where

$$F_0^L(y) = \sup_{z\in\Xi}\left[P(y|0,z)(1-p(z)) + L_{01}^{wst}(y,z)\right],$$

$$F_0^U(y) = \inf_{z\in\Xi}\left[P(y|0,z)(1-p(z)) + U_{01}^{sm}(y,z)\right],$$

$$F_1^L(y) = \sup_{z\in\Xi}\left[P(y|1,z)p(z) + L_{10}^{wst}(y,z)\right],$$

$$F_1^U(y) = \inf_{z\in\Xi}\left[P(y|1,z)p(z) + U_{10}^{sm}(y,z)\right],$$

$$F^L(y_0,y_1) = \sup_{z\in\Xi}\left[\begin{array}{c}\max\left\{(P(y_0|0,z)-1)(1-p(z)) + L_{10}^{wst}(y_1,z), 0\right\} \\ +\max\left\{L_{01}^{wst}(y_0,z) + (P(y_1|1,z)-1)p(z), 0\right\}\end{array}\right],$$

$$F^U(y_0,y_1) = \inf_{z\in\Xi}\left[\begin{array}{c}\min\left\{P(y_0|0,z)(1-p(z)), U_{10}^{sm}(y_1,z)\right\} \\ +\min\left\{U_{01}^{sm}(y_0,z), P(y_1|1,z)p(z)\right\}\end{array}\right],$$

$$F_\Delta^L(\delta) = \sup_{z\in\Xi}\left[\begin{array}{c}\sup_{y\in\mathbb{R}}\max\left\{P(y|1,z)p(z) - U_{01}^{sm}(y-\delta,z), 0\right\} \\ +\sup_{y\in\mathbb{R}}\max\left\{L_{10}^{wst}(y,z) - P(y-\delta|0,z)(1-p(z)), 0\right\}\end{array}\right],$$

$$F_\Delta^U(\delta) = 1 + \inf_{z\in\Xi}\left[\begin{array}{c}\inf_{y\in\mathbb{R}}\min\left\{P(y|1,z)p(z) - L_{01}^{wst}(y-\delta,z), 0\right\} \\ +\inf_{y\in\mathbb{R}}\min\left\{U_{10}^{sm}(y,z) - P(y-\delta|0,z)(1-p(z)), 0\right\}\end{array}\right].$$

Theorem 2

Theorem 2 *Under M.1 – M.5, and CPQD, sharp bounds on* $F_0(y_0)$, $F_1(y_1)$, *and* $F_\Delta(\delta)$ *are identical to those given in Theorem 1. Sharp bounds on* $F(y_0,y_1)$ *are obtained as follows: for* $(y_0,y_1)\in\mathbb{R}\times\mathbb{R}$,

$$F(y_0,y_1) \in \left[F^L(y_0,y_1), F^U(y_0,y_1)\right],$$

where

$$F_d(y) \in \left[F_d^L(y), F_d^U(y)\right],$$

$$F(y_0,y_1) \in \left[F^L(y_0,y_1), F^U(y_0,y_1)\right],$$

$$F_\Delta(\delta) \in \left[F_\Delta^L(\delta), F_\Delta^U(\delta)\right],$$