After providing general results in Sec. III we limit our attention to symmetric initial conditions and potentials where the tagged particle has no average drift. A general relation between the mean square displacement of the tagged particle and reflection probability of the non interacting particle is given in Eq. (37). Detailed calculations of the mean square displacement of the tagged particle then follow, for special choices of force fields and initial conditions, in Sec. IV. As we discuss in sub-section III D, in certain limits our problem is related to order statistics [37], a fact worth mentioning since it allows us to solve our problem and related ones using known methods. In Sec. V, we discuss non-Brownian kinds of motion and the Percus relation. A brief report of part of our results was recently published [21].

II. MODEL AND METHODS

In our model, $2N+1$ identical point particles with hard core particle-particle interactions are undergoing Brownian motion in one dimension, so particles cannot pass each other. The diffusion constant of particles free of interaction is D. As mentioned, towards the end of the paper we discuss the more general case where the dynamics between collision events is not necessarily Brownian. An external potential $V(x)$ acts on the particles. The system stretches from $-\overline{L}$ to $\overline{L}$; however, unless stated otherwise we will let $\overline{L} \to \infty$ and obtain a thermodynamic limit where $N/\overline{L}$ is fixed. We tag the center particle, which clearly has N particles to its left and N to it right. Initially the tagged particle is at the origin $x = 0$. The motion of a single particle in the absence of interactions with other particles is described by a single particle Green function $g(x, x_0, t)$, with the initial conditions $g(x, x_0, 0) = \delta(x - x_0)$. In the case of over damped Brownian motion the Green function is the solution of the Fokker-Planck equation [38]

$$\frac{\partial g(x, x_0, t)}{\partial t} = D \left[\frac{\partial^2}{\partial x^2} - \frac{1}{k_b T} \frac{\partial}{\partial x} F(x) \right] g(x, x_0, t), \quad (1)$$

where $F(x) = -V'(x)$ is the force field, T is the temperature and k_b is Boltzmann's constant. The initial conditions of N particles residing initially to the right (left) of the test particle are drawn from the probability density function (PDF) $f_R(x_0)$ ($f_L(x_0)$) respectively. We consider an ensemble of trajectories and average over trajectories and initial conditions (see details below). Our goal is to obtain the PDF of the position x_T of the tagged center particle $P(x_T)$ in the large N limit.

A. The Jepsen Line

A schematic diagram of the problem is presented in Fig. 1 for particles in a box. Initial positions of particles are given by x_0^j where $j = -N, \cdots, 0, \cdots N$ where j is the label of the interacting particles (see Fig. 1). The tagged particle whose coordinate is denoted with $x_T(t)$, is the center particle $j = 0$ (bold line in Fig. 1). Initially the tagged particle is at the origin $x_T(0) = 0$. Since particles do not pass each other, their order is clearly maintained, and the number of particles to the left and right of the tagged particle N is fixed.

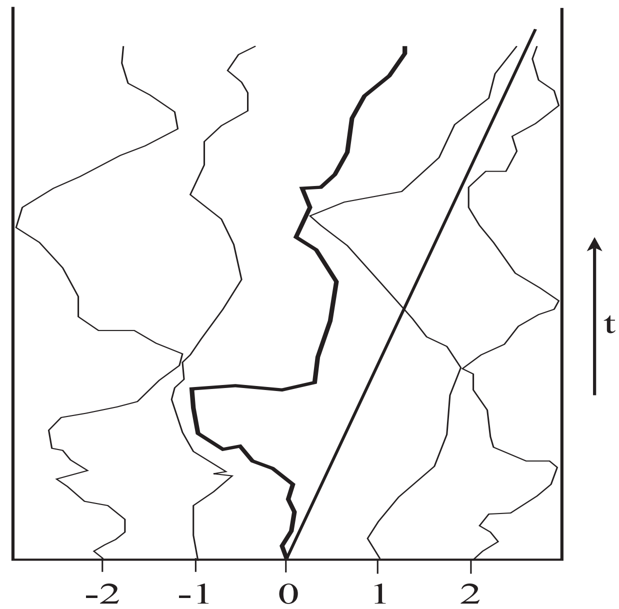

FIG. 1: Schematic motion of Brownian particles in a box, where particles cannot penetrate through each other. The center tagged particle label is 0, its trajectory is restricted by collisions with neighboring particles. The straight line is the Jepsen line, it follows vt as explained in the text. As explained in the text, in an equivalent non-interacting picture, we allow particles to pass through each other, and at time t we search for the position of the particle which has N particles to its right and N to its left (i.e. the center particle).

In Fig. 1 the straight line which starts at $x = 0$ is called the Jepsen line and follows $x(t) = vt$, where v is a test velocity [5, 6]. We label the interacting particles according to their initial position, increasing to the right (see Fig. 1). The tagged particle starts just to the right of the Jepsen line so at $t = 0$ the label of the particle to the right of the Jepsen line is zero. In this system we have $2N + 1$ particles. Hence initially we have N particles to the left of the Jepsen line and including the tagged particle $N + 1$ particles to the right.

Let $\tilde{\alpha}(t)$ be the label number of the first particle situated to the right of the Jepsen line. According to our rules, at $t = 0$ we have $\tilde{\alpha} = 0$, and then the random variable $\tilde{\alpha}$ will increase or decrease in steps of $+1$ or -1 according to:

i) if a particle crosses the Jepsen line from left to right $\tilde{\alpha} \to \tilde{\alpha} - 1$

ii) if a particle crosses the Jepsen line from right to left $\tilde{\alpha} \to \tilde{\alpha} + 1$. Thus the counter $\tilde{\alpha}$ is performing a ran-