Critical states of slow pattern in neuronal networks

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Outline

Outline

1. Motivation
   - The Basic Problem That We Studied
   - Previous Work
2. Our Results/Contribution
   - Main Results
   - Basic Ideas for Proofs/Implementation

Network Dynamics

- Network response correspond to specific neuronal parameter, including fire rate, degree of irregularity, spatiotemporal patterns in neuronal spike trains and neuronal critical dynamics.
- Explor the influence of simulation size of neuronal network as well as the community stuctural network. 2000, 5000, 10000, ..., 100 million.
- Synapase density and input heterogeneity.(to be confirmed)

Theoretical explanation

Mainly three aspects. . .

- Explain the mechanism underly the trainsition dynamics.
  - input current variablity analysis.
  - Mean-filed equation and hopf bifuraction
  - The real part of fixed point is decreasing.
- fit the network response with a simple f unction.

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Make Titles Informative.

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signle neuron dynamics

Orinignal model

$$C_j \dot{V}_j = - \sum_{A=L,E,I} g_A^j (V_j - E_A),$$

where $\frac{g_{E,I}^j}{g_L^j} = \tau_j \sum_m \sum_{k|t_m^k<t} \int_{-\infty}^t dt' a_{\tau_{decay}}(t-t')\,\delta(t'-t_m^k)$,
usually, $a_{\tau_{decay}}(t) = e^{-t/\tau_{decay}}/\tau_{decay}$, but here we use the limit that $\tau_{decay} \to 0$.

signle neuron dynamics

- excitatory and inhibitory currents must be fine-tuned to produce an average input below threshold. Specifically, if K and J represent the average number of input connections per neuron and synaptic efficacy, respectively, the difference between excitatory and inhibitory presynaptic inputs must be of the order of $1/KJ$.
- input fluctuations should be large enough to drive firing.

signle neuron dynamics

We can use the diffusion approximation and approximate the conductances as

$$\begin{aligned} \frac{g_E}{g_L} &= a\tau_L \left[Kr_E + \sqrt{Kr_E}\zeta_E \right], \\ \frac{g_I}{g_L} &= ag\tau_L \left[\gamma Kr_I + \sqrt{\gamma Kr_I}\zeta_I \right] \end{aligned} \tag{1}$$

signle neuron dynamics

Using the diffusion approximation, OU process, the CV equation can be reduced to

$$\tau \frac{dV}{dt} = -V + \mu + \sigma(V)\sqrt{\tau}\zeta \tag{2}$$

where

$$\begin{aligned} \tau^{-1} &= \tau_L^{-1} + aK\left(r_E + r_I g\gamma\right) \\ \mu &= \tau\left\{E_L/\tau_L + aK\left[r_E E_E + r_I g\gamma E_I\right]\right\} \\ \sigma^2(V) &= a^2 K\tau\left[r_E\left(V - E_E\right)^2 + g^2\gamma r_I\left(V - E_I\right)^2\right] \end{aligned} \tag{3}$$

under the assumption that $Ka >> 1$, we have

$$\tau \sim \frac{\tau_0^{\text{cond}}}{Ka}, \quad \mu \sim \mu_0^{\text{cond}}, \quad \sigma \sim \sqrt{a}\sigma_0^{\text{cond}} \tag{4}$$

we can conclude that:

1. μ is indenpendent of coupling strength, i.e, the synaptic efficacy a and degree k.
2. Increasing a modifies the drift force and the input noise, which increase proportionally to a and $\sqrt{a}$
3. incresing k increases the drift force as ka

In this case, $ka >> 1$, we called it strong coupling. However, if such assumpaiton not holds (weak coupling), the scaling function (4) is not ture.

