Using (5)

$$A = \left(I_{log}(\lambda^2) + 2b\right) \int_k^\Lambda \frac{1}{k^4(p-k)^2}$$
$$-b \ln\left(\frac{-p^2}{\lambda^2}\right) \int_k^\Lambda \frac{1}{k^4(p-k)^2}$$
$$-\left(I_{log}(\lambda^2) + 2b\right) \int_k^\Lambda \frac{1}{k^2(p-k)^4}$$
$$+b \int_k^\Lambda \frac{1}{k^2(p-k)^4} \ln\left(\frac{-k^2}{\lambda^2}\right) \tag{45}$$

The last integral is evaluated using the procedure described in the main text and yields

$$\frac{1}{p^2}\left[\tilde{I}_{log}(\tilde{\lambda}^{-2}) \ln\left(\frac{-p^2}{\lambda^2}\right) + 2b \ln\left(\frac{-p^2}{\lambda^2}\right)\right.$$
$$\left.+b \ln\left(\frac{-p^2}{\lambda^2}\right) \ln\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right)\right] \tag{46}$$

Upon insertion of (9) all terms cancel and $A = 0$.

VII. APPENDIX B - CANCELATION OF DIVERGENT PIECES

Here we collect the divergent pieces of $J_1(p)$, $J_2(p)$, $J_3(p)$ and $J_4(p)$. Putting together the results (5), (9) and (20) we have:

$$J_1(p) = \frac{1}{p^2}\left[I_{log}(\lambda^2) - b \ln\left(\frac{-p^2}{\lambda^2}\right) + 2b\right] \times$$
$$\left[-\tilde{I}^{(2)}_{log}(\tilde{\lambda}^{-2}) + b \ln\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right) + \frac{b}{2}\ln^2\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right) + \right.$$
$$\left.\ln\left(\frac{\bar{\tilde{\lambda}}^2}{\alpha^2}\right)\left(\tilde{I}_{log}(\tilde{\lambda}^{-2}) + b \ln\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right) + 2b\right)\right] \tag{47}$$

So

$$J_1^{div}(p) = \frac{1}{p^2}\left\{I_{log}(\lambda^2)\left[-\tilde{I}^{(2)}_{log}(\tilde{\lambda}^{-2}) + b \ln\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right)\right.\right.$$
$$+\frac{b}{2}\ln^2\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right) + \ln\left(\frac{\bar{\tilde{\lambda}}^2}{\alpha^2}\right)\left(\tilde{I}_{log}(\tilde{\lambda}^{-2})\right.$$
$$\left.\left.+b \ln\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right) + 2b\right)\right] + \left(\tilde{I}_{log}(\tilde{\lambda}^{-2}) \ln\left(\frac{\bar{\tilde{\lambda}}^2}{\alpha^2}\right)\right.$$
$$\left.\left.-\tilde{I}^{(2)}_{log}(\tilde{\lambda}^{-2})\right)\left(2b - b \ln\left(\frac{-p^2}{\lambda^2}\right)\right)\right\} \tag{48}$$

The other divergent pieces are

$$J_2^{div}(p) = \frac{1}{p^2}\left\{I_{log}(\lambda^2)\left[\tilde{I}_{log}(\tilde{\lambda}^{-2}) \ln\left(\frac{-p^2}{\alpha^2}\right)\right.\right.$$
$$\left.+2b \ln\left(\frac{-p^2}{\alpha^2}\right) + b \ln\left(\frac{-p^2}{\alpha^2}\right) \ln\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right)\right]$$
$$\left.+\tilde{I}_{log}(\tilde{\lambda}^{-2}) \ln\left(\frac{-p^2}{\alpha^2}\right)\left(2b - b \ln\left(\frac{-p^2}{\lambda^2}\right)\right)\right\} \tag{49}$$

$$J_3^{div}(p) = \frac{1}{p^2}\left\{I_{log}(\lambda^2)\left[\tilde{I}_{log}(\tilde{\lambda}^{-2}) \ln\left(\frac{-p^2}{\alpha^2}\right)\right.\right.$$
$$\left.+2b \ln\left(\frac{-p^2}{\alpha^2}\right) + b \ln\left(\frac{-p^2}{\alpha^2}\right) \ln\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right)\right]$$
$$+\tilde{I}_{log}(\tilde{\lambda}^{-2}) \ln\left(\frac{-p^2}{\alpha^2}\right)\left(2b - b \ln\left(\frac{\alpha^2}{\lambda^2}\right)\right)$$
$$\left.-\tilde{I}_{log}(\tilde{\lambda}^{-2}) \ln^2\left(\frac{-p^2}{\alpha^2}\right)\right\} \tag{50}$$

$$J_4^{div}(p) = \frac{1}{p^2}\left\{I_{log}(\lambda^2)\left(-\tilde{I}^{(2)}_{log}(\tilde{\lambda}^{-2})\right.\right.$$
$$+\frac{b}{2}\ln^2\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right) + b \ln\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right) + \ln\left(\frac{\bar{\tilde{\lambda}}^2}{\alpha^2}\right)\tilde{I}_{log}(\tilde{\lambda}^{-2})$$
$$\left.+b \ln\left(\frac{\bar{\tilde{\lambda}}^2}{\alpha^2}\right) \ln\left(\frac{-p^2}{\bar{\tilde{\lambda}}^2}\right) + 2b \ln\left(\frac{\bar{\tilde{\lambda}}^2}{\alpha^2}\right)\right)$$
$$\left.+\tilde{I}^{(2)}_{log}(\tilde{\lambda}^{-2})\left(-1 + \ln\left(\frac{\bar{\tilde{\lambda}}^2}{\alpha^2}\right)\right)\left(2b - b \ln\left(\frac{-p^2}{\lambda^2}\right)\right)\right\} \tag{51}$$

Using these results in $J^{div}(p) = J_1^{div}(p) + J_2^{div}(p) - J_3^{div}(p) - J_4^{div}(p)$ it is easy to see the cancellation of the divergent pieces.

VIII. APPENDIX C - FOURIER TRANSFORMS

The Fourier transform of any power of logarithmic functions [15] can be obtained through the Fourier transform of power functions. Our results are given in Minkowski space, for which we use [32]:

$$\int d^4x\, e^{ipx} \frac{1}{x^2}(-B^2x^2)^a = -i\frac{4\pi^2}{p^2}\left(\frac{-4B^2}{p^2}\right)^a \frac{\Gamma(1+a)}{\Gamma(1-a)}, \tag{52}$$