

Have no fear: Just the math you need to know for monitoring

**Lerna Ekmekcioglu
Sr Solutions Architect, AWS**

Thoughts and opinions are my own and do not represent that of my employer

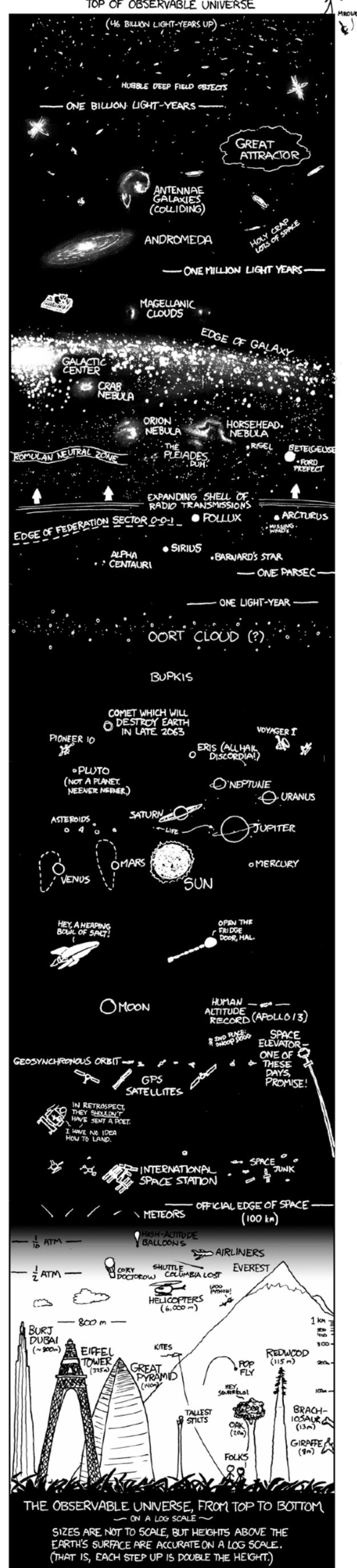
Agenda

- Visualizing the data: Logarithmic scale
- Summarizing the data:
 - Center
 - Spread
 - Relative standing

Visualizing the data: Logarithmic scale

Logarithms

[https://xkcd.com/482:](https://xkcd.com/482/) Height



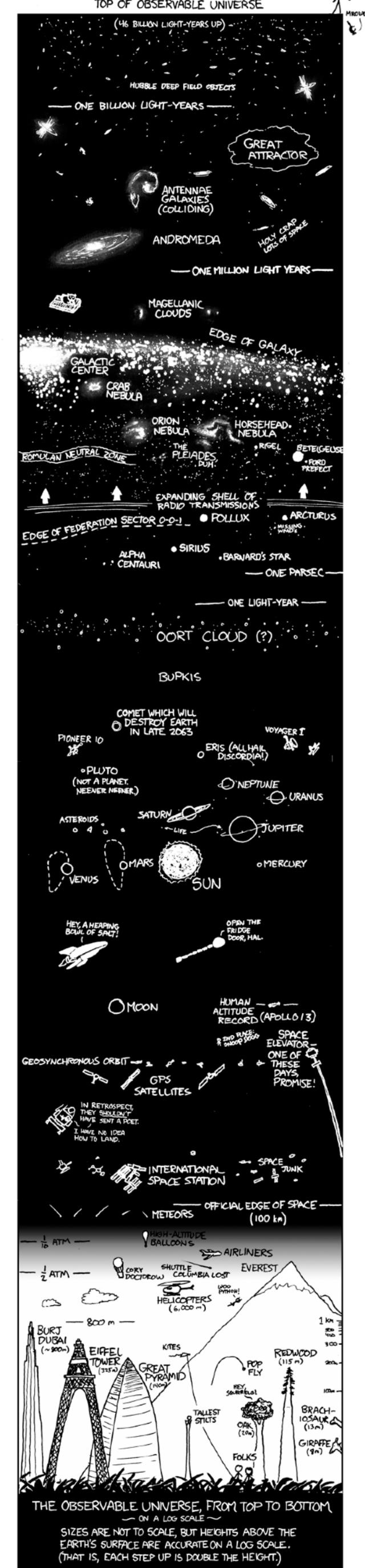
[https://xkcd.com/482:](https://xkcd.com/482/) Height, CC BY-NC 2.5

https://www.explainxkcd.com/wiki/index.php/482:_Height

Logarithms

The observable universe from top to bottom
~ on a log scale ~

...heights above the earth's surface are
accurate on a log scale.



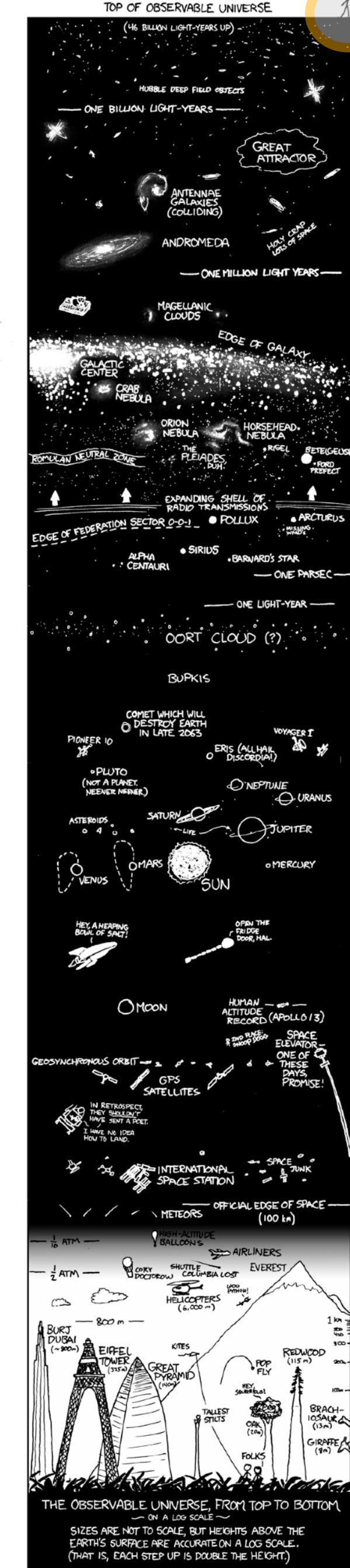
<https://xkcd.com/482>: Height, CC BY-NC 2.5

https://www.explainxkcd.com/wiki/index.php/482:_Height

Why logarithms?

