Critical states of slow pattern in neuronal networks

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Outline

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1. Motivation
   - The Basic Problem That We Studied
   - Previous Work

2. Our Results/Contribution
   - Main Results
   - Basic Ideas for Proofs/Implementation

Network Dynamics

- Network response correspond to specific neuronal parameter, including fire rate, degree of irregularity, spatiotemporal patterns in neuronal spike trains and neuronal critical dynamics.
- Explor the influence of simulation size of neuronal network as well as the community stuctural network. 2000, 5000, 10000, ..., 100 million.
- Synapase density and input heterogeneity.(to be confirmed)

Theoretical explanation

Mainly three aspects. . .

- Explain the mechanism underly the trainsition dynamics.
  - input current variablity analysis.
  - Mean-filed equation and hopf bifuraction
  - The real part of fixed point is decreasing.
- fit the network response with a simple f unction.

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Make Titles Informative.

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Outline

signle neuron dynamics

Orinignal model

$$C_j \dot{V}_j = - \sum_{A=L,E,I} g_A^j (V_j - E_A),$$

where $\frac{g^j_{E,I}}{g^j_L} = \tau_j \sum_m \sum_{k|t^k_m<t} \int_{-\infty}^t dt' \, a_{\tau_{decay}}(t-t')\,\delta(t'-t^k_m)$,
usually, $a_{\tau_{decay}}(t) = e^{-t/\tau_{decay}}/\tau_{decay}$, but here we use the limit that $\tau_{decay} \to 0$.

signle neuron dynamics

- excitatory and inhibitory currents must be fine-tuned to produce an average input below threshold. Specifically, if K and J represent the average number of input connections per neuron and synaptic efficacy, respectively, the difference between excitatory and inhibitory presynaptic inputs must be of the order of $1/KJ$.
- input fluctuations should be large enough to drive firing.

signle neuron dynamics

We can use the diffusion approximation and approximate the conductances as

$$
\begin{aligned}
\frac{g_E}{g_L} &= a\tau_L \left[Kr_E + \sqrt{Kr_E}\zeta_E \right], \\
\frac{g_I}{g_L} &= ag\tau_L \left[\gamma Kr_I + \sqrt{\gamma Kr_I}\zeta_I \right]
\end{aligned}
\tag{1}
$$

signle neuron dynamics

Using the diffusion approximation, OU process, the CV equation can be reduced to

$$\tau \frac{dV}{dt} = -V + \mu + \sigma(V)\sqrt{\tau}\zeta \tag{2}$$

where

$$\begin{aligned} \tau^{-1} &= \tau_L^{-1} + aK\left(r_E + r_I g\gamma\right) \\ \mu &= \tau\left\{E_L/\tau_L + aK\left[r_E E_E + r_I g\gamma E_I\right]\right\} \\ \sigma^2(V) &= a^2 K\tau\left[r_E\left(V - E_E\right)^2 + g^2\gamma r_I\left(V - E_I\right)^2\right] \end{aligned} \tag{3}$$

under the assumption that $Ka >> 1$, we have

$$\tau \sim \frac{\tau_0^{\text{cond}}}{Ka}, \quad \mu \sim \mu_0^{\text{cond}}, \quad \sigma \sim \sqrt{a}\sigma_0^{\text{cond}} \tag{4}$$

we can conclude that:

1. μ is indenpendent of coupling strength, i.e, the synaptic efficacy a and degree k.
2. Increasing a modifies the drift force and the input noise, which increase proportionally to a and $\sqrt{a}$
3. incresing k increases the drift force as ka

In this case, $ka >> 1$, we called it strong coupling. However, if such assumpaiton not holds (weak coupling), the scaling function (4) is not ture.

