Model Calibration

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1   2. Model Calibration and Testing

```
In [26]: %matplotlib inline
         from model.binomial import EuropeanVanillaModel, HullWhiteEuropeanModel, H
         from datetime import date
         import pandas as pd
         import numpy as np
         import matplotlib.pyplot as plt
         import seaborn as sns
```

GS Stock Prices (Nov 30, 2016 - Feb 27, 2017)   We chose these dates because they are between
dividend paying dates of *Nov 29,2016* and *Feb 28, 2017*

We'll only need closing price from the stock data and will compute the ratio of the closing price
for each day

$$ratio(p_i) = \frac{p_{i+1}}{p_i}$$

We use pandas DataFrames to work with out datasets and our data is from yahoo finance.

```
In [27]: stock_prices = pd.read_csv('dataset/gs-stock-prices.csv', index_col=0, par
         stock_prices['Ratio'] = [np.nan if i + 1 == len(stock_prices) else stock_p
         stock_prices.drop([col for col in stock_prices.columns if col not in ['Clo
         stock_prices.head(5)

Out[27]:                     Close      Ratio
         Date
         2016-11-30   219.289993   1.033472
         2016-12-01   226.630005   0.985571
```

```
2016-12-02   223.360001   1.023236
2016-12-05   228.550003   1.012382
2016-12-06   231.380005   1.018065
```

Now that we have our closing prices and associated ratios, we can start tuning our model using Hull-White. Using 03-17-2017 as our start date, we can determine the inital stock price s0. We determine that our time period T, is the number of days between 03/17/2017 and 06/16/2017 divided by days in a year. delta T (dt or ΔT) is determined by dividing our T period into equal intervals for n steps. We found that the current USD Libor 12 month rate is 1.81317%.

Choice of Stock: The Goldman Sachs Group, Inc. (GS)

Period of stock used for tuning: November 29th 2016~February 27th 2017

Calibration Method: Hull-White Model

Start Date (t=0):   03-17-2017

Initial Stock Price (t=0):   243.94

USD LIBOR - 12 months on Marth 17, 2017:   1.81317 %

GS Option Expiration Date: 06-16-2017

```python
In [28]: start_date = date(2017, 3, 17)
         expiration_date = date(2017,6,16)
         T = (expiration_date - start_date).days / 365
         n_steps = 16
         dt = T / n_steps
         s0 = 243.94
         r = .0181317
         n_training = len(stock_prices)

         def create_df(option_data, option):
             results = []
             for k in option_data['Strike']:
                 american = HullWhiteAmericanModel(k, T, dt, r, n_steps, option, n_
                 euro = HullWhiteEuropeanModel(k, T, dt, r, n_steps, option, n_trai
                 results.append((k, american.price(), euro.price()))
             df = pd.DataFrame(results, columns=['Strike', 'American', 'European'])
             df['Actual'] = option_data['Last Price']
             return df
```

Now we will read in the gs call option data and compute the theoretical option price using our Hull-White model for both European and American Options.
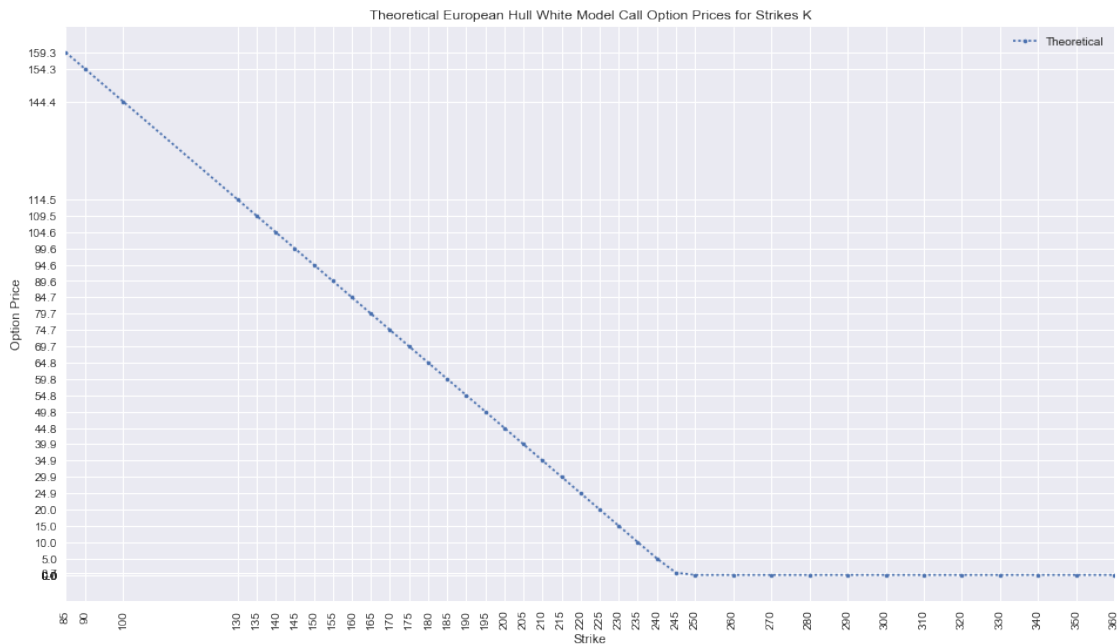
1.0.1 Call Option Prices

```
In [29]: gs_calls = pd.read_csv('dataset/gs-calls.csv')
         option = 'call'
         call_df = create_df(gs_calls, option)

In [30]: ax = call_df[['Strike','European']].plot(x='Strike',
                                                   y='European',
                                                   title='Theoretical European Hull White
                                                   figsize=(16,9),
                                                   xticks=call_df['Strike'],
                                                   yticks=call_df['European'],
                                                   kind='line',
                                                   rot=90,
                                                   style=[':'],
                                                   marker='.')
         ax.set_ylabel('Option Price')
         ax.legend(['Theoretical'])

Out[30]: <matplotlib.legend.Legend at 0x115f9d3c8>
```



```
In [31]: ax = call_df[['Strike','American', 'Actual']].plot(x='Strike',
                                                            title='Therotical Hull-White
                                                            figsize=(16,9),
                                                            xticks=call_df['Strike'],
                                                            kind='line',
                                                            rot=90,
```

```
                                          style=['g', 'r'],
                                          marker='.')
          ax.set_ylabel('Option Price')
          ax.legend(['Theoretical', 'Actual'])
```