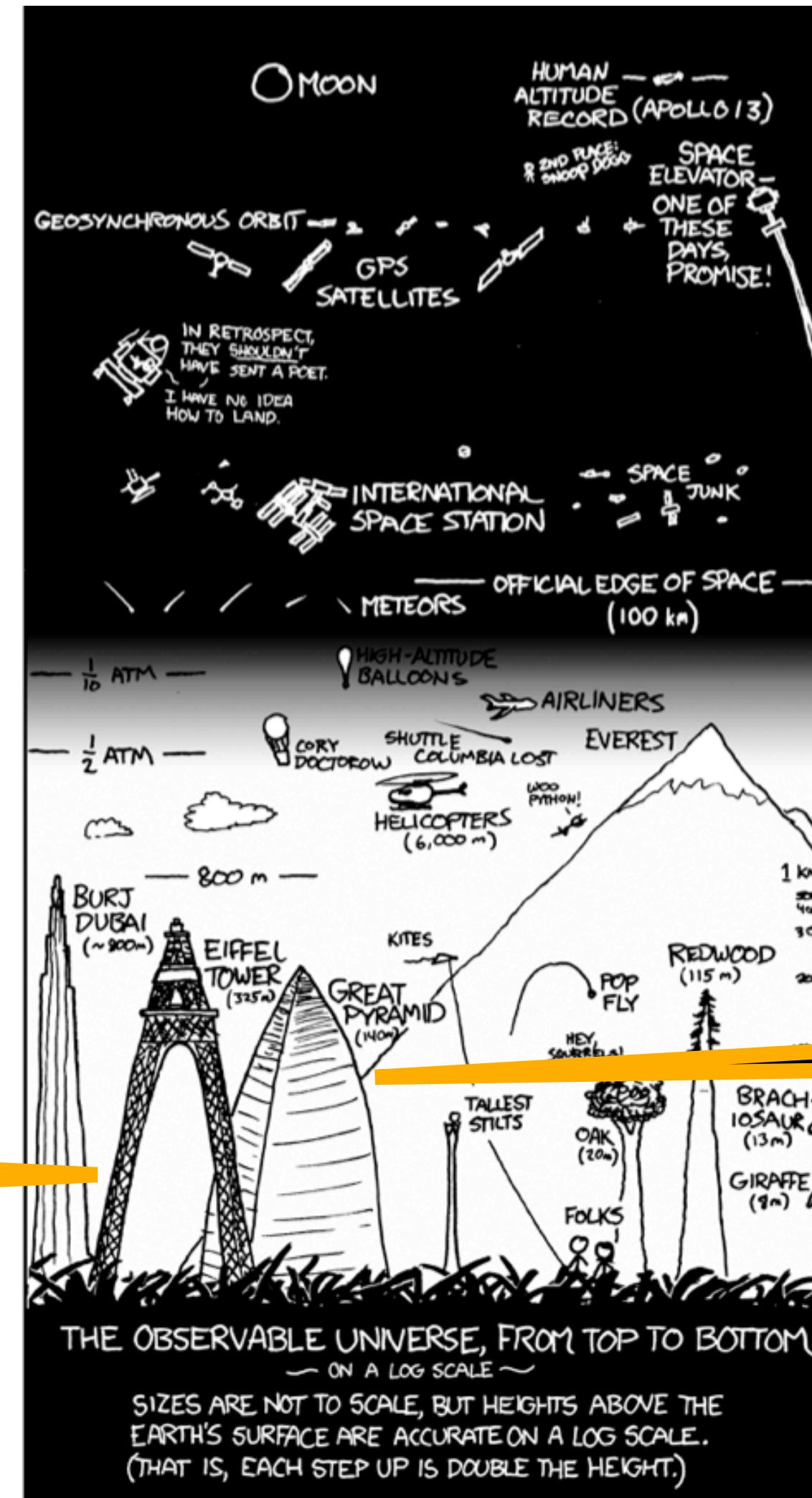
Display data over a very wide range of values

