C1 *The items in $I_{i,k}$ are only associated with attribute k, that is, their corresponding row vector in Q is e_k.*

C2 *Let $T_{I_{i,k}}$ be the corresponding T-matrix of this reduced simple-attribute model. The matrix $T_{I_{i,k}}$ is of full column rank.*

C3 *Let $\pi_{\boldsymbol{\alpha}}$ be the probability of latent class $\boldsymbol{\alpha}$. $\pi_{\boldsymbol{\alpha}} > 0, \boldsymbol{\alpha} = 1, 2, \ldots, M$.*

Then, the item parameters $p^k_{j\boldsymbol{\alpha}}$ and the latent class population $\pi_{\boldsymbol{\alpha}}$ are identifiable up to a permutation of the class labels.

The above theorem seemingly requires many single-attribute items. In practice, each of the subset $I_{i,k}$ usually contains very few, in fact most of the time, single items. However, notice that the matrix $T_{I_{i,k}}$ for the reduced single-attribute model contains d_k columns. In the case of binary attribute, it is sufficient to include a single item in each $I_{i,k}$; see the proof of Theorem 2. It may remain possibly sufficient to include a single item in each $I_{i,k}$ if the response to the item also takes more than two possible values. Generally speaking, we need to include sufficiently many items in each $I_{i,k}$ so that their responses contain information to differentiate different latent classes defined by attribute k. Furthermore, the construction of the T-matrices for the reduced model is much easier as $I_{i,k}$ often contains very few items and the matrix only contains d_k columns.

3.2 Identifiability of partial information structure

The previous subsection provides results on the identifiability of the item parameters and the attribute population. We now proceed to a discussion of the Q-matrix. The Q-matrix provides a qualitative description between the item-attribute relationship. The specific form an item response function takes depends on the model parameterization and the loading structure. As the aim of this study is to provide results applicable to general diagnostic classification models, we take a slightly different viewpoint and state the identifiability of the partial information structure for each item that is mathematically a more general concept than the Q-matrix.