problem, so that standard criticisms of Wald-type methods in the presence of identification problems (see Dufour (1997,2003)) do not apply in this case.

If the covariance matrix estimator can be modified so that it remains consistent and its rank converges to the appropriate asymptotic rank, then the asymptotic distribution of the modified Wald-type statistic (based on a generalized inverse of the covariance matrix) remains chi-square although with a reduced degrees-of-freedom number; see Andrews (1987). For example, Lutkepohl-Burda (1997) proposed such methods based on reducing the rank of the estimated covariance matrix by either using a form of randomization or setting "small eigenvalues" to zero. Such methods, however, effectively modify the test statistic and involve arbitrary truncation parameter for which no practical guidelines are available: in finite samples, the test statistic can become as small as one wishes leading to largely arbitrary results and unlimited power reductions.

Interestingly, except for a bound given by Sargan(1980) in a special case, the asymptotic distribution of Wald-type statistics in non-regular cases has not been studied. In this paper, we undertake this task and propose solutions to the problem that do not require modifying the test statistic. More specifically, the contributions of the paper can be summarized as follows.

First, we provide examples showing that Wald statistics in such non-regular cases can have several asymptotic distributions. We also show that usual critical values based on a chi-square distribution (with degrees-of-freedom equal to the number of constraints) can both lead to under-rejections