

is finite, then r_H is a simple root. Since g_{00} is differentiable on the interval $[0, \infty)$ we can expand it in a Taylor series and we obtain

$$g_{00}(r) = (r - r_H) \left[g'_{00}(r_H) + \mathcal{O}(r - r_H) \right].$$

Hence,

$$\lim_{r \rightarrow r_H} \frac{g_{00}(r)}{r - r_H} = g'_{00}(r_H)$$

and in order to show that r_H is a simple zero we need to prove that $g'_{00}(r_H) \neq 0$. Taking into account the fact that

$$g'_{00}(r_H) = \frac{4M}{\sqrt{\pi}r_H} \left[\frac{1}{r_H} \gamma \left(\frac{3}{2}, \frac{r_H^2}{4\theta} \right) - \gamma' \left(\frac{3}{2}, \frac{r_H^2}{4\theta} \right) \right]$$

where a prime denotes differentiation with respect to the horizon radius and comparing the above equation with the Eq. (5), we can see that $g'_{00}(r_H)$ vanishes if and only if $r_H = r_0$. This implies that $M = M_0$ which is at variance with the initial assumption. As a result we can conclude that $g'_{00}(r_H) \neq 0$ for $M > M_0$ and $r_H \neq r_0$.

III. A NEW TRANSFORMATION

We show that the singularities of (2) can be removed by a suitable coordinate transformation as in the case of the Reissner-Nordström solution. In order to do that we shall follow [15]. Like in the Kruskal approach [14] we introduce coordinates $u(t, r)$ and $v(t, r)$ such that the original metric goes over to

$$ds^2 = f^2(u, v)(dv^2 - du^2) - r^2(u, v)(d\vartheta^2 + \sin^2 \vartheta d\varphi^2) \quad (9)$$

with the requirement that $f^2 \neq 0$. This will happen if u and v satisfy the non homogeneous system of first order nonlinear partial differential equations

$$f^2(u, v) [(\partial_t v)^2 - (\partial_t u)^2] = g_{00}(r), \quad (10)$$

$$f^2(u, v) [(\partial_r v)^2 - (\partial_r u)^2] = -g_{00}^{-1}(r), \quad (11)$$

$$\partial_r u \partial_t u - \partial_r v \partial_t v = 0. \quad (12)$$

The next step is to find a suitable transformation of the variable r such that the above system becomes a homogeneous system of PDEs. If we multiply (11) by g_{00}^2 and we introduce a new