$C_J \sim (\phi - \phi_c)^{-1}$, which also diverges at jamming (see Materials and Methods Section III E). However, the fluctuations of E_J defined as $\langle (\Delta E_J)^2 \rangle = T_J^2 C_J \sim T_J$ has the same behavior as the fluctuations of volume and pressure, vanishing at the jamming transition $T_J \to 0^+$ [$A \to 0^+$ in Eq. (10)].

II. CONCLUSIONS

We have suggested that the concept of " thermalization " at a compactivity and angoricity in jammed systems is reasonable by the direct test of ergodicity. The numerical results indicate that the full canonical ensemble of pressure and volume describes the observables near the jamming transition quite well. From a static thermodynamic viewpoint, the jamming phase transition does not present critical fluctuations characteristic of second-order transitions since the fluctuations of several observables vanish approaching jamming. The lack of critical fluctuations is respect to the angoricity and compactivity under isotropic compression in the jammed phase $\phi \to \phi_c^+$, which does not preclude the existence of critical fluctuations when accounting for the full range of fluctuations in the liquid to the jammed phase transition from below ϕ_c. Thus, a critical diverging length scale might still appear as $\phi \to \phi_c^-$ [29, 30]. Our results suggest an ensemble treatment of the jamming transition. One possible analytical route to use this formalism would be to incorporate the coupling between volume and coordination number at the particle level found in [16] together with similar dependence for the stress to solve the partition function at the mean field level. This treatment would allow analytical solutions for the observables with the goal of characterizing the scaling law near the jamming transition.

III. MATERIALS AND METHODS

A. System Information

The systems used for both, ensemble generation and molecular dynamic simulation, are the same. They are composed of 30 spherical particles in a periodic boundary box. The particles have same radius $r = 5\mu m$ and interact via a Hertz normal repulsive force without