several occasions (see [7] and the references therein). Most studies in the literature impose the condition ϕ =const. (equivalent to R =const. in the original $f(R)$ theory) for ease of calculation, and compare static solutions with Solar System experiments. This point of view carries the risk of not exploring the richer variety of solutions with $\partial\phi/\partial t \neq 0$, which are certainly more generic than static ones.

CONCLUSIONS

Motivated by the recent attention to metric and Palatini $f(R)$ gravity theories revived to explain the cosmic acceleration without dark energy, we have considered spherical symmetry and the Jebsen-Birkhoff theorem in these theories and, by extension, in general scalar-tensor gravity.

Generalizing the Jebsen-Birkhoff theorem of General Relativity to situations with matter present allows one to understand the validity, or lack thereof, of this theorem in scalar-tensor gravity because the scalar-tensor field equations can be rewritten as effective Einstein equations with the Brans-Dicke like scalar acting as a form of effective matter. Using the Jordan frame description of scalar-tensor gravity, this effective matter distribution must be static in order for the Jebsen-Birkhoff theorem to be valid. This conclusion is not hard to obtain but is seems that it is necessary to formulate it explicitly in order to make progress with $f(R)$ gravity. The situation can be summarized as follows: if ϕ is static the spherically symmetric solution is locally static between horizons but not necessarily Schwarzschild-(anti-)de Sitter; if ϕ is constant the solutions is Schwarzschild-(anti-)de Sitter.

Since scalar-tensor gravity admits an Einstein frame description in which the scalar has canonical form except for the fact that it couples directly to matter, the result obtained in the Jordan frame must be recovered in the Einstein frame description, and we checked that this is indeed the case. The equivalence (at the classical level) between Jordan and Einstein frame breaks down when the conformal transformation (34) and (35) becomes ill-defined, and this occurrence allows one to understand the apparent contradiction between certain spherical solutions and Hawking's theorem on Brans-Dicke black holes. Shedding light onto this riddle certainly does not have deep new consequences (these non-Schwarzschild static solutions have now been known for a long time) but we are not aware of an explicit explanation in these terms in the literature.

Once the role of the Jebsen-Birkhoff theorem in scalar-tensor gravity is established, it is straightforward to understand that the validity of this theorem in Palatini $f(R)$ gravity is yet another manifestation of the non-dynamical character of the Brans-Dicke scalar present in this theory. Similarly, the failure of the theorem in metric $f(R)$ gravity reflects the dynamical nature of the scalar degree of freedom present in these theories. Most current studies of spherically symmetric solutions in metric $f(R)$ gravity focus on static solutions missing time-dependent solutions which are, without doubt, more generic than static ones (although there is at present no mathematically well-defined meaning of "generic"). To complicate the issue, metric $f(R)$ theories of current interest are designed to produce an effective time-varying cosmological constant in order to explain the present acceleration of the universe without dark energy, and it is expected that "generic" solutions (if a meaning can be assigned to this adjective) will be asymptotically Friedmann-Lemaitre-Robertson-Walker solutions violating the Jebsen-Birkhoff theorem. Some solutions of this kind are known in General Relativity [28, 58], scalar-tensor gravity [59] and in metric $f(R)$ gravity [60], but they are still not understood very well even in the context of General Relativity and it will be interesting to study them further in the future.

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* vfaraoni@ubishops.ca

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