THE DETERMINACY OF INFINITE GAMES WITH EVENTUAL PERFECT MONITORING

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Abstract. An infinite two-player zero-sum game with a Borel winning set, in which the opponent's actions are monitored eventually but not necessarily immediately after they are played, is determined. The proof relies on a representation of the game as a stochastic game with perfect information, in which Chance operates as a delegate for the players and performs the randomizations for them, and on Martin's Theorem about determinacy of such games.

1. Setup

Consider an infinite two-player zero-sum game that is given by a triple $\left(A,(P_n)_{n\in\mathbb{N}},W\right)$ where A is a finite set of *actions*, P_n is a partition of A^n for every $n\in\mathbb{N}$, the *information partition of stage* n, and $W\subseteq A^{\mathbb{N}}$ is a Borel set, the *winning set* of player 1. The game is played in stages: player 1 chooses an action $a_0\in A$; then player 2 chooses an action $a_1\in A$; then player 1 chooses an action $a_2\in A$, and so on, ad infinitum. Before choosing a_n, the player who plays at stage n receives some information about the actions of previous stages: Let $h=(a_0,a_1,\ldots,a_{n-1})$ be the *finite history* that consists of the actions played before stage n; then before choosing a_n, the player who plays at stage n observes the atom of P_n that contains h. Player 1 wins the game if the *infinite history* $(a_0,a_1,\ldots)$ is in W. When the action set and information partitions are fixed, I denote the game by $\Gamma(W)$.

A *behavioral strategy* $x=(x_n)_{n\in\mathbb{N}}$ of player 1 is a sequence $\{x_n:P_n\to\Delta(A)\}_{n=0,2,4,\ldots}$ of functions: At stage n, after observing the finite history $h=(a_0,a_1,\ldots,a_{n-1})$, player 1

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