Pairing and condensation in a resonant Bose-Fermi mixture

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We study by diagrammatic means a Bose-Fermi mixture, with boson-fermion coupling tuned by a Fano-Feshbach resonance. For increasing coupling, the growing boson-fermion pairing correlations progressively reduce the boson condensation temperature and make it eventually vanish at a critical coupling. Such quantum critical point depends very weakly on the population imbalance and for vanishing boson densities coincides with that found for the polaron-molecule transition in a strongly imbalanced Fermi gas, thus bridging two quite distinct physical systems.

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One of the main reasons behind the recent great interest in ultracold gases, is the possibility to reproduce physical systems relevant to other areas in physics, with a flexibility and a degree of tunability of physical parameters which is unimaginable in the original system of interest. The very same flexibility can also be used, however, to construct novel physical systems. A noticeable example of such novel systems, namely, resonant Bose-Fermi mixtures, will be here at issue. We will be interested in particular in the competition between boson-fermion pairing correlations and bosonic condensation, occuring in a mixture of single-component bosons and fermions when the boson-fermion pairing is made progressively stronger by means of a Fano-Feshbach resonance.

Initial theoretical studies of ultracold Bose-Fermi mixtures considered mainly non-resonant systems [1–5], where boson-fermion pairing is irrelevant, and studied, within mean-field like treatments, the tendency towards collapse or phase separation (as motivated by the first experimental results on non-resonant Bose-Fermi mixtures [6]). Bose-Fermi mixtures in the presence of a Fano-Feshbach resonance have been subsequently considered in [7–17], mostly for *narrow* resonances [7, 10–13, 16] and/or in specific contexts such as optical lattices, reduced dimensionality, zero temperature or vanishing densitiy of one component [8, 9, 14, 15, 17].

Most of current experiments on Bose-Fermi mixtures [19–23] appear however to be closer to the case of a *broad* resonance, which is characterized by the smallness of the effective range parameter r_0 of the interaction potential with respect to both the average interparticle distance and the scattering length [18]. Under these conditions, the resonant Bose-Fermi mixture is accurately described by a *minimal* Hamiltonian, made just by bosons and fermions mutually interacting via an attractive point-contact potential. This Hamiltonian is simpler and more "fundamental" in character than its counterpart for a narrow resonance because of the absence of any other parameter besides the strength of the boson-fermion interaction. In addition, its simplicity allows the implementation of more sophisticated many-body calculations (beyond mean-field).

We thus consider the following (grand-canonical) Hamiltonian:

$$H = \sum_s \int d\mathbf{r}\psi_s^\dagger(\mathbf{r})(-\frac{\nabla^2}{2m_s} - \mu_s)\psi_s(\mathbf{r}) + v_0 \int d\mathbf{r}\psi_B^\dagger(\mathbf{r})\psi_F^\dagger(\mathbf{r})\psi_F(\mathbf{r})\psi_B(\mathbf{r}) \qquad (1)$$

Here $\psi_s^\dagger(\mathbf{r})$, creates a particle of mass m_s and chemical potential μ_s at spatial position $\mathbf{r}$, where $s = B, F$ indicates the boson and fermion atomic species, respectively, while v_0 is the bare strength of the contact interaction (we set $\hbar = k_B = 1$ throughout this paper). Ultraviolet divergences associated with the use of a contact interaction in (1) are eliminated, as for two-component Fermi gases [24], by expressing the bare interaction v_0 in terms of the boson-fermion scattering length a via the (divergent) expression $1/v_0 = m_r/(2\pi a) - \int 2m_r/\mathbf{k}^2 d\mathbf{k}/(2\pi)^3$, where $m_r = m_B m_F/(m_B + m_F)$ is the reduced mass.

The Hamiltonian (1) does not contain explicitly the boson-boson interaction. Provided it is repulsive and non-resonant, this interaction yields in fact a mean-field shift, which consists in a simple redefinition of the boson chemical potential (attractive boson-boson interactions would lead to mechanical instability and are excluded from our consideration). S-wave interaction between fermions is finally excluded by Pauli principle.

The effective coupling strength in the many-body system is determined by comparing the boson-fermion scattering length a with the average interparticle distance $n^{-1/3}$ (with $n = n_B + n_F$, where n_B and n_F are the boson and fermion particle number density, respectively). In particular, we will use the same dimensionless coupling strength $(k_F a)^{-1}$ normally used for two-component Fermi gases, where the wave-vector $k_F \equiv (3\pi^2 n)^{1/3}$ (note that k_F coincides with the noninteracting Fermi wave-vector $k_F^0 = (6\pi^2 n_F)^{1/3}$ only for $n_B = n_F$).

The expected behavior of the system is clear in the two opposite limits of the boson-fermion coupling. In the weak-coupling limit, where the scattering length a is small and negative (such that $(k_F a)^{-1} \ll -1$), the