function $\lambda(s) \neq 0$ such that

$$\phi_1(s, t_0) = 0, \ \phi_2(s, t_0) = \lambda(s) \cosh\theta, \ \phi_3(s, t_0) = \lambda(s) \sinh\theta. \tag{4.3}$$

Secondly, since $S(T) = \omega T, \ \omega \neq 0,$

$$\theta(s) = -\int_{s_0}^{s} \tau ds + \theta_0, \tag{4.4}$$

where s_0 is the starting value of arc length and $\theta = \theta(s)$. In this paper, we assume $s_0 = 0$.

Combining (4.2), (4.3) and (4.4), we have the following theorem.

Theorem 6 *A spacelike curve $r(s)$ with timelike binormal is a line of curvature on the surface $P(s,t)$ if and only if the followings are satisfied*

$\theta(s) = -\int_{s_0}^{s} \tau ds + \theta(0),$
$u(s, t_0) = v(s, t_0) = w(s, t_0) \equiv 0,$
$\phi_1(s, t_0) \equiv 0 \ , \ \phi_2(s, t_0) = \lambda(s) \cosh\theta, \ \phi_3(s, t_0) = \lambda(s) \sinh\theta.$

Now, we analyse two different types of the marching-scale functions $u(s,t)$, $v(s,t)$ and $w(s,t)$ in the Eq. (4.1).

(i) If we choose

$u(s,t) = \sum_{k=1}^{p} a_{1k} l(s)^k U(t)^k$, $v(s,t) = \sum_{k=1}^{p} a_{2k} m(s)^k V(t)^k$ and $w(s,t) = \sum_{k=1}^{p} a_{3k} n(s)^k W(t)^k$

then, we can simply express the sufficient condition for which the curve $r(s)$ is a line of curvature on the surface $P(s,t)$ as