Out[31]: <matplotlib.legend.Legend at 0x1170a1240>

Plot of Theoretical American and European Call Prices Compared to Actual Option Price

```
In [32]: ax = call_df.plot(x='Strike',
                           title='Therotical American and European Hull-White and A
                           figsize=(16,9),
                           xticks=call_df['Strike'],
                           kind='line',
                           rot=90,
                           style=[':b', '-g', 'r'],
                           marker='.')
          ax.set_ylabel('Option Price')
          ax.legend(['American Theoretical', 'European Theoretical', 'Actual'])
```

Out[32]: <matplotlib.legend.Legend at 0x117d8f9b0>

Therotical American and European Hull-White and Actual Call Option Prices for Strikes K
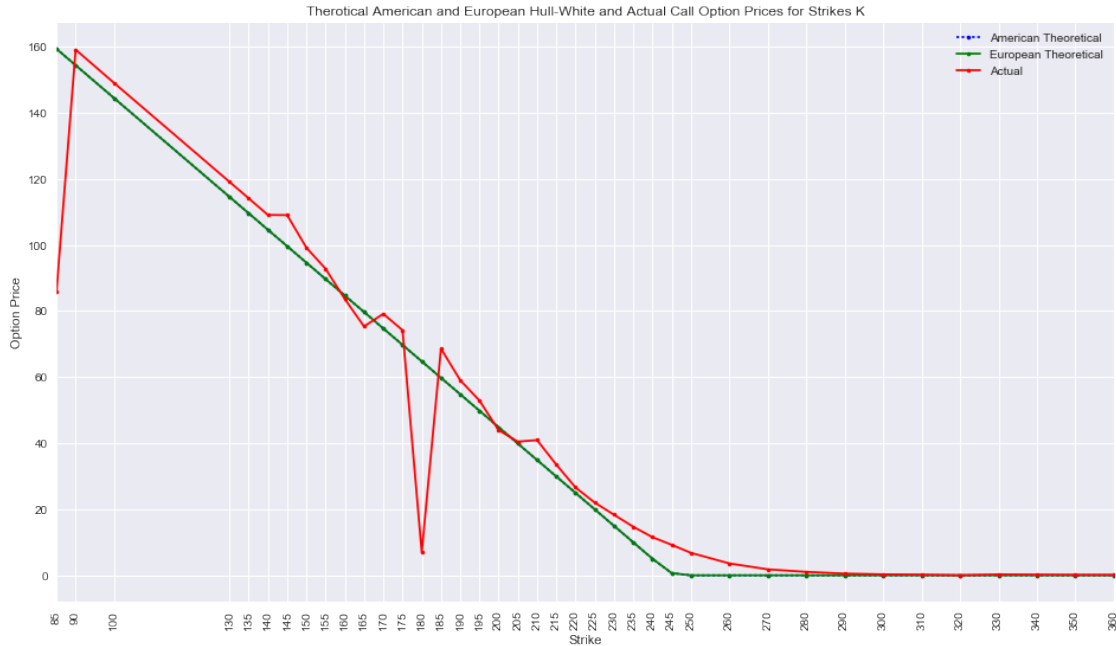
Findings From the graph above we see that the price of American call options is higher than that of European put options

