TRƯỜNG MARIE CURIE

Bài số	Thời gian làm bài	phút	Điểm hệ số		
Học sinh			Lớp		

	HT	/1
M	TỔNG	/10

Nhận xét của thầy, cô giáo (ký tên):

Problem 1.

$$L = X^T(\hat{y} - y)$$

$$\frac{\delta L}{\delta W} = \frac{\delta L}{\delta \hat{y}} \cdot \frac{\delta \hat{y}}{\delta z} \cdot \frac{\delta z}{\delta w}$$

$$\hat{y} = \frac{1}{1+e^{-z}} \quad \text{where } z = w_0 + w_1 x_1 + w_2 x_2 + \dots + w_n x_n \text{ then we can calculate.}$$

$$\frac{\delta y}{\delta z} = \frac{e^{-z}}{(1+e^{-z})^2} = \frac{1}{(1+e^{-z})} - \frac{1}{(1+e^{-z})^2}$$

$$= \hat{y} - \hat{y}^2 = \hat{y}(1-\hat{y}) \quad (1)$$

$$\frac{\delta z}{\delta W} = X \quad (2)$$

$$\frac{\delta L}{\delta \hat{y}} = X^T \quad (3)$$

From (1), (2), (3) =>

$$\frac{\delta L}{\delta W} = \frac{\delta L}{\delta \hat{y}} \cdot \frac{\delta \hat{y}}{\delta z} \cdot \frac{\delta z}{\delta W} = X^T \hat{y}(1-\hat{y})$$

Problem 5

a) Using result from Ex1 => Hessian of loss function

$$H_{jk} = \frac{\delta^2 L}{\delta w_j \cdot \delta w_k}$$

$$= -\sum_{i=1}^{N} - \frac{\partial y_i}{\partial w_k} x_{ij}$$

$$= \sum_{i=1}^{N} \cdot y_i (1-y_i) x_{ij} x_{ih}$$

$$\rightarrow H = \sum_{i=1}^{N} y_i (1-y_i) x_i^2$$

For $y_i \in [0,1]$, we have : $y_i(1-y_i) \in [0, 1/4]$. Therefore, $H \geq 0$ => Convex

b) MSE:

$$L = \frac{1}{N} \sum_{n=1}^{N} (\bar{y}_1 - y_i)^2$$

For

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial w}$$

$$= -2x(y \cdot \hat{y} - y\hat{y}^2 - \hat{y}^2 + \hat{y}^3)$$

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Học sinh	Lớp	M	TỔNG /10

Nhận xét của thầy, cô giáo (ký tên):

Second derivate

$$\frac{\partial^2 L}{\partial w^2} = -2x\left(y\frac{\partial \bar{y}}{\partial w} - y\frac{\partial y^2}{\partial \bar{y}}\cdot\frac{\partial \bar{y}}{\partial w} - \frac{\partial \bar{y}^2}{\partial y}\cdot\frac{\partial \bar{y}}{\partial w} + \frac{\partial y^3}{\partial \bar{y}}\cdot\frac{\partial \bar{y}}{\partial w}\right)$$

$$= -2x^2 y(1-\bar{y})(y - 2y\bar{y} - 2\bar{y} + 3\bar{y}^2)$$

Since $x^2 \cdot \bar{y}(1-\bar{y}) \geq 0$, consider only: $f(\bar{y})$

$$= -2x(y - 2y\bar{y} - 2\bar{y} + 3\bar{y}^2)$$

$$f(\bar{y}) = \begin{cases} 4\bar{y} - 6\bar{y}^2 = 2\bar{y}(2\bar{y} - 3\bar{y}) & y=0 \\ -2 + 8\bar{y} - 6y^2 = -2(3\bar{y} - 1)(y-1) & y= \end{cases}$$

$f(\bar{y}) \leq 0$ when $\frac{2}{3} \leq \bar{y} \leq 1$.

$f(\bar{y}) \leq 0$ when $0 \leq \bar{y} \leq \frac{1}{3}$.

=> Not Convex

YÊU CẦU HỌC SINH: *Chữ viết đẹp, đúng chính tả, trình bày hợp lý và sạch sẽ.*