(2.45) means that excitations in the near horizon throat region appear red shifted to an observer at infinity, $E_{\rm inf} = f^{-1/4}E_r$. Since it is the energy as measured by an observer at infinity that is important, in the limit $r \to 0$ the full Type IIB string theory must be kept. Nevertheless the higher energy modes from the string theory cannot escape the throat region without being redshifted away. We are thus left with supergravity in flat asymptopia and IIB string theory compactified on the near-horizon geometry, and the two are decoupled.

Stepping back we find that we have two pictures of the same low energy limit of one theory, Type IIB string theory. In the field theory picture we have two decoupled theories: supergravity in the far region and $\mathcal{N} = 4$ $SU(N)$ SYM on the D branes. In the geometry picture we also have two decoupled theories: supergravity in asymptotically flat space and Type IIB string theory in the throat region. Noticing that both pictures have identical asymptotic supergravity in them we boldly propose, in the spirit of Maldacena, that the other decoupled theories are also identical: Type IIB string theory compactified on the near horizon background of Eq. (2.45) is dual to $3+1D$ $\mathcal{N} = 4$ $SU(N)$ SYM.

The regions for which analytic tools exist for these two different pictures turn out to be completely incompatible. Comparing the Born-Infeld action of N coincident D p branes and that for an $SU(N)$ two form field strength yields

$$g^2_{SYM} = 2g_s(2\pi)^{p-2}l_s^{3-p} \;\Rightarrow\; g^2_{SYM} = 4\pi g_s \text{ for } p = 3. \tag{2.47}$$