As a result, we can also calculate the pressure field (2) in the liquid. The boundary conditions (3) and (4) in combination with the obvious relations (5) determine unambiguously the inertial properties of the interface described in terms of the effective density [25]

$$\rho_{\rm ef} = \rho_n + (\rho' - \rho_s)^2/\rho_s$$

in the sense that

$$\rho_s u_s^2/2 + \rho_n u_n^2/2 = \rho_{\rm ef} V^2/2\,.$$

To proceed further, we adopt the most simplifying assumptions [25] to describe the solid and its boundary. The solid is assumed to be always unstressed and all possible shearing components $\sigma_{i\neq k}$ of the stress tensor σ_{ik} are neglected. In other words, the stress tensor is isotropic, i.e., $\sigma_{ik} = -P'\delta_{ik}$, and we can define 'pressure' according to $P' = -\sigma_{ii}/3$ [27]. Then, from the formal point of view, the solid can be described as a liquid under pressure equal to P'.

The next boundary condition stems from the continuity of the momentum flux density across the interface. The momentum flux density in the superfluid [28] reads $P\delta_{ik} + \rho_n v_{n\,i} v_{n\,k} + \rho_s v_{s\,i} v_{s\,k}$. Then, we take $\sigma_{ik}\nu_k = -P'\nu_i$ into account, assume the small curvature of the interface $z = Z(x,\,y,\,t) = L(t) + \zeta(x,\,y,\,t)$, and use a frame that refers to the boundary

$$P + \rho_n(\boldsymbol{v}_n - \dot{Z}\boldsymbol{\nu})^2 + \rho_s(\boldsymbol{v}_s - \dot{Z}\boldsymbol{\nu})^2 - (P' + \rho'\dot{Z}^2) = \\ = \gamma_{ik}\partial Z^2/\partial r_i \partial r_k = \gamma_{ik}\partial\zeta^2/\partial r_i\partial r_k\,. \qquad (6)$$

Neglecting the quadratic terms in velocities gives the usual Laplace condition of mechanical equilibrium across the interface [25]. Here $\gamma_{ik}(\theta,\varphi) = \alpha\delta_{ik} + \partial\alpha^2/\partial\varphi_i\partial\varphi_k$ is the surface stiffness tensor [9, 10, 25] expressed in terms of surface tension $\alpha = \alpha(\theta,\varphi)$ depending on the angles between the crystalline orientation and the normal to the surface.

Let us turn now to the last boundary condition. It is a reasonable assumption that any motion of the interface accompanied also by the melting and growth of a solid will dissipate a certain amount of energy. Thus a finite velocity of the interface should produce some imbalance in the chemical potential difference $\mu - \mu'$ between the liquid and solid. The routine in various theories of the interfacial dynamics is an introduction of the so-called growth coefficient K which relates the interface growth rate with the difference in chemical potentials across the interface [24, 25]. Because of $u_n(t) \neq 0$ and $u_s(t) \neq 0$ we again have to take into account the squares of velocities which are always omitted in the linear perturbation theory of the interface being initially at rest. So, at the boundary we employ an effective relation

$$\dot{Z} = K\left[\mu + \frac{(\boldsymbol{v}_s - \dot{Z}\boldsymbol{\nu})^2}{2} - \left(\mu' + \frac{\dot{Z}^2}{2}\right)\right], \qquad (7)$$

where μ and μ' are the chemical potentials of the liquid and solid per unit mass. In a wide sense the growth coefficient here is a certain combination of all Onsager coefficients and the kinetic coefficients describing the near-surface dissipative processes. In general, the growth coefficient K can depend on the temperature as well as on the wave vector q. Usually, in the ballistic regime, when the mean free path l of excitations is large, the growth coefficient is independent of wave vector. In the opposite hydrodynamic limit $ql \ll 1$ the growth coefficient may depend on the wave vector approximately as $1/K \sim ql$ [25].

Lastly, we need an expression for the chemical potential difference. As usual, the reference point is the melting pressure P_c at which the chemical potentials μ and μ' coincide and the liquid-solid transition takes place. We take the necessary formulae for the superfluid from Ref. [28]. After expanding chemical potentials in the vicinity of the melting pressure, we obtain

$$\mu - \mu' = \\ = \sigma(T - T_\infty) + \frac{P - P_c}{\rho} - \frac{\rho_n}{\rho}\frac{(\boldsymbol{v}_n - \boldsymbol{v}_s)^2}{2} - \frac{P' - P_c}{\rho'}\,, \\ T - T_\infty = \frac{\rho_n}{\sigma\rho}\left(\frac{P_n - P_{n\,\infty}}{\rho_n} - \frac{P_s - P_{s\,\infty}}{\rho_s} - \frac{(\boldsymbol{v}_n - \boldsymbol{v}_s)^2}{2}\right),$$

where σ is the entropy and the quantities with index "∞" stand for the magnitudes taken far from the interface.

Now we are in position to find the equations which the interface dynamics obeys. Knowing velocity potentials ϕ_n and ϕ_s expressed via $u_{n,\,s}(t)$ and perturbation $\zeta_q(t)$, we can calculate the normal and superfluid velocities, pressure, and chemical potential difference. Next, we insert the quantities calculated at the interface into the boundary conditions (6) and (7) and eliminate the pressure P'. As a result of some algebraic formula manipulation linear in ζ_q, we obtain an equation consisting of the ζ_q-independent component and the other one linear in ζ_q. The first component gives an equation

$$V\frac{\rho'}{K} = \frac{\rho' - \rho}{\rho}\left[\Delta P - \rho g L)\right] + \rho_{\rm ef}\left(\dot{V}L + \frac{V^2}{2}\right),$$

which describes the undisturbed motion of the flat interface and relates overpressure $\Delta P(t) = P_\infty(t) - P_c$ to $V(t) = \dot{L}(t)$ in a complicated manner in order to support the necessary behavior of the growth rate. This equation does not have much interest for us.

The other equation obtained is the most significant one. It represents the equation for the linear dynamics of the interface perturbation $\zeta = \zeta_q(t)\exp(i\boldsymbol{qr})$ when the interface is subjected to an arbitrary driving acceleration $\dot{V}(t)$

$$\rho_{\rm ef}\frac{\ddot{\zeta}_q}{q} + \frac{\rho'}{K}\dot{\zeta}_q + \left[\gamma_{ik}q_i q_k + (\rho' - \rho)g - \rho_{\rm ef}\dot{V}(t)\right]\zeta_q = 0. \qquad (8)$$