Optimal reconstruction of the states in qutrits system

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Based on mutually unbiased measurements, an optimal tomographic scheme for the multiqutrit states is presented explicitly. Because the reconstruction process of states based on mutually unbiased states is free of information waste, we refer to our scheme as the optimal scheme. By optimal we mean that the number of the required conditional operations reaches the minimum in this tomographic scheme for the states of qutrit systems. Special attention will be paid to how those different mutually unbiased measurements are realized; that is, how to decompose each transformation that connects each mutually unbiased basis with the standard computational basis. It is found that all those transformations can be decomposed into several basic implementable single- and two-qutrit unitary operations. For the three-qutrit system, there exist five different mutually unbiased-bases structures with different entanglement properties, so we introduce the concept of physical complexity to minimize the number of nonlocal operations needed over the five different structures. This scheme is helpful for experimental scientists to realize the most economical reconstruction of quantum states in qutrit systems.

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I. INTRODUCTION

The quantum state of a system is a fundamental concept in quantum mechanics, and a quantum state can be described by a density matrix, which contains all the information one can obtain about that system. A main task for implementing quantum computation is to reconstruct the density matrix of an unknown state, which is called quantum state reconstruction or quantum state tomography[1, 2]. The technique was first developed by Stokes to determine the polarization state of a light beam [3]. Recently, Minimal qubit tomography process has been proposed by Řeháček *et al*, where only four measurement probabilities are needed for fully determining a single qubit state, rather than the six probabilities in the standard procedure [4]. But the implementation of this tomography process requires measurements of N-particle correlations[5].The statistical reconstruction of biphotons states based on mutually complementary measurements has been proposed by Bogdanov *et al*[6, 7]. Ivanov *et al* proposed a method to determine an unknown mixed qutrit state from nine independent fluorescence signals[8]. Moreva *et al* paid attention to experimental problem of the realization of the optimal protocol for polarization ququarts state tomography [9]. In 2009, Taguchi *et al* developed the single scan tomography of spatial three-dimensional (qutrits) state based on the effect of realistic measurement operators[10]. Allevi *et al* studied the implementation of the reconstruction of the Wigner function and the density matrix for coherent and thermal states by by switching on/off single photon avalanche photodetectors[11].

In order to obtain the full information about the system we need to perform a series of measurements on a large number of identically prepared copies of the system. These measurement results are not independent of each other, so there is redundancy in these results in the previously used quantum tomography processes [12], which causes a resources waste. If we remove this redundancy completely, the reconstruction process will become an optimal one. So, to design an optimal set of measurements for removing the redundancy is of fundamental significance in quantum information processing.

Mutually unbiased bases (MUBs) have been used in a variety of topics in quantum mechanics [13–36]. MUBs are defined by the property that the squared overlap between a vector in one basis and all basis vectors in the other bases are equal. That is to say the detection over a particular basis state does not give any information about the state if it is measured in another basis. Ivanović first introduced the concept of MUBs to the problem of quantum state determination [13], and proved the existence of such bases in the prime-dimension system by an explicit construction. Then it has been shown by Wootters and Fields that measurements in this special class of bases, i.e. mutually unbiased measurements (MUMs) provide a minimal as well as optimal way of complete specification of an unknown density matrix [14]. They proved that the maximal numbers of MUBs is $d+1$ in prime-dimension system. This result also applies to the prime-power-dimension system.

MUBs play a special role in determining quantum states, such as it forms a minimal set of measurement bases and provides an optimal way for determining a quantum state [13–16] etc. Recently an optimal tomographic reconstruction scheme was proposed by Klimov *et al* for the case of determining a state of multiqubit quantum system based on MUMs in trapped ions system [37]. However, the use of three-level systems instead of two-level systems has been proven to be securer against a symmetric attack on a quantum key distribution protocol with MUMs than the currently existing measurement protocol [38, 39]. Quantum tomography in high dimensional (qudit) systems has been proposed and the number of required measurements is $d^{2n}-1$ with d being the dimension of the qudit system and n being the number of the qudits [12]. This tomography process is not an optimal one,

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