confounding factors that predict the correct response probability other than $\boldsymbol{\alpha}$. This problem constitutes a substantial challenge to the evaluation of the likelihood function and consequently to the estimation of the person and item parameters.

To resolve this issue, we introduce a Markov network and explicitly model conditional dependence relationships among the items. The Markov network is specified by an undirected graph $G = (V, E)$ with a vertex set $V = \{1, \ldots, J\}$, of which each element is associated with a random variable, and an edge set $E \subset V \times V$, which characterizes pairwise relationships of the variables. In the present setup, the random variables of interest are the item response scores, $X_1, \ldots, X_J$. The graph models a conditional (in)dependence relationship between two random variables via an edge. That is, if X_j and $X_{j'}$ are conditionally independent given all the other variables, there is no edge between the vertices j and j' (i.e., $(j, j') \notin E$); if, on the other hand, the two variables are conditionally dependent, their associated vertices are linked by an edge $(j, j') \in E$. The graph is undirected in the sense that $(j, j') \in E$ if and only if $(j', j) \in E$. The relationships of all random variables may be implied by a symmetric design matrix $\mathbf{\Phi} = (\phi_{jj'} : j, j' = 1, \ldots, J)$, where $\phi_{jj'} = \phi_{j'j} = 0$ if $(j, j') \notin E$, and $\phi_{jj'} = \phi_{j'j} \neq 0$ if $(j, j') \in E$. The joint probability distribution of the random variables then has the form

$$f(X_1 = x_1, \ldots, X_J = x_J) = \frac{1}{z(\mathbf{\Phi})} \exp\left(\frac{1}{2}\mathbf{x}^\top \mathbf{\Phi}\mathbf{x}\right) \propto \exp\left(\frac{1}{2}\mathbf{x}^\top \mathbf{\Phi}\mathbf{x}\right), \tag{2}$$

where $z(\mathbf{\Phi}) = \sum_{\mathbf{x}\in\{0,1\}^J} \exp\left(\frac{1}{2}\mathbf{x}^\top \mathbf{\Phi}\mathbf{x}\right)$ is a normalizing constant that ensures the distribution sum to 1. The diagonal entry of the design matrix, ϕ_{jj}, shows a main effect of the variable X_j; the off-diagonal entry, $\phi_{jj'}$, quantifies an interaction effect between the variables X_j and $X_{j'}$. By convention, a vertex is not a neighbor of itself. Hence, the current study fixes all vertex-wise parameters ϕ_{jj}s at 0 in like manner and assumes that any nonzero values of the diagonal entries are traceable to the DCM of concern.

Incorporating the Markov network into the DCM, we obtain the graphical DCM where the joint probability function of $\mathbf{X} = (X_1, \ldots, X_J)$ is given by

$$f(\mathbf{X} = \mathbf{x} \mid \boldsymbol{\alpha}, \mathbf{B}, \mathbf{\Phi}) \propto \exp\left(\mathbf{x}^\top \mathbf{B}\boldsymbol{\alpha} + \frac{1}{2}\mathbf{x}^\top \mathbf{\Phi}\mathbf{x}\right). \tag{3}$$