



FIG. 1. (Color online) Number of atoms as a function of the chemical potential for the different states. The potential parameters are $\Omega = 0.1$, $k = \pi/5.37 \mu\text{m}^{-1}$ and $V_0 = 1.16 \times 10^{-12}$ eV. The insets show the densities of the different states for $\mu = 3 \times 10^{-12}$ eV.

symmetric branch to the linear limit ends at the eigenvalue ω_0 , corresponding to the ground state of the linear problem.

The second branch [branch (2)] starts from the first excited state in the linear limit, $\mu \rightarrow \omega_1$ as $N \rightarrow 0$. The wavefunctions of the states of this branch are *antisymmetric* and have a node at the center of the barrier, while their densities are symmetric. From this density symmetric one-soliton branch, two asymmetric one-soliton branches, namely branch (3) and its mirror image with respect to the $z = 0$ axis, bifurcate close to the linear limit, at $\mu \simeq 2.144 \times 10^{-12}$ eV – see the close-up inset. Let us define a local occupation number, i.e., number of atoms in the different wells, as the integral over the density up to the center of the barrier (see also Sec. V below). The occupation number in the well with the node is then smaller than the occupation number without the node. This can be explained by the fact that the state with one node is the first excited state, characterized by a higher energy than the one without a node, which is the ground state. Thus, in order to balance the chemical potential in both wells one needs a larger interaction energy (i.e., more atoms) in the well without a node. Note that the macroscopic quantum self-trapping (MQST) state as predicted in Ref [43] is not considered in what follows. The reason for that is that the MQST is a running phase state while our bifurcation analysis below focuses on the