Since $X_{\rm SPS}$ in this formula is calculated only from the pairs whose graph variables are one, it can be calculated with very short computational time as $\tilde{E}'$ and $\frac{\partial \tilde{E}'}{\partial\beta}$ in eqs. (32) and (33) are.

## V. THE CASE WHEN LONG-RANGE INTERACTIONS AND SHORT-RANGE INTERACTIONS COEXIST

When long-range interactions $\{V_{ij}^{\rm (L)}\}$ and short-range interactions $\{V_{ij}^{\rm (S)}\}$ coexist, we do not need to use the SCO method for short-range interactions because it does not reduce the computational time significantly. In other words, we can switch $V_{ij}^{\rm (L)}$ to 0 or $\bar{V}_{ij}^{\rm (L)}$ with $V_{ij}^{\rm (S)}$ unchanged. This can be realized by setting $\tilde{V}_{ij}^{\rm (L)}=0$ and $\tilde{V}_{ij}^{\rm (S)}=V_{ij}^{\rm (S)}$. In this case, we can decompose $E_{\rm const}$, $\tilde{E}$, $\frac{\partial\tilde{E}}{\partial\beta}$, and $X_{\rm SPS}$ in eqs. (27), (28), (29), and (40) as

$$E_{\rm const}=E_{\rm const}^{\rm (L)}+E_{\rm const}^{\rm (S)}, \tag{43a}$$

$$\tilde{E}=\tilde{E}^{\rm (L)}+\tilde{E}^{\rm (S)}, \tag{43b}$$

$$\frac{\partial\tilde{E}}{\partial\beta}=\frac{\partial\tilde{E}^{\rm (L)}}{\partial\beta}+\frac{\partial\tilde{E}^{\rm (S)}}{\partial\beta}, \tag{43c}$$

$$X_{\rm SPS}=X_{\rm SPS}^{\rm (L)}\times X_{\rm SPS}^{\rm (S)}, \tag{43d}$$

where the first terms and the second terms in the right hand sides denote contributions from long-range interactions and those from short-range interactions, respectively. As we have already mentioned, the first terms are given by eqs. (31), (32), (33), and (42), respectively. On the other hand, by substituting $\tilde{V}_{kl}=V_{kl}^{\rm (S)}$ and $\Delta V_{kl}^{*}=0$ into eqs. (27), (28), (29), and (40), we readily obtain

$$E_{\rm const}^{\rm (S)}=0, \tag{44a}$$

$$\tilde{E}^{\rm (S)}=-\sum\nolimits_{\langle kl\rangle}V_{kl}^{\rm (S)}, \tag{44b}$$

$$\frac{\partial\tilde{E}^{\rm (S)}}{\partial\beta}=0, \tag{44c}$$

$$X_{\rm SPS}^{\rm (S)}=\prod\nolimits_{\langle kl\rangle}\exp[(\beta_m-\beta_n)(V_{kl}(\boldsymbol{S}_k,\boldsymbol{S}_l)-V_{kl}(\boldsymbol{S}_k',\boldsymbol{S}_l'))]$$
$$=X_{\rm B}^{\rm (S)}, \tag{44d}$$

where we have used the fact that all the potentials are switched to $\tilde{V}_{kl}$. Note that $P_{kl}=1$ when $\tilde{V}_{kl}=V_{kl}$ and $\Delta V_{ij}^{*}=0$. These results are quite natural because they coincide with the results in the usual MC procedure.

## VI. NUMERICAL TESTS

### A. Internal energy and heat capacity measurements

In order to check the validity of the formulae for internal energy and heat capacity measurements, we perform MC simulations of a three dimensional magnetic dipolar system on a $L^3$ simple cubic lattice. The boundary condition is open. The Hamiltonian of the system is described as

$$\mathcal{H}=\mathcal{H}_{\rm dip}=D\sum_{i<j}\left[\frac{\boldsymbol{S}_i\cdot\boldsymbol{S}_j-3(\boldsymbol{S}_i\cdot\boldsymbol{r}_{ij})(\boldsymbol{S}_j\cdot\boldsymbol{r}_{ij})}{r_{ij}^3}\right], \tag{45}$$

where $\boldsymbol{S}_i$ is a classical Heisenberg spin of $|\boldsymbol{S}_i|=1$, $\boldsymbol{r}_{ij}$ is the vector spanned from a site $i$ to $j$ in the unit of the lattice constant $a$, $r_{ij}=|\boldsymbol{r}_{ij}|$, and $D=(g\mu_{\rm B}S)/a^3$. We hereafter call the system *purely dipolar system*.

In simulations with the SCO method, we regard each term in the right hand side of eq. (45) as $V_{ij}$. Since $\tilde{V}_{ij}=0$ in the SCO method, $\Delta V_{ij}$ defined by eq. (3) is equal to $V_{ij}$. Interaction $V_{ij}$ has the maximum value $2D/r_{ij}^3$ when $\boldsymbol{S}_i$ and $\boldsymbol{S}_j$ are anti-parallel along $\boldsymbol{r}_{ij}$. We therefore set $\Delta V_{ij}^{*}$, which should be equal to or greater than $\Delta V_{ij}$ over all $\boldsymbol{S}_i$ and $\boldsymbol{S}_j$, to $2D/r_{ij}^3$. By substituting these into eq. (2), we obtain

$$P_{ij}=\exp\left[\beta D\left\{\frac{\boldsymbol{S}_i\cdot\boldsymbol{S}_j-3(\boldsymbol{S}_i\cdot\boldsymbol{r}_{ij})(\boldsymbol{S}_j\cdot\boldsymbol{r}_{ij})-2}{r_{ij}^3}\right\}\right], \tag{46}$$

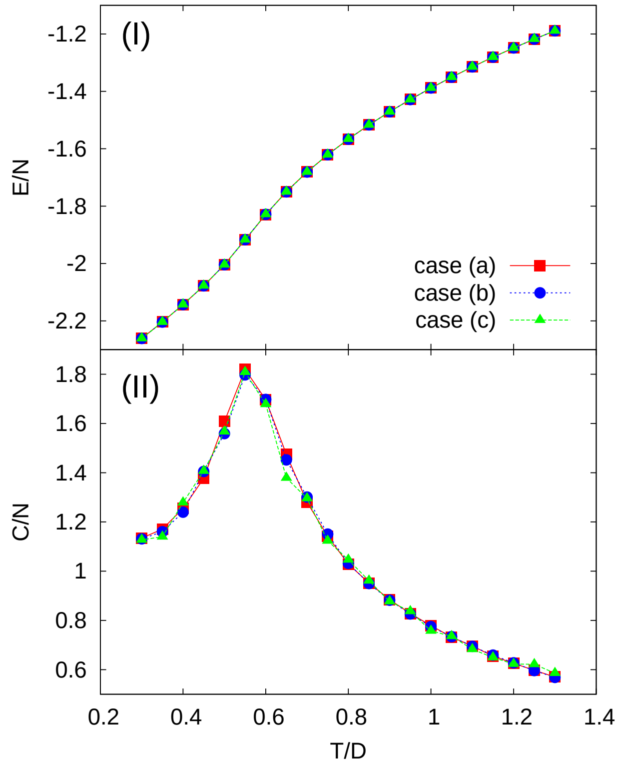


FIG. 1: (Color online) Temperature dependence of (I) internal energy $E/N$ and (II) heat capacity $C/N$ in the purely dipolar system. Simulated annealing method is used for the measurement. The size $N$ is $10^3$. Measurements are done in the three cases (see text).