external field "tickling" the charge, we find that

$$\begin{aligned} i\mathcal{M} &= -ie\bar{u}(p')\left(\mathcal{M}_0(p', p-k)\frac{i(\not{p} - \not{k} + m)}{(p-k)^2 - m^2}\gamma \cdot \epsilon^*(k)\right. \\ &\qquad \left. + \gamma \cdot \epsilon^*(k)\frac{i(\not{p}' + \not{k} + m)}{(p'+k)^2 - m^2}\mathcal{M}_0(p'+k, p)\right) u(p). \end{aligned} \tag{F.82}$$

In order to make contact with the classical limit we assume a soft photon: $|\vec{k}| \ll |\vec{p}' - \vec{p}|$. Then we can take

$$\mathcal{M}_0(p'+k, p) \approx \mathcal{M}_0(p', p-k) \approx \mathcal{M}_0(p', p), \tag{F.83}$$

and we can ignore the $\not{k}$ in the numerators.

Using Eq. (E.11) we have that Eq. (F.82) becomes

$$i\mathcal{M} = \bar{u}(p')\big[\mathcal{M}_0(p', p)\big]u(p) \cdot \left[e\left(\frac{p' \cdot \epsilon^*}{p' \cdot k} - \frac{p \cdot \epsilon^*}{p \cdot k}\right)\right]. \tag{F.84}$$

Summing over polarization states, integrating over phase space, and taking the conditional probability yields

$$\langle N_\gamma \rangle = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega}e^2 \sum_\lambda \left|\epsilon_\lambda \cdot \left(\frac{p'}{k \cdot p'} - \frac{p}{k \cdot p}\right)\right|^2. \tag{F.85}$$

For massless QED we have that in a diagram that satisfies the Ward identity $\sum_\lambda \epsilon^\lambda_\mu \epsilon^{*\lambda}_\nu = -g_{\mu\nu}$. Then we have that the energy radiated, to lowest order,