values of $\tilde{\vec{q}}$ are within the first Brillouin zone of the whole lattice. By using the rules of transition between the sums over the sublattices and the sum over the whole lattice one can obtain

$$A_{\vec{q}} = \left(1 - \frac{1}{\sqrt{2}}\right)\sigma\left(0\right) - \frac{1}{2^{5/2}}\sigma\left(\tilde{\vec{q}}\right) + \beta,$$
$$C_{\vec{q}} = -\frac{3i}{2^{5/2}}\sigma_{\mathrm{c}}(\tilde{\vec{q}})\,,\ D_{\vec{q}} = \frac{3}{2}\left(\sigma_{\mathrm{c}}(\vec{q}) + \frac{i}{2^{3/2}}\sigma_{\mathrm{c}}\left(\tilde{\vec{q}}\right)\right),$$
$$F_{\vec{q}} = \frac{1}{2}\left[\sigma\left(\vec{q}\right) - \frac{1}{2^{3/2}}\sigma\left(\tilde{\vec{q}}\right)\right]\,. \quad (19)$$

Thus, the Hamiltonian coefficients describing small oscillations of the AFM state do not include new dipole sums different from those for the FM case and could be expressed through the sums by means of cumbersome, but simple geometrical transformations. It is necessary to note that the Hamiltonian coefficients $F_{\vec{q}}$ and $D_{\vec{q}}$ in the terms like $a_{\vec{q}}b_{-\vec{q}}$, $a_{\vec{q}}b^{\dagger}_{\vec{q}}$, responsible for interaction between sublattices, are periodic relative to vectors of the reciprocal lattice of the whole system, i.e. they have lower symmetry than the coefficients responsible for interaction within the sublattice. However, it can be proofed that in final expressions for collective mode frequencies the translation symmetry of the reciprocal lattice of period $\pi/a\sqrt{2}$ is restored.

B. Dispersion relation.

For diagonalization of the Hamiltonian (17) one can use the generalized Bogolyubov $u-v$ transformation and introduce the creation and annihilation operators of magnons of two branches, $c^{\dagger}_{\vec{q}}$, $c_{\vec{q}}$ and $d^{\dagger}_{\vec{q}}$, $d_{\vec{q}}$, and the creation and annihilation operators of different branches commute, $[c_{\vec{q}}, d_{\vec{q}}] = 0$, $[c_{\vec{q}}, d^{\dagger}_{\vec{q}}] = 0$. For normal modes $\dot{c}_{\vec{q}} = -i\omega_{(-)}(\vec{q})c_{\vec{q}}$, $\dot{d}_{\vec{q}} = -i\omega_{(+)}(\vec{q})d_{\vec{q}}$, where $\omega_{(-)}(\vec{q})$ and $\omega_{(+)}(\vec{q})$ are the frequencies of magnon modes. The generalized Bogolyubov transform can be written as

$$a_{\vec{q}} = u_{\vec{q}}c_{\vec{q}} + v^{*}_{\vec{q}}c^{\dagger}_{-\vec{q}} + u'_{\vec{q}}d_{\vec{q}} + v'^{*}_{\vec{q}}d^{\dagger}_{-\vec{q}}\,,$$
$$b_{\vec{q}} = \xi_{\vec{q}}c_{\vec{q}} + \eta^{*}_{\vec{q}}c^{\dagger}_{-\vec{q}} + \xi'_{\vec{q}}d_{\vec{q}} + \eta'^{*}_{\vec{q}}d^{\dagger}_{-\vec{q}}\,. \quad (20)$$

Comparing the equations of motion for the operators $c_{\vec{q}}$, $d_{\vec{q}}$ (for example, $\dot{c}_{\vec{q}} = -i\omega_{(-)}(\vec{q})c_{\vec{q}}$) and the operators $a_{\vec{q}}$, $b_{\vec{q}}$ ($i\hbar\dot{a}_{\vec{q}} = [a_{\vec{q}}, \hat{H}]$), the system of equations for the coefficients is presented as a unitary transformation: