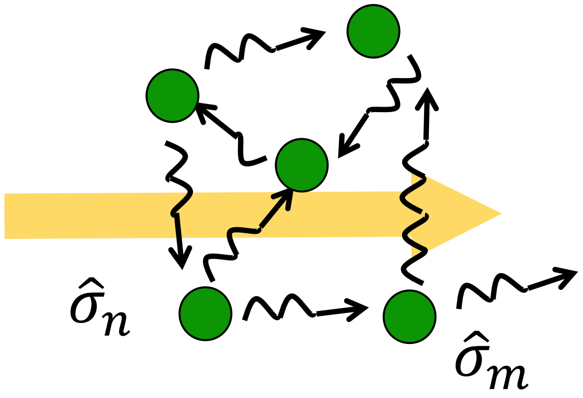
Collective atom-photon interaction: an introduction



Atom-Photon Interactions course, May 2022

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מכון ויצמן למדע
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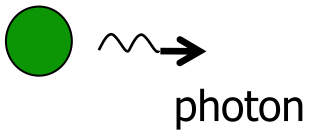
Single excited atom

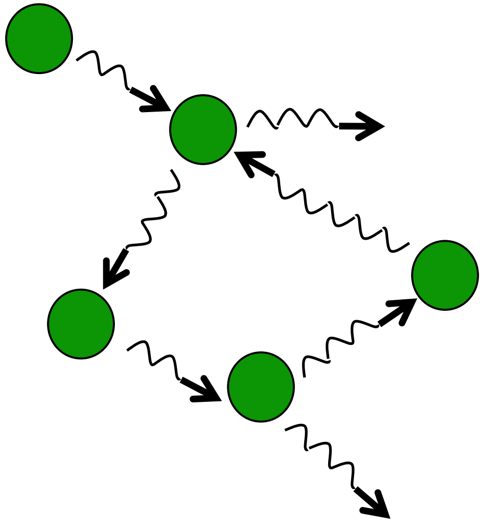


How do atoms radiate **together**?

Collectively!

(= multiple scattering)

(= dipole-dipole interactions [induced])

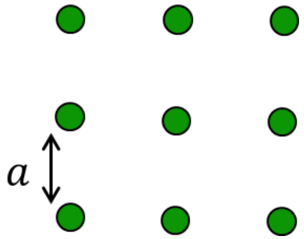


Collective light-matter interactions

- relevant:

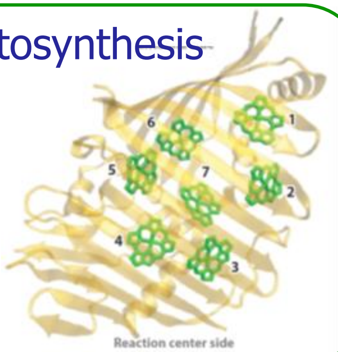
many systems
& applications

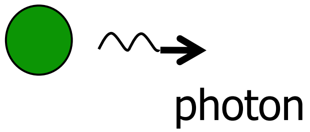
ultra-cold atoms



photonics

photosynthesis



Single excited atom 

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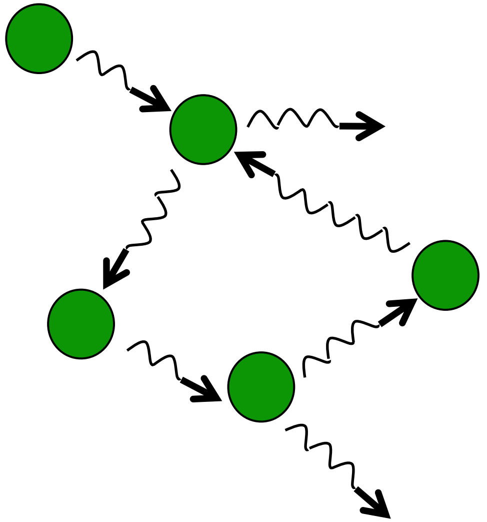


Collective light-matter interactions

- **relevant**

- <u>in general:</u> **unsolved** Q many-body problem, non-equilibrium, nonlinear

"superradiance"

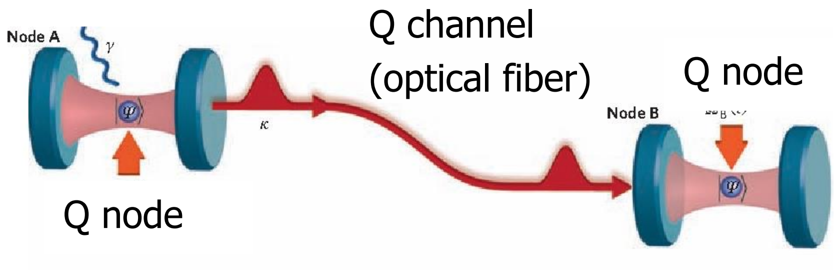
Q science & tech with atoms & photons

need: **Strong atom-photon interaction**

"strong" = 100% of photon power
(all quanta) absorbed/scattered by atoms

- Q memories, Q networks

Q state transfer to/via photons



Kimble, "The Quantum Internet" (2008)

- Q gates / entanglement

btwn atoms (assisted by light)

btwn photons (assisted by atoms)

(Weizmann, Dayan group)

- Q many-body Correlated states of atoms/photons

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need: **Strong atom-photon interaction**

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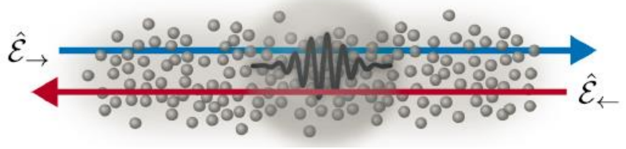
Typical platforms: bulk /macroscopic objects

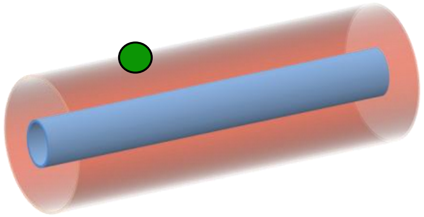
(1) **Large** atomic ensemble (~10^6)



credit: Murray & Pohl (2017)

WIS: Davidson, Firstenberg groups

(2) **Solid** dielectric: Cavity / waveguide



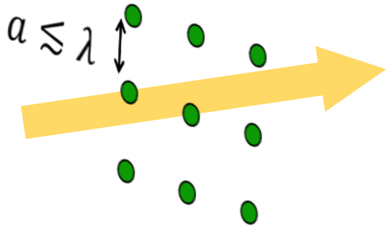
WIS: Dayan group

Alternatively:

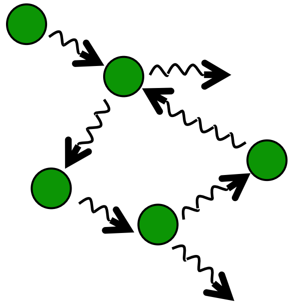
~ **dozens** of atoms in array

→ Strong atom-photon coupling

Collectivity + order



Collective atom-photon interaction



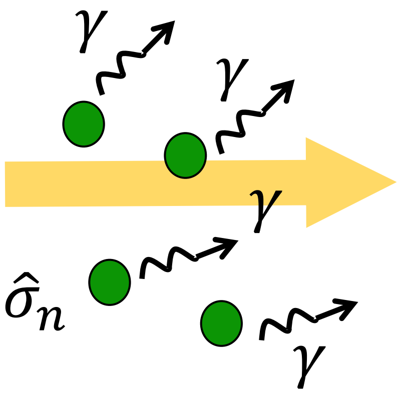
(a) General framework

(b) Example 1: **Atomic arrays** – collectivity + spatial order

(c) Example 2: **Dicke superradiance** – collectivity + nonlinearity

How do atoms radiate together?

