

For  $L = 2$ , we found  $\Phi_{[1]}^{\text{exact}} \propto Q_2$ , with the index  $[i]$  indicating the energy ordering in the full EXD spectrum (including both spurious and TI states). Since [see Eq. (34)]

$$Q_2 \propto (z_1 - z_c)(z_2 - z_c) + (z_1 - z_c)(z_3 - z_c) + (z_2 - z_c)(z_3 - z_c), \quad (39)$$

this result agrees with the findings of Refs. [25, 59] concerning ground states of bosons in the range  $0 \leq L \leq N$ .

For  $L = 3$ , one finds  $\Phi_{[1]}^{\text{exact}} \propto \Phi_3^{\text{RBM}}$ . Since [see Eq. (37)]

$$\Phi_3^{\text{RBM}} \propto (z_1 - z_c)(z_2 - z_c)(z_3 - z_c), \quad (40)$$

this result agrees again with the findings of Refs. [25, 59].

For  $L = 5$ , the single nonspurious state is an excited one,  $\Phi_{[2]}^{\text{exact}} \propto \Phi_3^{\text{RBM}} Q_2$ .

For  $L = 6$  ( $\nu = 1/2$ ), the ground-state is found to be

$$\begin{aligned} \Phi_{[1]}^{\text{exact}} &\propto -\frac{160}{9}\Phi_6^{\text{RBM}} + \frac{1}{4}Q_2^3 \\ &= (z_1 - z_2)^2(z_1 - z_3)^2(z_2 - z_3)^2, \end{aligned} \quad (41)$$

i.e., the bosonic Jastrow-Laughlin function for  $\nu = 1/2$  is equivalent to an RBM state that incorporates vibrational correlations.

For  $L \geq N(N - 1)$  (i.e.,  $\nu \leq 1/2$ ), the EXD yrast energies equal zero, and with increasing  $L$  the degeneracy of the zero-energy states for a given  $L$  increases. It is important that this nontrivial behavior is reproduced faithfully by the present method (see TABLE I).

### B. Three electrons

Although unrecognized, the solution of the problem of three spin-polarized electrons in the LLL using molecular trial functions was presented by Laughlin in Ref. [60]. Indeed, the main result of Ref. [60] [see Eq. (18) therein] were the following wave functions (we display the polynomial part only)

$$|k, m\rangle \propto \left[ \frac{(z_a + iz_b)^{3k} - (z_a - iz_b)^{3k}}{2i} \right] (z_a^2 + z_b^2)^m, \quad (42)$$

where the three-particle Jacobi coordinates are

$$z_c = \frac{z_1 + z_2 + z_3}{3}, \quad (43)$$

$$z_a = \left(\frac{2}{3}\right)^{1/2} \left[ \frac{z_1 + z_2}{2} - z_3 \right], \quad (44)$$

$$z_b = \frac{1}{\sqrt{2}}(z_1 - z_2). \quad (45)$$

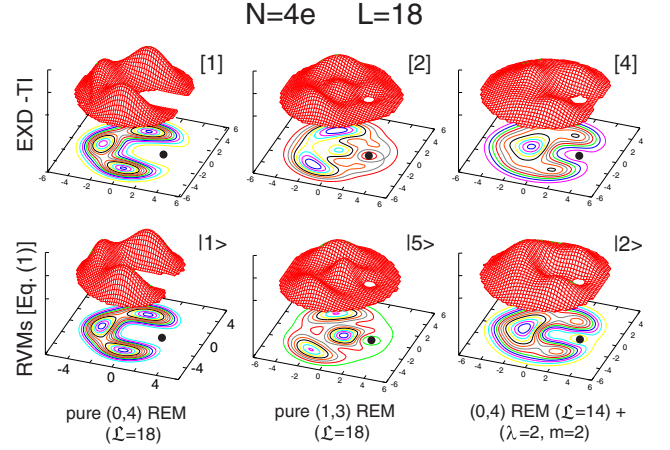


FIG. 3: (Color online) CPDs for  $N = 4$  LLL electrons with  $L = 18$  ( $\nu = 1/3$ ). Top row: The three lowest-in-energy EXD<sup>TI</sup> states (see TABLE II). Bottom row: The RVM trial functions associated with the largest expansion coefficients (underlined, see TABLE III) of these three EXD<sup>TI</sup> states in the correlated RVM basis. See the text for details. The solid dot denotes the fixed point  $\mathbf{r}_0$ . Distances in nm.

Expression (42) is precisely of the form  $\Phi_{3k}^{\text{REM}} Q_2^m$ , as can be checked after transforming back to Cartesian coordinates  $z_1, z_2$ , and  $z_3$ . Thus the wave functions  $|k, m\rangle$  of Ref. [60] describe both pure molecular rotations, as well as vibrational excitations, and they cover the translationally invariant LLL subspace. We note that the pairs of indices  $\{k, m\}$  are universal and independent of the statistics, i.e., the same for both bosons [Eq. (36)] and electrons [Eq. (42)], as can be explicitly seen through a comparison of TABLE I here and TABLE I in Ref. [60].

We further note that Laughlin did not present molecular trial functions for electrons with  $N > 3$ , or for bosons for any  $N$ . This is done in the present paper.

### C. Four electrons

For  $N = 4$  spin-polarized electrons, one needs to consider two distinct molecular configurations, i.e., (0, 4) and (1, 3). Vibrations with  $\lambda \geq 2$  must also be considered. In this case the RVM states are not always orthogonal, and the Gram-Schmidt orthogonalization is implemented.

Of particular interest is the  $L = 18$  case ( $\nu = 1/3$ ) which is considered [2] as the prototype of quantum-liquid states. However, in this case we found (see TABLE II) that the exact TI solutions are linear superpositions of the following seven RVM states [involving both the (0,4) and (1,3) configurations]:

$$\begin{aligned} |1\rangle &= \Phi_{18}^{\text{REM}}(0, 4), & |2\rangle &= \Phi_{14}^{\text{REM}}(0, 4)Q_2^2, \\ |3\rangle &= \Phi_{10}^{\text{REM}}(0, 4)Q_2^4, & |4\rangle &= \Phi_6^{\text{REM}}(0, 4)Q_2^6, \\ |5\rangle &= \Phi_{18}^{\text{REM}}(1, 3), & |6\rangle &= \Phi_{12}^{\text{REM}}(1, 3)Q_2^3, \\ |7\rangle &= \Phi_{15}^{\text{REM}}(1, 3)Q_3. \end{aligned} \quad (46)$$