$conv(\{x_{i_1},\ldots,x_{i_k}\}) \subset \bigcup_{j=1}^{k}\Gamma(x_{i_j})$, i.e., $\Gamma$ is a KKM map. By the convex KKM theorem, $\bigcap_{i=1}^{n}\Gamma(x_i)\neq\emptyset$, thus $\bigcap_{i=1}^{n}C_i\neq\emptyset$.

Assuming for a moment that $\bigcap_{x\in X}\Gamma(x)$ is contained in a compact subset $K$ of $Y$, then the conclusion $\bigcap_{x\in X}\Gamma(x)\neq\varnothing$ would follow at once from the characterization of compactness in terms of families of closed subsets having the finite intersection property.

Observe now that the restriction/compression map $\Gamma_0 : X_0 \longrightarrow 2^D$ defined by $\Gamma_0(x) := \Gamma(x)\cap D, x\in X_0$, has compact convex values and is also a KKM map. Indeed, for any subset $\{x_1,\ldots,x_n\}\subseteq X_0$, $conv(\{x_1,\ldots,x_n\})\subset (\bigcup_{i=1}^{n}\Gamma(x_i))\cap D = \bigcup_{i=1}^{n}\Gamma_0(x_i)$. Therefore, $\bigcap_{x\in X_0}\Gamma(x)\supseteq\bigcap_{x\in X_0}\Gamma_0(x)\neq\varnothing$. The conclusion follows immediately from the fact that $\bigcap_{x\in X}\Gamma(x)\subseteq\bigcap_{x\in X_0}\Gamma(x)$ is compact and non-empty. ■

**Remark 7** *(i) Theorem 6 is an extension to topological vector spaces of the elementary KKM theorem of Granas-Lassonde, stated in the context of super-reflexive Banach spaces [10].*

*(ii) Theorem 6 obviously follows from the KKM principle of Ky Fan [12] where the values of $\Gamma$ are not assumed to be convex. The latter requires, however, much more involved analytical or topological results. Indeed, the Ky Fan KKM principle is equivalent to Sperner's lemma, to the Brouwer fixed point theorem, and to the Browder-Ky Fan fixed point theorem (see e.g., [1, 2, 3]).*

*(iii) In this generality, the compactness condition in the KKM principle is due to Ky Fan [14]. It obviously extends the earlier compactness conditions: $Y$ is also compact, or all values of $\Gamma$ are compact, or a single value $\Gamma(x_0)$ is compact, or $\bigcap_{i=1}^{n}\Gamma(x_i)$ is compact for some finite subset $\{x_1,\ldots,x_n)$ of $X$.*

Naturally, the convex KKM theorem can be expressed as an equivalent fixed point property for what we call a *von Neumann relation.* Given a subset $A$ of a