Combining two factors of the form of the left–hand side of Eq. (73) with one factor of the form of the left–hand side of Eq. (74) we obtain a normalized generating functional involving both commuting and anticommuting integration variables. The term “supersymmetry” commonly used in that context is somewhat inappropriate. It stems from relativistic quantum field theory (Wess and Zumino, 1974a,b) where fermionic (i.e., anticommuting) and bosonic (i.e., commuting) fields are connected by a “supersymmetry”. That specific symmetry does not occur in the present context.

In a formal sense, the calculation of $\langle S^{\rm CN}\rangle$ and of the correlation function runs in parallel to the one for the replica trick. In content, the steps differ because of the simultaneous use of commuting and anticommuting variables. The steps are: Averaging the generating functional over the GOE, replacing the quartic terms in the integration variables generated that way with the help of a Hubbard–Stratonovich transformation, execution of the remaining Gaussian integrals over the original integration variables, use of the saddle–point approximation for the remaining integrals over the supermatrix σ. That matrix is introduced via the Hubbard–Stratonovich transformation and has both commuting and anticommuting elements. For $N \gg 1$ the saddle–point approximation is excellent and yields for $\langle S^{\rm CN}\rangle$ an asymptotic expansion in inverse powers of N, the leading term being given by the right–hand side of Eq. (56) when E is taken in the center of the GOE spectrum. When the same formalism is used for the S–matrix correlation function instead of S itself, the saddle–point changes into a saddle–point manifold. With the help of a suitable parametrization (Schäfer and Wegner, 1980) of σ, the integration over that manifold can be done exactly. The result is again valid to leading order in $(1/N)$. With $\varepsilon = E_2 - E_1$, one obtains (Verbaarschot *et al.*, 1985)

$$\langle S^{(\rm CN\ fl)}_{ab}(E_1)(S^{(\rm CN\ fl)}_{cd}(E_2))^*\rangle = \prod_{i=1}^{2}\int_0^{+\infty}{\rm d}\lambda_i\int_0^1{\rm d}\lambda \\ \times\frac{1}{8}\,\mu(\lambda_1,\lambda_2,\lambda)\ \exp\left\{-\frac{i\pi\varepsilon}{d}(\lambda_1+\lambda_2+2\lambda)\right\} \\ \times\prod_e\frac{(1-T_e\lambda)}{(1+T_e\lambda_1)^{1/2}(1+T_e\lambda_2)^{1/2}} \\ \times J_{abcd}(\lambda_1,\lambda_2,\lambda)\ . \quad (75)$$

The factor $\mu(\lambda_1,\lambda_2,\lambda)$ is an integration measure and given by

$$\mu(\lambda_1,\lambda_2,\lambda) = \frac{(1-\lambda)\lambda|\lambda_1-\lambda_2|}{\prod_{i=1}^2[((1+\lambda_i)\lambda_i)^{1/2}(\lambda+\lambda_i)^2]}\ , \quad (76)$$

while

$$J_{abcd}(\lambda_1,\lambda_2,\lambda) = (\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})T_aT_b \\ \times\left(\sum_{i=1}^2\frac{\lambda_i(1+\lambda_i)}{(1+T_a\lambda_i)(1+T_b\lambda_i)}+\frac{2\lambda(1-\lambda)}{(1-T_a\lambda)(1-T_b\lambda)}\right) \\ +\ \delta_{ab}\delta_{cd}T_aT_c\langle S^{\rm CN}_{aa}\rangle\langle(S^{\rm CN}_{cc})^*\rangle \\ \times\left(\sum_{i=1}^2\frac{\lambda_i}{1+T_a\lambda_i}+\frac{2\lambda}{1-T_a\lambda}\right) \\ \times\left(\sum_{i=1}^2\frac{\lambda_i}{1+T_c\lambda_i}+\frac{2\lambda}{1-T_c\lambda}\right) \quad (77)$$

describes the dependence of the correlation function on those channels which appear explicitly on the left–hand side of Eq. (75).

Eqs. (75) to (77) give the S–matrix correlation function in closed form, i.e., in terms of an integral representation. It does not seem possible to perform the remaining integrations analytically for an arbitrary number of channels and for arbitrary values of the transmission coefficients. In the way it is written, Eq. (75) is not suited very well for a numerical evaluation because there seem to be singularities as the integration variables tend to zero. Moreover, the exponential function oscillates strongly for large values of the λs. These difficulties are overcome by choosing another set of integration variables. Details are given in (Verbaarschot, 1986). For the case of unitary symmetry, formulas corresponding to Eqs. (75) to (77) were given in (Fyodorov *et al.*, 2005).

We turn to the physical content of Eqs. (75) to (77). The unitarity condition (1) must also hold after averaging for $S^{\rm CN}$. Using a Ward identity one finds that unitarity is indeed obeyed (Verbaarschot *et al.*, 1985). Except for the overall phase factors of the average S–matrices appearing on the right–hand side of Eq. (77), the S–matrix correlation function depends only on the transmission coefficients, as expected. Eqs. (75) to (77) apply over the entire GOE spectrum if d is taken to be the average GOE level spacing at $E = (1/2)(E_1 + E_2)$. That stationarity property enhances confidence in the result. When E is chosen in the center of the GOE spectrum, $E = 0$, then $\langle S^{\rm CN}\rangle$ is real, see Eq. (56), and the complex conjugate sign in the last term in Eq. (77) is redundant. Writing the product over channels in Eq. (75) as the exponential of a logarithm, expanding the latter in powers of λ_1, λ_2, and λ, and collecting terms, one finds that ε appears only in the combination $[(2i\pi\varepsilon/d)+\sum_c T_c]$. This fact was also established in the framework of the replica trick. It shows that Γ as defined by the Weisskopf estimate (11) defines the scale for the dependence of the correlation function on ε. Put differently, the energy difference ε appears universally in the dimensionless form ε/Γ (and not ε/d), and for $\{a, b\} = \{c, d\}$ the correlation function (75) has the form $f(1 + i\varepsilon/\Gamma; T_a, T_b; T_1, T_2, \ldots, T_\Lambda)$. The right–hand side of Eq. (77) is the sum of two terms. These correspond, respectively, to $\langle S^{(\rm CN\ fl)}_{ab}(E_1)\ (S^{(\rm CN\ fl)}_{ab}(E_2))^*\rangle$ and to $\langle S^{(\rm CN\ fl)}_{aa}(E_1)\ (S^{(\rm CN\ fl)}_{bb}(E_2))^*\rangle$ and are exactly the terms expected, see the introduction to Section V.

The limiting cases ($\Gamma \ll d$ and $\Gamma \gg d$) of Eqs. (75) to (77) were studied and compared with previous results (Verbaarschot, 1986). The case $\Gamma \ll d$ is obtained by expanding the result in Eqs. (75) to (77) in powers