The sum in (3.11) can be regularized by analytical continuation. We will write it in the following form, using symmetry properties with respect to $m \leftrightarrow -m$ and $j \leftrightarrow -j$:

$$\lim_{s\to 0^+}\sum_{m=0}^{\infty}{}' \sum_{j=0}^{\infty}{}' \frac{1}{\tilde{\varphi}^{1+s}} = \frac{1}{4}\lim_{s\to 0^+}\sum_{m,j\in\mathbb{Z}} \frac{1}{\tilde{\varphi}^{1+s}}$$
$$= \frac{1}{4} + \frac{1}{4}\mathcal{S}(\tau,p) + \frac{1}{2}\mathcal{K}(\tau). \quad (3.12)$$

Here we have defined

$$\mathcal{S}(\tau,p) = \lim_{s\to 0^+}\sum_{m,j\in\mathbb{Z}}{}'' \frac{1}{\varphi^{1+s}} = \mathcal{S}(p,\tau), \quad (3.13)$$

wherein the double prime on the summation mark means that the term $m = j = 0$ is explicitly excluded, and

$$\mathcal{K}(\tau) = \lim_{s\to 0^+}\sum_{j=1}^{\infty}\left[\frac{1}{\tilde{\varphi}^{1+s}} - \frac{1}{\varphi^{1+s}}\right]_{m=0}$$
$$= \sum_{j=1}^{\infty}\frac{j\tau - \sqrt{1+j^2\tau^2}}{j\tau\sqrt{1+j^2\tau^2}}. \quad (3.14)$$

Clearly, $\mathcal{K}(\tau)$ is finite for all $\tau > 0$.

The function $\mathcal{S}(\tau,p)$ may be regularized by use of the Chowla-Selberg formula [see e.g. Eq. (4.33) of Ref. [40]]

$$\sum_{m,j\in\mathbb{Z}}{}''(am^2+bmj+cj^2)^{-q} = 2\zeta(2q)a^{-q}$$
$$+ \frac{2^{2q}\sqrt{\pi}a^{q-1}\Gamma(q-\frac{1}{2})\zeta(2q-1)}{\Gamma(q)\Delta^{q-\frac{1}{2}}}$$
$$+ \frac{2^{q+\frac{5}{2}}\pi^q}{\Gamma(q)\Delta^{\frac{1}{2}(q-\frac{1}{2})}\sqrt{a}}\sum_{l=1}^{\infty} l^{q-\frac{1}{2}}\sigma_{1-2q}(l)$$
$$\times \cos(l\pi b/a)K_{q-\frac{1}{2}}(\pi l\sqrt{\Delta}/a), \quad (3.15)$$

where

$$\Delta = 4ac - b^2, \quad (3.16)$$
$$\sigma_w(l) = \sum_{\nu|l}\nu^w, \quad (3.17)$$

where ν are summed over the divisors of l and it is assumed that $\Delta > 0$. K is again the modified Bessel function of the second kind. The apparent pole as $q \to \frac{1}{2}$ now vanishes due to a cancellation between the first two terms of (3.15), and we find that letting $q = \frac{1}{2} + \frac{s}{2}$ and taking the limit $s \to 0^+$ (here $a = p^2, b = 0, c = \tau^2$)

$$\mathcal{S}(\tau,p) = \frac{2}{p}(\gamma - \ln\frac{4\pi p}{\tau}) + \frac{8}{p}\sum_{l=1}^{\infty}\sigma_0(l)K_0(2\pi l\tau/p) \quad (3.18)$$

where $\gamma = 0.577216...$ is Euler's constant. Now $\sigma_0(l)$ is simply the number of positive divisors of l, $\sigma_0(1) = 1, \sigma_0(2) = \sigma_0(3) = 2, \sigma_0(4) = 3$ etc. Note that Eq. (3.18) is valid for all τ; although it appears most convenient for large τ, it is, by the symmetry property seen in Eq. (3.13), equally useful for small τ.

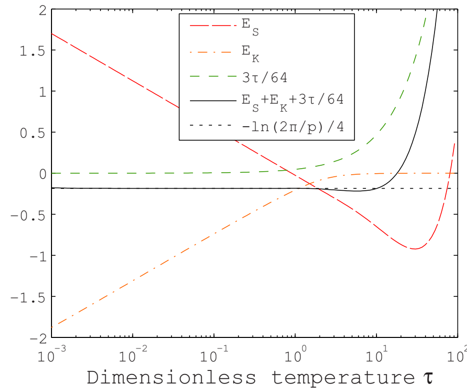

FIG. 2: The additional terms of the regularized energy in Eq. (3.20) which are subtracted from the double sum there in the case $p = 3$. Shown also is the sum of the three additional terms and their low-temperature asymptotic value from Eq. (4.11).

We finally write down the final, regularized energy of the wedge (and, simultaneously, cylinder) at finite T, using the convention used in Ref. [34]

$$\tilde{\mathcal{E}}(\tau,p,a) = \frac{1}{8\pi n a^2}e(\tau,p), \quad (3.19)$$

in terms of

$$e(\tau,p) = \frac{4\tau}{\pi}\sum_{m=0}^{\infty}{}'\sum_{j=0}^{\infty}{}'\tilde{e}_{m,j}(\tau,p)$$
$$- \frac{\tau}{64}(3 - 3\tau\partial_\tau + \tau^2\partial_\tau^2)[1 + 2\mathcal{K}(\tau) + \mathcal{S}(\tau,p)]. \quad (3.20)$$

with $\tilde{e}_{m,j}, \mathcal{K}$, and $\mathcal{S}$ given in Eqs. (3.5), (3.14) and (3.18), respectively. The differentiations with respect to τ are now straightforward, should the full expanded expression be desirable.

In Fig. 2 we plot the three additional terms in the second line of Eq. (3.20) where we have defined the shorthand

$$\hat{\mathcal{T}} = (3 - 3\tau\partial_\tau + \tau^2\partial_\tau^2); \quad (3.21a)$$
$$E_S(\tau,p) = \frac{\tau}{64}\hat{\mathcal{T}}\mathcal{S}(\tau,p); \quad E_K(\tau,p) = \frac{\tau}{32}\hat{\mathcal{T}}\mathcal{K}(\tau). \quad (3.21b)$$

Figure 3 shows a numerical calculation of $e(\tau, p = 3)$ as a function of τ along with its high and low τ asymptotes (see derivations in the following sections). The calculation was performed by "brute force" by truncating the sums after a number of terms, and has somewhat limited accuracy due to the large number of terms in the j sum in Eq. (3.20) required for small τ, scaling as τ^{-1}.