Biological plausible neuronal network

Consider a balanced E-I network with N neurons, in which 80% are excitatory neurons and the others inhibitory ones. Each neuron is equipped with a biological plausible neuronal model, leaky integrate and fire model,

$$C\frac{dV}{dt} = -g_l(V - V_l) + I_{syn} + I_{ext},$$

and the conductance-based synaptical filter

$$I_{syn}(t) = \sum_u g_u S_u(t)(V_{rev,u} - V)$$

$$S_{i,u}(t) = \frac{1}{\tau_u} e^{-t/\ \tau_u} * \sum_{n,j\in\partial^i} w_j \delta\left(t - t_j^n\right)$$

$$u = \{AMPA, NMDA, GABA_A, GABA_B\}.$$

Balance condition

Under the given network conditions and default neuron parameters, we need to seek for a group of appropriate parameters (g_u) to make the network self-sustaining in a stable firing state.
we can roughly estimate a gruop of parameter by adopting the first order diffusion approximation.

$$\frac{V_{th} - V_{reset}}{\sum_u \langle g_u S_u (V_u - V) \rangle - \langle g_l (V - V_l) \rangle} = \frac{1}{r_{\text{equilibrium}}},$$

and then we can dervie that

$$\begin{cases} 1g_{AMPA} + 5g_{NMDA} = 0.022 \\ 1g_{GABA_A} + 18g_{GABA_B} = 0.5 \end{cases} \tag{5}$$

Default Paramters

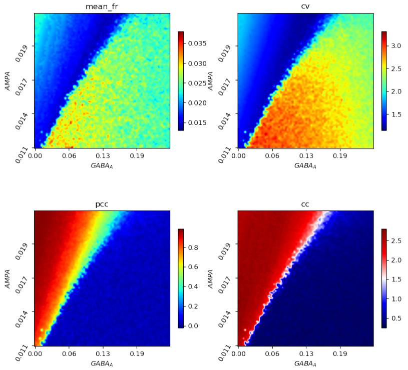
Symbol	Description	Value
N	Total number of neurons	2000
N_E	Total number of excitatory neurons	1600
N_I	Total number of inhibitory neurons	400
K_{in}	Mean of in-degrees	100
τ_{ref}	Refractory period	$5ms$
tau_u	decay time of receptors	$(8, 40, 10, 50)$
$V_{rev,u}$	reverse voltage	$(0, 0, -70, -100))$

Table: Default values of model parameters used in numerical simulations

Rich dynamics in parameter submainfold

We find rich dynamics in this small balanced E-I network.

- Obvious transition in the fire rate.
- CV is larger than 1, and multi stratification.
- Coherence coefficient reflects the spike coherence.
- Critical dynamics occurs in the stratification line paramete space.



avalanches pehenomena

Mainly due to E-I delay feedback. . .

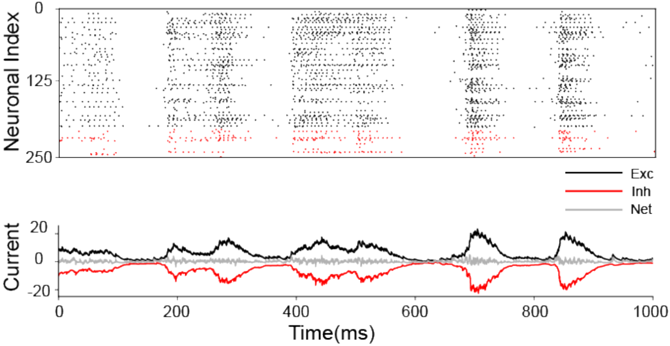


Figure: raster plot of slow dynamics

avalanches pehenomena

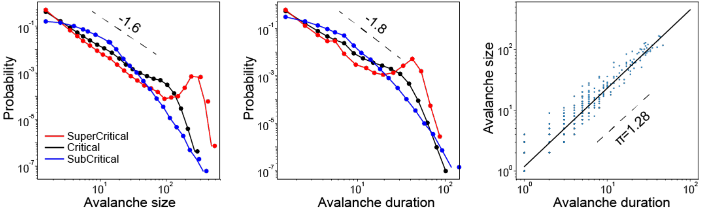


Figure: avalanches pehenomena

The slope of best fit powerlaw distribution for avalanche size is -1.6 and avalanche duration is -1.8