dilute ensemble



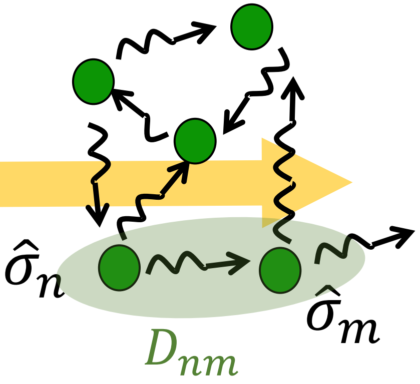
Individual-atom response:

atoms behave as independent emitters

→ emission to random directions

$\hat{\sigma}_n = |g\rangle_n\langle e|$

"dense" ensemble

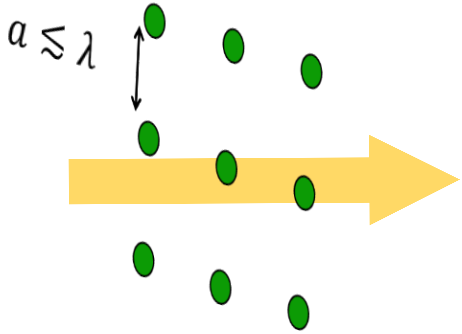


Collective response

(= multiple scattering = dipole-dipole)

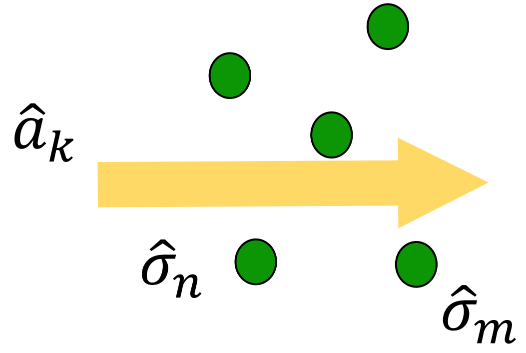
$D_{nm} =$ dipole-dipole kernel (photon Green's function)

Ordered array



Collective (can become strong!) + spatial order = ?

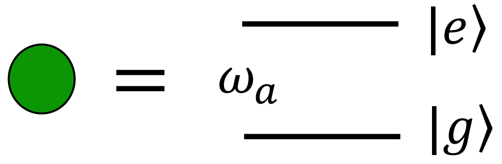
Model: atoms + photon modes



$$H = H_A + H_P + H_{AP}$$

"atoms": two-level emitters $n = 1, \ldots, N$ at positions $\mathbf{r}_n$

$$H_A = \hbar\omega_a \sum_{n=1}^{N} \hat{\sigma}_n^\dagger \hat{\sigma}_n \qquad \hat{\sigma}_n = |g\rangle_n\langle e|$$



"photons" = continuum of photon modes $\hat{a}_k$ (modes defined by geometry) $\{k\}$ = mode index

$$H_P = \sum_k \hbar\omega_k \hat{a}_k^\dagger \hat{a}_k$$

"atom-photon": dipole interaction (+rotating wave approx.)

mode profile

$$H_{AP} = -\sum_n \hat{E}(\mathbf{r}_n)\hat{\sigma}_n^\dagger d + h.c. \qquad \hat{E}(\mathbf{r}_n) = -i\sum_k \sqrt{\frac{\hbar\omega_k}{2\varepsilon_0}}\, u_k(\mathbf{r}_n)\hat{a}_k$$

Heisenberg-Langevin approach

[see Lehmberg PRA 2, 883 (1970) for a full derivation]

$$H = \hbar\omega_a \sum_{n=1}^{N} \hat{\sigma}_n^\dagger \hat{\sigma}_n + \sum_k \hbar\omega_k \hat{a}_k^\dagger \hat{a}_k - \sum_n [\hat{E}(\mathbf{r}_n)\hat{\sigma}_n^\dagger d + h.c.]$$

$$\hat{E}(\mathbf{r}_n) = -i\sum_k \sqrt{\frac{\hbar\omega_k}{2\varepsilon_0}} u_k(\mathbf{r}_n)\hat{a}_k$$

$$\hat{\sigma}_n = |g\rangle_n\langle e|$$

Heisenberg Eq. for atom n:

$$\partial_t \hat{\sigma}_n = \frac{i}{\hbar}[H, \hat{\sigma}_n]$$

Use: $[\hat{\sigma}_n^\dagger, \hat{\sigma}_n] = \hat{\sigma}_n^z$ $\quad \hat{\sigma}_n^z = |e\rangle_n\langle e| - |g\rangle_n\langle g|$

$$\partial_t \hat{\sigma}_n = -i\omega_a \hat{\sigma}_n - i\frac{d}{\hbar}\hat{\sigma}_n^z \hat{E}(\mathbf{r}_n)$$

Now wish to find Eq. for field $\hat{E}(\mathbf{r}_n)$ and insert it into the above

Heisenberg Eq. for field = Maxwell's Eqs. (=Hamilton's Eq. for field…)

$$\nabla \times \hat{E}(r,t) = -\partial_t \hat{B}(r,t) \qquad \nabla \times \hat{B}(r,t) = \frac{1}{c^2}\partial_t\left[\hat{E}(r,t) + \frac{1}{\varepsilon_0}\hat{P}(r,t)\right]$$

Polarization density

= dipoles (of atoms) per volume

$$\hat{P}(\mathbf{r},t) = \sum_n \hat{d}_n(t)\delta(\mathbf{r}-\mathbf{r}_n)$$

Freq. domain $\partial_t \to -i\,\omega$

$$\hat{d}_n = d\hat{\sigma}_n + \mathrm{h.c.}$$

$$\nabla \times \hat{E}(r,\omega) = i\,\omega \hat{B}(r,\omega) \qquad \nabla \times \hat{B}(r,\omega) = -i\frac{\omega}{c^2}\left[\hat{E}(r,\omega) + \frac{1}{\varepsilon_0}\hat{P}(r,\omega)\right]$$

Derive wave equation:

$$\nabla \times \nabla \times \hat{E} = i\,\omega \nabla \times \hat{B} = \frac{\omega^2}{c^2}\left[\hat{E} + \frac{1}{\varepsilon_0}\hat{P}\right]$$

$$\nabla \times \nabla \times \hat{E}(\mathbf{r},\omega) - \frac{\omega^2}{c^2}\hat{E}(\mathbf{r},\omega) = \frac{\omega^2}{c^2}\frac{1}{\varepsilon_0}\hat{P}(\mathbf{r},\omega) = \frac{\omega^2}{c^2}\frac{1}{\varepsilon_0}\sum_n \delta(\mathbf{r}-\mathbf{r}_n)d\hat{\sigma}_n(\omega)$$

Heisenberg Eq. for field: formal solution (1)

$$\underbrace{\nabla \times \nabla \times \hat{E}(\mathbf{r},\omega) - \frac{\omega^2}{c^2}\hat{E}(\mathbf{r},\omega)}_{\text{Linear operator (~ "Helmholtz" Eq. for the field)}} = \underbrace{\frac{\omega^2}{c^2}\frac{1}{\varepsilon_0}\sum_n \delta(\mathbf{r}-\mathbf{r}_n) d\hat{\sigma}_n(\omega)}_{\text{point sources ("dipoles") @ atom positions}}$$