435 yottameters = 46 billion light – years



Top of observable universe

Eiffel
Tower

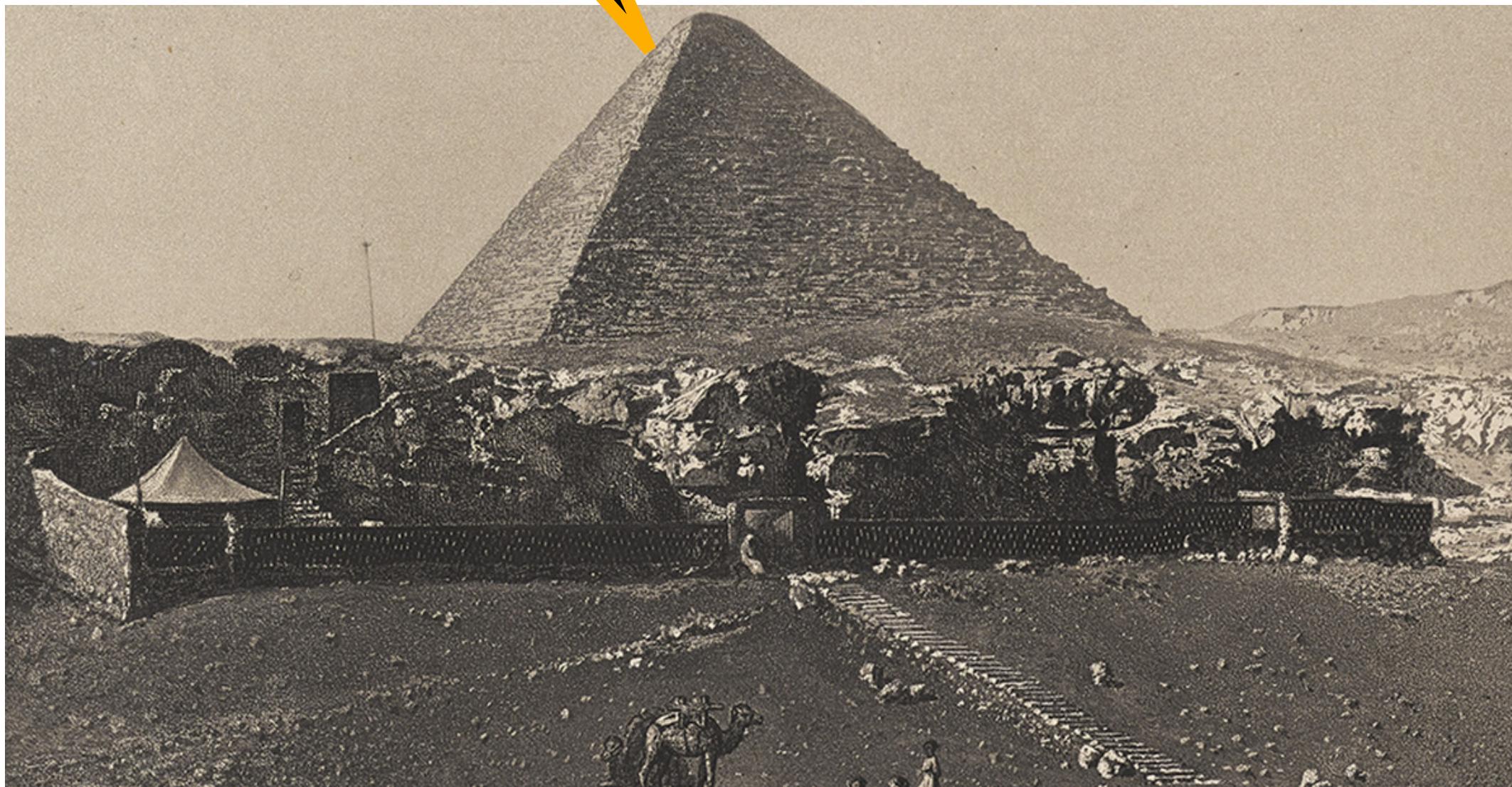


The Great
Pyramid

Grass

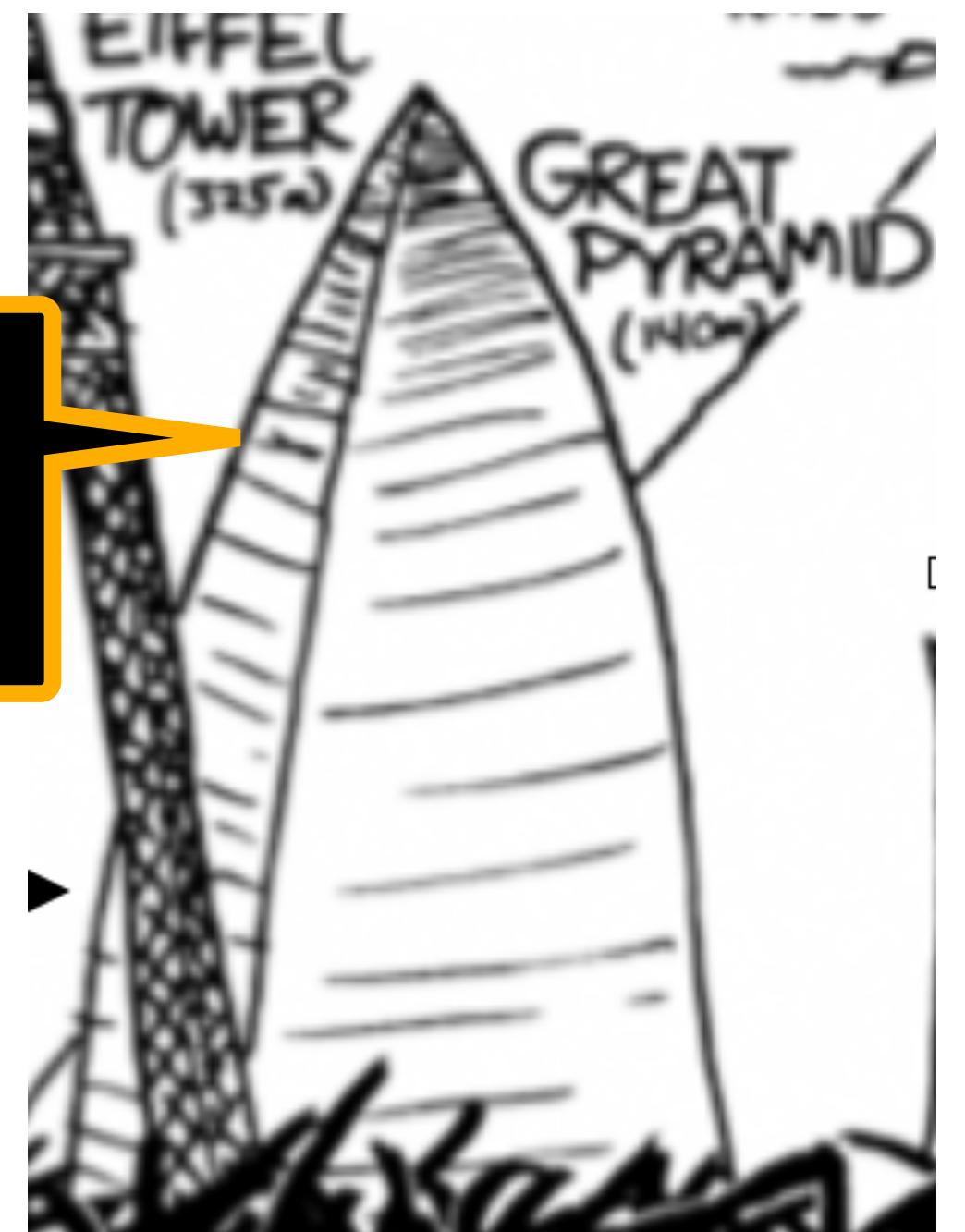
The Great Pyramid: linear vs log scale

Linear scale:
Pyramid



Daguerreotype of the Great Pyramid taken by Frédéric Goupil-Fesquet
on 20 November 1839

