hold for all $f \in I(X,\rho,R)$ and $x,y,z,w \in X$ such that $x\rho y$ and $z\rho w$.

$$f(x,y)\,e_{xy} = e_{xx} f e_{yy} \tag{3}$$

$$e_{xy}e_{zw} = \begin{cases} e_{x,w} & \text{if } y = z \text{ and } x\rho w \text{ in } X \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

In any situation where we refer to a generalized incidence ring we mean an associative ring with unity formed on the R-module of functions $I(X,\rho,R)$ where R is a ring with unity and ρ is a locally finite balanced relation on X. The operation of R on $I(X,\rho,R)$ does not play a significant role in our investigation. We reserve the term *generalized incidence algebra* for $I(X,\rho,R)$ where R is a commutative ring with unity and ρ is a locally finite balanced relation on X. If, additionally, ρ is a partial order then $I(X,\rho,R)$ is the usual incidence algebra over R (see [9]).

3 Good Gradings

DEFINITION 3.1 *Assume G is a semigroup and ρ is a relation on X.*

1. *Set* $\mathrm{Trans}(X) = \{(x,y,z) : x\rho y,\ y\rho z,\ x\rho z, \text{ and } x,y,z \in X\}$. *A* transitive triple in X *is an ordered triple in* $\mathrm{Trans}(X)$.

2. *We say* $\Phi : \rho \to G$ *is a* homomorphism *if* $\Phi(x,y)\,\Phi(y,z) = \Phi(x,z)$ *holds for any* $x,y,z \in X$ *such that* $(x,y,z) \in \mathrm{Trans}(X)$.

3. *If* $I(X,\rho,R)$ *is a G-graded generalized incidence ring then the grading is* good *if* e_{xy} *is homogeneous for all* $x,y \in X$ *such that* $x\rho y$.