A related question is whether an uninformed expert can produce forecasts that are close to the forecasts of the informed expert. In this section we show that the answer to these two questions might be different.

Recall that a distribution $\nu \in \Delta(S^{\mathbb{N}})$ *merges* with a distribution $\mu \in \Delta(S^{\mathbb{N}})$ if

$$\nu\left(s|s_0,\dots,s_{n-1}\right) - \mu\left(s|s_0,\dots,s_{n-1}\right) \xrightarrow[n\to\infty]{} 0$$

for μ-almost every realization $(s_0, s_1, s_2, \dots)$. This notion of merging was introduced by Kalai and Lehrer (1994) as a weakening of Blackwell and Dubins merging (1962). Sorin (1999) relates merging with several game theoretic models. Kalai et al. (1999) relate merging with certain classes of tests.

Say that a paradigm Γ is *learnable* if there exists $\nu \in \Gamma$ such that ν merges with every $\mu \in \Gamma$. If Γ is learnable then an uninformed expert can learn to predict, i.e., to provide forecasts that becomes arbitrary close to the forecasts of the informed expert.

The paradigm of all exchangeable theories is learnable: If $\Omega = \{0,1\}$ then the theory $\nu = \varepsilon_\lambda$ given in (4) where λ is the uniform distribution on $[0,1]$ merges with all exchangeable theories. However, by our Theorem 1, there exists a test that screens an uninformed expert from the informed expert based on the outcomes of the process.

The paradigm of all theories is not learnable, so that an uninformed expert cannot produce forecasts that are close to the informed expert's forecasts for every process, but, by Proposition 2, he can do as good as the informed expert in every prequential test.

Appendix A. Proof of Proposition 1

Let $F : \Delta(\Gamma) \otimes \Gamma \to [0,1]$ be given by $F(\zeta,\mu) = \zeta \otimes \mu\left(\left\{(\nu,x) \in \Gamma \times \Omega \colon x \in A_\nu^{(T)}\right\}\right)$. Then F is bilinear and, since T is finite, continuous in its second argument. It follows from Fan's Minimax Theorem (1953) that

$$\sup_\zeta \min_\mu F(\zeta,\mu) = \min_\mu \sup_\zeta F(\zeta,\mu), \tag{8}$$

where the suprema are over all $\zeta \in \Delta(\Gamma)$ and the minima are over all $\mu \in \Gamma$.