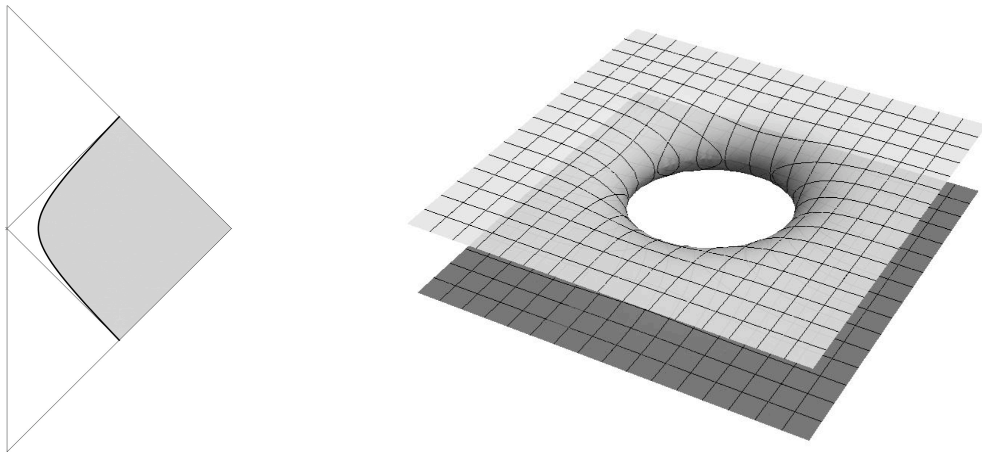

FIG. 1: Left: A $4d$ conformal diagram for Witten's bubble of nothing. The spacetime only exists outside of the bubble (shaded region). Right: The bubble surface is smooth, and located where the compactification volume, shown as the apparent vertical separation, degenerates to zero size.

In order for this geometry to represent an instability of the Kaluza – Klein vacuum, it must have the same asymptotics as the pure compactification, in particular the same (zero) value of Schwarzschild mass. This can be seen to be the case in a number of ways. The KK compactification possesses the eleven isometeries of Poincaré $\times U(1)$, which are broken down to the seven in $O(3,1) \times U(1)$ by the bubble of nothing (four translations are lost). This remaining symmetry is nevertheless larger than the five isometeries found in a generic Schwarzschild – Kaluza – Klein spacetime: $O(3) \times \mathbb{R} \times U(1)$, and so the mass of the bubble must be zero. Hence, barring any symmetry in place to prevent bubble nucleation, the KK vacuum is unstable.

III. BUBBLE OF NOTHING IN A $5d$ FLUX COMPACTIFICATION

In this section we will discuss the construction of a bubble of nothing geometry similar to that presented above, but with one crucial difference: we will stabilize the Kaluza – Klein radion. This is accomplished by introducing an axionic flux winding around the extra dimension. The simplest example of this type of flux compactification was described in [12] using the action for a complex scalar field given by

$$S = \int d^5x \sqrt{-g} \left(\frac{1}{2\kappa^2} R - \frac{1}{2} \partial_M \Phi \partial^M \bar{\Phi} - \frac{\lambda}{4} (\Phi\bar{\Phi} - \eta^2)^2 - \Lambda \right), \tag{7}$$