Characteristic in boundary line

if we dive into the characteristices of the network on the boundary line space

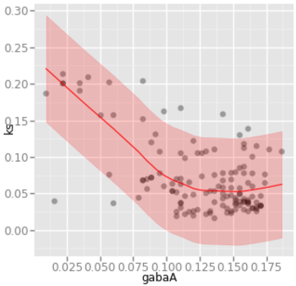


Figure: ks distance

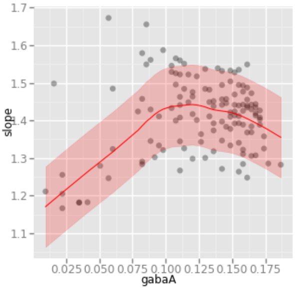
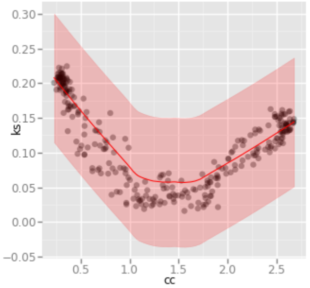
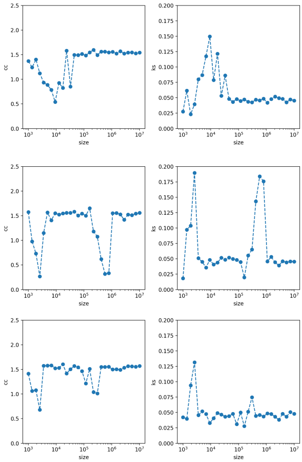


Figure: slope

robust on size

The corresponding relationship between *coherence coefficient* and *ks distance* is shown in the figure below





degree inluence

we perform experiments and summarize the main findings based on shifted exponential in- degree distributions.

