Here, $\theta(t)$ is the step function and we have set the volume equal to unity. The Fourier transform $\text{Im}\chi_{nn}(\mathbf{q}, \omega)$ of the imaginary part of the density response function is related to the dynamic structure factor $S(\mathbf{q}, \omega)$ by

$$\begin{aligned} \text{Im}\,\chi_{nn}(\mathbf{q}, \omega + i0^+) &= -n\pi[S(\mathbf{q}, \omega) - S(-\mathbf{q}, -\omega)] \\ &= -n\pi\, S(\mathbf{q}, \omega)(1 - e^{-\beta\hbar\omega}), \end{aligned} \tag{2}$$

where n is the density, $\beta = 1/T$ is the inverse temperature, and in the last step we have made use of the "detailed balance" relation, $S(\mathbf{q}, -\omega) = e^{-\beta\omega} S(\mathbf{q}, \omega)$ and also the inversion symmetry of the system which gives $S(-\mathbf{q}, \omega) = S(\mathbf{q}, \omega)$. Here and throughout this paper we set $\hbar = k_B = 1$. Equation (2) can also be re-written as

$$S(\mathbf{q}, \omega) = -\frac{1}{\pi n}[N^0(\omega) + 1]\text{Im}\,\chi_{nn}(\mathbf{q}, \omega + i0^+), \tag{3}$$

where $N^0(\omega) = (e^{\beta\omega} - 1)^{-1}$ is the Bose distribution function. The latter arises in both Bose and Fermi fluids because density fluctuations always obey Bose statistics. We see from (2) that $\text{Im}\chi_{nn}(\mathbf{q}, \omega)$ is the antisymmetric (frequency) part of the dynamic structure factor $S(\mathbf{q}, \omega)$.

The dynamic structure factor $S(\mathbf{q}, \omega)$ is measured by using inelastic scattering (e.g., neutron and Brillouin light scattering) of probe particles, which transfer momentum $\mathbf{q}$ and energy ω to the system. $S(\mathbf{q}, \omega)$ is discussed and calculated in all standard texts on many body physics (convenient reviews are given in [18, 19]). In contrast, the density response function $\chi_{nn}(\mathbf{q}, \omega)$ describes the density fluctuation induced by an external perturbing potential. As an example, in atomic gases, the imaginary part of the density response function is probed in two-photon Bragg scattering and also describes the density pulse produced by a blue-detuned laser. Brillouin and neutron scattering have been used extensively to study the collective modes in ^{4}He. These techniques cannot be applied to dilute atomic gases, however, since the gases are far too dilute to generate an appreciable signal. Instead, spectroscopic probes of dilute gases have successfully used Bragg scattering since this technique makes use of the stimulated light scattering processes induced by two counter-propagating laser beams, resulting in a strong enhancement of the signal.

In Bragg scattering experiments (see, e.g., [20]), by measuring the total momentum transferred to the sample, one measures the imaginary part of the density response function $\chi_{nn}(\mathbf{q}, \omega)$, where $\mathbf{q}$ and ω are the momentum and energy transferred by the stimulated absorption and emission of the photons [18]. If the Bragg pulse is short compared to $2\pi/\omega_z$,