The equations of motion are then any three of the four equations

$$-\frac{1}{B^2}+\frac{2B'}{rB}+\frac{B'^2}{B^2}+\frac{2C'}{rC}+\frac{2B'C'}{BC}+\frac{2B''}{B}+\frac{C''}{C} \;=\; \kappa^2\left(-\frac{1}{2}f'^2-\frac{n^2f^2}{2r^2C^2}-\frac{\lambda}{4}(f^2-\eta^2)^2-\Lambda\right)$$

$$-\frac{3}{B^2}+\frac{3B'}{rB}+\frac{3B'^2}{B^2}+\frac{3B'C'}{BC} \;=\; \kappa^2\left(\frac{1}{2}f'^2-\frac{n^2f^2}{2r^2C^2}-\frac{\lambda}{4}(f^2-\eta^2)^2-\Lambda\right)$$

$$-\frac{3}{B^2}+\frac{3B'^2}{B^2}+\frac{3B''}{B} \;=\; \kappa^2\left(-\frac{1}{2}f'^2+\frac{n^2f^2}{2r^2C^2}-\frac{\lambda}{4}(f^2-\eta^2)^2-\Lambda\right)$$

$$f''+\left(3\frac{B'}{B}+\frac{C'}{C}+\frac{1}{r}\right)f' \;=\; \frac{n^2f}{C^2r^2}+\lambda f(f^2-\eta^2)\ .$$

Because our ansatz is explicitly time-independent, the Lorentzian and Euclidean solutions are trivially related.

A. Asymptotic Solution of the full equations

Before numerically solving the near-bubble region, we determine the asymptotic values of all functions, which can be done exactly. The solution below exhibits the expected backreaction on the scalar modulus, the KK radion, and the vacuum energy density.

$$\Phi(x)=f_\infty e^{iny}\ , \tag{30}$$

$$ds^2=dr^2+\frac{1}{H^2}\sinh^2(Hr)(-dt^2+\cosh^2(t)d\Omega_2^2)+L^2dy^2\ , \tag{31}$$

where,

$$f_\infty^2=\eta^2-\frac{n^2}{\lambda L^2}=\frac{2\eta^2}{5}\left(1+\frac{3}{2}\Delta\right)\ , \tag{32}$$

$$L^2=-\frac{3n^2\eta^2}{4\Lambda}\left(1+\Delta\right)\ , \tag{33}$$

$$H^2=-\frac{4\kappa^2\Lambda}{15}\left(\frac{\frac{2}{3}+\Delta}{1+\Delta}\right)\ , \tag{34}$$

and we have introduced the parameter Δ,

$$\Delta=\sqrt{1+\frac{20\Lambda}{9\eta^4\lambda}}\ . \tag{35}$$

Note that $\Delta\to 1$ in the limit $\lambda\to\infty$, so we recover the pure flux compactification geometry described in previous section (See Eqs. (17) - (18)).