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Appendix A: Keldysh noise correlator calculation

From Eq. (41) we use a standard trigonometric identity in order to factorize the noise into contributions with τ and τ':

$$\begin{aligned} S^{\beta\beta'}(\Omega_1,\Omega_2) \;&=\; 2\frac{(e^*)^2\Gamma_0^2}{2\pi^2 a^2}\sum_{n=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty} J_n\left(\frac{e^*V_1}{\omega_{AC}}\right)J_m\left(\frac{e^*V_1}{\omega_{AC}}\right) \\ &\left[I_1(\Omega_1+\Omega_2,\omega)I_2^{\beta\beta'}(\Omega_1-\Omega_2,\omega_0,\omega)-I_3(\Omega_1+\Omega_2,\omega)I_4^{\beta\beta'}(\Omega_1-\Omega_2,\omega_0,\omega)\right]\;, \end{aligned} \tag{A1}$$

with:

$$\begin{aligned} I_1(\Omega_1+\Omega_2,\omega) \;&=\; \int_{-\infty}^{+\infty} d\tau' e^{i(\Omega_1+\Omega_2)\tau'/2}\cos\left(\frac{n-m}{2}\omega_{AC}\tau'\right) \\ I_2^{\beta\beta'}(\Omega_1-\Omega_2,\omega_0,\omega) \;&=\; \int_{-\infty}^{+\infty} d\tau e^{i(\Omega_1-\Omega_2)\tau/2}e^{2\nu G^{\beta\beta'}(\tau)}\cos\left(\left(\omega_0+\frac{n+m}{2}\omega_{AC}\right)\tau\right) \\ I_3(\Omega_1+\Omega_2,\omega) \;&=\; \int_{-\infty}^{+\infty} d\tau' e^{i(\Omega_1+\Omega_2)\tau'/2}\sin\left(\frac{n-m}{2}\omega_{AC}\tau'\right) \\ I_4^{\beta\beta'}(\Omega_1-\Omega_2,\omega_0,\omega) \;&=\; \int_{-\infty}^{+\infty} d\tau e^{i(\Omega_1-\Omega_2)\tau/2}e^{2\nu G^{\beta\beta'}(\tau)}\sin\left(\left(\omega_0+\frac{n+m}{2}\omega_{AC}\right)\tau\right)\;, \end{aligned} \tag{A2}$$

with the elements of the Keldysh Green's function for the chiral field:

$$G^{\beta\beta}(\tau) \;=\; -ln\left(1+\beta i\frac{\nu_F|\tau|}{a}\right) \tag{A3}$$

$$G^{\beta-\beta}(\tau) \;=\; -ln\left(1-\beta i\frac{\nu_F\tau}{a}\right)\;. \tag{A4}$$

I_1 and I_3 are expressed in terms of delta functions:

$$I_1=\frac{1}{2}\left(\delta(\Omega_1+\Omega_2+(n-m)\omega_{AC})+\delta(\Omega_1+\Omega_2-(n-m)\omega_{AC})\right)\;. \tag{A5}$$

$$I_3=\frac{1}{2i}\left(\delta(\Omega_1+\Omega_2+(n-m)\omega_{AC})-\delta(\Omega_1+\Omega_2-(n-m)\omega_{AC})\right)\;. \tag{A6}$$

Integrals $I_2^{\beta\beta'}$ and $I_4^{\beta\beta'}$ depend explicitly on the Keldysh indices β and β'. Here, we need two tabulated integrals:

$$\int_{-\infty}^{+\infty}\frac{\sin(\omega_0\tau)d\tau}{\left(\frac{a}{v_F}-i\eta\tau\right)^{\mu}} \;\approx\; i\pi\eta\,\mathrm{sgn}(\omega_0)\frac{|\omega_0|^{\mu-1}}{\mathbf{\Gamma}(\mu)} \tag{A7}$$

$$\int_{-\infty}^{+\infty}\frac{\cos(\omega_0\tau)d\tau}{\left(\frac{a}{v_F}-i\eta\tau\right)^{\mu}} \;\approx\; \pi\frac{|\omega_0|^{\mu-1}}{\mathbf{\Gamma}(\mu)}\;. \tag{A8}$$

The results for $I_2^{\beta\beta'}$ and $I_4^{\beta\beta'}$ are:

$$\begin{aligned} I_2^{\beta-\beta}=\frac{\pi}{2\Gamma(2\nu)}(\frac{a}{\nu_F})^{2\nu}&\left[\left(1-\beta\mathrm{sgn}\left(\frac{\Omega_1-\Omega_2}{2}-\omega_0-\frac{n+m}{2}\omega_{AC}\right)\right)\left|\frac{\Omega_1-\Omega_2}{2}-\omega_0-\frac{n+m}{2}\omega_{AC}\right|^{2\nu-1}\right. \\ &\left.+\left(1-\beta\mathrm{sgn}\left(\frac{\Omega_1-\Omega_2}{2}+\omega_0+\frac{n+m}{2}\omega_{AC}\right)\right)\left|\frac{\Omega_1-\Omega_2}{2}+\omega_0+\frac{n+m}{2}\omega_{AC}\right|^{2\nu-1}\right]\;. \end{aligned} \tag{A9}$$