Although this prior is still improper with respect to σ, its dependence on ϵ is different from that of the conditional Jeffreys' prior, Eq. (19). This demonstrates the potential importance of the compact subset normalization. The prior in Eq. (23) has a serious problem however. Suppose that the ϵ marginal of our evidence-based prior for ϵ and μ is $\exp(-\epsilon)/\sqrt{\pi\epsilon}$. It is then easy to verify that the resulting posterior is improper, since its ϵ marginal has the non-integrable form $\exp(-\epsilon)/\epsilon$. The cause of this problem is the choice of compact sets (22).

Fortunately it is not difficult to find a sequence of compact sets that will provide a proper posterior. Indeed, the σ dependence of the prior (19) suggests that the compact sets should be based on the parametrization $(\epsilon\sigma, \epsilon, \mu)$ rather than (σ, ϵ, μ) [18]. We therefore set:

$$\Theta_\ell \;=\; \Big\{(\sigma, \epsilon, \mu): \;\sigma \in [0, u_\ell/\epsilon],\; \epsilon \in [1/v_\ell, v_\ell],\; \mu \in [0, w_\ell]\Big\}, \tag{24}$$

where u_ℓ, v_ℓ, and w_ℓ are as before. Again using Eqs. (19), (20), and (21), we now find:

$$\pi_{R1}(\sigma \,|\, \epsilon, \mu) \;\propto\; \frac{\epsilon}{\sqrt{\epsilon\sigma + \mu}}, \tag{25}$$

which is identical to Jeffreys' prior for this problem and yields well-behaved posteriors. For future use, the subscript $R1$ on the left-hand side indicates that this reference prior was obtained with Method 1.

We now have all the ingredients needed to calculate the marginal reference posterior $\pi_{R1}(\sigma \,|\, n)$ for the cross section σ: the likelihood (16), the marginal nuisance prior (17), and the conditional reference prior (25). For calculating posterior summaries in terms of intervals and upper limits it is convenient to express the result as a tail probability:

$$\int_\sigma^\infty \pi_{R1}(\tau \,|\, n)\, d\tau \;=\; \int_{\frac{\sigma}{a+\sigma}}^1 \frac{u^{n+y}\,(1-u)^{x-\frac{1}{2}}}{B\big(n+y+1, x+\frac{1}{2}\big)}\;\frac{B_{\frac{b}{b+1}(1+\frac{u-1}{u}\frac{\sigma}{a})}\big(y+\frac{1}{2}, n+\frac{1}{2}\big)}{B_{\frac{b}{b+1}}\big(y+\frac{1}{2}, n+\frac{1}{2}\big)}\; du \tag{26}$$

where

$$B_z(u, v) \;\equiv\; \int_0^z t^{u-1}\,(1-t)^{v-1}\,dt \tag{27}$$

is the incomplete beta function, and $B(u, v) \equiv B_1(u, v) \;=\; \Gamma(u)\Gamma(v)/\Gamma(u+v)$.

2. *Application of Method 2 to the Single-Count Model*

In contrast with Method 1, Method 2 requires from the start that we specify the evidence-based prior for the effective integrated luminosity ϵ and the background contamination μ.