The standard simplicial approximation theorem from algebraic topology describes how continuous maps are approximated by simplicial maps that satisfy the star condition [35, §14]. Our simplicial deformation theorem applies to currents, which are more general objects than continuous maps. More importantly, we present explicit bounds on the expansion of mass of the current resulting from simplicial approximation. In his PhD thesis, Sullivan [42] considered deforming currents on to the boundary of convex sets in a cell complex, which are more general than the simplices we work with. But simplicial complexes admit efficient algorithms more naturally than cell complexes. We adopt a different approach for deformation from Sullivan and obtain new bounds on the approximations (see Section 2.5.2). Along with the multiscale simplicial flat norm, our deformation theorem also establishes how the optimal homologous chain problem and optimal bounding chain problem could be used on general continuous inputs by taking simplicial approximations, thus expanding widely the applicability of this family of techniques.

2.2 Definition of Simplicial Flat Norm

Consider a finite p-dimensional simplicial complex K triangulating the set Ω, where the simplices are oriented, with $p \geq d+1$. The set T is defined as the integer multiple of an oriented d-dimensional subcomplex of K, representing a rectifiable d-current with integer multiplicity. Let m and n be the number of d- and $(d+1)$-dimensional simplices in K, respectively. The set T is then represented by the d-chain $\sum_{i=1}^{m} t_i \sigma_i$, where σ_i are all d-simplices in K and t_i are the corresponding *weights*. We will represent this chain by the vector of weights $\mathbf{t} \in \mathbb{Z}^m$. We use bold lower case letters to denote vectors, and the corresponding letter with subscript to denote components of the vector, e.g., $\mathbf{x} = [x_j]$.