Figure 1 displays the temporal evolution of three respective temperatures for pumping electrons (a) and hot phonons (b). Before pumping, all three types of degrees of freedom (DoF) are in equilibrium, which is set at $T_e = T_{ph} = T_l = 0.01$. As shown in Fig. 1(a), when the pump pulse is absorbed by the electrons, the electronic temperature rises steeply around $\tau = 0$, and reaches the maximum after a small time delay. It then begins to drop at a time scale determined by the el-ph coupling strength, followed by a slower relaxation. Interestngly, we also observe a small kink in T_e at $T_e = T_c$, which entirely arises from the breakup of the Cooper pairs, resulting in the dramatic change in the rise rate of T_e. This kink was not captured in the early model [19]. Simultaneously, the hot phononic temperature T_{ph} rises smoothly via the energy exchange with electrons and then drops by exchanging energy with the cold lattice, causing the slight increase of T_l. When directly pumping phonons, the time dependence of the temperatures is similarly observed, including the kink in T_e, as shown in Fig. 1(b). Due to the large pumping width, the kink is more easily visible in this case.

Time-resolved spectral function: The time-resolved spectral function is defined as $A(\mathbf{k},\omega) \equiv -\frac{2}{\pi}\,\mathrm{Im}\mathcal{G}_{11}(\mathbf{k},\omega)$. Here $\mathcal{G}_{11}$ is the one-one component of the retarded Green's function $\hat{\mathcal{G}}(\mathbf{k},\omega)$, which is related to the self-energy by $\hat{\mathcal{G}}^{-1}(\mathbf{k},\omega) = \hat{\mathcal{G}}_0^{-1}(\mathbf{k},\omega) - \hat{\Sigma}(\mathbf{k},\omega)$, with $\hat{\mathcal{G}}_0^{-1}(\mathbf{k},\omega) = \omega\hat{\sigma}_0 - \Delta_{\mathbf{k}}\hat{\sigma}_1 - \xi_{\mathbf{k}}\hat{\sigma}_3$. $\hat{\sigma}_{0,1,2,3}$ are the unit and Pauli matrices. As is known [16], in the equilibrium state with $T_e = T_{ph}$, the self-energy can be evaluated more conveniently within the imaginary-time Green's function approach, in which a key step is to convert the Bose-Einstein distribution to the Fermi distribution, $n_B(i\omega_n \pm E_{\mathbf{k}-\mathbf{q}}) = -n_F(\pm E_{\mathbf{k}-\mathbf{q}})$ (with $\omega_n = (2n+1)\pi T$, $T = T_e = T_{ph}$). For the current situation, the temperatures of electrons and hot phonons are no longer tied to each other and the above conversions are not valid any more. To avoid this restriction, here we apply the double-time Green's function approach [27] to calculate $\hat{\Sigma}(\mathbf{k},\omega)$. The result is

$$\begin{aligned}\hat{\Sigma}(\mathbf{k},\omega) &= \frac{1}{N_L}\sum_{\mathbf{q}}|g_\nu(\mathbf{k},-\mathbf{q})|^2 \\ &\{[(\omega-\Omega_0)\Phi_1 - (\omega+\Omega_0)\Phi_2 + \Omega_0(\Phi_3+\Phi_4)]\hat{\sigma}_0 \\ &+[2E_{\mathbf{k}-\mathbf{q}}(\Phi_1-\Phi_3) + 2\Omega_0(\Phi_3-\Phi_4)]\hat{\sigma}_1 \\ &+[\xi_{\mathbf{k}}(\Phi_1-\Phi_2) + (\Omega_0\xi_{\mathbf{k}}/E_{\mathbf{k}})(\Phi_3-\Phi_4)]\hat{\sigma}_3\}, \quad (9)\end{aligned}$$

where $\Phi_1 = N(\Omega_0)/[(\omega-\Omega_0+E_{\mathbf{k}-\mathbf{q}})(\omega-\Omega_0-E_{\mathbf{k}-\mathbf{q}})]$, $\Phi_2 = N(-\Omega_0)/[(\omega+\Omega_0+E_{\mathbf{k}-\mathbf{q}})(\omega+\Omega_0-E_{\mathbf{k}-\mathbf{q}})]$, $\Phi_3 = f(-E_{\mathbf{k}-\mathbf{q}})/[(\omega+\Omega_0-E_{\mathbf{k}-\mathbf{q}})(\omega-\Omega_0-E_{\mathbf{k}-\mathbf{q}})]$, and $\Phi_4 = f(E_{\mathbf{k}-\mathbf{q}})/[(\omega+\Omega_0+E_{\mathbf{k}-\mathbf{q}})(\omega-\Omega_0+E_{\mathbf{k}-\mathbf{q}})]$.

