Lemma 2 *Under $M.1 - M.4$ and NSM, $P_0(y,1|z)$ and $P_1(y,0|z)$ are bounded as follows:*

$$P_0(y,1|z) \in \left[L_{01}^{wst}(y,z), U_{01}^{sm}(y,z)\right],$$
$$P_1(y,0|z) \in \left[L_{10}^{wst}(y,z), U_{10}^{sm}(y,z)\right],$$

where

$$L_{10}^{sm}(y,z) = \begin{cases} \left(\frac{\lim_{p(z)\to \underline{p}} P(y,1|z) - P(y,1|z)}{\overline{p} - p(z)}\right)(1-p(z)), & \text{for any } z \in \mathcal{Z} \setminus p^{-1}(\overline{p}), \\ 0, & \text{for } z \in p^{-1}(\overline{p}), \end{cases},$$
$$U_{01}^{sm}(y,z) = \begin{cases} \left(\frac{P(y,0|z) - \lim_{p(z)\to \underline{p}} P(y,0|z)}{p(z) - \underline{p}}\right)p(z), & \text{for any } z \in \mathcal{Z} \setminus p^{-1}(\underline{p}), \\ p(z), & \text{for } z \in p^{-1}(\underline{p}), \end{cases},$$

and these bounds are sharp.

Now, sharp bounds on marginal distributions of Y_0 and Y_1 are obtained by plugging the results in Lemma 2 into the counterfactual probabilities.

Note that under NSM, sharp bounds on the joint distribution and sharp bounds on the DTE are still obtained from Fréchet-Hoeffding bounds and Makarov bounds. To illustrate this, consider the case where $\rho_0 = \rho_1 = 0$ in the example (2).[13] This case satisfies NSM and NSM does not impose any restriction on the dependence between ν_0 and ν_1. Therefore, sharp bounds on the joint distribution and the DTE are obtained by the same token as in Subsection 3.1.

The specific forms of sharp bounds on marginal distributions of Y_0 and Y_1, their joint distribution, and the DTE under $M.1 - M.4$ and NSM are provided in Corollary 1 in Appendix.

3.3 Conditional Positive Quadrant Dependence

Unlike NSM, CPQD has no additional identifying power for the joint distribution and the DTE. In this subsection, I impose weak positive dependence between ε_0 and ε_1 conditional on U by considering CPQD as follows: for any $(e_0, e_1) \in \mathbb{R}^2$,

$$P[\varepsilon_0 \le e_0|u]\, P[\varepsilon_1 \le e_1|u] \le P[\varepsilon_0 \le e_0, \varepsilon_1 \le e_1|u]. \tag{6}$$

[13] Note that NSM restricts the sign of ρ_d as nonnegative for $d \in \{0,1\}$.