which is the first time at which the stochastic process hits the boundary, $\partial\Omega$, and is absorbed. Define a second stopping time, $\hat{t} = \min(\gamma_{\Omega}^{x,t}, t)$, which returns t if the process does not hit the boundary before time t, and returns the first time at which the stochastic process hits the boundary otherwise. For convenience, define:

$$k(\boldsymbol{x}) = \begin{cases} f(\boldsymbol{x}), & \text{if } \hat{t} = t, \\ g(\boldsymbol{x}), & \text{if } \hat{t} < t, \end{cases}$$

In this case the FKF takes the form

$$u(\boldsymbol{x}, t) = E\left\{ k\left(\boldsymbol{X}_{\hat{t}}^{x}\right) \exp\left(- \int_0^{\hat{t}} c\left(\boldsymbol{X}_s^x, t - s\right) ds \right) \right\}. \tag{4}$$

In the case of Neumann boundary conditions a similar modification can be used, in which the Brownian motion is reflected rather than absorbed (Pardoux and Răşcanu, 2014).

3. Obtaining FKF estimates via path space importance sampling

The law of the complementary SDE is rarely available in closed-form. We must resort to numerical simulation of the SDE in order to obtain a Monte Carlo estimate of the integrals in (3) and (4). Until recently this would require the use of a discrete time approximation, such as the Euler-Maruyama method, leading to bias in the FKF estimates. The development of path space importance sampling algorithms (commonly referred to as exact