1.0.2 Put Option Prices

```
In [33]: gs_puts = pd.read_csv('dataset/gs-puts.csv')
         option = 'put'
         put_df = create_df(gs_puts, option)

In [34]: ax = put_df[['Strike','European']].plot(x='Strike',
                                                  y='European',
                                                  title='Theoretical European Hull White
                                                  figsize=(16,9),
                                                  xticks=put_df['Strike'],
                                                  yticks=put_df['European'],
                                                  kind='line',
                                                  rot=90,
                                                  style=['b'],
                                                  marker='.')
         ax.set_ylabel('Option Price')
         ax.legend(['Theoretical'])

Out[34]: <matplotlib.legend.Legend at 0x11655f908>
```
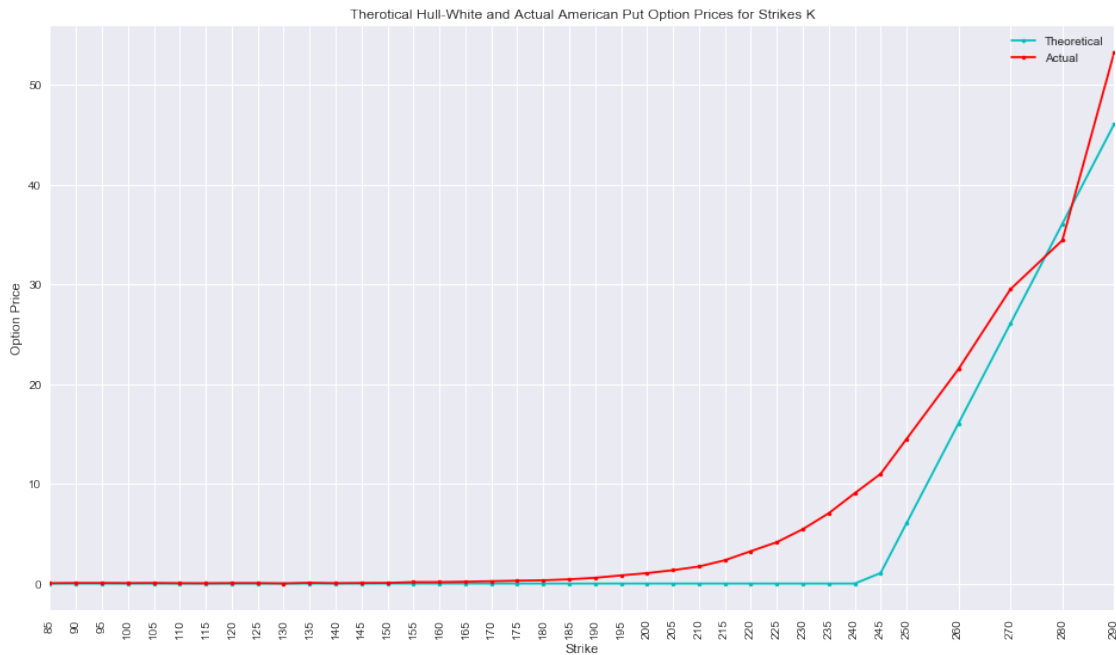
Theoretical European Hull White Model Put Option Prices for Strikes K
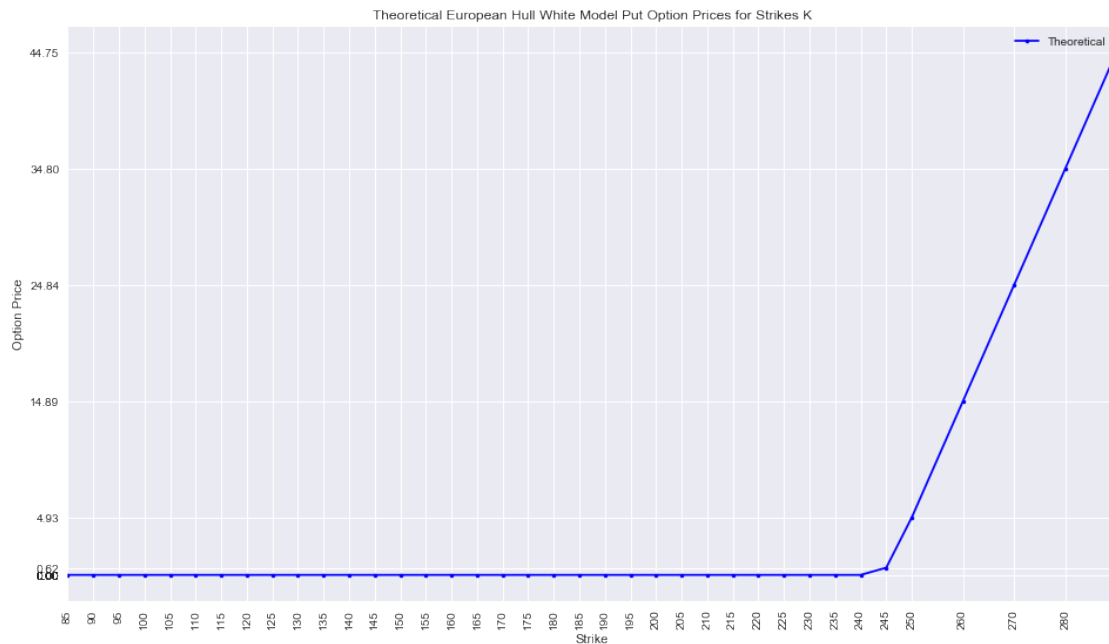
```
In [35]: ax = put_df[['Strike','American', 'Actual']].plot(x='Strike',
                                              title='Therotical Hull-White
                                              figsize=(16,9),
                                              xticks=put_df['Strike'],
                                              kind='line',
                                              rot=90,
                                              style=['c', 'r'],
                                              marker='.')
         ax.set_ylabel('Option Price')
         ax.legend(['Theoretical', 'Actual'])

Out[35]: <matplotlib.legend.Legend at 0x11813a5c0>
```

