binding energy is quite small at J/t = 0.3 but grows with increasing J/t. An extrapolation from finite size clusters to the infinite lattice however suggests that the pair state is no longer the groundstate at J/t =0.3, but an excited state with an energy of approximately 0.17t.[37] The inclusion of longer range interactions and hopping in the t-J model increases the energy of the pair state further and confirms the conclusion that for parameter values relevant to cuprates the groundstate of the cluster has two unbound holes in the nodal states.[38] Extending the calculations to the 32-site clusters with 4 holes, which corresponds to a doping of 1/8, shows all 4 holes entering into nodal states with no signs of pairing correlations.[39] In view of the prominent bound pair excited state for 2 holes, a low energy excited state with two of the holes in a bound state may also be expected here. However at present there is no information on this question to the best of our knowledge.

Leung and collaborators[37–39] concluded from these calculations that at low densities holes entered the nodal regions, possibly in pockets, and as a result there was no evidence for d-wave pairing correlations in the groundstate for realistic values of the parameters in t-J models. However the analysis presented here suggests a more optimistic conclusion. First we note that the nodal points in the 32-site cluster are very special, because exactly at these points the coupling in a Cooper channel to a d-wave Cooperon vanishes by symmetry. Thus if we interpret the d-wave pair excited state as evidence for a finite energy Cooperon in the t-J model and its extensions, then as the occupied holes at finite doping move out from the exact nodal points, a d-wave pairing attraction is generated through the coupling to this Cooperon, similar to the scenarios we discussed earlier. Note an earlier study for two holes on smaller clusters by Poilblanc and collaborators[40] concluded in favor of the interpretation of the 2-hole bound state as a quasiparticle with charge 2e and spin 0, which would be an actual carrier of charge under an applied electric field. In other words they concluded that a Cooperon is present in the strong coupling t-J model at low doping. A more detailed analysis of the origin of the pairing in this state was published recently by Maier et al.[41] Note the hole density in the case of 2 holes in a 32-site cluster is very low so that the superconducting order we are postulating should coexist with long range antiferromagnetic order. There is considerable evidence both numerical, in variational Monte Carlo calculations, and experimental, in favor of such coexistence, as discussed in the recent review by Ogata and Fukuyama.[9]

We conclude that there is strong evidence that the pairing mechanism in the present model is not confined to weak coupling and ladder lattices, but will also operate in the