

$$\begin{aligned}
J_5 &= s \int_{\Omega} |z|^2 \Delta_x (\Delta_y \varphi - \Delta_x \varphi) dx dy dt, \\
J_6 &= 2s^3 \int_{\Omega} |z|^2 \nabla_y \varphi \cdot \nabla_y \left(|\nabla_y \varphi|^2 - |\nabla_x \varphi|^2 \right) dx dy dt \\
&\quad - 2s^3 \int_{\Omega} |z|^2 \nabla_x \varphi \cdot \nabla_x \left(|\nabla_y \varphi|^2 - |\nabla_x \varphi|^2 \right) dx dy dt
\end{aligned}$$

and

$$\begin{aligned}
B_0 &= -2s \operatorname{Im} \int_{\Gamma_y} z \partial_t \bar{z} (\nabla_y \varphi \cdot \nu) dS_y dx dt \\
&\quad + 4s \operatorname{Re} \int_{\Gamma_y} (\partial_\nu z) (\nabla_x \varphi \cdot \nabla_x \bar{z} - \nabla_y \varphi \cdot \nabla_y \bar{z}) dS_y dx dt \\
&\quad + 2s \int_{\Gamma_y} (\partial_\nu \varphi) \left(|\nabla_y z|^2 - |\nabla_x z|^2 \right) dS_y dx dt \\
&\quad - 2s^3 \int_{\Gamma_y} (\partial_\nu \varphi) |z|^2 \left(|\nabla_y \varphi|^2 - |\nabla_x \varphi|^2 \right) dS_y dx dt \\
&\quad - 2s \int_{\Gamma_y} (\partial_\nu z) (\Delta_y \varphi - \Delta_x \varphi) \bar{z} dS_y dx dt + s \int_{\Gamma_y} \partial_\nu (\Delta_y \varphi - \Delta_x \varphi) |z|^2 dS_y dx dt \\
&\quad + 2s \operatorname{Im} \int_{\Gamma_x} z \partial_t \bar{z} (\nabla_x \varphi \cdot \nu) dS_x dy dt \\
&\quad + 4s \operatorname{Re} \int_{\Gamma_x} (\partial_\nu z) (\nabla_y \varphi \cdot \nabla_y \bar{z} - \nabla_x \varphi \cdot \nabla_x \bar{z}) dS_x dy dt \\
&\quad - 2s \int_{\Gamma_x} (\partial_\nu \varphi) \left(|\nabla_y z|^2 - |\nabla_x z|^2 \right) dS_x dy dt \\
&\quad + 2s^3 \int_{\Gamma_x} (\partial_\nu \varphi) |z|^2 \left(|\nabla_y \varphi|^2 - |\nabla_x \varphi|^2 \right) dS_x dy dt \\
&\quad + 2s \int_{\Gamma_x} (\partial_\nu z) (\Delta_y \varphi - \Delta_x \varphi) \bar{z} dS_x dy dt - s \int_{\Gamma_x} \partial_\nu (\Delta_y \varphi - \Delta_x \varphi) |z|^2 dS_x dy dt.
\end{aligned}$$

Next, we shall estimate J_k , $1 \leq k \leq 6$ and B_0 using the following elementary properties of the weight function:

$$\begin{aligned}
\partial_t \varphi &= (-2\gamma\beta t) \varphi, & \varphi_{x_i x_i} &= \gamma \varphi (2 + \gamma \psi_{x_i}^2), \\
\varphi_{x_i y_j} &= \gamma^2 \varphi \psi_{x_i} \psi_{y_j}, & \varphi_{x_i x_j} &= \gamma \varphi \left(\psi_{x_i x_j} + \gamma \psi_{x_i} \psi_{x_j} \right), \\
\varphi_{y_i y_j} &= \gamma \varphi \left(\psi_{y_i y_j} + \gamma \psi_{y_i} \psi_{y_j} \right), & \nabla_x \varphi &= \gamma \varphi \nabla_x \psi, \\
\nabla_y \varphi &= \gamma \varphi \nabla_y \psi, & \partial_t (\nabla_x \varphi) &= (-2\gamma^2 \beta t) \varphi \nabla_x \psi, \\
\partial_t (\nabla_y \varphi) &= (-2\gamma^2 \beta t) \varphi \nabla_y \psi, & \Delta_x \varphi &= \gamma \varphi \left(\Delta_x \psi + \gamma |\nabla_x \psi|^2 \right), \\
\Delta_y \varphi &= \gamma \varphi \left(\Delta_y \psi + \gamma |\nabla_y \psi|^2 \right), & \Delta_y \varphi - \Delta_x \varphi &= \gamma \varphi d_1(\psi) + \gamma^2 \varphi d_2(\psi),
\end{aligned}$$