a local attractor in initial condition space [19], even in the presence of linear metric fluctuations [20], and thus this model is free from the initial condition problem of new inflation. In the context of "real" particle physics theories such as supersymmetric models, gravitational effects often steepen the potential for values of $|\phi|$ beyond the Planck mass and therefore prevent slow-roll inflation.

One way to try to avoid this problem but maintain the success of chaotic inflation is to add a second scalar field ψ to the sector of the theory responsible for inflation and to invoke a potential of the form

$$V(\phi, \psi) \,=\, \frac{1}{2} m^2 \phi^2 + \frac{g^2}{2} \psi^2 \phi^2 + \frac{1}{4} \lambda \big(\psi^2 - v^2\big)^2 \,, \quad (5)$$

where g and λ are dimensionless coupling constants and v is the vacuum expectation value of ψ. For large values of $|\phi|$, the potential in ψ direction has a minimum at $\psi = 0$, whereas for small values of $|\phi|$, $\psi = 0$ becomes an unstable point. The reader can verify that in this model slow-rolling of ϕ does not require super-Planckian field values. This two field model is called "hybrid" inflation [21].

Let us return to the toy model of chaotic inflation with the potential (1). The slow-roll trajectory is given by

$$\dot{\phi} \,=\, -\frac{1}{2\sqrt{3\pi}} m m_{pl} \,, \quad (6)$$

and it is easy to see that the slow-roll conditions break down at the field value

$$\phi_c \,=\, \frac{m_{pl}}{2\sqrt{3\pi}} \,. \quad (7)$$

After the breakdown of slow-rolling, ϕ commences damped oscillatory motion about $\phi = 0$ and the time-averaged equation of state is that of cold matter ($p = 0$ where p denotes pressure). Asymptotically for large times $mt \gg 1$ the solution approaches

$$\phi(t) \,\rightarrow\, \frac{m_{pl}}{\sqrt{3\pi} mt} \sin(mt) \,. \quad (8)$$

This scalar field configuration will provide the classical background matter in the reheating phase.

III. INFLATON DECAY

A. Perturbative Decay

Reheating is a key part of inflationary cosmology. It describes the production of SM matter at the end of the period of accelerated expansion when the energy density is stored overwhelmingly in the oscillations of ϕ. Historically, reheating was first treated perturbatively [3].

We assume that the inflaton ϕ is coupled to another scalar field χ. Taking the interaction Lagrangian to be

$$\mathcal{L}_{\rm int} \,=\, -g \sigma \phi \chi^2 \,, \quad (9)$$

where g is a dimensionless coupling constant and σ is a mass scale, then the decay rate of the inflaton into χ particles is given by

$$\Gamma \,=\, \frac{g^2 \sigma^2}{8\pi m} \,, \quad (10)$$

where m is the inflaton mass.

In the approach of [3], the energy loss of the inflaton due to the production of χ particles was taken into account by adding a damping term to the inflaton equation of motion which in the case of a homogeneous inflaton field is

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} \,=\, -V^{'}(\phi) \,. \quad (11)$$

For small coupling constant, the interaction rate Γ is typically much smaller than the Hubble parameter at the end of inflation. Thus, at the beginning of the phase of inflaton oscillations, the energy loss into particles is initially negligible compared to the energy loss due to the expansion of space. It is only once the Hubble expansion rate decreases to a value comparable to Γ that χ particle production becomes effective. It is the energy density at the time when $H = \Gamma$ which determines how much energy ends up in χ particles and thus determines the "reheating temperature", the temperature of the SM fields after energy transfer.

$$T_R \,\sim\, (\Gamma m_{pl})^{1/2} \,. \quad (12)$$

Since Γ is proportional to the square of the coupling constant g which is generally very small, perturbative reheating is slow and produces a reheating temperature which can be very low compared to the energy scale at which inflation takes place.

There are two main problems with the perturbative decay analysis described above. First of all, even if the inflaton decay were perturbative, it is not justified to use the heuristic equation (11) since it violates the fluctuation-dissipation theorem: in systems with dissipation, there are always fluctuations, and these are missing in (11). For an improved effective equation of motion see e.g. [22].

The main problem with the perturbative analysis is that it does not take into account the coherent nature of the inflaton field. The inflaton field at the beginning of the period of oscillations is not a superposition of free asymptotic single inflaton states, but rather a coherently oscillating homogeneous field. The large amplitude of oscillation implies that it is well justified to treat the inflaton classically. However, the matter fields can be assumed to start off in their vacuum state (the red-shifting during the period of inflation will remove any matter particles present at the beginning of inflation). Thus, matter fields χ must be treated quantum mechanically. The improved approach to reheating initiated in [4] (see also [5]) is to consider reheating as a quantum production of χ particles in a classical ϕ background.