and, in particular, if they equal to zero, the model reduces to the ordinary closed FRW model.

The Hilbert action for the model, including the massless scalar field minimally coupled to gravity, is given by

$$S_H = \int \sqrt{-g}(R - \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu})d^4x, \tag{7}$$

where μ and ν runs from 1 to 4 and $\sqrt{-g} = N\sqrt{h}$. If one assume that the scalar field is spatially homogeneous, the Lagrangian for the model in terms of the Misner variables becomes

$$L = \frac{-6e^{3\alpha}}{N}(\dot{\alpha}^2 - \dot{\beta}_+^2 - \dot{\beta}_-^2) + \frac{Ne^{\alpha}}{2}V(\beta_+, \beta_-) - \frac{e^{3\alpha}}{2N}\dot{\phi}^2, \tag{8}$$

where

$$V(\beta_+, \beta_-) = e^{-8\beta_+} - 4e^{-2\beta_+}\cosh(2\sqrt{3}\beta_-) + 2e^{4\beta_+}(\cosh(4\sqrt{3}\beta_-) - 1). \tag{9}$$

For the Hamiltonian formulation of the theory, let's determine the conjugate momenta to the dynamical variables α, β_+, β_-, ϕ, and N:

$$P_\alpha = \frac{\partial L}{\partial \dot{\alpha}} = \frac{-12e^{3\alpha}\dot{\alpha}}{N}, \tag{10}$$

$$P_+ = \frac{\partial L}{\partial \dot{\beta}_+} = \frac{12e^{3\alpha}\dot{\beta}_+}{N}, \tag{11}$$

$$P_- = \frac{\partial L}{\partial \dot{\beta}_-} = \frac{12e^{3\alpha}\dot{\beta}_-}{N}, \tag{12}$$

$$P_\phi = \frac{\partial L}{\partial \dot{\phi}} = -\frac{e^{3\alpha}\dot{\phi}}{N}, \tag{13}$$

$$P_N = \frac{\partial L}{\partial \dot{N}} = 0. \tag{14}$$

Obviously, the variable N is not a canonical variable as its canonical conjugate momenta is zero. The canonical Hamiltonian can then be written as

$$H_c = \frac{e^{-3\alpha}N}{24}(-P_\alpha^2 + P_+^2 + P_-^2) - \frac{Ne^{\alpha}V(\beta_+, \beta_-)}{2} - \frac{Ne^{-3\alpha}{P_\phi}^2}{2}, \tag{15}$$

with the total Hamiltonian to be

$$H_t = H_c + \lambda P_N. \tag{16}$$

Since $P_N \approx 0$ is a primary constraint, preservation of this constraint over time yields a secondary constraint (called Hamiltonian constraint),

$$\mathcal{H} = \frac{e^{-3\alpha}}{24}[P_\alpha^2 - P_+^2 - P_-^2 + 12e^{4\alpha}V(\beta_+, \beta_-)] + \frac{e^{-3\alpha}{P_\phi}^2}{2} \approx 0. \tag{17}$$