→ Solution to point source (=dipole) @ $\mathbf{r}_n$ – Green's function: $G(\mathbf{r}-\mathbf{r}_n)$

$$\hat{E}(\mathbf{r},\omega) = \hat{E}_0(\mathbf{r},\omega) + \underbrace{\frac{\omega^2}{c^2}\frac{d}{\varepsilon_0}\sum_n G(\mathbf{r}-\mathbf{r}_n,\omega)\hat{\sigma}_n(\omega)}_{\text{Field from all "dipole" sources (=radiating atoms)}}$$

Total field

"Source free" solution
= "free" field (in the absence of atoms)
= vacuum fluctuations + incident laser

field propagator = Green's function $G(\mathbf{r}_1,\mathbf{r}_2) = \mathbf{e}_d{}^\dagger \cdot \overline{\overline{\boldsymbol{G}}}(\mathbf{r}_1,\mathbf{r}_2)\cdot \mathbf{e}_d$

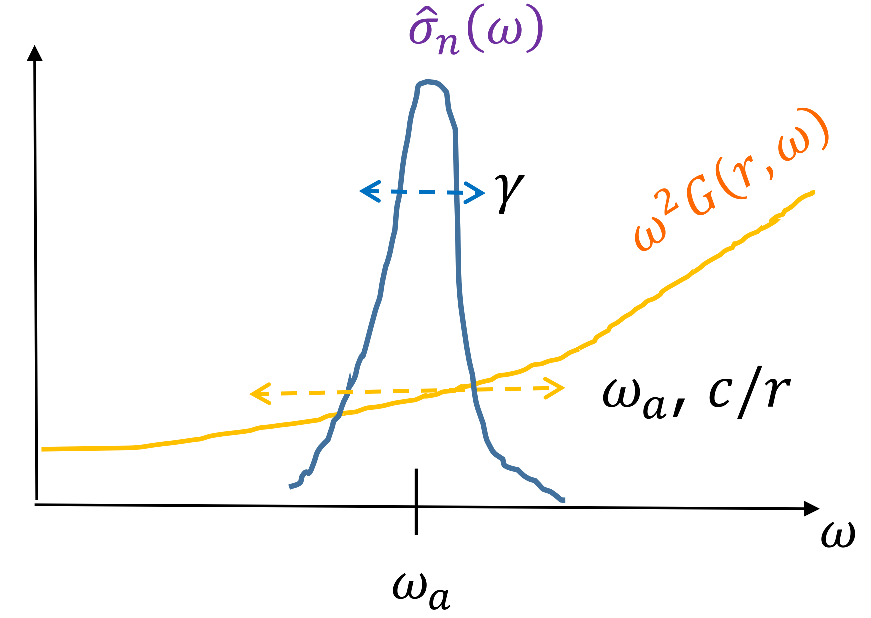
$$\mathbf{e}_i{}^\dagger \cdot \overline{\overline{\boldsymbol{G}}}(\mathbf{r}_1,\mathbf{r}_2)\cdot \mathbf{e}_j = G_{ij}(k,\mathbf{r}_1,\mathbf{r}_2) = \frac{e^{ikr}}{4\pi r}\left[\left(1+\frac{ikr-1}{k^2r^2}\right)\delta_{ij} + \left(-1+\frac{3-3ikr}{k^2r^2}\right)\frac{r^ir^j}{r^2}\right]$$

with $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$,
, $r = |\mathbf{r}|$ and $r^i = \mathbf{e}_i \cdot \mathbf{r}$.

$k = \omega/c$

Heisenberg Eq. for field: formal solution (2)

$$\hat{E}(\mathbf{r},\omega) = \hat{E}_0(\mathbf{r},\omega) + \frac{\omega^2}{c^2}\frac{d}{\varepsilon_0}\sum_n \hat{\sigma}_n(\omega) G(\mathbf{r}-\mathbf{r}_n,\omega)$$



Back to time-domain – perform IFT:

$$\hat{E}(\mathbf{r},t) = \hat{E}_0(\mathbf{r},t) + \frac{1}{c^2}\frac{d}{\varepsilon_0}\sum_n \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t}\hat{\sigma}_n(\omega)\ \underbrace{\omega^2 G(\mathbf{r}-\mathbf{r}_n,\omega)}$$

- Dominant "free" dynamics: oscillations at ω_a
- Slow dynamics due to radiation ~ decay/shift γ

Wide function around ω_a
Width ~ $\omega_a,\ c/r$

Markov approximation: $\omega_a,\ c/r \gg \gamma$

$$\hat{E}(\mathbf{r},t) \approx \hat{E}_0(\mathbf{r},t) + \frac{1}{c^2}\frac{d}{\varepsilon_0}\sum_n {\omega_a}^2 G(\mathbf{r}-\mathbf{r}_n,\omega_a) \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} e^{-i\omega t}\hat{\sigma}_n(\omega)$$

$$= \hat{E}_0(\mathbf{r},t) + \frac{1}{c^2}\frac{d}{\varepsilon_0}\sum_n {\omega_a}^2 G(\mathbf{r}-\mathbf{r}_n,\omega_a)\hat{\sigma}_n(t)$$

Heisenberg-Langevin Eq. for atoms (1)

$$\partial_t \hat{\sigma}_n = -i\omega_a \hat{\sigma}_n - i\frac{d}{\hbar}\hat{\sigma}_n^z \hat{E}(\mathbf{r}_n)$$

We got for field: $$\hat{E}(\mathbf{r}) = \hat{E}_0(\mathbf{r}) + \frac{1}{c^2}\frac{d}{\varepsilon_0}\sum_n {\omega_a}^2 G(\mathbf{r}-\mathbf{r}_n,\omega_a)\hat{\sigma}_n$$

→ Field "felt" by **atom** n, $\mathbf{r} = \mathbf{r}_n$:

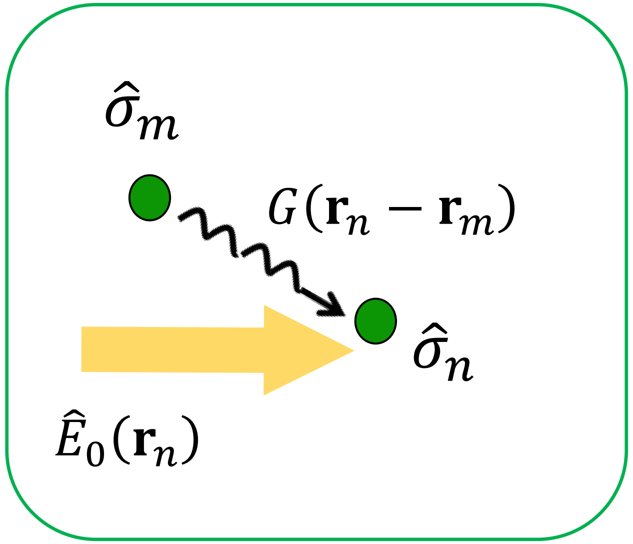
$$\hat{E}(\mathbf{r}_n) = \hat{E}_0(\mathbf{r}_n) + \frac{1}{c^2}\frac{d}{\varepsilon_0}\sum_m {\omega_a}^2 G(\mathbf{r}_n-\mathbf{r}_m)\hat{\sigma}_m$$

= "free" field (in the absence of atoms)
= vacuum fluctuations + incident laser

$$\hat{E}_0(\mathbf{r}_n) = -i\sum_k \sqrt{\frac{\hbar\omega_k}{2\varepsilon_0}}\, u_k(\mathbf{r}_n)\hat{a}_k(0)e^{-i\omega_k t}$$

