

In this section we consider the case in which  $M_c$  is smaller than roughly  $0.25 M_\odot$ , because this applies to KOI-81 and KOI-74 . (We include more massive cores in the calculations described in sections 3 and 4.) The primary is not a fully evolved giant at the time it fills its Roche lobe. Its mass will be close in value to  $M_1(0)$ , and its radius can be expressed as follows.

$$R_d = 0.85 M_*^{0.85} + \frac{3700 M_c^4}{1 + M_c^3 + 1.75 M_c^4} \quad (1)$$

For an isolated star, the value of  $M_*$  is simply the star’s initial mass. In the next paragraph we discuss its likely value in systems that undergo two-phase mass transfer. At the time when the primary first fills its Roche lobe,  $q = M_1/M_2 > 1$ . If  $q$  is larger than a critical value  $\eta$ , a common envelope will form and the core of the primary will spiral closer to the main-sequence companion. The final separation,  $a_f$  can be expressed in terms of its initial separation,  $a_i$ , the value of  $M_c$ , the stellar masses  $M_1$  and  $M_2$  at the time of Roche-lobe filling, and an efficiency parameter  $\alpha$ .<sup>1</sup>

$$a_f = a_i \left( \frac{M_c}{M_1} \right) \left[ 1 + \left( \frac{2}{\alpha f(q)} \right) \left( \frac{M_1 - M_c}{M_2} \right) \right]^{-1} \quad (2)$$

$f(q)$  represents the ratio between the radius of the donor and the orbital separation (Eggleton 1983). While the companion’s mass may not change significantly during the common envelope phase, it may have increased during the interval prior to it. This is because mass transfer may have begun in the form of a gravitationally focused wind as the primary expanded to fill its Roche lobe.

There is no single value of the critical mass ratio  $\eta$ . This is because mass transfer can be stabilized by a combination of factors that each assume values appropriate to a specific binary. These factors include the magnitude of winds ejected from the system, the specific angular momentum carried by winds, the role of radiation, and the donor’s adiabatic index. When a common envelope is avoided, stars with modest cores will donate mass during two phases. During phase 1, mass transfer occurs at a rate determined by the thermal time scale of the donor, as it attempts to adjust to mass loss. Once the masses have equalized, the donor has a chance to reestablish thermal equilibrium. The donor star may continue to lose mass, but as it comes into equilibrium with a smaller mass than its initial mass, and with a core that is still modest (less than roughly  $0.25 M_\odot$ ), it begins to shrink into its Roche lobe.

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<sup>1</sup>See, e.g., Webbink (2008) for details. Here we use  $\alpha$  to parameterize the both the binding energy of the primary’s envelope and the efficiency of ejecting the envelope from the system. High values of  $\alpha$  correspond to more efficient ejection and larger final orbits. An alternative approach which uses angular momentum in place of energy considerations can also be applied (Nelemans & Tout 2005). For the purposes of this paper we require only a formulation that parameterizes the resizing of the semimajor axis.