Replication of Li-Yorke Chaos Near a Homoclinic Orbit

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Abstract

We prove the presence of chaos near a homoclinic orbit in the modified Li-Yorke sense [10] by implementing chaotic perturbations. A Duffing oscillator is considered to show the effectiveness of our technique, and simulations that support the theoretical results are depicted. Ott-Grebogi-Yorke and Pyragas control methods are used to stabilize almost periodic motions.

Keywords: Li-Yorke chaos; Homoclinic orbit; Duffing oscillator; Almost periodic solutions

1 Introduction

The investigations of chaos theory for continuous-time dynamics started due to the needs of real world applications, especially with the studies of Poincaré, Cartwright and Littlewood, Levinson, Lorenz, and Ueda [16, 33, 36, 42, 48]. Chaotic dynamics has high effectiveness in various fields such as the analysis of electrical processes of neural networks, weather phenomena, mechanical systems, optimization and self-organization problems in robotics, and brain dynamics. The reason for that is the opportunities provided by the dynamical structure of chaos.

To explain the extension procedure of our paper, let us give the following information. It is known that if one considers the evolution equation $u' = L[u] + I(t)$, where $L[u]$ is a linear operator with spectra placed out of the imaginary axis of the complex plane, then a function $I(t)$ being considered as an input with a certain property (boundedness, periodicity, almost periodicity) produces through the equation the output, a solution with a similar property, boundedness/periodicity/almost periodicity. In particular, in our paper, we solved a similar problem when the linear system has eigenvalues with negative real parts and input is considered as a chaotic set of functions with a known type. Our results are different in the sense that the input and the output are not single functions, but *a collection of functions*. In other words, we prove that both the input and the output are *chaos* of the same type for the discussed equation. The way of our investigation is arranged in the well accepted traditional mathematical fashion, but with a new and a more complex way of arrangement of the connections between the input and the output. The

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