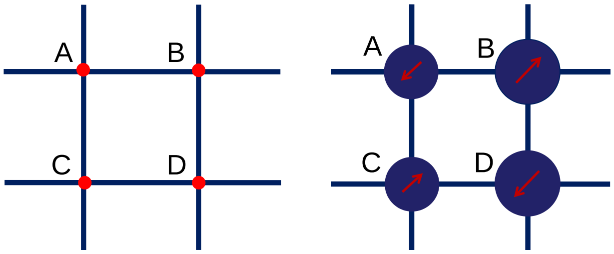

FIG. 1: (color online) Left panel: The unit cell of the graded Projected Entangled-Pair State representation of the ground-state wave functions. Right panel: The pattern of the coexisting charge and spin density wave order. Here, the radius of the circles are proportional to the fillings at sites, whereas the arrows inside the circles represent the directions and magnitudes of the spin density wave order parameters. This represents a vertical commensurate stripe state, which breaks the four-fold rotation symmetry and the translation symmetry in the horizontal direction for charge density wave order and in both directions for spin density wave order.

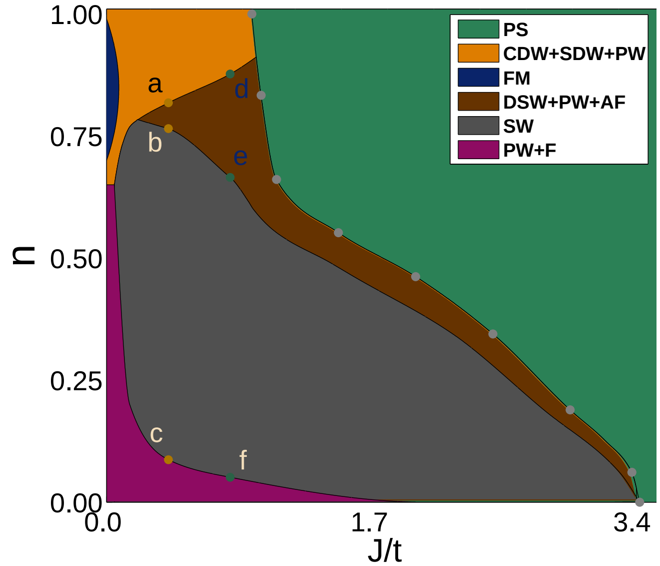

FIG. 2: (color online) The proposed ground-state phase diagram of the two-dimensional $t-J$ model. First, for $J/t \geq 0.95$, there is a line of phase separation (PS), whereas for $J/t \geq 0.95$, no PS occurs. Second, the homogeneous regime is divided into four different phases: one phase with charge and spin density wave order coexisting with a $p_x(p_y)$−wave superconducting state (CDW+SDW+PW), one phase with the symmetry mixing of $d+s$−wave superconductivity in spin-singlet channel and $p_x(p_y)$−wave superconductivity in spin-triplet channel in the presence of an anti-ferromagnetic background (DSW+PW+AF), one superconducting phase with an extended s−wave symmetry (SW), and one superconducting phase with a p−wave symmetry (PW+F), besides a possible Nagaoka's ferromagnetic state (FM). Here, a systematic computation has been performed for both $J/t = 0.4$ and $J/t = 0.8$ with the truncation dimension $\mathbb{D} = 6$, whereas the dash lines separating different phases are a guide for the eyes. For $J/t = 0.4$, the DSW+PW+AF phase occurs for fillings from $n = 0.818$ (denoted as point a) to $n = 0.765$ (denoted as point b), the SW phase occurs for fillings from $n = 0.765$ to $n = 0.087$ (denoted as point c), and the CDW+SDW+PW and PW+FM phases occur for fillings from $n = 1$ to $n = 0.818$ and from $n = 0.087$ to $n = 0$, respectively. For $J/t = 0.8$, the DSW+PW+AF phase occurs for fillings from $n = 0.877$ (denoted as point d) to $n = 0.665$ (denoted as point e), the SW phase occurs for fillings from $n = 0.665$ to $n = 0.051$ (denoted as point f), and the CDW+SDW+PW and PW+FM phases occur for fillings from $n = 1$ to $n = 0.877$ and from $n = 0.051$ to $n = 0$, respectively.

