The study of heat conduction is very important from both theoretical and experimental point of view. A traditional phenomenological approach to understand the thermal properties in solids is the Debye formula given by

$$\kappa = \sum_k C_k v_k l_k, \tag{1}$$

where κ is the thermal conductivity, C_k, v_k, l_k are the specific heat, the phonon group velocity and the phonon mean free path of mode k, respectively. In spite of its successfulness for qualitative explanation of heat conduction in dielectrics, quantitative predictions are hard to make from a microscopic viewpoint.

Recently, an increasing study of heat conduction in low dimensional Hamiltonian models may shed light on its microscopic understanding [1]. However, only a few integrable models can be solved rigourously [2]. Generally one has to rely on numerical simulations for non-integrable models. Thus it would be worthy to revisit the traditional kinetic approach by incorporating some microscopic consideration for the low dimensional non-integrable lattice systems.

According to the Debye formula, the thermal transport process in a nonlinear lattice is intrinsically relative to its dispersion relation and relaxation of normal modes (phonons). The existence of nonlinearity makes the definition of phonon delicate, and it is even harder in this case to quantify phonon transport from a first-principle way. To surmount the difficulties due to nonlinearity, the concept of "effective phonons" has been recently introduced to study heat conduction in dynamical models [3–6]. The basic idea consists in incorporating the nonlinearity into normal modes by renormalizing the harmonic frequency spectrum. In the serial studies [3–5], the authors apply the so-called effective phonon theory (EPT) to study heat conduction within the kinetic framework. In Ref. [6], the authors study heat conduction through a lattice consisting of two weakly coupled nonlinear segments via the self-consistent phonon theory (SCPT).

It is thus interesting to compare EPT and SCPT since they have both been applied to the field of heat conduction. In the present study, we will give a detailed comparison via the Debye formula as done in [3–5]. Our result shows the equivalence of SCPT and EPT in a large range of temperature. Considering the failure of the classical description at low temperature regime, we also compute quantum corrections to the thermal conductivity by extending our study to quantum regime, which gives qualitatively consistent results with