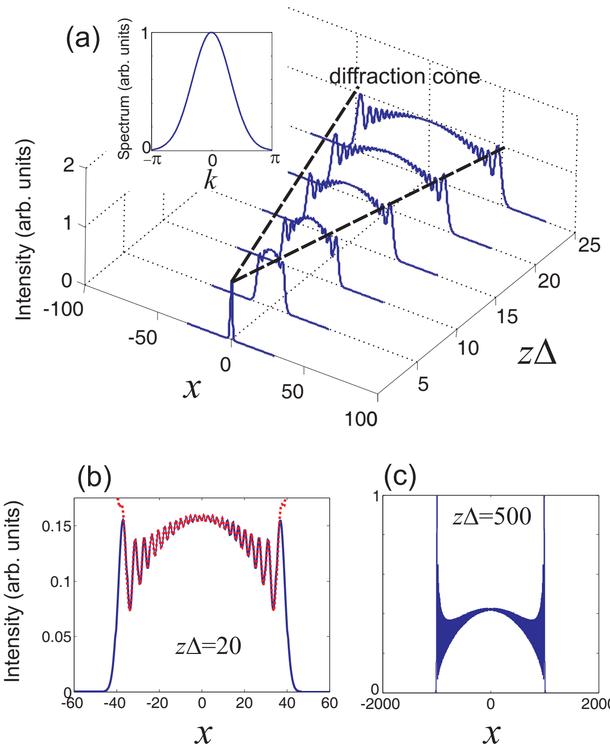

FIG. 1: (color online) Beam propagation in a one-dimensional tight binding lattice with nearest neighboring coupling terms, showing far-field properties of discrete diffraction. In (a) the intensity distributions $|\psi(x,z)|^2$ are plotted, in arbitrary units, for propagation distances $z=0$, $z=5/\Delta$, $z=10/\Delta$, $z=15/\Delta$, $z=20/\Delta$, and $z=25/\Delta$, where Δ is the coupling rate between adjacent waveguides. The inset in (a) shows the Gaussian spectrum $F(k)$ of the beam ($w_k=1.5$). In (c) the intensity distribution $|\psi(x,z)|^2$ is depicted for a propagation distance $z=20/\Delta$ as numerically calculated by Eq.(27) (solid curve) and by the approximate relation (33) (dotted curve, almost overlapped with the solid one). In (c) the beam intensity is plotted at a propagation distance $z=500/\Delta$, clearly showing the dominance of two peaks at the diffraction cone edges (self-collimation directions) and the onset of three different decay laws at $|\alpha|>2\Delta$, $\alpha=\pm 2\Delta$, and $|\alpha|<2\Delta$.

to the Fourier spectrum of the input (near-field) distribution. In this case, $g(k)=\alpha k-k^2/2$ has a unique saddle point at $k=k_0=\alpha$, with $g''(k_0)=-1\neq 0$; therefore, provided that the spectrum $F(k)$ has a nonvanishing component at $k=k_0$ and $F(k_0)$ does not diverge [25], according to the method of steepest descend one has

$$\psi_0(\alpha;z)\sim F(\alpha)\sqrt{\frac{2\pi}{z}}\exp[iz\alpha^2/2-i\pi/4] \tag{31}$$

as $z\to\infty$. From Eq.(31) we obtain the well-known result of paraxial diffraction theory that the amplitude $\psi_0(\alpha;z)$ of the beam decays as $\sim 1\sqrt{z}$ for any observation angle α [25], and that the far-field diffraction pattern is shaped as the Fourier spectrum $F(\alpha)$ of the near-field distribution. This scaling law may be referred to as the *normal* scaling law, in the sense that the beam intensity $I\propto|\psi|^2$ decays as $\sim 1/z$ whereas the beam spot size w_x increases asymptotically as $\sim z$ [see Eq.(17)], the product Iw_x being constant according to the power conservation law.

For the discrete diffraction problem, we prove now that the decay law is generally *slower* than $\sim 1/\sqrt{z}$ at the observation angles corresponding to self-collimation, and that the far-field pattern is peaked at such angles and does not reproduce the spectrum F of the near-field distribution. To this aim, let us observe that, according to the steepest descend method, the slowest decay term in the integral of Eq.(29) comes from the saddle points $g'(k_0)=0$ of largest order. In particular, if k_0 is a saddle point of order $n\geq 2$, i.e. $g(k)\simeq g(k_0)+[g^{(n)}(k_0)/n!](k-k_0)^n$ for k close to k_0 ($g^{(n)}(k_0)\neq 0$), the contribution of the saddle point to the integral in Eq.(29) for large values of z is given by [24]

$$\psi_0(\alpha;z)\sim\frac{F(k_0)}{|zg^{(n)}(k_0)|^{1/n}}(n!)^{\frac{1}{n}}\Gamma\left(\frac{1}{n}\right)\exp\left[izg(k_0)\pm i\frac{\pi}{2n}\right]. \tag{32}$$

Therefore, the decay law for $\psi_0(\alpha;z)$ scales as $\sim z^{-1/n}$, where n is the highest order of the saddle points of $g(k)$, provided that $F(k_0)\neq 0$. In the case of diffraction in a homogeneous continuous medium, the order of saddle point is always $n=2$. To determine n for the discrete diffraction problem, let us note that the dispersion curve $H_0(k)$ admits of at least a couple of inflection points, say at $k=\pm k_0$, at which $H_0''(k_0)=0$. These points correspond to the self-collimation directions introduced in Sec.II.C. Since $g'(k)=\alpha-H_0'(k)$, the inflection points $k=\pm k_0$ turn out to be also saddle points when the observation angle α is chosen equal to $H_0'(\pm k_0)$. Therefore, for the discrete diffraction problem the largest order n of saddle points is *at least* $n=3$, and the decay law of $\psi_0(\alpha;z)$, at the two observation angles $\alpha=\pm H_0'(k_0)$ corresponding to the self-collimation directions $\pm k_0$, is slowed down -as compared to continuous diffraction- to (at least) $\sim z^{-1/3}$. More generally, if the dispersion curve $H_0(k)$ of the lattice is engineered to achieve a very flat behavior near a self-collimation point $k=k_0$, with $g''(k_0)=g'''(k_0)=...=g^{(n-1)}(k_0)=0$ and $g^{(n)}(k_0)\neq 0$ ($n\geq 3$), the decay law of $\psi_0(\alpha;z)$ scales as $\sim z^{-1/n}$ at the observation angle $\alpha=H'(k_0)$. This scaling law of beam decaying in the discrete diffraction problem is therefore *anomalous*, in the sense that along the self-collimation directions the intensity decays slower that $1/z$, i.e. of the characteristic decay law that one might expect from power conservation arguments. This seemingly paradoxical circumstance may be solved by observing that, for an observation angle α different than any of the self-collimation directions, the decay of $\psi_0(\alpha;z)$ may be either normal (i.e. $\sim 1/\sqrt{z}$) or even *faster*. More precisely, for a fixed value of α in modulus larger than $\alpha_{max}=\max_k|H_0'(k)|$, the function $g(k)$ given by Eq.(30) does not have saddle points on the real axis, and $\psi_0(\alpha;z)$ decays as $\sim 1/z$. Conversely, for $|\alpha|<\alpha_{max}$ the equation $g'(k)=\alpha-H_0'(k)=0$ admits of at least one solution,