

model parameters and latent states (the unknown components for which we may seek inferences) as  $\Theta = \{\theta_1, \dots, \theta_d\}$ , which will be referred to as the parameters of  $\mathcal{M}$ .

An MCMC algorithm may be defined in terms of its sampling scheme over  $\Theta$ . Let  $b$  be any non-empty subset (“block”) of  $\Theta$ , and  $u \in U$  be any valid MCMC sampling (or “updating”) method such as slice sampling or conjugate Gibbs sampling (see Gilks, 2005, for a broad overview of MCMC sampling methods). We define a valid MCMC sampler  $\psi = u(b)$  as the application of  $u$  to  $b$ , where  $b$  satisfies any assumptions of  $u$  (*e.g.*, conjugacy). In addition to satisfying standard properties of Markov chains (see, for example, Robert and Casella, 2004), we define a valid MCMC algorithm as any set of samplers  $\Psi = \{\psi_1, \dots, \psi_k\}$ , where  $\psi_i = u_i(b_i)$  for  $i = 1, \dots, k$ , satisfying  $\cup_{i=1}^k b_i = \Theta$ ; that is, the MCMC algorithm updates each model parameter in at least one sampler. We represent the chain of samples generated from successive applications of  $\Psi$  as  $X^{(0)}, X^{(1)}, \dots$ , where sample  $X^{(i)}$  implies model state  $\Theta = X^{(i)}$ ,  $X^{(0)}$  is the set of initial values,  $X^{(i)} = (X_1^{(i)}, \dots, X_d^{(i)})$ , and  $X_k = \{X_k^{(0)}, X_k^{(1)}, \dots\}$  is the scalar chain of samples of  $\theta_k$  (for  $k = 1, \dots, d$ ).

This paper focuses attention on the restricted set of sampling methods  $U_0 = \{u_{\text{scalar}}, u_{\text{block}}\}$ , where  $u_{\text{scalar}}$  denotes univariate adaptive random walk Metropolis-Hastings sampling (hereafter, scalar sampling; Metropolis et al., 1953; Hastings, 1970), and  $u_{\text{block}}$  denotes the multivariate generalization of this algorithm (hereafter, block sampling; Haario, Saksman, and Tamminen, 1999), with the practical restriction that any  $\psi = u_{\text{block}}(b)$  satisfies  $|b| > 1$ . The  $u_{\text{scalar}}$  algorithm adaptively tunes the proposal scale, while  $u_{\text{block}}$  additionally tunes the proposal covariance (Roberts and Rosenthal, 2009). All scalar and block samplers asymptotically achieve the theoretically optimal scaling of proposal distributions (and therefore acceptance rates, and mixing) as derived in Roberts, Gelman, and Gilks (1997), and implement adaptation routines as set out in Shaby and Wells (2011).

For hierarchical model  $\mathcal{M}$  with parameters  $\Theta$ , our studies focus almost exclusively on the set of MCMC algorithms  $\Psi_{\mathcal{M}}$ , which contains all algorithms of the form  $\Psi = \{\psi_1, \dots, \psi_k\}$ , where  $\psi_i = u_i(b_i)$ , each  $u_i \in U_0$ , and the sets  $b_i$  form a partition of  $\Theta$ . We specifically name two algorithms in  $\Psi_{\mathcal{M}}$  which are boundary cases. The first consists of  $d$  scalar samplers:  $\Psi_{\text{scalar}} = \{\psi_1, \dots, \psi_d\}$ , where each  $\psi_i = u_{\text{scalar}}(\theta_i)$ . The second has a single block sampler for all parameters:  $\Psi_{\text{block}} = \{u_{\text{block}}(\Theta)\}$ . Implicit is the assumption that each  $\theta_i$  is continuous-valued, which is the case throughout this paper.