From this, we can see once again that all edges in the weak order graph must be black. Indeed, we can argue just as in Subsection 4.1.2 that for $i < n$, if α_i is a non-compact imaginary root, it must be of type I. And since the cross action of s_{α_n} is to interchange c_{n-1} with c_{n+2}, and c_n with c_{n+1}, s_{α_n} reverses one of the two above strings of four consecutive signs in the event that α_n is non-compact imaginary for Q_γ. Thus $s_{\alpha_n} \times Q_\gamma \neq Q_\gamma$, and so all non-compact imaginary roots are of type I.

5.2.3 Example

With this combinatorial description of the orbit structure and the weak ordering in hand, consider the example $n = 3$. There are 10 orbits. See Figure B.12 of Appendix B for the weak order graph.

As usual, the closed orbits are parametrized by the clans consisting only of signs. To obtain an S-fixed representative of each, we simply take $w \in S_{2n}$ to be the permutation which assigns $\{1, \ldots, n\}$, in ascending order, to the coordinates of the $+$ signs, and $\{n+1, \ldots, 2n\}$, also in ascending order, to the coordinates of the $-$ signs. The skew-symmetry of the clan dictates that this gives a signed element of S_{2n}, which corresponds to flag $\left\langle e_{w(1)}, \ldots, e_{w(2n)} \right\rangle \in X$. We then take the signed permutation in W which corresponds to this signed element of S_{2n}. This signed permutation is the one which assigns, for $i = 1, \ldots, n$, $i \mapsto \pm i$, depending on whether the sign in position i is a $+$ or a $-$.

Since our formulas for classes of closed orbits are a bit complicated, we give a couple of examples. For the orbit $(+,+,+,-,-,-)$, take $w = id$. Since $g(w) = 0$,

$$[Q_{(+,+,+,-,-,-)}] = \begin{vmatrix} c_2 & c_3 \\ c_0 & c_1 \end{vmatrix}.$$