by

$$f_{1\#}(T) - f_{0\#}(T) = \partial g_{\#}([0,1] \times T) + g_{\#}([0,1] \times \partial T).$$

Define the homotopy $g(\gamma, \mathbf{x}) = \gamma \mathbf{x} + (1-\gamma)\psi^1(\mathbf{x})$ for $\gamma \in [0,1]$. Then the homotopy formula gives

$$T - \psi^1_{\#}(T) = \partial g_{\#}([0,1] \times T) + g_{\#}([0,1] \times \partial T).$$

We define $R = g_{\#}([0,1] \times T)$ and $Q_1 = g_{\#}([0,1] \times \partial T)$. Then we get

$$T - \psi^1_{\#}(T) = \partial R + Q_1. \tag{2.18}$$

Finally, we map $\psi^1_{\#}(\partial T)$ forward to the $(d-1)$-skeleton of simplicial complex K with $\phi = \phi_{k+1}$ to get $\psi^2_{\#}(\partial T) = \phi_{\#}(\psi^1_{\#}(\partial T))$. For this purpose, consider the homotopy $h(\gamma, \mathbf{x})$ from $\psi^1_{\#}(\partial T)$ to $\psi^2_{\#}(\partial T)$, i.e.,

$$h(\gamma, x) = \gamma \psi^1_{\#}(\mathbf{x}) + (1-\gamma)\psi^2_{\#}(\mathbf{x}) \quad \text{for} \quad \gamma \in [0,1].$$

We define

$$P = \psi^1_{\#}(T) - h_{\#}([0,1] \times \psi^1_{\#}(\partial T)). \tag{2.19}$$

P is a d-current whose boundary ∂P is contained in the $(d-1)$-skeleton of K. Define $Q_2 = h_{\#}([0,1] \times \psi^1_{\#}(\partial T))$. Using the homotopy formula, we get

$$\begin{aligned}
\partial P &= \partial \Big(\psi^1_{\#}(T) - h_{\#}([0,1] \times \psi^1_{\#}(\partial T)) \Big) \\
&= \psi^1_{\#}(\partial T) - \partial h_{\#}([0,1] \times \psi^1_{\#}(\partial T)) \\
&= \psi^2_{\#}(\partial T) \subset (d-1)\text{-skeleton of } K.
\end{aligned}$$