MATLAB Exercise – Windows Comparisons

Program Directory: `matlab_gui\windows`
Program Name: `windows_GUI25.m`
GUI data file: `windows.mat`
Callbacks file: `Callbacks_windows_GUI25.m`
TADSP: Section 6.2.2, pp. 245-248, Problem 6.16

This MATLAB exercise computes and compares and contrasts the time and frequency responses of 6 L-point commonly used windows, namely the rectangular window, the triangular window, the Hanning window, the Hamming window, a modified Hamming window and the Blackman window. Pairs of windows can be compared as to time and frequency responses, and even all 6 windows can be displayed on the three graphics panels, if so desired.

Windows Comparisons – Theory of Operation

The time-domain responses of the 6 L-point windows used in this exercise are as follows:

1. Rectangular window:
$$w[n] = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{otherwise.} \end{cases}$$

2. Triangular window:
$$w[n] = \begin{cases} 2n/(L-1) & 0 \le n \le (L-1)/2 \\ 2 - 2n/(L-1) & (L+1)/2 \le n \le L-1 \\ 0 & \text{otherwise.} \end{cases}$$

3. Hann window:
$$w[n] = \begin{cases} 0.5 - 0.5\cos\left(\dfrac{2\pi n}{L-1}\right) & 0 \le n \le L-1 \\ 0 & \text{otherwise.} \end{cases}$$

4. Hamming window:
$$w[n] = \begin{cases} 0.54 - 0.46\cos\left(\dfrac{2\pi n}{L-1}\right) & 0 \le n \le L-1 \\ 0 & \text{otherwise.} \end{cases}$$

5. modified Hamming window:
$$w[n] = \begin{cases} 0.54 - 0.46\cos\left(\dfrac{2\pi n}{L-1}\right) & 0 \le n \le L-2 \\ 0 & \text{otherwise.} \end{cases}$$

6. Blackman window:
$$w[n] = \begin{cases} 0.42 - 0.5\cos\left(\dfrac{2\pi n}{L-1}\right) + 0.08\cos\left(\dfrac{4\pi n}{L-1}\right) & 0 \le n \le L-1 \\ 0 & \text{otherwise.} \end{cases}$$

It should be noted that the modified Hamming window of length L samples is actually of length $L-1$ samples since the $L-1$ sample is set to zero, as explained in Section 7.5.1, pp. 322-331 of TADSP.

The MATLAB exercise specifies the value of the window duration in samples, L, designs the six windows and plots (in pairs, singly or as a group) their time responses, on a common graphics panel. The log magnitude responses

of the user-selected windows (with normalizing the log magnitude response of each window to 0 dB at frequency 0 Hz) are plotted in a second graphics panel. Finally the exercise replots the log magnitude responses (in a third graphics panel) over a narrow band between 0 and $5 * f_s/L$ which enables the user to compare the effective bandwidths and peak sidelobe rejection of the set of user-selected windows.

