## IV. FIELDS WITH CURRENT-HELICITY

### A. Steep rotation law

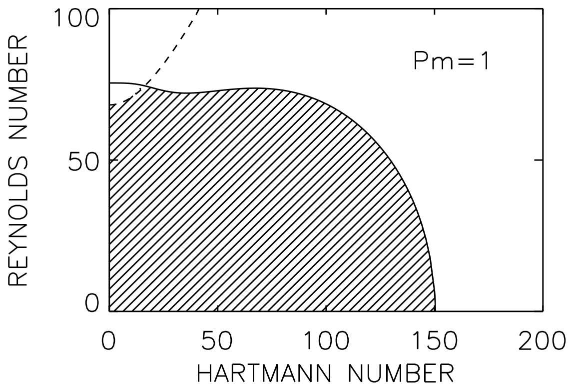


FIG. 5: Tayler instability (TI) of a toroidal field under the influence of differential rotation with stationary outer cylinder. The solid curve is $m = 1$, the dotted curve is $m = 0$. $\mu_B = 1$, $\mu_\Omega = 0$. $\mathrm{Pm} = 1$.

We begin by considering the stability of purely toroidal fields, and differential rotation profiles with a stationary outer cylinder. There are then three classical results known: First, in the absence of any fields, axisymmetric Taylor vortices arise at $\mathrm{Re} = 68$, and nonaxisymmetric instabilities at $\mathrm{Re} = 75$. Second, in the absence of any rotation, $m = 1$ Tayler instabilities arise at $\mathrm{Ha} = 150$. Figure 5 shows how these results are linked when both Ha and Re are non-zero. For Ha very small, the axisymmetric Taylor vortex mode is stabilized, whereas the nonaxisymmetric mode is eventually destabilized, and connects smoothly to the pure Tayler instability.

The next step is to add a uniform axial field to the azimuthal field, with (say) positive polarity. The background field then has a positive helicity, that is, it spirals to the right. If the axial field is weak, e.g. with $\beta = 100$, then the marginal instability curves (Fig. 6, top) strongly resemble the map for $\beta \to \infty$ (Fig. 5). The main differences are i) the slightly smaller Hartmann number of the toroidal field, and ii) the splitting of the spiral modes $m = 1$ and $m = -1$ into two curves with different helicity (R and L). The left-hand modes require a greater rotation than the right-hand modes. For background fields with positive helicity, we thus find that the right spirals are preferred, whereas for background fields with negative helicity, R and L would be exchanged, and the left spirals would be preferred.

For $\beta = 10$ the differences between the L and R modes for given $m$ increase, so that the pure Tayler instability exists only as the 1R mode. The 1L mode no longer connects to $\mathrm{Re} = 0$, and is not the most unstable mode anywhere in the given domain (Fig. 6, middle).

The 1R mode also dominates for $\beta$ of order unity. There is, however, an interesting particularity in this case. For very slow rotation, a 2R mode reduces the stability domain. For

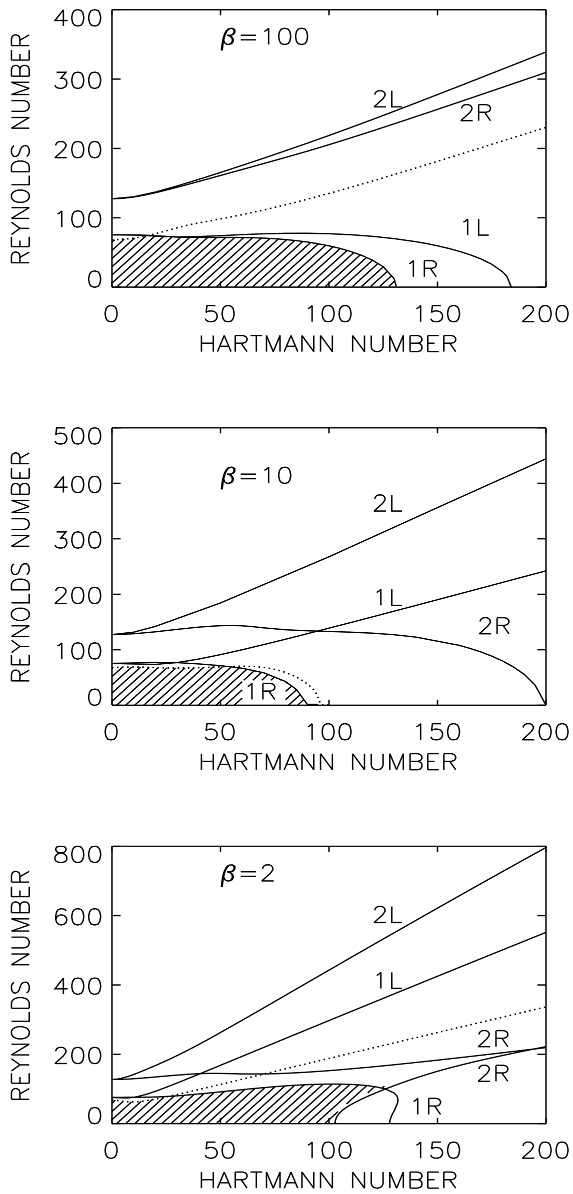


FIG. 6: Instability curves for magnetic fields with positive current helicity. The curves are marked with their azimuthal wave numbers $m$. The shaded areas are the stable regions. Without magnetic fields the curves always start at $\mathrm{Re} = 68$ for the $m = 0$ Taylor vortices. Note the dominance of the axisymmetric modes (dotted) also for weak magnetic fields, unless for very high $\beta$ the field becomes nearly toroidal. $\mathrm{Pm} = 1$, $\mu_B = 1$, $\mu_\Omega = 0$.

$\mathrm{Re} \simeq 0$, and in a limited range of Ha ($\mathrm{Ha} \simeq 100...130$), this mode forms the first instability (see [13]). A small amount of differential rotation, however, brings the system back to the 1R instability.

For models with helical fields and steep rotation laws (with stationary outer cylinder), we indeed find the expected splitting between right and left spiral instabilities. If the axisymmetric background field is right-handed, then the first unstable