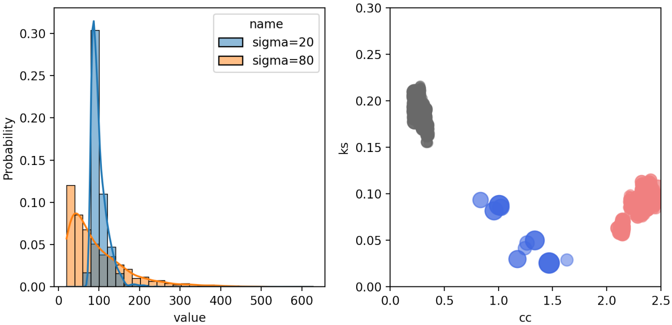


Figure: degree influence

parameter heterogeneity

- the raidus is from 1 to 15
- along the boundary line

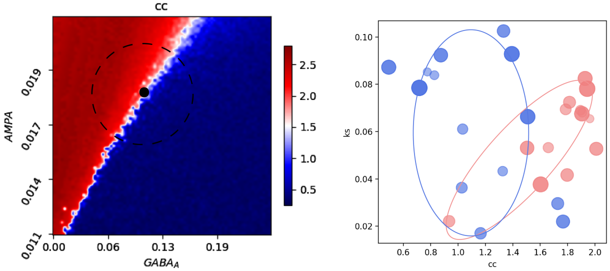


Figure: parameter heterogeneity

Criticality in large-scale network

Fit to function $s \cdot tanh(ax + by + c) + t$

- in the small blcok, slop is 1.6
- in the large-scale block, slop is 1.6

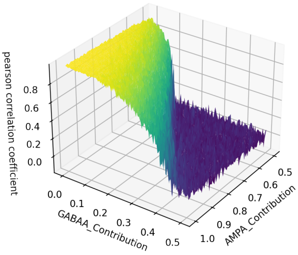


Figure: size=2k

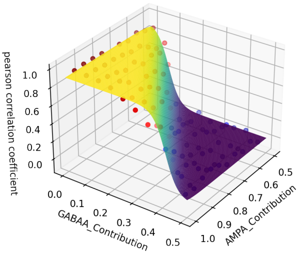


Figure: size=100m

Criticality in large-scale network

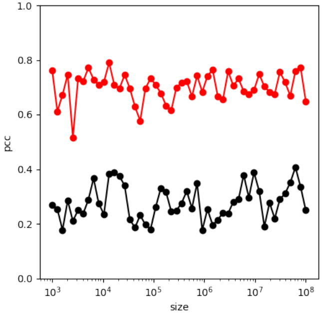


Figure: pcc with respect to different sizes

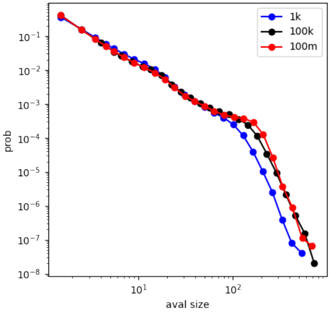


Figure: avalanches distribution

Criticality in large-scale network

Some simulation case from a fixed parameter of critical space.

- The avalanche size increases with the size of the simulation.

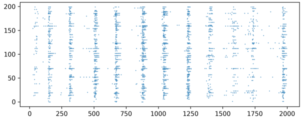


Figure: size=10k

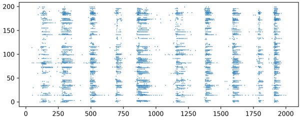
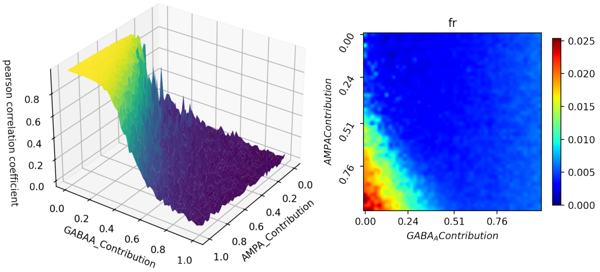


Figure: size=100m

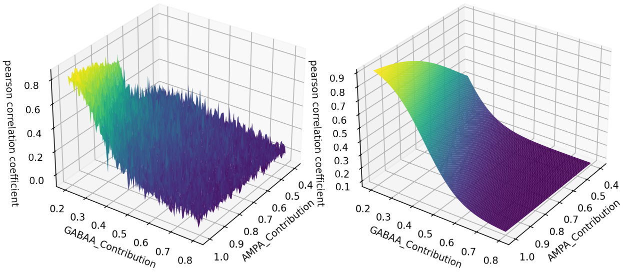
Dependence on connection density

grid search on network of 2000 neurons with 300 in-connections. x and y range is $[0, 1]$, each 50 points.



Dependence on connection density

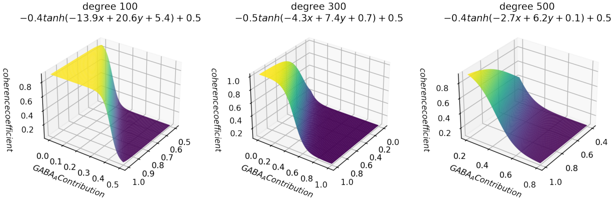
grid search on network of 2000 neurons with 500 in-connections.



As we can see, the plane is much more smooth than its in degree=100 and the one-dimensional linear submanifold disappears.

Dependence on connection density

If all case fit to function $s \cdot tanh(ax + by + c) + t$. we can find that its slope is smaller with the degree increasing.



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Synaptic drive

The full model:

$$C_j \dot{V}_j = -\sum_A g^j_{L,A}(V_j - E_A)$$

$$g^j_A = c_A \sum_m \sum_{k|t^k_m<t} \int_{-\infty}^{t} dt'\, a_{\tau_{decay}}\,(t-t')\,\delta(t'-t^k_m),$$

where $A = AMPA, NMDA, GABA_A, GABA_B$.
Herein, only the single neuron properties will be considered.
We aim to explore the influence of restricted parameter c_A to the synaptic drive and membrane response.

