

FIG. 15: Left: z = n (q - 1) vs K with  $K > K^*$  for the  $L \times L \times L$ -lattice, L = 4, 6, 8, 12, 16, 32, 64 (downwards). Right: y = n (p - 1) vs K with  $K > K^*$  for the  $L \times L \times L$ -lattice, L = 4, 6, 8, 12, 16, 32, 64 (upwards). In both plots the red line indicates location of  $K_c$  and the points are the locations of  $K^*$ .

## D. 4D-lattices

In the case of 4-dimensional lattices we have sampled data of magnetisation distributions for L = 4, 6, 8, 10, 12, 16. Figure 16 shows some of these magnetisation distributions for L = 12 near  $K^*$  together with fitted p, q-binomial distributions. The fit is quite good, considerably better than for 2D and 3D, in the whole range of selected temperatures. Though it is hard to distinguish the fitted curves from the magnetisation curves, there is a small deviation near the middle. How should z at  $K^*$  depend on n? Actually, taking the data at face-value they are rather well-fitted to the simple formula  $z = -6.5 - 0.45 \log n$ . However, for the 4D-lattice we have  $\gamma=\gamma'=1,\ \beta=1/2$  and  $\nu=\nu'=1/2$ . This gives that  $1 + \gamma/d\nu = 3/2$  and  $1 - \beta/d\nu = 3/4$ . Moreover, according to<sup>21</sup> there should be a correction to this. They calculated, using renormalization group techniques, that the susceptibility should scale as  $L^2 \sqrt{\log L}$  near  $K_c$ . This means that  $\sigma_2$  should scale as  $n^{3/2} \sqrt{\log n}$ . From (112) we see that we have to choose  $\lambda_2=-2$ , with  $\lambda_1=0$  and  $\lambda_3=-6$ , to obtain this. In the left plot of figure 17 we have set  $z = -1.2 - 2 \log \log n - 6 \log \log \log n$  and plotted it versus  $\log \log n$ . The curve would then behave as a limit curve rather than as a fitted curve. The choice of coefficient  $\lambda_0 = -1.2$  is only supported by the human eye as a guide rather than any theory and herein lies a problem. With this choice the coefficient of (112) is about 0.558. However, dividing the measured  $\sigma_2$  at the different  $K^*$  with  $n^{3/2}\sqrt{\log n}$  gives values close to 0.15. This discrepancy could be due to several sources; e.g. the expression in (112)