and the ratio between the anti-ferromagnetic coupling and the hoping constant, respectively. At half filling, $n = 1$, the $t-J$ model reduces to the two-dimensional Heisenberg spin 1/2 model. Therefore, a long range anti-ferromagnetic order exists [24]. In this case, our algorithm yields the ground-state energy per site, $e = -1.1675J$, for the truncation dimension $\mathbb{D} = 4$, and $e = -1.1683J$, for the truncation dimension $\mathbb{D} = 6$. This is comparable to the best QMC simulation result: $e = -1.1694J$ [24, 25], although the anti-ferromagnetic Néel order moment is 0.36 versus 0.31. Away from half filling, the model exhibits quite different behaviors for small and large values of the anti-ferromagnetic coupling J. For $J/t \geq 0.95$, there is a line of PS between the Heisenberg anti-ferromagnetic state without hole and a hole-rich state, whereas for $J/t \leq 0.95$, no PS occurs. This agrees qualitatively with the results based on the HTE [11], the VMC method [13], and the DMRG [14]. Note that our result for the transition point $J_c = 3.45t$ at low electron density is quite close to the exact value $J_c = 3.4367t$ [15]. Here, the simulation has been performed for the truncation dimension $\mathbb{D} = 6$.

We point out that a discrepancy exists concerning the PS boundary of the $t-J$ model. In Ref. [9], a combination of analytic and numerical calculations is used to establish the existence of PS at all super-exchange interaction strengths. Although the simulation based on the Green's function Monte Carlo method supports this scenario [10], many others [11–14] found that the model phase separates only for J/t larger than some finite critical value around $J/t \sim 1$. That is, PS occurs *only* outside the physically realistic parameter region of the $t-J$ model. With the observation that no significant shift with the truncation dimension $\mathbb{D}$ increasing from $\mathbb{D} = 4$ [21] to $\mathbb{D} = 6$ is found, we conclude that *PS does not occur for* $J/t \leq 0.95$.

In the homogeneous regime, the two-dimensional $t-J$ model exhibits very rich physics. Away from half filling, both the $d+s$−wave superconductivity in the spin-singlet channel and the $p_x(p_y)$−wave superconductivity in the spin-triplet channel, and/or charge and spin density wave order, occur in different doping regimes for a fixed J/t. Here, a superconducting state is characterized by a superconducting order parameter $\Delta \equiv \langle\hat{\Delta}\rangle$, with $\hat{\Delta}$ defined as follows: For s-wave, $\hat{\Delta}_s = 1/(4\sqrt{2})\, c^+_{i_x,i_y\uparrow}[c^+_{i_x+1,i_y\downarrow} + c^+_{i_x-1,i_y\downarrow} + c^+_{i_x,i_y+1\downarrow} + c^+_{i_x,i_y-1\downarrow}] - [\uparrow\leftrightarrow\downarrow]$; for d-wave, $\hat{\Delta}_d = 1/(4\sqrt{2})\, c^+_{i_x,i_y\uparrow}[c^+_{i_x+1,i_y\downarrow} + c^+_{i_x-1,i_y\downarrow} - c^+_{i_x,i_y+1\downarrow} - c^+_{i_x,i_y-1\downarrow}] - [\uparrow\leftrightarrow\downarrow]$; for p_x-wave, $\hat{\Delta}_{p_x} = \hat{\Delta}_{p_x+} - \hat{\Delta}_{p_x-}$, with $\hat{\Delta}_{p_x\pm} = 1/2\left(c^+_{i_x,i_y\uparrow}c^+_{i_x\pm1,i_y\uparrow}, [c^+_{i_x,i_y\uparrow}c^+_{i_x\pm1,i_y\downarrow} + c^+_{i_x,i_y\downarrow}c^+_{i_x\pm1,i_y\uparrow}]/\sqrt{2}, c^+_{i_x,i_y\downarrow}c^+_{i_x\pm1,i_y\downarrow}\right)$, and a similar definition of $\hat{\Delta}_{p_y}$ for p_y-wave.

Charge and spin density wave order coexisting with a