

On the other hand, (Hecht *et al.*, 2001) employ algebraic *Ansätze* for the elements in Eq. (VI.86) whose form is known even on the nonperturbative domain. They are determined by studies of meson properties (Burden *et al.*, 1996; Maris and Roberts, 1998; Roberts, 1996) and exhibit behavior that is broadly consistent with that of QCD’s  $n$ -point functions; viz., the ultraviolet power-laws are explicitly expressed but the logarithms are suppressed in order to achieve a level of simplicity. Hence, following the path described above, (Hecht *et al.*, 2001) delivers a representation of the pion’s distribution functions at an infrared resolving scale through the definitions

$$u_v^\pi(x; Q_0) := \frac{9}{2} F_1^+(x; Q_0), \quad d_v^\pi(x; Q_0) := 9 F_1^-(x; Q_0). \quad (\text{VI.91})$$

How should Eqs. (VI.91) be understood? In (Hecht *et al.*, 2001), the computation is interpreted within the setting of a rainbow-ladder DSE truncation. Hence, sea-quark contributions are absent because they cannot appear without nonperturbative dressing of the quark-gluon vertex (Chang *et al.*, 2009; Chang and Roberts, 2009; Cloët and Roberts, 2008). Thus Eqs. (VI.91) describe valence-quark distribution functions and one should have

$$\int_0^1 dx u_v^\pi(x; Q_0) = 1 = \int_0^1 dx \bar{d}_v^\pi(x; Q_0); \quad (\text{VI.92})$$

viz., the  $\pi^+$  contains one, and only one,  $u$ -valence-quark and one  $\bar{d}$ -valence-quark. Note that one has  $\bar{d}_v^{\pi^+}(x; Q_0) = u_v^{\pi^+}(x; Q_0) = d_v^{\pi^-}(x; Q_0)$  in this calculation.

It is a deficiency of this and kindred calculations that the model’s resolving scale is not determined *a priori*. That can be overcome by calculating the moments of the pion’s distribution via Eq. (VI.33), in which case the resolving scale is identical to the renormalization point used in computing the dressed Schwinger functions. The cost of that approach, however, is a loss of direct knowledge about the distribution’s pointwise evolution. In (Hecht *et al.*, 2001),  $Q_0$  was chosen so that the computed distribution, when evolved to  $Q'_0 = 2 \text{ GeV}$  using leading-order formulae, produced first and second moments in agreement with those reported in (Sutton *et al.*, 1992a). This yields<sup>19</sup>  $Q_0 = 0.54 \text{ GeV}$  so that Eqs. (VI.92) are satisfied with a valence-quark mass  $\tilde{M} = 0.30 \text{ GeV} \approx M(Q_0)$ .

It is apparent that this procedure views the distributions defined in Eqs. (VI.91) as infrared boundary value input for the valence-quark evolution equations. This is strictly valid

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<sup>19</sup> The quantitative similarity between this and the mass-scale for LO-evolution in Sec. V.C is noteworthy.