Almost Periodicity in Chaos

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Abstract

Periodicity plays a significant role in the chaos theory from the beginning since the skeleton of chaos can consist of infinitely many unstable periodic motions. This is true for chaos in the sense of Devaney [1], Li-Yorke [2] and the one obtained through period-doubling cascade [3]. Countable number of periodic orbits exist in any neighborhood of a structurally stable Poincaré homoclinic orbit, which can be considered as a criterion for the presence of complex dynamics [4]-[6]. It was certified by Shilnikov [7] and Seifert [8] that it is possible to replace periodic solutions by Poisson stable or almost periodic motions in a chaotic attractor. Despite the fact that the idea of replacing periodic solutions by other types of regular motions is attractive, very few results have been obtained on the subject. The present study contributes to the chaos theory in that direction.

In this paper, we take into account chaos both through a cascade of almost periodic solutions and in the sense of Li-Yorke such that the original Li-Yorke definition is modified by replacing infinitely many periodic motions with almost periodic ones, which are separated from the motions of the scrambled set. The theoretical results are valid for systems with arbitrary high dimensions. Formation of the chaos is exemplified by means of unidirectionally coupled Duffing oscillators. The controllability of the extended chaos is demonstrated numerically by means of the Ott-Grebogi-Yorke [9] control technique. In particular, the stabilization of tori is illustrated.

Keywords: Chaos with almost periodic motions; Li-Yorke chaos; Cascade of almost periodic solutions; Control of chaos; Stabilization of tori

1 Introduction

The transition between regular and chaotic motions has been a field of interest among scientists since the second half of the twentieth century. Investigations of chaos for continuous-time dynamics started with the studies of Poincaré [10], Cartwright and Littlewood [11], Levinson [12], Lorenz [13], and Ueda [14]. Chaos theory has many applications in various disciplines such as neuroscience, economics, mechanics, electronics, meteorology, and medicine [15].

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