

where

$$\begin{aligned}
I_1 &= -4s \operatorname{Re} \int_{\Omega} i \partial_t z (\nabla_y \varphi \cdot \nabla_y \bar{z} - \nabla_x \varphi \cdot \nabla_x \bar{z}) dx dy dt, \\
I_2 &= -2s \operatorname{Re} \int_{\Omega} i \partial_t z (\Delta_y \varphi - \Delta_x \varphi) \bar{z} dx dy dt, \\
I_3 &= -4s \operatorname{Re} \int_{\Omega} \Delta_y z (\nabla_y \varphi \cdot \nabla_y \bar{z} - \nabla_x \varphi \cdot \nabla_x \bar{z}) dx dy dt, \\
I_4 &= -2s \operatorname{Re} \int_{\Omega} \Delta_y z (\Delta_y \varphi - \Delta_x \varphi) \bar{z} dx dy dt, \\
I_5 &= 4s \operatorname{Re} \int_{\Omega} \Delta_x z (\nabla_y \varphi \cdot \nabla_y \bar{z} - \nabla_x \varphi \cdot \nabla_x \bar{z}) dx dy dt, \\
I_6 &= 2s \operatorname{Re} \int_{\Omega} \Delta_x z (\Delta_y \varphi - \Delta_x \varphi) \bar{z} dx dy dt, \\
I_7 &= -4s^3 \operatorname{Re} \int_{\Omega} \left(|\nabla_y \varphi|^2 - |\nabla_x \varphi|^2 \right) z (\nabla_y \varphi \cdot \nabla_y \bar{z} - \nabla_x \varphi \cdot \nabla_x \bar{z}) dx dy dt, \\
I_8 &= -2s^3 \operatorname{Re} \int_{\Omega} \left(|\nabla_y \varphi|^2 - |\nabla_x \varphi|^2 \right) z (\Delta_y \varphi - \Delta_x \varphi) \bar{z} dx dy dt,
\end{aligned}$$

and \bar{z} is the conjugate of z .

Now, we shall estimate the terms I_k , $1 \leq k \leq 8$, using the integration by parts and the condition $z(x, y, \pm T) = 0$. Then we have

$$\begin{aligned}
I_1 &= -4s \operatorname{Re} \int_{\Omega} i \partial_t z \nabla_y \varphi \cdot \nabla_y \bar{z} dx dy dt + 4s \operatorname{Re} \int_{\Omega} i \partial_t z \nabla_x \varphi \cdot \nabla_x \bar{z} dx dy dt \\
&= -2s \operatorname{Im} \int_{\Omega} z \partial_t (\nabla_y \varphi) \cdot \nabla_y \bar{z} dx dy dt - 2s \operatorname{Im} \int_{\Gamma_y} z \partial_t \bar{z} (\nabla_y \varphi \cdot \nu) dS_y dx dt \\
&\quad + 2s \operatorname{Im} \int_{\Omega} z \Delta_y \varphi \partial_t \bar{z} dx dy dt + 2s \operatorname{Im} \int_{\Omega} z \partial_t (\nabla_x \varphi) \cdot \nabla_x \bar{z} dx dy dt \\
&\quad + 2s \operatorname{Im} \int_{\Gamma_x} z \partial_t \bar{z} (\nabla_x \varphi \cdot \nu) dS_x dy dt - 2s \operatorname{Im} \int_{\Omega} z \Delta_x \varphi \partial_t \bar{z} dx dy dt. \quad (3.14)
\end{aligned}$$

In (3.14), we used the equality $\operatorname{Re}(iz) = -\operatorname{Im}(z)$ and $\operatorname{Im}(z) - \operatorname{Im}(\bar{z}) = 2\operatorname{Im}(z)$, where $\operatorname{Im}(z)$ denotes the imaginary part of $z \in \mathbb{C}$.

$$\begin{aligned}
I_2 &= -2s \operatorname{Re} \int_{\Omega} i \partial_t z \bar{z} (\Delta_y \varphi - \Delta_x \varphi) dx dy dt \\
&= -2s \operatorname{Im} \int_{\Omega} \partial_t \bar{z} z (\Delta_y \varphi - \Delta_x \varphi) dx dy dt. \quad (3.15)
\end{aligned}$$