$\eta_k = s_k - \tau/2$. One can find an integer j_k with $1 \leq j_k \leq n$ such that

$$|h_{j_k}(x(s_k)) - h_{j_k}(\widetilde{x}(s_k))| \geq \frac{L_2}{\sqrt{n}} \|x(s_k) - \widetilde{x}(s_k)\| > \frac{L_2 \epsilon_0}{\sqrt{n}}. \tag{7.23}$$

Using (7.22) one can obtain for $t \in [\eta_k, \eta_k + \tau]$ that

$$|h_{j_k}(x(s_k)) - h_{j_k}(\widetilde{x}(s_k))| - |h_{j_k}(x(t)) - h_{j_k}(\widetilde{x}(t))| < \frac{L_2 \epsilon_0}{2\sqrt{n}}.$$

Moreover, inequality (7.23) yields

$$|h_{j_k}(x(t)) - h_{j_k}(\widetilde{x}(t))| > |h_{j_k}(x(s_k)) - h_{j_k}(\widetilde{x}(s_k))| - \frac{L_2 \epsilon_0}{2\sqrt{n}} > \frac{L_2 \epsilon_0}{2\sqrt{n}}$$

for all $t \in [\eta_k, \eta_k + \tau]$. Since there exist numbers $r_1, r_2, \ldots, r_n \in [\eta_k, \eta_k + \tau]$ satisfying

$$\left\| \int_{\eta_k}^{\eta_k + \tau} [h(x(s)) - h(\widetilde{x}(s))] \, ds \right\| = \tau \left(\sum_{k=1}^{n} [h_k(x(r_k)) - h_k(\widetilde{x}(r_k))] \right)^{1/2},$$

we have that

$$\begin{aligned} \left\| \int_{\eta_k}^{\eta_k + \tau} [h(x(s)) - h(\widetilde{x}(s))] \, ds \right\| & \geq \tau \left| h_{j_k}(x(r_{j_k})) - h_{j_k}(\widetilde{x}(r_{j_k})) \right| \\ & > \frac{\tau L_2 \epsilon_0}{2\sqrt{n}}. \end{aligned}$$

Let $\zeta \in \Theta$ be an arbitrary sequence. Making use of the relations

$$\phi_{x,\zeta}(t) = \phi_{x,\zeta}(\eta_k) + \int_{\eta_k}^{t} [A\phi_{x,\zeta}(s) + f(\phi_{x,\zeta}(s), s) + \nu(s, \zeta) + h(x(s))] \, ds$$

and

$$\phi_{\widetilde{x},\zeta}(t) = \phi_{\widetilde{x},\zeta}(\eta_k) + \int_{\eta_k}^{t} [A\phi_{\widetilde{x},\zeta}(s) + f(\phi_{\widetilde{x},\zeta}(s), s) + \nu(s, \zeta) + h(\widetilde{x}(s))] \, ds,$$

it can be deduced that

$$\begin{aligned} \|\phi_{x,\zeta}(\eta_k + \tau) - \phi_{\widetilde{x},\zeta}(\eta_k + \tau)\| & \geq \left\| \int_{\eta_k}^{\eta_k + \tau} [h(x(s)) - h(\widetilde{x}(s))] \, ds \right\| \\ & - \|\phi_{x,\zeta}(\eta_k) - \phi_{\widetilde{x},\zeta}(\eta_k)\| \\ & - (\|A\| + L_f) \int_{\eta_k}^{\eta_k + \tau} \|\phi_{x,\zeta}(s) - \phi_{\widetilde{x},\zeta}(s)\| \, ds. \end{aligned}$$

Hence,

$$\max_{t \in [\eta_k, \eta_k + \tau]} \|\phi_{x,\zeta}(t) - \phi_{\widetilde{x},\zeta}(t)\| > \frac{\tau L_2 \epsilon_0}{2[2 + \tau(\|A\| + L_f)]\sqrt{n}}.$$