Biological plausible neuronal network

Consider a balanced E-I network with N neurons, in which 80% are excitatory neurons and the others inhibitory ones. Each neuron is equipped with a biological plausible neuronal model, leaky integrate and fire model,

$$C\frac{dV}{dt} = -g_l(V - V_l) + I_{syn} + I_{ext},$$

and the conductance-based synaptical filter

$$I_{syn}(t) = \sum_u g_u S_u(t)(V_{rev,u} - V)$$

$$S_{i,u}(t) = \frac{1}{\tau_u} e^{-t/\ \tau_u} * \sum_{n,j\in\partial^i} w_j \delta\left(t - t_j^n\right)$$

$$u = \{AMPA, NMDA, GABA_A, GABA_B\}.$$

Balance condition

Under the given network conditions and default neuron parameters, we need to seek for a group of appropriate parameters (g_u) to make the network self-sustaining in a stable firing state.
we can roughly estimate a gruop of parameter by adopting the first order diffusion approximation.

$$\frac{V_{th} - V_{reset}}{\sum_u \langle g_u S_u (V_u - V) \rangle - \langle g_l (V - V_l) \rangle} = \frac{1}{r_{\text{equilibrium}}},$$

and then we can dervie that

$$\begin{cases} 1g_{AMPA} + 5g_{NMDA} = 0.022 \\ 1g_{GABA_A} + 18g_{GABA_B} = 0.5 \end{cases}$$

Default Paramters

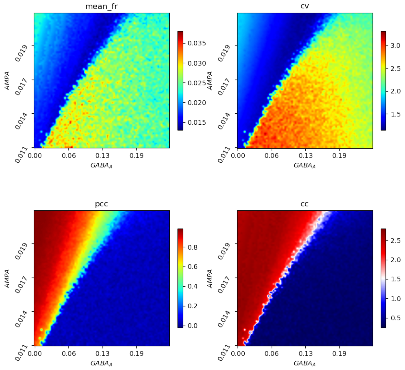
Symbol	Description	Value
N	Total number of neurons	2000
N_E	Total number of excitatory neurons	1600
N_I	Total number of inhibitory neurons	400
K_{in}	Mean of in-degrees	100
τ_{ref}	Refractory period	$5ms$
tau_u	decay time of receptors	$(8, 40, 10, 50)$
$V_{rev,u}$	reverse voltage	$(0, 0, -70, -100))$

Table: Default values of model parameters used in numerical simulations

Rich dynamics in parameter submainfold

We find rich dynamics in this small balanced E-I network.

- Obvious transition in the fire rate.
- CV is larger than 1, and multi stratification.
- Coherence coefficient reflects the spike coherence.
- Critical dynamics occurs in the stratification line paramete space.



avalanches pehenomena

Mainly due to E-I delay feedback. . .

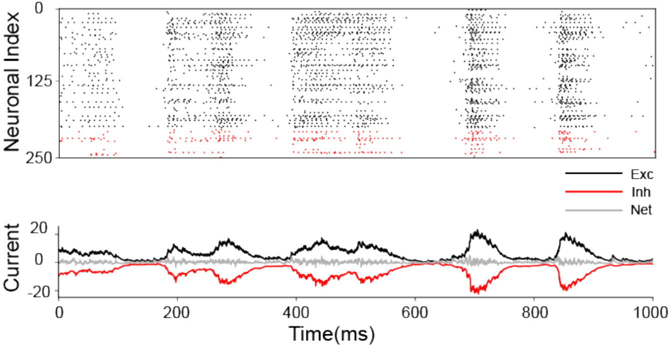


Figure: raster plot of slow dynamics

avalanches pehenomena

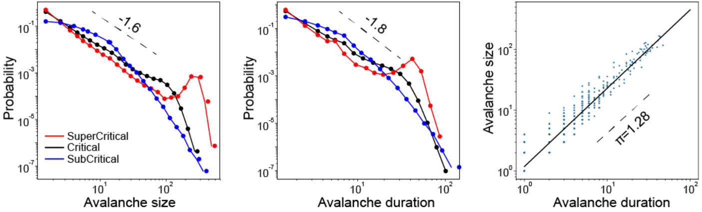


Figure: avalanches pehenomena

The slope of best fit powerlaw distribution for avalanche size is -1.6 and avalanche duration is -1.8

