of the colorless $SU(N_c)$ state of composite bag to the colorless states of other color-flavor symmetries such as $U(1)^{N_c}$, orthogonal $O_{(S)}(N_c)$ and symplectic $Sp(N_c)$ symmetry groups. These symmetry groups are restricted to an additional unimodular-like constraint. These symmetry groups are related to each other either by the decomposition or by the reduction. In each symmetry group, the color confinement (colorless) is guaranteed by projecting only the color-singlet-state wave-function though every symmetry group represents the composite color and flavor degrees of freedom in a different way. The unimodular-like constraint is imposed in order to be consistent with QCD. This assumption of the bag's internal symmetry modification takes us to consider the possibility of the phase transition from a specific symmetry group to another one. For instance, the hadronic matter which is populated by the colorless $SU(N)$ state bags can transmute to another matter that is dominated either by the colorless orthogonal $O_{(S)}(N)$ state bags or by the colorless symplectic $Sp(N)$ state bags or even by colorless $U(1)^{N_c}$ state bags.

The effective Coulomb Vandermonde potential is induced by the symmetry group constraint that projects the bag's color-singlet state. It has been shown that the effective Coulomb Vandermonde potential is regulated in a nontrivial way in the extreme hot and dense nuclear bath. The characteristic modification of the Vandermonde potential within $SU(N_c)$ symmetry causes the third order Gross-Witten hadronic (Hagedorn) transition [6]. The physics around Gross-Witten point neighbourhood is very rich. The emergence of new class of hadronic matter (i.e. Hagedorn states) is relevant to the tri-critical point [7]. The authors of Refs. [8, 9] have shown the existence of a critical chemical potential μ_c such that for $T > 0$, the physical properties for the low-lying spectrum are unaffected by the chemical potential $|\mu| < \mu_c$. The Gross-Witten hadronic (Hagedorn) transition has received much attention in vast fields such as the weak-strong phase transition in AdS/CFT [10] and the search for quark-gluon plasma (see for example [11–13]).

It is reasonable to expand the quark-gluon bag's internal structure to incorporate the color, flavor and angular-momentum degrees of freedom $(N_c, N_f, L) \to (N, \cdots)$ in an appropriate configuration in order to maintain the system's internal symmetry invariance in particular under the extreme hot and dense conditions. The value N ($\equiv N_c$) is the number of the symmetry group Hamiltonian invariant charges (see for example [14]). The unimodular-like constraint is essential in QCD-like theory. It is considered in all the symmetry groups which are considered in the present work. The symmetry group is defined by the $N_{fun} = N$