where $q^a = (\rho_d + p_d)v_d^a$ is the energy-flux of DE observed by the observer rested in the DM frame. In the above equation, we have already neglected the second order terms of v_d^a and assumed that the energy-flux velocity is far less than the speed of light $v_d^a \ll 1, \gamma \sim 1$. In the spherical model, we define the top-hat radius as the radius of the boundary of DM. When DE does not trace DM, DE will not be bounded inside the top-hat radius and will get out of the spherical region. We assume that the leakage of DE is still spherically symmetric and from the Birkhoff theorem, in the spherical region, the Raychaudhuri's equation takes the same form as Eqs. (24). For the energy density conservation law, we have,

$$\nabla_a T^{ab}_{(\lambda)} = Q^b_{(\lambda)} \tag{33}$$

where Q^b is the coupling vector as illustrated in Eqs. 2 and "λ" denotes DE and DM respectively. The timelike parts of the above equation $u_b Q^b_{(\lambda)}$ give

$$\begin{aligned} \dot{\rho}_m^{cluster} + 3h\rho_m^{cluster} &= 3H(\xi_1\rho_m^{cluster} + \xi_2\rho_d^{cluster}) \\ \dot{\rho}_d^{cluster} + 3h(1+w)\rho_d^{cluster} &= -\vartheta(1+w)\rho_d^{cluster} - 3H(\xi_1\rho_m^{cluster} + \xi_2\rho_d^{cluster}) \end{aligned} \tag{34}$$

where $\vartheta = \nabla_x v_d$. The external term incorporating ϑ in the DE density evolution indicates the energy loss caused by the leakage of DE out of the spherical region.

For the spacelike part, only DE has non-zero spatial component and $h_b^a\ Q_d^b$ gives

$$\dot{q}^a_{(d)} + 4hq^a_{(d)} = 0 \tag{35}$$

where $q^a_{(d)}$ is the dark energy-flux. Assuming that the energy and pressure are distributed homogenously, we obtain,

$$\dot{\vartheta} + h(1-3w)\vartheta = 3H(\xi_1\Gamma + \xi_2)\vartheta \tag{36}$$

where $\Gamma = \rho_m^{cluster}/\rho_d^{cluster}$and we have already used Eqs. (34) and kept linear order terms of ϑ. From Eqs. (36), if ϑ vanishes initially, ϑ will keep to be zero at all the time during the evolution, DE will fully trace DM. However, in most cases, even in linear region there is a small difference between v_d and v_m that the initial condition for ϑ is non-zero. We take the initial conditions for ϑ as $\vartheta \sim k(v_d - v_m) \sim -\delta_{mi}/200 < 0$, which is obtained from the prediction of linear equation with $k = 1\text{Mpc}^{-1}$ at $z_i = 3200$. ϑ is much smaller than 1, $|\vartheta| << 1$, and the negative value of ϑ means that at the initial moment DM expanded faster than that of DE.

Defining $\zeta_m = \rho_m^{cluster}/\rho_m$, $\zeta_d = \rho_d^{cluster}/\rho_d$ and converting the time derivative from $\frac{d}{dt}$ to $\frac{d}{da}$ we have the evolution