Characteristic in boundary line

if we dive into the characteristices of the network on the boundary line space

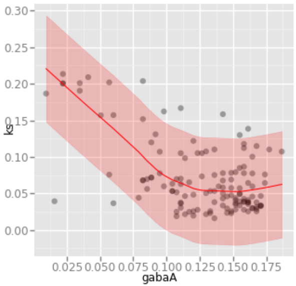


Figure: ks distance

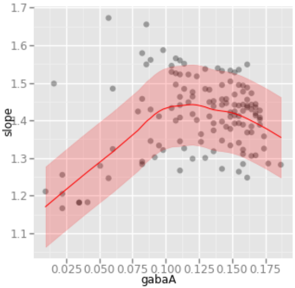
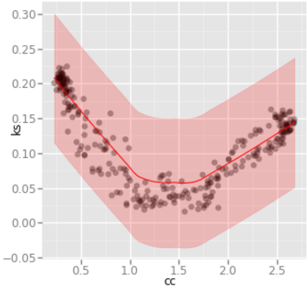
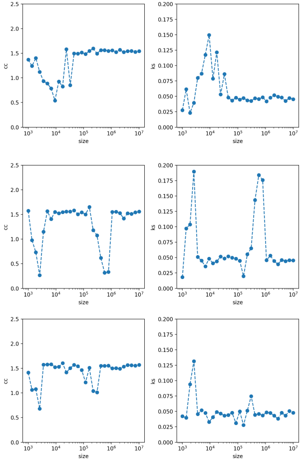


Figure: slope

robust on size

The corresponding relationship between *coherence coefficient* and *ks distance* is shown in the figure below





degree inluence

we perform experiments and summarize the main findings based on shifted exponential in- degree distributions.

