In this paper we extend the work in reference [5] by exploring the growth rate of the height difference between the two free surfaces, for a wider range of frequencies. We obtain a differential equation for the change of the measured growth rates with the dimensionless acceleration of the vibrations.

2 Experimental setup

For the vibrational system we used a function generator (GW Instek SFG-2110) linked to an audio amplifier (Crown XLS 202) feeding two coupled 1000 Watts loudspeakers. For the granular material we used glass spheres with diameter between 250 and 300 μm and bulk static density $\rho_g = 1440\, kg/m^3$. The top of the container was made permeable to air but not to grains (using a 125 μm nylon mesh). The container is a U-shaped tube 200 mm tall, with square internal cross section of 400 mm^2. The total mass of grains used was 124 g, chosen to completely fill one of the U-tube branches.

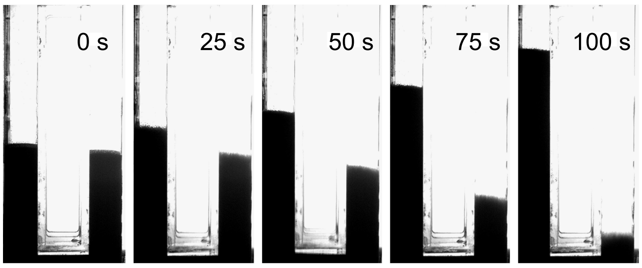

Fig. 1 Snapshot sequence of the partially filled U-tube in a typical experiment. The grains migrate to the branch of the tube with the initially taller granular column. The images shown correspond to an experiment for which the U-tube was shaken at a frequency of 20 Hz and $\Gamma = 4.5$. The increase of the growth velocity with time can be readily appreciated.

Images of the experiments were recorded using a digital camera (Pixelink PL-B741F). Filming against a back light (see Fig. 1), allowed us to analyze the images using a commercial software to obtain the difference in height Δh between the two free surfaces as a function of time.

The excitation signal used was a sine wave. The maximum amplitude of oscillation that could be reached was $(11.1 \pm 0.1)\, mm$. The excitation was controlled considering the maximum dimensionless acceleration defined as $\Gamma = A(2\pi f)^2/g$, where g denotes the acceleration of gravity, A the amplitude of the oscillations and f the frequency of the oscillations. The value of Γ was varied from 1 to 5.75. Lower values did not generate any instability and higher values of this quantity were not possible to achieve due to the limitations of the equipment.

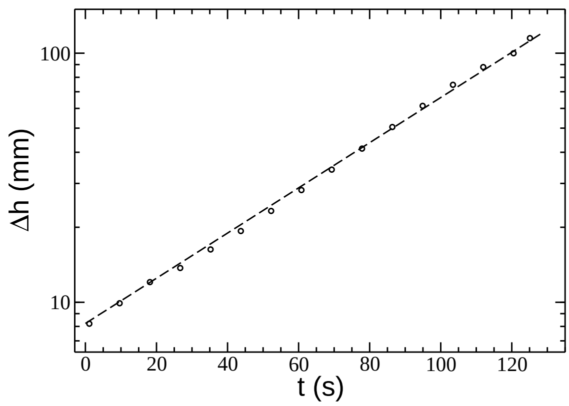

Fig. 2 Typical time dependence of the height difference Δh, between the free surfaces at the two branches of the tube. This plot is for $f = 15\, Hz$ and $\Gamma = 2.75$.

3 Results

The height difference Δh between the granular level in both branches of the U-tube grows exponentially with time (see Fig. 2). The growth rate γ is determined from the fit of the exponential $\Delta h = \Delta h_0 e^{(\gamma t)}$, where Δh_0 is the initial height difference and t is the time.

The graph of the growth rate γ versus dimensionless Γ is shown in Fig.3. The horizontal axis begins at $\Gamma = 1$, because for $\Gamma < 1$, $\gamma = 0$. For each curve the oscillation frequency is fixed and the amplitude is varied. For all frequencies investigated there was a seemingly continuous transition from zero growth at the lowest amplitudes to a finite γ at higher amplitudes. It is apparent that for the larger frequencies (15 Hz and 20 Hz) the growth rate reaches a saturation value, for sufficiently large amplitudes. For three of the intermediate frequencies (10 Hz, 12.5 Hz and 15 Hz) we see that γ reaches a point where it changes abruptly to zero at a threshold of Γ (see arrows in Fig. 3). This suggests the occurrence of a first order phase transition, but more work needs to be done to characterize this observed abrupt transition. For the lowest frequency measured (7.5 Hz) we could not determine whether there is a saturation region or a transition from high nonzero growth rate to zero growth at high amplitudes because of the limitations of our equipment.

Our experimental data is well described by a Boltzmann sigmoidal curve for γ versus Γ. This fitting relation is given by eq. (1), and it has four fitting parameters: (a) The higher asymptote γ_{max} (characteristic value for the higher growth rates) (b) The size of the