in powers of , i.e. $\theta^<(\mathbf{x},t) \approx \theta^<(\mathbf{r},t) + \xi\nabla\theta^<(\mathbf{r},t) + \frac{1}{2}\xi_\alpha\xi_\beta\frac{\partial\theta^<(\mathbf{r},t)}{\partial r_\alpha \partial r_\beta}$. Then

$$\frac{g}{a^2}\cos(\theta^<(\mathbf{r},t))\theta_1^>(\mathbf{r},t) \approx \frac{C_2}{8\pi}\frac{g^2}{\lambda}\frac{\delta\Lambda}{\Lambda}\triangle\theta - \frac{C_1}{4\pi}\frac{g^2}{\lambda^2\mu}\frac{\delta\Lambda}{\Lambda}\ddot{\theta}, \tag{A17}$$

where

$$\frac{C_1}{\Lambda^2} = \int d^2\xi\, J_0(\Lambda\xi), \quad \frac{C_2}{\Lambda^4} = \int d^2\xi\, \xi^2 J_0(\Lambda\xi). \tag{A18}$$

When we need to substitute these expressions back into Eq. (A9), we find that the term containing $\triangle\theta$ renormalizes the coupling λ as

$$\lambda \to \lambda + \frac{C_2}{8\pi}\frac{g^2}{\lambda}\frac{\delta\Lambda}{\Lambda}. \tag{A19}$$

In addition there is an extra term proportional to $\ddot{\theta}$ generated in Eq. (A9), which renormalizes μ:

$$\frac{1}{\mu} \to \frac{1}{\mu} + \frac{C_1}{4\pi}\frac{g^2}{\lambda^2\mu}\frac{\delta\Lambda}{\Lambda}. \tag{A20}$$

Finally we restore the cutoff by rescaling $\mathbf{k} \to \mathbf{k}(1 - \delta\Lambda/\Lambda)$, $\mathbf{r} \to \mathbf{r}(1+\delta\Lambda/\Lambda)$, $t \to t(1+\delta\Lambda/\Lambda)$, and $p \to p(1-\delta\Lambda/\Lambda)$. This rescaling additionally renormalizes the coupling g: $g \to g(1+2\delta\Lambda/\Lambda)$. Combining this result with Eqs. (A11), (A19), (A20) we find the following renormalization group equations:

$$\frac{dg}{dl} = g\left(2 - \frac{1}{4\pi v_0}\frac{1}{K}\right) \tag{A21}$$

$$\frac{dK}{dl} = \frac{1}{16\pi K}\frac{g^2}{v^2}(C_2 + 2C_1), \tag{A22}$$

$$\frac{dv}{dl} = \frac{g^2}{16\pi v K^2}(C_2 - 2C_1). \tag{A23}$$

where $l = \ln\Lambda$. We can read off from Eq. A23, that if the system contains a fixed velocity, for example in relativistic systems, we need to have $C_2 = 2C_1$, to enforce that the velocity is invariant under the flow.

Note that if the initial system is already close to the critical point then the RG equations above simplify to

$$\frac{dg}{dl} \approx g\left(2 - \frac{1}{4\pi\lambda}\right), \tag{A24}$$

$$\frac{d\lambda}{dl} \approx \frac{C_2}{8\pi}\frac{g^2}{\lambda}, \tag{A25}$$

which are equivalent to Eqs. (40) and (41). Note that a more complete set of RG equations (A21) - (A23) has the same universal predictions of the dynamical phase transitions and exponential divergence of the time scales as the simplified equations above. Also note that the real RG equations bear close analogy to the flow equations in imaginary time characterizing the equilibrium Kosterlitz-Thouless transition[35]. Thus the non-equilibrium KT transition discussed here is characterized by exponentially divergent length and time scales. Physically these long scales characterize very slow process of vortex unbinding and equilibration at long distances. Note that the RG equations (A21) - (A23) also implicitly take into account renormalization of the temperature in the system. This comes from the fact that creating vortex-antivortex pairs removes the energy from the phonon degrees of freedom. We are going to investigate this issue in more detail in a separate publication.

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