A. Optimal Problem Formulation in High SINR Regime

Assuming perfect knowledge of channel gains at a centralized controller, the optimal solution for (1) can be computed in the high SINR regime by an exhaustive search over all possible combinations of the allocations. For each possible allocation, optimum powers can be computed by transforming (1) into a GP. Note that the power allocation problem is in itself a known non-convex problem for the general SINR regime [14]. However, in the high SINR regime the problem becomes a convex GP problem. For a given set of allocation variables and considering a high SINR regime, the objective function in (1) can be rewritten as follows:

$$\underset{p_{n,k,l}}{\text{maximize}} \quad \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{N} \alpha_{n,k,l}\log_2\left(\frac{p_{n,k,l}h_{n,k,l}}{\sigma^2+I_{n,l}}\right) \tag{5}$$

Maximizing the SINRs is equivalent to minimizing the interference to signal ratio:

$$\underset{p_{n,k,l}}{\text{minimize}} \quad \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{N} \alpha_{n,k,l}\log_2\left(\frac{\sigma^2+I_{n,l}}{p_{n,k,l}h_{n,k,l}}\right) \tag{6}$$

Equivalently, (1) can be reformulated for high SINR regime and given allocation variables as follows:

$$\begin{aligned} \underset{p_{n,k,l}}{\text{minimize}} \quad & \log_2 \prod_{l=1}^{L}\prod_{k=1}^{K}\prod_{n=1}^{N}\left(\frac{\sigma^2+I_{n,l}}{p_{n,k,l}h_{n,k,l}}\right)^{\alpha_{n,k,l}} \\ \text{subject to} \quad & \sum_{n=1}^{N}\alpha_{n,k,l}p_{n,k,l} \leq \mathrm{P}_{k,\max}, \quad \forall k, \forall l \end{aligned} \tag{7}$$

Note that the numerator in (7) is a posynomial and the denominator is a monomial, hence (7) is a GP problem in standard form that can be solved optimally through efficient interior point methods [13] after performing the logarithmic transformation of variables [14]. However, even for small dimensions, it is not recommendable to compute the optimal solution, due to the huge computational complexity $O(K^{LN})$ associated with the exhaustive search based subcarrier allocation phase. In addition, the GP based power allocation method discussed above has two restrictions: high-SINR assumption and centralized time-consuming computations. Due to the mentioned facts, there is a need to develop bounds and sub-optimal resource allocation schemes for multi-cell OFDMA networks.