are chiral; each ring of disks, or disks that share a common radius, beginning with the fourth ring (the fourth farthest from the center), can be angularly oriented in more than one distinct fashion relative to the preceding ring. By reorienting rings, the degenerate packings for a given k can be generated from one another. Figure 7 depicts curved hexagonal packings for $N = 60$, 90 and 126.

DLP optimal packings for N disks are equivalent to the densest packings of $N + 1$ disks enclosed in an encompassing disk when one of the disks is fixed at the center. For $N = 6$, 18, 36, 60 and 90 ($k = 1 \ldots 5$), we find that the curved hexagonal packings are the only DLP optimal packings, in support of the conjecture of Lubachevsky and Graham that curved hexagonal packings are the densest packings up to $k = 5$. Further, for $N = 126$ ($k = 6$), we also find that curved hexagonal packings are the only DLP optimal packings, indicating that though there are packings of 127 unconstrained disks within an encompassing disk denser than the curved hexagonal packings (as were found by Lubachevsky and Graham [19]), the curved hexagonal packings remain the densest packings of 126 disks around a fixed central disk. This is not the case for $N = 168$ ($k = 7$), as we find DLP optimal packings with higher density, such as the $N = 168$ packing to be shown later in the top panel of Fig. 16.

C. Wedge hexagonal packings

Another class of packings, previously unidentified, contain a subset of disks with centers arranged on the sites of the triangular lattice and the remainder arranged in six "wedges". We hereafter term such packings *wedge hexagonal* packings. Wedge hexagonal packings are not DLP optimal packings when arranged symmetrically (point group D_{6h}); however, minor deviations from perfect symmetry in a wedge hexagonal packing can produce a DLP optimal packing. Figure 8 depicts such DLP optimal packings for $N = 84$, 120 and 162. Lines to guide the eye have been drawn on the three optimal packings in Fig. 8.

In a wedge hexagonal packing, the subset of disks with centers arranged on the sites of the triangular lattice contains two parts; a regular hexagonal core of hexagonal number $3k(k + 1) + 1$ disks, with $k \geq 3$ odd; and six 'branches' composed of $(pk - a)$ disks, with $p \geq 2$ and $a \geq 1$ integers, extending from each of the vertices of the core regular hexagon. The branches are k disks wide and p disks long, with a of the farthest disks removed such that the end of the branch approximates a circle (as opposed to the point of a triangle). The