at zero field is due to electronic scattering while the origin of the other half remains to be clarified. Finally, we note that the observed effect of the magnetic field on the $2D$ peak is not specific to monolayer graphene; it should be analogous for multilayer graphene and graphite.

Acknowledgments

Part of this work has been supported by EC MTKD-CT-2005-029671, EuromagNetII, PICS-4340, ANR-08-JCJC-0034-01 and ANR-06-NANO-019 projects. P.K. is financially supported by the EU under FP7, contract no. 221515 'MOCNA'. M. A. thanks the Cariplo Foundation (project QUANTDEV), MICINN (Project MOSAICO) and JCYL SA052A07 for support.

Appendix A: Quasiclassical calculation of the two-phonon Raman matrix element in a uniform magnetic field

1. Classical action for a Dirac particle

The two-phonon Raman matrix element is given by a loop of four electronic Green's functions. The latter will be taken in the quasiclassical approximation. The most important ingredient of the quasiclassical Green's function is the classical action. Thus, we first discuss the classical action, then we give the explicit expression for the electronic Green's function, and finally, we perform the calculation of the Raman matrix element. In fact, the first two steps have been already made by Carmier and Ullmo[32], but we include them here for the sake of completeness of the presentation, and to fix the notations.

We start with the classical equation of motion of a charge in the magnetic field $\mathbf{B}$:

$$\frac{d\mathbf{p}}{dt} = \frac{e}{c}\,\mathbf{v}\times\mathbf{B} \tag{A1}$$

For a Dirac electron with energy ϵ the velocity $\mathbf{v}(t) = v\mathbf{n}(t)$ and momentum $\mathbf{p}(t) = (\epsilon/v)\mathbf{n}(t)$ are expressed in terms of a unit vector, $|\mathbf{n}| = 1$. [If instead of electrons with positive and negative energies one prefers to work with positive-energy electrons and holes, then $\mathbf{p}(t) = |\epsilon/v|\mathbf{n}(t)$, so the action given below should be multiplied by $\operatorname{sgn}\epsilon$]. Eq. (A1) can be obtained by variation of the action functional

$$S[\mathbf{r}(t)] = \int dt\left(\frac{\epsilon}{v}\,|\dot{\mathbf{r}}| + \frac{e}{c}\,\mathbf{A}(\mathbf{r})\cdot\dot{\mathbf{r}}\right) \tag{A2}$$

keeping the ends $\mathbf{r}_1, \mathbf{r}_2$ of the trajectory fixed. Indeed, upon integration by parts

$$\begin{aligned}\delta S \;&=\; \int dt\left[-\frac{\epsilon}{v}\,\frac{d}{dt}\frac{\dot{x}_i}{|\dot{\mathbf{r}}|} + \frac{e}{c}\,\dot{x}_j\left(\frac{\partial A_j}{\partial x_i} - \frac{\partial A_i}{\partial x_j}\right)\right]\delta x_i \\ &\quad + \mathbf{p}_2\cdot\delta\mathbf{r}_2 - \mathbf{p}_1\cdot\delta\mathbf{r}_1,\end{aligned} \tag{A3}$$

where indices $i, j = x, y, z$ label the Cartesian components, and the momentum is defined as

$$\mathbf{p} \equiv \frac{\epsilon}{v}\,\frac{\dot{\mathbf{r}}}{|\dot{\mathbf{r}}|} + \frac{e}{c}\,\mathbf{A}(\mathbf{r}). \tag{A4}$$

If we now define the function $S(\mathbf{r}, \mathbf{r}')$ as the action on the classical trajectory, corresponding to the motion from $\mathbf{r}'$ to $\mathbf{r}$ according to Eq. (A1), we obtain $\nabla S = \mathbf{p}$ at the end of the trajectory. Thus, this function satisfies the Hamilton-Jacobi equation

$$\left|\nabla S(\mathbf{r},\mathbf{r}') - \frac{e}{c}\,\mathbf{A}(\mathbf{r})\right| = \left|-\nabla' S(\mathbf{r},\mathbf{r}') - \frac{e}{c}\,\mathbf{A}(\mathbf{r}')\right| = \frac{|\epsilon|}{v}. \tag{A5}$$

Also, $\partial S/\partial\epsilon$ is equal to the time it takes to go along the trajectory.

In a uniform magnetic field, $\mathbf{n}$ is precessing with a constant frequency $\omega = -(eB/c)(v^2/\epsilon)$, and the trajectories are circles of the radius $R = v/|\omega|$. Two particular points $\mathbf{r}$ and $\mathbf{r}'$ can be connected either by two trajectories corresponding to short and long arcs (plus an integer number of full rotations), or by no trajectories at all if the distance between the points is greater than the circle diameter, $|\mathbf{r} - \mathbf{r}'| > 2R$. Let us assume $\mathbf{B}$ to be along the z axis and choose the gauge $\mathbf{A}(\mathbf{r}) = (B/2)[\mathbf{e}_z \times \mathbf{r}]$. The action along the short/long arc is given by

$$\begin{aligned}S_\pm(\mathbf{r},\mathbf{r}') = {}& \frac{\epsilon R}{2v}\left[\vartheta_\pm(\mathbf{r},\mathbf{r}') + \sin\vartheta_\pm(\mathbf{r},\mathbf{r}')\right] \\ & - \frac{eB}{2c}\,[\mathbf{r}\times\mathbf{r}']_z,\end{aligned} \tag{A6}$$

$$\vartheta_+(\mathbf{r},\mathbf{r}') = 2\arcsin\frac{|\mathbf{r}-\mathbf{r}'|}{2R}, \tag{A7}$$

$$\vartheta_-(\mathbf{r},\mathbf{r}') = 2\pi - 2\arcsin\frac{|\mathbf{r}-\mathbf{r}'|}{2R}. \tag{A8}$$

Here $\vartheta_\pm(\mathbf{r},\mathbf{r}')$ is the angular size of the short/long arc. We also introduce $\vartheta_\pm^{(j)}(\mathbf{r},\mathbf{r}') = 2\pi j + \vartheta_\pm(\mathbf{r},\mathbf{r}')$. We denote by $\mathbf{n}_\pm$ the unit tangent vector to the short/long arc at the point $\mathbf{r}$ (Fig. 6):

$$\begin{aligned}\mathbf{n}_\pm(\mathbf{r},\mathbf{r}') \;&=\; \left[\mathbf{e}_z\times\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\right]\sin\frac{\vartheta_\pm(\mathbf{r},\mathbf{r}')}{2}\operatorname{sgn}\omega \\ &\quad + \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\cos\frac{\vartheta_\pm(\mathbf{r},\mathbf{r}')}{2}.\end{aligned} \tag{A9}$$

The tangent determines the direction of the kinematic momentum:

$$\nabla S_\pm(\mathbf{r},\mathbf{r}') - \frac{e}{c}\mathbf{A}(\mathbf{r}) = \frac{\epsilon}{v}\,\mathbf{n}_\pm(\mathbf{r},\mathbf{r}'), \tag{A10}$$

$$-\nabla' S_\pm(\mathbf{r},\mathbf{r}') - \frac{e}{c}\mathbf{A}(\mathbf{r}') = -\frac{\epsilon}{v}\,\mathbf{n}_\mp(\mathbf{r},\mathbf{r}'). \tag{A11}$$