with a, b and c being arbitrary complex numbers. Multiplying the mass matrix of Eq. (8) by I_{ij} following $I_{ij} m_F^{\rm dia} I_{ij}^\dagger$, $I_{ij} m_F^{\rm dia} I_{ij}$ and $I_{ij} m_F^{\rm dia} I_{ij}^T$, one can find that the resulted new matrices are still diagonal. For illustration, we explicitly express $(\bar{m}_F^{\rm dia})_{ij} \equiv I_{ij} m_F^{\rm dia} I_{ij}^T$ as

$$(\bar{m}_F^{\rm dia})_{00} = \begin{pmatrix} a^2 m_{f1} & 0 & 0 \\ 0 & b^2 m_{f2} & 0 \\ 0 & 0 & c^2 m_{f3} \end{pmatrix}, \ (\bar{m}_F^{\rm dia})_{12} = \begin{pmatrix} a^2 m_{f2} & 0 & 0 \\ 0 & b^2 m_{f1} & 0 \\ 0 & 0 & c^2 m_{f3} \end{pmatrix},$$

$$(\bar{m}_F^{\rm dia})_{23} = \begin{pmatrix} a^2 m_{f1} & 0 & 0 \\ 0 & b^2 m_{f3} & 0 \\ 0 & 0 & c^2 m_{f2} \end{pmatrix}, \ (\bar{m}_F^{\rm dia})_{31} = \begin{pmatrix} a^2 m_{f3} & 0 & 0 \\ 0 & b^2 m_{f2} & 0 \\ 0 & 0 & c^2 m_{f1} \end{pmatrix}. \quad (11)$$

We find that besides the diagonal form is obtained, the new matrices may not have the mass hierarchy as shown in Eq. (8). Moreover, by the multiplications of $I_{ij} \times I_{mn}$, more possible patterns can be found. Consequently, a nontrivial and interesting relation between $Y^F_{1(2)}$ and $Y^F_{2(1)}$ indeed exists and FCNC free at the tree level can be realized in the THDM without imposing symmetry. In order to give a general expression, we formulate the new diagonal matrix as

$$\bar{m}_F^{\rm dia} \equiv I_{\rho\sigma} m_F^{\rm dia} \tilde{I}_{\rho\sigma} = V_L^F \bar{I}^F_{L\rho\sigma} \frac{v}{\sqrt{2}} \bar{Y}^F_\alpha \tilde{\bar{I}}^F_{R\rho\sigma} V_R^{F\dagger}, \quad (12)$$

where $I_{\rho\sigma}$ could be any one of the matrices shown in Eq. (10) or their combinations, $\tilde{I}_{\rho\sigma}$ could be $I^\dagger_{\rho\sigma}$, or $I_{\rho\sigma}$ or $I^T_{\rho\sigma}$, $\bar{I}^F_{\chi\rho\sigma} = V_\chi^{F\dagger} I_{\rho\sigma} V_\chi^F$ with $\chi = L(R)$ being the helicity projection operator. Hence, if we set $\bar{Y}^F_{2(1)} = \bar{I}^F_{L\rho\sigma} \bar{Y}^F_{1(2)} \tilde{\bar{I}}^F_{R\rho\sigma}$, our purpose to find the solution to diagonalizing $\bar{Y}^F_{1(2)}$ and $\bar{Y}^F_{2(1)}$ simultaneously has been achieved. It is worth mentioning that although there are no FCNCs at the tree level, however, due to no symmetry protection, the FCNCs could be induced by radiative corrections, sketched in Fig. 1(a). Nevertheless, due to the soft Z_2 or $U(1)$ breaking term, the similar radiative corrections also occur in the type-II THDM, illustrated in Fig. 1(b). Although the loop-suppressed FCNCs could have interesting impacts on rare decays [5, 24], here we only pay attention to the leading effects on tree processes.

We now move forward to the charged Higgs interactions with fermions. Although the elements in Eq. (11) do not show the regular hierarchy in masses of fermions, however, due to a, b and c being arbitrary complex numbers, we can reparameterize $\bar{m}_F^{\rm dia}$ to be

$$\bar{m}_F^{\rm dia} = \boldsymbol{\eta}_F m_F^{\rm dia} \quad (13)$$