DSP2 - Practice Homework

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Exercise 1

No, $\mathscr{H}$ does not define an LTI because it is note time-invariant. With $x[n] = \delta[0]$:

$$\mathscr{H}[x][n] = 0 \cdot \delta[0] \neq 1 \cdot \delta[1] = \mathscr{H}[x][n-1]$$

Exercise 2

(a) $x[n]$ is the convolution of $x_1[n]$ and $x_2[n]$ with

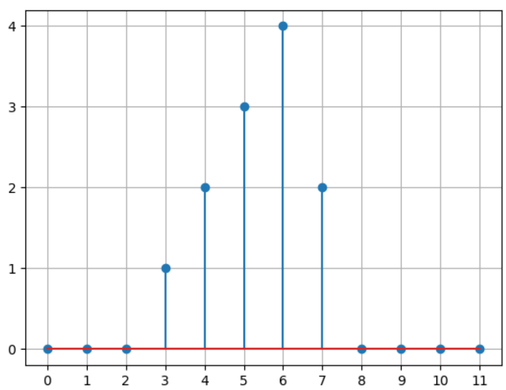
$$x_1[n] = x_2[n] = rect\left[\frac{n}{M-1}\right]$$

(b) Using the convolution theorem in combination with the known DTFT of the rect function we get:

$$\begin{aligned} X(e^{j\omega}) &= X_1(e^{j\omega}) \cdot X_2(e^{j\omega}) \\ &= \left(\frac{sin\left((M-1)\,\omega/2\right)}{sin(\omega/2)}\right)^2 \end{aligned}$$

Exercise 3

Because $u[n-4] = \sum_{i=4}^{\infty} \delta[n-i]$, we can write the impulse response of $u[n-4]$ as the sum of impulse responses because $u[n-4] = \sum_{k=4}^{\infty} \delta[n-k]$. This results in the following sketch:



Exercise 4

Output of $\mathscr{H}_1$:

$$x_1[n] = 3\left(\frac{-1}{4}\right)^n u[n-2]$$

Output of $\mathscr{H}_2$:

$$\begin{aligned} x_2[n] = x_1[n] * u[n+2] &= \sum_{k=-\infty}^{\infty} x_1[k]u[k-n-2] \\ &= \sum_{k=n+2}^{\infty} x_1[k] = \sum_{k=n+2}^{\infty} 3\left(\frac{-1}{4}\right)^k u[k-2] \end{aligned}$$

with $k' = k - n - 2$:

$$= 3\sum_{k'=0}^{\infty}\left(\frac{-1}{4}\right)^{(k'+n+2)} u[k'+n] = 3\left(\frac{-1}{4}\right)^{n+2}\sum_{k'=0}^{\infty}\left(\frac{-1}{4}\right)^{k'} u[n]$$

Using the formula for geometric series this results in:

$$x_2[n] = 3\left(\frac{-1}{4}\right)^{n+2}\left(\frac{4}{5}\right) u[n] = \frac{12}{5}\left(\frac{-1}{4}\right)^{n+2} u[n]$$

Calculating $x_4[n]$ with $x_2[n] = x_3[n]$:

$$x_4[n] = x_3[n] * \delta[n-1] = x_3[n-1] = \frac{12}{5}\left(\frac{-1}{4}\right)^{n+1} u[n-1]$$

This results in:

$$\begin{aligned} y[n] = x_3[n] + x_4[n] &= \frac{12}{5}\left(\frac{-1}{4}\right)^{n+2} u[n] + \frac{12}{5}\left(\frac{-1}{4}\right)^{n+1} u[n-1] \\ &= \begin{cases} 0, n < 0 \\ \frac{3}{20}, n = 0 \\ \frac{9}{5}\left(\frac{-1}{4}\right)^{n+1}, n > 0 \end{cases} \end{aligned}$$

The whole system is causal because its impulse response is 0 for $n < 0$. It is also BIBO stable because the geometric series is absolutely summable.

Exercise 5

(a) Linear, time-invariant, BIBO stable but not causal.

(b) BIBO stable and causal, but not linear or time-invariant.

(c) Linear, time-invariant, BIBO stable but not causal.

Exercise 6