Taken together, the last two claims give us the curvature as a function of arclength. This determines γ up to translations and rotations. ■

4.8 Numerical experiments

In this section, we consider a numerical curve reconstruction for the situation in which $g(s,r)$ is known for a given radius r but no derivative information is available. This reconstruction is more strict than the scenarios of sections 4.5–4.7. Our motivation is to explore whether any γ can be uniquely and practically reconstructed with this limited information.

We consider $\gamma_a(\bar{s}) \in \mathcal{P}^N$, the set of *simple* polygons of N ordered vertices $\{(x_1, y_1), \ldots, (x_N, y_N)\}$ parameterized by the set $\{\bar{s}_k\}_{k=1}^N$ with $\bar{s}_k = k/N$ as

$$\begin{aligned} x_k &= \sum_{j=0}^{m-1} a_{1,j} \cos(2\pi j \bar{s}_k/N) + a_{2,j} \sin(2\pi j \bar{s}_k/N), \\ y_k &= \sum_{j=0}^{m-1} a_{3,j} \cos(2\pi j \bar{s}_k/N) + a_{4,j} \sin(2\pi j \bar{s}_k/N), \end{aligned} \tag{4.1}$$

for some coefficients $a_{i,j} \in \mathbb{R}$. In this way, the polygon γ is a discrete approximation of a C^∞ curve. The sides of $\gamma_a(\bar{s})$ are not necessarily of equal length.

We take the vector signature $g_a(\bar{s}, r) \in \mathbb{R}^N$ to be the discrete area densities of $\gamma_a(\bar{s})$ computed at each vertex. Given such a signature for fixed radius r and fixed partition $\bar{s}$, we seek a^* satisfying

$$\begin{aligned} a^* \in \quad & \arg\min_{b \in \mathbb{R}^{4m}} \|g_b(\bar{s}, r) - g_a(\bar{s}, r)\|_2^2 \\ & \text{s.t. } \gamma_b \in \mathcal{P}^N \end{aligned} \tag{4.2}$$