can produce only the terms proportional to even powers of the product $\mathbf{kq}$ being not responsible for the gyrotropy properties.

The off diagonal elements of $\hat{\boldsymbol{\tau}}(\mathbf{k})$ have the mixed parity. So, the dispersive terms proportional to the odd powers of the product $\mathbf{kq}$ can arise only from the part of eqn. (12) containing the off diagonal elements of $\hat{\boldsymbol{\tau}}(\mathbf{k})$ matrices. Hence, for the calculation of gyrotropy of conductivity, only the last two lines in eqn. (12) consisting of interband terms are important. They are equal to

$$\frac{\partial\gamma_l}{\partial k_i}\frac{\partial\gamma_m}{\partial k_j}\{\tau_{l,+-}\tau^*_{m,+-}[G_-G_+ - F_-F_+^\dagger] + \tau_{l,-+}\tau^*_{m,-+}[G_+G_- - F_+F_-^\dagger]\}. \quad (15)$$

Using the identity

$$\tau_{l,+-}\tau^*_{m,+-} = \tau^*_{l,-+}\tau_{m,-+} = \delta_{lm} - \hat{\gamma}_l\hat{\gamma}_m + ie_{lmn}\hat{\gamma}_n,$$

where $\hat{\gamma}_l = \gamma_l/|\boldsymbol{\gamma}|$, one can rewrite the above expression as

$$\frac{\partial\gamma_l}{\partial k_i}\frac{\partial\gamma_m}{\partial k_j}\{(\delta_{lm} - \hat{\gamma}_l\hat{\gamma}_m)[G_-G_+ + G_+G_- - F_-F_+^\dagger - F_+F_-^\dagger] + ie_{lmn}\hat{\gamma}_n[G_-G_+ - G_+G_- - F_-F_+^\dagger + F_+F_-^\dagger]\}. \quad (16)$$

Starting this point we need the explicit form of spin-orbit coupling vector $\boldsymbol{\gamma}(\mathbf{k})$. Its momentum dependence is determined by the crystal symmetry.[9,17] For the cubic group $G = O$, which describes the point symmetry of $Li_2(Pd_{1-x},Pt_x)_3B$, the simplest form compatible with the symmetry requirements is

$$\boldsymbol{\gamma}(\mathbf{k}) = \gamma_0\mathbf{k}, \quad (17)$$

where γ_0 is a constant. For the tetragonal group $G = C_{4v}$, which is relevant for $CePt_3Si$, $CeRhSi_3$ and $CeIrSi_3$, the spin-orbit coupling is given by

$$\boldsymbol{\gamma}(\mathbf{k}) = \gamma_\perp(k_y\hat{x} - k_x\hat{y}) + \gamma_\parallel k_xk_yk_z(k_x^2 - k_y^2)\hat{z}. \quad (18)$$

The gyrotropy current $\mathbf{j}_g$, which is linear with respect to the wave vector $\mathbf{q}$, originates from the last term in the eqn. (16). One can show that for the tetragonal crystal with the symmetry group $G = C_{4v}$, for the electric field lying in the basal plane the linear in the component of wave vector $\mathbf{q}$ part of conductivity is absent. In that follows we continue calculation for the metal with cubic symmetry where $\hat{\gamma} = \hat{k}\ sign\gamma_0$. We put $\hat{\gamma} = \hat{k}$ taking γ_0 as a positive constant. Thus, we obtain for gyrotropy current[21]

$$j_{gi}(\omega_n, \mathbf{q}) = ie_{ijl}\frac{e^2\gamma_0^2}{c}I_lA_j(\omega_n, \mathbf{q}), \quad (19)$$

$$I_l = \int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l \times \mathrm{T}\sum_{m=-\infty}^{\infty}[G_+(K_+)G_-(K_-) - F_+(K_+)F_-^\dagger(K_-) - G_-(K_+)G_+(K_-) + F_-(K_+)F_+^\dagger(K_-)]. \quad (20)$$

Let us find first the gyrotropy conductivity in the normal state.

