$$j_w^*(c_d^S(N_Y X) \cap [Y]) = c_d^S(N_Y X|_{wB}) \cap j_w^*([Y]) = c_d^S(N_Y X|_{wB}) \cap [wB],$$

where d is the codimension of Y in X. Here we have used some basic facts of intersection theory regarding pushforwards and pullbacks, for which the standard reference is [Ful98]. Note that we are able to use the self-intersection formula because Y is smooth, and hence $E \times^S Y$ is regularly embedded in $E \times^S X$.

On the other hand,

$$i_w^*(i_*([Y])) = i_w^*(\alpha \cap [X]) = \alpha|_{wB} \cap i_w^*([X]) = \alpha|_{wB} \cap [wB].$$

Then in $H_S^*(X)$, we have

$$\alpha|_{wB} = c_d^S(N_Y X|_{wB}).$$

Thus computing the restriction of the class α at each S-fixed point amounts to computing $c_d^S(N_Y X|_{wB}) \in H_S^*(\{\text{pt.}\}) \cong \mathbb{C}[X_1, \ldots, X_r]$. We want to compute this Chern class explicitly, as a polynomial in the X_i. Note that the S-equivariant bundle $N_Y X|_{wB}$ is simply a representation of the torus S, and its top Chern class is the product of the weights of this representation. We now compute these weights.

The S-module $N_Y X|_{wB}$ is simply $T_w X / T_w Y$, so we determine the weights of S on $T_w X$ and $T_w Y$, then subtract the weights of $T_w Y$ from those of $T_w X$. It is standard that

$$T_w X = \mathfrak{g}/\mathrm{Ad}(w)(\mathfrak{b}).$$

Since B has been taken to correspond to the negative roots, the weights of S on $T_w X$ are the restrictions of the following weights of T on $T_w X$:

$$\Phi \setminus w\Phi^- = w\Phi^+.$$