where τ_2 acts on color indices and $\epsilon_{ab} = -\epsilon_{ba}$ on flavor. The resulting BEC is composed of weakly interacting qq baryons with $J^P = 0^+$.

For $\mu \geq \mu_o$ leading-order χPT predicts a smooth rotation of the chiral condensate $\langle\bar\psi\psi\rangle$ into the superfluid condensate $\langle qq\rangle$ as μ increases [5]. In addition there is a quantitative prediction for quark density:

$$n_q(\mu) = 8N_f f_\pi^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right), \qquad (2)$$

where the parameters μ_o and f_π suffice to specify χPT at this order. It is possible to develop the thermodynamics of the system at $T = 0$ more fully, to extract pressure and energy density [6]:

$$p_{\chi PT} = \int_{\mu_o}^{\mu} n_q d\mu \;=\; 4N_f f_\pi^2 \left(\mu^2 + \frac{\mu_o^4}{\mu^2} - 2\mu_o^2\right); \quad (3)$$

$$\varepsilon_{\chi PT} = -p + \mu n_q \;=\; 4N_f f_\pi^2 \left(\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 2\mu_o^2\right); \quad (4)$$

$$(T_{\mu\mu})_{\chi PT} = \varepsilon - 3p \;=\; 8N_f f_\pi^2 \left(-\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 4\mu_o^2\right). \quad (5)$$

Note that the trace of the stress-energy tensor $(T_{\mu\mu})_{\chi PT} < 0$ for $\mu > \sqrt{3}\mu_o$.

These model results should be contrasted with those of another paradigm for cold dense matter, namely a degenerate system of weakly interacting (thus presumably deconfined) quarks populating a Fermi sphere up to some maximum momentum $k_F \approx E_F = \mu$:

$$n_{SB} = \frac{N_f N_c}{3\pi^2}\mu^3; \; \varepsilon_{SB} = 3p_{SB} = \frac{N_f N_c}{4\pi^2}\mu^4. \qquad (6)$$

In this picture superfluidity arises from condensation of diquark Cooper pairs from within a layer of thickness Δ centred on the Fermi surface; hence

$$\langle qq\rangle \propto \Delta\mu^2. \qquad (7)$$

Since p_{SB} eventually exceeds $p_{\chi PT}$ as μ increases, the degenerate system must be the more thermodynamically stable at high density. In fact, since both n_q and ε are discontinuous at the point where $p_{SB} = p_{\chi PT}$, this naive treatment predicts the resulting deconfining transition is first order [6].

These considerations have motivated us to pursue lattice simulations of QC$_2$D beyond the BEC regime, using $N_f = 2$ flavors of Wilson fermion. The quark action is

$$S = \sum_{i=1,2} \bar\psi_i M \psi_i + \kappa j[\psi_2^{tr}(C\gamma_5)\tau_2\psi_1 - h.c.], \qquad (8)$$

with

$$M_{xy} = \delta_{xy} - \kappa \; \sum_\nu \; \Big[(1-\gamma_\nu)e^{\mu\delta_{\nu 0}}U_\nu(x)\delta_{y,x+\hat\nu} + (1+\gamma_\nu)e^{-\mu\delta_{\nu 0}}U_\nu^\dagger(y)\delta_{y,x-\hat\nu}\Big]. \quad (9)$$

A conventional Wilson action was used for the glue fields. Further details can be found in [6].

Since Wilson fermions do not have a manifest chiral symmetry, we have little to say at this stage about this aspect of the physics, which at high quark density should be of secondary importance for phenomena near the Fermi surface; they do however carry a conserved baryon charge due to the U(1)$_B$ symmetry $\psi \mapsto e^{i\alpha}\psi$, $\bar\psi \mapsto \bar\psi e^{-i\alpha}$. Our initial runs on a $8^3 \times 16$ lattice with $\beta = 1.7$, $\kappa = 0.178$ corresponding to lattice spacing $a = 0.230(5)$fm, $m_\pi a = 0.79(1)$ and $m_\pi/m_\rho = 0.779(4)$ have been described in [6]. In this Letter we present data from runs on an approximately matched $12^3 \times 24$ lattice with $\beta = 1.9$, $\kappa = 0.168$ corresponding to $a = 0.186(8)$fm, $m_\pi a = 0.68(1)$ and $m_\pi/m_\rho = 0.80(1)$. The physical scale is set by equating the observed string tension at $\mu = 0$ to $(440\text{MeV})^2$. Note that the physical temperature T is approximately 54(1)MeV for the smaller lattice, and 44(2)MeV for the larger. We used a standard HMC algorithm – the only novelty is the inclusion of a diquark source term (proportional to j in Eqn.(8)); this mitigates the impact of IR fluctuations in the superfluid regime and also enables the algorithm to change the sign of det$M$ for a single flavor, thus maintaining ergodicity. All results presented here were obtained with $ja = 0.04$; ultimately the physical limit $j \to 0$ must be taken.

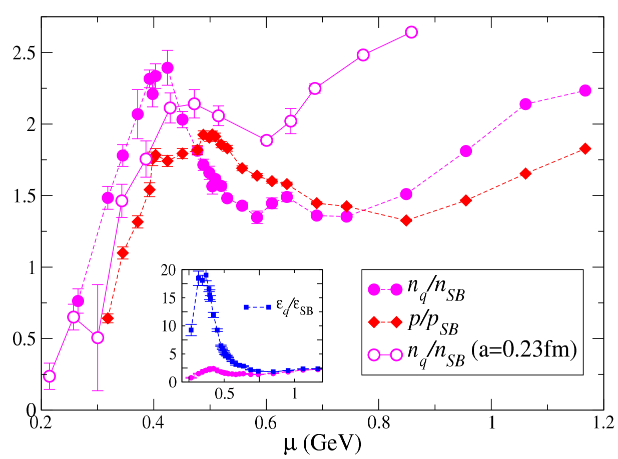

FIG. 1: (Color online) n_q/n_{SB} and p/p_{SB} vs. μ for QC$_2$D. Inset shows $\varepsilon_q/\varepsilon_{SB}$ for comparison.

Fig. 1 shows results for quark density and pressure as functions of μ, plotted as ratios of the same quantities evaluated for free massless quarks on the same lattice [6]. In the $j \to 0$ limit the onset is expected at $\mu_o a = 0.34$ corresponding to $\mu_o \simeq 360$MeV; the observation of $n_q, p > 0$ for $\mu < \mu_o$ is an artifact of working with $j \neq 0$. Beyond onset the ratio n_q/n_{SB} rises to a peak at $\mu \approx 400$MeV, then falls to a plateau beginning at $\mu_Q \approx 530$MeV, which continues until $\mu_D \approx 850$MeV where it starts to rise again. If following the arguments presented above we associate the plateau with the setting