applied for a time τ. This induces controlled rotations of the dark spins, resulting in an entangled state

$$\left(|0\rangle|\uparrow ... \uparrow\rangle + |1\rangle|\varphi_1 ... \varphi_N\rangle\right)/\sqrt{2}, \tag{2}$$

where $|\varphi_i\rangle \equiv \cos(\varphi_i)|\uparrow\rangle - i\sin(\varphi_i)|\downarrow\rangle$ with $\varphi_i = \lambda_i\tau$. This state is then used to sense the magnetic field. We assume for now $\kappa = 0$ and define $\theta_d = \xi\int_0^\tau dt\, b(t)$. Under the action of the magnetic field the state evolves to $\left(|0\rangle|\uparrow \dots\rangle + |1\rangle|\psi_1 \dots \psi_N\rangle\right)/\sqrt{2}$ with

$$\begin{aligned}|\psi_i\rangle = &\cos\varphi_i|\uparrow\rangle_i - i\sin\varphi_i e^{2i\theta_d}|\downarrow\rangle_i \\ &\approx e^{i2\theta_d sin^2\varphi_i}|\varphi_i\rangle + \theta_d\tfrac{\sin(2\varphi_i)}{2}\left|\varphi_i^\perp\right\rangle,\end{aligned}$$

where $\left\langle\varphi_i|\varphi_i^\perp\right\rangle = 0$. The central spin is then flipped by a π-pulse and another control operation with H_{int} along x is applied. If the field (and thus θ_d) were zero the interaction between central and dark spins would then be refocused, corresponding to decoupling the sensor spin from the environment as in a spin-echo. For metrology purposes, to first order, the effect of a small field is to introduce a phase difference $\Phi \approx 2\theta_d\sum_i \sin^2(\varphi_i)$ between the states of the dark spins, depending on the state of the central spin (while the terms $\propto \left|\varphi_i^\perp\right\rangle$ only contribute to second order in θ_d). After the sensor spin is rotated around the x-axis, this yields an additional contribution to the probability of finding the spin in the $|1\rangle$-state: $P_1 \approx (1 + \Phi + \theta_s)/2 + O(b^2)$ (where we reintroduced the phase acquired by the sensor spin alone, $\theta_s = \kappa\int_0^\tau dt\, b(t)$, which can be simply added as H_b^S commutes with the rest of the Hamiltonian).

While the signal is enhanced by a factor $\propto \Phi$, the quantum projection noise remains the same as we still read out one spin only. The minimum field that can be measured in a total time T is then:

$$\delta b_{min} = \sqrt{\frac{\tau}{T}}\frac{1}{\Phi + \theta_s} \approx \frac{1}{n\xi\sqrt{T\tau}}, \tag{3}$$

where n is the total number of dark spins. The linear scaling in n of the phase Φ can be achieved in principle for any distribution of λ_i's, since we can always choose a duration τ such that $\left\langle\sin^2(\lambda_i\tau)\right\rangle \geq \frac{1}{2}$, leading to order one contribution from each spin. Thus we are able to perform Heisenberg-limited spectroscopy despite the fact that the precise form of the entangled state (2) is uncontrolled, and may not even be known to us [20]. This considerably relaxes the requirements for entanglement enhanced spectroscopy as compared to known strategies involving squeezed or GHZ-like states. We next discuss two experimental implementations that approximate this idealized scheme: quantum clocks with trapped ions and spin-based magnetometry.

To reach high precision in quantum clocks, the ions must posses several characteristics: a stable clock transition, a cooling cycling transition, good initial state preparation and reliable state detection. It is then convenient to use two species of ions in the same Paul trap [24, 30]: The *spectroscopy* ions (e.g. $^{27}Al^+$) provide the clock transition while the *logic* ion (e.g. $^9Be^+$) fulfills the other requirements. Although inspired by a similar idea as the one proposed here, experiments using two ion species have been so far limited to just one spectroscopy ion [24, 28–30]; with our method the number of spectroscopy (*dark*) ions can be increased.

Specifically, by using multichromatic gates [21] one can implement the Hamiltonian H_{int} in Eq. (1) [26] (such multichromatic gates are known to be much more robust to heating noise than Cirac-Zoller gates [21] that have been used so far). We can then use the method presented here to transfer the phase difference due to the detuning of several spectroscopy ions onto a single logic ion, which can be read out by fluorescence. In principle, we achieve Heisenberg-limited sensing of the clock transition without individual addressability of the spectroscopy ions or producing GHZ states. Importantly, we achieve this even if the spectroscopy ions have different couplings to the logic ion, which will be the case in a trap with different ion species due to the absence of a common center of mass mode.

We next briefly discuss the effects of decoherence on this method. It has been argued (see e.g. [31]), that the coherence time reduction for a n-spin entangled state reduces the sensitivity to roughly that of spectroscopy performed on n individual spin, scaling as $\sqrt{n}$, as opposed to the ideal scaling $\propto n$ derived here (See Ref. [32] for a different scaling in the case of atomic clocks). Our scheme would obtain an improvement $\propto\sqrt{n}$ with respect to current experimental realizations where only a single ancillary ion is available, even in the decoherence model of Ref. [31], where each spin undergoes individual Markovian dephasing. Furthermore, this decoherence model is not so relevant in present setups, as technical noise during the gates and imperfect rotations are dominant for traps with many ions. Our method is highly robust to static repetitive imperfections during the gates, leading to further improvement depending on the exact noise specifications [26].

In many physical situations short bursts of controlled rotations, as used above, are not available. Instead, the couplings between the central and dark spins are always on and their exact strength is unknown. Examples of such systems are solid-state spin systems used for magnetometry [3, 4, 12, 33]. Still, it is possible to achieve nearly Heisenberg-limited metrology even for these systems.

Specifically, we will consider magnetic sensing using a single Nitrogen Vacancy (NV) center in diamond [8], surrounded by *dark* spins associated with Nitrogen electronic impurities [10, 11]. We focus on NV centers since their electronic spins (S=1) can be efficiently initialized into the $S_z = 0$ state by optical pumping and measured via state selective fluorescence. By applying an exter-