Consider now the original matrix $G(y)$, denote it $G^1(y)$ with $q_1 = q$ and employ the construction recursively until for some m it ends: $\Sigma_{v=1}^{m} \bar{r}_v = q$.

Denote by $\bar{S}^v$ the matrix $\begin{bmatrix} I_{\bar{r}_1+...+\bar{r}_{v-1}} & \\ & S^v \end{bmatrix}$ and define $S = \bar{S}^m ... \bar{S}^1$. Set $a = (\alpha_1, ..., \alpha_q) = (\bar{k}_1, ..., \bar{k}_1, ..., \bar{k}_m, ..., \bar{k}_m)$, where each $\bar{k}_v$ enters $\bar{r}_v$ times. Then for this a and S

$$\lim_{\lambda \to \infty} [diag(\lambda^{\alpha_i}) S G(y/\lambda)]$$

is a finite matrix $\bar{G}(y) = \begin{bmatrix} \bar{G}^1(y) \\ \vdots \\ \bar{G}^m(y) \end{bmatrix}$; if $\sum_{i=1}^{q} \alpha_i = \bar{\alpha}$, then CLDR property holds, if $\sum_{i=1}^{q} \alpha_i < \bar{\alpha}$ then the limit matrix has deficient rank. ■

Proof of Theorem 4.1. Consider $y_T^* = \lambda_T y_T$ and the quadratic form similar to (21)

$$W(y_T^*, g_{\bar{\theta}}, \lambda_T, A\hat{V}_T A') = \lambda_T^2 g'_{\bar{\theta}}(y_T^*/\lambda_T)[G_{\bar{\theta}}(y_T^*/\lambda_T) A\hat{V}_T A' G'_{\bar{\theta}}(y_T^*/\lambda_T)]^{-1} g_{\bar{\theta}}(y_T^*/\lambda_T).$$

From Assumption 2.3 if $\lambda = \lambda_T$ and $\theta = \hat{\theta}_T$ then the probability limit of corresponding $V^{-\frac{1}{2}} A^{-1} y_T^*$ is Z with distribution $Q(\bar{\theta})$; from Assumption 2.4 $\hat{V}_T = V + o_p(1)$. From (20) and convergence it follows that

$$\begin{aligned} diag(\lambda_T^{\alpha_i}) S G_{\bar{\theta}}(y_T^*/\lambda_T) &= \bar{G}_{\bar{\theta}}(y_T^*) + O_p(1/\lambda_T); \\ diag(\lambda_T^{\alpha_i}) S \lambda g_{\bar{\theta}}(y_T^*/\lambda_T) &= \bar{g}_{\bar{\theta}}(y_T^*) + O_p(1/\lambda_T). \end{aligned} \quad (32)$$