

fidelity, see Ref. 19. The first couple of cumulants are given by

$$\begin{aligned}\langle\langle Q^2 \rangle\rangle &= 2P_1^{\text{eq}}P_2^{\text{eq}} \frac{V^2(G_1 - G_2)^2[(\Gamma t - 1) + e^{-\Gamma t}]}{\Gamma^2}, \\ \langle\langle Q^3 \rangle\rangle &= 6P_1^{\text{eq}}P_2^{\text{eq}}(P_2^{\text{eq}} - P_1^{\text{eq}}) \\ &\quad \times \frac{V^3(G_1 - G_2)^3[(\Gamma t - 2) + (\Gamma t + 2)e^{-\Gamma t}]}{\Gamma^3}.\end{aligned}\quad (24)$$

F. Symmetric fluctuator

A special situation is given when the two-level fluctuator is symmetric, $\Delta = 0$, i.e., $P_1^{\text{eq}} = P_2^{\text{eq}} = 1/2$. Then the characteristic function assumes the simple form

$$\log \chi_{\gg}(\lambda) = \frac{\Gamma t}{2} \left[\sqrt{1 - \lambda^2 V^2 (G_1 - G_2)^2 / \Gamma^2} - 1 \right] \quad (25)$$

for long times. In the short time limit, the generating function

$$\chi_{\ll}(\lambda) = \cos[\lambda V (G_1 - G_2) t / 2] \quad (26)$$

becomes periodic. In both cases, due to the symmetry of the states x_1 and x_2 , only the even cumulants are nonvanishing.

IV. A PAIR OF TWO-LEVEL FLUCTUATORS

Needless to say, the mapping of Sec. III A is not restricted to a single two-level fluctuator. It can be generalized to an arbitrary number of states whose dynamics is governed by classical rate equations described by a Fokker-Planck Hamiltonian \mathbf{h} . To illustrate this concept, the example of two classical, uncorrelated two-level fluctuators coupled to a wire is discussed in the following. We restrict ourselves to the case where the dynamics of the two-level systems is completely independent of each other such that $\mathbf{h} = \mathbf{h}^\alpha + \mathbf{h}^\beta$; here and in the following, we denote quantities involving only the first (second) fluctuator with a superscript $\alpha(\beta)$, e.g., $\mathbf{h}^\alpha = \mathbf{h}^\alpha \otimes \mathbf{1}^\beta$. Written explicitly in the basis $\{|1\rangle^\alpha \otimes |1\rangle^\beta, |2\rangle^\alpha \otimes |1\rangle^\beta, |1\rangle^\alpha \otimes |2\rangle^\beta, |2\rangle^\alpha \otimes |2\rangle^\beta\}$, the Fokker-Planck Hamiltonian reads

$$\mathbf{h} = \begin{pmatrix} \gamma_{21}^\alpha + \gamma_{21}^\beta & -\gamma_{12}^\alpha & -\gamma_{12}^\beta & 0 \\ -\gamma_{21}^\alpha & \gamma_{12}^\alpha + \gamma_{21}^\beta & 0 & -\gamma_{12}^\beta \\ -\gamma_{21}^\beta & 0 & \gamma_{21}^\alpha + \gamma_{12}^\beta & -\gamma_{12}^\alpha \\ 0 & -\gamma_{21}^\beta & -\gamma_{21}^\alpha & \gamma_{12}^\alpha + \gamma_{12}^\beta \end{pmatrix}. \quad (27)$$

One is tempted to think that the independent dynamics of the two subsystems would lead to independent statistics, such that the characteristic function of the full counting statistics is given by the product of the individual characteristic functions. Indeed, this is the generic case for two-level fluctuators coupling to a qubit, where the combined effect leads to $1/f$ noise.^{11,13,15–17}

However, here, this argument is only valid when the interaction with the wire is “linear” such that the effects of the individual subsystems simply add up, in formula $\mathbf{G} = \mathbf{G}^\alpha + \mathbf{G}^\beta$. Having the model of the Fig. 1 in mind, this assumption is incorrect as a (quantum) point contact does not react linearly on changes in the gate potential and, therefore, the noise of individual fluctuators does not simply add up. In the following, we first treat the (simple) case of linear interaction and then comment on the correlation which arises in the general case by applying perturbation theory in the nonlinearity.

Introducing the reference conductance $G_0 = G_0^\alpha + G_0^\beta$ as well as the induced changes $\Delta G^x = G_1^x - G_0^x$ due to the fluctuator $x = \alpha, \beta$, the conductance matrix for linear interaction is given by

$$\mathbf{G} = G_0 \mathbf{1}_4 + \text{diag}(0, \Delta G^\alpha, \Delta G^\beta, \Delta G^\alpha + \Delta G^\beta); \quad (28)$$

the increase of conductance in the case when both fluctuators are in state x_2 is the sum of the respective increases when only one of the fluctuators is in state x_2 , that is to say, the effects of the two fluctuators simply add up. In this case, the characteristic function assumes the form $\chi(\lambda) = \chi^\alpha(\lambda)\chi^\beta(\lambda)$ and the cumulants $\langle\langle Q^n \rangle\rangle$ become a sum of cumulants generated by the two individual subsystems.

In the case of a general (diagonal) matrix \mathbf{G} , the solution of the problem involves the determination of the roots of a polynomial of fourth degree and the characteristic function does not separate, even though the time evolution of the two fluctuators is completely independent of each other. To be more explicit, we want to show how a small perturbation $\Delta G \ll \Delta G^\alpha + \Delta G^\beta$ in \mathbf{G}_{44} destroying the linearity (additivity) leads to correlations which can be arbitrary large for long times. We define $\chi^{\text{corr}}(\lambda) = \chi(\lambda) / \chi^\alpha(\lambda)\chi^\beta(\lambda)$ as the part of the characteristic function which describes the correlation between the action of the individual subsystems. In the long-time limit, to first order in ΔG , we can apply standard perturbation theory to find the correction to the lowest eigenvalue of Eq. (27). The cumulant generating function for the correlation is given by

$$\log \chi_{\gg}^{\text{corr}}(\lambda) = i\lambda V \Delta G t \prod_{x=\alpha,\beta} \frac{(\Gamma^x + i\lambda V \Delta G^x) P_2^{\text{eq},x} - \hat{h}_-^x}{\hat{h}_+^x - \hat{h}_-^x}. \quad (29)$$

The average transmitted charge changes according to

$$\Delta\langle Q \rangle = P_2^{\text{eq},\alpha} P_2^{\text{eq},\beta} V \Delta G t, \quad (30)$$

with $P_2^{\text{eq},\alpha} P_2^{\text{eq},\beta}$ the probability to be in the state $|2\rangle^\alpha \otimes |2\rangle^\beta$ and $V \Delta G$ the change in the current. Less trivial, the correlation contribution to the noise

$$\Delta\langle\langle Q^2 \rangle\rangle = 4V \Delta\langle Q \rangle \left[\frac{P_1^{\text{eq},\alpha}}{\Gamma^\alpha} \Delta G^\alpha + \frac{P_1^{\text{eq},\beta}}{\Gamma^\beta} \Delta G^\beta \right] \quad (31)$$

depends both on ΔG^α and ΔG^β .