

changes, and so does the set  $\mathcal{N}(\mathcal{S}') - \mathcal{S}'_{\mathcal{I}}$ . In addition, the span of a subset of  $\{h'_1, \dots, h'_r\}$  can change after the operation of  $V$ , though the span of the full set remains unchanged. This means that if we add less than the maximum amount of entanglement to a code, we must optimize over such unitary row operations. Since the group  $\mathcal{S}'_{\mathcal{I}}$  and the set  $\mathcal{N}(\mathcal{S}') - \mathcal{S}'_{\mathcal{I}}$  remain the same under the operation of type 1 unitary row operators that operate on the  $h'_j$  for  $j \notin T$ , it suffices to assume that the operation  $V$  consists of type 1 unitary row operators that operate only on the  $h'_j$  for  $j \in T$ .

Let  $M_V$  be a  $c \times r$  matrix such that the  $i$ -th row of  $M_V$  is the  $t_i$ -th row of  $M_Z$  for  $i = 1, \dots, c$ . It is obvious that some  $M_V$ 's have the same effect on the row space of  $H'$ . For example, if  $c = 2$ ,  $\{g'_1 g'_2, g'_2, \dots, g'_r, h'_1, h'_1 h'_2\}$  and  $\{g'_1, g'_2, \dots, g'_r, h'_1, h'_2\}$  are two different sets of generators but they generate the same space and hence their corresponding EAQEC codes have the same minimum distance. Therefore, a distinct unitary row operation  $V$  is assumed to be represented by a matrix  $M_V$  in reduced row echelon form.

**Theorem 3.** The operation of  $V$  is equivalent to applying a series of type 1 unitary row operators on the  $h'_j$  for  $j \in T$ . In addition, there are

$$N(r, c) \triangleq \sum_{l_c=0}^{r-c} \sum_{l_{c-1}=0}^{l_c} \sum_{l_{c-2}=0}^{l_{c-1}} \dots \sum_{l_1=0}^{l_2} 2^{c(r-c) - \sum_{i=1}^c l_i}$$

distinct unitary row operations.

*Proof.* The total number of distinct unitary row operations  $N(r, c)$  is determined as follows. If we begin with matrices of the form

$$\begin{bmatrix} 1 & 0 & \dots & 0 & \square & \dots & \square \\ 0 & 1 & \dots & 0 & \square & \dots & \square \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & \square & \dots & \square \end{bmatrix},$$