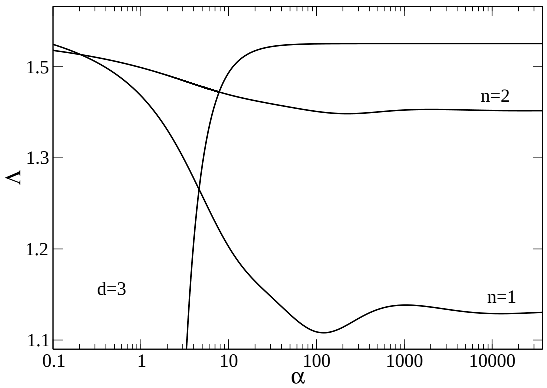

FIG. 30: Energy Λ as a function of α for the first two branches in $d = 3$. For $\alpha \to +\infty$, the energy of the first branch undergoes damped oscillations around the value $\Lambda_s \simeq 1.13$. We have taken $\mu = 1$.

Introducing the dimensionless variables defined previously, recalling that $r = \xi R/\alpha$ and $\mu = k_0 R$, and introducing the normalized energy (57), we obtain

$$\Lambda = -\frac{d}{2\eta} + \frac{1}{2\eta^2}\frac{1}{\alpha^{d-2}}\int_0^\alpha \left(\frac{d\psi}{d\xi}\right)^2 \xi^{d-1} d\xi + \frac{\mu^2}{2\eta^2}\frac{1}{\alpha^d}\int_0^\alpha (\psi + \beta m\Phi_0)^2 \xi^{d-1} d\xi. \tag{78}$$

Using the expressions of κ and λ following Eq. (68), we find that

$$\beta m \Phi_0 = -\frac{\lambda}{\kappa^2}, \tag{79}$$

so that, finally,

$$\Lambda = -\frac{d}{2\eta} + \frac{1}{2\eta^2}\frac{1}{\alpha^{d-2}}\int_0^\alpha \left(\frac{d\psi}{d\xi}\right)^2 \xi^{d-1} d\xi + \frac{\mu^2}{2\eta^2}\frac{1}{\alpha^d}\int_0^\alpha \left(\psi - \frac{\lambda\alpha^2}{\mu^2}\right)^2 \xi^{d-1} d\xi. \tag{80}$$

This equation gives the relation between the energy Λ and α for the n-th branch. In Figs. 26, 27, 29 and 30, we plot the energy Λ as a function of α for the first two branches $n = 1$ and $n = 2$ in different dimensions of space $d = 1, 2, 3$. The discussion is similar to the one given in Sec. III D. We have also represented the branch corresponding to the homogeneous solution. Using Eq. (87) and $\eta = \alpha^2/d$, its equation is given by $\Lambda = -d^2/(2\alpha^2) + d/(2\mu^2)$.

E. The entropy and the free energy

Finally, we relate α to the entropy S and to the free energy F. The entropy is given by

$$S = \frac{d}{2} N k_B \ln T - k_B \int \frac{\rho}{m} \ln \frac{\rho}{m}\, d\mathbf{r}. \tag{81}$$

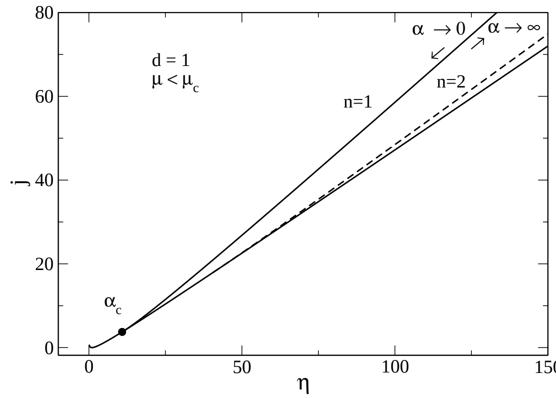

FIG. 31: Free energy $\frac{J}{Nk_B}$ as a function of the inverse temperature η for $d = 1$. We have taken $\mu = 1 < \mu_c$.

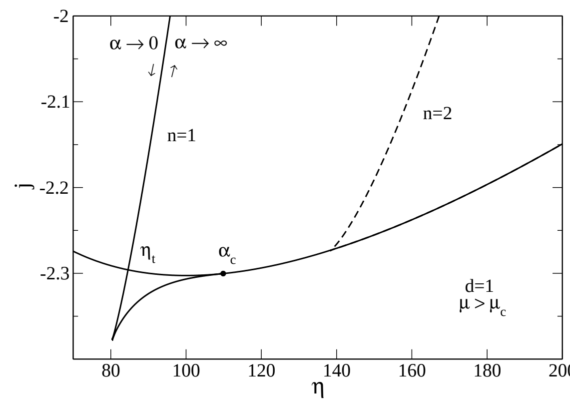

FIG. 32: Free energy $\frac{J}{Nk_B}$ as a function of the inverse temperature η for $d = 1$. We have taken $\mu = 10 > \mu_c$. The free energies of the homogeneous phase and inhomogeneous phase become equal at $\eta = \eta_t(\mu)$. This corresponds to a first order phase transition in the canonical ensemble marked by the discontinuity of the slope $J'(\beta) = -E$.

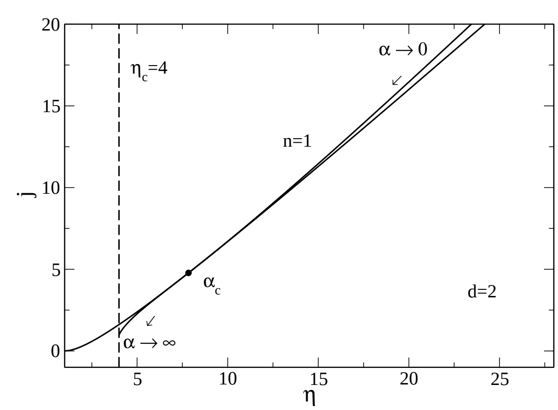

FIG. 33: Free energy $\frac{J}{Nk_B}$ as a function of the inverse temperature η for $d = 2$. We have taken $\mu = 1$.