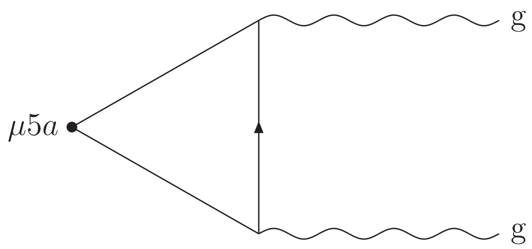

Figure 1.6: One of the diagrams that leads to the axial vector anomaly in QCD

independent U(1) symmetries for both the left and right handed symmetries. These phases always cancel out since the fermions are always bilinear. The presence of the non-zero VEV (1.33) also breaks this classical $U(1)_L \otimes U(1)_R$ symmetry to leave a single $U(1)_V$ symmetry. And once again, Goldstone's theorem applies: a spontaneously broken symmetry implies the presence of a massless boson. Here the broken symmetry has one only one generator, so we'd expect one massless boson. A look at the known QCD mass spectrum shows that the only suitable candidate, in terms of quantum numbers, is the $\eta'(958)$ meson. However it has a disappointingly high mass. Even allowing for the small masses of the quarks, we'd expect our boson to have a mass similar to that of the pions.

The explanation of this dichotomy puzzled physicists for a long while, and wasn't explained until 1986 [31]. It is quite easy to show [14, 32] that due to the necessity of regularization and renormalization (in particular triangle diagrams such as figure 1.6) the classical axial symmetry of QCD ($\partial_\mu j^{\mu 5} = 0$) is broken by quantum effects to

$$\partial_\mu j^{\mu 5} = -\frac{e^2}{16\pi^2} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \tag{1.40}$$

The obvious question to ask is what is the source of such an anomaly, and the answer lies in the complicated field of instantons [31–34]. There are