Critical behaviour of the Ising $S = 1/2$ and $S = 1$ model on $(3, 4, 6, 4)$ and $(3, 3, 3, 3, 6)$ Archimedean lattices

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We investigate the critical properties of the Ising $S = 1/2$ and $S = 1$ model on $(3, 4, 6, 4)$ and $(3^4, 6)$ Archimedean lattices. The system is studied through the extensive Monte Carlo simulations. We calculate the critical temperature as well as the critical point exponents γ/ν, β/ν and ν basing on finite size scaling analysis. The calculated values of the critical temperature for $S = 1$ are $k_BT_C/J = 1.590(3)$ and $k_BT_C/J = 2.100(4)$ for $(3, 4, 6, 4)$ and $(3^4, 6)$ Archimedean lattices, respectively. The critical exponents β/ν, γ/ν and $1/\nu$ for $S = 1$ are $\beta/\nu = 0.180(20)$, $\gamma/\nu = 1.46(8)$ and $1/\nu = 0.83(5)$ for $(3, 4, 6, 4)$ and 0.103(8), 1.44(8) and 0.94(5) for $(3^4, 6)$ Archimedean lattices. Obtained results differ from the Ising $S = 1/2$ model on $(3, 4, 6, 4)$, $(3^4, 6)$ and square lattice. The evaluated effective dimensionality of the system for $S = 1$ are $D_{\text{eff}} = 1.82(4)$ for $(3, 4, 6, 4)$ and $D_{\text{eff}} = 1.64(5)$ for $(3^4, 6)$.

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I. INTRODUCTION

The Ising model [1, 2] remains probably the most cited model in statistical physics. Today, the *ISI Web of Knowledge* abstracting and indexing service returns over eleven thousands records for the query on "Ising" for time span from 1996 to 2010. For *Inspec* database (for years 1969-2010) this number is almost doubled and reaches 19 thousands for *Scopus* database (for data range 1960-2010). The latter means that during the last half of century ≈ 380 papers refer to the Ising model every year. The *Google* search engine indicates over 279 thousands web pages which contain "Ising model" phrase.

The beauty and the popularity of this model lies in both its simplicity and possible applications from pure and applied physics, via life sciences to social sciences. In the way similar to the percolation phenomenon, the Ising model is one of the most convenient way of numerical investigations of second order phase transitions.

In the simplest case, the Ising model may be used to simulate the system of interacting spins which are placed at the nodes of graphs or regular lattices. In its basic version only two values of the spin variable are available, i.e. $S = -\frac{1}{2}$ and $S = +\frac{1}{2}$. This is the classical Ising $S = \frac{1}{2}$ model. For a square lattice this model defines the universality class of phase transitions with analytically known critical exponents which describe the system behaviour near the critical point. The critical point separates two — ordered and disordered — phases.

One of possible generalisation of the Ising model is to enlarge the set of possible spin values (like in the Potts model [3, 4]). The Ising $S = 1$ model corresponds to three possible spin values, i.e. $S \in \{-1, 0, +1\}$, Ising $S = \frac{3}{2}$ allows for four spin variables $S \in \{\pm\frac{3}{2}, \pm\frac{1}{2}\}$, *etc.* The Ising $S \neq \frac{1}{2}$ model on various networks and lattices may form universality classes other than the classical square lattice Ising model.

The spin models for $S = 1$ were extensively studied by several approximate techniques in two and three dimensions and their phase diagrams are well known [5–11]. The case $S > 1$ has also been investigated according to several procedures [12–18]. The Ising model $S = 1$ on directed Barabási–Albert network was studied by Lima in 2006 [19]. It was shown, that the system exhibits first-order phase transition. The result is qualitatively different from the results for this model on a square lattice, where a second-order phase transition is observed.

In this paper we study the Ising $S = 1$ model on two Archimedean lattices (AL), namely on $(3, 4, 6, 4)$ and $(3^4, 6)$. The topologies of $(3, 4, 6, 4)$ and $(3^4, 6)$ AL are presented in Fig. 1. Critical properties of these lattices were investigated in terms of site percolation in Ref. [20]. Topologies of all eleven existing AL are given there as well. Also the critical temperatures for Ising $S = \frac{1}{2}$ model [21] and voter model [22] on those AL were estimated numerically.

Here, with extensive Monte Carlo simulations we show that the Ising $S = 1$ model on $(3, 4, 6, 4)$ and $(3^4, 6)$ AL exhibits a second-order phase transition with critical exponents that *do not* fall into universality class of the square lattice Ising $S = \frac{1}{2}$ model.

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