a combination of a 755 nm filter with a 40 nm pass band with a long-pass filter at 715 nm for F2. The 13 peaks, labeled a through m in Fig. 6, were used to extract the transition frequencies. The peaks were acquired with both increasing and decreasing scans of f_{rep}, resulting in 26 individual peaks. By use of the procedure described above, the extracted center-of-gravity two-photon frequency and the hyperfine A constant for the $8S_{1/2}$ state were found to be 729 009 798.844(38) MHz and 219.137(19) MHz. The χ^2 was 31 for 26 peaks. Here the uncertainties are determined from the χ^2 function, as described in Section IV A, and do not include any systematic uncertainties.

The $6S_{1/2} \rightarrow 6P_{3/2} \rightarrow 8S_{1/2}$ transition is shown in Fig. 4. For this spectrum, one of the counter-propagating laser beams was filtered with an interference filter centered at 850 nm with a 10 nm pass band. This resulted in an average intensity of ≈ 60 W/cm^2 in the interaction region, spread over ≈ 4000 optical modes. This light served to excite the first step of the two-photon transition at 852 nm. Based on the transmission profile of the filter we estimate that we have ≈ 10 mW/cm^2 of light in the optical mode resonant with the $6S_{1/2} \rightarrow 6P_{3/2}$ transition. The second stage of the transition was selected by use of a combination of a 800 nm short-pass filter and a 780 nm long-pass filter and provided ≈ 70 W/cm^2 over ≈ 8000 modes. Using the procedure described above we fit the 14 peaks labeled in the figure and find a center-of-gravity frequency of 729 009 798.863(29) MHz and a hyperfine coupling constan 219.133(17) MHz, where again, the uncertainties are determined purely from the χ^2 function. The value of the χ^2 was 64 for 28 peaks (14 peaks from both the increasing and decreasing frequency scans) prior to the normalization. As described in Section IV A the uncertainties are increased to make the χ^2 consistent with the expected value. The resulting frequencies are in good agreement with the value extracted from the transition through the $6P_{1/2}$ state.

We analyzed in detail the systematic effects that could lead to a shift of the measured frequencies from the true transition frequencies using the transitions to the $8S_{1/2}$ state through the $6P_{3/2}$ state. Systematic shifts might be more significant for excitation through the $6P_{3/2}$ state, because of the smaller hyperfine structure shifts of the $6P_{3/2}$ state, relative to the $6P_{1/2}$ state. In addition, the signal-to-noise ratio of the spectrum excited through the $6P_{3/2}$ state was higher than that excited through the $6P_{1/2}$ state, as an artifact of the filters used in the measurements.

We considered four possible sources of systematic effects: ac Stark shifts, Zeeman shifts, pressure shifts, and errors arising from misalignment of the counter-propagating laser beams. The associated uncertainties are summarized in Tab. I and described below.

To evaluate the systematics we focused on the peaks labeled d, e, and f in Fig. 4. A typical data set and fit to the peaks is shown in Fig. 5. The effect of ac Stark shifts on the extracted frequencies was investigated by varying the optical powers. Neutral density filters were used to reduce the power in both beams by approximately the same amount. Data were collected at three optical powers ranging from the maximum power, which was used for collection of the full spectrum shown in Fig. 4, to 60% of the maximum power. The three peaks used in this analysis correspond to the $6S_{1/2}\,(F=4) \rightarrow 6P_{3/2}\,(F'=3,4,5) \rightarrow 8S_{1/2}\,(F''=4)$ transitions. Because the final state is the same for all three peaks, it is not possible to extract both the center-of-gravity frequency and the hyperfine A constant from these data. In order to place a limit on possible effects due to the ac Stark shifts on the center-of-gravity frequency, we fixed the hyperfine A constant to the value determined from the analysis of the full spectrum and then extracted a center-of-gravity frequency at each power by finding the minimum in the χ^2 function as defined by Eq. (10). The center-of-gravity frequencies as a function of power were fitted to a straight line. We used the slope of the line to look for any possible dependence of the center-of-gravity frequency on the power. We find the slope to be $-43(123)$ kHz/P, where P is the normalized operating power. Combining the value of the slope with its uncertainty we arrive at a maximum possible shift of 166 kHz at the the nominal operating power, $P = 1$. We take this value as an estimate of the systematic uncertainty from the ac Stark effect. Because we fix the hyperfine A constant this approach provides an upper limit on the center-of-gravity frequency. Therefore, we do not attempt to apply a correction to the data. To determine the possible effect on the hyperfine splitting we fix the center-of-gravity frequency to the value determined from the analysis of the full spectrum and extract the hyperfine A constant from the data. Fitting the data for the A constant we found a slope of $-22(72)$ kHz/P, resulting in an uncertainty of 94 kHz.

For two-photon transitions between S states with $\Delta M_F = 0$ there is no linear Zeeman shift. However, the laser polarization is not perfectly linear and these states are step-wise resonant through the intermediate P states, which are magnetically sensitive. In addition, the power on the first stage of the transition is near saturation, so there may be some optical pumping that could couple with an external magnetic field to lead to asymmetric line shapes. To investigate the effect of Zeeman-shifts due to imperfect cancelation of stray magnetic fields, data were collected without the compensation coils and also with the current through the compensation coils reversed, thus doubling the residual magnetic field. Again, the peaks labeled d, e, and f in Fig. 4 were used. The value of the hyperfine A constant was fixed and the center-of-gravity frequency was extracted as described above. We see no evidence for a systematic shift as the magnetic field is increased (Fig. 7). The standard deviation of the data is 70 kHz, which we take as the uncertainty associated with the magnetic field. A similar analysis for the hyperfine A constant (with the center-of-gravity frequency fixed) also shows no evidence of any shift with the magnetic field.