K_1. It is actually a bit easier to see (equivalently) that πg^{-1} conjugates K_1 to K_2. Suppose that $k \in K_1$, so that

$$I_{p,2q+1,p} k I_{p,2q+1,p} = k.$$

Then since g commutes with $I_{p,2q+1,p}$, we have

$$(gI_{p,2q+1,p}g^{-1})k(gI_{p,2q+1,p}g^{-1}) = k,$$

so

$$I_{p,2q+1,p}(g^{-1}kg)I_{p,2q+1,p} = g^{-1}kg.$$

Now, since $\pi I_{p,2q+1,p}\pi = I_{2p,2q+1}$, and since $\pi^2 = \mathrm{Id}$, we have

$$(\pi I_{p,2q+1,p}\pi)(\pi g^{-1}kg\pi)(\pi I_{p,2q+1,p}\pi) = \pi g^{-1}kg\pi,$$

so

$$I_{2p,2q+1}(\pi g^{-1}kg\pi)I_{2p,2q+1} = \pi g^{-1}kg\pi.$$

This says that $\pi g^{-1}kg\pi \in K_2$. Thus $g\pi$ conjugates K_2 to K_1.

Now, with that established, given a representative $F_\bullet$ of the K_2-orbit on X given by some symmetric $(2p, 2q+1)$-clan, to get a representative of the K_1-orbit corresponding to that same clan, we just act on the flag $F_\bullet$ by the matrix $g\pi$ to get the new flag $F'_\bullet = g\pi F_\bullet$. The flag $F_\bullet$ is isotropic with respect to the diagonal form, so the flag $F'_\bullet$ is isotropic with respect to the anti-diagonal form.

Let us look at a small example which illustrates the method just described for finding an isotropic representative of the K'-orbit Q_γ corresponding to a symmetric $(2p, 2q+1)$-clan γ. Take $p = q = 1$, so that $n = 2$, and so that we are dealing with $G = SO(5, \mathbb{C})$, $K = S(O(2, \mathbb{C}) \times O(3, \mathbb{C}))$. Take the symmetric $(2, 3)$-clan $\gamma = (1, -, +, -, 1)$. None of the