using one (or two) dimensional gas [15–19]. This model is an alternate approximation to deal the color-singlet state of quark gas. The quarks are treated as classical particles but their non-Abelian interactions are introduced by the exact Coulomb gauge potential. This model has been considered to study the order of the deconfinement phase transition. The effective potential in that model is dominated by a linear potential and it differs from the effective Vandermonde potential that emerges from the symmetry group invariance Haar measure. Furthermore, the density of states for the classical quark gas is not given in these studies.

The hadronic phase transition has been studied using the bootstrap model [20, 21]. The continuous hadronic mass spectrum is an exponentially increasing mass spectrum and it resembles the bootstrap mass density, namely, $\rho(m) \sim c\, m^{-\alpha}\, e^{b\,m}$ [22–25]. The grand potential density is reduced to

$$\begin{aligned} \frac{\Omega}{V} &= -\frac{\partial}{\partial V}\left(T \ln Z\right), \\ &= -T \int_{m_0}^{\infty} dm\, \rho(m)\, \frac{\partial}{\partial V}\left[\ln Z(m, \beta, \cdots)\right]. \end{aligned} \tag{1}$$

The simple and reasonable approximation is the Boltzmann gas with the canonical ensemble that is given by,

$$\begin{aligned} \ln Z &= \int d^3\vec{r} \int \frac{d^3\vec{k}}{(2\pi)^3} \Lambda e^{-\beta\epsilon(p,m,r)}, \\ &\approx V \left(\frac{m\,T}{2\pi}\right)^{3/2} \Lambda \, \exp\left(-\frac{m}{T}\right), \end{aligned} \tag{2}$$

where Λ is the thermodynamic fugacity and $\epsilon(p, m, r)$ is the energy of constituent particle. For the simplicity, the free energy $\epsilon(p, m) = \sqrt{\vec{p}^2 + m^2}$ is usually adopted. When the critical point of the deconfinement phase transition is reached, the Hagedorn mass spectrum leads to the following results: The grand potential density diverges for the exponent $\alpha \leq 5/2$. This means that the instant phase transition to explosive quark-gluon plasma does not exist for this class of mass spectrum. The deconfinement phase transition turns to be smooth cross-over one. On the other hand, the system likely undergoes first order deconfinement phase transition when the exponent becomes $\alpha > 7/2$ because the grand potential density and its derivative are finite at the critical point making the grand potential density continuous and making its first derivative with respect to the thermodynamic ensemble discontinuous. When the exponent α which appears in the mass spectral density runs over $5/2 < \alpha \leq 7/2$,