$\lambda_i$, $i=1,...,d^2-1$, are the generators of the $SU(d)$ algebra with $Tr\{\lambda_i\lambda_j\}=2\delta_{ij}$, $r_i(\rho)=\frac{1}{2}Tr\{\rho\lambda_i(1)\otimes I\}$, $s_j(\rho)=\frac{1}{2}Tr\{\rho I\otimes\lambda_j(2)\}$, $n_{ij}(\rho)=\frac{1}{4}Tr\{\rho\lambda_i(1)\otimes\lambda_j(2)\}$, $P_+$ stands for the projection operator to $|\psi_+\rangle$, $N(P_+)$ is similarly defined to $N(\rho)$, $N^T$ stands for the transpose of $N$, $||N||_{KF}=Tr\sqrt{NN^\dagger}$ is the Ky Fan norm of $N$. This upper bound was used to improve the distillation protocol proposed in [16]. Here we show that the upper bound in (8) is different from that in (15) by an example.

**Example 1:** We consider the bound entangled state [17]

$$\rho(a)=\frac{1}{8a+1}\begin{pmatrix} a & 0 & 0 & 0 & a & 0 & 0 & 0 & a \\ 0 & a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a & 0 & 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 & a & 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 & 0 & a & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1+a}{2} & 0 & \frac{\sqrt{1-a^2}}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a & 0 \\ a & 0 & 0 & 0 & a & 0 & \frac{\sqrt{1-a^2}}{2} & 0 & \frac{1+a}{2} \end{pmatrix}. \tag{17}$$

From Fig. 1 we see that for $0\leq a<0.572$, the upper bound in (8) is larger than that in (15). But for $0.572<a<1$ the upper bound in (8) is always lower than that in (15), i.e. the upper bound (8) is tighter than (15) in this case.

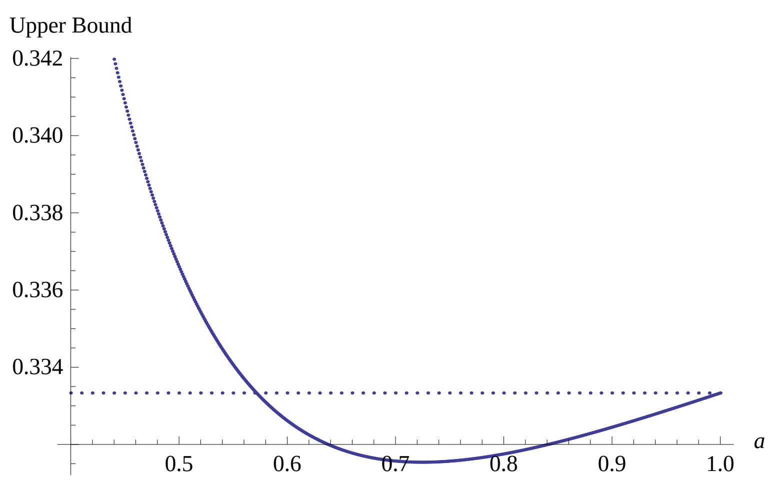


FIG. 1: Upper bound of $\mathcal{F}(\rho)$ from (8) (solid line) and upper bound from (15) (dashed line).

By using the operator norm, we have further