function vanishes. When isospin is a good symmetry, one has

$$H_{\pi^+}^{u+d} := H_{\pi^+}^u + H_{\pi^+}^d = H_{\pi^-}^{u+d} = H_{\pi^0}^{u+d}, \tag{VI.57}$$

$$H_{\pi^+}^{u-d} := H_{\pi^+}^u - H_{\pi^+}^d = -H_{\pi^-}^{u-d}, \tag{VI.58}$$

$$H_{\pi^0}^{u-d} \equiv 0\,. \tag{VI.59}$$

The analogues of Eqs. (VI.49) and (VI.50) are correct, and, moreover,

$$\int_{-1}^{1} dx\, H_\pi^q(x,\xi,t;Q_0) = F_\pi^q(t)\,,\; F_\pi^u(t) - F_\pi^d(t) = 2F_\pi(t)\,. \tag{VI.60}$$

2. Lattice QCD

An operator expression for the pion's quark distribution function is given in Eq. (VI.31). We have seen in Sec. VI.B.1 that it is generally true; namely, in a spinless hadron "h" with total momentum k, the distribution function is given by

$$q^h(x;Q_0) = \frac{1}{4\pi}\int_{-\infty}^{\infty} d\xi^- \mathrm{e}^{\frac{i}{2}xk^+\xi^-} \langle h(k)|\,\bar\psi_q(0)\gamma^+\psi_q(\xi^-)|h(k)\rangle. \tag{VI.61}$$

This expression is usually understood as representing the distribution in light-cone gauge; i.e., $A_a^+ = 0$, where A_a^μ is the gluon field. Equation (VI.61) can readily be generalized to bound-states with spin.

It is straightforward to modify Eq. (VI.61) so that the expectation value is gauge invariant and yet unchanged in light-cone gauge; viz., one introduces a path-ordered exponential (Wilson line)

$$\mathcal{E}(\xi^-) := \mathbf{P}\exp\left\{\tfrac{i}{2}g\int_{\xi^-}^{0} dz^-\, \frac{\lambda^a}{2} A_a^+(0,z^-,\vec{0}_\perp)\right\} \tag{VI.62}$$

as follows

$$q^h(x;\mathcal{E};Q_0) := \frac{1}{4\pi}\int_{-\infty}^{\infty} d\xi^- \mathrm{e}^{\frac{i}{2}xk^+\xi^-} \langle h(k)|\,\bar q(0)\gamma^+\mathcal{E}(\xi^-)q(\xi^-)|h(k)\rangle. \tag{VI.63}$$

With Eq. (VI.63) one has a gauge-invariant expectation value of a bilocal operator evaluated along a light-like line. It is mathematically precise and provides the pointwise behavior of the distribution function. However, it cannot be evaluated using the numerical approach of lattice-regularized QCD.

Lattice methods can, however, be used to evaluate expectation values of local operators. Consider therefore

$$\langle x^n\rangle_{Q_0}^h := \int_0^1 dx\, x^n\,[q^h(x;\mathcal{E};Q_0) - (-1)^n \bar q^h(x;\mathcal{E};Q_0)]\,. \tag{VI.64}$$