Thus we need only describe the situation for α_n. This root is complex for Q_γ (and $s_{\alpha_n} \cdot Q_\gamma \neq Q_\gamma$) if and only if c_n and c_{n+1} are unequal natural numbers, with the mate for c_n to the left of the mate for c_{n+1}. In this case, $s_{\alpha_n} \cdot Q_\gamma = Q_\delta$, where δ is obtained from γ by interchanging c_n and c_{n+1}.

On the other hand, α_n is non-compact imaginary for Q_γ if and only if c_n and c_{n+1} are opposite signs. In this case, $s_{\alpha_n} \cdot Q_\gamma = Q_{\delta'}$, where δ' is obtained from γ by replacing c_n and c_{n+1} by a pair of matching natural numbers. In this case, α_n is of type I, since the cross action of s_{α_n} is to interchange the opposite signs in positions $n, n+1$, so that $s_{\alpha_n} \times Q_\gamma \neq Q_\gamma$.

4.2.3 Example

With the parametrization and ordering spelled out, consider the example $n = 2$. There are 11 orbits. The weak order graph appears as Figure B.10 of Appendix B.

To obtain a representative of each closed orbit, we use the method of [Yam97, Theorem 3.2.11]. In the case of closed orbits, whose clans once again consist only of signs, this amounts to the following: Letting $\gamma = (c_1, \ldots, c_{2n})$, choose a permutation $\sigma \in S_{2n}$ with the following properties:

1. If $i \leq n$ and $c_i = +$, $\sigma(i) \leq n$.

2. If $i \leq n$ and $c_i = -$, $\sigma(i) > n$.

3. For $i = 1, \ldots, n$, $\sigma(2n+1-i) = 2n+1-\sigma(i)$.

Having chosen such a σ, the flag $F_\bullet = \langle v_1, \ldots, v_{2n} \rangle$, with $v_i = e_{\sigma(i)}$, is a representative of Q_γ. Note that any representative so obtained is S-fixed, so it is straightforward to apply Proposition 4.2.1 to compute the class $[Q_\gamma]$. Divided difference operators (scaled by factors of $\frac{1}{2}$ where appropriate) then give the remaining formulas. The results are given in Table B.10 of Appendix B.