ing behavior at inverse lattice spacings below $a^{-1} \approx 2.5$ GeV, the difference between the predicted value for f_{B_s} at $a \approx 0.11$ fm given $\mathcal{O}(a^2)$ scaling and the value of f_{B_s} that they obtain in the continuum limit using data within the scaling region is only about 25%, which is again close to our power-counting estimate. Thus we expect that naive power-counting should lead to a reasonable estimate of the discretization error in the $SU(3)$-breaking ratios.

For the error estimates in this subsection, we evaluate the strong coupling constant at the lattice scale, $\alpha_s^{\overline{\mathrm{MS}}}(1/a) \sim 1/3$. We choose $\Lambda_{\mathrm{QCD}} = 500$ MeV because the typical QCD scale that enters heavy-light quantities tends to be larger than for light-light quantities, as indicated by fits to moments of inclusive B-decays using the heavy-quark expansion [74]. Fortunately, some of the finite lattice-spacing effects cancel in the ratios f_{B_s}/f_{B_d} and ξ. This can be seen by the fact that, although the APE and HYP data differ by about 15–20% for the individual decay constants and matrix elements, they agree within statistical errors for the ratios. In $SU(3)$-breaking quantities, errors must be proportional to the difference in quark masses $(m_s - m_d)$. Dimensional analysis therefore suggests that contributions to the total discretization error are suppressed by the factor $(\widetilde{m}_s - \widetilde{m}_d)/\Lambda_{\mathrm{QCD}} \sim 1/5$, where we use $\widetilde{m}_s$ and $\widetilde{m}_d$ to denote the renormalized quark masses in the $\overline{\mathrm{MS}}$ scheme [6] (as opposed to the bare lattice quark masses) in this subsection and the next. The observed size of $SU(3)$-breaking effects in the B-meson decay constants $(f_{B_s}/f_{B_d} - 1)$ and in the $B-$mixing matrix elements $(\xi - 1)$ are consistent with this expectation. Discretization errors in f_{B_s}/f_{B_d} and ξ can arise from both the actions and the operators. We estimate each source of error separately, and add them in quadrature to obtain the total discretization error.

None of the actions that we are using are $\mathcal{O}(a^2)$-improved. Therefore the leading discretization errors from the domain-wall fermion action and Iwasaki gauge action are of $\mathcal{O}(a^2\Lambda_{\mathrm{QCD}}^2)$. When combined with the $SU(3)$-breaking suppression factor, this leads to discretization errors in the ratios f_{B_s}/f_{B_d} and ξ of $\mathcal{O}(a^2\Lambda_{\mathrm{QCD}}^2 \times (\widetilde{m}_s - \widetilde{m}_d)/\Lambda_{\mathrm{QCD}}) \sim 1.7\%$. The leading heavy-quark discretization errors from the static action are also of $\mathcal{O}(a^2\Lambda_{\mathrm{QCD}}^2)$. Hence heavy-quark discretization errors also contribute $\sim 1.7\%$ to the total error in the ratios. Because we improve the heavy-light axial current used to compute the decay constant through $\mathcal{O}(\alpha_s ap)$, the leading discretization errors from the heavy-light current are of $\mathcal{O}(\alpha_s am)$, $\mathcal{O}(\alpha_s^2 a\Lambda_{\mathrm{QCD}})$, and $\mathcal{O}(a^2\Lambda_{\mathrm{QCD}}^2)$.[4] When combined with the $SU(3)$-breaking suppression fac-

[4] There are also discretization errors from mixing with operators of other chiralities that are proportional