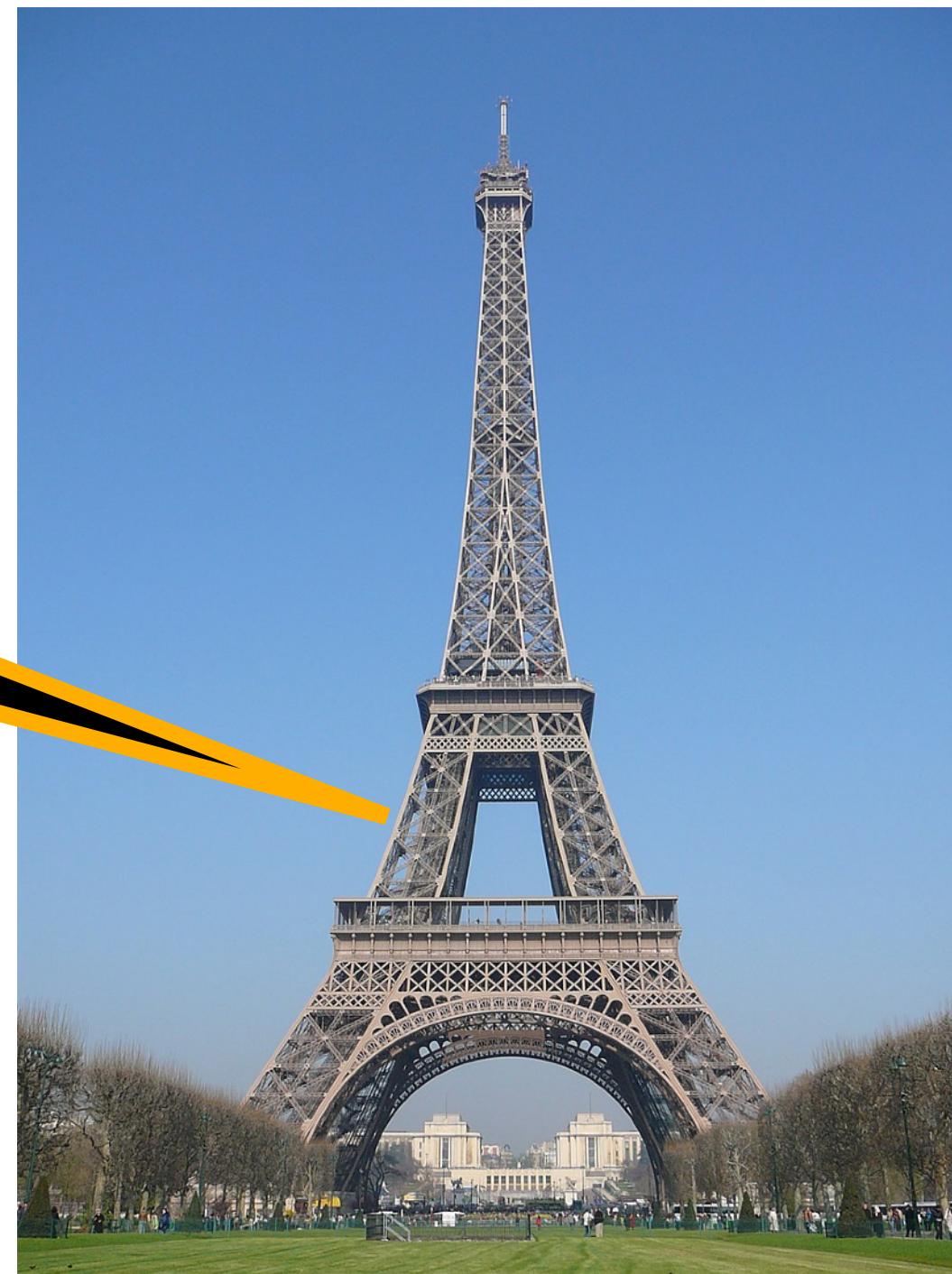
Log scale:
Stretched up



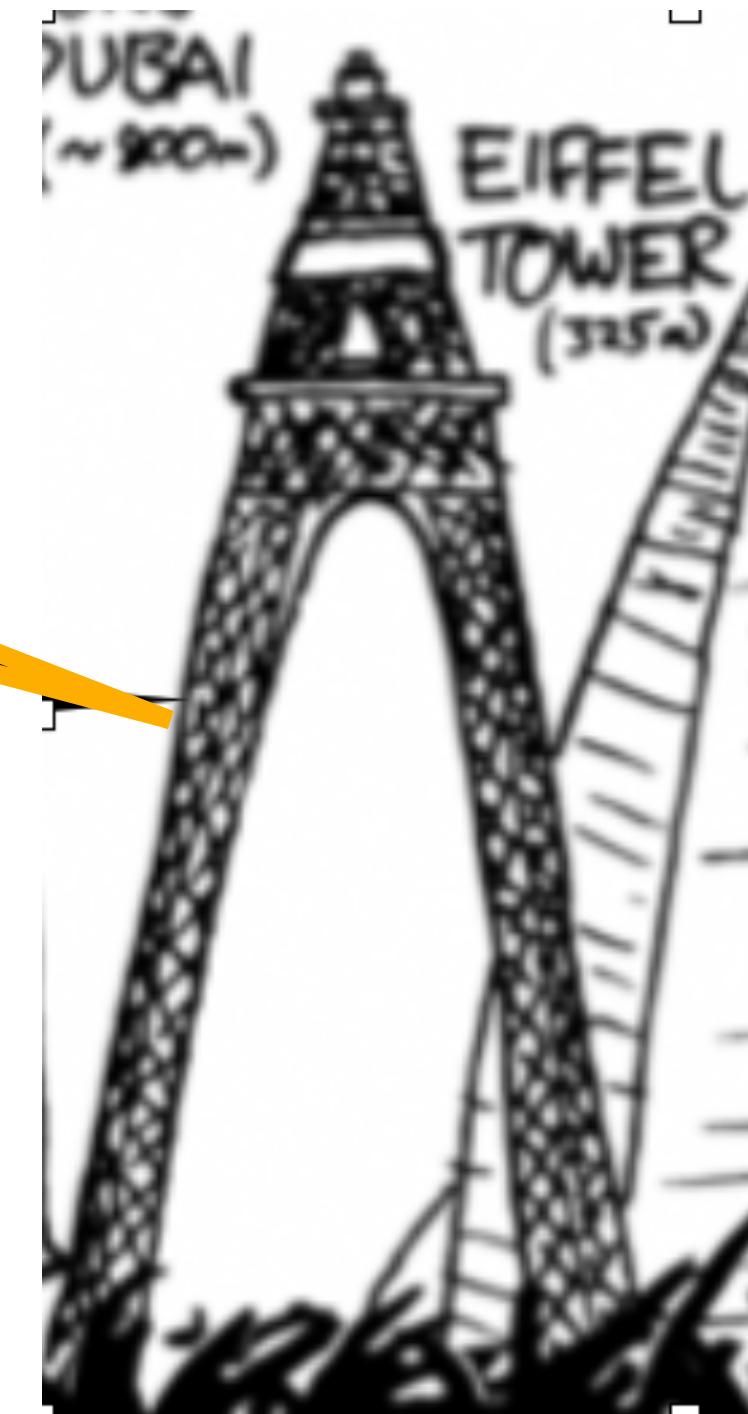
The Eiffel Tower: linear vs log scale

