is analogous to the phase transition from the discrete low-lying mass spectrum particles to the highly excited and massive Hagedorn states (i.e. continuous high-lying mass spectrum). The Hagedorn states are given by the mass spectrum of the color-singlet state composite (colorless) where the constituent quarks and gluons are represented by the $SU(N_c)$ symmetry group representation and these states are hadrons [58–61]. This kind of matter should not be immediately interpreted as the deconfined quark-gluon plasma. The critical Gross-Witten point is the threshold point where the Hagedorn states emerge in the system. The internal color structure is known to be essential in the phase transition to the quark-gluon plasma [58, 59, 62–64]. The deconfinement phase transition can either take place immediately when unstable Hagedorn states are produced or as a subsequent process when the metastable Hagedorn phase eventually undergoes the phase transition to quark-gluon plasma. The Hagedorn phase may undergo multiple intermediate transitions before the deconfinement phase transition is eventually reached.

The effective Vandermonde potential plays a crucial role in the intermediate phase transition processes from the hadronic phase and quark-gluon plasma. This potential is back reacted to the heat and compression of the nuclear matter. When the invariance Haar measure is regulated, another analytical solution with different characteristic properties emerges. Hence, Vandermonde determinant is regulated in a nontrivial way and the action can be expanded over the group fundamental variables around the stationary Fourier color points up to the quadratic terms. Fortunately, the saddle Fourier color points are convened around the origin and fortunately this simplifies the problem drastically. Despite the complexity of the action due to the realistic physical situation that is involved, there will be an easy way to find the quadratic expansion around the group saddle points. The resultant integral is evaluated using the standard Gaussian quadrature over the group (i.e. color) variables. Hence, beyond the Gross-Witten point the partition function can approximated by the quadratic Taylor expansion. over the group variables The quadratic expansion of the quark and anti-quark ensemble around the saddle points is reduced to the following Gaussian-like function

$$Z_{q\bar{q}}(\beta, V) = \exp\left[a_{q\bar{q}}^{(0)} - \frac{1}{2}a_{q\bar{q}}^{(2)}\sum_{i=1}^{N_c}\theta_i^2\right]. \tag{25}$$

This regulated ensemble $Z_{q\bar{q}}(\beta, V)$ leads to a continuous high-lying mass spectrum. The