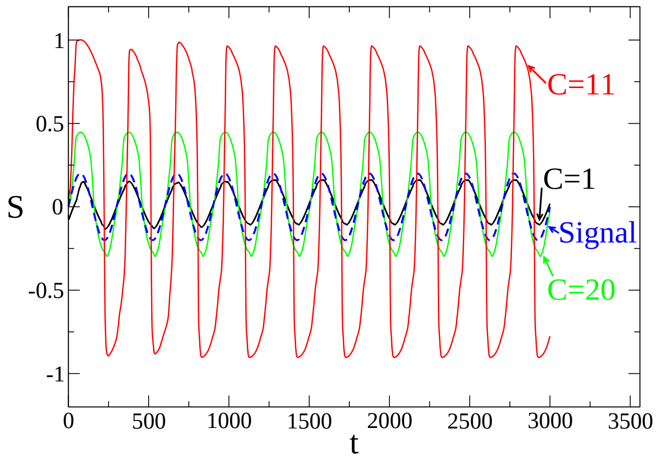

FIG. 7: Coupling-induced resonance, as revealed by the resonant trajectory at optimal $C = 11$. Values of N, A and T as in Fig. 3.

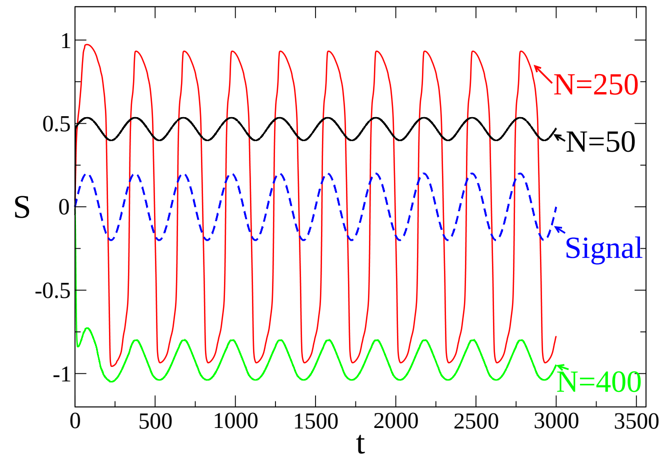

FIG. 9: System size induced resonance, as revealed by the resonant trajectory at optimal $N = 250$. Values of T, A and C as in Fig. 3.

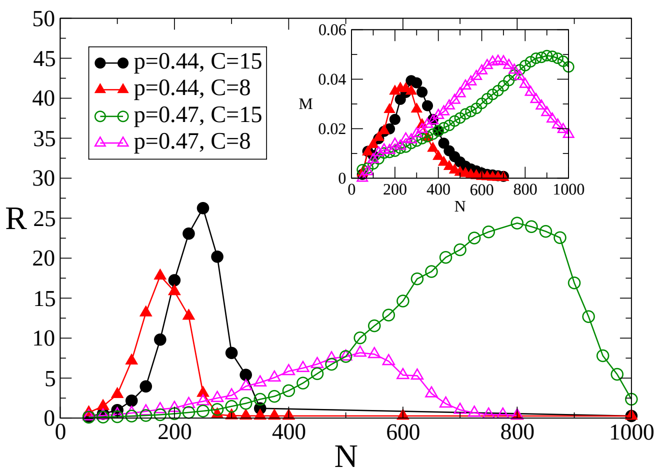

FIG. 8: System-size induced resonance. Main plot: the response R shows a maximum as a function of the number of units, that follows the same pattern as the maximum M ($K = 0.2$) (inset). Since N decreases the influence of a single neighbor, and C increases it, when the coupling intensity is larger, the optimal system size increases. Values of T and A as in Fig. 3.

of a larger system is best amplified at a higher probability of repulsive links. As the fraction of repulsive links must not exceed the fraction of positive ones, there can be a limit on how large can a system be, to be able to amplify a signal. The same behavior, focusing on the number of neighbors was found in a previous study of an Ising-like network model [28].

IV. SPECTRAL ANALYSIS

We have already commented that the optimal probability of repulsive links drives the system to a glassy phase. Anderson [34, 35] has proposed a connection between a glass and a delocalization-localization transition, relating the existence of many metastable states with a localization of modes. From this proposal, we retain the idea to work in the eigenspace of the interaction matrix, and to look for the fraction of repulsive links where mode localization becomes significant. This approach has the virtue of not only identifying the steady states, but also to shed light onto how the reaction to perturbations is sustained and spreads along the system, depending on the fraction of repulsive links. In this manner, we hope to locate the region where multistability is expected, and also to understand the mechanism of response to external perturbations.

Following [27], let us define the eigenvalues Q_α and (normalized) eigenvectors $e^\alpha = (e_1^\alpha, \ldots, e_N^\alpha)$ of the Laplacian matrix [29] J'_{ij} :

$$J'_{ij} = J_{ij} - \delta_{ij} \sum_{k=1}^{N} J_{kj}, \tag{5}$$

$$\sum_{j=1}^{N} J'_{ij} e_j^\alpha = Q_\alpha e_i^\alpha. \tag{6}$$

The effect of the competitive interactions can be described by the so-called participation ratio of eigenvector e^α, defined as $\mathrm{PR}_\alpha = 1/\sum_{i=1}^{N} [e_i^\alpha]^4$. It quantifies the number of components that participate significantly in each eigenvector. A state α with equal components has $\mathrm{PR}_\alpha = N$, and one with only one component has $\mathrm{PR}_\alpha = 1$. When $\mathrm{PR}_\alpha = 1$ on a fraction f of elements, and 0 elsewhere, then $\mathrm{PR}_\alpha = f$, which justifies its name. More precisely, we will define "localized" modes as the ones whose participation ratio is less than $0.1N$. Our first observation (Fig. 10) is that at the optimal region