$$= \frac{2C(\omega_n, \mathbf{q})}{1 + g(\mathbf{q})C(\omega_n, \mathbf{q}) - 2\sum_i \Gamma_i^2(\mathbf{q})C(\omega_n, , \mathbf{q})D_i^0(\omega_n, \mathbf{q})}. \tag{3.13}$$

We have assumed that the couplings $g(\mathbf{q}, \mathbf{k}, \mathbf{k}')$ and $\Gamma_i(\mathbf{k}, \mathbf{q})$ are such that we can ignore their dependence on $\mathbf{k}$ and $\mathbf{k}'$. Here $C(\omega_n, \mathbf{q})$ is the Cooper bubble:

$$C(\omega_n, \mathbf{q}) = 2\int \frac{dk_x dk_y}{4\pi^2}\left[\frac{f(\epsilon_{A1}(\mathbf{k}+\mathbf{q})) - f(-\epsilon_{A1}(-\mathbf{k}))}{i\omega_n - \epsilon_{A1}(\mathbf{k}+\mathbf{q}) - \epsilon_{A1}(-\mathbf{k})} + (\epsilon_{A1} \leftrightarrow \epsilon_{A2})\right]. \tag{3.14}$$

Here $\epsilon_{A_{1/2}}(k)$ are the bare dispersions of the $A_{1/2}$ quasi-particles. As $T \to 0$, $C(\omega_n, q=0)$ develops a logarithmic divergence: $C(\omega_n, q=0) \approx \sqrt{m_{||}m_{\perp}}\log(\frac{\epsilon_F+\mu}{T})$.

The pair susceptibility for the Cooperons fields has a similar RPA form:

$$\chi_i^{RPA}(\omega_n, \mathbf{q}) = \langle T\phi_i(q,\tau)\phi_i^\dagger(q,0)\rangle \;=\; D_i^0(\omega_n, k) + (D_i^0(\omega_n, k))^2\Gamma_i^2(q)\chi_{QPA}^{RPA}(\omega_n, \mathbf{q})$$

$$= \frac{D_i^0(\omega_n, k) + g(q)C(\omega, q) - 2D_{\tilde{i}}^0(\omega_n, \mathbf{q})C(\omega_n, q)\Gamma_{\tilde{i}}^2(q)}{1 + g(\mathbf{q})C(\omega_n, \mathbf{q}) - 2\sum_i \Gamma_i^2(\mathbf{q})C(\omega_n, , \mathbf{q})D_i^0(\omega_n, \mathbf{q})}. \tag{3.15}$$

where $\tilde{A} = B, \tilde{B} = A$. The superconducting instability occurs when the denominator in Eqns. (3.3 and 3.5) vanishes at $\omega_n = 0, q = 0$, that is

$$C(0,0)\left(g(0) - 2\sum_i \Gamma_i^2(0)\frac{v_{Fi}}{\Delta_i^2 - 4\mu^2}\right) - 1 = 0. \tag{3.16}$$

We note that this vanishing occurs simultaneously in all channels. If $g > 0$ (though inter-ladder Coulomb repulsion is repulsive, the interactions between quasi-particles on a given ladder is attractive leaving the sign of g indeterminate) the instability occurs only when the chemical potential approaches sufficiently close to $\Delta_A/2$ so that the resulting effective interaction becomes attractive. This chemical potential corresponds to minimal doping at which the superconductivity appears. Taking $\Delta_A \ll \Delta_B$, the corresponding transition temperature takes the form

$$T_c = \max\{T_{c1}, T_{c2}\}; \quad T_c \approx \epsilon_{Fi}\exp\frac{1}{\sqrt{m_{||i}m_{\perp i}}}\left[\frac{2\Gamma_A^2(0)v_{FA}}{(\Delta_A^2 - 4\mu^2)} - g(0)\right]^{-1}, \tag{3.17}$$

where $\epsilon_{Fi} = \frac{\gamma_i^2}{4}\frac{(t_i^{eff}(0))^2}{t_i^{eff}(0) - 2\mu}$. If we suppose that μ and t_i^{eff} are such that we only have hole pockets, the density of dopants is equal to

$$x(\mu) = \frac{\kappa_0}{2^{7/2}\pi^2|t_i^{eff}(0)|^{1/2}}\frac{(2\Delta_i + |t_i^{eff}(0)| + 2\mu)(-2\Delta_i + |t_i^{eff}(0)| + 2\mu)}{\sqrt{|t_i^{eff}(0)| + 2\mu}} \tag{3.18}$$