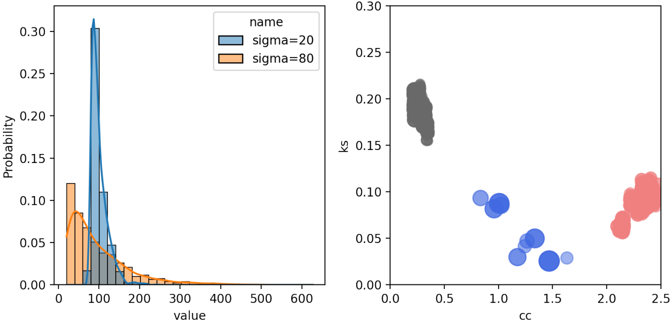


Figure: degree influence

parameter heterogeneity

- the raidus is from 1 to 15
- along the boundary line

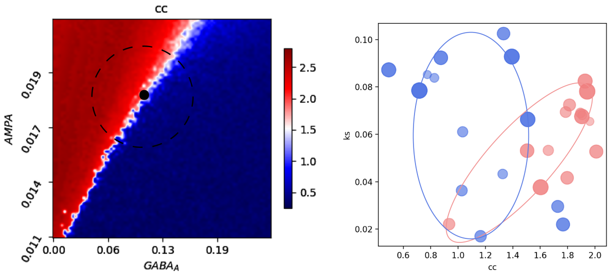


Figure: parameter heterogeneity

Criticality in large-scale network

Fit to function $s \cdot tanh(ax + by + c) + t$

- in the small blcok, slop is 1.6
- in the large-scale block, slop is 1.6

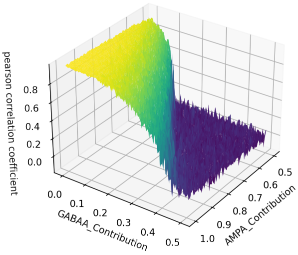


Figure: size=2k

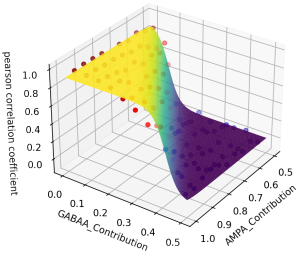


Figure: size=100m

Criticality in large-scale network

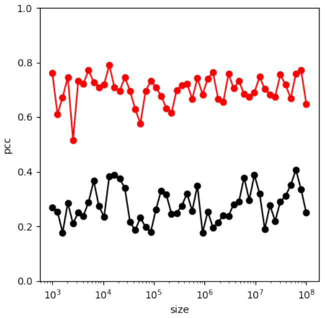


Figure: pcc with respect to different sizes

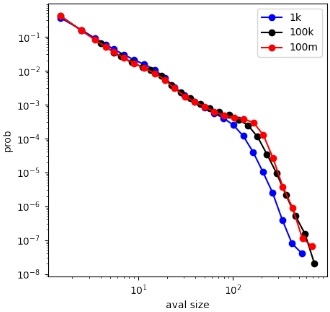


Figure: avalanches distribution

Criticality in large-scale network

Some simulation case from a fixed parameter of critical space.

- The avalanche size increases with the size of the simulation.

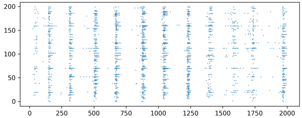


Figure: size=10k

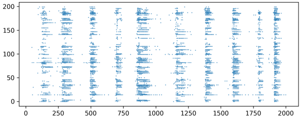
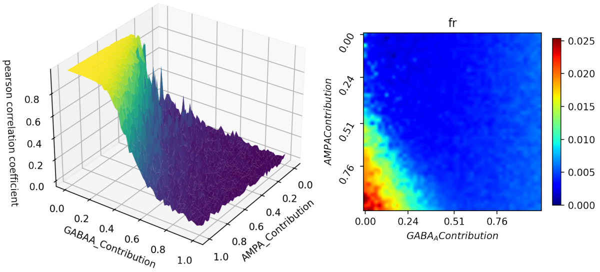


Figure: size=100m

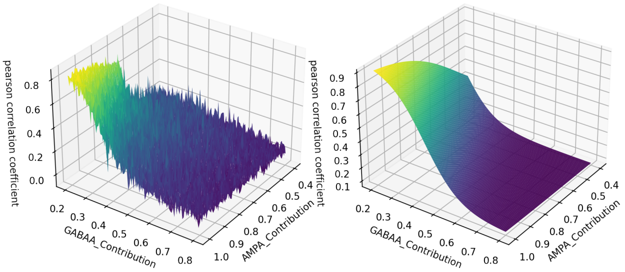
Dependence on connection density

grid search on network of 2000 neurons with 300 in-connections. x and y range is $[0, 1]$, each 50 points.



Dependence on connection density

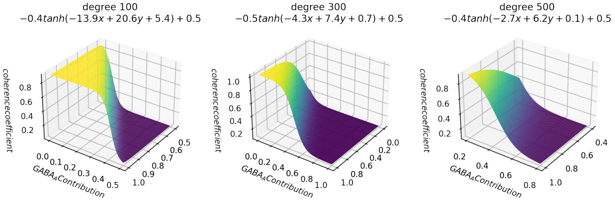
grid search on network of 2000 neurons with 500 in-connections.



As we can see, the plane is much more smooth than its in degree=100 and the one-dimensional linear submanifold disappears.

Dependence on connection density

If all case fit to function $s \cdot tanh(ax + by + c) + t$. we can find that its slope is smaller with the degree increasing.



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Summary

- The first main message of your talk in one or two lines.
- The second main message of your talk in one or two lines.
- Perhaps a third message, but not more than that.

- Outlook
  - Something you haven't solved.
  - Something else you haven't solved.

For Further Reading I

A. Author.
Handbook of Everything.
Some Press, 1990.

S. Someone.
On this and that.
Journal of This and That, 2(1):50–100, 2000.