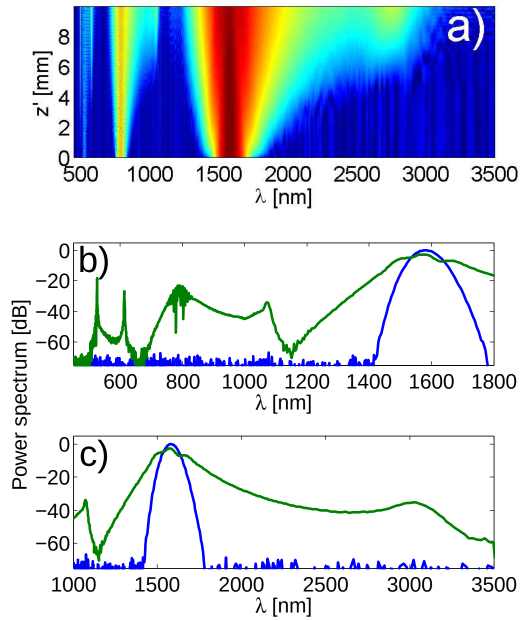

FIG. 2: Propagation of a femtosecond pulse into a PPLN sample. a) Evolution of the power spectrum (in dB) from numerical solution of Eq. (11). b) Power specrum at the crystal output in the visible and NIR range. c) Power specrum at the crystal output in the infrared range. The division into separate spectral region is made to facilitate the comparison with experimental data [10]. The initial pulse has gaussian shape and the parameters are $T = 50fs$, $I = 15GW/cm^2$, $\lambda_{in} = 1580nm$, $\lambda_0 = 2\pi c/\omega_0 = 700nm$, $d_{33} = \chi^{(2)}_{LN}/2 = 27pm/V$

mismatched SHG. When the FF broadening reaches the first order quasi phase matching wavelength at around $2\mu m$, the more efficient conversion process generates a spike at around $1\mu m$. Figure 2 b) shows the visible and the near infrared (NIR) part of the spectrum at the crystal output. We can see a broadband second and third harmonic of the broadened laser spectrum, and the presence of some spikes given by the quasi phase matching of high order spatial harmonics of the grating. We verified that the two spikes at the third harmonic correspond to the third and fifth order QPM for the process $\omega + 2\omega \to 3\omega$. We can also see a spectral overlap between the harmonics of the broadened laser spectrum, that can be exploited to achieve carrier-envelope-offset phase slip stabilization [10], that is of paramount importance for frequency metrology applications.
Figure 2c) shows the infrared spectrum at the output. This spectrum exhibits more than an octave spanning between $1300nm$ and $3000nm$ at the $-40dB$ spectral power lever with respect to the peak power level. The spectral components near the zero GVM wavelength around $3000nm$ are generated more efficiently.
All the features described above compares surprisingly well with the experimental results of Langrock et al. [10], even if we simulate a slightly different environment. In fact we use a bulk PPLN sample and not a RPE PPLN waveguide. The effect of waveguide is to slightly modify the power levels and the crystal dispersion: a detailed simulation of the real set-up is out of the scope of this Letter. It is worth noting that numerical modelling of such phenomena without our model is an irksome job since (i) time domain Maxwell equation solvers require a prohibitive computational effort and (ii) coupled wave approaches cannot be used in the presence of overlapping among frequency bands of different field components.

In conclusion we have derived a robust nonlinear envelope equation describing the propagation in dispersive quadratic materials. Thanks to a proper formal definition of the complex envelope, it is possible to treat pulses of arbitrary frequency content. A proper definition of envelope is crucial for second order nonlinearities, due to the generation of frequency components around zero. Computationally it is possible to accurately evolve optical pulses of arbitrarily wide band over a meter scale physical distance, which is a few order of magnitude longer than those accessible by Maxwell equation solvers.

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