on the contrary, it produces a general equation that allows us to compute the correction terms in θ enclosed into $G(A_i, \pi_i, A_0, \theta)$, given in Eq. (2.21). To compute the correction term linear in θ, namely, $\mathcal{G}^{(1)}$, we pick up the following terms from the general relation (2.21) given by,

$$\int_x \left[\partial_l^w \left(m^2 A^l(x)\delta^{(3)}(\vec{x}-\vec{w}) + \frac{1}{2}F_{ij}(x)\frac{\partial F^{ij}(x)}{\partial A^l(w)}\right) + \frac{\partial \mathcal{G}^{(1)}(x)}{\partial\theta(w)}\right] = 0. \tag{3.24}$$

After a straightforward calculation, the correction term linear in θ is,

$$\mathcal{G}^{(1)} = -m^2\partial^i A_i\theta. \tag{3.25}$$

Substituting this result into the symplectic potential (3.15), we have that

$$\begin{aligned}\tilde{V}^{(0)} &= \frac{1}{2}{\pi_i}^2 + \frac{1}{4}F_{ij}^2 + \frac{1}{2}\,m^2\,{A_i}^2 - A_0(\partial_i\pi^i + \frac{1}{2}\,m^2\,A_0)\\ &- m^2\partial^i A_i\theta.\end{aligned} \tag{3.26}$$

However, the invariant formulation of the Proca model was not obtained yet because the contraction of the zero-mode (3.19) with the symplectic potential above does not generate a null value. Due to this, higher order correction terms in θ must be computed. For the quadratic term, we have,

$$\int_x \left[\partial_l^w \left(-m^2\theta(x)\partial_x^l\delta^{(3)}(\vec{x}-\vec{w})\right) + \frac{\partial\mathcal{G}^{(2)}(x)}{\partial\theta(w)}\right] = 0, \tag{3.27}$$

and after a direct calculation, we can write that,

$$\mathcal{G}^{(2)} = +\frac{1}{2}\,m^2\,(\partial_i\theta)^2\ . \tag{3.28}$$

Then, the first-order Lagrangian becomes,

$$\tilde{\mathcal{L}} = \pi^i\dot{A}_i + \dot{\theta}\Psi - \tilde{V}^{(0)}, \tag{3.29}$$

where the symplectic potential is

$$\begin{aligned}\tilde{V}^{(0)} &= \frac{1}{2}{\pi_i}^2 + \frac{1}{4}F_{ij}^2 + \frac{1}{2}\,m^2\,{A_i}^2 - A_0(\partial_i\pi^i + \frac{1}{2}\,m^2\,A_0)\\ &- m^2\partial^i A_i\theta + \frac{1}{2}m^2\ \ (\partial_i\theta)^2.\end{aligned} \tag{3.30}$$

Since the contraction of the zero-mode ($\bar{\nu}^{(0)}$) with the symplectic potential above does not produce a new constraint, a hidden symmetry is revealed.

To complete the gauge invariant reformulation of the Abelian Proca model, the infinitesimal gauge transformation will be computed also. In agreement with the symplectic formalism, the zero-mode $\bar{\nu}^{(0)}$ is the generator of the infinitesimal gauge transformation ($\delta\mathcal{O} = \varepsilon\bar{\nu}^{(0)}$), given by,

$$\begin{aligned}\delta A_i &= -\partial_i\varepsilon,\\ \delta\pi_i &= 0,\\ \delta A_0 &= 0,\\ \delta\theta &= \varepsilon,\end{aligned} \tag{3.31}$$

where ε is an infinitesimal time-dependent parameter. Indeed, for the above transformations the invariant Hamiltonian, identified as being the symplectic potential $\tilde{V}^{(0)}$, changes as

$$\delta\mathcal{H} = 0. \tag{3.32}$$

Now we will investigate the result from the Dirac point of view. The chains of primary constraints computed from the Lagrangian (3.29) are

$$\begin{aligned}\phi_1 &= \pi_0,\\ \chi_1 &= \partial_i\pi^i + \pi_\theta.\end{aligned} \tag{3.33}$$

Next, these constraints will be introduced into the invariant Hamiltonian (3.30) through the Lagrange multipliers and so it can be rewritten as

$$\tilde{V}^{(0)}_{primary} = \tilde{V}^{(0)} + \lambda_1\phi_1 + \gamma_1\chi_1. \tag{3.34}$$

The time stability condition for the primary constraint ϕ_1 requires a secondary constraint, such as

$$\phi_2 = \partial_i\pi^i + m^2 A_0, \tag{3.35}$$

and no more constraint appears from the time evolution of ϕ_2. Now, the total Hamiltonian is written as

$$\tilde{V}^{(0)}_{total} = \tilde{V}^{(0)} + \lambda_1\phi_1 + \lambda_2\phi_2 + \gamma_1\chi_1. \tag{3.36}$$

Since the time evolution of ϕ_1 just allows us to obtain the Lagrange multiplier λ_2, and the constraint χ_1 has no time evolution $\dot{\chi}_1 = 0$, no more constraints arise. Hence, the gauge invariant model has three constraints (ϕ_1, ϕ_2, χ_1). The nonvanishing Poisson brackets among these constraints are,