hold in any part of the parameter space and using the corresponding critical values may lead to a severely oversized test.

Example 3.3. *Suppose that $g(\theta) = \theta_1^2 + ... + \theta_p^2$; then $G(\theta) = [2\theta_1, ..., 2\theta_p]$ and*

$$W_T = T \frac{\left(\Sigma_{i=1}^p \hat{\theta}_i^2 \right)^2}{4 \Sigma_{i=1}^p \hat{\theta}_i^2}.$$

Then the limit distribution is that of $\frac{1}{4} \|Z\|^{\frac{1}{2}}$; under normality this is $\frac{1}{4}\chi_p^2$; it is a pivotal distribution even though non-standard. If p is large enough, the χ_1^2 will not provide an upper bound.

In the case of more than one restriction in addition to all the non-standard features that can arise for a single restriction it is also possible that the test statistic diverges even under H_0.

Example 3.4. *Suppose that $q = p = 2$ and $g(\theta) = \left[\theta_1^2 : \theta_1 \theta_2^2\right]'$. Then*

$$G(\theta) = \begin{bmatrix} 2\theta_1 & 0 \\ \theta_2^2 & 2\theta_1\theta_2 \end{bmatrix};$$

it follows that

$$W_T = T \frac{4\hat{\theta}_1^2 + \hat{\theta}_2^2}{16}.$$

Then if (i) $\bar{\theta}_1 = \bar{\theta}_2 = 0$ the asymptotic distribution is $\frac{1}{4}Z_1^2 + \frac{1}{16}Z_2^2$ and thus under normality is a linear combination of two independent χ_1^2 and is bounded by $\frac{1}{4}\chi_2^2$. However, if (ii) $\bar{\theta}_1 = 0$, but $\bar{\theta}_2 \neq 0$ the null still holds, but as $T \to \infty$ the Wald statistic diverges to $+\infty$.