



Figure 4: The stabilization of the 2-periodic solution of (5.15) corresponding to the 2-periodic orbit  $\kappa^{(1)} \approx 0.34459$ ,  $\kappa^{(2)} \approx 0.90497$  of (5.11). The value  $\varepsilon = 0.072$  is used and the control is switched on at  $t = \zeta_{25}$ .

different, and this reveals the presence of periodic motions in the quasi-minimal set.

## 6 Conclusions

The unpredictable function as an unpredictable point of the Bebutov dynamics has been defined, and chaos in the quasi-minimal set of the function is verified. This is the first time in the literature that the existence of an unpredictable solution for a quasi-linear ordinary differential equation is proved. Moreover, through simulations it is demonstrated that cycles and non-cyclic Poisson stable orbits can coexist in a quasi-minimal set.

The concept of unpredictable solutions can be useful for finding more delicate features of chaos in systems with complicated dynamics. Researches based on unpredictable functions may pave the way for the functional analysis of chaos to involve the operator theory results. Hopefully, our approach will give a basis for a deeper comprehension and possibility to unite different appearances of chaos. In this framework, the results can be developed for partial differential equations, integro-differential equations, functional differential equations, evolution systems, etc.

## References

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