will use this connection in Section V to obtain the effective space-time gauge action and compare it with the large N result.

From the expression (7) for the two-point function we determine the path- and field-dependent action for Z-particle propagation in the CP^{N-1} model to be (up to an overall constant which determines the scale of the proper time parameter τ)

$$S \propto \int_0^T d\tau \left(\left(\frac{dx^\mu}{d\tau} \right)^2 - \lambda(x) - \frac{dx^\mu}{d\tau} A_\mu(x) \right) \tag{8}$$

The string theoretic cutoff procedure described above consists of identifying the path action (8) as the $l \to 0$ limit of the worldsheet action for an open bosonic string coordinate $X(\sigma, \tau)$,

$$S_{string} = S_{bulk} + S_{boundary} \tag{9}$$

The bulk term is the usual CFT for a bosonic string coordinate, defined inside a rectangular area R of width l and length T in the complex $z \equiv \tau + i\sigma$ plane.

$$S_{bulk} = \int_0^T d\tau \int_0^l d\sigma \left(\left(\frac{dX^\mu}{d\tau} \right)^2 + \left(\frac{dX^\mu}{d\sigma} \right)^2 \right) = \int_R d^2 z \partial_a X^\mu \partial^a X_\mu \tag{10}$$

The worldsheet action also includes two boundary terms, corresponding to the spacetime fields $\lambda(X)$ and $A_\mu(X)$ interacting with one or both ends of the string, The most physically sensible formulation for our purposes is to put the Z-particle charge that interacts with the background fields at one end of the string ($\sigma = 0$) and impose Neumann boundary conditions at the other end ($\sigma = l$). (Physically, the charged particle still follows a definite path $x_\mu(\tau) = X_\mu(0, \tau)$ but now has a string attached to it.) Thus we take the boundary term to be along the real axis,

$$S_{boundary} = \int_0^T d\tau \left[\lambda(X) + \frac{dX^\mu}{d\tau} A_\mu(X) \right]_{\sigma=0} \tag{11}$$

(9)-(11) defines the worldsheet action for the coordinates of an open bosonic string in background tachyon and U(1) gauge fields.

The string theoretic cutoff for the CP^{N-1} model may also be understood by writing the one-loop vacuum amplitude for an open string in a spectral representation which sums over intermediate open string states. This spectral analysis also exposes the essential role of the open string tachyon, and its connection to the instability of the $\lambda = 0$ vacuum seen in the large N field theory. Let us begin by considering the one-loop vacuum string amplitude $G(x, x)$ without background fields (i.e. $\lambda = A_\mu = 0$). This amplitude has the form

$$G = \text{const.} \times \int \frac{d^2 p}{(2\pi)^2} \sum_i \frac{1}{p^2 + m_i^2} = \text{const.} \times \int \frac{d^2 p}{(2\pi)^2} \int d\tau \, e^{-\tau p^2} Z(e^{-\tau/4\alpha'}) \tag{12}$$