perbolic space H_3, whose boundary is the physical transverse space $\mathbb{R}^2$. Using Poincaré coordinates,

$$ds^2(H_3) = \frac{dr^2 + ds^2(\mathbb{R}^2)}{r^2} \ ,$$

we identify L with the geodesic distance between two points that are separated by $l_\perp$ along $\mathbb{R}^2$ and have radial coordinates r and $\bar{r}$. The variable S measures the local energy squared of the scattering process in AdS. The Greek indices μ and τ in (2) label tangent directions to H_3, which are the physical polarizations of the AdS gauge field dual to the conserved current j^a. The functions

$$\Psi^{ab\mu\tau}(r) = \psi_{in}^{a\mu}(r)\, \psi_{out}^{b\tau}(r)\,, \qquad \Phi(\bar{r}) = \phi_{in}(\bar{r})\, \phi_{out}(\bar{r})\,,$$

are given by the product of the radial part of the incoming and outgoing dual AdS fields. These functions are non-normalizable because they are produced by a plane wave source created by the dual operator at the boundary.

We shall consider a black disk model defined by a phase shift in the impact parameter representation (2) given by

$$\Big[1 - e^{i\chi(S,L)}\Big]^{\mu}_{\ \tau} = \Theta\big(L_s(S) - L\big)\, \delta^\mu_{\ \tau} \ ,$$

where the radius L_s of the disk increases with energy as

$$L_s(S) \approx \omega \log S \ . \tag{3}$$

Note that the size of the disk is independent of the dual AdS gauge field polarization, so that this simple model is characterized by the single parameter $\omega > 0$. To motivate this model, in section V we assume Reggeon exchange and consider the limiting cases corresponding to the BFKL Pomeron at weak coupling and $\mathcal{N} = 4$ SYM at strong coupling. We then use geometric scaling observed in DIS at low x to phenomenologically fix ω.

III. DEEP INELASTIC SCATTERING

The total DIS cross section, and corresponding hadron structure functions, are related to the hadronic tensor

$$W^{ab}(k_j) = i \int d^4y\, e^{ik_1\cdot y} \langle k_2 | \mathrm{T}\left\{ j^a(y) j^b(0) \right\} | k_2 \rangle \,,$$

where j^a is the electromagnetic current and $|k_2\rangle$ is the target hadron state of momentum k_2. We define the virtuality $Q^2 = k_1^2$, target mass $M^2 = -k_2^2$ and Bjorken

$$x = -\frac{Q^2}{2k_1 \cdot k_2} \approx \frac{Q^2}{s} \ .$$

Lorentz invariance and conservation restricts W^{ab} to

$$W^{ab} = \left(\eta^{ab} - \frac{k_1^a k_1^b}{k_1^2}\right)\Pi_1 + \frac{2x}{Q^2}\left(k_2^a + \frac{k_1^a}{2x}\right)\left(k_2^b + \frac{k_1^b}{2x}\right)\Pi_2 \,.$$

The structure functions F_i satisfy $2\pi F_i = \mathrm{Im}\,\Pi_i$.

At zero momentum transfer we can use the representation (2) to write the hadronic tensor as

$$W^{ab} \approx 4\pi i s \int \frac{dr}{r^2}\frac{d\bar{r}}{\bar{r}^2}\, \Psi^{ab\ \ \tau}_{\ \ \mu}(r)\ \Phi(\bar{r}) \times \int_{|\ln \bar{r}/r|}^{+\infty} dL \sinh L \left[e^{i\chi(S,L)} - 1\right]^{\mu}_{\ \tau} \ , \tag{4}$$

where we did the angular integral in the impact parameter $l_\perp$ and traded $|l_\perp|$ in the radial integration for the AdS impact parameter L. Note that $\Phi(\bar{r}) = |\phi(\bar{r})|^2$, where now $\phi(\bar{r})$ is the radial part of the normalizable AdS wave function dual to the state $|k_2\rangle$. This wave function is localized in the IR around $\bar{r} \sim 1/M$. Its explicit form in the IR region, where space is no longer AdS, will not be important in what follows, because we shall consider a hard probe localized near the AdS boundary.

The black disk model of the previous section gives

$$W^{ab} \approx -2\pi i s \int \frac{dr}{r^2}\frac{d\bar{r}}{\bar{r}^2}\, \Psi^{ab\ \ \mu}_{\ \ \mu}(r)\ \Phi(\bar{r}) \times \left[(sr\bar{r})^{\omega} + (sr\bar{r})^{-\omega} - \frac{r}{\bar{r}} - \frac{\bar{r}}{r}\right] . \tag{5}$$

At very low x the first term dominates and we have

$$W^{ab} \approx -2\pi i s^{1+\omega} \int \frac{dr}{r^{2-\omega}}\, \Psi^{ab\ \ \mu}_{\ \ \mu}(r) \int \frac{d\bar{r}}{\bar{r}^{2-\omega}}\, \Phi(\bar{r})\,. \tag{6}$$

To give the explicit form of Ψ it is convenient to write

$$k_1 = \left(\sqrt{s}, -\frac{Q^2}{\sqrt{s}}, 0\right) \ , \qquad k_2 = \left(\frac{M^2}{\sqrt{s}}, \sqrt{s}, 0\right) \ , \tag{7}$$

in light-cone coordinates $(+,-,\perp)$. Then, following [18],

$$\Psi^{ab\ \ \mu}_{\ \ \mu}(r) = -\frac{\pi^2}{6} C\, r^2 \int_0^\infty dudv\, e^{-u-v-\frac{Q^2r^2}{4u}-\frac{Q^2r^2}{4v}} \times \begin{pmatrix} \frac{sr^2}{4uv} & \frac{v-1}{u} & 0 \\ \frac{u-1}{v} & \frac{4(u-1)(v-1)}{sr^2} & 0 \\ 0 & 0 & \mathbb{I} \end{pmatrix} , \tag{8}$$

where the matrix elements are also ordered by the light-cone coordinates. In particular, we have

$$\Psi^{ij\ \ \mu}_{\ \ \mu}(r) = -\delta^{ij}\frac{\pi^2}{6}\, CQ^2r^4K_1^2(Qr)\,,$$
$$\Psi^{++\ \ \mu}_{\ \ \ \ \mu}(r) = -\frac{\pi^2}{6}\, Csr^4K_0^2(Qr)\,,$$

where i,j run over the transverse space $\mathbb{R}^2$ directions and K is the Bessel function of the second kind. The constant C is determined by the conformal two point function

$$\langle j^a(y) j^b(0) \rangle = C\, \frac{y^2\eta^{ab} - 2y^a y^b}{(y^2 + i\epsilon)^4} \ .$$

By dimensional analysis the integral over $\bar{r}$ in (6) is

$$\int \frac{d\bar{r}}{\bar{r}^{2-\omega}}\, \Phi(\bar{r}) = \frac{h(\omega)}{M^{1+\omega}} \,,$$