

was considered recently theoretically and with simulations in [27, 28]. It should be noted that the meaning of collision in such a model should be taken with care. For a random walk on a lattice with waiting times on each lattice point, one can envision several collision rules. For example one might allow two particles to occupy the same site at a given time, or one may consider a mechanism where a particle once hopping into a trap already occupied will eject the particle previously residing in the trap. Or a particle is allowed to jump only into an empty site, as is usually assumed. These type of collision rules might yield behaviors different than ours. For example a particle stuck with a very large sojourn time, might be ejected by another particle, hence one can imagine a situation where some form of interaction causes the particles to move faster. In our work the collision implies that we can let particles go past one another as if they were non interacting (so on a lattice two particles may occupy the same point at the same time) and eventually we look for the center particle. Even more interesting will be to investigate interacting particles in systems with quenched disorder, since the latter, when strong enough, is known to lead to non Gaussian sub-diffusion [45, 46]. There the simple formula Eq. (89) is generally not expected to hold. Indeed in [34] single file motion of a tagged particle in the Sinai model was considered, the results are much richer when compared with Eq. (89).

VI. DISCUSSION

The one dimensional problem of motion of a tagged particle interacting via hard core interactions with other particles was solved using the Jepsen line. The formalism we developed treats both Brownian and non Brownian motion in between collision events, is suited for rather general external fields acting on the particles, for open and closed system, and handles also different types of initial conditions. Following others we have mapped the

problem onto a non interacting problem using the Jepsen line. The motion of the tagged particle belongs to the general problem of order statistics. The problem reduces to considering a list of $2N+1$ random variables and finding the distribution of the variable which has N variables smaller than it and N larger corresponding to center particle (note that the right most particle we will have an extreme value problem). Classical theory of order statistics deals however with the case where all the random variables have identical distributions. In contrast in the exclusion process under consideration, particles have non identical distribution. Thus except for two cases: i) all the particles initially on the same position and ii) equilibrium state, the problem deals with non identically distributed random variables (since the initial condition are non identical). While we treated the problem of symmetric potential and symmetric initial condition for the center particle in detail, it is left for future work to consider non-symmetric potential fields, non symmetric initial conditions, and the dynamics of the particle in the tails of the packet. We believe that the methods developed here with some modifications can treat these cases too.

Acknowledgment EB is supported by the Israel Science Foundation, and RS by the NSF. We thank Ludvig Lizana and Tobias Ambjörnson for sharing with us their data in Fig. 5 and for providing helpful insight. We also thank M. Lomholt and A. Taloni for useful discussions.

VII. APPENDIX A

In this Appendix we use Boltzmann's distribution for the interacting system to find the PDF of the tagged center particle in equilibrium. The multi dimensional PDF for $2N+1$ interacting particles, in the presence of an external binding field $V(x)$, with $V(x) = V(-x)$, acting on all of them is

$$P(x_{-N}, \dots, x_{-1}, x_0, x_1, \dots, x_N) = \frac{1}{Z_{2N+1}} \exp \left[- \sum_{j=-N}^N \frac{V(x_j)}{k_b T} \right] \theta(x_{-N+1} - x_{-N}) \theta(x_{-N+2} - x_{-N+1}) \dots \theta(x_N - x_{N-1}) \quad (93)$$

where Z_{2N+1} is a normalizing factor, and $\theta(x)$ is a step function: $\theta(x) = 0$ if $x < 0$, $\theta(x) = 1$ for $x \geq 0$. The center tagged particle is $x_0 = x_T$. To find the PDF of x_T in equilibrium, which we call $P^{\text{eq}}(x_T)$, we must integrate Eq. (93) over all coordinates besides $x_0 \rightarrow x_T$

$$P^{\text{eq}}(x_T) = \frac{\exp \left[- \frac{V(x_T)}{k_b T} \right]}{Z_{2N+1}} \times \int_{-\infty}^{x_T} dx_{-N} \int_{x_{-N}}^{x_T} dx_{-N+1} \dots \int_{x_{-2}}^{x_T} dx_{-1} \exp \left[- \frac{\sum_{j=-N}^{-1} V(x_j)}{k_b T} \right] \int_{x_T}^{\infty} dx_1 \int_{x_1}^{\infty} dx_2 \dots \int_{x_{N-1}}^{\infty} dx_N \exp \left[- \frac{\sum_{j=1}^N V(x_j)}{k_b T} \right]. \quad (94)$$