or through the end mirror of the cavity. Note that in the latter case the emitted photon can in principle be detected via photon counting, provided it can be distinguished from the coherent component exiting the cavity (see Section III). If the coupling is large enough the excitation can be exchanged a number of times between the cavity and the mechanics before being lost. Such an oscillation will modulate the detection rate for the photons leaving the cavity and will hence leave an unambiguous signature of the coherent exchange of energy between the cavity and the mechanical resonator. Eventually this detection rate will return to zero as the opto-mechanical system returns to the steady state.

II. MASTER EQUATION FOR CASCADED SYSTEMS.

The interaction picture master equation describing the interaction between the system is

$$\begin{aligned}\frac{d\rho}{dt} &= -i\Delta[a^\dagger a,\rho] - i\omega_m[b^\dagger b,\rho] - ig[(a+a^\dagger)(b+b^\dagger),\rho] + \kappa\mathcal{D}[a]\rho + \gamma\mathcal{D}[c]\rho \\ &\quad +\mu(\bar{n}+1)\mathcal{D}[b]\rho + \mu\bar{n}\mathcal{D}[b^\dagger]\rho - i\Delta[c^\dagger c,\rho] + \sqrt{\gamma\kappa}\left([c\rho,a^\dagger]+[a,\rho c^\dagger]\right)\end{aligned} \tag{6}$$

where the interaction picture is defined by the coherent driving laser. Here, μ is decay rate of the mechanical system resonator, $\bar{n}$ is the mean thermal excitation of the mechanical environment at frequency ω_m and $c, c^\dagger$ are the annihilation and creation operators for the field of the source cavity.

To demonstrate a successful state transfer we calculate the value of $\langle a^\dagger a\rangle$ as the count rate for the photon emitted from the cavity is proportional to this quantity. As the Hamiltonian is at most quadratic in the field amplitude operators, a closed system of equations can be obtained for the second order moments. To this end, we define the correlation matrix

$$C(t) = \langle \vec{A}(t)\vec{A}^T(t)\rangle \tag{7}$$

where $\vec{A}^T = (a(t), a^\dagger(t), b(t), b^\dagger(t))$. This obeys the following system of equations,

$$\frac{dC(t)}{dt} = KC(t) + C(t)^T K^T - \sqrt{\gamma\kappa}N(t) \tag{8}$$

where

$$K = \begin{pmatrix} -\tilde{\kappa} & 0 & -ig & -ig \\ 0 & -\tilde{\kappa}^* & ig & ig \\ -ig & -ig & -\tilde{\mu} & 0 \\ ig & ig & 0 & -\tilde{\mu}^* \end{pmatrix} \tag{9}$$