Here $\sigma_t\left(\bar{\varepsilon}, \bar{Z}\right)$ is the transport scattering cross section from Eq. (8) for the effective electron energy $\bar{\varepsilon}$ and an effective distance $\bar{Z}$ between the traps and the OS interface, $n_T^+ = \int_0^D N_T^+(Z)\,dZ$ is the 2D density of charged traps, and $v(\bar{\varepsilon})$ is the electron velocity which corresponds with $\bar{\varepsilon} = m_c v^2/2$. Comparing Eq. (12) and (16) we find

$$\bar{Z}^{2/3} = \frac{\int_0^D N_T^+(Z)\,Z^{2/3}dZ}{\int_0^D N_T^+(Z)\,dZ} = \frac{\int_0^D N_T^+(Z)\,Z^{2/3}dZ}{n_T^+} \quad (17)$$

for this factor in $\sigma_t\left(\bar{\varepsilon}, \bar{Z}\right) = c_\sigma \bar{\varepsilon}^{-1/3}\bar{Z}^{2/3}$.

In order to calculate the resistivity ρ the knowledge of n_T^+ is not necessary, n_T^+ cancels out with the denominator of $\bar{Z}^{2/3}$ within $\sigma_t\left(\bar{\varepsilon}, \bar{Z}\right)$, AM do not use it. As mentioned in the introduction we are interested in the metal-insulator transition depending on the electron density n_s, i. e. we want to know the temperature behavior of ρ as a function of n_s. In this context n_T^+ is very useful in order to understand on the basis of Eq. (16) that it contributes the main variations to the resistivity $\rho(n_s, T)$ whereas $\sigma_t\left(\bar{\varepsilon}, \bar{Z}\right)$ and $v(\bar{\varepsilon})$ show only weak dependence on n_s and T. The benefit of Eq. (16) against (12) is, that the physical meaning of the terms is immediately clear.

The integral in the numerator and that in the denominator of $\bar{Z}^{2/3}$ can be treated in quite the same way, so we define

$$\Omega_j \equiv \int_0^D N_T^+(Z)\,Z^j dZ = \\ = N_T \int_0^D \frac{Z^j dZ}{\frac{1}{2}\exp\left(-\frac{E_T(Z)-\mu}{k_B T}\right)+1}. \quad (18)$$

In the last step we followed AM and assumed that the trap density is constant within the oxide, respectively in the region where $p_+(Z)$ does not vanish. Now we can write

$$\sigma_t\left(\bar{\varepsilon}, \bar{Z}\right) = c_\sigma \bar{\varepsilon}^{-1/3}\bar{Z}^{2/3} = c_\sigma \bar{\varepsilon}^{-1/3}\frac{\Omega_{2/3}}{\Omega_0}, \quad (19)$$

$$n_T^+ = \Omega_0, \quad (20)$$

$$\rho = \frac{\sqrt{2m_c}\,c_\sigma \bar{\varepsilon}^{1/6}\Omega_{2/3}}{n_s e^2}. \quad (21)$$

To be able to calculate the integral which corresponds with $\Omega_{2/3}$ AM expanded the electrostatic energy $\varepsilon_T(Z)$ into a Taylor series about the point Z_m where it reaches its maximum ε_m. This procedure is called saddle-point approximation.

$$Z_m = D\sqrt{\frac{\varepsilon_D}{eV_{\mathrm{ins}}}}, \quad (22)$$

$$\varepsilon_m = -2\sqrt{eV_{\mathrm{ins}}\varepsilon_D}, \quad (23)$$

$$\varepsilon_e(Z) \simeq \varepsilon_m - \varepsilon_D \frac{D}{Z_m^3}(Z - Z_m)^2, \quad (24)$$

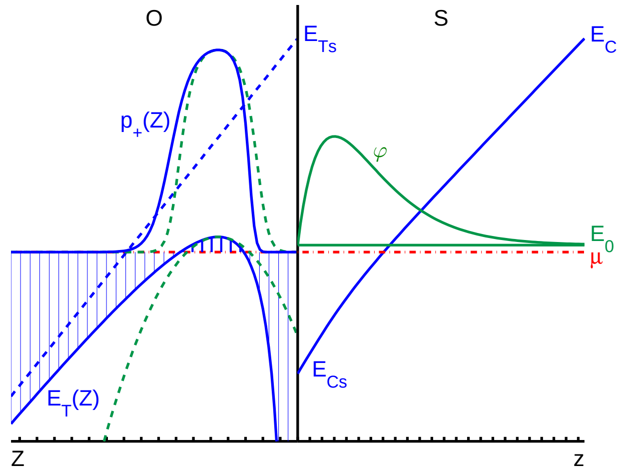

Figure 3. Oxide-semiconductor interface. Trap energy $E_T(Z)$ (full line), its Taylor approximation (dashed line), and the resulting probabilities $p_+(Z)$ in arbitrary units with μ as zero point.

see Fig. 2 and 3. Now (18) can be written as

$$\Omega_j \simeq N_T \int_0^D \frac{Z^j dZ}{\frac{1}{2}\exp\left(\frac{\mu_{E_0}-\varepsilon_{Ts0}-\varepsilon_m+\varepsilon_D\frac{D}{Z_m^3}(Z-Z_m)^2}{k_B T}\right)+1}, \quad (25)$$

$$\varepsilon_{Ts0} = E_{Ts} - E_0 = \text{const.} \quad (26)$$

AM assume that the energy E_{Ts} relative to the ground state energy E_0 is constant, but we believe that rather the conduction band edge at the interface E_{Cs} has to be used as reference energy, i. e. $\varepsilon_{TsCs} = E_{Ts} - E_{Cs} = \text{const.}$ This issue will be further treated in section VI.

The integrand is a peak around Z_m which drops off exponentially on both sides. In order to come to the same result as AM, we further apply the following approximations: (i) Because of the exponential decrease one can integrate from $-\infty$ to ∞. (ii) The integrand is dominated by the denominator, so $Z^j \simeq Z_m^j$ can be set in the numerator. Now the integrand is symmetric around Z_m and with help of the substitution $\mathcal{Z} = \frac{\varepsilon_D D}{Z_m^3 k_B T}(Z - Z_m)^2$ one gets

$$\Omega_j = N_T Z_m^{j+3/2}\sqrt{\frac{k_B T}{\varepsilon_D D}} \\ \times \int_0^\infty \frac{\mathcal{Z}^{-1/2}d\mathcal{Z}}{\exp\left(\frac{\mu_{E_0}-\varepsilon_{Ts0}-\varepsilon_m}{k_B T} - \ln 2 + \mathcal{Z}\right)+1}. \quad (27)$$

This corresponds to the integral in equation (9c) in Ref. 26 ($\mathcal{Z}$ corresponds to x^2). We brought it into the form above as it corresponds now to a Fermi-Dirac integral[32]

$$\mathcal{F}_k(\eta) = \frac{1}{\Gamma(k+1)}\int_0^\infty \frac{\mathcal{Z}^k d\mathcal{Z}}{\exp(\mathcal{Z}-\eta)+1}, \quad (28)$$