single energy level ε near the Fermi level of the lead while other levels in the dot do not contribute to the processes described below. The state of the system is specified by the occupation probability $p(t)$ of having an electron in the dot. The lead plays the role of a thermal bath at temperature T and chemical potential μ. Electrons are assumed to thermalize instantaneously upon tunnelling into the lead.

When the energy level ε is modulated by an external agent according to a given protocol, a certain amount of energy, positive or negative, flows into the system in the form of work and/or heat. In the case of an occupied level, an amount of work equal to $(\varepsilon_f - \mu_f) - (\varepsilon_i - \mu_i)$ is delivered to the system, where the subscripts f and i refer to final and initial values. When the electrons at energy level ε tunnel in (out), an amount of heat equal to $Q = \varepsilon - \mu$ ($Q = -\varepsilon + \mu$) is extracted from the bath.

The basic problem that we address is the finite-time performance of this engine as it runs through the following four standard stages of a Carnot cycle (also see figure 1):

I *Isothermal process*
The quantum dot is in contact with a cold lead at temperature T_c and chemical potential μ_c. The energy level is raised from ε_0 to ε_1 according to a certain protocol during a time interval of duration τ_c. Both work and heat are exchanged during this process.

II *Adiabatic process*
The quantum dot is disconnected from the cold lead, and the quantum level is shifted from ε_1 to a new level ε_2. Since the quantum dot is thermodynamically isolated, the population of the level does not change during this process. Hence, there is no heat exchange. However, the change of the energy level releases a corresponding amount of work. We assume that the operation time of this step is very short, in particular negligibly small compared to that of the isothermal processes.

III *Isothermal process*
The dot is connected to the hot lead at temperature T_h and chemical potential μ_h. The energy level is lowered from ε_2 to ε_3 based on another protocol during a time interval of length τ_h. Just as in step I, both heat and work are exchanged.

IV *Adiabatic process*
The system is again disconnected from the lead and the level is restored from ε_3 to the initial level ε_0, at the cost of a corresponding amount of work. Afterwards, the dot is reconnected to the cold lead. Again, we assume that the operation time of this process is negligibly small.

The above procedure defines one cycle of the thermal engine, requiring a total time $\tau_c + \tau_h$. The protocols in steps I and III must be designed in such a way that

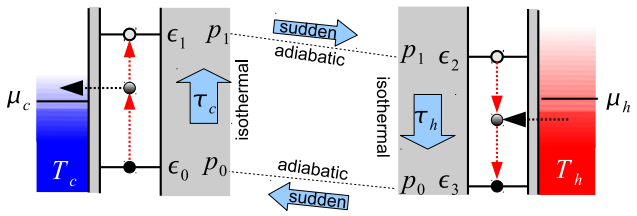

FIG. 1: A Carnot cycle of the model heat engine consisting of a single-level quantum dot interacting with a lead through a tunnleing junction.

the thermodynamic state of the system, in our case the occupation probability p of the quantum level, returns to the same initial value after every cycle. Since there is no change in occupation probability during the adiabatic stages II and IV, the change in occupation probability from, say p_0 to p_1, during process I, must necessarily be compensated by a change back from p_1 to p_0 during process III.

The time evolution of the occupation probability $p(t)$ for the state of the quantum dot in contact with a lead at temperature β^{-1} ($k_b = 1$) obeys the following quantum master equation:

$$\dot{p}(t) = -\omega_a(t)p(t) + \omega_b(t)[1 - p(t)], \tag{1}$$

where the ω_a and ω_b are transition rates. In the wide-band approximation, these rates are given by

$$\omega_a = \frac{C}{\mathrm{e}^{-\beta[\varepsilon(t)-\mu(t)]} + 1} \tag{2a}$$

$$\omega_b = \frac{C}{\mathrm{e}^{+\beta[\varepsilon(t)-\mu(t)]} + 1}, \tag{2b}$$

where C is a rate constant. Noting that raising the energy level is equivalent to lowering the chemical potential, we introduce an effective energy level $\epsilon \equiv \varepsilon - \mu$. The master equation (1) can now be rewritten as

$$\dot{p}(t) = -Cp(t) + \frac{C}{\mathrm{e}^{\beta\epsilon(t)} + 1}. \tag{3}$$

The effective level varies along the Carnot cycle as $\epsilon_0 = \varepsilon_0 - \mu_c \to \epsilon_1 = \varepsilon_1 - \mu_c \to \epsilon_2 = \varepsilon_2 - \mu_h \to \epsilon_3 = \varepsilon_3 - \mu_h$. Note that the change in the chemical potential is included in the jump of the effective level during processes II and IV.

We next turn to the thermodynamic description of the model. We use the convention that heat entering the system is (like work) positive. The internal energy of the system at time t is

$$\mathcal{E}(t) = \mathcal{U}(t) - \mu\mathcal{N}(t) = \epsilon(t)p(t), \tag{4}$$

where

$$\mathcal{U}(t) = \varepsilon(t)p(t), \quad \mathcal{N}(t) = p(t). \tag{5}$$