Let us first suppose the function is constant. In that case, $f(q) = \pm 1$ necessarily, so the projection in (5) can be simplified to

$$\hat{\Pi}_{\Delta q_0} |Q\rangle = \pm |q_0\rangle , \qquad (6)$$

and then Alice measures with total certainty her initial quadrature state $|q_0\rangle$. In the other case, when the function is balanced, the output quadrature would have any value but q_0, yielding a zero measurement.

In practice, the preparation of nonideal computational states and finite precision in measurement pose severe problems for the experimental realization of this scheme, specially because the constant or balanced nature of the function would not be as straightforward to differentiate as in the idealized case presented above, the outcome of the measurement would not be as certain. Additionally, this model does not address the question of the implementation of the oracle, but suggests that using some kind of *hybrid* setting (i.e. one that combines discrete and continuous states,) one may improve the algorithm.

III. DEUTSCH-JOZSA ALGORITHM IN A HYBRID SETTING

In the hybrid computer proposed in Ref. [11], necessary and sufficient conditions are given to build up a universal set of gates operating in both a qubit and a CV state. Actually, the construction of such set is completed with just one interaction gate between both states, because universality is achieved by the repeated application of such interaction gate and the single-mode gates of each setting.

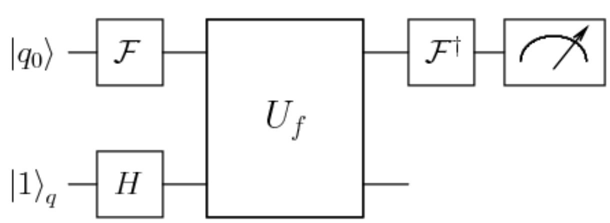

Figure 2. Quantum circuit for the Deutsch-Jozsa algorithm in a hybrid setting.

Let us consider the Deutsch-Jozsa problem in a hybrid setting. We use a qunat state to store Alice's query and a qubit to store the result of Bob's evaluation. The quantum circuit is given in figure 2. We keep the continuous part to implement the Fourier gate, which is much more efficient than the discrete superposition achieved with consecutive Hadamard gates [9]; but we also use a discrete register in which it is easier to store the answer given by the oracle.

A. Optical implementation of the algorithm

The all-optical setting we present here starts with an ideal[15] qunat state $|q_0\rangle$ in Alice's register, and a dual-rail qubit in Bob's register. The latter requires two paths or *rails* through which a single photon travels, a $|0\rangle_q$ qubit is encoded if the lower path is populated $|0,1\rangle$, and a $|1\rangle_q$ qubit is obtained when the upper rail carries the photon [3]. For the particular case of this algorithm, Bob produces a $|1\rangle_q$ qubit before the computation.

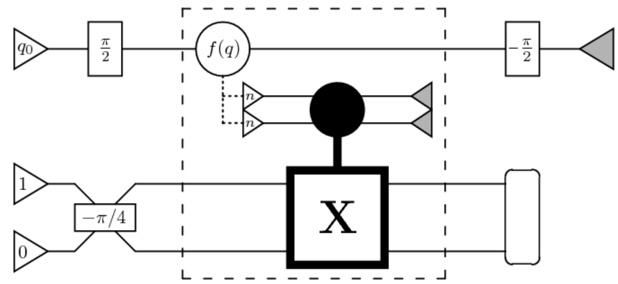

Figure 3. Optical circuit of the hybrid Deutsch-Jozsa algorithm. The explicit implementation of the oracle is shown.

Having prepared both registers, Alice and Bob produce superpositions on each of them. For the CV state, she uses a phase shifter with $\varphi = \pi/2$ to implement the Fourier gate [6, 7]. On the other hand, Bob performs the Hadamard gate using a 50–50 beam splitter ($\theta = \pi/4$.) The optical circuit is shown explicitly in figure 3. The resulting state just before the oracle operation is then

$$\left(\frac{1}{\sqrt{\pi}}\int \mathrm{d}q\, e^{2iq_0 q}\,|q\rangle\right) \otimes \frac{1}{\sqrt{2}}(|0\rangle_q - |1\rangle_q). \qquad (7)$$

In this setting, we require that the action of the oracle transforms the state (7) into

$$\left(\frac{1}{\sqrt{\pi}}\int \mathrm{d}q\, e^{2iq_0 q}(-1)^{f(q)}\,|q\rangle\right) \otimes \mathbf{H}\,|1\rangle_q\,, \qquad (8)$$

in analogy to the action of the oracle in the CV setting. For now, it is all we are saying about the oracle, the complete analysis is given in the next section.

As usual, Bob's register is left unchanged after the oracle operation, and Alice performs one last superposition using an inverse Fourier gate (via a phase shifter with $\varphi = -\pi/2$.) Actually, these final steps are identical to those in the CV implementation, i.e. steps (3) through (6). Once more, if the function is constant, Alice measures her initial state $|q_0\rangle$ with total certainty. In contrast, she measures zero if the function is balanced.

B. Operation of the Oracle

Now, the crucial step of the algorithm occurs in the oracle, where the registers interact with each other. The