Field from all "dipole" sources
(=radiating atoms)

field propagator = $G(\mathbf{r}_n - \mathbf{r}_m)$



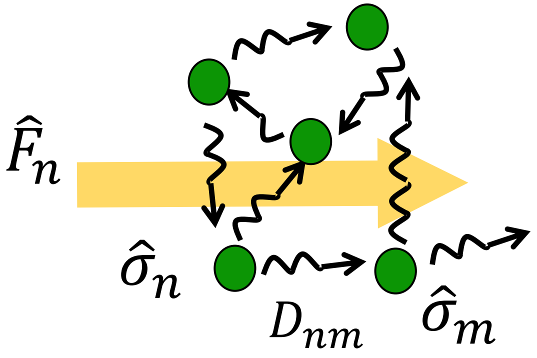
Heisenberg-Langevin Eq. for atoms (2)

$$\partial_t \hat{\sigma}_n = -i\omega_a \hat{\sigma}_n - i\hat{\sigma}_n^z \frac{d}{\hbar}\left[\hat{E}_0(\mathbf{r}_n) + \frac{{\omega_a}^2}{c^2}\frac{d}{\varepsilon_0}\sum_m G(\mathbf{r}_n - \mathbf{r}_m)\hat{\sigma}_m\right]$$

$$D_{nm} = -i\frac{3}{2}\gamma\lambda G(\mathbf{r}_n - \mathbf{r}_m) \qquad \gamma = \frac{d^2 {\omega_a}^3}{3\pi\varepsilon_0 \hbar c^3}$$

$$\lambda = 2\pi c/\omega_a$$

$$\boxed{\partial_t \hat{\sigma}_n = -i\omega_a \hat{\sigma}_n + \hat{\sigma}_n^z\left[\hat{F}_n + \sum_m D_{nm}\hat{\sigma}_m\right]}$$



→ Collective response

(= multiple scattering
= dipole-dipole)

quantum noise (vacuum) + incident laser

$$\hat{F}_n = -i\frac{d}{\hbar}\hat{E}_0(\mathbf{r}_n) = -d\sum_k \sqrt{\frac{\omega_k}{2\varepsilon_0\hbar}}\, u_k(\mathbf{r}_n)\hat{a}_k(0)e^{-i\omega_k t}$$

Dipole-dipole coupling (with all atomic dipoles)

$$D_{nm} = \gamma_{nm}/2 + i\Delta_{nm}$$

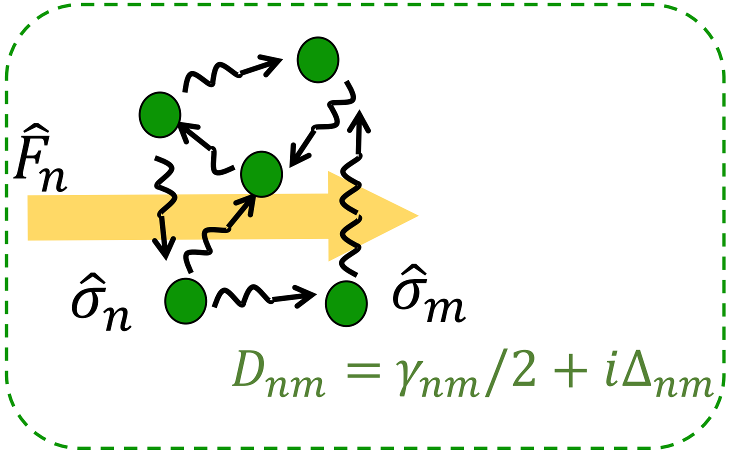
dipole-dipole kernel
(photon Green's function)

$$\partial_t \hat{\sigma}_n = -\left(i\omega_a + \frac{\gamma}{2}\right)\hat{\sigma}_n + \hat{\sigma}_n^z\left[\hat{F}_n + \sum_{m\neq n} D_{nm}\hat{\sigma}_m\right]$$

$$\mathrm{Re}\, D_{nn} = \gamma/2$$

$$\mathrm{Im}\, D_{nn} + \omega_a \to \omega_a$$

Many-body physics of quantum emitters (atoms)



Heisenberg-Langevin Eq. for atoms:

$$\partial_t \hat{\sigma}_n = \frac{i}{\hbar}\left[H_{\text{eff}}, \hat{\sigma}_n\right] + \hat{\sigma}_n^z \hat{F}_n$$

Effective Hamiltonian (non-Hermitian): **collectivity**

$$H_{\text{eff}} = \hbar\left(\omega_a - i\frac{\gamma}{2}\right)\sum_{n=1}^{N}\hat{\sigma}_n^\dagger\hat{\sigma}_n + \hbar\sum_{n=1}^{N}\sum_{m\neq n}\left(\Delta_{nm} - i\frac{\gamma_{nm}}{2}\right)\hat{\sigma}_n^\dagger\hat{\sigma}_m$$

Quantum noise
(has to exist due to dissipation)

$$\hat{F}_n = -d\sum_k\sqrt{\frac{\omega_k}{2\varepsilon_0\hbar}}u_k(\mathbf{r}_n)\hat{a}_k(0)e^{-i\omega_k t}$$

Dipole-dipole interaction
(reversible excitation exchange)

Collective radiation
(dissipation)

Equivalent description: quantum master equation

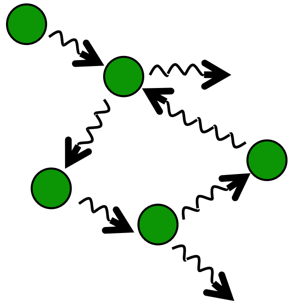
$$\partial_t\hat{\rho} = -\frac{i}{\hbar}\left(H_{\text{eff}}\hat{\rho} - \hat{\rho}H_{\text{eff}}^\dagger\right) + \sum_{n,m}\gamma_{nm}\hat{\sigma}_n\hat{\rho}\hat{\sigma}_m^\dagger$$

Quantum noise: "jump term"

Output light

$$\hat{E}(\mathbf{r}) = \hat{E}_0(\mathbf{r}) + \frac{1}{c^2}\frac{d}{\varepsilon_0}\sum_n \omega_a{}^2 G(\mathbf{r} - \mathbf{r}_n, \omega_a)\hat{\sigma}_n$$

<u>Collective</u> atom-photon interaction



(a) General framework

(b) Example 1: **Atomic arrays** – collectivity + spatial order

(c) Example 2: **Dicke superradiance** – collectivity + nonlinearity

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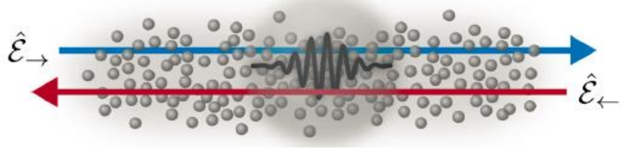

credit: Murray & Pohl (2017)

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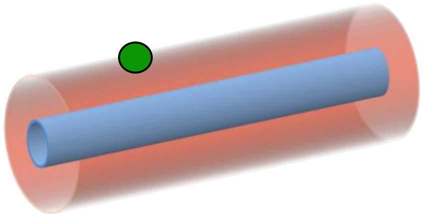


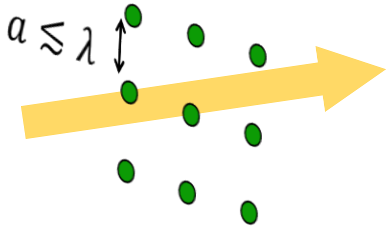
WIS: Dayan group

Alternatively:

~ **dozens** of atoms in array

→ Strong atom-photon coupling

Collectivity + order



Collective response of ordered atom array (linear regime)

$$\partial_t \hat{\sigma}_n = -i\omega_a \hat{\sigma}_n + \hat{\sigma}_n^z\left[\hat{F}_n + \sum_m D_{nm}\,\hat{\sigma}_m\right]$$

