

Finally, applying to Eq. (2.12) the inverse Laplace transform defined as $h(t) = \mathcal{L}^{-1}\{h_s\} = (1/2\pi i) \int_{c-i\infty}^{c+i\infty} ds e^{st} h_s$ (c is chosen to be larger than the real parts of all singularities of h_s), we find the first

$$\langle Y_t \rangle = l \mathcal{L}^{-1} \left\{ \frac{p_s}{s(1-p_s)} \right\} \quad (2.13)$$

and the second

$$\langle Y_t^2 \rangle = l^2 \mathcal{L}^{-1} \left\{ \frac{p_s^2 + p_s}{s(1-p_s)^2} \right\} \quad (2.14)$$

moments of Y_t , which in turn determine the variance of the particle position:

$$\sigma^2(t) = \langle Y_t^2 \rangle - \langle Y_t \rangle^2. \quad (2.15)$$

III. CONDITIONS OF ANOMALOUS DIFFUSION

As it follows from the waiting time probability density (2.9), the m th moment of the waiting time, $\overline{\tau^m} = \int_0^\infty d\tau \tau^m p(\tau)$ ($m = 1, 2, \dots$), can be written in the form

$$\overline{\tau^m} = \int_{-g_0}^{g_0} \int_{-g_0}^{g_0} dg dg' u(g) u(g') \left(\frac{\kappa \nu l + g - g'}{\kappa(f + g)} \right)^m. \quad (3.1)$$

Since the probability density $u(g)$ is normalized on the interval $[-g_0, g_0]$, from Eq. (3.1) it follows that $\overline{\tau^m} \leq (\nu l + 2g_0/\kappa)^m / (f - g_0)^m$. Thus, if $f > g_0$ then all these moments are finite, and so in this case the classical central limit theorem for sums of a random number of random variables [17] is applied to Y_t . This implies that $\sigma^2(t) \propto t$ as $t \rightarrow \infty$, i.e., at $f > g_0$ the biased diffusion of particles is normal, and the rescaled probability density $\mathcal{P}(\psi, t) = \sigma(t) P(\langle Y_t \rangle + \sigma(t)\psi, t)$ in the long-time limit tends to the probability density $\mathcal{P}(\psi, \infty) = (2\pi)^{-1/2} e^{-\psi^2/2}$ of the standard normal distribution.

It is clear from the above that the anomalous long-time behavior of the variance $\sigma^2(t)$ is expected at $\overline{\tau^2} = \infty$ when the mentioned central limit theorem becomes inapplicable. The condition $\overline{\tau^2} = \infty$ implies $f = g_0$ that, in accordance with (2.4), yields $\tau_{\min} = \nu l / 2g_0$ and $\tau_{\max} = \infty$. Since the divergence of $\overline{\tau^2}$ occurs when $p(\tau)$ at $\tau \rightarrow \infty$ tends to zero slowly enough, next we assume that

$$p(\tau) \sim \frac{a}{\tau^{1+\alpha}} \quad (3.2)$$

($\tau \rightarrow \infty$) with $a > 0$ and $\alpha \in (0, 2]$. Thus, the biased diffusion in a randomly layered medium is expected to be anomalous if both conditions, $f = g_0$ and $\alpha \in (0, 2]$, hold. The former guarantees that the waiting time τ can be arbitrarily large ($\tau_{\max} = \infty$), and so it is a