therefore

$$d_L^{(m)} = \sum_{m=0}^{2^L-1} h_m \tag{21}$$

for the Potts model, and

$$d_L^{S(m)} = \sum_{m=0}^{2^L-1} h_m^S \tag{22}$$

for the site percolation.

The partitions can again be coded by means of a sequence of integers. A detailed description of the coding algorithm can be found in [18]. The magnetic sectors of the transfer matrix now have the dimensions $d_L^{(m)} \times d_L^{(m)}$ and $d_L^{S(m)} \times d_L^{S(m)}$ for the Potts model and site percolation, respectively, and are much larger than those of the non-magnetic sectors. The magnetic sector of the transfer matrix can also be converted into a product of sparse matrices in the same way as in the case of the non-magnetic sector.

The inverse magnetic correlation length is given by

$$\frac{1}{\xi_h(v,L)} = \zeta \ln\left(\frac{\lambda_0}{\lambda_0'}\right), \tag{23}$$

where λ_0 and λ_0' are the leading eigenvalues of the transfer matrix in the non-magnetic and magnetic sector respectively, ζ is a geometrical factor which is the ratio between the unit of L and the thickness of a layer added by the transfer matrix. The magnetic scaled gap then follows.

According to finite-size scaling theory [25] and Cardy's conformal mapping [26], $X_h(v,L)$ can be expanded as

$$X_h(v,L) = X_h + atL^{y_t} + buL^{y_u} + \ldots, \tag{24}$$

where X_h is the magnetic scaling dimension, t is the deviation from the critical point, and u the irrelevant field. Here, y_t is the thermal renormalization exponent, y_u the leading irrelevant renormalization exponent, and a and b are unknown constants.

We substitute (24) into the finite-size scaling equation connecting lattices of sizes L and $L-1$,

$$X_h(v,L) = X_h(v,L-1), \tag{25}$$

and denote the solution of (25) by $v_c(L)$, which has the expansion

$$v_c(L) = v_c + a'uL^{y_u-y_t} + \cdots, \tag{26}$$