Then, from Eq. (37), we derive an expression for the electric charge as a function of the model parameters:

$$\frac{1}{e^2} = \frac{2\pi R}{g_5^2} + \frac{1}{\tilde{g}^2} + \frac{1}{\tilde{g}'^2}. \tag{44}$$

The actual profiles and masses can of course only be obtained by specifying the warp factor $b(y)$. However, it is possible to write, in general, the equations from (14) to (32)) in a more compact form. In fact, equations of motion (15), (23) and (29) all have the same form, $\hat{D}f = -m^2 f$. This is a second order ODE, so it admits two independent solutions. Following ref. [66], we can introduce two convenient linear combinations $C(y, m_n)$ and $S(y, m_n)$ ("warped sine and cosine") such that

$$C(0,m) = 1, \quad \partial_y C(0,m) = 0; \qquad S(0,m) = 0, \quad \partial_y S(0,m) = m \tag{45}$$

with $m \neq 0$ (we have already seen that there is a single massless mode and that its profile is constant). In the limit of a flat extra dimension, these functions reduce to the ordinary sine and cosine.

Thanks to the Neumann BCs on the $y = 0$ brane (17), (24), (30), the vector profiles $f^a_{L,Rn}$ are all proportional to $C(y, m_n)$. The eigenvalues, that is the physical masses of the vector fields $m_{L\,n}$, $m_{R\,n}$ and $m_{N\,n}$, are then fixed by the BCs on the IR brane (18), (25) and (31). For the three sectors we can easily derive three eigenvalue equations:

Left charged:

$$\tfrac{\tilde{g}^2}{g_5^2} C'(\pi R, m_{L\,n}) - (b(\pi R)\ m_{L\,n}^2 - \tfrac{\tilde{g}^2 \tilde{v}^2}{4})\, C(\pi R, m_{L\,n}) = 0 \tag{46}$$

Right charged:

$$C(\pi R, m_{R\,n}) = 0 \tag{47}$$

Neutral:

$$\begin{aligned} &\Big(\tfrac{\tilde{g}^2}{g_5^2} C'(\pi R, m_{N\,n}) - (b(\pi R)\ m_{N\,n}^2 - \tfrac{\tilde{g}^2 \tilde{v}^2}{4})\, C(\pi R, m_{N\,n})\Big) \cdot \\ &\Big(\tfrac{\tilde{g}'^2}{g_5^2} C'(\pi R, m_{N\,n}) - (b(\pi R)\ m_{N\,n}^2 - \tfrac{\tilde{g}'^2 \tilde{v}^2}{4})\, C(\pi R, m_{N\,n})\Big) \\ &= \tfrac{\tilde{g}^2 \tilde{g}'^2 \tilde{v}^4}{16} C(\pi R, m_{N\,n})^2 \end{aligned} \tag{48}$$

In section VI we will make extensive use of these equations for specific choices of the warp factor and of the parameters of the models to obtain explicit examples of the KK spectrum.