inhibitory interactions of the shunting type. Define the $r-$neighborhood of the cell C_{ij} as

$$N_r(i,j) = \{C_{hl} : \max(|h-i|, |l-j|) \leq r,\ 1 \leq h \leq m,\ 1 \leq l \leq n\}.$$

The dynamics of a cell C_{ij} are described by the following nonlinear ordinary differential equation:

$$\frac{dx_{ij}}{dt} = -a_{ij}x_{ij} - \sum_{C_{hl}\in N_r(i,j)} C_{ij}^{hl} f(x_{hl}(t))x_{ij} + \mathscr{L}_{ij}(t), \tag{1.1}$$

where x_{ij} is the activity of the cell C_{ij}; $\mathscr{L}_{ij}(t)$ is the external input to the cell C_{ij}; the constant $a_{ij} > 0$ represents the passive decay rate of the cell activity; $C_{ij}^{hl} \geq 0$ is the coupling strength of post-synaptic activity of the cell C_{hl} transmitted to the cell C_{ij}; the positive and continuous function $f(x_{hl})$ represents the output or firing rate of the cell C_{hl}.

The existence and stability of periodic, almost-periodic and anti-periodic solutions of SICNNs have been widely studied in the literature [16]-[25]. Moreover, chaos in SICNNs were considered in the papers [26]-[30]. However, to the best of our knowledge, the existence of homoclinic and heteroclinic solutions of SICNNs has not been investigated before in the literature. Motivated by the lack of mathematical methods for the investigation of homoclinic and heteroclinic motions in SICNNs, we propose the results of the present study. Both continuous and discontinuous external inputs in rectangular form are utilized in our model.

The generation of homoclinic and heteroclinic motions in systems of differential equations by means of discontinuous perturbations was first realized by Akhmet [31] on the basis of functional spaces. The paper [31] was concerned with the presence of homoclinic motions embedded in the chaotic attractor of a relay system (see [32, 33] for details about relay systems). Similar results for impulsive differential equations were obtained in the studies [34, 35], and the formation of homoclinic and heteroclinic motions in economic models was considered within the scope of the paper [36]. Based on the definitions given in [31], in the present study, we provide the descriptions of homoclinic and heteroclinic motions for the multidimensional dynamics of SICNNs in the functional sense, and rigorously prove that the discontinuous inputs give rise to the generation of homoclinic and heteroclinic motions.

One of the techniques for the confirmation of chaos in neural networks is to determine homoclinic and heteroclinic motions. A geometric construction of a transversal homoclinic orbit for a nonlinear neuron model with time delays was provided by Chen [37] in order to show the existence of chaos. Moreover, Li et al. [38] proved the existence of a saddle periodic orbit in the dynamics of a small Hopfield neural network with a memristive synaptic weight, and verified the existence of a hyperchaotic invariant set via a homoclinic intersection of its stable and unstable manifolds by using the Smale-Birkhoff homoclinic theorem. Zou et al. [39], on the other hand, presented a method for finding homoclinic and heteroclinic motions in the three-cell autonomous cellular neural network, which has chaotic behavior similar to that