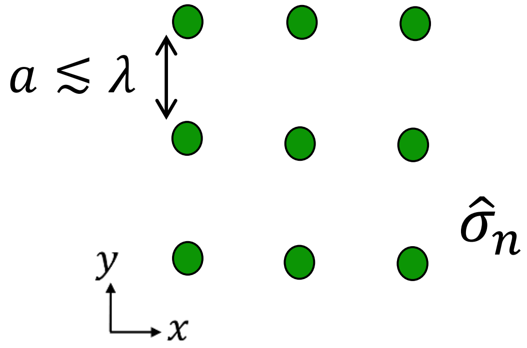
$$\hat{F}_n = -\,i\,\frac{d}{\hbar}\hat{E}_0(\mathbf{r}_n)$$

Linear response: # excited atoms << # of atoms $\quad \hat{\sigma}_n^z = |e\rangle_n\langle e| - |g\rangle_n\langle g| \approx -1$

$$\partial_t \hat{\sigma}_n \approx -i\omega_a \hat{\sigma}_n - \sum_m D_{nm}\hat{\sigma}_m - \hat{F}_n$$

$$D_{nm} = -i\frac{3}{2}\gamma\lambda G(\mathbf{r}_n - \mathbf{r}_m)$$

Spatial order (array): translational invariance (discrete) – move to Fourier (discrete):



$$\hat{\sigma}_{\mathbf{k}_\perp} = \sum_n e^{-i\mathbf{k}_\perp\cdot\mathbf{r}_n}\hat{\sigma}_n \qquad \partial_t \hat{\sigma}_{\mathbf{k}_\perp} = -\left[i\left(\omega_a + \Delta_{\mathbf{k}_\perp}\right) + \frac{\Gamma_{\mathbf{k}_\perp}}{2}\right]\hat{\sigma}_{\mathbf{k}_\perp} - \hat{F}_{\mathbf{k}_\perp}$$

→ Collective dipole modes [normal modes in lattice, e.g. 2D array $\mathbf{k}=(k_x,k_y)$]

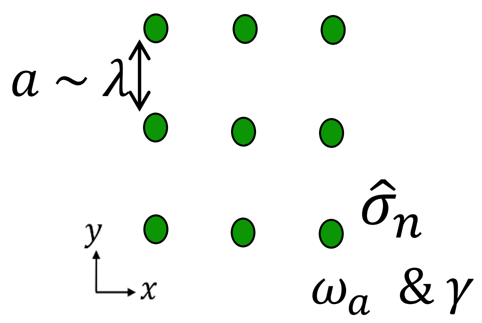
collective resonance & width $\quad \omega_a + \Delta_{\mathbf{k}_\perp}$ & $\Gamma_{\mathbf{k}_\perp}$

$$\frac{\Gamma_{\mathbf{k}_\perp}}{2} - i\Delta_{\mathbf{k}_\perp} = \sum_n e^{-i\mathbf{k}_\perp\cdot\mathbf{r}_n} D_{nm}$$

ES, Wild, Lukin, Yelin, PRL (2017)

ES, Lukin, Yelin, AAMO (2019)

Collective response of ordered atom array (linear regime)

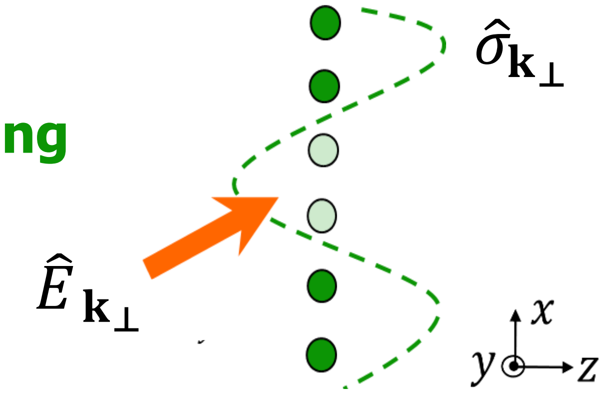


Collective dipole modes:

$$\hat{\sigma}_{\mathbf{k}_\perp} = \sum_n e^{-i\mathbf{k}_\perp \cdot \mathbf{r}_n} \hat{\sigma}_n$$

collective resonance & width $\quad \omega_a + \Delta_{\mathbf{k}_\perp}$ & $\Gamma_{\mathbf{k}_\perp}$

Coupling to light:



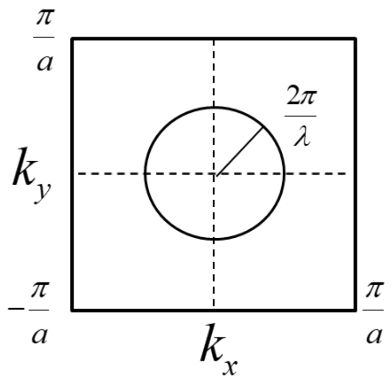
For subwavelength array: $a \lesssim \lambda$ $\quad \mathbf{k}_\perp = (k_x, k_y)$ conserved → **Directional coupling**

Out-coupling of light, determined by: $k_z = \sqrt{(2\pi/\lambda)^2 - \mathrm{k}_\perp{}^2}$

→ Two types of "surface dipoles":

$k_\perp < (2\pi/\lambda)$ "scattering" modes $k_z \in \mathrm{Re}$ (propagating to free-space)

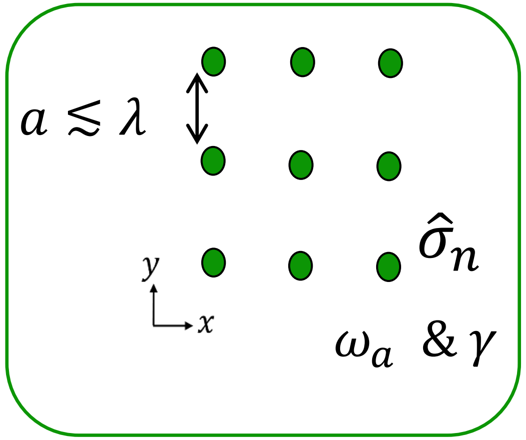
$k_\perp > (2\pi/\lambda)$ "confined" modes $k_z \in \mathrm{Im}$ (guided; propagating on surface)
"Sub-radiant" modes (no decay) $\Gamma_{\mathbf{k}_\perp} = 0$ Exist for: $a < \lambda/2$ (1D array)
$a < \lambda/\sqrt{2}$ (2D array)



ES, Wild, Lukin, Yelin, PRL (2017)

Light scattering (linear regime)

$\mathbf{k}_\perp=(k_x,k_y)$ conserved



Solve for atoms (take as classical dipoles, no "hats" – same as quantum in linear regime):

