

functions considered to a bounded  $\bar{W}$ . If the features of interest are associated with low-frequency components only, then if the functions are restricted to a bounded space the low-frequency part can be identified and is well-posed.

## 5 Implications for estimation

### 5.1 Plug-in non-parametric estimation

Solutions in Table 5 for the equations that express the unknown functions via known functions of observables give scope for plug-in estimation. As seen e.g. in the example of Model 4, 4 and 4a are different expressions that will provide different plug-in estimators for the same functions.

The functions of the observables here are characteristic functions and Fourier transforms of density-weighted conditional expectations and in some cases their derivatives, that can be estimated by non-parametric methods. There are some direct estimators, e.g. for characteristic functions. In the space  $S^*$  the Fourier transform and inverse Fourier transform are continuous operations thus using standard estimators of density weighted expectations and applying the Fourier transform would provide consistency in  $S^*$ ; the details are provided in Zinde-Walsh (2012). Then the solutions can be expressed via those estimators by the operations from Table 5 and, as long as the problem is well-posed, the estimators will be consistent and the convergence will obtain at the appropriate rate. As in An and Hu (2012), the convergence rate may be even faster for well-posed problems in  $S^*$  than the