in contrast to the lossless limit which corresponds to an infinite value for σ_0.

Another correction to the Casimir expressions is associated with the effect of thermal fluctuations [22, 23] which is correlated to the effect of imperfect reflection [24]. Bostrom and Sernelius have remarked that the small non zero value of γ had a significant effect on the force evaluation at $T \neq 0$ [25]. This remark has led to a blossoming of contradictory papers (see references in [26–28]). The current status of Casimir experiments appears to favor predictions obtained with $\gamma = 0$ rather than those corresponding to the expected $\gamma \neq 0$ (see Fig.1 in [29]). Note that the ratio between the prediction at $\gamma = 0$ with that at $\gamma \neq 0$ reaches a factor 2 at the limit of large temperatures or large distances, although it is not possible to test this striking prediction with current experiments which do not explore this domain.

At this point, it is worth emphasizing that microscopic descriptions of the Casimir interaction between two metallic bulks lead to predictions agreeing with the lossy Drude model rather than the lossless plasma model at the limit of large temperatures or large distances [30–32]. At the end of this discussion, we thus have to face a worrying situation with a lasting discrepancy between theory and experiment. This discrepancy may have various origins, in particular artefacts in the experiments or inaccuracies in the calculations. A more subtle but maybe more probable possibility is that there exist yet unmastered differences between the situations studied in theory and the experimental realizations.

The role of geometry

The geometry of Casimir experiments might play an important role in this context. Precise experiments are indeed performed between a plane and a sphere whereas calculations are often devoted to the geometry of two parallel planes. The estimation of the force in the plane-sphere geometry involves the so-called *Proximity Force Approximation* (PFA) [33] which amounts to averaging over the distribution of local inter-plate distances the force calculated in the two-planes geometry, the latter being deduced from the Lifshitz formula [34, 35].

This trivial treatment of geometry cannot reproduce the rich interconnection expected to take place between the Casimir effect and geometry [36]. In the plane-sphere geometry in particular, the PFA can only be valid when the radius R is much larger than the separation L [37]. But even if this limit is met in experiments, the PFA does not tell one what is its accuracy for a given value of L/R or whether this accuracy depends on the material properties of the mirror. Answers to these questions can only be obtained by pushing the theory beyond the PFA, which has been done in the past few years (see references in [38–42]). In fact, it is only very recently that these calculations have been done with plane and spherical metallic plates coupled to electromagnetic vacuum [43], thus opening the way to a comparison with experimental studies of PFA in the plane-sphere geometry [44].

Another specific geometry of great interest is that of surfaces with periodic corrugations. As lateral translation symmetry is broken, the Casimir force contains a lateral component which is smaller than the normal one, but has nevertheless been measured in dedicated experiments [45]. Calculations beyond the PFA have first been performed with the simplifying assumptions of perfect reflection [46] or shallow corrugations [47–49]. As expected, the PFA was found to be accurate only at the limit of large corrugation wavelengths. Very recently, experiments have been able to probe the beyond-PFA regime [50, 51] and it also became possible to calculate the forces between real mirrors with deep corrugations [52]. More discussions on these topics will be presented below.

Introduction to the scattering approach

The best tool available for addressing these questions is the scattering approach. We begin the review of this approach by an introduction considering the two simple cases of the Casimir force between 2 scatterers on a 1-dimensional line and between two plane and parallel mirrors coupled through specular scattering to 3-dimensional electromagnetic fields [53].

The first case corresponds to the quantum theory of a scalar field with two counterpropagating components. A mirror is thus described by a 2x2 $S-$matrix containing the reflection and transmission amplitudes r and t. Two mirrors form a Fabry-Perot cavity described by a global $S-$matrix which can be evaluated from the elementary matrices S_1 and S_2 associated with the two mirrors. All $S-$matrices are unitary and their determinants are shown to obey the simple relation

$$\begin{aligned} ln \det S &= \ln \det S_1 + \ln \det S_2 + i\Delta \\ \Delta &= i \ln \frac{d}{d^*} \quad , \quad d(\omega) = 1 - r_1 r_2 \exp\left(\frac{2i\omega L}{c}\right) \end{aligned} \tag{3}$$

The phaseshift Δ associated with the cavity is expressed in terms of the denominator d describing the resonance effect. The sum of all these phaseshifts over the field modes leads to the following expression of the Casimir free energy $\mathcal{F}$

$$\begin{aligned} \mathcal{F} &= -\hbar \int \frac{\mathrm{d}\omega}{2\pi} N(\omega)\Delta(\omega) \\ N(\omega) &= \frac{1}{\exp\left(\frac{\hbar\omega}{k_\mathrm{B}T}\right) - 1} + \frac{1}{2} \end{aligned} \tag{4}$$

Here N is the mean number of thermal photons per mode, given by the Planck law, augmented by the term $\frac{1}{2}$ which represents the contribution of vacuum [53].