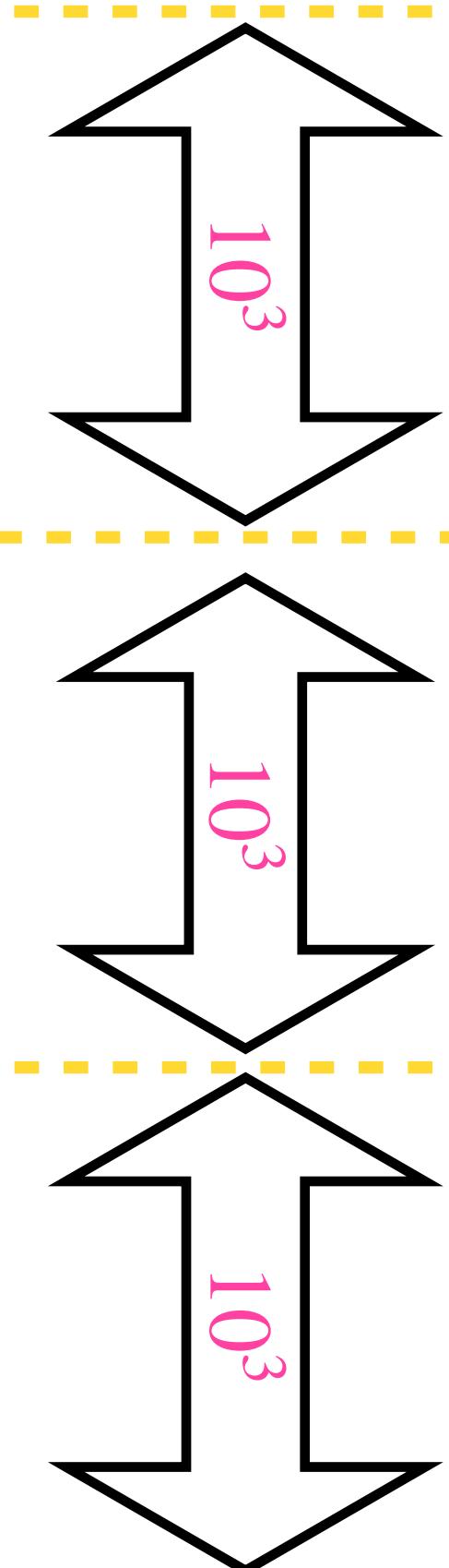
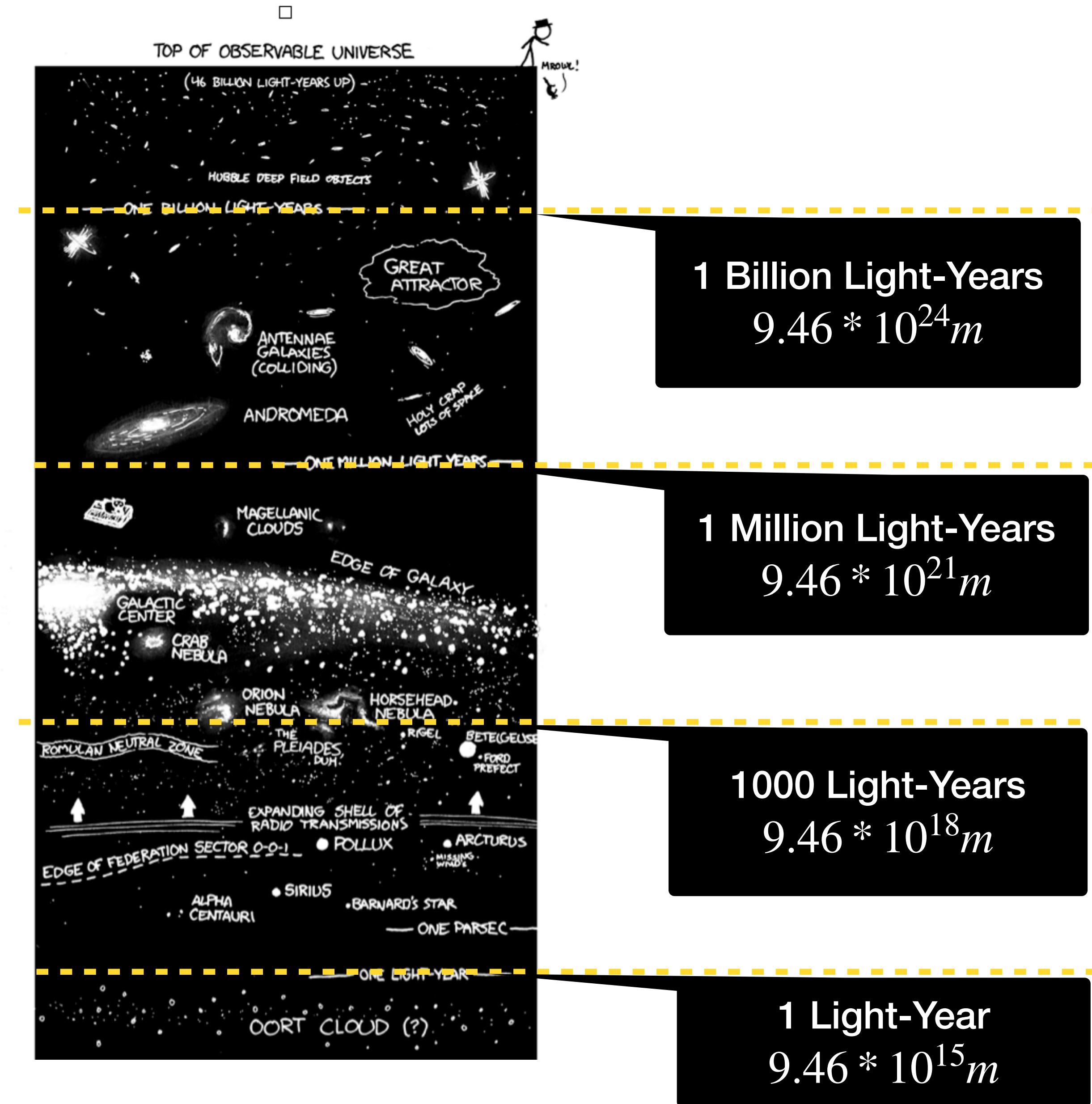
Linear scale:
Exponential
Curve

