are obtained by Fréchet-Hoeffding bounds as follows: for any $(y_0, y_1) \in \mathbb{R}^2$,

$$\begin{aligned}
&\max\{P(y_0|0,z) + P_1(y_1|0,z) - 1, 0\} \\
&\leq P(Y_0 \leq y_0, Y_1 \leq y_1|0,z) \\
&\leq \min\{P(y_0|0,z), P_1(y_1|0,z)\}.
\end{aligned}$$

Since $P_1(y_1|0,z)$ is only partially identified, sharp bounds on $P(Y_0 \leq y_0, Y_1 \leq y_1|0,z)$ are obtained by taking the union over all possible values of $P_1(y_1|0,z)$. Therefore, sharp bounds on $P(Y_0 \leq y_0, Y_1 \leq y_1, D = 0|z) = P(Y_0 \leq y_0, Y_1 \leq y_1|0,z)(1 - p(z))$ are derived as follows:

$$\begin{aligned}
&\max\left\{P(y_0, 0|z) + L_{10}^{wst}(y,z) - (1 - p(z)), 0\right\} \\
&\leq P(Y_0 \leq y_0, Y_1 \leq y_1, D = 0|z) \\
&\leq \min\left\{P(y_0, 0|z), U_{10}^{wst}(y,z)\right\}.
\end{aligned}$$

Similarly,

$$\begin{aligned}
&\max\left\{L_{01}^{wst}(y,z) + (P(y_1|1,z) - 1)p(z), 0\right\} \\
&\leq P(Y_0 \leq y_0, Y_1 \leq y_1, D = 1|z) \\
&\leq \min\left\{U_{01}^{wst}(y,z), P(y_1|1,z)p(z)\right\}.
\end{aligned}$$

By (12), sharp bounds on $P(Y_0 \leq y_0, Y_1 \leq y_1)$ are obtained by taking the intersection of the bounds over all values of $z \in \Xi$,

$$\begin{aligned}
F^L(y_0, y_1) = \sup_{z \in \Xi} &\left\{\max\left\{(P(y_0|0,z) - 1)(1 - p(z)) + L_{10}^{wst}(y_1, z), 0\right\}\right. \\
&\left.+ \max\left\{L_{01}^{wst}(y_0, z) + (P(y_1|1,z) - 1)p(z), 0\right\}\right\}, \\
F^U(y_0, y_1) = \inf_{z \in \Xi} &\left\{\min\left\{P(y_0|0,z)(1 - p(z)), U_{10}^{wst}(y|z)\right\}\right. \\
&\left.+ \min\left\{U_{01}^{wst}(y_0, z), P(y_1|1,z)p(z)\right\}\right\}.
\end{aligned}$$