TABLE I: Summary of the topological charge of the free space Hamiltonian and the low-energy spectrum of fermions localized around a vortex line (or a boundary where the pairing gap changes its sign). v is a parameter dependent velocity.

	even parity pairing		odd parity pairing	
	topological charge	mid-gap state dispersion	topological charge	mid-gap state dispersion
$m=0$	$N_R = -N_L = \frac{\Delta_0}{2\lvert\Delta_0\rvert}$	$E=\pm v p_z$	$N_R = N_L = \frac{\Delta_0}{2\lvert\Delta_0\rvert}$	$E=\pm v p_z$
$m\neq 0$	$N=0$	$E=\pm v\sqrt{m^2+p_z^2}$	$\begin{cases} N=\frac{\Delta_0}{\lvert\Delta_0\rvert} \\ N=0 \end{cases}$	$E=\pm v p_z \quad (m^2<\mu^2+\Delta_0^2)$ none $\quad (m^2>\mu^2+\Delta_0^2)$

left-handed quarks. In the presence of the small quark mass or chiral condensate, such localized quarks become gapped but their mass gap vm is much smaller than the energy gap of bulk quarks Δ.[7] Furthermore, the mass gap at high density $vm \sim m\,(\Delta/\mu)^2 \ln(\mu/\Delta)$ [see Eq. (27)] is parametrically smaller than the masses of pseudo-Nambu-Goldstone bosons $\sim m\,(\Delta/\mu)$ in the CFL phase [36], which are important to the transport properties and neutrino emissivity of the CFL quark matter [24]. Whether such new low-energy degrees of freedom localized around the vortex line have some impact on the physics of rotating neutron/quark stars is an important problem and should be investigated in a future work.

Our results for the odd parity pairing might be irrelevant to the color superconductivity of quarks. Nevertheless it would be interesting to consider if the topological phase transition found in this paper can be realized in condensed matter systems where the 2D Dirac fermions appear.

Finally, Table I reveals the intriguing connection between the nonzero topological charge of the free space Hamiltonian and the existence of localized gapless fermions in the presence of a vortex. We also elucidated the existence of gapless surface fermions localized at a boundary when two phases with different topological charges are connected. The mathematical proof of these correspondences remains an open question.

Note added.—After the submission of this paper, there appeared a paper by S. Yasui, K. Itakura, and M. Nitta [37] which has some overlap with the present paper. In their paper, a non-Abelian vortex in the CFL phase is also considered.

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[7] Supposing $\mu \sim 500$ MeV, $\Delta \sim 50$ MeV, and $m \sim 10$ MeV and using Eq. (27), we can estimate the mass gap to be as small as $vm/\Delta \sim 5\times 10^{-3}$.

Appendix A: Derivations of solutions for even parity pairing

Here we outline how the solutions to the Bogoliubov–de Genne equation (18) for the even parity pairing are derived. We introduce notations

$$\Phi(r,\theta,z) = \begin{pmatrix} F_R \\ F_L \\ G_R \\ G_L \end{pmatrix} e^{ip_z z}, \tag{A1}$$

where $F_{R(L)}$ and $G_{R(L)}$ are right-handed (left-handed) two-component fields. We first make an ansatz

$$F_R(r,\theta) = \begin{pmatrix} f_{R\uparrow}(r) \\ e^{i\theta} f_{R\downarrow}(r) \end{pmatrix} e^{-\int_0^r |\Delta(r')|dr'} \tag{A2}$$

and

$$G_R(r,\theta) = \begin{pmatrix} e^{-i\theta} g_{R\uparrow}(r) \\ g_{R\downarrow}(r) \end{pmatrix} e^{-\int_0^r |\Delta(r')|dr'} \tag{A3}$$

and the same for $R\to L$ so that they are exponentially localized in the x-y plane. Then we look for $f_{R(L)}$ and $g_{R(L)}$ that are regular at origin and independent of $|\Delta(r)|$. $|\Delta(r)|$ can be eliminated from the equations by imposing

$$g_R = -i\sigma_1 f_R \qquad \text{and} \qquad g_L = i\sigma_1 f_L. \tag{A4}$$

Now f_R and f_L satisfy the following four sets of equations:

$$\begin{pmatrix} -\mu & \frac{1}{i}\left(\partial_r + \frac{1}{r}\right) \\ \frac{1}{i}\partial_r & -\mu \end{pmatrix} f_R = 0 \tag{A5a}$$

$$\begin{pmatrix} p_z - E & 0 \\ 0 & -p_z - E \end{pmatrix} f_R + m f_L = 0 \tag{A5b}$$

$$\begin{pmatrix} \mu & \frac{1}{i}\left(\partial_r + \frac{1}{r}\right) \\ \frac{1}{i}\partial_r & \mu \end{pmatrix} f_L = 0 \tag{A5c}$$

$$\begin{pmatrix} -p_z - E & 0 \\ 0 & p_z - E \end{pmatrix} f_L + m f_R = 0. \tag{A5d}$$

We can find consistent solutions in two cases; $m = p_z = E = 0$ [Eqs. (21) and (22)] and $\mu = 0$ [Eq. (29)].