

Then  $W(y_T^*, g_{\bar{\theta}}, \lambda_T, A\hat{V}_T A') =$

$$\begin{aligned} & [\bar{g}_{\bar{\theta}}(y_T^*) + O_p(1/\lambda_T)]' \left\{ [\bar{G}_{\bar{\theta}}(y_T^*) + O_p(1/\lambda_T)] A[V + o_p(1)] A' [\bar{G}_{\bar{\theta}}(y_T^*) + O_p(1/\lambda_T)]' \right\}^{-1} \\ & \times [\bar{g}_{\bar{\theta}}(y_T^*) + O_p(1/\lambda_T)]. \end{aligned}$$

(a) If CLDR holds then  $W_T$  by continuity of the determinants of polynomials and polynomial matrices converges to

$$[\bar{g}_{\bar{\theta}}(Z)]' \left\{ [\bar{G}_{\bar{\theta}}(Z)] A V A' [\bar{G}_{\bar{\theta}}(Z)]' \right\}^{-1} [\bar{g}_{\bar{\theta}}(Z)];$$

substituting the reparametrized functions for  $A = V^{-\frac{1}{2}}$  we get the result.

(b) Follows by continuity of the determinants of polynomial matrices and (24). ■

**Proof of Theorem 5.1.** Consider the asymptotically equivalent statistic:

$$\begin{aligned} & Z' \bar{G}(Z)' \Lambda [\bar{G}(Z) \bar{G}(Z)']^{-1} \Lambda \bar{G}(Z) Z \\ = & Z' \bar{G}(Z)' (\bar{G}(Z) \bar{G}(Z)')^{-\frac{1}{2}} \\ & \times \left[ (\bar{G}(Z) \bar{G}(Z)')^{\frac{1}{2}} \Lambda (\bar{G}(Z) \bar{G}(Z)')^{-\frac{1}{2}} (\bar{G}(Z) \bar{G}(Z)')^{-\frac{1}{2}} \Lambda (\bar{G}(Z) \bar{G}(Z)')^{\frac{1}{2}} \right] \\ & \times (\bar{G}(Z) \bar{G}(Z)')^{-\frac{1}{2}} \bar{G}(Z) Z \\ \leq & \left\| (\bar{G}(Z) \bar{G}(Z)')^{-\frac{1}{2}} \bar{G}(Z) Z \right\|^2 \left\| (\bar{G}(Z) \bar{G}(Z)')^{\frac{1}{2}} \Lambda (\bar{G}(Z) \bar{G}(Z)')^{-\frac{1}{2}} \right\| \\ & \times \left\| (\bar{G}(Z) \bar{G}(Z)')^{-\frac{1}{2}} \Lambda (\bar{G}(Z) \bar{G}(Z)')^{\frac{1}{2}} \right\| \\ \leq & \|\Lambda\|^2 \|Z\|^2 \sim \frac{1}{(1 + i_0)^2} \chi_p^2, \end{aligned}$$