N-slit interference: Path integrals, Bohmian trajectories.

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Path integrals give a possibility to compute in details routes of particles from particle sources through slit gratings and further to detectors. The path integral for a particle passing through the Gaussian slit results in the Gaussian wavepacket. The wavepackets prepared on N slits and superposed together give rise to interference pattern in the near-field zone. It transforms to diffraction in the far-field zone represented by divergent principal rays, at that all rays are partitioned from each other by $(N-2)$ subsidiary rays. The Bohmian trajectories in the near-field zone of N-slit gratings show wavy behavior. And they become straight in the far-field zone. The trajectories show zigzag behavior on the interference Talbot carpet (ratio of particle wavelength to a distance between slits are much smaller than 1 and $N \gg 1$). Interference from the the N-slit gratings is simulated by scattering monochromatic neutrons (wavelength=0.5 nm). Also we have considered simulation of interference fringes arising at scattering on an N-slit grating of fullerene molecules (according to the real experiment stated in e-print 1001.0468).

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I. INTRODUCTION.

Classical mechanics operates with point particles behavior of which is found unambiguously from its variational principles [1]. Initially, we begin with the action integral:

$$S = \int_{t_0}^{t_1} L(\vec{\mathbf{q}}, \dot{\vec{\mathbf{q}}}; t) dt \qquad (1)$$

Here $L(\vec{\mathbf{q}}, \dot{\vec{\mathbf{q}}}; t)$ is a Lagrangian function equal to difference of kinetic energy and potential energy of the particle. Dynamical variables $\vec{\mathbf{q}}$ and $\dot{\vec{\mathbf{q}}}$ are generalized coordinate and velocity of the particle. Ones proclaim, that the action S remains constant along an optimal path of the movement particle. It is the least action principle. According to this principle, finding of the optimal path adds up to solution of the extremum problem $\delta S = 0$. This solution leads to Hamilton-Jacobi equation (HJ-equation):

$$-\frac{\partial S}{\partial t} = H(\vec{\mathbf{q}}, \vec{\mathbf{p}}; t). \qquad (2)$$

Here $H(\vec{\mathbf{q}}, \vec{\mathbf{p}}; t) = (\vec{\mathbf{p}}\dot{\vec{\mathbf{q}}}) - L(\vec{\mathbf{q}}, \dot{\vec{\mathbf{q}}}; t)$ is the Hamilton function and $\vec{\mathbf{p}} = \nabla S$ is a particle momentum. It is worth mentioning the optical-mechanical analogy [1] of particular solutions of the HJ-equation. It brings to light on deep parallels between mechanical trajectories, optical rays, and even fluid streams fall under these parallels.

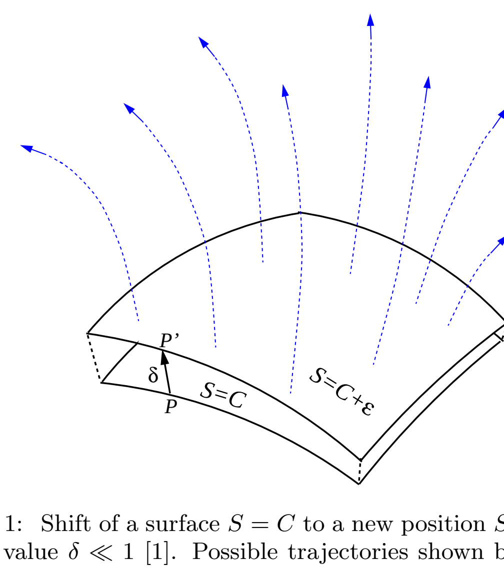

FIG. 1: Shift of a surface $S = C$ to a new position $S = C + \varepsilon$ on a value $\delta \ll 1$ [1]. Possible trajectories shown by dotted blue curves intersect the surfaces perpendicularly.

Please note, that the gradient of the function S, that is, ∇S, is directed normally to its surface S = const. Consider two nearby surfaces $S = C$ and $S = C + \varepsilon$, see Fig. 1. Let us trace a normal from an arbitrary point P of the first surface up to its intersection with the second surface at point P'. Hereupon, make another shift

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