II. DYNAMICS OF COSMIC STRINGS

The dynamics of a cosmic string, whose width can be neglected, is described by the Nambu-Goto action,

$$S = -\mu \int d^2\zeta \sqrt{-\det(\gamma_{ab})}. \tag{1}$$

where ζ^a $(a = 0, 1)$ are coordinates on the world sheet of the cosmic string, $\gamma_{ab} = g_{\mu\nu} x^\mu_{,a} x^\nu_{,b}$ ($x^\mu_{,a} = \frac{\partial x^\mu}{\partial \zeta^a}$) is the induced metric on the world sheet, and μ is the tension of the string. The energy-momentum tensor is

$$T^{\mu\nu}(x) = \mu \int d^2\zeta \sqrt{-\det(\gamma_{ab})}\gamma^{ab} x^\mu_{,a} x^\nu_{,b} \delta^4(x - X(\zeta)), \tag{2}$$

where $X = X(\zeta)$ is embedding of the world sheet on the background metric. If the background space-time is Minkowski one, we can select the coordinate system $(\zeta^0, \zeta^1) = (\tau, \sigma)$ which satisfies the gauge conditions

$$\tau = t \text{ (physical time)}, \qquad x_{,\tau} \cdot x_{,\sigma} = 0, \qquad x^2_{,\tau} + x^2_{,\sigma} = 0. \tag{3}$$

The time scale of a GW burst is much shorter than the Hubble expansion, hence we consider an individual burst event on the Minkowskian background. The general solution of the equation of motion derived from the action (1) is

$$x^\mu = \frac{1}{2}(a^\mu(u) + b^\mu(v)), \ \mathbf{a}'^2(u) = \mathbf{b}'^2(v) = 1 \tag{4}$$

where $u = \sigma + t, v = \sigma - t$. We call $a(u)$ $(b(v))$ the left (right)-moving mode. Then, Eq. (2) can be rewritten in terms of $a(u)$ and $b(v)$,

$$T^{\mu\nu}(k) = \frac{\mu}{4}(I^\mu_+(k) I^\nu_-(k) + I^\mu_-(k) I^\nu_+(k)), \tag{5}$$

$$I^\mu_+(k) = \int du a'^\mu(u) e^{ik\cdot a(u)/2}, \ I^\mu_-(k) = \int dv b'^\mu(v) e^{ik\cdot b(v)/2}, \tag{6}$$

where $T^{\mu\nu}(k)$ is the Fourier transform of the $T^{\mu\nu}(x)$, i.e. $T^{\mu\nu}(k) = \int d^4x T^{\mu\nu}(x) e^{ik\cdot x}$.

III. DISTRIBUTION FUNCTION OF KINKS

Kinks can be defined as discontinuities of $\mathbf{a}'$ or $\mathbf{b}'$. They are produced when two infinite strings collide and reconnect because $\mathbf{a}'$ and $\mathbf{b}'$ on the new infinite string are created by