On the instability of Reissner-Nordström black holes in de Sitter backgrounds

Vitor Cardoso*
CENTRA, Departamento de Física, Instituto Superior Técnico,
Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal &
Department of Physics and Astronomy, The University of Mississippi, University, MS 38677-1848, USA

Madalena Lemos† and Miguel Marques‡
CENTRA, Departamento de Física, Instituto Superior Técnico,
Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

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Recent numerical investigations have uncovered a surprising result: Reissner-Nordström-de Sitter black holes are unstable for spacetime dimensions larger than 6. Here we prove the existence of such instability analytically, and we compute the timescale in the near-extremal limit. We find very good agreement with the previous numerical results. Our results may me helpful in shedding some light on the nature of the instability.

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I. INTRODUCTION

In physics, stability of a given configuration (solution of some set of equations), is a useful criterium for relevance of that solution. Unstable configurations are likely not to be realizable in practice, and represent an intermediate stage in the evolution of the system. Nevertheless, the instability itself is of great interest, since an understanding of the mechanism behind it may help one to better grasp the physics involved. In particular, it is of interest to be able to predict which other systems display similar instabilities, or even have a deeper understanding of the physics behind the instability (why is the system unstable? is there some fundamental principle behind the instability?).

In General Relativity, the Kerr family exhausts the black hole solutions to the electro-vac Einstein equations. Kerr black holes are stable, and can therefore describe astrophysical objects. However, there are many instances of instabilities afflicting objects with an event horizon, such as the Gregory-Laflamme [1], the ultra-spinning [2] or superradiant instabilities [3] and other instabilities of higher-dimensional black holes in alternative theories [4, 5](for a review see Ref. [6]).

Konoplya and Zhidenko (hereafter KZ) recently studied small perturbations in the vicinity of a charged black hole in de Sitter background, a Reissner-Nordström de Sitter black hole (RNdS) [7]. Their (numerical) results show that when the spacetime dimensionality $D > 6$, the spacetime is unstable, provided the charge is larger than a given threshold, determined by KZ for each D. Because the results are so surprising (the mechanism behind it is not yet understood), we set out to to investigate this instability and hopefully understand it better. Our results can be summarized as follows: (i) we can prove analytically the existence of unstable modes for charge Q higher than a certain threshold. (ii) in the near-extremal regime, we are able to find an explicit solution for the unstable modes, determining the instability timescale analytically. We hope that our incursion in this topic helps to better understand the physics at work.

II. EQUATIONS

This work focuses on the higher dimensional RNdS geometry, described by the line element

$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2 d\Omega_n^2\,, \tag{1}$$

where $d\Omega_n^2$ is the line element of the n sphere and

$$f = 1 - \lambda\, r^2 - \frac{2M}{r^{n-1}} + \frac{Q^2}{r^{2n-2}}\,. \tag{2}$$

the background electric field is $E_0 = q/r^n$, with q the electric charge. The quantities M and Q are related to the physical mass M and charge q of the black hole [8], and λ to the cosmological constant. The spacetime dimensionality is $D = n + 2$.

The above geometry possesses three horizons: the black-hole Cauchy horizon at $r = r_a$, the black hole event horizon is at $r = r_b$ and the cosmological horizon is at $r = r_c$, where $r_c > r_b > r_a$, the only real, positive zeroes of f. For convenience, we set $r_b = 1$, i.e., we measure all quantities in terms of the event horizon r_b. We thus get

$$2M = 1 + Q^2 - \lambda\,, \tag{3}$$

*Electronic address: vitor.cardoso@ist.utl.pt
†Electronic address: madalena.dal@gmail.com
‡Electronic address: miguel.e.marques@gmail.com