We also have the the inequality given by

$$r_{k+1} < r_k \tag{22}$$

This is obvious, as the r_{k+1} is dominated more strongly by the particles near the inner edge than r_k. It can also be proved mathematically using Hölders inequality or Jensen's inequality.

From here, it follows that

$$\sqrt{\frac{f_k}{f_{k+1}}} < r_{k+1} \tag{23}$$

And this inequality persists when we take averages. Hence the claim. Also, in the limit $k \to \infty$, both definitions would converge to the same value (before averaging the limit is exclusively dominated by the innermost particle), but the one based on the ratios converges faster (what we called R_0). Hence the definition R_0 is the most numerically economic calculation of the radius of the configuration, as promised earlier. For completeness, we show now the plot in figure 5 with the variance of the data set included.

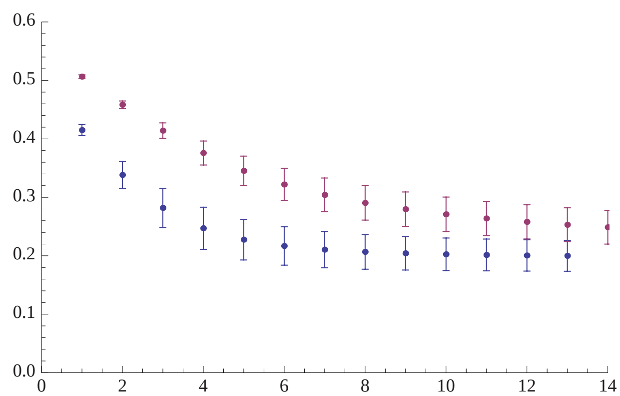

FIG. 6: Plot of R_{0k} and $r_k^{(r)}$ with the variance of sample included: this would indicate the error of a single measurement on the quantum system. It essentially accounts for the quantum fluctuations. This is the same data set used to generate figure 5.

As should be recognized from the analysis above, the choice of variable to measure a geometric property in a model of emergent geometry with quantum fluctuations can receive very large corrections due to the fluctuations. This depends on how we average over configurations. This is determined in essence by the observable that one chooses to probe the system with. Although this is not surprising, the fact that fluctuations can be rather large