$$\Delta E^{\mathrm{SE}(G2)}_{\mathrm{SQED}} = 2 \sum_{n_1}{}' \frac{1}{\varepsilon_{Pa} - \varepsilon_{n_1}} \Big[\langle Pa|\Sigma(\varepsilon_{Pa})|n_1\rangle \langle n_1|T_0|Pa\rangle \langle PaPb|I'(\Delta)|QaQb\rangle$$
$$+ \langle Pa|\Sigma(\varepsilon_{Pa})|n_1\rangle \langle n_1 Pb|I'(\Delta)|QaQb\rangle \Big(\langle Qb|T_0|Qb\rangle - \langle Pb|T_0|Pb\rangle \Big) \Big]$$
$$+2\, \langle Pa|\Sigma(\varepsilon_{Pa})|Pa\rangle \sum_{n_1}{}' \Bigg[\frac{\langle PaPb|I'(\Delta)|n_1 Qb\rangle \langle n_1|T_0|Qa\rangle}{\varepsilon_{Qa} - \varepsilon_{n_1}}$$
$$+ \frac{\langle PaPb|I'(\Delta)|Qa n_1\rangle \langle n_1|T_0|Qb\rangle}{\varepsilon_{Qb} - \varepsilon_{n_1}} \Bigg] \,, \tag{28}$$

$$\Delta E^{\mathrm{SE}(G3)}_{\mathrm{SQED}} = \langle Pa|\Sigma(\varepsilon_{Pa})|Pa\rangle \langle PaPb|I''(\Delta)|QaQb\rangle \Big(\langle Qb|T_0|Qb\rangle - \langle Pb|T_0|Pb\rangle \Big) \,. \tag{29}$$

Equations (19)–(29) possess ultraviolet (UV) divergences. Taking into account the mass counterterm and employing Eq. (17) we find that $\Delta E^{\mathrm{SE}(B)}_{\mathrm{SQED}}$ has a non-zero UV-divergent part,

$$\Delta E^{\mathrm{SE}(B)}_{\mathrm{SQED}}(\mathrm{UV}) = 2\, B^{(1)} \sum_{n_1}{}' \frac{\langle Pa|T_0|n_1\rangle \langle n_1 Pb|I(\Delta)|QaQb\rangle}{\varepsilon_{Pa} - \varepsilon_{n_1}} \,. \tag{30}$$

By the end of the next subsection we will show that the sum of all the UV-divergent terms is zero.

B. ”Modified vertex” diagrams

For the irreducible parts of the diagrams C and F we have,

$$\Delta E^{\mathrm{SE}(C)}_{\mathrm{SQED}} = \Delta E^{\mathrm{SE}(C1)}_{\mathrm{SQED}} + \Delta E^{\mathrm{SE}(C2)}_{\mathrm{SQED}} \,, \tag{31}$$

$$\Delta E^{\mathrm{SE}(C1)}_{\mathrm{SQED}} = 2\, \frac{i}{2\pi} \int \mathrm{d}\omega \sum_{n_{1,2,3}}^{\varepsilon_{n_3} \neq \varepsilon_{Pa}} \frac{\langle Pa n_2|I(\omega)|n_1 n_3\rangle \langle n_1|T_0|n_2\rangle}{(\varepsilon_{Pa} - \omega - u\varepsilon_{n_1})(\varepsilon_{Pa} - \omega - u\varepsilon_{n_2})} \frac{\langle n_3 Pb|I(\Delta)|QaQb\rangle}{(\varepsilon_{Pa} - \varepsilon_{n_3})} \,, \tag{32}$$

$$\Delta E^{\mathrm{SE}(C2)}_{\mathrm{SQED}} = 2\, \frac{i}{2\pi} \int \mathrm{d}\omega \sum_{n_{1,2,3}}^{\varepsilon_{n_3} \neq \varepsilon_{Qa}} \frac{\langle Pa n_2|I(\omega)|n_1 n_3\rangle \langle n_1 Pb|I(\Delta)|n_2 Qb\rangle}{(\varepsilon_{Pa} - \omega - u\varepsilon_{n_1})(\varepsilon_{Qa} - \omega - u\varepsilon_{n_2})} \frac{\langle n_3|T_0|Qa\rangle}{(\varepsilon_{Qa} - \varepsilon_{n_3})} \,, \tag{33}$$

$$\Delta E^{\mathrm{SE}(F)}_{\mathrm{SQED}} = 2\, \frac{i}{2\pi} \int \mathrm{d}\omega \sum_{n_{1,2,3}}^{\varepsilon_{n_3} \neq \varepsilon_{Qb}} \frac{\langle Pa n_2|I(\omega)|n_1 Qa\rangle \langle n_1 Pb|I(\Delta)|n_2 n_3\rangle}{(\varepsilon_{Pa} - \omega - u\varepsilon_{n_1})(\varepsilon_{Qa} - \omega - u\varepsilon_{n_2})} \frac{\langle n_3|T_0|Qb\rangle}{(\varepsilon_{Qb} - \varepsilon_{n_3})} \,. \tag{34}$$

Since these diagrams have one vertex inside the self-energy loop, the corresponding expressions have the following structure of the ω-dependent denominators: $(\Delta_1 - \omega)^{-1}(\Delta_2 - \omega)^{-1}$. All the reducible terms that have similar structure are denoted as $\Delta E^{\mathrm{SE}(H)}_{\mathrm{SQED}}$,

$$\Delta E^{\mathrm{SE}(H)}_{\mathrm{SQED}} = \Delta E^{\mathrm{SE}(H1)}_{\mathrm{SQED}} + \Delta E^{\mathrm{SE}(H2)}_{\mathrm{SQED}} + \Delta E^{\mathrm{SE}(H3)}_{\mathrm{SQED}} \,, \tag{35}$$