$1 \leq j_0 \leq n$, such that

$$|g_{j_0}(x(s_0)) - g_{j_0}(\overline{x}(s_0))| \geq \frac{L_g}{n} \|x(s_0) - \overline{x}(s_0)\| > \frac{L_g \epsilon_0}{n}. \tag{10.17}$$

On the other hand, making use of the inequality (10.16) it can be verified for all $t \in [\theta, \theta + \tau]$ that

$$\begin{aligned}
&|g_{j_0}(x(s_0)) - g_{j_0}(\overline{x}(s_0))| - |g_{j_0}(x(t)) - g_{j_0}(\overline{x}(t))| \\
&\leq |(g_{j_0}(x(t)) - g_{j_0}(\overline{x}(t))) - (g_{j_0}(x(s_0)) - g_{j_0}(\overline{x}(s_0)))| \\
&< \frac{L_g \epsilon_0}{2n}.
\end{aligned}$$

Therefore, by means of (10.17), we have that the inequality

$$|g_{j_0}(x(t)) - g_{j_0}(\overline{x}(t))| > |g_{j_0}(x(s_0)) - g_{j_0}(\overline{x}(s_0))| - \frac{L_g \epsilon_0}{2n} > \frac{L_g \epsilon_0}{2n} \tag{10.18}$$

is valid for $t \in [\theta, \theta + \tau]$.

One can find numbers $s_1, s_2, \ldots, s_n \in [\theta, \theta + \tau]$ such that

$$\begin{aligned}
\int_\theta^{\theta+\tau} [g(x(s)) - g(\overline{x}(s))]\, ds = \Big(&\tau [g_1(x(s_1)) - g_1(\overline{x}(s_1))], \tau [g_2(x(s_2)) - g_2(\overline{x}(s_2))], \\
&\ldots, \tau [g_n(x(s_n)) - g_n(\overline{x}(s_n))]\Big).
\end{aligned}$$

By using the inequality (10.18), we attain that

$$\left\| \int_\theta^{\theta+\tau} [g(x(s)) - g(\overline{x}(s))]\, ds \right\| \geq \tau |g_{j_0}(x(s_{j_0})) - g_{j_0}(\overline{x}(s_{j_0}))| > \frac{\tau L_g \epsilon_0}{2n}.$$

The relation

$$y(t) - \overline{y}(t) = (y(\theta) - \overline{y}(\theta)) + \int_\theta^t [f(y(s)) - f(\overline{y}(s))]\, ds + \int_\theta^t \mu [g(x(s)) - g(\overline{x}(s))] ds, \ t \in [\theta, \theta + \tau]$$

yields

$$\begin{aligned}
&\|y(\theta + \tau) - \overline{y}(\theta + \tau)\| \geq |\mu| \left\| \int_\theta^{\theta+\tau} [g(x(s)) - g(\overline{x}(s))] ds \right\| \\
&- \|y(\theta) - \overline{y}(\theta)\| - \int_\theta^{\theta+\tau} L_f \|y(s) - \overline{y}(s)\|\, ds \\
&> \frac{|\mu|\, \tau L_g \epsilon_0}{2n} - \|y(\theta) - \overline{y}(\theta)\| - \int_\theta^{\theta+\tau} L_f \|y(s) - \overline{y}(s)\|\, ds.
\end{aligned}$$