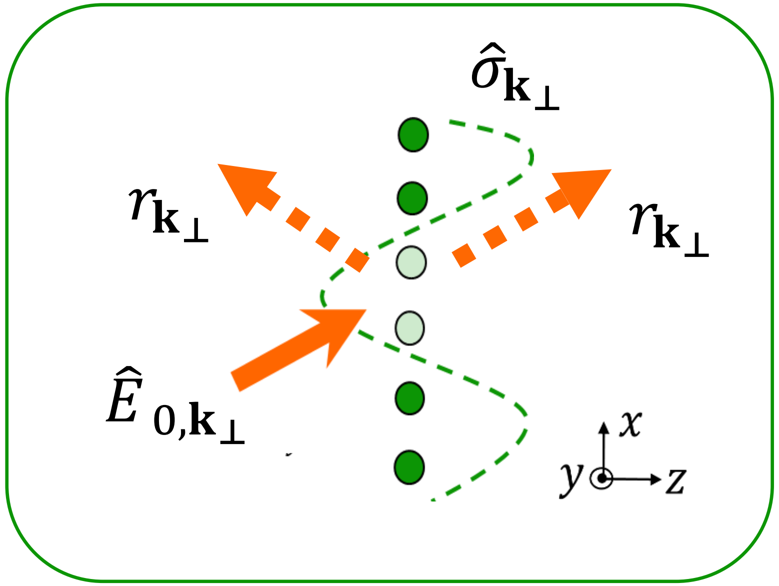
$$\partial_t \sigma_{\mathbf{k}_\perp} = -\left[i\left(\omega_a + \Delta_{\mathbf{k}_\perp}\right) + \frac{\Gamma_{\mathbf{k}_\perp}}{2}\right]\hat{\sigma}_{\mathbf{k}_\perp} + E_{0,\mathbf{k}_\perp}\frac{id}{\hbar}$$

$$\hat{\sigma}_{\mathbf{k}_\perp} = \sum_n e^{-i\mathbf{k}_\perp\cdot\mathbf{r}_n}\hat{\sigma}_n$$

$$\frac{\Gamma_{\mathbf{k}_\perp}}{2} - i\Delta_{\mathbf{k}_\perp} = \sum_n e^{-i\mathbf{k}_\perp\cdot\mathbf{r}_n} D_{nm}$$

→ Steady state:

$$\sigma_{\mathbf{k}_\perp} = \frac{E_{0,\mathbf{k}_\perp}}{\left[i\left(\omega_a + \Delta_{\mathbf{k}_\perp}\right) + \frac{\Gamma_{\mathbf{k}_\perp}}{2}\right]}\frac{id}{\hbar} \quad \rightarrow \hat{\sigma}_n = \sum_{\mathbf{k}_\perp} e^{i\mathbf{k}_\perp\cdot\mathbf{r}_n}\hat{\sigma}_{\mathbf{k}_\perp}/N$$



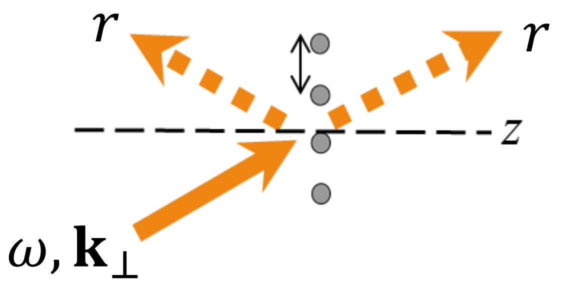
Plug into Eq. for the field: $\hat{E}(\mathbf{r}) = \hat{E}_0(\mathbf{r}) + \frac{1}{c^2}\frac{d}{\varepsilon_0}\sum_n \omega_a{}^2 G(\mathbf{r}-\mathbf{r}_n,\omega_a)\hat{\sigma}_n$

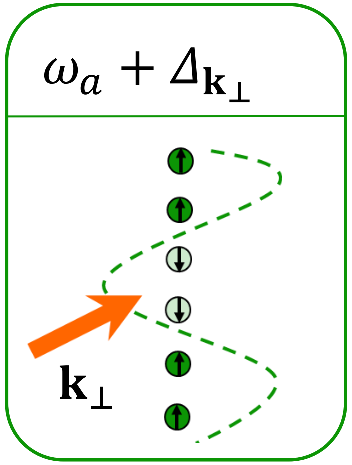
→ Perform FT to $E_{\mathbf{k}_\perp}\dots$ →

$$\boxed{E_{\mathbf{k}_\perp} = E_{0,\mathbf{k}_\perp}\left(e^{ik_z z} + r_{\mathbf{k}_\perp} e^{ik_z|z|}\right)}$$

ES, Wild, Lukin, Yelin, PRL (2017)

Atom array → platform for Q optics



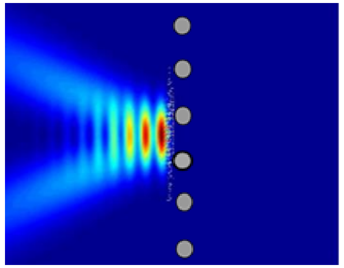


Light scattering: directional, reflectivity r

$$r = -\frac{i\Gamma/2}{(\omega - \omega_a - \Delta) + i\Gamma/2}$$

For light @ **collective resonance** $\omega = \omega_a + \Delta_{\mathbf{k}}$: $r = -1$

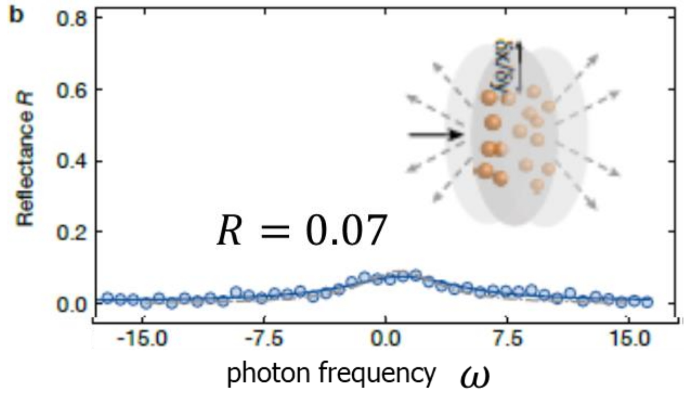
"Perfect" mirror (tunable, robust)



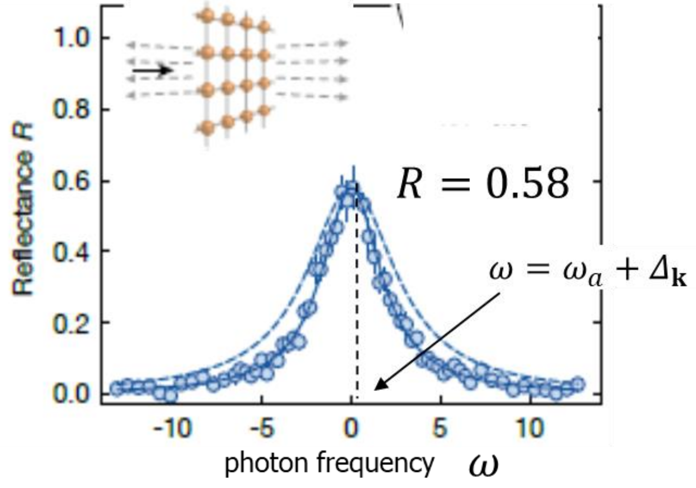
ES, Wild, Lukin, Yelin, PRL (2017)
Bettles, Gardiner, Adams, PRL (2016)

Confirmation of theory: 2D optical lattice [Bloch group, MPQ] Rui,…,Gross, Bloch, Nature 2020

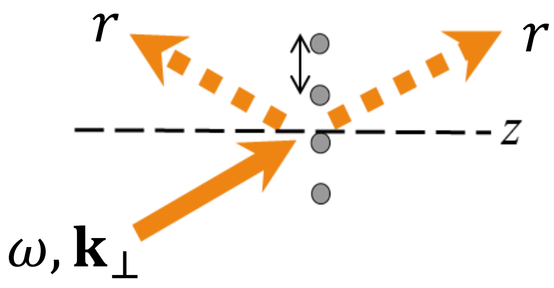
disordered 2D atom cloud

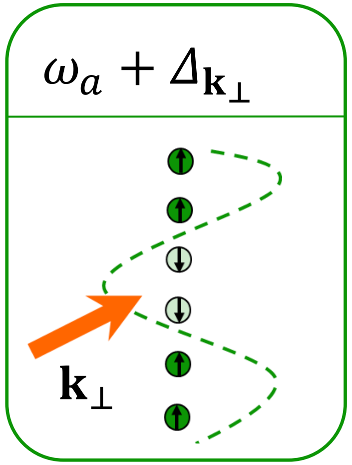


ordered 2D atom array
(optical lattice)



Atom array → platform for Q optics





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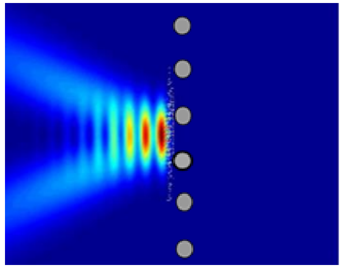


ES, Wild, Lukin, Yelin, PRL (2017)
Bettles, Gardiner, Adams, PRL (2016)
Rui,…,Gross, Bloch, Nature (2020)

Key point: **collectivity + order → strong & directional light-matter int.** **← reflectivity**

More applications: → new platform for Q science & tech.

