tation to obtain the asymptotics of the likelihood ratio process. This asymptotics is then used along with the Neyman-Pearson lemma and Le Cam's third lemma to establish a simple analytical formula for the maximal possible asymptotic probability of correct signal detection holding the asymptotic probability of false detection fixed. Section 6 concludes. All proofs are relegated to the Appendix.

2 Likelihood ratios

As mentioned above, we assume that data consist of n independent observations of p-dimensional complex-valued Gaussian vectors $X_t \sim N_{\mathbb{C}}(0, \Sigma)$. This means that $X_t = Y_t + \mathrm{i}Z_t$, where i denotes the imaginary unit, and the joint density of (Y_t, Z_t) at (y, z) equals $\frac{1}{(2\pi)^p \det \Sigma} \exp\left\{-\operatorname{tr}\left[\Sigma^{-1}(y + \mathrm{i}z)(y - \mathrm{i}z)'\right]\right\}$ (see, for example, Goodman, 1963). Further, we assume that the covariance matrix Σ equals $\sigma^2\left(I_p + VH\bar{V}'\right)$, where $H = \operatorname{diag}(h_1, ..., h_r)$ quantifies the sizes of the covariance spikes. Our goal is to study the asymptotic power of tests of $H_0 : h_1 = ... = h_r = 0$ against $H_1 : h_i > 0$ for some $i = 1, ..., r$.

If σ^2 is specified, the model is invariant with respect to unitary transformations and the maximal invariant statistic is λ, the vector of the first $m = \min\{n, p\}$ eigenvalues of XX'/n, where $X = [X_1, ..., X_n]$. Therefore, we consider tests based on λ. If σ^2 is unspecified, the model is invariant with respect to the unitary transformations and multiplications by non-zero scalars, and the maximal invariant is the vector of normalized eigenvalues $\mu = \left(\mu_1, ..., \mu_{m-1}\right)$, where $\mu_j = \lambda_j / (\lambda_1 + ... + \lambda_p)$. Hence, we consider tests based on μ. Note that the distribution of μ does not depend on σ^2, whereas if σ^2 is specified, we can always normalize λ dividing it by σ^2. Therefore, in what follows, we will assume without loss of generality that $\sigma^2 = 1$.

Let $h = (h_1, ..., h_r)$, and let us denote the joint density of $\lambda_1, ..., \lambda_m$ as $p_\lambda(x; h)$, $x = (x_1, ..., x_m) \in (\mathbb{R}^+)^m$ and that of $\mu_1, ..., \mu_{m-1}$ as $p_\mu(y; h)$, $y = (y_1, ..., y_{m-1}) \in$