

where $p(0 \rightarrow 1) = dt/\tau_c$ and $p(1 \rightarrow 1) = 1 - p(1 \rightarrow 0) = 1 - dt/\tau_e$ and also $P_{00}(t) = 1 - P_{01}(t)$. This leads to a simple differential equation: $dP_{01}(t)/dt = \tau_c^{-1} - (\tau_e^{-1} + \tau_c^{-1})P_{01}(t)$. If $P_{00}(0) = 1$, its solution is $P_{01}(t) = \tau_e(\tau_e + \tau_c)^{-1} [1 - \exp(-t/\tau_{eq})]$, where $1/\tau_{eq} = 1/\tau_e + 1/\tau_c$. Similar results can be performed leading to $P_{11}(t) = \frac{\tau_e}{\tau_e + \tau_c} [\tau_e + \tau_c \exp(-t/\tau_{eq})]$. Unless fluctuations on the current amplitude, whose average Δ depends on other microscopic factors of the device, this probability corresponds to the autocorrelation of the system $A(t) = \langle \sigma(0)\sigma(t) \rangle$, where $\sigma(t)$ corresponds to the state of the trap (occupied $\sigma = 0$ or empty $\sigma = 1$). In frequency domain this exponential decay of autocorrelation for one trap is described by Lorentzians, since the power spectrum density, i.e., the fourier transform of the autocorrelation is

$$S(f) = \int_{-\infty}^{\infty} e^{-2\pi f t i} A(t) dt = \frac{4\Delta^2}{(\tau_e + \tau_c)(\tau_{eq}^2 + 2\pi f^2)}$$

The known $1/f$ noise results from a sum of these Lorentzians (a contribution of the many traps in device). For more details about the origin of $1/f$ noise, see for example [7],[4], [13], [14].

So, coming back to relaxation phenomena, and starting from $n(0) = 0$ (all traps empty), we can calculate the average density of occupied traps at time t

$$\begin{aligned} \langle n(t) \rangle &= \left\langle \sum_{k=0}^{N_{tr}} \sigma_i(t) \right\rangle \\ &= \sum_{k=0}^{N_{tr}} k \Pr(k|n(0), t) \end{aligned} \tag{1}$$

where $\Pr(k|n(0), t)$ is the probability of just k traps are occupied at time t , with $k = 0 \dots N_{tr}$. But the traps have different constants τ_e and τ_c and from that, we write

$$\Pr(k|n(0), t) = \sum_{C_k} \prod_{i=1}^k P_{01}(\tau_c^{(d_i)}, \tau_e^{(d_i)}; t) \prod_{i=k+1}^{N_{tr}} P_{00}(\tau_c^{(d_i)}, \tau_e^{(d_i)}; t)$$

with C_k denoting every subset $\{d_1, d_2, \dots, d_k\}$ from $\{1, 2, \dots, n\}$. But $\{\tau_c^{(d_i)}, \tau_e^{(d_i)}\}_{i=1}^{N_{tr}}$ are statistically independent and identically distributed, and we have :

$$\begin{aligned} \overline{\Pr(k|n(0), t)} &= \sum_{C_k} \overline{P_{01}(\tau_c, \tau_e; t)^k} \cdot \overline{P_{00}(\tau_c, \tau_e; t)^{N_{tr}-k}} \\ &= \binom{n}{k} \overline{P_{01}(\tau_c, \tau_e; t)^k} \cdot \overline{P_{00}(\tau_c, \tau_e; t)^{N_{tr}-k}} \end{aligned}$$