A. Solution of the linearized EL equations

Let us first consider the linearized equations (41)- (43). For the linearized equations we shall simplify the notation, $\vartheta^{(1)} \equiv \vartheta, \eta^{(1)} \equiv \eta, \phi^{(1)} \equiv \phi$. For the electrostatic part we obtain the linearized equations of the well known form (the spatial dependence of permittivity leads to nonlinear contributions to the EL equations)

$$\frac{\bar{\epsilon}}{4\pi}\frac{d^2\psi(z)}{dz^2} + e\phi(z) = 0 \tag{51}$$

$$e\psi(z) + \frac{kT\phi(z)}{\bar{\rho}_c} = 0 \tag{52}$$

which together give

$$\phi(z)'' = \kappa^2\phi(z), \tag{53}$$

where

$$\kappa^2 = \frac{4\pi e^2\bar{\rho}_c}{kT\bar{\epsilon}}. \tag{54}$$

Note that κ is the inverse Debye length in units of the molecular size. Solution of Eq. (53) with the boundary condition Eq. (46) has the well known form

$$\phi(z) = -\kappa\sigma e^{-\kappa z}. \tag{55}$$

The equations for (η, ϑ) are formally the same as in the Landau theory for a mixture near the demixing critical point, and can be written in the form

$$\mathbf{v}'' = \mathbf{M}\mathbf{v} \tag{56}$$

where the matrix $\mathbf{M} = (M_{ij})$ with the indices 1 and 2 corresponding to s and ρ respectively, is defined in (33), and the vector $\mathbf{v}$ is defined below Eq.(26). In the semi-infinite systems in the one-phase state $\vartheta(z), \eta(z) \to 0$ for $z \to \infty$, and from the Ansatz:

$$\vartheta = t_1\exp(-\lambda_1 z) + t_2\exp(-\lambda_2 z) \tag{57}$$

$$\eta = n_1\exp(-\lambda_1 z) + n_2\exp(-\lambda_2 z) \tag{58}$$