the inefficient HD of the value x_I:

$$\mathcal{W}_{\mathrm{retr}}^{[x_I;\phi]}(x_\phi, p_\phi) \underset{e_n \to +\infty}{\sim} \frac{1}{\pi\sqrt{1+e_n\, s(\eta)}} \exp\left[-\frac{\left(x_\phi - x_I/\sqrt{\eta}\right)^2}{s(\eta)} - \frac{p_\phi^2}{1/s(\eta)+e_n}\right] \tag{31}$$

We recognize a squeezed coherent state with an amplitude $\alpha_I = x_I/\sqrt{2\eta}$ and a squeezing parameter $s(\eta) = (1-\eta)/\eta$, but with a very large excess noise e_n following the conjugate quadratue p_ϕ.

We can therefore say that an HD performs non-classical measurements in the same sense as squeezed states are non-classical states. Indeed, the squeezing of the pre-measurement state (31) is ensured for a detection efficiency such that $\eta > 50\,\%$.

In the continuous-wave (cw) regime, the detection efficiency η can currently reach values around 98% [29], corresponding to a really impressive squeezing level $s_{\mathrm{dB}} \simeq -17\,\mathrm{dB}$. At our knowledge, one of best levels recorded in the cw regime [30] is $-10\,\mathrm{dB}$, corresponding to an efficiency around 90%. Obviously, the pre-measurement state retrodicted from such a detection has a lower purity also corresponding to the projectivity of the HD.

VI. CONCLUSION

In this paper, we have shown that the retrodiction of pre-measurement states, in addition to the usual predictive approach, leads to a complete study of the measurement apparatus as an implementation of tests checking propositions about the system interacting with it. These propositions simply correspond to POVM elements describing the behavior of its responses in the predictive approach. When a certain proposition is checked, the pre-measurement state retrodicted from this one highlights the behavior of the apparatus in terms of states, which is the main language of quantum physics. We have also proposed an experimental procedure, essentially based on the retrodictive approach, allowing the direct tomography of pre-measurement states for making such a translation. We hope that this experimental procedure will be realized for probing each response of a measurement device. These results, or those from the predictive version [4], could then be interpreted in the framework of our