We now present conditions under which the empirical projection onto a tensor-product wavelet basis is stable wpa1. Here the projection operator is

$$P_{K,n}h(x) = b^K(x)' \left(\frac{B'B}{n}\right)^{-} \frac{1}{n}\sum_{i=1}^{n} b^K(X_i)h(X_i)$$

where the elements of b^K span the tensor products of d univariate spaces $\text{Wav}(K_0, [0,1])$. The following Theorem states simple sufficient conditions for $\|P_{K,n}\|_\infty \lesssim 1$ wpa1.

Theorem 5.2 *Let conditions stated in Theorem 5.1 hold. Then $\|P_{K,n}\|_\infty \lesssim 1$ wpa1 provided that either (a), (b), or (c) is satisfied:*

(a) $\{X_i\}_{i=1}^n$ are i.i.d. and $\sqrt{(K \log n)/n} = o(1)$

(b) $\{X_i\}_{i=1}^n$ are exponentially β-mixing and $\sqrt{K(\log n)^2/n} = o(1)$, or

(c) $\{X_i\}_{i=1}^n$ are algebraically β-mixing at rate γ and $\sqrt{(K \log n)/n^{\gamma/(1+\gamma)}} = o(1)$.

6 Brief review of B-spline and wavelet sieve spaces

We first outline univariate B-spline and wavelet sieve spaces on $[0,1]$, then deal with the multivariate case by constructing a tensor-product sieve basis.

B-splines B-splines are defined by their order $r \geq 1$ (or degree $r-1 \geq 0$) and number of interior knots $m \geq 0$. Define the knot set

$$0 = t_{-(r-1)} = \ldots = t_0 \leq t_1 \leq \ldots \leq t_m \leq t_{m+1} = \ldots = t_{m+r} = 1\,. \tag{31}$$

We generate a L^∞-normalized B-spline basis recursively using the De Boor relation (see, e.g., Chapter 5 of DeVore and Lorentz (1993)) then appropriately rescale the basis functions. Define the interior intervals $I_1 = [t_0, t_1), \ldots, I_m = [t_m, t_{m+1}]$ and generate a basis of order 1 by setting

$$N_{j,1}(x) = 1_{I_j}(x) \tag{32}$$