

product pairs of subspaces of their Fourier transforms.

The classical pairs of spaces (Schwartz, 1966) are the convolution pair (S^*, O_C^*) and the corresponding product pair (S^*, O_M) , where O_C^* is the subspace of S^* that contains rapidly decreasing (faster than any polynomial) generalized functions and O_M is the space of infinitely differentiable functions with every derivative growing no faster than a polynomial at infinity. These pairs are important in that no restriction is placed on one of the generalized functions that could be any element of space S^* ; the other belongs to a space that needs to be correspondingly restricted. A disadvantage of the classical pairs is that the restriction is fairly severe, for example, the requirement that a characteristic function be in O_M implies existence of all moments for the random variable. Relaxing this restriction would require placing constraints on the other space in the pair; Zinde-Walsh (2012) introduces some pairs that incorporate such trade-offs.

In some models the product of a function with a component of the vector of arguments is involved, such as $d(x) = x_k a(x)$, then for Fourier transforms $Ft(d)(s) = -i \frac{\partial}{\partial s_k} Ft(a)(s)$; the multiplication by a variable is transformed into $(-i)$ times the corresponding partial derivative. Since the differentiation operators are continuous in S^* this transformation does not present a problem.

Assumption 2. *The functions $a \in A, b \in B$, are such that (A, B) form a convolution pair in S^* .*

Equivalently, $Ft(a), Ft(b)$ are in the corresponding product pair of spaces.