to achieve objective 2 by continuing outward using the Maxwell equations. However, these functions do not have completeness properties, item 1. And there is no smoothing, item 5, to overcome the amplification of noise due to numerical differentiation.

In the case of midplane fitting, one approach would be to attempt to employ expressions that relate the on-axis gradients to midplane data and its derivatives. For example, in the case of midplane symmetry, there are the relations

$$C^{[0]}_{m,c}(z) = 0, \tag{15a}$$

$$C^{[0]}_{1,s}(z) = B_y(x=0, y=0, z), \tag{15b}$$

$$C^{[0]}_{2,s}(z) = \frac{1}{2}\frac{\partial B_y}{\partial x}\bigg|_{(0,0,z)}, \tag{15c}$$

$$C^{[0]}_{3,s}(z) = \frac{1}{6}\frac{\partial^2 B_y}{\partial x^2}\bigg|_{(0,0,z)} + \frac{1}{24}\frac{\partial^2 B_y}{\partial z^2}\bigg|_{(0,0,z)}, \text{etc.} \tag{15d}$$

By repeatedly differentiating these relations with respect to z, one can obtain the $C^{[n]}_{m,s}(z)$ for $n > 0$. In general, the determination of $C^{[n]}_{m,s}(z)$ requires the computation of $m + n - 1$ derivatives. Although this approach achieves objective 2, since all relevant quantities are subsequently computed in terms of on-axis gradients, in the case where data is available only at grid points it presupposes the ability to compute very high-order derivatives by high-order numerical differentiation. This is generally impossible, due to the high noise sensitivity associated with high-order numerical differentiation, because there is no intrinsic smoothing, item 5.

Another approach is to use an analytic functional form, with free parameters, that is known to satisfy the the 3-dimensional Laplace equation for all parameter values. These parameters can be adjusted so that the field derived from this representation well approximates the field at various grid points. (These grid points could be in the midplane, but could be out of the midplane as well.) This representation can then be repeatedly differentiated to provide the required field derivatives. However, commonly this fitting procedure has no known completeness/convergence property, item 1. In some cases Fourier series representations with known completeness properties are used. But with Fourier series representations, an artificial periodicity is imposed in the transverse horizontal direction. As a result, the Fourier coefficients for the field expansions, call them a_n, can fall off at best as $(1/n^2)$[4]. Correspondingly, the Fourier coeficients in the associated expansion for ψ fall off at best as $(1/n^3)$. As a result, repeated differentiation produces nonconvergent series, and there is no analyticity, item 3. Whatever representation is used, there is again no intrinsic smoothing to overcome the amplification of noise due to numerical differentiation.

A. Use of Field Data on Surface of Circular Cylinder

All three-dimensional electromagnetic codes calculate all three components of the field on some three-dimensional grid. Also, such data is in principle available from actual field measurements. In this subsection we will describe how to compute the on-axis gradients from such field data using the surface of a circular cylinder [5]. Once these gradients are known, we may use (11) and (12) to compute the associated scalar and vector potentials.

Consider a circular cylinder of radius R, centered on the z-axis, fitting within the bore of the beam-line element in question, and extending beyond the fringe-field regions at the ends of the beam-line element. The beam-line element could be any straight element such as a solenoid, quadrupole, sextupole, octupole, etc., or it could be wiggler with little or no net bending. See Fig. 1, which illustrates the case of a wiggler.

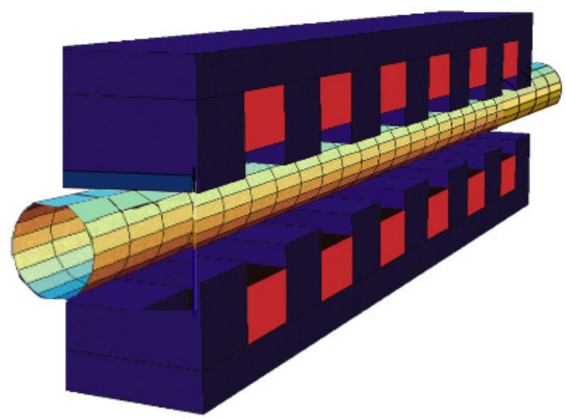

FIG. 1: A circular cylinder of radius R, centered on the z-axis, fitting within the bore of a beam-line element, in this case a wiggler, and extending beyond the fringe-field regions at the ends of the beam-line element.

Suppose the magnetic field $\boldsymbol{B}(x, y, z) = \boldsymbol{B}(\rho, \phi, z)$ is given on a grid, and this data is interpolated onto the surface of the cylinder using values at the grid points near the surface. Next, from the values on the surface, compute $B_\rho(R, \phi, z)$, the component of $\boldsymbol{B}(\rho, \phi, z)$ *normal* to the surface. The major remaining task is to compute the on-axis gradients from a knowledge of $B_\rho(R, \phi, z)$. See Fig. 2. At this point we note that the functions $\exp(ikz)\sin(m\phi)$ and $\exp(ikz)\cos(m\phi)$ form a complete set over the surface of the circular cylinder.

Let $\tilde{B}_\rho(R, \phi, k)$ be the Fourier transform of $B_\rho(R, \phi, z)$ given by the integral

$$\tilde{B}_\rho(R, \phi, k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dz\, e^{-ikz} B_\rho(R, \phi, z). \tag{16}$$

Because $\boldsymbol{B}$ decays rapidly in the fringe field region, $B_\rho(R, \phi, z)$ is absolutely integrable along the z-axis, and