Eiffel Tower

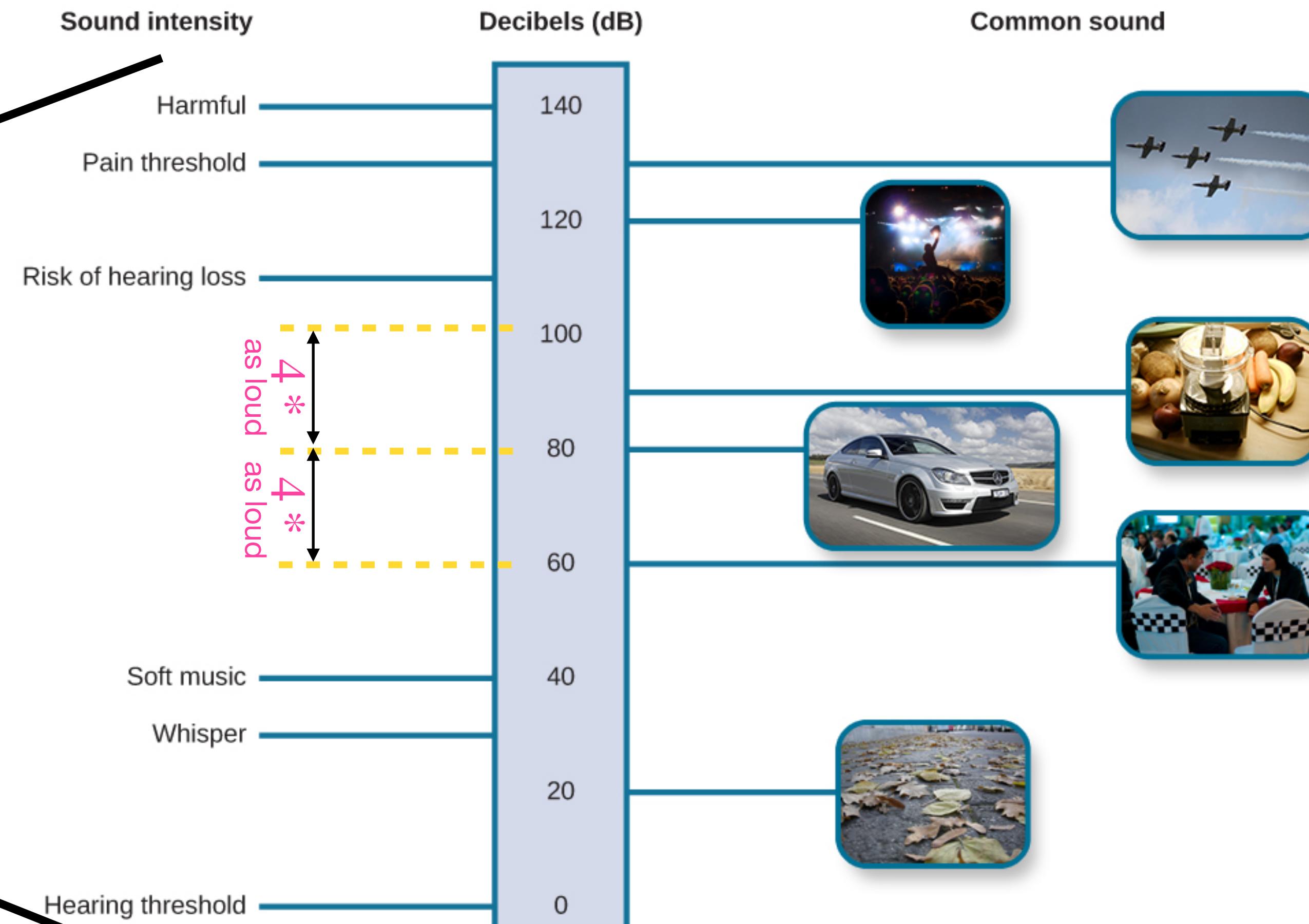


Log scale:
Straight line



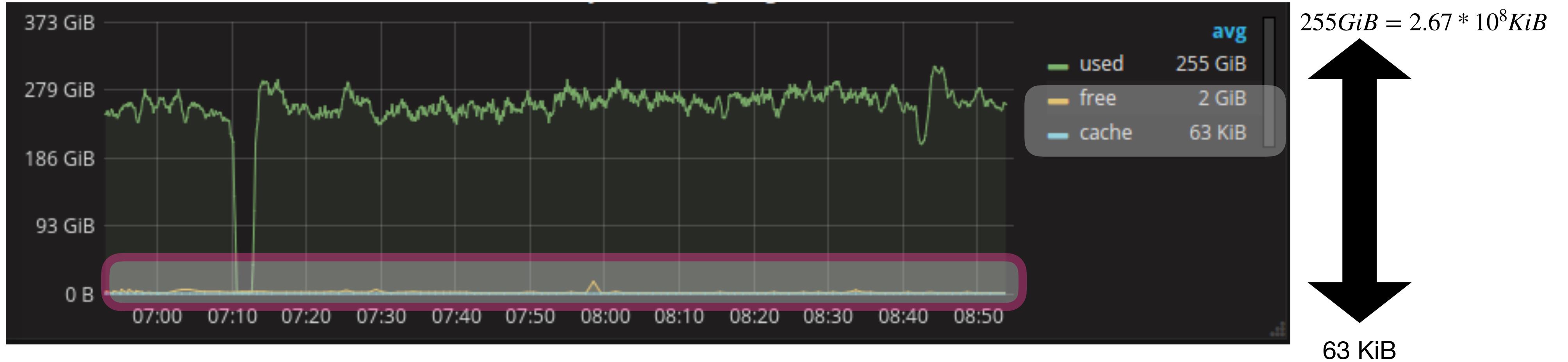


Ears perceive sounds at log scale

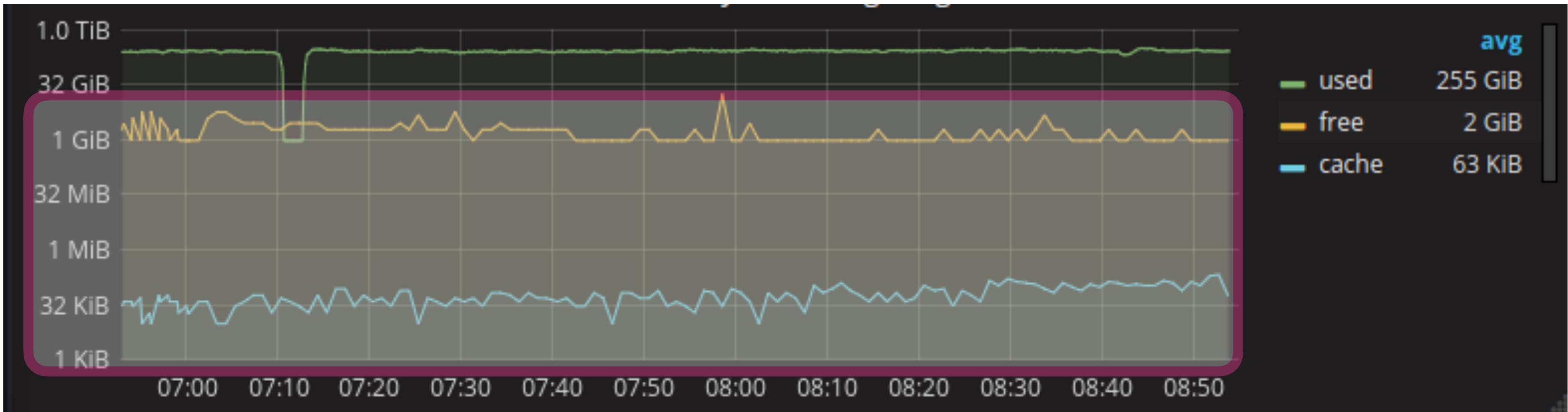


Gustav Fechner
(1801-1887)

Linear



Log Scale



Origins of logarithm

Large calculations required for:

- Astronomy
- Celestial navigation of ships



Mathematicians pain points in 1600s

“...nothing is more tedious, ..than the great delays suffered in the tedium of lengthy multiplications and divisions,

.. I have found an amazing way of shortening ..
multiplications, and divisions .. by .. addition, subtraction,

”
...

Description of the Wonderful Law of Logarithms, 1614, John Napier

John, as a gnome



John Napier
(1550-1617)

The wonderful laws of logarithms

Turn multiplications into additions

$$\log(x * y) = \log(x) + \log(y)$$

$$\log(10^2 * 10^3) = \log 10^2 + \log 10^3$$

Turn divisions into subtractions

$$\log\left(\frac{x}{y}\right) = \log(x) - \log(y)$$

$$\log(10^3/10^2) = \log 10^3 - \log 10^2$$

Henry Briggs' first table of common logarithms, Logarithmorum Chilias, 1617

Prima, from 1617.

