and the relative depth of the minima at points Γ and M is defined by a field value. Thus, all symmetrical points of the Brillouin zone, Γ, X and M are occupied by minima or maxima. For this reason the saddle points, to which standard logarithmic peculiarities of density of states correspond, are moved inside of the Brillouin zone along a direction like (1,0).

The relative position of minima is defined by a value of the magnetic field. For the lower branch at weak fields ($h < 3$) the absolute minimum of the function $\omega_-(\vec{k})$ is located at a point like M. The parabolic dependence $\omega(\vec{q})$ corresponds to this minimum, and the latter defines a standard peculiarity of the density of states like a finite discontinuity (see Fig. 7a,b). At such values of the field the peculiarity connected with the point Γ ($\vec{q} = 0$) lies within the lower band frequency and manifests itself as a derivative jump $D(\omega)$ (for example, at $\omega = 0.6\omega_a$ for $h = 2.5$) and it is almost invisible in the figures. For higher fields ($h > 3$) the frequency minimum at the point Γ ($\vec{q} = 0$) becomes deeper, i.e. the local minimum value of the lower branch of the spectrum ($\omega_-(\mathrm{M})$) lies higher than the frequency of long-wave oscillations $\omega_-(0)$, (see Fig. 7). In this case a linear behavior of the density of states $D(\omega) \sim (\omega - \omega_0)\Theta(\omega - \omega_0)$ is clearly seen at the lower edge of the frequency band for the lower branch. In this case, within the band there is one more singularity of the density of states, which is a finite jump of the density of states, determined by a local parabolic minimum at the point M $(1, 1)$, see Fig. 7c.

One of the most interesting singularities of the spectrum of the lower branch is observed in the special case of zero field. First, in this case the global maxima at the points X (1,0) have a non-parabolic dependence $\omega(\vec{q})$, $\omega(\vec{q}) = \omega_{\max} - c|\Delta\vec{q}|$, where $\Delta\vec{q}$ is a deviation of $\vec{q}$ from the point X. For this reason, the upper edge of the energy band is characterized be non-standard linear Van Hove singularity, $D(\omega) = C(\omega_{\max} - \omega) \cdot \Theta(\omega_{\max} - \omega)$. On the other hand in the expansion of $\omega(\vec{q})$ in the center of the Brillouin zone the linear term in q is absent, and there is a specific saddle point with four-fold symmetry having a non-analytic behavior, see Eq. (24). However the analysis shows, that this saddle point leads to the standard logarithmic Van Hove singularity. For small, but finite value of the external field in the zone center there is a linear non-analytical behavior of $\omega(\vec{q}) \approx \omega_0 + \alpha|\vec{q}|$, which is typical for the systems having dipole interaction. While at the point $\vec{q} = 0$ there is a minimum and a non-analytic saddle point splits into four standard saddle points moved towards the points X.