approach is to treat Q as part of the model parameters and to consider it as a usual estimation problem. This is often difficult from the computational aspect in that Q is a discrete matrix living on a high dimensional space, in particular, $Q \in \{0,1\}^{J \times K}$. Even with a reasonably small number of items and a few attributes, this space is often too large to explore thoroughly by any existing numerical method as the dimension grows exponentially fast with both J and K. Estimators developed based on this idea, even though theoretically sound, often suffer from substantial computational overhead. One of such instances is the maximum likelihood estimator of Q. Generally speaking, optimizing a discrete and nonlinear function over $\{0,1\}^{J \times K}$ is computationally intensive and sometimes infeasible. This approach does not take advantage of the special structures of the Q-matrix and further of the likelihood function.

A different approach is to cast the Q-matrix estimation in the context of variable selection. Consider the item response function $f_j(y|\theta, \boldsymbol{\alpha})$. If both the response Y^j and the latent variable $\boldsymbol{\alpha}$ were observed, then the estimation of Q is a regular variable selection problem. In most situations, f_j takes the form of a generalized linear model, in which the responses to items are the dependent variables, the attributes play the role of covariates, and the item parameters θ are the regression coefficients. Thus, the Q-matrix estimation is equivalent to a variable selection problem. However, in the context of latent class models, the covariates $\boldsymbol{\alpha}$'s are all missing and therefore the task is, rigorously speaking, to select latent variables. Chen et al. [3] took this viewpoint and developed estimation methods for the Q-matrix via regularized likelihood.

The last approach is similar to the previous one, but is more generic and is the primary focus of the current analysis. The introduction of the Q-matrix suggests that a single item usually does not provide information to differentiate all dimensions of the attribute profile. In particular, $q_{jk} = 0$ means that item j is irrelevant to attribute k. Under the setting of latent class models (not necessarily possessing a specific parameterization), this corresponds to an item-specific *partial information structure*. Each particular attribute profile $\boldsymbol{\alpha}$ in the DCM parameterization corresponds to one latent class. If an item does not differentiate all dimensions of $\boldsymbol{\alpha}$, then some distinct attribute