This page tabulates the base-10 logarithms of the numbers 0 to 67 to fourteen decimal places.

	Logarithmi.	Logarithmi.	Log.
1	00000,00000,00000	34	15314,78917,04226
2	03010,29995,66398	35	15440,68044,35028
3	04771,21254,71966	36	15563,02500,76729
4	06020,59991,32796	37	15682,01724,06700
5	06989,70004,33602	38	15797,83596,61681
6	07781,51250,38364	39	15910,64607,02650
7	08450,98040,01426	40	16020,59991,32796
8	09030,89986,99194	41	16127,83856,71974
9	09542,42509,43932	2	16232,49290,39790
10	10000,00000,00000	43	16334,68455,57959
11	10413,92685,15823	4	16434,52676,48619
12	10791,81246,04762	45	16532,12513,77534
13	11139,43352,30684	6	16627,57831,68157
14	11461,28035,56782	47	16720,97857,93572
15	11760,91259,05568	8	16812,41237,37559
16	12041,19982,65592	49	16901,96080,02851
17	12304,48921,37827	50	16989,70004,33602
18	12552,72505,10331	51	17075,70176,09794
19	12787,53600,95283	2	17160,03343,63480
20	13010,29995,66398	53	17242,75869,60079
21	13222,19294,73392	4	17323,93759,82297
22	13424,22680,82221	55	17403,62689,49424
23	13617,27836,01759	6	17481,88027,90620
24	13802,11241,71161	57	17558,74853,67249
25	13979,40008,67204	8	17634,27993,56294
26	14149,73347,97082	59	17708,52011,64214
27	14313,63764,15899	60	17781,51250,38364

Image from microfilm of copy in the British Museum.

<https://commons.wikimedia.org/wiki/>

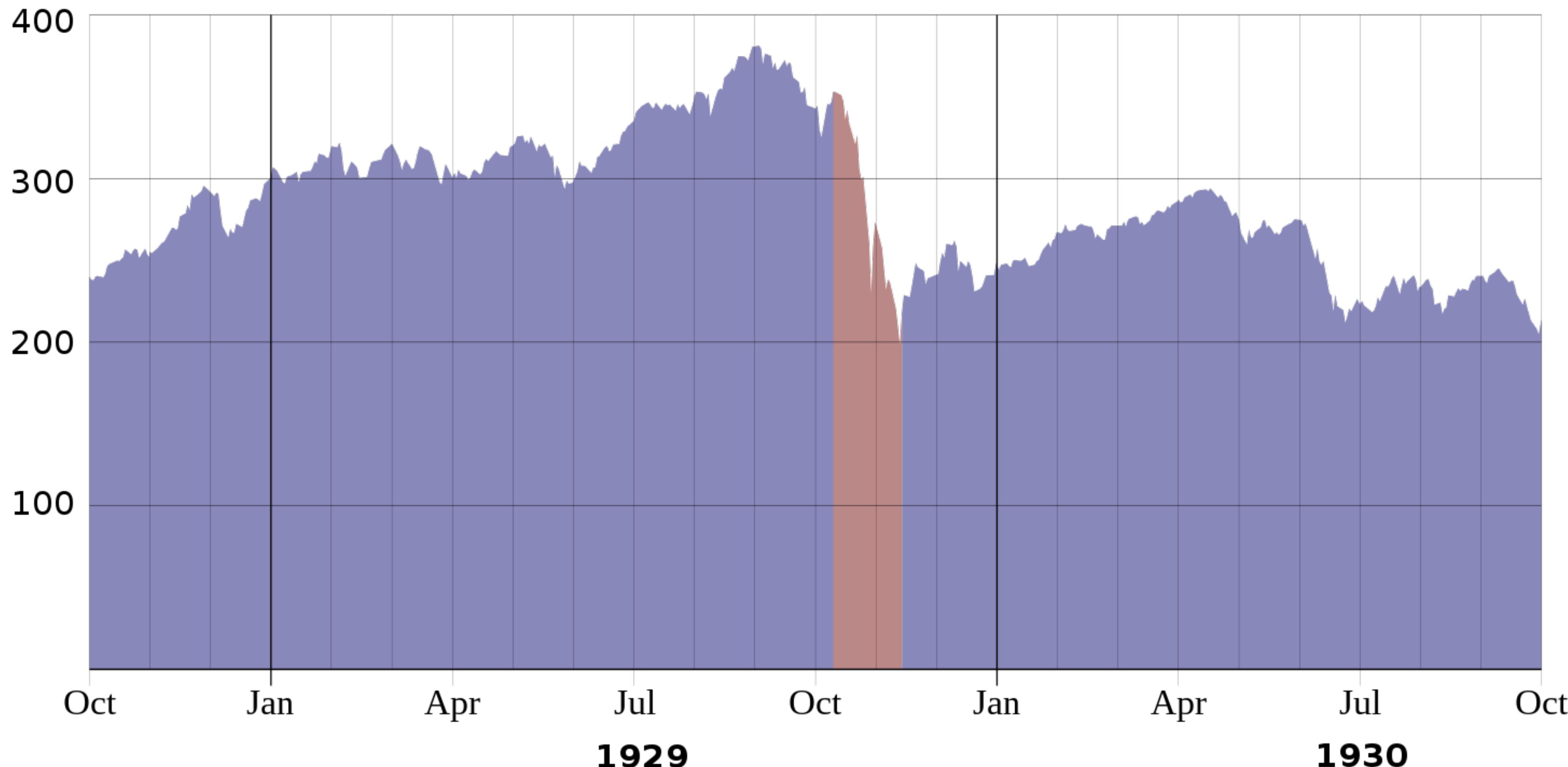
File:Logarithmorum Chilias Prima page 0-67.jpg

Why logarithmic scale?

