advantage of the localization theorem. However, the equivariant fundamental classes of K-orbit closures in fact live in K-equivariant cohomology $H_K^*(G/B)$. (In the event that K is disconnected, this should be interpreted as $H_{K^0}^*(G/B)$, where K^0 denotes the identity component of K.) Indeed, for a K-orbit closure Y, the S-equivariant class $[Y]_S$ is simply the image $\pi^*([Y]_K)$ under the pullback by the natural map

$$\pi : E \times^S (G/B) \to E \times^K (G/B).$$

It is a basic fact about equivariant cohomology that this pullback is injective, and embeds $H_K^*(G/B)$ in $H_S^*(G/B)$ as the W_K-invariants ([Bri98]). Thus $H_K^*(G/B)$ is a subring of $H_S^*(G/B)$, and the S-equivariant fundamental classes of K-orbit closures live in this subring.

Now, $H_K^*(G/B)$ is, by definition, the cohomology of the space $E \times^K (G/B)$, and this space is easily seen to be isomorphic to the fiber product $BK \times_{BG} BB$. (The argument is identical to that given in the proof of Proposition 1.2.1 to show that $E \times^S (G/B) \cong BS \times_{BG} BB$ — simply replace S by K.)

Now, suppose that X is a scheme, and that $V \to X$ is a complex vector bundle of rank n. In type A, no further structure on V is presumed, while in types BCD, V is assumed to be equipped with an orthogonal (BD) or symplectic (C) form. In any event, we have a classifying map $X \xrightarrow{\rho} BG$ such that V is the pullback $\rho^*(\mathcal{V})$, where $\mathcal{V} = E \times^G \mathbb{C}^n$ is a universal vector bundle over BG, with $\mathbb{C}^n$ carrying the natural representation of G.

For any closed subgroup H of G, $BH \to BG$ is a fiber bundle with fiber isomorphic to G/H. A lift of the classifying map ρ to BH corresponds to a reduction of structure group to H of the bundle V. Such a reduction of structure group can often be seen to amount to some additional structure on V. For instance, in type A, reduction of the structure group of V from $GL(n, \mathbb{C})$ to the Borel subgroup B of upper-triangular matrices is well-known to be equivalent to V being equipped with a complete flag of subbundles. (In Types BCD, this