

Let us follow the discussion of [582], where they begin by considering the simplest model which involves a pNGB θ which comes from the breaking of a global $SO(2)$ symmetry. After integrating out the “radial” degree of freedom and pushing the cutoff of this non-linear sigma model to the point where the interactions become strongly coupled, namely $\Lambda \sim 4\pi f$. The inflaton, Φ , can be parameterized as

$$\Phi = \begin{pmatrix} \cos(\theta/f) & \sin(\theta/f) \\ -\sin(\theta/f) & \cos(\theta/f) \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \times \frac{f}{\sqrt{2}} \quad (206)$$

Let us consider the tree-level potential to be:

$$V = \lambda (\sigma^T \sigma - v^2)^2 + \frac{g_1}{4} (\sigma^T \Phi)^2 + \frac{g_2}{4} (\sigma^T \tau_1 \Phi)^2 \quad (207)$$

where $\sigma^T = (\sigma_1 \sigma_2)$ and τ_1 is the first Pauli matrix. Let us consider a simple situation when $g_1, g_2 = g \neq 0$. From expanding out the Φ s in the potential, one finds:

$$V = \frac{gf^2}{4} (\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2 \cos(2\theta/f)) \quad (208)$$

Now computing the one-loop corrections to the mass of θ , the authors of Ref. [582] obtained that there is no one-loop quadratic divergent contribution to a θ mass. This is because θ only couples to the combination $\sigma_1\sigma_2$ making it impossible to close a loop with only one vertex. There is a logarithmic divergence at one loop proportional to $g_1g_2 = g^2$

$$\begin{aligned} V_{1-loop} &= \frac{g^2}{128\pi^2} \log\left(\frac{\Lambda^2}{m_\theta^2}\right) (\Phi^T \tau_1 \Phi)^2 + \dots \\ &= \frac{g^2 f^4}{128\pi^2} \log\left(\frac{\Lambda^2}{m_\theta^2}\right) \cos^2(2\theta/f) + \dots \end{aligned} \quad (209)$$

The value of Λ could be as large as M_P or below, but the corrections to the potential is only logarithmic dependent. The pNGB inflaton could also originate in SUSY inflation models and in extra dimensional models [582, 584, 585].

Some of the above mentioned challenges can be addressed if inflation is explicitly embedded within an observable sector. One such example of inflaton is the SM Higgs in a non-SUSY context.