

work model Kauffman (1969, 1993). Thus, our approach contains the standard Boolean network model as a special case.

*Regulation at the protein level:* In the next example we study regulation on the transcription factor level Fig. 3b. As opposed to the first example, here we have proteins and protein compounds regulating transcription; this includes the assembly rule, i.e. the regime change on transcription factor level. The only non-zero matrix elements are those corresponding to arrows in Fig. 3b and the only relevant dynamic modes are the cycles  $A_{\mathcal{P}}$  and  $B$  in Fig. 2. The characteristic polynomial (3) now reduces to  $P(\lambda) = -\lambda^2(\lambda^3 - A_{\mathcal{P}}\lambda^2 - B)$ . While finding the roots of this cubic polynomial can be cumbersome, finding the stability condition is in this case straightforward. At  $\lambda = 1$  we obtain the stability condition  $B + A_{\mathcal{P}} = 1$ . Furthermore, since the largest eigenvalue  $\Lambda$  of  $A$ ,  $A_{\mathcal{P}}$  and  $B$  are all continuous increasing functions of the associated matrix elements of  $A$ , then  $\Lambda < 1$  for  $B + A_{\mathcal{P}} < 1$  and  $\Lambda > 1$  when  $B + A_{\mathcal{P}} > 1$ . So in this case, the effective control parameter for stability is given by  $\theta = B + A_{\mathcal{P}}$ . In particular, assuming constant neighborhood and membership sizes and synchronous updates, from (4) and (5) we obtain

$$\theta = s_{\mathcal{G}}K_1^2\rho_{\mathcal{P}}^+M_1\rho_{\mathcal{G}}^{M_1-1} + s_{\mathcal{P}}K_2^2\rho_{\mathcal{G}}^{M_1} \quad (8)$$

Notice that this formula consistently solves the following subtlety: instead of speaking of gene-gene regulation networks as in the previous example, one could actually distinguish between the gene and its products (proteins), and construct a network where single gene products regulate gene transcription. This would be a more accurate description of the biological reality, should, however, lead to the same results, since de facto we do not change any inter-