The final result for the real time noise correlator is then:

$$S^{\beta\beta'}(t,t') = \frac{(e^*)^2}{4\pi^2 a^2} e^{2\nu G^{\beta\beta'}(t-t')} \left(A(t)A^*(t') + A^*(t)A(t')\right) \ , \tag{36}$$

where we introduced the chiral green function of the bosonic fields:

$$G^{\beta\beta'}(t,t') = \langle T_K\{\phi_r(t^\beta)\phi_r(t'^{\beta'})\}\rangle - \frac{1}{2}\langle T_K\{\phi_r(t^\beta)^2\}\rangle - \frac{1}{2}\langle T_K\{\phi_r(t'^{\beta'})^2\}\rangle \ . \tag{37}$$

The double Fourier transform of this quantity, which will allow to relate it to the noise correlator, reads:

$$S^{\beta\beta'}(\Omega_1,\Omega_2) = \int\int dtdt' e^{i(\Omega_1 t+\Omega_2 t')} S(t,t') \ . \tag{38}$$

We now specify the periodic voltage modulation, which allows to write the tunneling amplitude in terms of a series of Bessel functions J_n:

$$A(t) = \Gamma_0 \sum_{n=-\infty}^{+\infty} e^{i(\omega_0+n\omega_{AC})t} J_n\left(\frac{e^*V_1}{\hbar\omega_{AC}}\right) \ , \tag{39}$$

which gives the Fourier transform of non-symmetrised noise:

$$\begin{aligned} S^{\beta\beta'}(\Omega_1,\Omega_2) \ &= \ \frac{(e^*)^2\Gamma_0^2}{2\pi^2a^2} \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} J_n\left(\frac{e^*V_1}{\omega_{AC}}\right) J_m\left(\frac{e^*V_1}{\omega_{AC}}\right) \\ &\times \int\int dtdt' e^{i(\Omega_1 t+\Omega_2 t')} e^{2\nu G^{\beta\beta'}(t,t')} \cos(\omega_0(t-t') + \omega_{AC}(nt-mt')) \ . \end{aligned} \tag{40}$$

Next, it is convenient to perform a change of variable $\tau = t-t'$ and $\tau' = t+t'$:

$$\begin{aligned} S^{\beta\beta'}(\Omega_1,\Omega_2) \ &= \ 2\frac{(e^*)^2\Gamma_0^2}{2\pi^2a^2} \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} J_n\left(\frac{e^*V_1}{\omega_{AC}}\right) J_m\left(\frac{e^*V_1}{\omega_{AC}}\right) \\ &\times \int\int d\tau d\tau' e^{i(\Omega_1-\Omega_2)\tau/2} e^{i(\Omega_1+\Omega_2)\tau'/2} e^{2\nu G^{\beta\beta'}(\tau)} \cos\left(\left(\omega_0 + \frac{n+m}{2}\omega_{AC}\right)\tau + \frac{n-m}{2}\omega_{AC}\tau'\right) \ . \end{aligned}$$

Using standard trigonometric identities, one can write this expression as a product of separate integrals over τ and τ'. Integrals over τ contain the (zero temperature) Green's function of the chiral fields and can be expressed in terms of Gamma function. The result has the form:

$$\begin{aligned} S^{\beta\beta'}(\Omega_1,\Omega_2) \ &= \ 2\frac{(e^*)^2\Gamma_0^2}{2\pi^2a^2} \sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} J_n\left(\frac{e^*V_1}{\omega_{AC}}\right) J_m\left(\frac{e^*V_1}{\omega_{AC}}\right) \\ &\left[I_1(\Omega_1+\Omega_2,\omega) I_2^{\beta\beta'}(\Omega_1-\Omega_2,\omega_0,\omega) - I_3(\Omega_1+\Omega_2,\omega) I_4^{\beta\beta'}(\Omega_1-\Omega_2,\omega_0,\omega)\right] \ . \end{aligned} \tag{41}$$

The integrals I_1, $I_2^{\beta\beta'}$, I_3, $I_4^{\beta\beta'}$ are defined and computed in the Appendix. The final result for the 4 Keldysh matrix elements of the noise correlator is:

$$\begin{aligned} S^{\beta-\beta}(\Omega_1,\Omega_2) = 2\frac{(e^*)^2\Gamma_0^2}{4\pi^2a^2} &\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} J_n\left(\frac{e^*V_1}{\omega_{AC}}\right) J_m\left(\frac{e^*V_1}{\omega_{AC}}\right) \frac{\pi}{\Gamma(2\nu)}\left(\frac{a}{\nu_F}\right)^{2\nu} \times \\ &\Big[(1-\beta\,\mathrm{sgn}\,(\Omega_1+\omega_0+n\omega_{AC}))\left|\Omega_1+\omega_0+n\omega_{AC}\right|^{2\nu-1}\delta(\Omega_1+\Omega_2+(n-m)\omega_{AC}) \\ &+ \ (1-\beta\,\mathrm{sgn}(\Omega_1-\omega_0-n\omega_{AC}))\left|\Omega_1-\omega_0-n\omega_{AC}\right|^{2\nu-1}\delta(\Omega_1+\Omega_2-(n-m)\omega_{AC})\Big] \ , \end{aligned} \tag{42}$$

$$\begin{aligned} S^{\beta\beta}(\Omega_1,\Omega_2) = 2\frac{(e^*)^2\Gamma_0^2}{4\pi^2a^2} &\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} J_n\left(\frac{e^*V_1}{\omega_{AC}}\right) J_m\left(\frac{e^*V_1}{\omega_{AC}}\right) \frac{\pi}{\Gamma(2\nu)}\left(\frac{a}{\nu_F}\right)^{2\nu} \frac{e^{-\beta i\pi\nu}}{\cos(\pi\nu)} \times \\ &\times\left[\left|\Omega_1+\omega_0+n\omega_{AC}\right|^{2\nu-1}\delta(\Omega_1+\Omega_2+(n-m)\omega_{AC}) + \left|\Omega_1-\omega_0-n\omega_{AC}\right|^{2\nu-1}\delta(\Omega_1+\Omega_2-(n-m)\omega_{AC})\right] \ . \end{aligned} \tag{43}$$