

In (3.17), we used the equality $\operatorname{Re} \bar{z} \nabla_y z = \frac{1}{2} \nabla_y (|z|^2)$.

$$\begin{aligned}
I_5 &= 4s \operatorname{Re} \int_{\Omega} \Delta_x z \nabla_y \varphi \cdot \nabla_y \bar{z} dx dy dt - 4s \operatorname{Re} \int_{\Omega} \Delta_x z \nabla_x \varphi \cdot \nabla_x \bar{z} dx dy dt \\
&= -4s \operatorname{Re} \int_{\Omega} \sum_{j=1}^m \sum_{i=1}^n \varphi_{y_i x_j} \bar{z}_{y_i} z_{x_j} dx dy dt - 2s \int_{\Omega} \sum_{i=1}^m \Delta_y \varphi |\nabla_x z|^2 dx dy dt \\
&\quad - 2s \int_{\Gamma_y} (\partial_{\nu} \varphi) |\nabla_x z|^2 dS_y dx dt + 4s \operatorname{Re} \int_{\Gamma_x} (\partial_{\nu} z) \nabla_y \varphi \cdot \nabla_y \bar{z} dS_x dy dt \\
&\quad + 4s \operatorname{Re} \int_{\Omega} \sum_{i,j=1}^n \varphi_{x_i y_j} \bar{z}_{x_i} z_{y_j} dx dy dt - 2s \int_{\Omega} \Delta_x \varphi |\nabla_x z|^2 dx dy dt \\
&\quad - 2s \int_{\Gamma_x} (\partial_{\nu} \varphi) |\nabla_x z|^2 dS_x dy dt \\
&\quad + 4s \operatorname{Re} \int_{\Gamma_x} (\partial_{\nu} z) \nabla_x \varphi \cdot \nabla_x \bar{z} dS_x dy dt.
\end{aligned} \tag{3.18}$$

In (3.18), we used the equality $\operatorname{Re} z_{x_j} \bar{z}_{x_j y_i} = \frac{1}{2} (|z_{x_j}|^2)_{y_i}$.

$$\begin{aligned}
I_6 &= 2s \operatorname{Re} \int_{\Omega} \Delta_x z (\Delta_y \varphi - \Delta_x \varphi) \bar{z} dx dy dt \\
&= -2s \int_{\Omega} (\Delta_y \varphi - \Delta_x \varphi) |\nabla_x z|^2 dx dy dt \\
&\quad + s \int_{\Omega} \Delta_x (\Delta_y \varphi - \Delta_x \varphi) |z|^2 dx dy dt \\
&\quad - s \int_{\Gamma_x} \partial_{\nu} (\Delta_y \varphi - \Delta_x \varphi) |z|^2 dS_x dy dt \\
&\quad + 2s \operatorname{Re} \int_{\Gamma_x} (\partial_{\nu} z) (\Delta_y \varphi - \Delta_x \varphi) \bar{z} dS_x dy dt.
\end{aligned} \tag{3.19}$$