only valid in the large field limit $\phi \gg \phi_*$, since in the small field limit, the potential is not bounded from below and should be completed.

C. Non-SUSY models involving several fields

1. *Original hybrid inflation*

The most studied multi-field inflation model is the hybrid inflation first discussed in Ref. [147] (and studied extensively in [512]) as a model that differs from chaotic inflation on two main properties; it ends inflation with a waterfall triggered by a Higgs (not necessarily the SM Higgs) field coupled to the inflaton and it does not necessarily require an extremely small coupling to account for the normalization of the power-spectrum. The model is based on the potential given by [147]

$$V(\phi,\psi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\left(\psi^2 - M^2\right)^2 + \frac{\lambda'}{2}\phi^2\psi^2 \ , \tag{176}$$

where ϕ is the inflaton and ψ is the Higgs-type field. λ and λ' are two positive coupling constants, m and M are two mass parameters. It is the most general form (omitting a quartic term $\lambda''\phi^4$) of renormalizable potential satisfying the symmetries: $\psi \leftrightarrow -\psi$ and $\phi \leftrightarrow -\phi$. Inflation is assumed to be realized in the false-vacuum along the $\psi = 0$ valley and ends with a tachyonic instability for the Higgs-type field. The critical point of instability below which the potential develops non-vanishing minimum is at

$$\phi_{\rm c} = M\sqrt{\frac{\lambda}{\lambda'}}\,. \tag{177}$$

The system then evolves toward its true minimum at $V = 0$, $\langle\phi\rangle = 0$, and $\langle\psi\rangle = \pm M$ [48].

The inflationary valley, for $\langle\psi\rangle = 0$, is usually assumed to be where the last 60 e-foldings take place. This is supported by numerical and analytical simulations [206, 514, 536–538], where the fine-tuning of the initial conditions were discussed. In Ref. [206] it was found that when the initial VEV of the inflaton, ϕ, is sub-Planckian, a subdominant but non-negligible part of the initial conditions for the phase space leads to a successful inflation, i.e. around less than 15% depending on the model parameters. Initial conditions with super-Planckian

[48] The hybrid inflation models were also considered in Refs. [163, 164, 316, 317, 529–534] in the context of large extra dimensions at TeV scale [535].