neously the two conditions

$$\kappa_a^2 \sin k_0 = \kappa_0 \mathrm{Im}(E_a) \quad (72)$$

$$(\kappa_a^2 - 2\kappa_0^2) \cos k_0 = \kappa_o \mathrm{Re}(E_a). \quad (73)$$

Physically, the condition $R \rightarrow \infty$ implies the existence of an outgoing wave in the lattice that is sustained by the amplifying complex potential at the impurity site. Such a divergence of $R(k)$ is the signature of a spectral singularity of H. In fact, taking into account that the energy of incident/reflected waves is $\mathcal{E}_0 = -2\kappa_0 \cos k_0$, it can be easily shown that Eq.(73) is equivalent to Eq.(55), whereas Eq.(72) is equivalent to Eq.(54) with the upper (positive) sign on the right hand side. Therefore, for an amplifying impurity site [$\mathrm{Im}(E_a) > 0$], the condition $R \rightarrow \infty$ is equivalent to the appearance of a spectral singularity. This equivalence extends, to our scattering problem on a truncated lattice, the general result shown by Mostafazadeh [9], suggesting to interpret spectral singularities of a non-Hermitian Hamiltonian as resonances with vanishing width. However, for an absorbing potential energy at the impurity site, i.e. for $\mathrm{Im}(E_a) < 0$, our lattice model indicates that the appearance of spectral singularities has a different physical interpretation. In fact, for $\mathrm{Im}(E_a) < 0$ the reflection probability $R(k)$ is bounded from above and smaller than one, which prevents $R(k)$ to diverge. However, in this case $R(k)$ can vanish at wave numbers $k = k_0$ (with $0 \leq k_0 \leq \pi$) that satisfy simultaneously the two conditions [see Eq.(71)]

$$\kappa_a^2 \sin k_0 = -\kappa_0 \mathrm{Im}(E_a) \quad (74)$$

$$(\kappa_a^2 - 2\kappa_0^2) \cos k_0 = \kappa_o \mathrm{Re}(E_a). \quad (75)$$

Note that Eq.(75) is equivalent to Eq.(55), whereas Eq.(74) is equivalent to Eq.(54) with the lower (negative) sign on the right hand side. Therefore, for an absorbing impurity site [$\mathrm{Im}(E_a) < 0$], the appearance of a spectral singularity is equivalent to the vanishing of the reflection probability R: an ingoing plane wave with wave number k_0 is fully absorbed by the impurity site at the lattice edge. An an example, Fig.4 shows the behavior of $R(k)$ for an amplifying [Fig.4(a)] and for an absorbing [Fig.4(b)] impurity site for $\mathrm{Re}(E_a) = 0$ and $\kappa_a/\kappa_0 = 1$. The different curves in the figures refer to different values of $\mathrm{Im}(E_a)/\kappa_0$. Note that, at the value of $\mathrm{Im}(E_a)/\kappa_0$ corresponding to the appearance of the spectral singularity, a divergence and a zero in the $R(k)$ curve are observed in Figs.4(a) and 4(b), respectively. Figure 5 shows the behavior of $R(k)$ as in Fig.4, but for $\mathrm{Re}(E_a) = 0$ and $\kappa_a/\kappa_0 = \sqrt{2}$. In this case there are two spectral singularities at energies given by Eq.(57), which explain the two peaks [Fig.5(a)] or dips [Fig.5(b)] in the reflectance curve $R(k)$. By increasing $|\mathrm{Im}(E_a)|/\kappa_0$, the two peaks (or dips) get closer each other, until they coalesce at $|\mathrm{Im}(E_a)|/\kappa_0 = 2$ (curve 3 [50]).

Figures 6 and 7 show two examples of wave packet reflection from the lattice boundary for an amplifying (Fig.6) and an absorbing (Fig.7) impurity site. The figures show a snapshot of $|c_n(t)|$ as obtained by numerical analysis of Eqs.(65-67) assuming at $t = 0$ a broad Gaussian distribution of site occupation amplitudes, i.e. $c_n(0) = \exp[-(n - n_0)^2/\Delta n^2 - ikn]$, where Δn is the wave packet width, k the mean wave packet momentum, and $n_0 \gg \Delta n$ the wave packet center of mass. The large (diverging) amplification of the reflected wave packet in Fig.6(b), and the almost absence of wave packet reflection in Fig.7(b), are clearly visible when the energy $\mathcal{E} = -2\kappa_0 \cos k$ of the incoming wave packet attains the spectral singularity point $\mathcal{E}_0 = 0$.

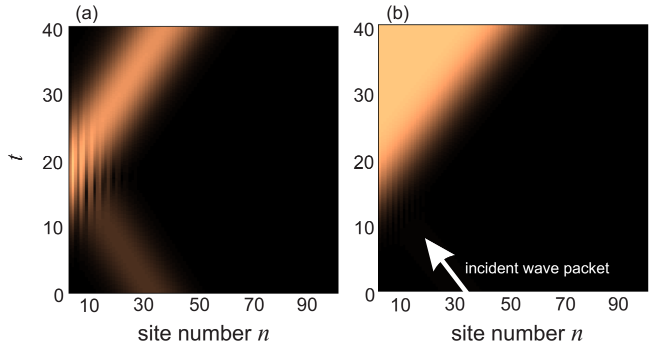

FIG. 6: (color online) Wave packet reflection in a semi-infinite lattice with an amplifying impurity site for $\kappa_0 = \kappa_a = 1$, $\mathrm{Re}(E_a) = 0$ and $\mathrm{Im}(E_a) = 1$. The initial wave packet is Gaussian- shaped, with peak amplitude $|c_n(0)| = 1$ at $n = n_0 = 30$, width $\Delta n = 12$ and momentum $k = 3\pi/10$ in (a), and $k = \pi/2$ in (b) (corresponding to the spectral singularity energy $\mathcal{E}_0 = 0$). In (b)the incident wave packet, indicated by an arrow, is not visible owing to the large amplification of the reflected wave.

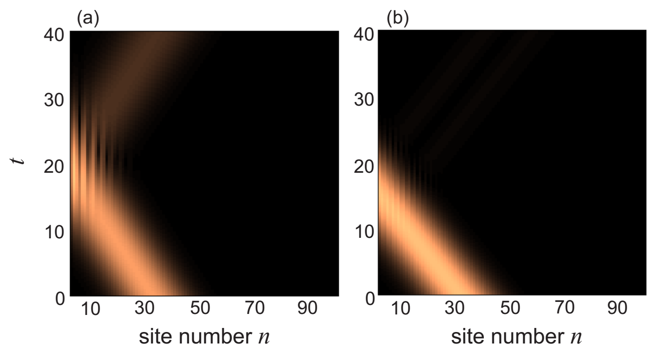

FIG. 7: (color online) Same as Fig.6, but for an absorbing impurity site $\mathrm{Im}(E_a) = -1$ (other parameters as in Fig.6).

The interplay between spectral singularities and decay dynamics, discussed in Sec.II.D, is exemplified in Figs.8 and 9. The system of Eqs.(65-67) has been numerically integrated with the initial condition $c_a(0) = 1$ and $c_n(0) = 1$. Note that, in the photonic realization of the semi-infinite lattice model of Ref.[37], such an initial condition simply corresponds to initial excitation of the boundary waveguide. The behavior of the site occupation probability $P(t) = |c_a(t)|^2$ for an amplifying and for an absorbing impurity site is depicted in Figs.8 and 9, respectively. Note that, according to the gen-