

of solving these convolution equations in spaces of generalized functions: existence of a unique solution (identification) and continuity of the mapping from the known functions to the solution (well-posedness).

The usual blueprint for deconvolution in function spaces works as follows. Assume: $g * f = w$ holds, convolution exists, e.g. for functions in L_1 , say, densities. Fourier transform (Ft) is defined: for the function g , $Ft(g) = \int g(x) \exp(ix^T \zeta) dx$; this is a characteristic function if g is a density. Exchange formula applies: for Fourier transforms $\gamma = Ft(g)$; $\phi = Ft(f)$; $\varepsilon = Ft(w)$ a convolution is transformed into product:

$$g * f = w \implies \gamma \phi = \varepsilon.$$

Also, if additionally Fourier transform exists for $h_k(x) = x_k g(x)$, then $Ft(h_k) = -i \frac{\partial}{\partial \xi_k} \gamma(\xi)$; denote the derivative $\frac{\partial}{\partial \xi_k} \gamma(\xi)$ by γ'_k .

If w and f in equation (2) (thus also ε and ϕ) are known and $\phi \neq 0$ solve the algebraic equation : $\gamma = \phi^{-1} \varepsilon$, then apply the inverse Fourier transform, Ft^{-1} , to obtain g ,

$$g = Ft^{-1}(\gamma).$$

When the functions that enter the equations are estimated based on available data on the observables, the solutions will be stochastic and for establishing consistency well-posedness of the solutions becomes crucial; Carrasco et al (2007), An and Hu (2012) discuss well-posedness that applies to similar problems in various normed spaces, mostly in spaces of integrable functions,