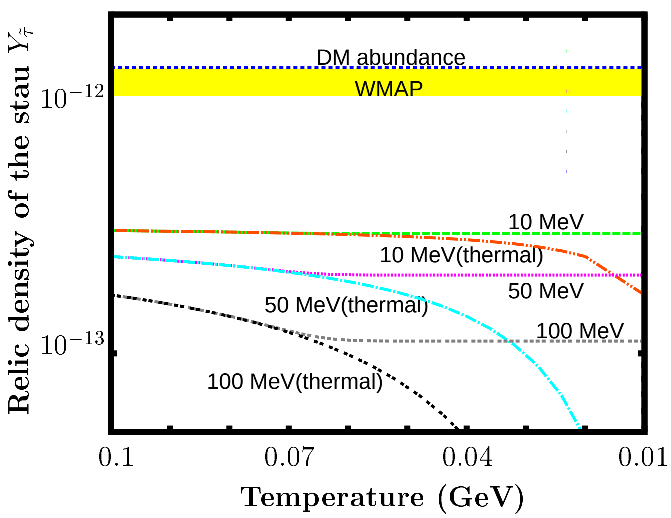

FIG. 3: The evolution of the number density of negative charged stau. Each line attached [δm] shows the actual evolution of the number density of stau, while the one atattched [δm(thermal)] shows its evolution under the equilibrium determined given by Eq. (13) and the total relic abundance. Yellow band represents the allowed region from the WMAP observation at the 2σ level [1].

B. Stau relic density at the BBN era

Next, we solve the Boltzmann equations (18), (19), and (20) numerically, and obtain the ratio of the stau number density to the total number density of stau and neutralino. Fig. 3 shows the evolution of the number density of stau as a function of the universe temperature. Here we took $m_{\tilde{\tau}} = 350$ GeV, $\sin\theta_\tau = 0.8$, and $\gamma_\tau = 0$ and chose $\delta m = 10$ MeV, 50 MeV, and 100 MeV as sample points. Each line attached [δm] shows the actual evolution of the number density of stau, while the one atattched [δm(thermal)] shows its evolution under the equilibrium determined by Eq. (13) and the total relic abundance. Horizontal dotted line represents the relic density of DM, which is the total abundance calculated above. We took it as a initial condition of total value for the calulation of the number density ratio. Yellow band represents the allowed region from the WMAP observation at the 2σ level [1].

The number density evolution of stau is qualitatively understood as follows. As shown in Fig. 3, the freeze-out temperature of stau almost does not depend on δm. It is determined by the exchange processes Eq. (10), whose magnitude $\langle\sigma v\rangle Y_{\tilde{\tau}} Y_\gamma$ is governed by the factor $e^{-(m_\tau-\delta m)/T}$, where m_τ represents the tau lepton mass. The freeze-out temperature of the stau density $T_{f(\mathrm{ratio})}$ is given by $(m_\tau - \delta m)/T_{f(\mathrm{ratio})} \simeq 25$ as in Eq. (9), since the cross section of the exchange process is of the same magnitude as weak processes. Thus $T_{f(\mathrm{ratio})}$ hardly depends on δm. In contrast, the ratio of the number density between stau and neutralino depends on δm according to Eq. (13), $n_{\tilde{\tau}}/n_{\tilde{\chi}} \sim \exp(-\delta m/T)$, since they follow the Boltzmann distribution before their freeze-out. Thus, the relic density of stau strongly depends on δm.

Here, we comment on the dependence of the stau relic density $n_{\tilde{\tau}^-}$ on other parameters such as $m_{\tilde{\tau}}$, θ_τ, and γ_τ. The number density of the negatively charged stau is expressed in terms of the total relic density N by

$$n_{\tilde{\tau}^-} = \frac{N}{2(1+e^{\delta m/T_{f(\mathrm{ratio})}})} . \qquad (23)$$

Here, the freeze-out temperature $T_{f(\mathrm{ratio})}$ hardly depends on these pararameters. This is because the cross section of the exchange processes are changed by these parameters at most by factors but not by orders of magnitudes, and the $T_{f(\mathrm{ratio})}$ depends logarithmically on $\langle\sigma v\rangle$ as shown in Eq. (9). On the other hand, the total relic density N is proportional to $m_{\tilde{\tau}}$ as in Eq. (22). The value of N is also affected by the left-right mixing θ_τ as seen in Fig. 2 since the annihilation cross section depends on this parameter. In contrast, γ_τ scarcely affects the relic density, since this parameter appears in the annihilation section through the cross terms of the contributions from the left-handed stau and the right-handed one, and such terms always accompany the suppression factor of $m_\tau/m_{\tilde{\tau}}$ compared to the leading contribution. Thus the relic number density of stau $n_{\tilde{\tau}}$ strongly depends on $m_{\tilde{\tau}}$ and θ_τ while scarecely depends on γ_τ.

We comment on the generality of our method to calculate the density of exotic heavy particles that coannihilate with other (quasi)stable particles: we calculate the total number density of these particles and then calculate the ratio among them by evaluating the exchange processes such as Eq. (10). This method of calculation can be found versatile in various scenarios including the catalyzed BBN and the exotic cosmological structure formation [31–33].

C. Long-lived stau and BBN

After the number density of stau freezes out, stau decays according to its lifetime [3], or forms a bound state with a nuclei in the BBN era. Their formation rate has been studied in literatures [14, 16, 17]. The bound states modify the predictions of SBBN, and make it possible to solve the ^{7}Li problem via internal conversion processes in the bound state [14–16].

In Fig. 4 we show parameter regions that are consistent with the observed abundances of the DM and of the light elements. We calculate the relic density of stau by varying the value of δm with the values of $m_{\tilde{\tau}} = 350$ GeV, $\sin\theta_\tau = 0.8$, and $\gamma_\tau = 0$. With these parameters, the allowed region is shown inside the dotted oval. We see that there are allowed regions at $Y_{\tilde{\tau}^-} \sim 10^{-13} - 10^{-12}$ for $\delta m \lesssim 130$ MeV to solve the ^{7}Li problem at 3σ. On the other hand, it is found that the observational ^{6}Li to ^{7}Li ratio excludes $Y_{\tilde{\tau}^-} \gtrsim 10^{-15}$ and $\delta m \lesssim 100$ MeV. We will explain this feature as follows.