- Quantum network
- Quantum memory
- Topological photonics
- 1D Subradiant modes
- Q nonlinear optics

- atom-photon coupler (ES, Wild et al. 2017)
- optomechanics (ES et al., several works),
- Cavity QED (ES, Wild et al. 2020)
- Q info, metasurface (Bekenstein,…ES,… et al.)
- 2D waveguide QED (Patti,…ES,… et al.)

See works by many groups (2017-2021):
Yelin+Lukin, Chang, Asenjo, Zoller, Malz/Cirac, Ruostekoski, Ritsch, Pohl, Sheremet,…

Early works: Ritsch & Zoubi (1D subradiance) Ruostekoski (numerical evidence)

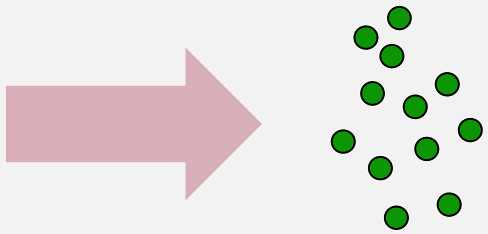
Atom array → platform for Q optics

Key point: **collectivity + order → strong & directional light-matter int.** **← reflectivity**

More applications:

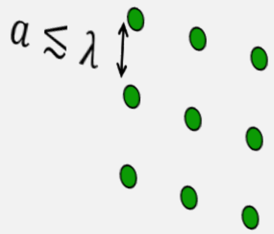
Q memory: state of light mapped to atoms

Manzoni,…,Chang, NJP (2018)
Asenjo, Moreno…,Chang, PRX (2017)



Typically (atomic ensemble):

10^6



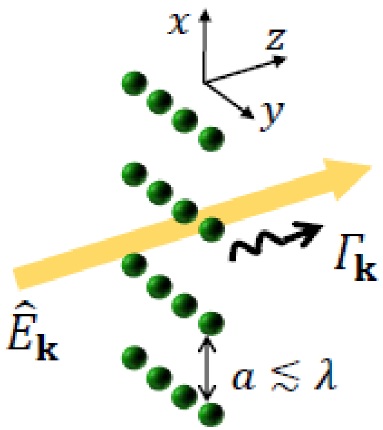
Collectivity + order

16 atoms

memory fidelity ~ reflectivity

(instead of Optical Depth)

Example 1: atom array



~ dozens atoms, single layer

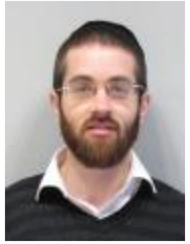
novel platform for Q info/technologies

Current related projects:

1. Q memory for photons

2. Q gates for photons

Yakov Solomons
(PhD student)

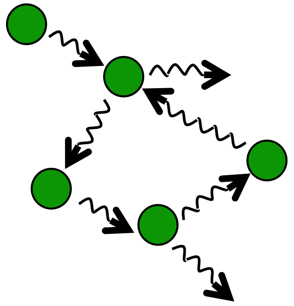


3. Q entanglement from light to atoms

Roni Ben Maimon
(PhD student)

Experiment: collaboration with Davidson & Firstenberg groups

Collective atom-photon interaction



(a) General framework

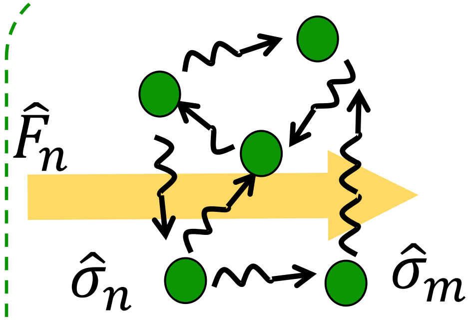
(b) Example 1: **Atomic arrays** – collectivity + spatial order

(c) Example 2: **Dicke superradiance** – collectivity + nonlinearity

Dicke superradiance (1)

Recall: general picture of collective atom-photon

$$H_{\text{eff}} = \hbar \sum_{n=1}^{N} \sum_{m=1}^{N} \left(\cancel{\Delta_{nm}} - i\frac{\gamma_{nm}}{2} \right) \hat{\sigma}_n^\dagger \hat{\sigma}_m \quad \longrightarrow \quad -i\hbar\frac{\gamma}{2} \sum_{n=1}^{N} \sum_{m=1}^{N} \hat{\sigma}_n^\dagger \hat{\sigma}_m$$



$D_{nm} = \gamma_{nm}/2 + i\Delta_{nm}$

$$\partial_t \hat{\rho} = -\frac{i}{\hbar}\left(H_{\text{eff}}\hat{\rho} - \hat{\rho}H_{\text{eff}}^\dagger\right) + \sum_{n,m} \gamma_{nm} \hat{\sigma}_n \hat{\rho} \hat{\sigma}_m^\dagger$$

1. **"dense" atomic ensemble**:

inter-atomic distance << λ: $\gamma_{nm} \approx \gamma_{nn} = \gamma$

$[\, \boldsymbol{\gamma_{nm}} = 3\gamma\lambda \mathrm{Im}[\mathrm{G}(\mathbf{r}_n - \mathbf{r}_m)] \approx 3\gamma\lambda \mathrm{Im}[\mathrm{G}(0)] = \boldsymbol{\gamma} \,]$

2. ignore dipole-dipole (for now)

$\Delta_{nm} \rightarrow 0$

→ Permutation symmetry (field does not distinguish between different atoms)

→ Collective operator $D^- = \sum_n \hat{\sigma}_n \quad \longrightarrow \quad H_{\text{eff}} = -i\hbar\frac{\gamma}{2} D^+ D^-$

→ All atoms = one "big" dipole/spin

$$\left(\frac{\mathrm{d}\rho}{\mathrm{d}t}\right)_{\text{real}} = -\frac{\Gamma}{2}[D^+ D^-, \rho]_+ + \Gamma D^- \rho D^+$$

Dicke superradiance (2)

→ Permutation symmetry → Collective operator $D^- = \sum_n \hat{\sigma}_n$

$$\left(\frac{d\rho}{dt}\right)_{\text{real}} = -\frac{\Gamma}{2}[D^+D^-, \rho]_+ + \Gamma D^- \rho D^+$$

addition of N spin-1/2:

$|ee\ldots e\rangle = |J = \frac{N}{2}, M = \frac{N}{2}\rangle$

$|J, M\rangle \propto (D^-)^{J-M} |J, J\rangle$

J=N/2, M=-J,…,J

→ *2J+1=N+1* states

instead of 2^N…

Project to *J,M* basis $\rho_M = \langle J, M|\rho|J, M\rangle$

$$d\rho_M/dt = -\Gamma(J+M)(J-M+1)\rho_M + \Gamma(J+M+1)(J-M)\rho_{M+1}$$

Dicke superradiance (3)

