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[63] It is known that this approximation becomes exact for systems with long-range interactions in a proper thermodynamic limit $N \to +\infty$ [42]. For systems with short-range interactions (e.g. a screened Newtonian potential), we shall still use a mean field approximation although it may not be exact. One motivation of our approach is that the Keller-Segel model in biology is formulated in the mean field approximation even if the degradation of the chemical is large. The incorrectness of the mean field approximation as the interaction becomes short-range is interesting but will not be considered in this paper.

[64] We cannot impose the boundary conditions (33) for the Poisson equation (41) since the integration of this equation $\oint \nabla c \cdot d\mathbf{S} = -\lambda M \neq 0$ implies that $\nabla c \cdot \mathbf{n} \neq 0$ on the boundary of the domain.

[65] This interpretation also holds for the chemotactic problem. We have seen that the (mean field) Keller-Segel model has an effective thermodynamical structure associated with the canonical ensemble. Furthermore, we can derive kinetic models of chemotaxis in which the evolution of the cells (or bacteria) is described in terms of coupled stochastic equations [51, 52]. In that sense, the cells behave as Brownian particles in interaction as in Secs. II 2 and II 3, and the canonical structure of this model is clear.

[66] In many papers, only fully stable states forming the strict caloric curve are indicated. We think that clarity is gained when the full series of equilibria is shown. Then, we can see where the stable, metastable and unstable branches are located and how they are connected to each other. This also allows us to use the Poincaré theory of linear series of equilibria to settle their stability without being required to study an eigenvalue equation associated with the second order variations of the thermodynamical potential [18, 31].

[67] This is, in fact, just a quasistationary state that forms on a timescale that is short with respect to the Hubble time so that the expansion of the universe can be neglected or treated adiabatically. Indeed, if we allow for the time variation of the scale factor $a(t)$, it is simple to see that there is no statistical equilibrium state in a strict sense.