the $H^{\pm}$-mediated transition matrix element combined with SM contribution for $\bar{P} \to \ell \bar{\nu}_\ell$ is obtained as

$$\mathcal{M}^{SM+H^{\pm}}_{\bar{P}\to\ell\bar{\nu}_\ell} = -i\frac{G_F}{\sqrt{2}} V_{ij} m_\ell f_P r^\ell_P (\bar{\ell}\nu_\ell)_{S-P} \tag{16}$$

with

$$r^\ell_P = 1 + \frac{\eta^*_\ell m^2_P}{m^2_{H^{\pm}}} \frac{\eta^*_{U_i} m_{U_i} - \eta_{D_j} m_{D_j}}{m_{U_i} + m_{D_j}} \,. \tag{17}$$

We learn that the $H^{\pm}$-mediated contribution is only associated with the factor r^ℓ_P and it depends on the species of lepton due to the appearance of η_ℓ. Since η_{U_i}, η_{D_j} and η_ℓ are all free parameters, in order to make the results be more predictive, we can adopt a simple scenario. As mentioned earlier, η_{U_i} and $\eta_{D_i,\ell}$ play the role of $\cot\beta$ and $\tan\beta$ in the type-II THDM, respectively. If the new $H^{\pm}$-mediated effects would like to satisfy the constraints of current data such as $b \to s\gamma$, it is plausible to set $|\eta_{U_i}| \ll |\eta_{D_i}| \approx |\eta_\ell|$ [25]. As a consequence, r^ℓ_P could be simplified by

$$r^\ell_P \approx 1 - \frac{m^2_P}{m^2_{H^{\pm}}} \frac{m_{D_j}}{m_{U_i} + m_{D_j}} |\eta_{D_j}|^2 e^{i\phi^\ell_{D_j}} \,. \tag{18}$$

Intriguingly, in this plain scenario we see that the dependence of lepton flavor in r^ℓ_P can be ascribed to the phase factor $\phi^\ell_{D_j}$. Since $\phi^\ell_{D_j}$ are the new physical phases, in general, they cannot be rotated away. If we enforce $\eta_{D_i} = \tan\beta$, we see that the magnitude of charged Higgs effects is the same as that in type-II THDM. In other words, apart from the new phase factor $\phi^\ell_{D_j}$, we do not introduce a new enhanced factor.

In order to display the new physics effects numerically, we investigate the influence of charged Higgs on R_K for $K_{\ell 2}$, on $f_{D_s} r^\ell_{D_s}$ for $D_s \to \ell^+ \nu_\ell$ decays and on BR for $B \to \ell^+ \nu_\ell$, respectively. Using Eqs. (9) and (16), the ratio of $\Gamma(K^{\pm}_{e2})$ to $\Gamma(K^{\pm}_{\mu 2})$ can be expressed by

$$R_K = R^{\rm SM}_K \left(1 + \frac{m^2_K}{m^2_{H^{\pm}}} \frac{m_s}{m_u + m_s} \eta^2_s \Delta c^{\mu e}_s \right) \tag{19}$$

with $\Delta c^{\mu e}_s = \cos(\phi^\mu_s) - \cos(\phi^e_s)$, where because of the second term in the brackets being much smaller than unity, we have neglected the terms whose the order is higher than $m^2_K \eta^2_s / m^2_{H^{\pm}}$. The resulted numerical values as a function of $\eta_s / m_{H^{\pm}}$ and $\Delta c^{\mu e}_s$ are presented in Fig. 2. The values in the figure denote the ratio $R^{\rm Exp}_K / R^{\rm SM}_K$. We see clearly that due to the lepton flavor dependent phases, $H^{\pm}$-mediated contributions could modify the SM prediction and be still consistent with current data.