For simplicity, let us first look at the density of states (DOS), which is defined by $\rho(\omega) = \frac{1}{N_L}\sum_{\mathbf{k}} A(\mathbf{k},\omega)$. For direct pumping of electrons, we calculate the DOS at time sequences $\tau = -100$, -2.7, 2.8, and 600, respectively. The results are plotted in Fig. 2 (a). At the initial time

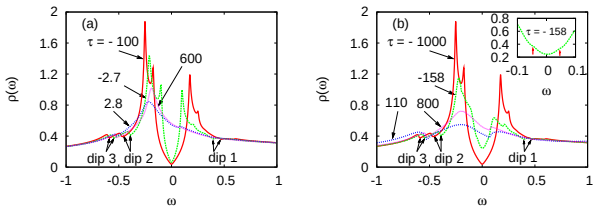

FIG. 2: (Color) Time evolution of the density of states for selectively exciting electrons (a) and phonons (b), respectively.

$\tau = -50$ where $T_e = 0.01$ and the system is at superconducting state [$\Delta_0(T_e) = 0.19$], we observe two signatures of the el-ph coupling located at $\pm(\Omega_0 + \Delta_0(T_e))$, as shown in Fig. 2 (dip 1 and dip 2). They are symmetric with respect to $\omega = 0$, but dip 2 is much stronger. It is related to the fact that the normal-state van Hove singularity is located below the Fermi energy and as such the coherent peak at $-\Delta_0(T_e)$ has stronger intensity than that at $\Delta_0(T_e)$. In addition, dip 3 occurs near the characteristic energy $-(E_M + \Omega_0)$, where E_M is the quasiparticle energy at $\mathbf{k} = (\pi, 0)$ or equivalent wavevector points in Brillouin zone. It arises from the van Hove singularity at $\mathbf{k} = (\pi, 0)$. The signature at the energy $E_M + \Omega_0$ is much weaker also because of the van Hove singularity location in the normal state. At $\tau = -2.7$ where $T_e = 0.041$ and $\Delta_0(T_e) \approx 0.1$, similar behaviors are observed. However, at $\tau = 2.8$, $T_e = 1.05$ and thus only the signature of the normal-state el-ph coupling (dip 3) can be observed. At $\tau = 600$, a similar structure of the normal-state el-ph coupling is observed.

For direct pumping of phonons, we choose different time sequences $\tau = -1000$, -158, 110, and 800 to evaluate the DOS. The results are displayed in Fig. 2(b). From Fig. 2(b), we observe that the dips suggesting the el-ph couplig in both normal and supercondcting states becomes stronger than those for selectively pumping electrons, as expected. In particular, we observe two new small dips at $\omega = \pm 0.05$ for $\tau = -158$ (at which $T_e = 0.041$ and $T_{ph} = 0.163$), both of which arise from the poles of $\Sigma_{\mathbf{k}}(\omega)$ at $\omega = \pm(\Omega_0 - \Delta_0(T_e))$, in addition to those observed in the case of electrons being excited directly. This is because the hot phononic temperature becomes very high while the electronic temperature is still cold and the contributions from the terms as weighted by $N(\pm\Omega_0)$ are significantly enhanced. Figure. 2(b) clearly shows that, as time elapses from $\tau = -1000$, -158, to 110, the peaks move toward the *zero energy*, while from $\tau = 110$ to 800, the locations of these two peaks remain unchanged. This can be understood by considering that, after pumping, T_e rises with the time delay, while $\Delta_0(T_e)$ decreases and rapidly vanishes when T_e reaches T_c.

Finally we numerically evaluate the time-resolved spectral function for the two excitations discussed above at those time sequences chosen for calculating the DOS. In