same line of reasoning, we could calculate the number of degrees of freedom of S_4, that is, of the system made up by n particles with fixed relative distances, but which is, unlike S_3, immerse in a four dimentional space.

In this four-dimentional case, four coordinates are needed to locate the center of mass of each particle, which makes $4n$ coordinates for the set of n particles. And the number of constraint equations is the same as for S_3

In principle, the number of degrees of freedom should be the same as for a tetra-dimensional rigid body. And in four dimensions there are ten degrees of freedom for the rigid body: four coordinates are needed to locate its center of mass and there are six possible rotation angles. Now, in the case of the n particles with fixed distances, we need $4n$ coordinates to locate the particles´ centers of mass, while the number of distances is still $\frac{n(n-1)}{2}$. And the number of possible rotation angles is obtain observing that a "hiperplane" can be defined with four points and that the number of diferent ways in which pairs may be chosen from a group of $n-4$ particles is given by:

$$C_{n-4}^{2}=\frac{(n-4)!}{2!(n-6)!}=\frac{(n-4)(n-5)}{2}, \qquad (8)$$

for $n \geq 4$.

Then, the number of degrees of freedom of S_4 is

$$N_4 = 4n-\frac{n(n-1)}{2}+\frac{(n-4)(n-5)}{2}= 10, \qquad (9)$$

when $n \geq 4$,

and

$$N_4 = 4n - \frac{n(n-1)}{2}, \qquad (10)$$

when $2 \leq n \leq 5$,

since the number of possible φ_i is equal to zero for these values of n.

That N_4 is equal to ten for any value of n less than or equal to four is consistent with the fact that ten is also the number of degrees of freedom of a rigid body in four-dimentional space (four coordinates are needed to locate the center of mass, and six more to describe the orientation of the body. Indeed, our procedure works for the four-dimentional as it does for the three-dimensional case. Moreover, we believe that it works for the general case. We propose that for a system of n particles with fixed relative distances, immerse in a space of D dimensions, the number of degrees of freedom is given by:

$$N_D= Dn-\frac{n(n-1)}{2}+\frac{(n-D)(n-D-1)}{2} \qquad (11)$$

$$=\frac{D(D+1)}{2}, \text{when } n \geq D,$$

and by:

$$N_D = Dn - \frac{n(n-1)}{2}, \qquad (12)$$

when $2 \leq n \leq D+1$.