The diffusion approximation

The number of pulses that arrive within the timescalesτ_A will be approximately gaussian distributed. Thus, we can constitute this Ornstein-Uhlenbeck process.

$$\tau_A \frac{dg_A}{dt} \simeq g_{A0} - g_A + \sqrt{2}\sigma_A \xi_A(t) \quad (6)$$

where the gaussian white noise ξ_A has a zero mean and relatively independent.

$$\langle \xi_A(t) \rangle = 0 \quad \left\langle \xi_A(t) \xi_A\left(t'\right) \right\rangle = \tau_A \delta\left(t - t'\right) \quad (7)$$

The average conductance $g_{A,0}$ and the standard deviation σ_A are related to the variables c_A, τ_A and $\mathcal{R}$ through

$$g_{A0} = c_A \tau_A \mathcal{R}_A, \quad \sigma_A = c_A \sqrt{\frac{\tau_A \mathcal{R}_A}{2}} \quad (8)$$

The diffusion approximation

we sole the above OU equation, then derive:

$$g_A(t) = g_A(0)e^{-t/\tau_A} + g_{A0}\left(1 - e^{-t/\tau_A}\right) + \sqrt{2}\sigma_e \int_0^t \frac{1}{\tau_A} e^{-(t-s)/\tau_A} dB_s, \tag{9}$$

and

$$g_{AF}(t) \equiv g_A(t) - g_{A0} \simeq \sqrt{2}\sigma_A \int_0^t \frac{1}{\tau_A} e^{-(t-s)/\tau_A} dB_s \tag{10}$$

$$g_{AF}(t) \sim \mathcal{N}\left(0, 2\sigma_A^2 \int_0^t (\frac{1}{\tau_A} e^{-(t-s)/\tau_A})^2 dt\right) \text{ (Ito isometry)}, \tag{11}$$

if $t \to \infty$,

$$p_D(g_A) = \frac{1}{\sqrt{2\pi\sigma_A^2}} \exp\left(-\frac{(g_A - g_{A0})^2}{2\sigma_A^2}\right) \tag{12}$$

The diffusion approximation

The criterion for validity of the approximation is $\sigma_A/g_{A0} << 1$. If we thake consideration of the eq(5) and we can derive the total conductance drive. Thus, using excitation as an example,

$$\frac{dg_e}{dt} \simeq \frac{g_0^{(1)}}{\tau_1} + \frac{g_0^{(2)}}{\tau_2} - (\frac{\tau_1 + \tau_2}{\tau_1 \tau_2}) g_e + \sqrt{2}(\frac{\tau_1 \sigma_1 + \tau_2 \sigma_1}{\tau_1 \tau_2}) \xi_e(t),$$

similarly, we can conclude that the excitatory conducatance obey the gaussian disritbuion.

Membrane response

By separating the synaptic conductances into components $g_0^{(i)}, i = 1, 2, 3, 4$, and fluctuating components $g_F^{(i)}, i = 1, 2, 3, 4$.

$$C\frac{dV}{dt} = -g_0 \left(V - E_0\right) - \sum_{i=1}^{4} g_F^{(i)} \left(V - E_i\right) \tag{13}$$

where

$$g_0 = g_L + \sum g_0^{(i)} \quad \text{and} \quad E_0 = \frac{1}{g_0}\left(g_L E_L + \sum g_0^{(i)} E_i\right) \tag{14}$$

effective membrane time constant: $\tau_0 = C/g_0$

Membrane response

the voltage-dependent component of the drive can be expanded around the equilibrium potential E_0

$$g_F^i \left(V - E_i \right) = g_F^i \left(E_0 - E_i \right) + g_F^i \left(V - E_i \right) \tag{15}$$

If we drope the multiplicative noise term, that is

$$\begin{cases} C \dfrac{dV}{dt} \simeq -g_0 \left(V - E_0 \right) + \sum g_F^i \left(E_i - E_0 \right) \\ \tau_i \dfrac{dg_F^o}{dt} \simeq g_F^i + \sqrt{2} \sigma_i \xi_i(t) \end{cases} \tag{16}$$

where, ξ_1 and ξ_2 is totally correlated, ξ_3 and ξ_4 is totally correlated.