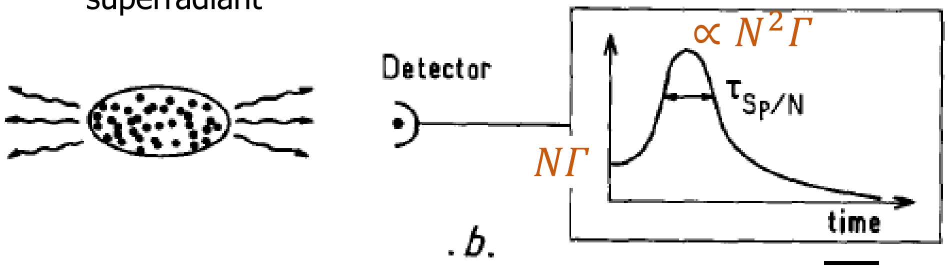
Permutation symmetry: Project to *J=N/2, M* basis $\rho_M = \langle J, M | \rho | J, M \rangle$

$$d\rho_M/dt = -\Gamma(J+M)(J-M+1)\rho_M + \Gamma(J+M+1)(J-M)\rho_{M+1}$$

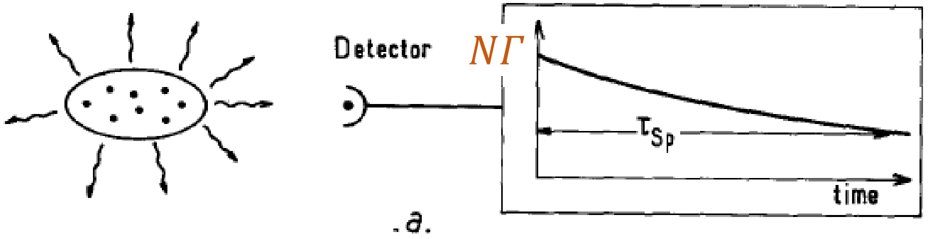
Maximal decay rate for M=0:

$$\Gamma J(J+1) \propto N^2 \Gamma$$

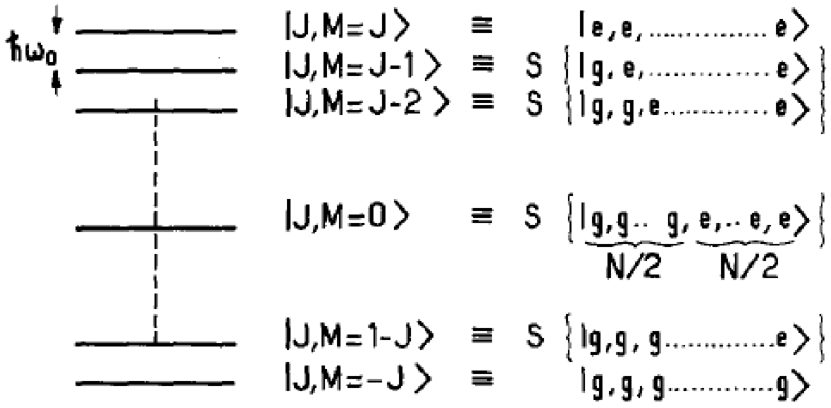
superradiant



independent atoms



Emission process:



Going down the ladder of J,M states

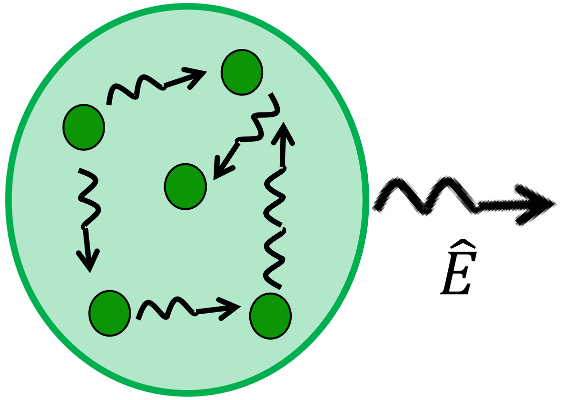
Gross and Haroche, Physics Reports 1982

Example: coherently radiating spin states (CRSS)

Radiation from dense atomic cloud

$N \gg 1$ atoms: “big atom/antenna”

$$D^- = \sum_n \hat{\sigma}_n$$



Ori Somech
(MSc student)

Question: Is there a macroscopic classical-antenna limit? **No classical limit!**

surprising result: **Classical radiation** is emitted **only if** the **atoms** are **quantum-entangled**

Theory: introduce new many-body entangled atomic state: **CRSS**

1. Eigenstate of SU(2) lowering operator (j=N/2 representation) $D^- | \alpha \rangle = \alpha | \alpha \rangle$

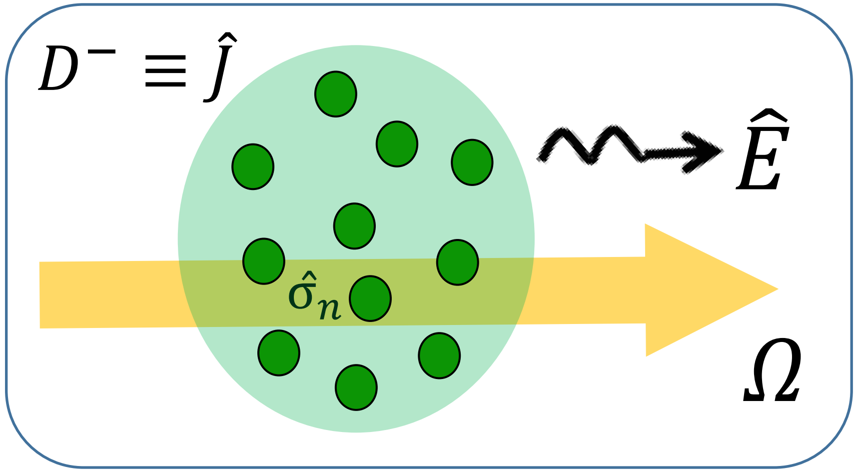
2. Exists for $N \gg 1$, exhibits spin squeezing entanglement, explains Dicke phase transition

NEW projects: opens many directions:

Q metrology, Q phase transitions and magnetism, superradiant lasers

CRSS are physical: Steady-state superradiance

Resonant laser drive Ω + collective dissipation to photon reservoir $\hat{E}$



→ Master equation for atoms/macro-spin

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}\left(\hat{H}_{\mathrm{nh}}\,\hat{\rho} - \hat{\rho}\hat{H}^{\dagger}_{\mathrm{nh}}\right) + \gamma \hat{J}\hat{\rho}\hat{J}^{\dagger},$$

$$\hat{H}_{\mathrm{nh}} = \hbar\left(\Delta - i\frac{\gamma}{2}\right)\hat{J}^{\dagger}\hat{J} - \hbar\left(\Omega\hat{J}^{\dagger} + \Omega^{*}\hat{J}\right)$$

$\gamma =$ collective decay

$\Delta =$ dipole-dipole shift

Lindblad form master eq.:

Steady state is a pure state iff it is eigenstate of $\hat{J}$ and $\hat{H}_{\mathrm{nh}}$

→ CRSS is e.s. of $\hat{J}$ $\quad \hat{J}|\alpha\rangle = \alpha|\alpha\rangle$

→ CRSS is e.s. of $\hat{H}_{\mathrm{nh}}$ for $\boxed{\alpha = \dfrac{\Omega}{\Delta - i\gamma/2}}$ →

CRSS underlies

driven-dissipative superradiance