A. Superconductivity in the dilute Fermion Gas with varying Interactions: Theory and Experiments for Cold Atoms

The advent of the technique of cooling atoms in atomic traps has generated (among other things) a new class of Fermi-liquid in which the particle density ν is low but the inter-particle interactions U can be varied from very weak to very strong. The *effective* particle-particle interaction in the low density limit is completely specified by the scattering length a so that all physical properties are functions of the dimensionless coupling strength

$$\kappa = \frac{1}{k_f a}, \tag{30}$$

where k_f is the magnitude of the fermi-vector. The theoretical deduction of the effective interaction in this situation is simpler than for other fermi-liquids. The upper cut-off of interaction energies is the Fermi-energy. The essentially exact calculations of T_c/E_f possible for this problem provide a measure of the highest value to be expected from e-induced s-wave pairing for the complicated situations. It is not coincidental that the maximum T_c/θ_D realized in e-ph superconductors approaches the highest value in the calculations summarized below.

The t-matrix approximation for effective interactions, which is exact in the low density limit, gives that the the scattering length a is given in terms of the parameters of the Hubbard model by [5]

$$\frac{m}{4\pi a} = U^{-1} + \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{1}{2\epsilon(\mathbf{k})} \equiv U^{-1} - U_*^{-1}. \tag{31}$$

Here U is the interaction parameter in the Hubbard model, < 0 to give s-wave pairing, $\epsilon(\mathbf{k}) = k^2/2m$, and $U_*^{-1} = -\pi l_0 m$; $2\pi/l_0$ is the upper cut-off in the integral over $\mathbf{k}$ introduced to avoid the ultraviolet divergence.

The weak-coupling limit (BCS-limit), when the attractive interaction strength is negligible compared to the kinetic energy is given by $\kappa \to -\infty$ and the opposite or Bose molecular limit by $\kappa \to +\infty$. In between is the unitarity limit $\kappa \to 0$ where $a \to \infty$. Universal results are to be expected for physical properties for all low density attractive interaction models in these three limits. These limits are realized in experiments by tuning the interactions through the *Feshbach resonance*.

Monte-carlo methods have been used to calculate [5] T_c/E_f both for interactions for free fermions around the unitarity limit as well as for attractive interactions in the Hubbard