Membrane response

Then we can rewrite the eq16 as:

$$\frac{dV}{dt} \simeq -\frac{1}{\tau_0}(V - E_0) + I_F \tag{17}$$

and we have

$$I_F(t) = V(t) - E_0$$
$$\simeq \sum \sqrt{2}\left(\frac{\sigma_i}{g_0}\right)\frac{(E_i - E_0)}{(\tau_i - \tau_0)}\int_0^\infty ds\left(e^{-s/\tau_i} - e^{-s/\tau_0}\right)\xi_i(t-s) \tag{18}$$

Membrane response

The distribution predicted for the voltage is the gaussian

$$p_0(V) = \frac{1}{\sqrt{2\pi\sigma_V^2}} \exp\left(-\frac{(V - E_0)^2}{2\sigma_V^2}\right) \tag{19}$$

where, for the case where there are no correlations between excitation and inhibition, the variance is

$$\begin{aligned}
\sigma_V^2 = & \sum_{i=1}^{4} \left(\frac{\sigma_i}{g_0}\right)^2 (E_i - E_0)^2 \frac{\tau_i}{(\tau_i + \tau_0)} \\
& + 2\left(\frac{\sigma_1\sigma_2}{g_0^2}\right)(E_1 - E_0)(E_2 - E_0)\frac{\sqrt{\tau_1\tau_2}\,(2\tau_1\tau_2 + \tau_1\tau_0 + \tau_2\tau_0)}{(\tau_1 + \tau_2)(\tau_1 + \tau_0)(\tau_2 + \tau_0)} \\
& + 2\left(\frac{\sigma_3\sigma_4}{g_0^2}\right)(E_3 - E_0)(E_4 - E_0)\frac{\sqrt{\tau_3\tau_4}\,(2\tau_3\tau_4 + \tau_3\tau_0 + \tau_4\tau_0)}{(\tau_3 + \tau_4)(\tau_3 + \tau_0)(\tau_4 + \tau_0)}
\end{aligned}$$

Membrane response

Increasing $GABA_A$ modifies the equilibrium value E_0, the drift force and the input noise.

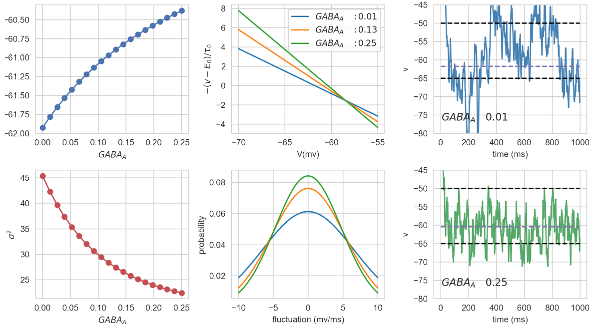

Figure: Effects of GABAA on the friing behavior of isolated conductance-based neuron

input-output correlation

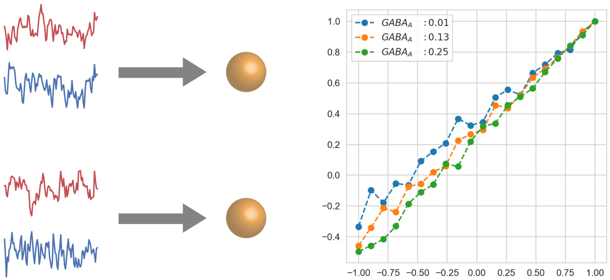

Figure: The relationship between the input and output correlation coefficients for an LIF neuron

Summary

- The first main message of your talk in one or two lines.
- The second main message of your talk in one or two lines.
- Perhaps a third message, but not more than that.

- Outlook
  - Something you haven't solved.
  - Something else you haven't solved.

For Further Reading I

A. Author.
Handbook of Everything.
Some Press, 1990.

S. Someone.
On this and that.
Journal of This and That, 2(1):50–100, 2000.