Therotical Hull-White and Actual American Put Option Prices for Strikes K

Plot of Theoretical American and European Put Prices Compared to Actual Option Price

```
In [38]: ax = put_df.plot(x='Strike',
                           title='Therotical American and European Hull-White and Ac
                           figsize=(16,9),
                           xticks=put_df['Strike'],
                           kind='line',
                           rot=90,
                           style=['c', 'b', 'r'],
                           marker='.')
         ax.set_ylabel('Option Price')
         ax.legend(['American Theoretical', 'European Theoretical', 'Actual'])

Out[38]: <matplotlib.legend.Legend at 0x118c25320>
```

Therotical American and European Hull-White and Actual American Put Option Prices for Strikes K
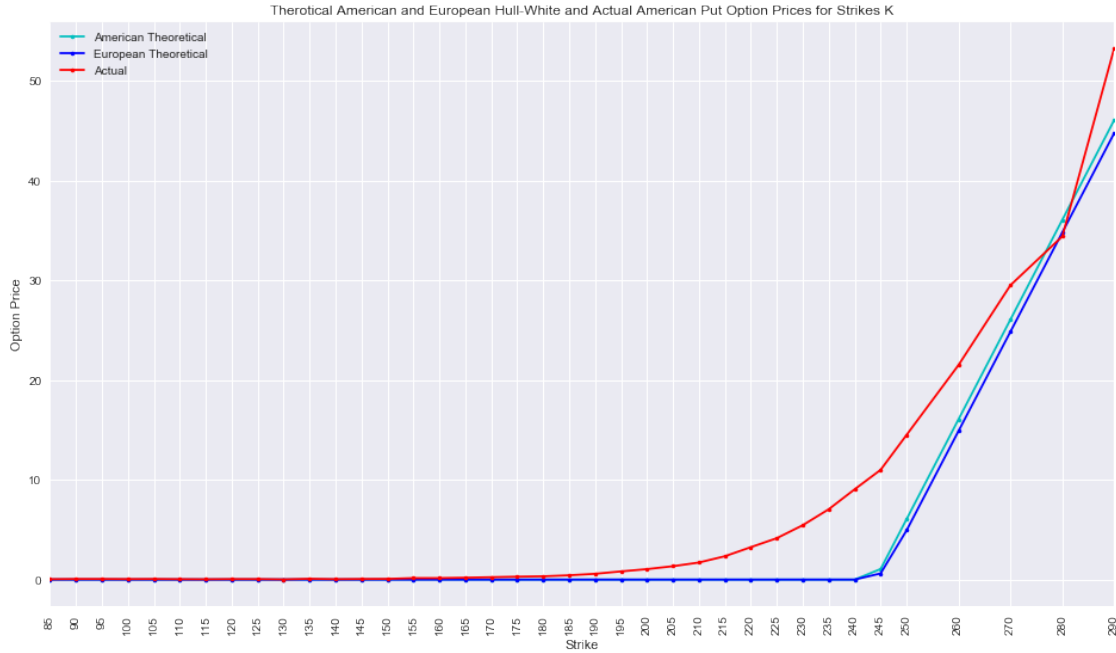
Findings From the graph above we see that the price of the American option for put options is higher than that of European put options

1.0.3 Conclusion

The option prices we computed using our model are very similar to the actual option prices. Although our theoretical prices are not exactly the same as the actual prices, our predictions follow a similar trend. For Call Options, we found that our European and American prices were very similar. For Put options we found that American prices clearly cost more.