(ii) If $\eta \in \mathcal{H}$ is heteroclinic to ζ^1, $\zeta^2 \in \mathcal{H}$, then $\varphi_\eta(t) \in \mathcal{D}$ is heteroclinic to $\varphi_{\zeta^1}(t)$, $\varphi_{\zeta^2}(t) \in \mathcal{D}$;

(iii) If $\mathcal{H}$ is hyperbolic, then the same is true for $\mathcal{D}$.

The next section is devoted to a numerical example, which supports the results of Theorem 3.1.

4 An Example

Consider the following system,

$$\begin{aligned} y_1^{\Delta}(t) &= -(11/60)y_1(t) + (1/30)y_2(t) + 0.05\sin(\pi t/4) + g(t,\zeta), \\ y_2^{\Delta}(t) &= -(1/20)y_1(t) - (1/15)y_2(t) + 0.003\arctan(y_1(t)), \end{aligned} \tag{4.3}$$

where t belongs to the time scale $\mathbb{T}_0 = \bigcup_{k=-\infty}^{\infty}[\theta_{2k-1}, \theta_{2k}]$, $\theta_k = 4k + (5+(-1)^k)/2$ and $g(t,\zeta) = \zeta_k$ for $t \in [\theta_{2k-1}, \theta_{2k}]$, $k \in \mathbb{Z}$, such that $\zeta = \{\zeta_k\}_{k\in\mathbb{Z}}$, $\zeta_0 \in [0,1]$, is a sequence generated by the logistic map

$$\zeta_{k+1} = F(\zeta_k), \tag{4.4}$$

where $F(\upsilon) = 3.9\upsilon(1-\upsilon)$. It is worth noting that the map (4.4) is chaotic in the sense of Li-Yorke (Li & Yorke, 1975), and the unit interval $[0,1]$ is invariant under the iterations of the map (Hale & Koçak, 1991).

System (4.3) is in the form of (1.1), where

$$A = \begin{pmatrix} -\dfrac{11}{60} & \dfrac{1}{30} \\ -\dfrac{1}{20} & -\dfrac{1}{15} \end{pmatrix}, \quad f(t,y_1,y_2) = \begin{pmatrix} 0.05\sin\left(\dfrac{\pi}{4}t\right) \\ 0.003\arctan(y_1) \end{pmatrix}.$$

The conditions $(C1)-(C6)$ are valid for system (4.3) with $N = 2.62$, $\lambda = 1/12$, $M_f = 0.0503$, $L_f = 0.003$, $\bar{\kappa} = 5$ and $\bar{\delta} = 3$.

Since $t+8 \in \mathbb{T}_0$ whenever $t \in \mathbb{T}_0$ and $f(t+8,y_1,y_2) = f(t,y_1,y_2)$ for all $t \in \mathbb{T}_0$, y_1, $y_2 \in \mathbb{R}$, system (4.3) is Li-Yorke chaotic according to the results of (Akhmet & Fen, 2015). Moreover, one can verify that if $\zeta = \{\zeta_k\}_{k\in\mathbb{Z}}$ is a p-periodic solution of (4.4) for some natural number p, then the corresponding bounded solution $\varphi_\zeta(t)$ of (4.3) is $8p$-periodic. The y_1 and y_2 coordinates of the solution of (4.3) with $y_1(0) = 3$, $y_2(0) = -2$, $\zeta_0 = 0.292$ is shown in Figure 1. It is seen in Figure 1 that the solution behaves chaotically.

Now, we will demonstrate that homoclinic and heteroclinic motions take place in the chaotic dynamics of (4.3). Consider the function $\gamma_1(\upsilon) = \dfrac{1}{2}\left(1+\sqrt{1-\dfrac{4\upsilon}{3.9}}\right)$. It was mentioned in (Avrutin et al., 2015) that the orbit

$$\eta = \left\{\ldots, \gamma_1^3(\eta_0), \gamma_1^2(\eta_0), \gamma_1(\eta_0), \eta_0, F(\eta_0), F^2(\eta_0), F^3(\eta_0), \ldots\right\},$$