Local Coulomb versus Global Failure Criterion for Granular Packings

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Contacts at the Coulomb threshold are unstable to tangential perturbations and thus contribute to failure at the microscopic level. How is such a local property related to global failure, beyond the effective picture given by a Mohr-Coulomb type failure criterion? Here, we use a simulated bed of frictional disks slowly tilted under the action of gravity to investigate the link between the avalanche process and a global generalized isostaticity criterion. The avalanche starts when the packing as a whole is still stable according to this criterion, underlining the role of large heterogeneities in the destabilizing process: the clusters of particles with fully mobilized contacts concentrate *local* failure. We demonstrate that these clusters, at odds with the pile as a whole, are also *globally* marginal with respect to generalized isostaticity. More precisely, we observe how the condition of their stability from a local mechanical proprety progressively builds up to the generalized isostaticity criterion as they grow in size and eventually span the whole system when approaching the avalanche.

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Understanding the failure of granular packings is of tremendous importance from both practical and theoretical aspects. Practically, avalanches are clearly of special interest for industrial and natural processes. From a more fundamental point of view, the mechanical rigidity of granular packings is related to the recently explored field of rheology close to dynamical arrest [1–3], as well as to the nature of the jamming transition for frictional particles [4–7]. Despite many studies both from a continuum and a microscopic point of view (see, for example, [8–17]), the mechanisms of failure in frictional granular media are still unclear.

Macroscopically, the application of the well known Coulomb criterion [18] requires the knwowledge of an effective friction coefficient, which remains out of reach of most recent developments. From a microscopic perspective, Maxwell derived a global stability criterion based on counting the number of independent contact force components, which has to exceed the number of degrees of freedom for a packing to be mechanically stable [19]. Recently, this isostaticity criterion has been generalized for frictional packings by including contacts exactly *at* the Coulomb threshold in the above counting argument [20, 21]. Such fully mobilized contacts are prone to tangential slipping, and it was indeed shown by Staron et al. [13, 15] that they play a key role in the destabilization process. A frictional packing of N particles with mean contact number z and mean number of fully mobilized contacts per particle n_m has only $Ndz/2 - Nn_m$ independent force components, due to the additional restrictions on the tangential forces. For this packing to be stable, this number has to exceed $Nd(d+1)/2$, the number of degrees of freedom and the the generalized isostaticity criterion in d dimensions reads:

$$z \geq z_{iso}^{\mu} + \frac{2n_m}{d} \equiv z_{iso}^{gen}, \qquad (1)$$

where $z_{iso}^{\mu} = d + 1$ is the isostatic value for $\mu = \infty$ frictional packings [20]. It can be represented in a (z, n_m) phase diagram (see Figure 1) where a line of marginal stability divides the stable from the unstable regions of phase space. Recent molecular dynamics simulations using an isotropic compression protocol [20] have shown that frictional packings unjam (in the sense that $p \to 0$) close to the generalized isostaticity line. The final state is characterized by $z(\mu)$ and $n_m(\mu)$ and the linear response properties of these packings suggest that it is the distance to the line of marginal stability, $\delta z^{gen} = z - z_{iso}^{gen}$, which controls stability [22].

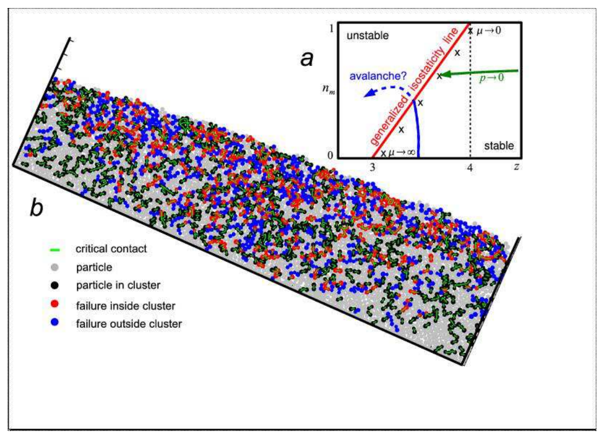

FIG. 1: *a* - Generalized isostaticity phase diagram in 2D. In red is the line of marginal stability (Eq. 1). Frictional packings under isotropic compression unjam – when compression is released (schematic green arrow) – at different positions close to this line (crosses) depending on friction μ [20, 22]. Does a pile slowly inclined under gravity towards avalanche follow the hypothesized blue line? *b* - The system during the avalanche, at an inclination of 24°, for the details of the coloring scheme please refer to Fig. 3.

These promising results, regarding both the global stability criterion and the details of the microscopic mecha-