III. GYROTROPY CONDUCTIVITY IN THE NORMAL STATE

Substituting the Green function in the eqn. (20) and performing summation over the Matsubara frequencies, we obtain

$$I_l = \int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l\left\{\frac{f(\xi_-(\mathbf{k}_-)) - f(\xi_+(\mathbf{k}_+))}{i\hbar\omega_n + \xi_-(\mathbf{k}_-) - \xi_+(\mathbf{k}_+)} - \frac{f(\xi_+(\mathbf{k}_-)) - f(\xi_-(\mathbf{k}_+))}{i\hbar\omega_n + \xi_+(\mathbf{k}_-) - \xi_-(\mathbf{k}_+)}\right\}. \quad (21)$$

Here $f(\xi_\pm(\mathbf{k}_\pm))$ is the Fermi distribution function and $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{q}/2$. By changing the sign of momentum $\mathbf{k} \to -\mathbf{k}$ in the first term under integral and making use that $\xi_\lambda(\mathbf{k})$ is even function of $\mathbf{k}$, we come to

$$I_l = 2\int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l\frac{[\xi_+(\mathbf{k}_+) - \xi_-(\mathbf{k}_-)][f(\xi_+(\mathbf{k}_+)) - f(\xi_-(\mathbf{k}_-))]}{(\xi_+(\mathbf{k}_+) - \xi_-(\mathbf{k}_-))^2 - (i\hbar\omega_n)^2}. \quad (22)$$

Analytical continuation of this expression from the discrete set of Matsubara frequencies into entire half-plane $\omega > 0$ is performed by the usual substitution $i\omega_n \to \omega + i/\tau$.

We shall work at frequencies smaller when the band splitting $\hbar\omega < \gamma_0k_F$ far from the resonance region $\hbar\omega \approx \gamma_0k_F$ but still in the collisionless limit $\omega\tau > 1$ where one can decompose the integrand in powers of ω^2:

$$I_l = 2\int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l[f(\xi_+(\mathbf{k}_+)) - f(\xi_-(\mathbf{k}_-))] \times\left\{\frac{1}{\xi_+(\mathbf{k}_+) - \xi_-(\mathbf{k}_-)} + \frac{(\hbar\omega)^2}{(\xi_+(\mathbf{k}_+) - \xi_-(\mathbf{k}_-))^3}\right\}, \quad (23)$$

The frequency independent term in eqn. (23) corresponds to the current density $\nu\mathbf{B}$ introduced in eqn. (4). We are interested in linear in $\mathbf{q}$ part of density of current. Expanding the integrand up to the first order in $\frac{\partial\xi_\pm}{\partial\mathbf{k}}\mathbf{q}$ one can prove by direct calculation that this term vanishes. Thus, in the normal state $\nu = 0$ as it should be in gauge invariant theory (see Section V and[14]). The frequency dependent term determines the current

$$j_i^g(\omega, \mathbf{q}) = ie_{ijl}\frac{e^2\gamma_0^2}{c}\hbar q_m(\hbar\omega)^2I_{lm}A_j(\omega, \mathbf{q}), \quad (24)$$

where

$$I_{lm} = \int\frac{d^3k}{(2\pi\hbar)^3}\hat{k}_l\left[-3\frac{f(\xi_+) - f(\xi_-)}{(\xi_+ - \xi_-)^4}\left(\frac{\partial\xi_+}{\partial k_m} + \frac{\partial\xi_-}{\partial k_m}\right) + \frac{1}{(\xi_+ - \xi_-)^3}\left(\frac{\partial f(\xi_+)}{\partial\xi_+}\frac{\partial\xi_+}{\partial k_m} + \frac{\partial f(\xi_-)}{\partial\xi_-}\frac{\partial\xi_-}{\partial k_m}\right)\right]. \quad (25)$$

After substitution of the Fourier component of the vector potential by the Fourier component of an electric field $\mathbf{A} = c\mathbf{E}/i\omega$, we obtain

$$j_i^g(\omega, \mathbf{q}) = e_{ijl}e^2\hbar^3\gamma_0^2\omega q_mI_{lm}E_j(\omega, \mathbf{q}). \quad (26)$$