Using orthogonality of the Legendre polynomials it can be expressed as

$$C_\alpha = \frac{2}{a_\perp\sqrt{\pi L}}\left[\sum_{n=0}^{\infty}\left(\frac{\cot\sqrt{\epsilon_\alpha-\lambda n}}{(\epsilon_\alpha-\lambda n)^{3/2}} + \frac{1}{(\epsilon_\alpha-\lambda n)\sin^2\sqrt{\epsilon_\alpha-\lambda n}}\right)\right]^{-1/2}, \qquad (10)$$

where the rescaled energy ϵ_α is given by $\epsilon_\alpha \equiv \lambda E_\alpha/(2\hbar\omega_\perp)$ and $\lambda \equiv (L/a_\perp)^2$ is the aspect ratio.

For the Fermi-Huang interaction the eigenenergy equation (4) attains the form

$$\frac{\partial}{\partial r}\left[r\sum_{n,l}\frac{\langle\rho|n\rangle\langle z|l\rangle\langle n|0\rangle\langle 0|0\rangle}{E-E_{nl}}\right]_{r=0} = \frac{1}{V} \qquad (11)$$

In the limit $r\to 0$, the eigenstate is spherically symmetric. This allows us to deal with the z-axis only (see [6]). Substitution of the noninteracting wavefunctions (6) and (7) and energies (8) with the subsequent summation over l leads to

$$\sqrt{\lambda}\frac{\partial}{\partial z}\left[z\sum_{n=0}^{\infty}\frac{\cos\left(2\sqrt{\epsilon_\alpha-\lambda n}(z/L-1/2)\right)}{\sqrt{\epsilon_\alpha-\lambda n}\sin\sqrt{\epsilon_\alpha-\lambda n}}\right]_{z=0} = \frac{a_s}{a_\perp}. \qquad (12)$$

This sum contains both the regular part and the irregular one, the latter being proportional to z^{-1}. The regular part can be extracted using the identity

$$\lim_{z\to 0}\left[\sum_{n=0}^{\infty}(\lambda n-\epsilon)^{-1/2}\exp(-2\sqrt{\lambda n-\epsilon}\,z/L) - \frac{L}{\lambda z}\right] = -\frac{1}{\sqrt{\lambda}}\zeta\left(\frac{1}{2},-\frac{\epsilon}{\lambda}\right) \qquad (13)$$

(see [6]) where $\zeta(\nu,\alpha)$ is the Hurwitz zeta function (see, e. g., [7, 8]. Finally we arrive at the transcendental equation for the eigenenergies (4) in [1]. The summands in the sums Eqs. (3) and (4) in [1] and in Eq. (10) decay exponentially with n, leading to the fast converging series. Note that the imaginary parts of the two terms in the left hand side of Eq. (4) in [1] cancel each other automatically.

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