

there are $\frac{n(n-1)}{2}$ different equations. These equations represent the system's restrictions; they are the constraint equations of the system.

Einstein must have thought that he would obtain the number of degrees of freedom of S_3 merely by subtracting the number of constraint equations from $3n$:

$$N_3 = 3n - \frac{n(n-1)}{2} \quad (2)$$

If (2) is solved for $n > 4$, it will be seen that the values of N_3 differ from those obtained when S_3 was viewed as a single body. Why does this happen? Maybe because it is not all appropriate to consider the collection of particles with rigid links as one body. Or more likely, because the count of the degrees of freedom of S_3 by consideration of the individual particles was not done correctly. Which ever the reason may be, we will soon find out.

As it turns out, there is something definitely wrong with (2), and it is that

$$3n - \frac{n(n-1)}{2} \approx -\frac{n^2}{2} < 0, \quad (3)$$

for $n \gg 1$,

which is absurd.

Einstein did notice this flaw, because in his book, instead of (2) he has:

$$N_3 = \frac{n(n-1)}{2} - 3n \quad (4)$$

We cannot think of any physical or mathematical justification for this change of signs, and although it removes the problem of getting a negative value of N_3 when $n \gg 1$, it brings up a new problem.

In the limit when n tends to infinity, the system S_3 is equivalent to a rigid body of continuous mass. So it would be expected that if the limit of N_3 is taken when n tends to infinity, this limit should be equal to six. But this does not hold true for N_3 as defined in (4); the limit when n tends to infinity diverges.

Einstein introduced, as a footnote, the following correction:

$$N_3 = \frac{n(n-1)}{2} - 3n + 6 \quad (5)$$

Nonetheless, the limit when n tends to infinity of the modified N_3 is still undefined, so (5) cannot be the correct expression for N_3 either.

When we took up the task of developing an accurate expression for N_3 , we did not take off from where Einstein left the problem, but instead, we directed our attentions back to (2), which is the expression that Einstein must have come up with originally, in spite of the fact that it doesn't appear in his book. We did so because, as incorrect as it may be, there is a consistent line of thinking behind expression (2), which there is not behind expressions (4) or (5).

Expression (3) gave us a hint of where the