earlier theoretical studies [6, 7], Taylor and co-workers [8] have recently given detailed predictions for the first and second sound breathing oscillations in a trapped Fermi gas at unitarity. The coupled differential equations of the Landau two-fluid description were solved variationally, with results which agreed with analytic predictions at $T \to 0$ and $T \to T_c$. So far, only an isotropic trap has been studied.

In [8], it was shown that (as in superfluid ^{4}He) the frequencies of first and second sound in unitary Fermi gases were quite well approximated by assuming that the solutions of the two-fluid equations corresponded to pure uncoupled density and temperature waves. This is in sharp contrast to the situation in dilute, weakly interacting Bose gases, where first and second sound involve both density and temperature oscillations [9]. Naïvely, the first result above would lead one to believe that second sound in a unitary Fermi gas would have very small weight in the density response function. However, Arahata and Nikuni [10] have shown that in a uniform Fermi gas at unitarity, a density disturbance can have first and second sound pulses of comparable magnitude at temperatures of order $0.8T_c$.

The purpose of this paper is to provide a systematic study of the density response function $\chi_{nn}(\mathbf{q}, \omega)$ of a uniform superfluid atomic gas in the hydrodynamic regime [11, 12], as described by the non-dissipative Landau two-fluid equations (see sections II and III).