### 0.0.1 1. Introduction

Experimental findings[1-4] in recent years have revived the interest in multiferroics. They showed that magnetic and ferroelectric orders are closely related[5-8]. What is more intriguing is that only certain types of magnetic orders, namely helical spins and frustrated spins, can be coupled to ferroelectricity[9]. It is this fascinating interplay between ferroelectric and magnetic orders that has attracted many researchers. There were already models based on the Ginzburg-Landau theory[10,11] that provide instructive physical description of the systems. As for the microscopic mechanism, there are currently two schools of theories. One of them proposed that the electric polarization and the anomaly of dielectric constant come from atomic displacements. The displacements or phonons are in turn, coupled to spins[12-14]. Though proposed for systems of orthorhombic structure, it is more readily applied to multiferroics of hexagonal structures, such as $HoMnO_3$, as there is experimental evidence of atomic displacements from neutron scattering data[15]. The second school of theory proposed a new possibility: electric polarization coming from electronic wave function and thus density distribution. Katsura et. al.(KNB)[16] predicted that the magnetoelectric effect can be induced by "spin current"[17]. The coupling between "spin current" and internal electric field has the same form as that of Dzyaloshinskii-Moriya interaction (DM)[18,19] or AC-effect[20] where the motion of a magnetic moment is coupled to electric field. In this latter theory, the atomic displacement is not essential. On the other hand, spin-orbit interaction is indispensable in generating electric dipole moments.

The "spin current" model, though a bright idea, needs additional substantiation in order to be applied to physical systems. Jia[21] et. al. gave a detailed calculation of this model. Their results showed that the "spin current" model is able to explain at least semi-quantitatively many experimental data. This