and

$$\begin{aligned}
&\frac{d}{dt}\langle a^\dagger a\rangle = G\langle a^\dagger\sigma + a\sigma^\dagger\rangle - \kappa\langle a^\dagger a\rangle + \eta(\langle a^\dagger\rangle + \langle a\rangle),\\
&\frac{d}{dt}\langle \sigma^\dagger\sigma\rangle = -G\langle a^\dagger\sigma + a\sigma^\dagger\rangle - \gamma\langle\sigma^\dagger\sigma\rangle,\\
&\frac{d}{dt}\langle a^\dagger\sigma + a\sigma^\dagger\rangle = i\Delta\langle a^\dagger\sigma - a\sigma^\dagger\rangle\\
&\quad + 2G\langle a^\dagger a\sigma_z + \sigma^\dagger\sigma\rangle - \frac{\kappa+\gamma}{2}\langle a^\dagger\sigma + a\sigma^\dagger\rangle\\
&\quad + \eta(\langle\sigma\rangle + \langle\sigma^\dagger\rangle),\\
&\frac{d}{dt}\langle a^\dagger\sigma - a\sigma^\dagger\rangle = i\Delta\langle a^\dagger\sigma + a\sigma^\dagger\rangle\\
&\quad - \frac{\kappa+\gamma}{2}\langle a^\dagger\sigma - a\sigma^\dagger\rangle + \eta(\langle\sigma\rangle - \langle\sigma^\dagger\rangle).
\end{aligned} \tag{29}$$

Here $\Delta = \omega_c - \omega_a$ is the cavity–atom detuning.

In order to get insight into the effect of the atom on the guided cavity field, we use the procedures of Ref. [12] to linearize Eqs. (28) and (29). For this purpose, we assume that the probe field is so weak that the excited state is hardly occupied and there is at most one photon in the cavity. In this case, we have [12]

$$\begin{aligned}
\langle a\sigma_z\rangle &= -\langle a\rangle,\\
\langle a^\dagger a\sigma_z\rangle &= -\langle a^\dagger a\rangle.
\end{aligned} \tag{30}$$

With the help of the above formulae, we can linearize Eqs. (28) and (29) and solve them in the steady-state regime. The results are [12]

$$\begin{aligned}
&\langle a\rangle = -\frac{\eta}{D}(i\Delta_a - \gamma/2),\\
&\langle\sigma\rangle = -\frac{\eta}{D}G,\\
&\langle a^\dagger a\rangle = \frac{\eta^2}{|D|^2}(\Delta_a^2 + \gamma^2/4),\\
&\langle\sigma^\dagger\sigma\rangle = \frac{\eta^2}{|D|^2}G^2,\\
&\langle a^\dagger\sigma + a\sigma^\dagger\rangle = -\frac{\eta^2}{|D|^2}G\gamma,\\
&\langle a^\dagger\sigma - a\sigma^\dagger\rangle = -2i\frac{\eta^2}{|D|^2}G\Delta_a,
\end{aligned} \tag{31}$$

where

$$D = G^2 + \kappa\gamma/4 - \Delta_c\Delta_a - i(\Delta_c\gamma + \Delta_a\kappa)/2. \tag{32}$$