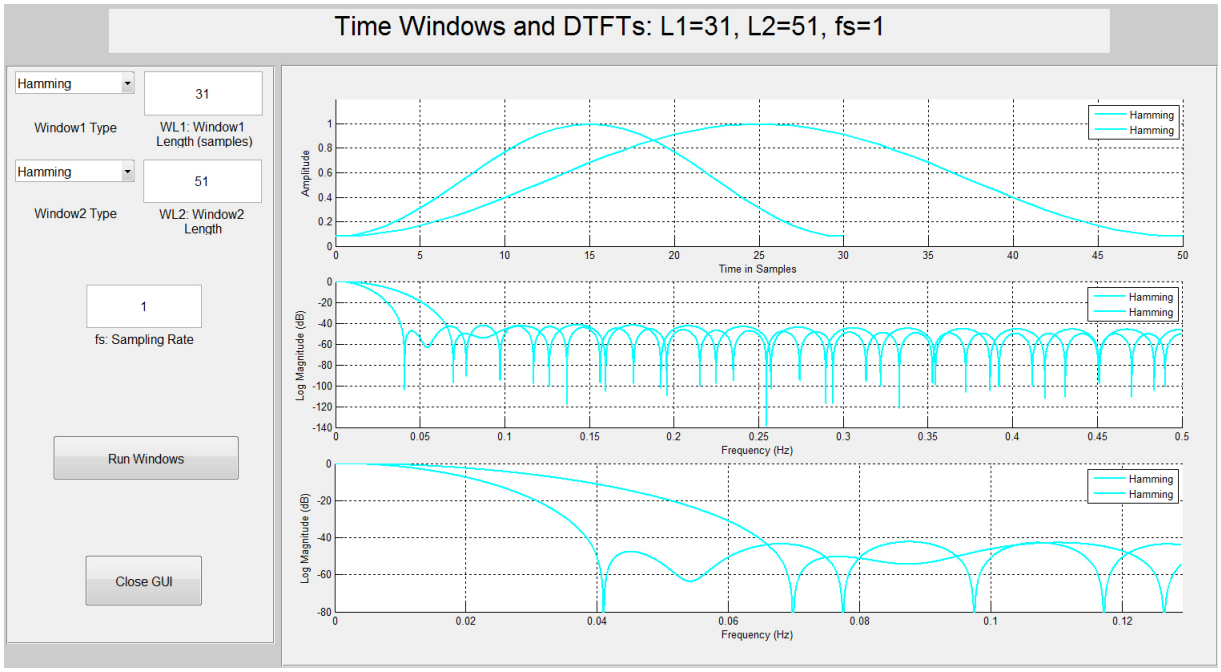

Figure 1: Time and frequency responses (log magnitude) for comparing a 31-point and a 51-point Hamming window; upper graphics panel shows the pair of time domain responses; the middle graphics panel shows the pair of log magnitude responses for a full frequency scale (0-fs/2 Hz), and the bottom graphics panel shows the pair of log magnitude frequency responses on a reduced frequency scale.

Windows Comparison – GUI Design

The GUI for this exercise consists of two panels, three graphics panels, 1 title box and 7 buttons. The functionality of the two panels is:

1. one panel for the graphics display,
2. one panel for parameters related to defining the windows to be plotted and compared.

The set of three graphics panels is used to display the following:

1. the time response of each of the user-selected windows; the windows can be compared in pairs or as a group of 6,
2. the log magnitude response(s) (normalized to a peak of 0 dB) for the chosen windows,
3. the log magnitude response(s) for the chosen windows on a zoomed in frequency scale.

The title box displays the information about the user-designated window parameters. The functionality of the 7 buttons is:

1. a popupmenu button specifying the first window type to be compared for analysis; the choices are one of the six windows or all six windows as a group,

2. an editable button which specifies the value of `WL1`, the first window duration in samples; (default is `WL1`=31 samples),

3. a popupmenu button specifying the second window type to be compared for analysis; the choices are again one of the six windows, or all six windows as a group,

4. an editable button which specifies the value of `WL2`, the second window duration in samples; (default is `WL2`=31 samples),

5. an editable button which specifies the value of `fs`, the sampling rate of the digital processing; (default is `fs`=1 sample per second),

6. a pushbutton to 'Run' the exercise with the selected window parameters,

7. a pushbutton to close the GUI.

Windows Comparison – Scripted Run

A scripted run of the program 'windows_GUI25.m' is as follows:

1. run the program 'windows_GUI.m' from the directory 'matlab_gui\windows',

2. using the popupmenu button, select a Hamming window as the first window,

3. using the editable button, choose a first window length, `WL1`, of 31 samples, the default value of first window length,

4. using the popupmenu button, again select a Hamming window as the second window,

5. using the editable button, choose a second window length, `WL2`, of 51 samples, which is somewhat larger than the default value of second window length,

6. using the editable button, choose a sampling rate of the `fs`=1 sample per second, the default value,

7. hit the 'Run' button to compute the time and frequency response(s) of the selected window(s) and plot the time response(s) in the upper graphics panel, the log magnitude frequency response(s) in the middle graphics panel, and the log magnitude frequency response(s) on a reduced frequency scale, in the bottom graphics panel. The exercise normalizes the log magnitude responses of each window to 0 dB, at frequency 0 Hz, so as to be able to compare the log magnitude responses of the different windows,

8. experiment with different values of window lengths, sampling rate, and window types to see the impact on the window characteristics,

9. hit the 'Close GUI' button to terminate the run.

An example of the graphical output obtained from this exercise is shown in Figure 1. The default values of WL1=31, WL2=51, $f_s = 1$, were used in this plot, with both windows being Hamming windows of different lengths. The differences in time and frequency responses clearly illustrate the properties of different windows.

Windows Comparison – Issues for Experimentation

1. run the scripted exercise above, and answer the following:

   - using the expanded log magnitude frequency response plot, determine the bandwidth of each of the 6 windows; (the bandwidth is defined as the frequency in the transition band which has the same log magnitude as the peak log magnitude in the stopband)
   - express the bandwidth of each of the 6 windows in terms of normalized width, namely f_s/L where f_s is the sampling rate of the window, and L is the window length in samples
   - what is the peak ripple in the stopband (in dB relative to the log magnitude response at zero frequency) for each of the 6 windows

2. change both window lengths to `WL1`=`WL2`=101 samples and repeat the processing of the previous step

   - how stable are the peak ripples and the bandwidths of the various windows as `WL1` and `WL2` vary from `WL1`=`WL2`=31 to `WL1`=`WL2`=101 samples?

   - set both window lengths to `WL1`=`WL2`=41 samples and experiment with each of the 6 windows. Record, in each case, the width of the main lobe and the height of the maximum side lobe.
   - decrease both window lengths to `WL1`=`WL2`=31 samples and repeat the measurements of the previous step.
   - increase both window lengths to `WL1`=`WL2`=51 samples and again repeat the measurements of main lobe width and side lobe level.
   - compare the results of the previous three steps. How do the side lobe amplitudes change with window length? How do the main lobe widths vary with window length?
   - compare the Hamming window of length 31 to the modified Hamming window of the same length. Note any differences in main lobe width, side lobe amplitude, and spacing of the zeros (the sharp dips in the plot). See the overlap-add GUI exercise for the implications of this behavior.