$\text{Li}_\nu(z)$, where $z = e^{\beta\mu}$ is a fugacity, using $\int d^d\mathbf{k} = 2\pi^{d/2}/\Gamma(d/2)\int dk k^{d-1}$ and the integrals

$$\begin{aligned} \int_0^\infty dx\, \frac{z x^{\nu-1}}{e^x - z} &= \Gamma(\nu)\text{Li}_\nu(z) \\ \int_0^\infty dx\, \frac{z x^{\nu-1}}{e^x + z} &= -\Gamma(\nu)\text{Li}_\nu(-z) \end{aligned} \tag{21}$$

valid for $\text{Re}(\nu) > 0$. The result, as expected, is proportional to T/λ_T^d:

$$\mathcal{F} = -sT\left(\frac{mT}{2\pi}\right)^{d/2}\text{Li}_{(d+2)/2}(sz) \tag{22}$$

There are two important limits to consider. For Bose gases near Bose-Einstein condensation, physically the interesting limit is $\mu/T \to 0$. Since $\text{Li}_\nu(1) = \zeta(\nu)$, where ζ is Riemann's zeta function, this leads us to define the scaling functions $c_d(\mu/T)$:

$$\begin{aligned} \mathcal{F} &= -\frac{\pi m T^2}{12}\, c_2(\mu/T) \qquad (d=2) \\ \mathcal{F} &= -\frac{\zeta(5/2) m^{3/2} T^{5/2}}{(2\pi)^{3/2}}\, c_3(\mu/T) \qquad (d=3) \end{aligned} \tag{23}$$

where we have used $\zeta(2) = \pi^2/6$. With the above normalizations, $c_d = 1$ for a single free boson when $\mu/T = 0$.

The above formulas are well-defined for fermions at zero chemical potential. Using

$$-\text{Li}_\nu(-1) = \left(1 - \frac{1}{2^{\nu-1}}\right)\zeta(\nu) \tag{24}$$

one finds as $\mu/T \to 0$:

$$c_2 = \frac{1}{2}, \qquad c_3 = 1 - \frac{1}{2\sqrt{2}} \qquad \text{(free fermions)} \tag{25}$$

It should be pointed out that the coefficients c_d are analogous to the Virasoro central charge for relativistic systems in $d = 1$, as discussed in [27].

The other interesting limit is $T/\mu \to 0$, i.e. $z \to \infty$ or $z \to 0$, depending on the sign of the chemical potential. Here the scaling forms are naturally based on the