Since, in our example, the system has only one bound state, with appropriate values of $M$ and $\beta$, the diagonalization of $H$ results in one negative eigenvalue $E_0$ and $M-1$ positive eigenvalues $E_j$ $(j = 1, ...., M-1)$. The corresponding wave functions

$$\Psi_j = \sum_{m=0}^{M-1} a_m^j \phi_m \quad j = 1, ...., M-1 \ , \tag{26}$$

are approximate solutions of $(H - E_j)\Psi_j = 0$ in the interaction region. As $r \to \infty$ they go to zero exponentially and therefore they do not represent physical scattering states. The negative energy $E_0$ and the first three positive energy eigenvalues ($E_j$, $j = 1, 3$) are shown in Fig. 1 as a function of $\beta$ for $M = 40$. We observe the plateau already reached by $E_0$ for the values of $\beta$ showed in the figure. Furthermore, we observe the monotonic behavior of the positive eigenvalues towards zero as $\beta$ decreases. The corresponding eigenvectors can be used to compute the integral relations of Eq. (21) and to calculate the second order estimate of the phase-shifts $\delta_j$ at the specific energies $E_j$. This analysis is shown in Table I in which the non linear parameter $\beta$ of the Laguerre basis has been chosen to be 1.2 fm$^{-1}$. In the first row of the table the ground state energy is given for different values of the number $M$ of Laguerre polynomials. The stability of $E_0$ at the level of 1 keV is achieved already with $M = 20$. For a given value of $M$, $E_j$, with $j = 1, 2, 3$, are the first three positive eigenvalues. The eigenvectors corresponding to positive energies approximate the scattering states at these specific energies. Since the lowest scattering state appears at zero energy, none of the positive eigenvalues can reach this value for any finite values of $M$. We observe (see Fig. 1) that the eigenvalues diminish as $M$ increases. Defining $k_j^2 = \frac{m}{\hbar^2} E_j$, the second order estimate for the phase shift at each energy and at each value of $M$ is obtained as

$$\begin{aligned}
&-\frac{m}{\hbar^2} < \Psi_j | H - E | F_j >= B_j \qquad \text{with} \qquad F_j = \sqrt{\frac{k_j}{4\pi}} \frac{\sin(k_j r)}{k_j r} \\
&\frac{m}{\hbar^2} < \Psi_j | H - E | \tilde{G}_j >= A_j \quad \text{with} \qquad \tilde{G}_j = f_{reg} \sqrt{\frac{k_j}{4\pi}} \frac{\cos(k_j r)}{k_j r} \\
&[\tan\delta_j]^{2^{nd}} = \frac{B_j}{A_j}.
\end{aligned} \tag{27}$$

On the other hand, as we are considering the $A = 2$ system, at each specific energy value $E_j$ the phase shift $\tan\delta_j$ can be obtained by solving the Schrödinger equation numerically. The two values, $[\tan\delta_j]^{2^{nd}}$ and $\tan\delta_j$, are given in the Table I at the corresponding energies as a function of $M$. We observe that, as $M$ increases, the relative difference between the