which can be put in standard Sturm-Liouville form. Analyzing that Sturm-Liouville eigenvalue problem is straightforward, since in particular it is known that the eigenvalues are bounded from below, and that the eigenfunction corresponding to the smallest eigenvalue has no zeros within the interval. Therefore, the question of stability in practical terms becomes the search for a solution to the Kaluza-Klein equation such that it contains no nodes. Its associated eigenvalue will be the lightest possible eigenvalue and, if positive, the system will have no classical instabilities.

In the present case, however, the Kaluza-Klein problem is a system of coupled differential equations. Consequently, matrix Sturm-Liouville techniques are required. In order to analyze stability further, one must extend the theory of oscillations and the concept of nodes of solutions to a higher dimensional problem. Such an analysis, although rather involved, is underway, and will be presented in a future work.

VI. DISCUSSION AND OUTLOOK

Braneworld theories generally lead to scalar degrees of freedom that propagate in the extra-dimensional bulk. Understanding the vacuum structure of these models in the presence of bulk scalar fields is therefore a prerequisite to fully appreciating their phenomenological possibilities. Furthermore, bulk scalars may provide a useful way to localize fermions and build braneworld models purely with field theory (e.g., fat branes and soft walls).

In this work, we have studied the vacuum structure of braneworld models with one warped extra dimension and multiple bulk scalar fields. In particular we have focused on static configurations along the extra space coordinate where one of the fields–with Dirichlet boundary conditions–acquires a nontrivial kink-like profile. To find these solutions one needs to solve both the Einstein and the scalar field equations. In the limit of a flat 5D metric and weak gravity such solutions are known to exist, and the problem of finding all possible static configurations as well as determining their perturbative stability has been addressed and solved [40, 41]. Here we have built upon this previous work to determine how warping along the extra dimension effects the existence and stability of these kink-like solutions.

When considering a fixed warped background, it was sufficient to look for nontrivial solutions for a single scalar field. In this case, neglecting any backreaction of the scalar field on the gravitational dynamics, we found that such kink-like solutions do indeed exist. As