the 'standard basis' and $\{|+\rangle, |-\rangle\}$ as the 'dual basis'.

After all N qubits in the cluster state are prepared in the $|+\rangle^{\otimes N}$ state, nearest-neighbor qubits then interact via the two-qubit controlled-Z operations, denoted CZ. Here Z is the Pauli 'phase' operator. The other Pauli operators are the 'flip' operator X, the 'flip+phase' operator $Y = XZ$, and the identity operator $\mathbb{1}$. This operation is represented in the two-qubit standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ as $\mathrm{CZ} = \mathrm{diag}\{1, 1, 1, -1\}$. The unitary operation CZ is independent of whether the line in Fig. 1(a) is horizontal or vertical.

Fig. 1(b) shows that the horizontal vs vertical symmetry is in fact broken by the coupling axis of the charge qubit, which is represented in Fig. 1(b) as an excess charge in the left or right quantum dot. Although the charge-qubit coupling axis could be aligned independently of the orientation of the overall qubit lattice, we will treat the case that the charge-qubit coupling axis is in the x-direction. This is a physically reasonable case, and extending to the case of arbitrary alignment is involved but not difficult.

The charge qubit can be created as a semiconductor ddot structure[7]. Other alternatives exist such as the superconducting charge qubit[8] or a pair of dangling bonds on a surface[9]. In any case, the logical states typically correspond either to the left- and right-well occupancy by the excess charge or, alternatively, to the cases of symmetric or antisymmetric charge states between the two dots of a ddot charge qubit.

For coherently evolving charge qubits, Schrödinger's equation can be used to describe the dynamics, and the potential in Schrödinger's equation is depicted in Fig. 1(c). Here we treat the standard basis as corresponding to left- and right-occupancy; the dual basis then corresponds to the symmetric and antisymmetric charge-occupancy states.

The quantum dots are engineered so that each potential well has only one bound energy state for the excess electron. Due to the Pauli exclusion principle, the number of electrons in each well is either zero or one or else two electrons with opposite spins. The case of two excess electrons in one double-dot structure should be energetically forbidden by Coulomb repulsion between the two electrons to preserve the integrity of the charge qubit.

III. MODELING THE DYNAMICS

The goal is to have one excess electron per closely-spaced quantum dot pair, but the general picture is that each quantum dot can have one or two electrons. The restriction of one excess electron must emerge as an energetically favorable configuration rather than be imposed by fiat. The full second-quantized description of the electrons in the array of quantum dots is given by the extended Hubbard model (EHM).

A. The extended Hubbard model

The EHM applies to an array of quantum dots whose locations in a two-dimensional array are denoted by lattice coordinates. For $\hat{c}_{ij}$ the annihilation operator at dot site (i, j), $\hat{c}_{ij}^\dagger$ the conjugate creation operator, and $\hat{n}_{ij} = \hat{c}_{ij}^\dagger \hat{c}_{ij}$ the number operator, the dynamics of the charge-qubit cluster state can conveniently be described by the extended Hubbard Hamiltonian[10] (As spin is conserved, we can, without loss of generality, assume fixed spin and ignore this degree of freedom)

$$\hat{H} = \sum_{i,j} E\hat{n}_{ij} + \hat{V} + \sum_{i,j,i',j'} W_{ij,i'j'}\hat{n}_{ij}\hat{n}_{i'j'} - T_{ij,i'j'}\left(\hat{c}_{ij}^\dagger \hat{c}_{i'j'} + \text{h.c.}\right). \tag{1}$$

Here E is the effective on-site energy for each site (i, j), which can vary due to local field effects. $T_{ij,i'j'}$ is the coherent tunneling rate between sites (i, j) and (i', j'). $W_{ij,i'j'}$ is the Coulomb repulsion energy between sites (i, j) and (i', j'). Finally, for $\widehat{\Delta n}_{ij,i'j'} := \hat{n}_{ij} - \hat{n}_{i'j'}$ the number-difference operator between sites (i, j) and (i', j'), the potential bias operator is

$$\hat{V} = \frac{1}{2}\sum_{i,j,i',j'} V_{ij,i'j'}\widehat{\Delta n}_{ij,i'j'} \tag{2}$$

with $V_{ij,i'j'}$ the inter-site $(i, j) \leftrightarrow (i', j')$ potential difference.

In fact Eq. (1) describes not just nearest-neighbor interactions but interactions between all dots with all other dots, where the inter-dot couplings $T_{ij,i'j'}$ and $W_{ij,i'j'}$ are suitably chosen. For charge qubits corresponding to closely spaced dot pairs, $T_{ij,i'j'}$ can be neglected for all but the ddot of a given charge qubit. Also $W_{ij,i'j'}$ is only significant between charge qubits. The interdot (possibly screened) Coulomb repulsion is neglected for the ddots of a charge qubit because $W_{ij,ij}$ is sufficiently large to prevent both dots from being simultaneously excessively charged.

B. Single-qubit gates

Eq. (1) is a second-quantized Hamiltonian. To bridge this Hamiltonian over to the multi-qubit description, we restrict the Hilbert space, upon which the first-quantized version of the Hamiltonian acts, to the case of a single excess electron in each double well. Note here that a dot has lattice coordinates expressed here as (i_1, j_1) and dot 2 is at (i_2, j_2).

For an array with close proximity between dots of the ddot pair, the resultant charge-qubit ddot pair can be treated as a point-like object in the quantum computing architecture. This ddot charge qubit has a point-like coordinate designated by $(\mathfrak{m}, \mathfrak{n})$ where the change of font is