is lower than the worst lower bound through the improved lower bound on $P_0(y, 1|z)$ and improved upper bound on $P_1(y, 0|z)$ by MTR, while it still poses the Makarov upper bound. On the other hand, the lower bound under MTR is higher than the worst lower bound obtained from the Makarov lower bound because of the direct effect of MTR on the lower bound on the joint distribution. Remember that the lower bound on the joint distribution is not affected by the improved components of the bounds on counterfactual probabilities: the improved lower bound on $P_0(y, 1|z)$ and improved upper bound on $P_1(y, 0|z)$. Lastly, under NSM and MTR both the lower bound and the upper bound improve through counterfactual probabilities $U_{01}^{sm}(y, z)$ and $L_{10}^{sm}(y, z)$, respectively which are improved by NSM compared to the bounds under MTR only.

I also obtained sharp bounds on the potential outcomes distributions and the DTE for $z \in \{2, 1.5, 1, .5\}$ to see how the support of the instrument affect the identification region. Tables 3, 4, and 5 document the identification regions of F_0, F_1, and F_Δ, respectively, under NSM and MTR for these different values of z. As expected, as the support of the instrument gets larger, the identification regions of the marginal distributions and the DTE become more informative. Table 5 shows the identification regions of the DTE for different values of $\rho = \{-.25, -.5, -.75\}$. Since the true DTE does not depend on the value of ρ, one can see from Table 5 how the size of correlation between the outcome heterogeneity and the selection heterogeneity affects the identification region of the DTE for the fixed true DTE. As shown in Table 5, the identification region becomes tighter as ρ approaches 0. That is, the weaker endogeneity with the smaller absolute value of ρ helps identification of the DTE. This is readily understood from the extreme case. If $\rho = 0$ where the treatment selection is independent of potential outcomes Y_0 and Y_1, marginal distributions of potential outcomes are exactly identified, which clearly leads to tighter bounds on the DTE.

6 Conclusion

In this paper, I established sharp bounds on marginal distributions of potential outcomes, their joint distribution, and the DTE in triangular systems. To do this, I explored various types of restrictions to tighten the existing bounds including stochastic monotonicity between each outcome unobservable and the selection unobservable, conditional positive quadrant dependence between two outcome unobservables given the selection unobservable, and the monotonicity of the potential