tra to some extent, we modify the synchrotron emission procedure of Section 3.3 for secondary particles in high Landau levels ($n \geq 3$). In every step 10 photons are emitted, each with a weighting factor of $0.1\Delta N_\epsilon$ (rather than one photon with a weighting factor of ΔN_ϵ, as before). Each photon has a different energy $\epsilon_{\perp,i}$ [chosen randomly according to Eq. (35)], so that the total energy lost by the secondary particle becomes $\gamma_\perp m_e c^2 = 0.1\sum_{i=1}^{10}\epsilon_{\perp,i}$. We do not apply this procedure to the synchrotron emission from secondary particles in Landau levels $n = 1$ or $n = 2$, as it would not gain anything; each of the 10 photons emitted would have the same value of $\epsilon_{\perp,i}$.

4 RESULTS

In this section we present the results of our simulations of photon- and electron-initiated cascades (Sections 4.1 and 4.2, respectively), for a variety of different surface field strengths, rotation periods, field geometries, and initial energies of the primary particle. For each type of cascades, we present the "final" spectra of the cascade photons and pairs as they cross the light cylinder and escape from the magnetosphere. For the electron-initiated cascades we also show the spectra at several intermediate stages (i.e., the spectra of all photons and pairs that cross the height $r = 1.2R$, $2R$, $5R$, etc.). The photon spectra are plotted over the energy range 10 keV-1 TeV, since for energies $\lesssim 1$ keV the thermal photons dominate the spectra while above ~ 1 TeV fewer than one photon is produced per primary electron. We are particularly interested in the pair multiplicities, i.e., the total number of cascade electrons + positrons produced per primary particle. We use n_E to denote the number of electrons and positrons per "primary" photon and N_E to denote the number per primary electron; the two multiplicities are related by

$$N_E = N_0 \times n_E\,, \tag{47}$$

where N_0 is the number of photons produced by the primary electron (see Section 2.2). From our numerical results we infer various empirical relations for each cascade; quantitative arguments for the validity of several of these relations are given in Appendix C.

We first present our results for photon-initiated cascades (see Section 3.4), as they are simpler and aid us in our discussion of the results for the full cascade (initiated by a primary electron).

4.1 Results: photon-initiated cascades

Our results for photon-initiated cascades are presented in Figs. 6–9. We consider primary photons with energies in the range of 10^3–10^5 MeV; for $B_{p,12} = 1$–1000, the primary electron should emit very few photons (via either resonant ICS or curvature radiation) above this energy range (see Section 2). Unless otherwise stated, the primary photon is emitted from near the surface, in the direction tangent to the last open field line. Thus the radius of curvature near the point of emission is $\mathcal{R}_c \simeq 9\times10^7 P_0^{1/2}$ cm for dipole fields [Eq. (14)].

We find significant differences in the behavior of the cascades at magnetic field strengths below and above $B_{\rm crit} \simeq 3\times10^{12}$ G [Eq. (B17)]. At low fields $B \lesssim B_{\rm crit}$, the primary photon can pair produce if [Eq. (C2); see also Hibschman & Arons 2001a]

$$\epsilon_0 > \epsilon_{\rm min} \sim 3000 B_{p,12}^{-1}\mathcal{R}_8 \text{ MeV}\,, \tag{48}$$

where $\mathcal{R}_8$ is the radius of curvature $\mathcal{R}_c$ in units of 10^8 cm, evaluated at the surface along the last open field line. Strong cascades, where more than one electron-positron pair is produced, typically occur at energies ~ 10 times $\epsilon_{\rm min}$. For ϵ_0 in the range from $\epsilon_{\rm min}$ to $\sim 10^5$ MeV, we find that the multiplicities of photons and e^+e^- particles produced in the cascade are

$$n_\epsilon \sim \frac{\epsilon_0}{500 \text{ MeV}}\mathcal{R}_8^{-1} \tag{49}$$

and

$$n_E \sim \frac{\epsilon_0}{10^4 \text{ MeV}} B_{p,12}\mathcal{R}_8^{-1}\,, \tag{50}$$

respectively. These results are (largely) independent of the hot spot model used. When ICS is inactive, the cascade electron/positron has final energy (after it has finished radiating synchrotron photons) extending from [Eq. (C3)]

$$E_{\rm max} \sim 0.1 B_{p,12}\epsilon_0 \tag{51}$$

(for the first pair produced) down to $\sim 0.1 B_{p,12}\epsilon_{\rm min}$ for the lowest-energy pairs, and the total energy of the pairs is [Eq. (C4)]

$$\mathcal{E}_{\rm tot} \sim 2E_{\rm max} + 0.1 B_{p,12}\epsilon_{\rm min} n_E \ln\left(\frac{0.075\epsilon_0}{\epsilon_{\rm min}}\right)\,. \tag{52}$$

When ICS is active from a hot spot (Section 3.3), the number of pairs produced does not change, since the photons produced through ICS at these field strengths have energies $\sim B_{p,12}^2 T_6^{-1}$ MeV [Eq. (8)] and can not pair produce. The total pair energy $\mathcal{E}_{\rm tot}$ decreases, however, since the ICS process transfers energy from the pairs to photons. Although resonant ICS is most important for electrons and positrons at $\gamma_{\rm crit} \simeq \epsilon_c/kT$ [Eq. (7); see Section 2.2], we find in these cascades that all electrons and positrons with energies in the range of

$$\begin{aligned} E_{\rm RICS} &\sim (0.3-30)\gamma_{\rm crit} m_e c^2 \\ &\simeq (20-2000) B_{p,12} T_6^{-1} \text{ MeV} \end{aligned} \tag{53}$$

are strongly affected. Thus hot surface spots with higher T tend to lower $\mathcal{E}_{\rm tot}$ more. As expected, we find that photon splitting does not affect the cascade at these field strengths (see Section 3.2). The photon and pair cascade spectra for $B_{p,12} = 1$ are shown in Fig. 6, both when ICS is inactive and when ICS is active from a "warm spot" ($T_6 = 1$, $\theta_{\rm spot} = 0.3$).

At high fields ($B \gtrsim B_{\rm crit}$), a primary photon injected from the surface will pair produce when

$$\epsilon_0 > \epsilon_{\rm min} \sim 200\,\mathcal{R}_8 \text{ MeV}\,, \tag{54}$$

largely independent of field strength. When ICS is inactive, almost all of the cascade energy resides in the pairs; i.e., $\mathcal{E}_{\rm tot} \simeq \epsilon_0$. The pair cascade will be very weak regardless of photon energy, with $n_E < 10$ and $n_\epsilon = 0$ or 1 (i.e., at most one photon escapes the magnetosphere without pair production). This is because the $e^\pm$ pairs are produced exclusively through the $(jk) = (00)$ or (01) channel (see Section 3.2), so that at most one synchrotron photon is emitted per pair. For $B_{p,12} \gtrsim 20$, photon splitting causes all pairs to be produced