

We will at times view signed permutations of $\{1, \dots, n\}$ as being embedded in a larger symmetric group (usually S_{2n} or S_{2n+1}). To avoid any confusion in terminology, an element $\sigma \in S_m$ will be called a “signed element of S_m ” if and only if it has the property that

$$\sigma(m+1-i) = m+1-\sigma(i)$$

for $i = 1, \dots, m$. Signed permutations of $\{1, \dots, n\}$ can be embedded as signed elements of S_{2n} as follows: Given a signed permutation π , define the first n values of the signed element $\sigma \in S_{2n}$ by

$$\sigma(i) = \begin{cases} \pi(i) & \text{if } \pi(i) > 0 \\ 2n+1-|\pi(i)| & \text{if } \pi(i) < 0, \end{cases}$$

and then define the remaining values of σ to be what they are required to be: $\sigma(2n+1-i) = 2n+1-\sigma(i)$.

Embedding signed permutations in S_{2n+1} works very similarly. Define the first n values of the signed element $\sigma \in S_{2n+1}$ by

$$\sigma(i) = \begin{cases} \pi(i) & \text{if } \pi(i) > 0 \\ 2n+2-|\pi(i)| & \text{if } \pi(i) < 0, \end{cases}$$

then insist that $\sigma(2n+2-i) = 2n+2-\sigma(i)$ for $i = 1, \dots, n$. Note that this forces $\sigma(n+1) = n+1$.

We will also deal often with flags, i.e. chains of subspaces of a given vector space V . A flag

$$\{0\} \subset F_1 \subset F_2 \subset \dots \subset F_{n-1} \subset F_n = V$$

will often be denoted by F_\bullet . When we wish to specify the components F_i of a given flag F_\bullet .