where the symbols Θ_i^q denote the quark parts of the gravitational form factors of Eq. (49). A monopole fit has been undertaken, yielding $m_\sigma = 0.89(27)(9)$ MeV and $A_{22}(0) = -0.076(5)$ (*cf.* Table C.8 on p. 105 of Ref. [64]). By using several parameterizations it has been noted that the precise form of the function cannot be pinned down from the data unambiguously due to large uncertainties in $A_{22}(t)$. In the following we will use the low-energy theorem

$$A_{22}(0) = -\frac{1}{4}A_{20}(0), \tag{62}$$

which provides a relatively accurate fixing of A_{22} at the origin and greatly helps the regression analysis.

In terms of the form factors related to the Generalized Parton Distributions (GPD) of Ref. [63, 64], the quark contribution to the gravitational form factor reads

$$\Theta_\pi^q(t) = -4tA_{22}^q(t) \tag{63}$$

Due to the multiplicative QCD evolution one has

$$\Theta_\pi^q(t,\mu) = \langle x\rangle_q^\pi(\mu)\Theta_\pi(t), \tag{64}$$

where the (valence) quark momentum fraction depends on the renormalization scale μ. Its leading-order perturbative evolution reads

$$R = \frac{\langle x\rangle_q(\mu)}{\langle x\rangle_q(\mu_0)} = \left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\gamma_1^{(0)}/(2\beta_0)}, \tag{65}$$

where the anomalous dimension is $\gamma_1^{(0)}/(2\beta_0) = 32/81$ for $N_F = N_c = 3$. The QCD running coupling constant is equal to

$$\alpha(\mu) = \frac{4\pi}{\beta_0\log(\mu^2/\Lambda^2_{\rm QCD})}, \tag{66}$$

where we take $\Lambda_{\rm QCD} = 226$ MeV for $N_c = N_f = 3$.

Phenomenologically, it is known from the Durham group analysis [66], based mainly on the E615 Drell-Yan data [67] and the model assumption that the sea quarks carry $10-20\%$ fraction of the momentum, that $\langle x\rangle_{u+d}^\pi = 0.47(2)$ at the scale $\mu = 2$ GeV. The analysis of the Dortmund group [68], based on the assumption that the momentum fraction carried by the valence quarks in the pion coincides with that in the nucleon, yields $\langle x\rangle_{u+d}^\pi = 0.4$ at $\mu = 2$ GeV. The lattice data at the lattice spacing $a_{\rm lat} = 0.1$fm as well as a recent chiral quark model calculation [69] support this view.

We recall that the large-N_c analysis implies sums of monopoles in the form factors. The largest available momentum transfer, $t = -4$ GeV^2, obtained in Ref. [64], suggests that some information on the contribution of the excited states might be extracted. Therefore, following the approach already used for the electromagnetic pion form factor in Ref. [70] (see also [71]), we have attempted a Regge-like fit,

$$\Theta_\pi(t) = tf_b(t), \tag{67}$$

including infinitely many states, of the form

$$f_b(t) = \frac{B(b-1, \frac{M^2-t}{a/2})}{B(b-1, \frac{M^2}{a/2})}, \tag{68}$$

with $B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ denoting the Euler Beta function. The function (68) fulfills the normalization condition

$$f_b(0) = 1. \tag{69}$$

For $x \gg y$ one has $B(x,y) \sim \Gamma(y)x^{-y}$, hence in the asymptotic region of $M^2 - t \gg (b-1)a$ we find

$$f_b(t) \sim \frac{\Gamma\left(\frac{M^2}{a/2} + b - 1\right)}{\Gamma\left(\frac{M^2}{a/2}\right)}\left(\frac{a/2}{M^2-t}\right)^{b-1}. \tag{70}$$

The result for $a = 1.31$ is

$$\langle x\rangle_{u+d}^\pi = 0.52(3),\ m_\sigma = 495^{+250}_{-135}\ \text{MeV},\ b = 2.24^{+1.56}_{-0.55}. \tag{71}$$

As we can see, the result is fully compatible with the monopole ($b = 2$) and at present the large errors in the lattice data wash out any insight from the excited scalar spectrum, despite the large momenta[3].

Thus, we restrict ourselves to a simple monopole fit

$$\Theta_\pi^q(t) = \langle x\rangle_{u+d}^\pi \frac{t\,m_\sigma^2}{m_\sigma^2 - t}, \tag{72}$$

which yields $\chi^2/\text{DOF} \sim 2.4$. Actually the large χ^2 is due to incompatible values of nearby points. In such a situation, in order to obtain reliable estimate of the model parameters, we rescale the errors by a factor of 1.5 to make them mutually compatible. Moreover, we enforce the low energy theorem, Eq. (62) as a constraint – a possibility not directly considered in Ref. [63, 64]. As a result we get $\chi^2/\text{DOF} \simeq 1$ (after the mentioned rescaling of the data errors) and the optimum values

$$\langle x\rangle_{u+d}^\pi = 0.52(2) \qquad m_\sigma = 445(32)\ \text{MeV}. \tag{73}$$

In Fig. 3 we present the corresponding correlation ellipse. The gravitational form factor $\Theta_0(t)$ at the optimum values of the parameters (73) is presented in Fig. 4.

B. Nucleon

The scalar component of the nucleon gravitational form factor is

$$\langle N(p')|\Theta(0)|N(p)\rangle = \bar u(p')u(p)\Theta_N(q^2), \tag{74}$$

[3] We note that a similar fit [71] to the vector form factor using $a = 1.2(1)$ GeV^2 yields $m_\rho = 775(15)$ MeV and $b = 2.14(7)$, which can be distinguished from a simple monopole fit at the two-standard-deviation level.