TABLE I: Population probabilities P_n of the ^1S bound states of helium after the β decay of a T^- anion. Also given are the corresponding energies E_n (in atomic units) obtained in the present work.

n	E_n (this work)[a]	P_n(%) (this work)	P_n(%) (Ref. 17)	P_n(%) (Ref. 18)
1	$-$**2.903**4572	22.98998	22.993764	19.147
2	$-$**2.1459**527	46.86960	46.867404	21.149
3	$-$**2.0612**659	0.01320	0.135	0.27
4	$-$**2.03358**41	0.18363	0.21	0.143
5	$-$**2.02117**49	0.09220	–	0.07
6	$-$**2.01456**04	0.05262	–	0.039
7	$-$**2.01062**35	0.03275	–	0.024
8	$-$**2.00809**09	0.02175	–	0.016
9	$-$**2.00636**79	0.01522	–	0.011
10	$-$**2.00514**07	0.01111	–	0.008
11	$-$**2.00423**55	0.00848	–	–
12	$-$**2.00355**07	0.00666	–	–
13	$-$**2.00302**05	0.00513	–	–
14	$-$**2.002**5951	0.00469	–	–
15	$-$**2.0022**570	0.00273	–	–
$\sum P_n$		70.30975	70.206168	40.877

[a]The bold digits agree with the results in Ref. 25.

angle θ can in principle be chosen arbitrarily within $0^\circ \leq \theta \leq 45^\circ$. In the limit of an infinite basis all observables calculated with the aid of complex scaling should become independent of θ. Since only finite basis sets can be applied in practice, only approximate eigenstates can be obtained that may depend on θ. The angle θ can thus be understood as a variational parameter that modifies the adopted basis as can be seen from the inverse relation between basis-set exponents and the scaling angle discussed, e. g., in [29]. A diagonalization of the Hamiltonian (8) in the basis described by the Eqs. (4) and (5) yields the complex-scaled energies $E_j(\theta)$ and wavefunctions $\Psi_j(\theta)$ where the latter are still defined by Eq. (6), but with complex coefficients $c_{jk}(\theta)$.

With the aid of the complex-scaled energies and wavefunctions the transition-probability density into the electronic continuum can be extracted from the complex-scaled resolvent according to [9]

$$P(E,\theta) = \frac{1}{\pi}\mathrm{Im}\left\{\sum_k \frac{\langle\Psi_i^{T^-}(\theta^*)|\Psi_k^{He}(\theta)\rangle\langle\Psi_k^{He}(\theta^*)|\Psi_i^{T^-}(\theta)\rangle}{E_k^{He}(\theta)-E}\right\}. \quad (9)$$

The $\langle\Psi(\theta^*)|$ is the biorthonormal eigenstate to $|\Psi(\theta)\rangle$. It is obtained from the latter by a transposition and complex conjugation of the angular part, while the radial part is only transposed but not complex conjugated. The sum over k includes all complex-scaled eigenstates and eigenvalues calculated by solving the generalized complex symmetric, but non-hermitian eigenvalue problem. As discussed above in the limit of exact eigenstates the density $P(E,\theta)$ becomes independent of the complex-scaling parameter θ. A variation of θ for approximate eigenstates provides the possibility to determine an optimal θ_{opt} with highest stability. The best approximation of $P(E,\theta)$ is then obtained according to

$$\left.\frac{\partial P(E,\theta)}{\partial\theta}\right|_{\theta_{\mathrm{opt}}} = \mathrm{min.} \rightarrow P(E) := P(E,\theta_{\mathrm{opt}}). \quad (10)$$

Furthermore, the θ dependence of the spectra gives an indication for the convergence of the results.

III. RESULTS

In Table I the calculated transition probabilities for 15 ^{1}S bound states of He are listed. The results reveal that almost every second T^- decay will end in the first excited state of He. The next probable final state is the He ground state with nearly 23%. With a summed probability of 0.45% the higher excited He states are rarely populated after β decay of T^-. The sum over all calculated bound states yields 70.3%. The summation over all calculated states (discrete and discretized continuum states) yields the expected value of 100.00%, since the same basis is used for initial and all final states, but indicates the proper numerical implementation. The excellent agreement of the energy eigenvalues at the order of μhartree with the very accurate data in [25] assures on the other hand the high quality of the basis set adopted in the present work and its ability to describe many states simultaneously with high precision. A closer view on the energies shows that the degree of accuracy of the present results follows the expected trends. First, the accuracy increases with n, since the importance of correlation decreases, if the state becomes more asymmetric and the two electrons have smaller spatial overlap. For even higher values of n the states become increasingly diffuse and thus it is very difficult to describe them properly without running into numerically caused linear dependencies.

A comparison to the final-state probabilities reported by Frolov [17] and Harston and Pyper [18] is also given in Table I. Especially for the highly populated ground and first excited states the results of this work confirm the expectedly very accurate results of Frolov [17] that were obtained with explicitly correlated basis functions. The agreement for the third excited state ($n = 4$) is, however, less good, and for the second excited state ($n = 3$) there is even an order of magnitude difference. All attempts to improve the basis set for this state failed to yield a better agreement. This could be an indication for a typographical error (a missing zero after the decimal point) in [17].

The comparison with the results in [18] that were obtained with a relativistic MCDF (multi-configuration Dirac-Fock) method shows on the other hand pronounced