established once cSDW order fades away. Figure 7 then provides evidence for a quantum phase transition that seperates a hidden ferromagnetic state at weak Hund's rule coupling from a cSDW state at strong Hund's rule coupling. Notice that the putative quantum phase transition at $J_0 \cong -1.35\, J_1^{\perp}$ (see Fig. 5) occurs at a Hund's rule exchange coupling that is almost a factor-of-2 times larger than the classical prediction $J_{0c} = -0.8\, J_1^{\perp}$ listed in Table I. Last, the bottom graph in Fig. 7 shows the evolution of the relevant order parameters in the case of true Néel order with intervening hidden Néel order. Here, we have chosen Heisenberg exchange coupling constants $J_1^{\parallel} > 0, J_1^{\perp} = 0$, and $J_2^{\parallel} = 0.25\, J_1^{\parallel} = J_2^{\perp}$, which lies at maximum frustration when Hund's rule is obeyed. The common inflection point at $J_0 \cong -1.2 J_1^{\parallel}$ is consistent with a quantum phase transition at a value of Hund's rule coupling that is noticably larger than the classical prediction of $J_{0c} = -J_1^{\parallel}$ listed in Table I.

IV. DISCUSSION AND CONCLUSIONS

Recent inelastic neutron scattering measurements on the parent compound to ferro-pnictide superconductors $CaFe_2As_2$ have uncovered well-defined spin-wave excitations in the cSDW phase throughout virtually the entire Brillouin zone.[14] Instead of softening at the wavenumber $(\pi/a, \pi/a)$ that corresponds to Néel order as predicted by the conventional J_1-J_2 model over the square lattice, they show a local maximum there. Figure 8 displays a fit of the spin-wave spectrum obtained by inelastic neutron scattering on $CaFe_2As_2$ to the prediction for the linear spin-wave dispersion of a square lattice of spin-1 iron atoms, $(\Omega_+\Omega_-)^{1/2}$, at the quantum critical point that seperates the cSDW phase from hidden ferromagnetic (Néel) order [see Eq. (12) and Fig. 2]. The spin per orbital is set to $s = 1/2$ and the following Heisenberg exchange coupling constants are used: $J_{0c} = -57.0$ meV, $J_1^{\parallel(\perp)} = 0$, $J_1^{\perp(\parallel)} = 115.8$ meV, and $J_2^{\parallel} = 43.7\,\text{meV} = J_2^{\perp}$. These imply a ratio $J_2/J_1 = 0.75$ between the next-nearest-neighbor and the nearest-neighbor Heisenberg exchange constants deep inside the cSDW, where Hund's rule is obeyed. Notice that the measured spin-wave spectrum terminates before it reaches the predicted Goldstone modes at zero momentum that correspond to hidden magnetic order. This is consistent with Fig. 3, which displays how the spectral weight of low-energy spin-waves about zero momentum is dwarfed by that (15) of low-energy spinwaves about momentum $(\pi/a, 0)$ at the quantum critical point that seperates the cSDW phase from hidden magnetic order.