

Evidently, ν and μ have identical marginal distributions: $\nu_{X_i} = \mu_{X_i}$, and also identical joint distributions of any consecutive pairs: $\nu_{X_i, X_{i+1}} = \mu_{X_i, X_{i+1}}$. Therefore

$$\nu_{X_i}(\{0\}) = \mu_{X_i}(\{0\}) \quad (100)$$

and

$$\nu_{X_i, X_{i+1}}(\{x_i = 0, x_{i+1} \neq 0\}) = \mu_{X_i, X_{i+1}}(\{x_i = 0, x_{i+1} \neq 0\}). \quad (101)$$

Since $\mu \in \Lambda(\gamma, q, c)$, we have $\nu \in \Lambda(\gamma, q, c)$. Let $\{X_i\}$ follow distribution μ and $\{Z_i\}$ follow distribution ν . Then

$$I(Z_1; Z_2^\infty) = I(Z_1; Z_2) + I(Z_1; Z_3^\infty | Z_2) \quad (102)$$

$$= I(Z_1; Z_2) \quad (103)$$

$$= I(X_1; X_2) \quad (104)$$

$$\leq I(X_1; X_2^\infty) \quad (105)$$

where equality holds if and only if $\{X_i\}$ is a first-order Markov process. By (11) and (105), $L(\nu) \geq L(\mu)$. So for any μ which maximizes $L(\mu)$, ν can be generated from μ by (99) with $L(\nu) \geq L(\mu)$. $L(\mu)$ must be maximized by a first-order Markov process.

Property (c): Suppose ν is a stationary first-order Markov process, sufficiently denote as $\nu = \{\mathcal{X}, P_{X_2|X_1}\}$, where \mathcal{X} is the state space of ν and $P_{X_2|X_1}$ is the transition probability distribution. Define a new first-order Markov process $\bar{\nu}$ from ν as follows.

Definition 1: Let $\bar{\nu}$, defined on the same state space \mathcal{X} as ν , be a first-order Markov process denoted by $(\mathcal{X}, P_{Z_2|Z_1})$, where

$$P_{Z_2|Z_1}(z_2|z_1) = \begin{cases} \alpha & z_1 = 0 \ z_2 = 0, \\ 1 - \beta & z_1 \neq 0 \ z_2 = 0, \\ \frac{1 - \alpha}{\eta} P_X(z_2) & z_1 = 0 \ z_2 \neq 0, \\ \frac{\beta}{\eta} P_X(z_2) & z_1 \neq 0 \ z_2 \neq 0, \end{cases} \quad (106)$$