2. α is non-compact imaginary for Q.

In case (1) above, the map $\pi_\alpha|_Y$ has degree 1 (i.e. is birational). In case (2), $\pi_\alpha|_Y$ is birational if α is non-compact imaginary type I, and has degree 2 if α is non-compact imaginary type II.

Proof. We only briefly sketch what is involved in the proof. A more detailed exposition can be found in [RS90, Section 4].

One first establishes a correspondence between K-orbits on $Z_\alpha(Q)$ and the orbits of $K(g,\alpha) := K \cap gP_\alpha g^{-1}$ on the fiber $\pi_\alpha^{-1}(gP_\alpha)$. This fiber being isomorphic to $\mathbb{P}^1$, there are only a few possibilities for the orbit structure. This structure depends on the image of $h : K(g,\alpha) \to \mathrm{Aut}(\mathbb{P}^1)$ determined by the action of $K(g,\alpha)$. In the event that α is complex for the orbit Q, this image contains a non-trivial unipotent subgroup, and there are two orbits on the $\mathbb{P}^1$ fiber: one dense orbit and one fixed point. The dense orbit corresponds to Q itself if $l(s_\alpha a\theta(s_\alpha)) = l(a) - 2$, but to an orbit one dimension higher in the event that $l(s_\alpha a\theta(s_\alpha)) = l(a) + 2$.

If α is non-compact imaginary, then one of two cases occurs. In case 1, the image $h(K(g,\alpha))$ is a maximal torus. In this case, there are three orbits on the fiber - one dense orbit and two fixed points. In case 2, $h(K(g,\alpha))$ is the normalizer of a maximal torus. In this case, there are two orbits - one dense orbit and a two-point orbit. Each point of the two-point orbit is fixed by the identity component $K(g,\alpha)^0$, and the two points are permuted by $K(g,\alpha)$. Which case we are in depends on whether α is type I or type II. If α is type I, we are in case 1, and if α is type II, we are in case 2.

These various cases give us information about the K-orbits on $Z_\alpha(Q)$, and we can see what the degree of $\pi_\alpha|_Y$ over its image is in each case. Indeed, when α is complex and $l(s_\alpha a\theta(s_\alpha)) = l(a) + 2$, or when α is non-compact imaginary type I, Q corresponds to a 1-point orbit on $\mathbb{P}^1$ (the lone fixed point in the former case, and one of the two fixed points in the latter). When α is non-compact imaginary type II, Q corresponds to the two-point