



Figure 4.6: Notation and conventions

forced by the TGL condition, one of these points of intersection must be on the right side of the circle and one must be on the left side.

With reference to figure 4.6 we define $s^+(s)$ and $s^-(s) \in [0, L)$ so that $\gamma(s^+(s))$ is the point of intersection on the right and $\gamma(s^-(s))$ is the point of intersection on the left. The notation is motivated by the fact that $0 < s^-(s) < s < s^+(s) < L$ in general due to our convention that γ traverses $\partial\Omega$ counterclockwise. The only case where this is not true is when $\gamma(L) = \gamma(0)$ is in the disk but even then it will hold for a suitably shifted $\hat{\gamma}$ that starts at some point outside the current disk.

The quantities $\theta_1(s)$ and $\theta_2(s)$ are the angles that the rays from the origin to the right and left points of intersection, respectively, make with the positive x axis. We can assume $\theta_1(s) \in (-\frac{\pi}{2}, \frac{\pi}{2})$ and $\theta_2(s) \in (\frac{\pi}{2}, \frac{3\pi}{2})$.

We define $\nu_1(s)$ as the angle between the vector $\gamma(s^+(s)) - \gamma(s)$ and the vector $\lim_{t \downarrow s^+(s)} \gamma'(t)$, the one-sided tangent to $\partial\Omega$ at the point of intersection on the right. That is, we are measuring the angle between the outward normal to the disk at the point of intersection and the actual direction γ is going as it exits the disk. We define