in Chapter 5. The renormalization can be implemented using the methods discussed in Chapter 1. However, in order to perform the renormalization in a self-consistent fashion, renormalization-induced contributions must be included. In practical terms this means that the explicit $\mathcal{O}(\epsilon)$ terms in the leading order perturbative contribution must be calculated. These terms can be calculated using the loop integration methods discussed in Chapter 2. After the correlation function has been renormalized, the imaginary part is needed for the QSR analysis. A closed form expression for the imaginary part can be determined using methods discussed in Chapter 2.

Mass predictions were successfully extracted for all positive parity diquarks. However, mass predictions could not be extracted for any negative parity diquarks due to instabilities in those sum rules. The scalar and axial vector charm-light diquark masses were found to be $1.86 \pm 0.05\,\text{GeV}$ and $1.87 \pm 0.10\,\text{GeV}$, respectively. These mass predictions are degenerate within uncertainty as expected by heavy quark symmetry and in excellent agreement with the constituent charm-light diquark mass of $1.93\,\text{GeV}$ predicted by Maiani *et al.* [84]. Similarly, the scalar and axial vector bottom-light diquark masses were both found to be $5.08{\pm}0.04\,\text{GeV}$, which is in reasonable agreement with the mass of $5.20\,\text{GeV}$ determined by Ali *et al.* [10]. Therefore, these heavy-light diquark mass predictions support interpreting the $X(3872)$ and the $Y_b\,(10890)$ as tetraquarks. This QCD-based test supports the constituent diquark model of tetraquarks, and provides indirect support for the tetraquark interpretation of the charged heavy quarkonium-like states $Z_c^{\pm}\,(3895)$, $Z_b^{\pm}(10610)$ and $Z_b^{\pm}(10650)$.

The research presented in this chapter will contribute to the ongoing effort to understand the $X(3872)$, $Y_b\,(10890)$ and the electrically charged heavy quarkonium-like states. There are several technical challenges that are involved in calculating the next-to-leading order perturbative contributions to the heavy-light diquark correlation function. Although only the imaginary part of the correlation function is required for the QSR analysis, the entire correlation function must be calculated in order to properly deal with the gauge invariance and renormalization issues that arise in this calculation. The loop integration techniques discussed in Chapter 2 are essential for this. In order to verify that the heavy-light diquark correlation function is gauge invariant, and hence is suitable for use in a QSR analysis, the entire correlation function must be calculated in a general covariant gauge. In addition, the