TABLE I: Known $c\bar{s}$ mesons organized according to s_ℓ^P and J^P; the measured masses are indicated. Allowed possibilities for $D_{sJ}(3040)$ are displayed as $\mathbf{D_{sJ}(3040)}$?.

s_ℓ^P	$\frac{1}{2}^-$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^-$
$(n=1)$					
$J^P = s_\ell^P - \frac{1}{2}$	$D_s(1965)$ (0^-)	$D_{sJ}(2317)$ (0^+)	$D_{s1}(2536)$ (1^+)	(1^-)	$\mathbf{D_{sJ}(3040)}$?(2^-)
$J^P = s_\ell^P + \frac{1}{2}$	$D_s^*(2112)$ (1^-)	$D_{sJ}(2460)$ (1^+)	$D_{s2}^*(2573)$ (2^+)	$\mathbf{D_{sJ}(3040)}$?(2^-)	$D_{sJ}(2860)$?(3^-)
$(n=2)$					
$J^P = s_\ell^P - \frac{1}{2}$	(0^-)	(0^+)	$\mathbf{D_{sJ}(3040)}$?(1^+)	(1^-)	(2^-)
$J^P = s_\ell^P + \frac{1}{2}$	$D_{sJ}(2710)$ (1^-)	$\mathbf{D_{sJ}(3040)}$?(1^+)	(2^+)	(2^-)	(3^-)

($D_{sJ}(2860)$ has been assigned to the $s_\ell = \dfrac{5}{2}$ doublet, with a question mark since confirmation is needed).

Some indications about the masses of these states come from potential model calculations. For example, in Ref.[11] the spectrum of heavy-light mesons is computed in the framework of a relativistic quark model (RQM), with results:

$$\begin{aligned} M(\tilde{D}_{s1})^{(RQM)} &= 3114 \text{ MeV} \\ M(\tilde{D}'_{s1})^{(RQM)} &= 3165 \text{ MeV} \\ M(D_{s2})^{(RQM)} &= 2953 \text{ MeV} \\ M(D^{*\prime}_{s2})^{(RQM)} &= 2900 \text{ MeV} . \end{aligned} \tag{3}$$

Notice that, if the identification of $D_{sJ}(2860)$ as the $J^P_{s_\ell} = 3^-_{5/2}$ state were experimentally confirmed, this would disfavor the assignment of $D_{sJ}(3040)$ to its spin partner $D^{*\prime}_{s2}$ with $J^P_{s_\ell} = 2^-_{5/2}$, since a mass inversion in a spin doublet seems unlikely. For a similar reason, one would also disfavor the identification of $D_{sJ}(3040)$ with D_{s2}, although in that case the two mesons would belong to different doublets.

The four classifications for $D_{sJ}(3040)$ in Table I can be discussed computing the allowed strong decays. To this purpose, we work in the heavy quark limit in which the various spin doublets are described by effective fields: H_a for $s_\ell^P = \frac{1}{2}^-$ ($a = u, d, s$ is a light flavour index); S_a and T_a for $s_\ell^P = \frac{1}{2}^+$ and $s_\ell^P = \frac{3}{2}^+$, respectively; X_a and X'_a for the doublets corresponding to orbital angular momentum $\ell = 2$, i.e. $s_\ell^P = \frac{3}{2}^-$ and $s_\ell^P = \frac{5}{2}^-$:

$$\begin{aligned} H_a &= \frac{1+\slashed{v}}{2}[P^*_{a\mu}\gamma^\mu - P_a\gamma_5] \\ S_a &= \frac{1+\slashed{v}}{2}\left[P'^\mu_{1a}\gamma_\mu\gamma_5 - P^*_{0a}\right] \\ T_a^\mu &= \frac{1+\slashed{v}}{2}\Bigg\{P^{\mu\nu}_{2a}\gamma_\nu \\ &- P_{1a\nu}\sqrt{\frac{3}{2}}\gamma_5\left[g^{\mu\nu} - \frac{1}{3}\gamma^\nu(\gamma^\mu - v^\mu)\right]\Bigg\} \\ X_a^\mu &= \frac{1+\slashed{v}}{2}\Bigg\{P^{*\mu\nu}_{2a}\gamma_5\gamma_\nu \\ &- P^{*\prime}_{1a\nu}\sqrt{\frac{3}{2}}\left[g^{\mu\nu} - \frac{1}{3}\gamma^\nu(\gamma^\mu - v^\mu)\right]\Bigg\} \\ X'^{\mu\nu}_a &= \frac{1+\slashed{v}}{2}\Bigg\{P^{\mu\nu\sigma}_{3a}\gamma_\sigma \\ &- P^{*\prime\alpha\beta}_{2a}\sqrt{\frac{5}{3}}\gamma_5\Bigg[g^\mu_\alpha g^\nu_\beta - \frac{1}{5}\gamma_\alpha g^\nu_\beta(\gamma^\mu - v^\mu) \\ &- \frac{1}{5}\gamma_\beta g^\mu_\alpha(\gamma^\nu - v^\nu)\Bigg]\Bigg\} \end{aligned} \tag{4}$$

with the various operators annihilating mesons of four-velocity v which is conserved in strong interaction processes (the heavy field operators contain a factor $\sqrt{m_P}$ and have dimension 3/2).

Let us consider decays with the emission of a light pseudoscalar meson. The octet of light pseudoscalar mesons is introduced considering the fields: $\xi = e^{\frac{i\mathcal{M}}{f_\pi}}$, $\Sigma = \xi^2$, and the matrix $\mathcal{M}$ containing π, K and η fields ($f_\pi = 132$ MeV):

$$\mathcal{M} = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \tag{5}$$

At the leading order in the heavy quark mass and light meson momentum expansion, the decays $F \to HM$ ($F = H, S, T, X, X'$ and M a light pseudoscalar meson) are described by the Lagrangian interaction terms [12]:

$$\begin{aligned} \mathcal{L}_H &= g\,Tr\Big[\bar{H}_a H_b \gamma_\mu\gamma_5\mathcal{A}^\mu_{ba}\Big] \\ \mathcal{L}_S &= h\,Tr\Big[\bar{H}_a S_b \gamma_\mu\gamma_5\mathcal{A}^\mu_{ba}\Big] + h.c. \\ \mathcal{L}_T &= \frac{h'}{\Lambda_\chi}Tr\Big[\bar{H}_a T^\mu_b(iD_\mu\slashed{\mathcal{A}} + i\slashed{D}\mathcal{A}_\mu)_{ba}\gamma_5\Big] + h.c. \\ \mathcal{L}_X &= \frac{k'}{\Lambda_\chi}Tr\Big[\bar{H}_a X^\mu_b(iD_\mu\slashed{\mathcal{A}} + i\slashed{D}\mathcal{A}_\mu)_{ba}\gamma_5\Big] + h.c. \\ \mathcal{L}_{X'} &= \frac{1}{{\Lambda_\chi}^2}Tr\Big[\bar{H}_a X'^{\mu\nu}_b\big[k_1\{D_\mu, D_\nu\}\mathcal{A}_\lambda \\ &+ k_2(D_\mu D_\lambda\mathcal{A}_\nu + D_\nu D_\lambda\mathcal{A}_\mu)\big]_{ba}\gamma^\lambda\gamma_5\Big] + h.c. \end{aligned} \tag{6}$$