Using the Baker-Hausdorff formula,

$$e^{-iG}Ae^{iG} = A - i[G,A] + \frac{(-i)^2}{2!}[G,[G,A]] + \frac{(-i)^3}{3!}[G,[G,[G,A]]] + \ldots$$

and $[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k$, after some calculations we find

$$\hat{\mathbf{\Sigma}}' = \hat{\mathbf{\Sigma}}\cos 2I + \frac{\mathbf{I}(\hat{\mathbf{\Sigma}}\cdot\mathbf{I})}{I^2}(1-\cos 2I) + \frac{\hat{\mathbf{\Sigma}}\times\mathbf{I}}{I}\sin 2I$$

where $I = |\mathbf{I}| = |\int_C e^{i\gamma\cot\theta}\cos\theta d\varphi|$. Hence

$$\hat{\mathbf{B}}' = -(m + \beta k + \hat{\mathbf{\Sigma}}\cdot\mathbf{D}')\frac{\mathbf{k}}{k^3}$$

where

$$\mathbf{D}' = \mathbf{D}\cos 2I + \frac{\mathbf{I}(\mathbf{D}\cdot\mathbf{I})}{I^2}(1-\cos 2I) - \frac{\mathbf{D}\times\mathbf{I}}{I}\sin 2I.$$

Therefore,

$$\mathbf{B} = -(m + \beta k + \sigma D_3')\frac{\mathbf{k}}{k^3}$$

with

$$D_3' = D_3\cos 2I + \frac{I_3(\mathbf{D}\cdot\mathbf{I})}{I^2}(1-\cos 2I)$$

which reduces to the result of [27] in the absence of dislocation, namely, the field of a magnetic monopole of charge $m+\sigma$ situated at the origin of the momentum space.

The equations of motion of the beam in the presence of momentum space Berry curvature have been derived repeatedly for various particle beams (photons [14–16, 20–23], phonons [40–42] and electrons [24–26, 43]). We have

$$\dot{\mathbf{k}} = k\nabla \ln n$$
$$\dot{\mathbf{r}} = \frac{\mathbf{k}}{k} + \mathbf{B}\times\dot{\mathbf{k}}$$

where dot denotes derivative with respect to the beam length. These differ from the standard ray equations of the geometrical optics by the term involving the Berry curvature, which yields the beam displacement

$$\delta\mathbf{r} = -\int_C (m + \beta k + \sigma D_3')\frac{\mathbf{k}\times d\mathbf{k}}{k^3}. \tag{7}$$

The displacement, which results in the splitting of beams with different polarizations and/or orbital angular momentums, is orthogonal to the beam direction and produces a current