predictions in SM4 compared to SM, enhanced or diminished by a factor of $\mathcal{O}(3)$. Note also that enhanced branching fractions correspond to a large CP asymmetry in $B_s \to \psi\phi$ and smaller branching fractions correspond to smaller asymmetry. The corresponding upper limit on the branching fractions are given by,

$$\begin{aligned} Br(B_s \to \mu^+\mu^-) &< \ 8.0 \times 10^{-9} \qquad m_{t'} = 400\ GeV, \\ &< 1.2\ \times 10^{-8}, \qquad m_{t'} = 600\ GeV, \\ Br(B_s \to \tau^+\tau^-) &< \ 1.8 \times 10^{-6} \qquad m_{t'} = 400\ GeV, \\ &< 2.4\ \times 10^{-6}, \qquad m_{t'} = 600\ GeV. \end{aligned} \tag{92}$$

However, when $S_{\psi\phi}$ is close to its SM value i.e when the CP violating phase, $\phi^s_{t'}$, of $V_{t's}$ is close to zero, the branching fractions reduce from their SM value since $|C^{\rm tot}_{10}|$ and δ' in eq. 91 are reduced from its SM value due to destructive interference with SM4 counterpart.

G. Branching fraction $B \to X_s\nu\bar{\nu}$

The decays $B \to X_s\nu\bar{\nu}$ are the theoretically cleanest decays in the field of rare B-decays. They are dominated by the same Z^0-penguin and box diagrams involving top quark exchanges which we encounter in the case of $K_L \to \pi^0\nu\bar{\nu}$, since the change of the external quark flavors has no impact on the $m_{t/t'}$ dependence, the later is fully described by the function $X(x_{t/t'})$ which includes the NLO corrections. The charm contribution is negligible here. The effective Hamiltonian for the decay $B \to X_s\nu\bar{\nu}$ is given by

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\Theta_w}\left(V^*_{tb}V_{ts}X(x_t) + V^*_{t's}V_{t'd}X(x_{t'})\right)(\bar{b}s)_{V-A}(\bar{\nu}\nu)_{V-A} + h.c. \tag{93}$$

with

$$X(x) = \frac{x}{8}\left[\frac{2+x}{x-1} + \frac{3x-6}{(x-1)^2}\ln x\right] \tag{94}$$

The calculation of the branching fractions for $B \to X_s\nu\bar{\nu}$ can be done in the spectator model corrected for short distance QCD effects. Normalizing it to $Br\big(B \to X_c\nu\bar{\nu}\big)$ and summing over three neutrino flavors one finds [56, 109]

$$\begin{aligned} \frac{Br\big(B \to X_s\nu\bar{\nu}\big)}{Br\big(B \to X_c e\bar{\nu}\big)} &= \frac{3\alpha^2}{4\pi^2\sin^4\Theta_W}\frac{\bar{\eta}}{f(z)\kappa(z)}\frac{1}{|V_{cb}|^2}\Big|\lambda_t X(x_t) + \lambda_{t'}X(x_{t'})\Big|^2 \\ &= \frac{\tilde{C}^2\bar{\eta}}{|V_{cb}|^2 f(z)\kappa(z)}, \end{aligned} \tag{95}$$