is a strategy profile for which another finite number of linear inequalities (inequalities that assert that this profile is an ϵ-equilibrium) is satisfied. More formally, Theorem 2.5 is a first order sentence in the language of real numbers with addition (without multiplication)[16].

However, the proof of Theorem 2.5 is based on Nash's theorem of existence of mixed equilibrium, and therefore on Brouwer's fixed point theorem. It may seem dubious that an appeal to such powerful theorems is required (note that the much more trivial Proposition 2.3 clearly does not rely on a fixed point argument). To show the non-triviality of the bound on the Lipschitz constant in Theorem 2.5, we show below that in fact Theorem 2.5 also implies Nash's theorem via an elementary argument. A similar argument appears in Schmeidler [20, pp. 298-299] for the non-atomic setup.

Claim. Assume Theorem 2.5. Then every finite normal-form game admits a mixed strategy Nash equilibrium.

Sketch of Proof. Fix a normal form game G with n players and strategy sets $A_1, \ldots, A_n$. Let m be such that $|A_i| \leq m$ for every i, and fix $\epsilon > 0$. Let L be a sufficiently large integer and consider the game G' with $n \cdot L$ players divided into groups $(T_1, \ldots, T_n)$ of size L each, where players in T_i have strategy sets A_i. In the game G' every n-tuple of players $(t_1, \ldots, t_n)$ where $t_i \in T_i$ play the game G, and each player must use the same strategy in all the games in which he participates. The payoff to a player is the average of all the payoffs he receives. If δ is the Lipschitz constant of the original game G, then the Lipschitz constant of G' is at most δ/L. Thus, the game G' has nL players and for sufficiently large L its Lipschitz constant is smaller than $\epsilon/\sqrt{8nL\log(2mnL)}$, and therefore by Theorem 2.5 G' admits a pure ϵ-equilibrium. If $\mu_i \in \Delta(A_i)$ is the distribution of strategies played by players in T_i according to the pure ϵ-equilibrium profile of G', then $(\mu^1, \ldots, \mu^n)$ is a mixed ϵ-equilibrium in G. Thus, we proved that G admits a mixed ϵ-equilibrium for every $\epsilon > 0$. An accumulation point of these ϵ-equilibria is a mixed Nash equilibrium of G. $\square$