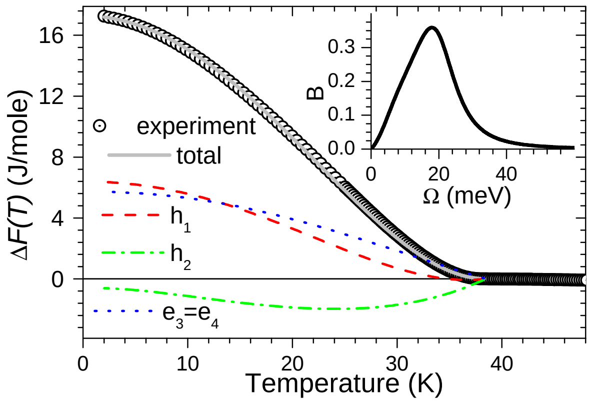

FIG. 4: Free-energy difference between normal and superconducting states, experimentally obtained (open circles) and calculated from the four-band Eliashberg model (light gray line). The dashed, dash-dotted, and dotted lines represent the partial contributions of the individual bands. Inset: Spin-fluctuation coupling function $B(\Omega)$.

namical spin susceptibility of $\mathrm{Ba(Fe_{1-x}Co_x)_2As_2}$ [27, 28]. Here λ_{ij} is the coupling constant for pairing of electrons in bands i and j. For our calculations we use the intraband coupling matrix elements λ_{ii} = 0.2 in order to take account of the weak electron-phonon contribution [1], for interband repulsion $\lambda_{12} = \lambda_{34} = 0$ due to the symmetry of the wave functions, and $\lambda_{13} = \lambda_{14} = -1.0$, $\lambda_{23} = \lambda_{24} = -0.2$ ($\lambda_{ji} = \lambda_{ij}N_i^{h,e}(0)/N_j^{h,e}(0)$). The corresponding densities of states are taken as $N_1^h(0)$ = 29 Ry^{-1}, $N_2^h(0)$ = 43 Ry^{-1} for the hole bands, and as $N_3^e(0) = N_4^e(0)$ = 8.5 Ry^{-1} for the two equivalent electron bands [14]. The chosen parameters, $\lambda_{ij}(i \neq j)$ and $N_i^{h,e}(0)$, allow the best simultaneous fit to the experimental values of T_c, the superconducting gaps, and the temperature dependence of the free-energy difference, yielding T_c = 38.5 K, and the following gap values: Δ_1^h = -8.5 meV, Δ_2^h = -3.6 meV for hole bands, and $\Delta_3^e = \Delta_4^e$ = 9.2 meV for electronic ones. The effective coupling constant averaged over all bands, $\lambda^{av} = \sum_{ij} N_i^{h,e}(0)\lambda_{ij}/N_{tot}$, has a value of 1.9, remarkably close to the coupling constant reported for $\mathrm{Pb_{0.8}Bi_{0.2}}$ [19] and in agreement with the conclusions of our phenomenological analysis presented in Fig. 3. The consistency with the experiment tolerates some variation of the parameters and remains satisfactory for $\hbar\Omega_{SF}^{\max}$ within the 10 - 20 meV energy range [14]. A shift of $\hbar\Omega_{SF}^{\max}$ to lower energy leads to a larger values of λ^{av} with $N_i^{h,e}(0)$ approaching the results of density functional theory (DFT) [17]. This accounts for the strong renormalization of the Sommerfeld constant $\gamma_K^N/\gamma_{DFT} \sim 5$.

The superconductivity-induced free-energy difference, $\Delta F(T)$, was calculated by using the expressions in Ref. 21. The result is presented in Fig. 4, along with the partial band contributions to $\Delta F(T)$ which demonstrate that the superconductivity in the second hole band has an induced origin. Fig. 4 also shows that the result of the model calculation is in fairly good agreement with the free-energy difference obtained by integrating the experimentally measured $\Delta C_{el}(T)$, while some deviations between the calculated and observed specific heat can be seen in Fig. 2. Some such deviations are expected because feedback effects of superconductivity on the bosonic spectral function, which lead to the formation of a temperature-dependent "resonant mode" in the spin fluctuation spectrum below T_c [27], have not been considered in the calculations.

In summary, the specific heat of hole-doped $\mathrm{Ba_{0.68}K_{0.32}Fe_2As_2}$ was measured on crystals of superior quality and phase purity. Our α-model fit to the electronic part of the specific heat shows that there are two groups of superconducting gaps with magnitudes of 11 and 3.5 meV. The magnitude and shape of the superconducting jump anomaly indicate strong coupling with intermediate bosons, in good agreement with an Eliashberg analysis of a spin-fluctuation model.

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