For the magnetic moment of the neutron in units of nuclear magnetons, we thus find

$$\mu_n^{\text{conn}}(m_\pi = 390\,\texttt{MeV}) = -1.63(10)(4)(5)\,[\mu_N].$$

We have appended a superscript to reflect that our computation includes only connected contributions. The three uncertainties quoted are from: (i) statistics and fitting, (ii) the systematic due to the fit window, and (iii) conversion to units of physical nuclear magnetons. For (i), we take the largerst value of the uncertainty from the two fits to the field-strength dependence (which both gave the same value for μ^{latt}). There are additional sources of systematic uncertainty that we have not unaccounted for, namely the effects of finite lattice spacing and finite lattice volume.

To convert the lattice electric polarizability to physical units, we compare the fit function in lattice units, Eq. (21), to the energy in physical units, Eq. (3), and find

$$\alpha_E = \frac{e^2}{2\pi} a_t a_s^2\, \alpha_E^{\text{latt}} = 0.0776(58) \times \alpha_E^{\text{latt}} \times 10^{-4}\texttt{fm}^3, \tag{26}$$

where the uncertainty arises from scale setting, and is specifically three times the uncertainty in the lattice spacing. For the neutron electric polarizability, we thus find

$$\alpha_E^{n\,\text{conn}}(m_\pi = 390\,\texttt{MeV}) = 3.3(1.5)(2)(3) \times 10^{-4}\texttt{fm}^3,$$

taking the central value and uncertainties from fit II. The notation and sources of uncertainty on the electric polarizability are as for the magnetic moment.

B. Proton

For the proton, we perform a similar analysis. Fits to proton correlation functions are carried out using a two-state fit function. This allows us to remove excited state contamination. We perform simultaneous time-correlated fits to both boost projected proton correlation functions using the fit function

$$\begin{aligned}\mathcal{G}_\pm(t,n) \;=\; & Z(n)\left[1 \pm \tilde{\mu}^{\text{latt}}(n)\,\frac{\mathcal{E}^{\text{latt}}}{\xi}\right] D\left(t, E(n)^2\left[1-\left(\tilde{\mu}^{\text{latt}}(n)\,\frac{\mathcal{E}^{\text{latt}}}{\xi}\right)^2\right] \mp \frac{\mathcal{E}^{\text{latt}}}{\xi}, \frac{\mathcal{E}^{\text{latt}}}{\xi}\right) \\ & + Z'(n)\left[1 \pm \tilde{\mu}'^{\,\text{latt}}(n)\,\frac{\mathcal{E}^{\text{latt}}}{\xi}\right] D\left(t, E'(n)^2\left[1-\left(\tilde{\mu}'^{\,\text{latt}}(n)\,\frac{\mathcal{E}^{\text{latt}}}{\xi}\right)^2\right] \mp \frac{\mathcal{E}^{\text{latt}}}{\xi}, \frac{\mathcal{E}^{\text{latt}}}{\xi}\right),\end{aligned} \tag{27}$$

with $D(x, E^2, \mathcal{E})$ as the relativistic propagator function given in Eq. (12). As the overall amplitudes $Z(n)$ and $Z'(n)$ enter the fit function linearly, we utilize variable projection to eliminate them from the simultaneous fits. For each value of the electric field $\mathcal{E}$ (or equivalently the integer n), there are then four parameters in the fit: the ground state rest energy, $E(n)$, the ground state anomalous magnetic coupling, $\tilde{\mu}(n)$, as well the rest energy

netons, $\mu_N^{\text{latt}} = \frac{e}{2M}$, with M as the lattice value of the nucleon mass. With these units, there is no uncertainty from scale setting, however, they introduce additional pion mass dependence of the extracted moment.