BCH code, the $[[15,7,5;7]]$ and the $[[15,7,5;6]]$ EAQEC codes have 4 more information qubits at the cost of 3 and 2 more ebits, respectively. The $[[15,7,6;8]]$ EAQEC code has 4 more information qubits and a higher minimum distance at the cost of 4 more ebits. In addition, the $[[15,7,6;8]]$ EAQEC code is not equivalent to any known standard quantum stabilizer code.

On the other hand, the classical linear quaternary $[15,9,5]$ code and $[15,8,6]$ code in Grassl's table can be used to construct a $[[15,9,5;6]]$ EAQEC code and a $[[15,9,6;8]]$ EAQEC code by the construction of [10], respectively. These two codes are better than the $[[15,7,6;8]]$ EAQEC code we obtained. This may be because these codes were not fully optimized, but BCH codes in any case need not give the best possible EAQEC codes, even using the encoding optimization procedure.

$\square$

Example 8. We applied the random search algorithm to Gottesman's $[[8,3,3]]$ code [4] and the $[[13,1,5]]$ quantum QR code [2, 18], and the results are shown in Table 8 and Table 9, respectively. By the construction of [10], the $[8,3,5]$, $[13,3,9]$, $[13,4,8]$, $[13,5,7]$ classical linear quaternary codes in Grassl's Table can be transformed to $[[8,2,5;4]]$, $[[13,3,9;10]]$, $[[13,0,8;5]$, $[[13,1,7;4]]$ EAQEC codes, respectively. Hence the $[[8,3,5;5]]$, $[[13,1,11;11]]$, $[[13,1,11;10]]$, $[[13,1,9;9]]$, $[[13,1,9;8]]$ EAQEC codes are new, and are not equivalent to any standard quantum stabilizer code. $\square$

Table 8: Optimization over Gottesman's $[[8,3,3]]$ code

c	d_{opt}	d_{std}	$2^{2ck}N(r,c)$
5	5	4	1.07×10^9
4	4	4	5.20×10^7
3	4	3	4.06×10^7
2	3	3	6.34×10^6