$S(\rho||\sigma)$ reduces to $\ln 2 - \ln x$, which is less than $\ln 2$ if $x \neq 1$. Therefore σ on $(o_1^{(+)}, o_2^{(+)}, o_3^{(-)})$ is not CSS of $|\beta_1\rangle\langle\beta_1|$. By same way one can show that σ on $(o_1^{(-)}, o_2^{(+)}, o_3^{(+)})$ or $(o_1^{(-)}, o_2^{(-)}, o_3^{(-)})$ is not CSS of $|\beta_1\rangle\langle\beta_1|$, which completes the proof.

Theorem 2. *The CSS of the any Bell-diagonal state ρ corresponds to the crossing point between the nearest surface of $\mathcal{L}$ from ρ and the straight line ℓ, which connects ρ and the nearest vertex of $\mathcal{T}$ from ρ.*

Proof. If σ is CSS of ρ, the CSS of $\tilde{\rho} = x\rho + (1-x)\sigma$ is also σ[12]. Let ρ be $\rho = |\beta_1\rangle\langle\beta_1|$. Then, theorem 1 implies that σ can be any point on the surface $(o_1^{(+)}, o_2^{(-)}, o_3^{(+)})$. Let $\tilde{\rho}$ belong to the small tetrahedron $(v_1, o_1^{(+)}, o_2^{(-)}, o_3^{(+)})$. Note that $\tilde{\rho}$ corresponds to a internally dividing point of the line segment $\overline{\rho\sigma}$. Since Eq.(1.6) implies that the set of the entangled states which have same CSS should be represented by the straight line, the only possible σ as CSS of $\tilde{\rho}$ is a crossing point between a line $\overline{\rho\tilde{\rho}}$ and the surface $(o_1^{(+)}, o_2^{(-)}, o_3^{(+)})$, which completes the proof for the Bell-diagonal states.

By making use of the Theorem 2 one can always find the CSS σ if ρ is a Bell-diagonal state. Fig. 2 shows how to find the CSS for the Bell-diagonal state. First, extend the line segment between ρ and the point corresponding to the nearest vertex of $\mathcal{T}$. Second, compute the coordinate of the crossing point between the line and the nearest surface of the octahedron $\mathcal{L}$. Finally, find the CSS which corresponds to the crossing point. This complete the reverse process of Ref.[22].

III. GEOMETRICAL DEFORMATION OF $\mathcal{T}$ AND $\mathcal{L}$

When the Bloch vectors $\boldsymbol{r}$ and $\boldsymbol{s}$ are non-zero, the geometrical objects $\mathcal{T}$ and $\mathcal{L}$ should be deformed. In this section we will discuss how $\mathcal{T}$ and $\mathcal{L}$ are deformed. In order to perform the following analysis analytically we consider in this paper the case where $\boldsymbol{r}$ and $\boldsymbol{s}$ are parallel to each other. It is worthwhile noting that if $\boldsymbol{r}$ and $\boldsymbol{s}$ are x- or y-direction, one can make them to be z-directional via the appropriate local-unitary transformation. For example, if they are x-direction, $\rho' = (U \otimes U)\rho(U \otimes U)^\dagger$ with

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$