Single-transverse spin asymmetries (SSAs) in hadronic processes, such as in the single inclusive hadron production in single transversely-polarized nucleon-nucleon scattering, $p^{\uparrow}p \rightarrow hX$, have attracted much interests from both experimental and theoretical sides in the last few years, and great progress has been made in understanding the underlying physics [1]. Although it is a simple observable, defined as the spin asymmetry when one flips the transverse spin of one of the hadrons involved in the scattering: $A_N = (d\sigma(S_\perp) - d\sigma(-S_\perp))/(d\sigma(S_\perp) + d\sigma(-S_\perp))$, it is far more complicated to explain in the fundamental theory of strong interaction. It usually represents a correlation between the transverse polarization vector $S_\perp$ of one of the hadrons and the transverse momentum $P_{h\perp}$ of the final-state hadron in the differential cross section. For example, in the process $p^{\uparrow}p \rightarrow hX$, it is the correlation between the polarization vector $S_\perp$ of the incoming nucleon and the transverse momentum $P_{h\perp}$ of the final-state hadron that generates the asymmetry. In this paper, we will focus on the SSAs in these processes, especially for the neutral mesons, π^0 and η production, motivated by the recent striking experimental observations of large SSAs for them by the STAR collaboration at RHIC experiments [2, 3]. In particular, at forward direction of the polarized proton, it was found that the SSA for η meson is much larger than that for π^0 meson. This also confirmed previous experimental observations from the fixed target experiments [4]. In addition to these STAR results, both BRAHMS and PHENIX collaborations at RHIC have observed large single transverse spin asymmetries in charged and neutral meson production in the forward rapidity region [5, 6].

In the QCD framework, there have been mainly two approaches to study the SSAs in high energy scattering processes: the transverse momentum dependent (TMD) parton distribution and fragmentation function approach [7–15] and the twist-three quark-gluon correlation approach in the collinear factorization [16–21]. In the TMD approach, it is required to have an additional hard momentum scale besides the transverse momentum $P_{h\perp}$ of the hadron, such as Q^2, the momentum transfer square of the virtual photon in semi-inclusive deep inelastic scattering (SIDIS) or Drell-Yan lepton pair production in pp collisions, to study the transverse momentum dependence in these processes. On the other hand, the twist-three approach is more appropriate in the processes that all momentum scales are much larger than the nonperturbative scale $\Lambda_{\rm QCD}$ [17]. It has been shown that these two approaches are consistent with each other for the SIDIS and Drell-Yan processes in the intermediate transverse momentum region where they both apply [22, 23].