ature phase cannot be described using an expansion in terms of the low–temperature order parameter. A recent very detailed simulation study of a family of three–dimensional dimer models with ordered ground states and high–temperature Coulomb phases found a rich variety of continuous and discontinuous phase transitions, including double phase transitions where monopole excitations condense out of the Coulomb phase to give a conventional paramagnet at intermediate temperatures[52].

The present understanding of these phenomena is that the gauge field associated with Coulomb phase is minimally coupled to a matter field which condenses in the ordered phase, following an Anderson–Higgs mechanism[50–52]. In fact it is also possible to study zero temperature (quantum) phase transitions from Coulomb to ordered phases in three–dimensional *quantum* dimer models[56]. These can in principle be continuous, occurring through the condensation of monopole excitations in the Coulomb phase[57], but numerical simulations suggest that the transition is first order[37].

Less is known about transitions in spin models, but one interesting scenario exists for a continuous transition in an extended Heisenberg model on a pyrochlore lattice from a Coulomb phase to a four–sublattice ordered state[53]. This transition is found to be in the same universality class as a uniaxial ferroelectric with dipolar interactions, for which the upper critical dimension is three[58]. This makes possible to continuous transitions with mean-field exponents (up to log corrections) — a scenario which closely resembles the transition from plateau-liquid into vector-quadrupole phase discussed below. Generically, however, transitions from Coulomb liquids into ordered states seem to be first order[53], a fact which may be explained by interactions between fluctuations of associated gauge field[54].

We conclude by noting that the complex forms of order which can occur in Heisenberg models on the pyrochlore lattice as a result of the interplay between farther–neighbor interactions and thermal fluctuations are also a topic of current interest[26]. In finite magnetic field, these lead to a half–magnetization plateau which can be tuned at will between different forms of order[27]. A similar fluctuation driven plateau, but with a uniquely defined form of order, is also expected to occur for the edge sharing tetrahedra of the FCC lattice[59].

We now return to the model in question.

B. Transition from plateau–liquid to ordered $uuud$ state

For $h \simeq 4$, $T \lesssim b$, Eq. (1) exhibits the plateau–liquid state described in Sec. III C [cf. Fig. 2(b)]. Inclusion of a FM third–neighbor interaction J_3 [Eq. (2)] causes it to order at low temperatures. We consider first the conventional limit where both $|J_3|$ and b are "large", choosing parameters $J_3 = -0.06$ and $b = 0.6$. In this case there is strongly first order transition from paramagnet to four–sublattice plateau state for $T_N \approx 0.70$. This can be seen very clearly in simulation results for the heat capacity and the order parameter $\lambda^{\mathsf{global}}_{\mathsf{T}_2}$, and its susceptibility $\chi^{\mathsf{global}}_{\mathsf{T}_2}$, presented in Fig. 19. If we now decrease $|J_3|$, the transition temperature T_N must also decrease,

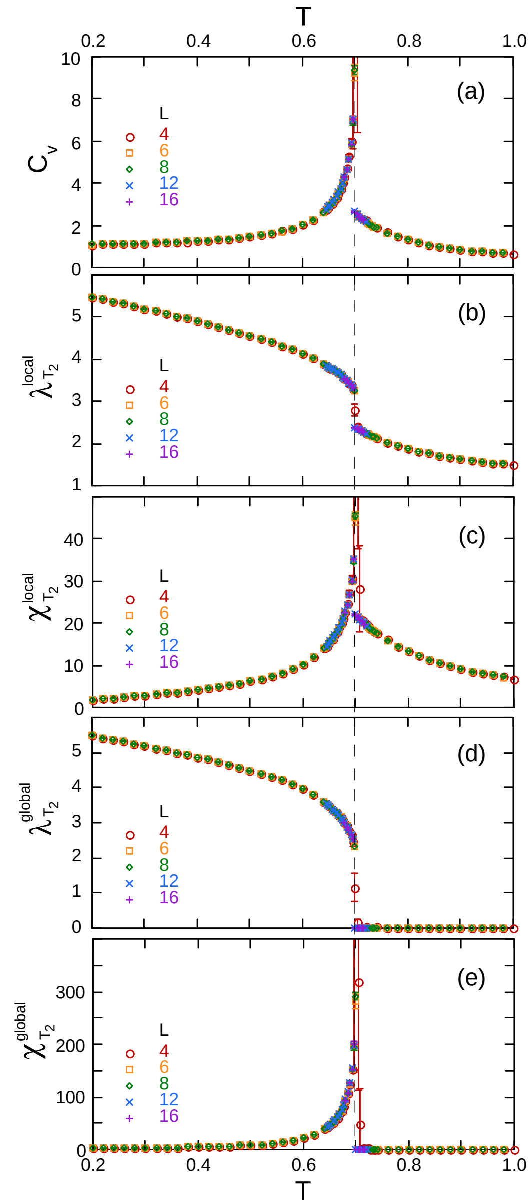

FIG. 19: (Color online) Temperature dependence of (a) heat capacity defined by Eqs. (31), (b) and (c) the related measure of *local* correlation $\lambda^{\mathsf{local}}_{\mathsf{T}_2}$ and its susceptibility defined by Eqs. (10) and (11), and (d) and (e) the *global* order parameter $\lambda^{\mathsf{global}}_{\mathsf{T}_2}$ and its susceptibility defined by Eqs. (8) and (9) for $b = 0.6$, $h = 4$ and a range of values of system size. The results are for $J_3 = -0.06$, showing a single first order transition into the ordered phase at $T_N = 0.70(1)$ (indicated by the vertical dashed line).

and for sufficiently small $|J_3|$ it will become smaller than the crossover temperature $T^* \approx b$ associated with the plateau liquid.

In this case, there are anomalies in thermodynamic quantities at two distinct temperatures as demonstrated in Fig. 20. There is a broad maximum in $\chi^{\mathsf{local}}_{\mathsf{T}_2}$ at $T^* \approx 0.6$ [Fig. 20(c)], signaling the onset of the plateau liquid state, accompanied by a broad peak in the heat capacity C_v at a slightly lower temper-