Chapter 1

Introduction

1.1 Overview

This dissertation applies and extends geometric measure theory tools used for currents and densities. In particular, the flat norm is used to measure currents and provides a useful metric in surface space. This notion is discretized to obtain the multiscale simplicial flat norm and a simplicial deformation theorem (Chapter 2, based on [24]) which approximates currents with chains on a simplicial complex via small deformations (as measured by the flat norm).

The multiscale simplicial flat norm can be computed efficiently and, for integral inputs, has guaranteed integral minimizers in several important cases (in particular, for codimension 1 chains). This statement is stronger than what was known for the continuous case (where the statement was for codimension 1 boundaries). Bridging the gap between these statements and extending the discrete results to the continuous case is the goal of Chapter 3 (based on [25]) where it is shown for 1-currents in $\mathbb{R}^2$ with a framework for establishing the result in general assuming suitable triangulation results.

Lastly, the notion of nonasymptotic densities (also known as the integral area invariant) is developed in the plane in Chapter 4 (based on [27]) where uniqueness questions are addressed in light of a certain useful regularity condition (tangent cone graph-like).

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