expectation value is evaluated with the initial state, ρ_A, of the device. In other words, it is an operator that can be used for expressing the energy transferred to B in terms of the state of A. Second, only the average of Q can be given a physical meaning. It has been shown that no quantum observable can be defined for the work done or the heat transferred during a process that captures the detailed distribution of these quantities[20]. However, if only the average ΔE_B is needed, then Eq. (8) is the correct definition, which eventually yields the HTO Q. On the other hand, Q cannot be used for computing the higher moments of the energy transferred to B. For example, even though the average energy transferred to B is given by $\mathrm{tr}\,\rho_A Q$, the quantity $\mathrm{tr}\,\rho_A Q^2$ or the averages of other nonlinear functions of Q do not have the anticipated meaning. Averages of such nonlinear functions of energy change of B should be computed separately following the approach in Ref. 20. This crucial property of the HTO must always be kept in mind.

Once the HTO is computed, the average total work that needs to be done for carrying out the operation can be obtained as

$$W_{\mathrm{tot}} = \mathrm{tr}\left(\mathcal{E}(\rho_A)H'_A - \rho_A H_A + \rho_A Q\right) \ . \tag{10}$$

As a result, for investigating the average energies transferred during the realization of the operation $\mathcal{E}$, the only nontrivial quantity to be determined is the heat dumped to the bath. This is the main reason why this article concentrates solely on the HTO.

The isometry condition $U_{AB}^\dagger U_{AB} = \mathbb{1}_{AB}$ and the fact that the bath is in thermal equilibrium imposes some restrictions on the operator Q. The investigation of these restrictions is the primary subject of this article. The main question that needs to be answered is the following: for any given quantum operation $\mathcal{E}$, hermitian operator Q and temperature value T, is it possible to find a bath B (namely a Hamiltonian H_B) and an isometry U_{AB} such that both Eq. (4) and Eq. (9) are satisfied? We will say that Q is a possible HTO for $\mathcal{E}$ when such a bath and isometry exist. Our main problem is then to find all necessary and sufficient conditions for Q to be a possible HTO for $\mathcal{E}$ at temperature T. These conditions will enable us to discuss the thermodynamics of quantum information processing (for given $\mathcal{E}$) without specifying anything about the bath, the bath-device coupling and the specific way the quantum operation is realized.

Before embarking on the investigation of this problem, it will be convenient to list the following basic properties of the HTOs.