with

$$\mathrm{g}^{<}_{\alpha,\alpha}(t_1,t_2)=\frac{1}{N}\sum_j \mathrm{g}^{<}_{\alpha,j}(t_1,t_2)=\int_{-\infty}^{+\infty}\mathrm{d}\varepsilon_\alpha \mathrm{f}_\alpha(\varepsilon_\alpha)2\mathrm{i}n_\alpha \exp[-\mathrm{i}\varepsilon_\alpha(t_1-t_2)],$$

where $\mathrm{f}_\alpha(\varepsilon_\alpha)$ is the Fermi distribution functions of the left and right leads, which have different chemical potentials under a voltage bias. For the present case of zero temperature the lesser self–energy may be recast in terms of the Heaviside step function $\theta(x)$ as

$$\Sigma^{<}_{0,0,\alpha}(t_1,t_2)=\mathrm{i}\Gamma_\alpha\int_{-\infty}^{+\infty}\frac{\mathrm{d}\varepsilon_\alpha}{2\pi}\theta\left(\epsilon_{\mathrm{F}\alpha}+\tfrac{1}{2}\hbar\omega-\varepsilon_\alpha\right)\theta(t_1)\theta(t_2)\exp[-\mathrm{i}\varepsilon_\alpha(t_1-t_2)]\,, \tag{17}$$

where $\Sigma^{r,(a),(<)}_{0,0,\alpha}(t_1,t_2)$ are all non-zero only when both times (t_1,t_2) are positive $t_1,t_2>0$ and $\epsilon_{\mathrm{F}\alpha}$ is the Fermi energy on each of leads.

The density matrix $\rho_{n,n}(t,t)$ can be calculated by using Eqs. (12, 13, 14, 15, 17) in Eq. (16) at $t=t'$ and $n=n'$ as

$$\begin{aligned}\rho_{n,n}(t,t)=-\mathrm{i}\sum_{\alpha,m,k}\int_0^t\int_0^t \mathrm{d}t_1\mathrm{d}t_2\Phi_{n,m}\Phi^*_{0,m}\exp[-\mathrm{i}(\varepsilon_m-\mathrm{i}\Gamma)(t-t_1)]\\ \times\{\mathrm{i}\Gamma\int_{-\infty}^{\epsilon_{\mathrm{F}\alpha}}\frac{\mathrm{d}\varepsilon_\alpha}{2\pi}\exp[-\mathrm{i}\varepsilon_\alpha(t_1-t_2)\,\}\Phi_{0,k}\Phi^*_{n,k}\exp[-\mathrm{i}(\varepsilon_k+\mathrm{i}\Gamma)(t_2-t)],\end{aligned}$$

Although $\mathrm{g}^{r,(a)}(t_1,t_2)$ is non-zero for $t<0$, it is never required due to the way it combines with $\Sigma^{r,(a),(<)}_{0,0,\alpha}(t_1,t_2)$. By carrying out the time integrations, the resulting expression is written as

$$\begin{aligned}\rho_{n,n}(t,t)=\frac{\Gamma}{2\pi}\sum_{\alpha,m,k}\int_{-\infty}^{\epsilon_{\mathrm{F}\alpha}}\mathrm{d}\varepsilon_\alpha\frac{\Phi_{n,m}\Phi^*_{0,m}\Phi_{0,k}\Phi^*_{n,k}}{(\varepsilon_\alpha-\varepsilon_k-\mathrm{i}\Gamma)(\varepsilon_\alpha-\varepsilon_m+\mathrm{i}\Gamma)}\\ \times\{1+\exp[\mathrm{i}(\varepsilon_k-\varepsilon_m+2\mathrm{i}\Gamma)t]-\exp[-\mathrm{i}(\varepsilon_\alpha-\varepsilon_k-\mathrm{i}\Gamma)t]-\exp[\mathrm{i}(\varepsilon_\alpha-\varepsilon_m+\mathrm{i}\Gamma)t]\}\end{aligned}$$

The integral over the energy in the above equation is carried out[45]. The final result for the density matrix is written as

$$\rho_{n,n}(t,t)=\frac{\Gamma}{2\pi}\sum_{m,k}\frac{\Phi_{n,m}\Phi^*_{m,0}\Phi_{0,k}\Phi^*_{n,k}}{\varepsilon_k-\varepsilon_m+2\mathrm{i}\Gamma}[Y^L_{mk}+Y^R_{mk}+Z^L_{mk}+Z^R_{mk}], \tag{18}$$

where we have added the contribution from the right and the left leads, which can be written in terms of α as