

Identifying the connected components of *all*  $K$ -orbits would amount to a parametrization of the  $\tilde{K}$ -orbits on  $X$ . Given such a parametrization, along with a description of the weak closure order, one could find formulas for the classes of all  $\tilde{K}$ -orbit closures, which would then give formulas for the classes of irreducible components of  $K$ -orbit closures. Formulas for  $K$ -orbit closures would follow by adding the formulas for the irreducible components. One might say that this would be a more complete solution to the problem at hand.

We justify our approach as follows: First, the  $K$ -orbits on  $X$  are simpler to parametrize, since they are precisely the intersections of  $K' = GL(2p, \mathbb{C}) \times GL(2q + 1, \mathbb{C})$ -orbits on the type  $A$  flag variety  $X'$  with the smaller flag variety  $X$ . Such intersections need not be single  $\tilde{K}$ -orbits on  $X$ ; some are, while others (e.g. the closed orbits) are unions of two  $\tilde{K}$ -orbits. It is not completely obvious how to determine precisely which intersections are single  $\tilde{K}$ -orbits, and which are not.

Second, due to the fact that the  $K$ -orbits are intersections of  $K'$ -orbits on  $X'$  with  $X$ , formulas for the classes of their closures pull back to Chern class formulas for degeneracy loci which admit identical linear algebraic descriptions to those in the type  $A$  case, but which involve a vector bundle equipped with a quadratic form and an *isotropic* flag of subbundles. So from this standpoint, our type  $B$  formulas have similar applications to those obtained in the type  $A$  case, where the symmetric subgroup in question was connected.

The author acknowledges, however, that it would be nice to have the  $\tilde{K}$ -orbit picture sorted out, since formulas for these classes would pull back to formulas for irreducible components of such degeneracy loci, giving more refined information. Thus describing the combinatorics of those orbits would likely be a worthwhile question to consider. However, we do not attempt to solve the problem in this generality here.

With all of these preliminary comments made, we turn our attention now to the ( $S$ -stable) connected components of closed  $K$ -orbits on  $X$ . As stated, these coincide with the closed  $\tilde{K}$ -orbits on  $X$ . We will denote the Weyl group for  $\tilde{K}$  (alternatively, for  $K^0$ , or for  $\text{Lie}(K)$ )