Chapter 1

Introduction and Preliminaries

Let G be a connected, complex, simple algebraic group of classical type. Let θ be a (holomorphic) involution of G - that is, θ is an automorphism of G whose square is the identity. Fix $T \subseteq B$, a θ-stable maximal torus and Borel subgroup of G, respectively. Let $K = G^\theta$ be the subgroup of elements of G which are fixed by θ. Such a subgroup of G is referred to as a *symmetric subgroup.*

K acts on the flag variety G/B with finitely many orbits ([Mat79]), and the geometry of these orbits and their closures plays an important role in the theory of Harish-Chandra modules for a certain real form $G_{\mathbb{R}}$ of the group G — namely, one containing a maximal compact subgroup $K_{\mathbb{R}}$ whose complexification is K. For this reason, the geometry of K-orbits and their closures have been studied extensively, primarily in representation-theoretic contexts.

Their role in the representation theory of real groups aside, K-orbit closures can be thought of as generalizations of Schubert varieties, and, in principle, any question one has about Schubert varieties may also be posed about K-orbit closures. With this in mind, we note here that our work is motivated by earlier work of Fulton ([Ful92, Ful96b, Ful96a]) on Schubert loci in flag bundles, their role as universal degeneracy loci of maps of flagged