small τ_0 (= large a/b) we have [16]

$$Ce_m(\tau) \approx \text{const.} \times \cos\left(\sqrt{2q-\alpha_m}\tau\right) \tag{C12a}$$

$$Se_m(\tau) \approx \text{const.} \times \sin\left(\sqrt{2q-\beta_m}\tau\right) \tag{C12b}$$

so that

$$\sqrt{2q-\alpha_m}\tau_0 = \frac{\pi n}{2} \tag{C13}$$

because $\alpha_m = \beta_m$ for large m. Here n is odd or even for the Ce or Se modes, respectively. We conclude therefore that

$$2q - \alpha_m = \left(\frac{\pi n a}{2b}\right)^2 \tag{C14}$$

has to be large and justify large $q \sim m^2$.

2. Evaluating the determinant

Each mode contributes

$$\det\left(\Delta^{-1/2}\right) = \prod_{n,m} \lambda_{n,m}^{-1/2} = \mathrm{e}^{\sum_{n,m} \ln\left(\lambda_{n,m}^{-1/2}\right)} . \tag{C15}$$

Since a is large, the sum over m can be replaced by an integral over $\omega = m/a$ like in Ref. [18] and for large $q^{1/2} \sim m \sim a/b$, we have

$$\lambda_{n,m}^{1/2} = rf\left(\frac{\omega}{r}\right), \tag{C16}$$

where

$$r = \frac{\pi n}{2b}, \tag{C17}$$

so that

$$\sum_m \ln\left(\lambda_{n,m}^{-1/2}\right) = -a\int_0^\infty \mathrm{d}\omega\, \ln\left[rf\left(\frac{\omega}{r}\right)\right]. \tag{C18}$$

For large r the integral on the right-hand side is proportional to r and we can get the coefficient of proportionality by differentiating with respect to r. This gives

$$\sum_m \ln\left(\lambda_{n,m}^{-1/2}\right) = -ar\int_0^\infty \mathrm{d}x\, \left[1 - xf'(x)/f(x)\right] \tag{C19}$$