On the p, q-binomial distribution and the Ising model

P. H. Lundow* and A. Rosengren†

Condensed Matter Theory, Department of Theoretical Physics,

AlbaNova University Center, KTH, SE-106 91 Stockholm, Sweden

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Abstract

A completely new approach to the Ising model in 1 to 5 dimensions is developed. We employ p, q-binomial coefficients, a generalisation of the binomial coefficients, to describe the magnetisation distributions of the Ising model. For the complete graph this distribution corresponds exactly to the limit case $p = q$. We take our investigation to the simple d-dimensional lattices for $d = 1, 2, 3, 4, 5$ and fit p, q-binomial distributions to our data, some of which are exact but most are sampled. For $d = 1$ and $d = 5$ the magnetisation distributions are remarkably well-fitted by p, q-binomial distributions. For $d = 4$ we are only slightly less successful, while for $d = 2, 3$ we see some deviations (with exceptions!) between the p, q-binomial and the Ising distribution. We begin the paper by giving results on the behaviour of the p, q-distribution and its moment growth exponents given a certain parameterization of p, q. Since the moment exponents are known for the Ising model (or at least approximately for $d = 3$) we can predict how p, q should behave and compare this to our measured p, q. The results speak in favour of the p, q-binomial distribution's correctness regarding their general behaviour in comparison to the Ising model. The full extent to which they correctly model the Ising distribution is not settled though.