II. THE MODEL OF THE EDGE AND SPIN-ORBIT COUPLING

Let us consider a sample with 2DEG created within a GaAs-based quantum well. The electrical current in the plane of the 2DEG will be driven along the y direction. We will be interested in the physics close to one of the two edges of the sample along which the current flows. The coordinate in the direction perpendicular to the edge will be x, $x > 0$ corresponding to the region where the 2DEG is present (see Fig. 1).

We will describe the electronic states for electrons in the 2DEG within the effective mass approximation, treating the electrons as noninteracting quasiparticles. Qualitative changes in our results introduced within a self-consistent mean-field treatment will be studied afterwards. The length scale characterizing the electrons is given by their Fermi wavelength. It typically attains values[33] $\lambda_F = \sqrt{2\pi/n_{2D}} \sim 2.5a_B^*$, where n_{2D} is the two-dimensional electronic density and $a_B^* = 9.79$nm is the effective Bohr radius in GaAs. Since this is about two orders of magnitude larger than the interatomic distances, we can employ the effective mass theory and approximate the form of the confining potential with a simple functional form.

In our work will assume that the confining edge potential, $V(x)$, is independent of the coordinate y, directed along the edge. Within our analytical derivations we model $V(x)$ as an abrupt step of height ΔV, $V(x) = V_\theta(x) = \Delta V\theta(-x)$, where $\theta(x)$ is the unit step function. The actual value of the step is much larger than the Fermi energy of the 2DEG. This is frequently used to set the potential step to infinity. However, here it is essential that the step is finite as the whole spin-orbit coupling is nonzero only in the region of the nonzero gradient of the confining potential. Since this value should be of the order of the work function of the electrons in the 2DEG, we fix this value to $\Delta V = 2.7\text{eV} \sim 230\text{Ha}^*$. One of the results of our work is the demonstration that the current induced spin polarization is rather independent of this value so that we do not need to be concerned with its precise numerical value, as long as it is large ($>> E_F$) but finite value.

Once we establish the analytical result for this model of abrupt potential step, we will also consider more general forms: step with a linear slope on a distance d, $V_d(x)$ (used in Fig. 1), and a partially self-consistent, density-dependent model with a small triangular-shaped dip, $V_n(x)$, close to the edge. In the absence of this dip, the decrease in the density to zero at the edge in the 2DEG results in a slight local depletion of electrons there. A small negative value of the depth of the dip, $-V_0$, that is found self-consistently results in a charge neutral edge which is the physically expected situation.

The essential part of our model is the spin-orbit (SO) coupling term in the electrons' Hamiltonian. In general, several different types of SO interactions can be similar in magnitude: the Rashba-Bytchkov, the Dresselhaus or the impurity-induced SO coupling. However, since they are all small perturbations to the effective Hamiltonian, we can consider them separably and superpose their outcomes in the sense of first order perturbation theory in terms of SO parameter appearing in the SO interaction. In our paper we will consider a special case of the impurity-induced SO coupling for the special case when the "impurity" is the edge potential only. In general, the SO contribution to the Hamiltonian takes the form[27]

$$\hat{V}^{SO} = \alpha_E \vec{\sigma} \cdot \left(\vec{k} \times \nabla V(x)\right), \tag{2}$$

where α_E characterizes the strength of the SO coupling, $\vec{\sigma}$ are Pauli matrices, $\vec{k}$ is the operator of electron's momentum and $V(x)$ is the effective one-electron potential energy. Within our study the latter corresponds to various models of the edge potential: $V_\theta(x)$, $V_d(x)$ or $V_n(x)$. The strength of the SO coupling in GaAs is known to be approximately $\alpha_E \approx 5.3\text{Å}^2 = 5.53 \times 10^{-4}\text{a}_B^*$ which is 10^6 times larger than the strength of the SO coupling in vacuum[26].

Within the framework of this model it is very easy to understand the origin of the current-induced spin polarization. The gradient of the potential energy is directed only in the x direction, the momentum is confined only within the xy plane so that the only nonzero component of the spin operator comes from its z component due to the mixed product form in Eq. 2. Specifically, the SO interaction is simply a spin-dependent potential

$$\hat{V}^{SO} = -\alpha_E \sigma_z q V'(x) \tag{3}$$

with a well defined quantum number for the projection of the spin in the z direction, σ_z, attaining values ± 1, and q is the y component of the electron's momentum (Fig. 1). Hence, the spin-orbit term is a potential energy which, for states with positive momentum in the $+y$ direction (the current), is smaller (larger) in the spatial region of negative derivative of the edge potential, $V'(x) < 0$, for electrons with their spin down (up). Since close to $x = 0$ we have $V'(x) = -\Delta V\delta(x) < 0$, we expect that close to the edge we will find majority of spin-down electrons. Of course, in equilibrium for zero total current density there will be equal number of electron with $q > 0$ and $q < 0$ so that in this special case zero total spin polarization will be observed.

III. SPIN POLARIZATION AND THE SPIN-DEPENDENT PHASE-SHIFT

In our previous work[22], we have given estimates for the total induced spin per unit length of the edge using arguments based on wavepacket propagation. We considered a single-electron wavepacket state, $\psi^{q,\sigma}_{k,\Delta k}(t)$, for a specific momentum in the y direction q, spin σ, with momentum k of uncertainty Δk,

$$\begin{aligned}\psi^{q,\sigma}_{k,\Delta k}(t) \;=\; & \int_{k-\Delta k/2}^{k+\Delta k/2} \frac{dk}{\sqrt{2\pi\Delta k}} \left(e^{-ikx} + r^{q,\sigma}(k)e^{ikx}e^{iqy}\right) \\ & \times e^{-i(k^2/2+q^2/2)t},\end{aligned} \tag{4}$$

where $r^{q,\sigma}(k)$ is the reflection amplitude of the eigenstate with energy $E = (1/2)(k^2 + q^2)$. For large negative times only the incoming part (moving from the right in the Fig. 1) of this