autonomous system (3.5) with the autonomous equation

$$x' = \overline{F}(x, \mu_\infty), \tag{3.6}$$

where $\overline{F} : \mathbb{R}^m \times \mathbb{R} \to \mathbb{R}^m$ is a continuous function in all of its arguments.

We suppose that system (3.5) ((3.6)) admits a chaotic attractor, let us say a set in $\mathbb{R}^m$ for (3.6). Fix x_0 from the attractor, and take a solution $x(t)$ of (3.6) with $x(0) = x_0$. Since x_0 is a member of the chaotic attractor, we will call $x(t)$ a chaotic solution [1, 3, 13, 66].

There exists a compact set $\Lambda \subset \mathbb{R}^m$ such that the trajectories of the chaotic solutions of (3.5) ((3.6)) lie inside Λ for all t. If we denote $M_H = \max_{x\in\Lambda} \|H(x)\|$, then one can verify that $\sup_{t\in\mathbb{R}} \left\|\phi_{x(t)}(t)\right\| \leq \dfrac{N(M_G + M_H)}{\omega}$ for each chaotic solution $x(t)$ of (3.5) ((3.6)).

The following conditions are needed throughout the section.

(C3) There exists a positive number L_2 such that $\|H(x_1) - H(x_2)\| \leq L_2 \|x_1 - x_2\|$ for all $x_1,\ x_2 \in \Lambda$;

(C4) There exists a positive number L_3 such that $\|H(x_1) - H(x_2)\| \geq L_3 \|x_1 - x_2\|$ for all $x_1,\ x_2 \in \Lambda$;

(C5) There exists a positive number L_4 such that $\|F(t, x_1, \mu_\infty) - F(t, x_2, \mu_\infty)\| \leq L_4 \|x_1 - x_2\|$ for all $t \in \mathbb{R}$, $x_1, x_2 \in \Lambda$.

The next subsection is concerned with the extension of sensitivity.

3.1 Extension of Sensitivity

Let us describe the sensitivity feature for system (3.5) as well as its replication by system (1.1).

System (3.5) is called sensitive if there exist positive numbers ϵ_0 and Δ such that for an arbitrary positive number δ_0 and for each chaotic solution $x(t)$ of (3.5), there exist a chaotic solution $\overline{x}(t)$ of the same system, $t_0 \in \mathbb{R}$ and an interval $J \subset [t_0, \infty)$ with a length no less than Δ such that $\|x(t_0) - \overline{x}(t_0)\| < \delta_0$ and $\|x(t) - \overline{x}(t)\| > \epsilon_0$ for all $t \in J$.

We say that system (1.1) replicates the sensitivity of (3.5) if there exist positive numbers ϵ_1 and $\overline{\Delta}$ such that for an arbitrary positive number δ_1 and for each chaotic solution $x(t)$ of (3.5), there exist a chaotic solution $\overline{x}(t)$ of (3.5), $t_0 \in \mathbb{R}$ and an interval $J^1 \subset [t_0, \infty)$ with a length no less than $\overline{\Delta}$ such that $\left\|\phi_{x(t)}(t_0) - \phi_{\overline{x}(t)}(t_0)\right\| < \delta_1$ and $\left\|\phi_{x(t)}(t) - \phi_{\overline{x}(t)}(t)\right\| > \epsilon_1$ for all $t \in J^1$.

The next assertion is about the sensitivity feature of system (1.1).

Lemma 3.1 *Under the conditions $(C1) - (C5)$, system (1.1) replicates the sensitivity of (3.5).*

We omit the proof of Lemma 3.1 since it can be proved in a very similar way to Lemma 5.1 [32].

We will handle the presence of chaos through a cascade of almost periodic motions in system (1.1) in the following subsection.