

Table 2: Model comparison using the conditional log-likelihood ($\ln L$), Akaike information criterion (AIC) and Bayesian information criterion (BIC) in Simulation Study 1.

| | 25TH QUANTILE | 50TH QUANTILE | 75TH QUANTILE |
|--------|-----------------------|-----------------------|-----------------------|
| | ($\ln L$, AIC, BIC) | ($\ln L$, AIC, BIC) | ($\ln L$, AIC, BIC) |
| FBQROR | (−338, 688, 710) | (−340, 691, 714) | (−338, 688, 710) |
| BQROR | (−346, 701, 720) | (−340, 690, 708) | (−348, 706, 724) |

Akaike information criterion or AIC (Akaike, 1974) and the Bayesian information criterion or BIC (Schwarz, 1978) for both the FBQROR and BQROR models. Higher conditional log-likelihood is preferable, while lower values of AIC/BIC indicates better fitting model. As seen from Table 2, the conditional log-likelihood for the FBQROR model is identical to the BQROR model at the median, but higher at the other two considered quantiles. However, the FBQROR model has an extra shape parameter and so to rule out the possibility of higher log-likelihood arising due to additional parameters (i.e., overfitting), we compare the models using AIC and BIC. These two measures introduce different penalty terms to account for the number of model parameters. Based on AIC/BIC, there is strong evidence that the FBQROR model provides a better fit at the 25th and 75th quantiles, but there is some evidence in favor of BQROR model at the 50th quantile. The poorer fit at the first and third quartiles reflects the rigidity of the AL distribution, since $p_0 = 0.25$ (0.75) forces the AL distribution to be positively (negatively) skewed.

4.2. Simulation Study 2

Once again we estimate the FBQROR and BQROR models with a simulated data, but now the errors are generated from a chi-square distribution such that the resulting distribution for the continuous latent variable z is positively skewed. In particular, 300 observations are generated from the model $z_i = x_i' \beta + \epsilon_i$, where covariates are sampled from a standard uniform distribution $Unif[0, 1]$, $\beta = (3, -7, 5)'$ and ϵ is generated from $\chi^2(4) - 4$, i.e., a demeaned chi-square distribution. The discrete response variable y is obtained from z based on cut-point vector $\xi = (0, 3, 6)$, which yields 74 (24.67%), 110 (36.67%), 65 (21.67%) and