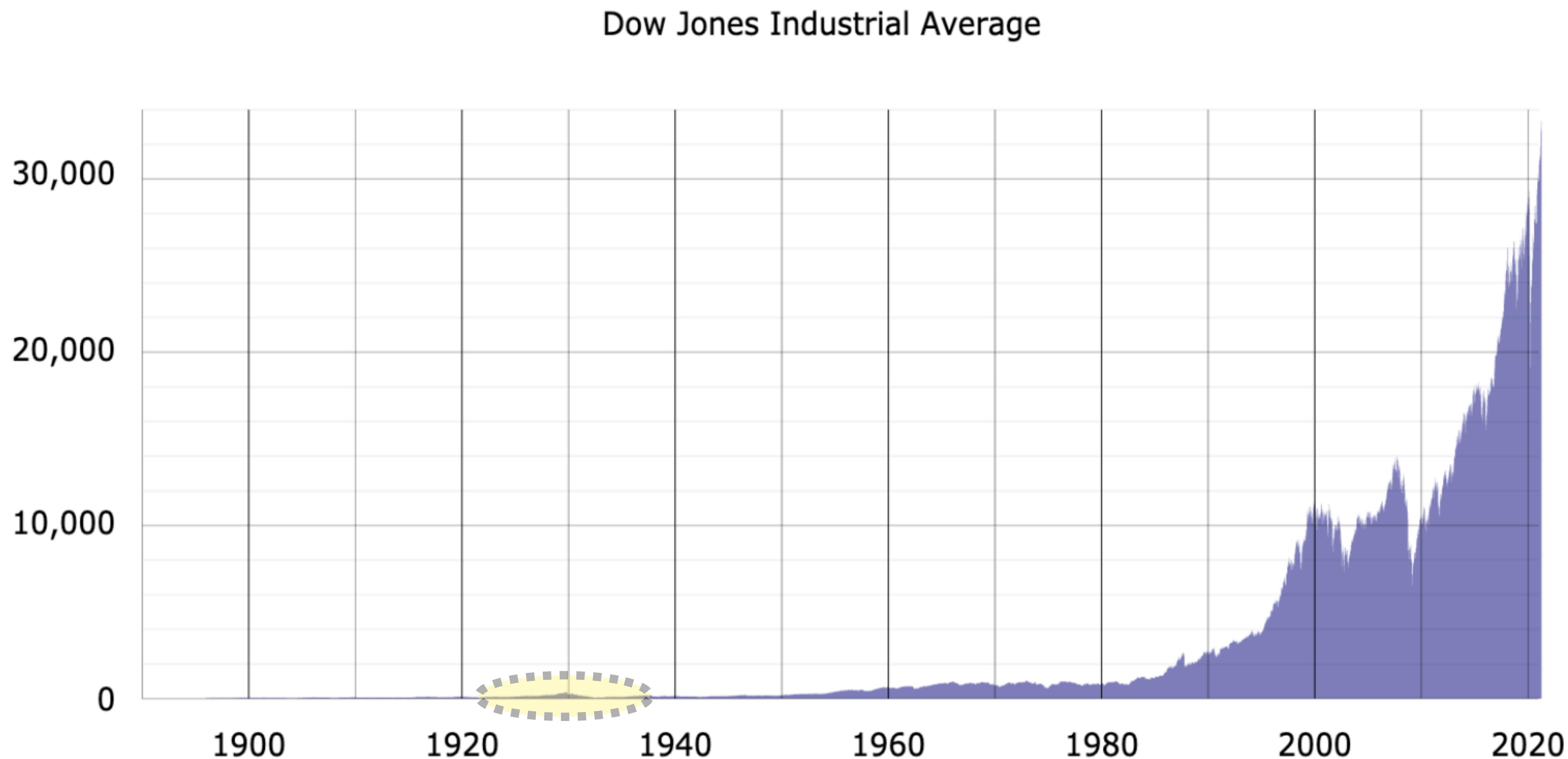
- Display data over a very wide range of values in a compact way
- Display relative change instead of absolute quantity of change

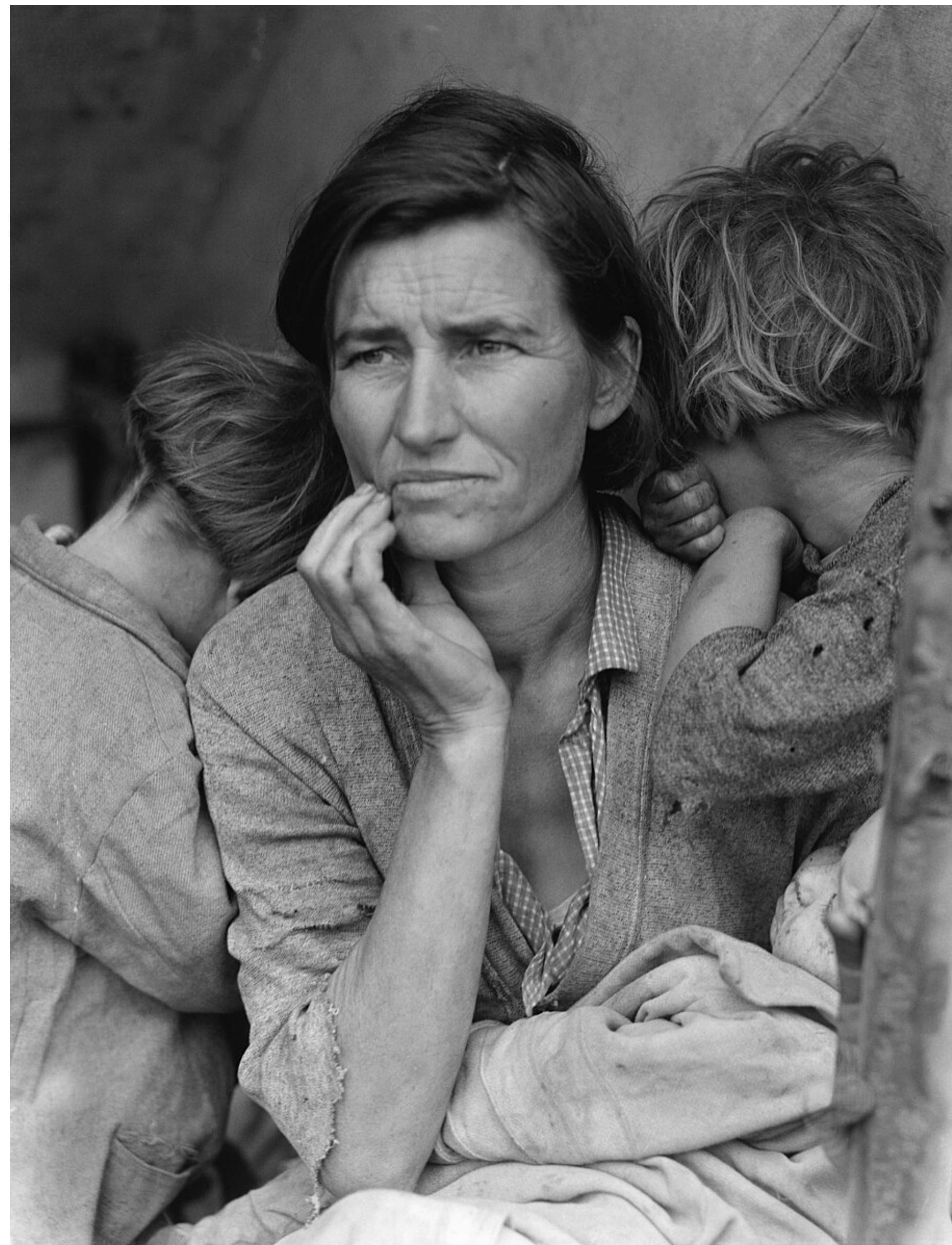
1929 Wall Street Crash

Wall Street Crash on the Dow Jones Industrial Average, 1929

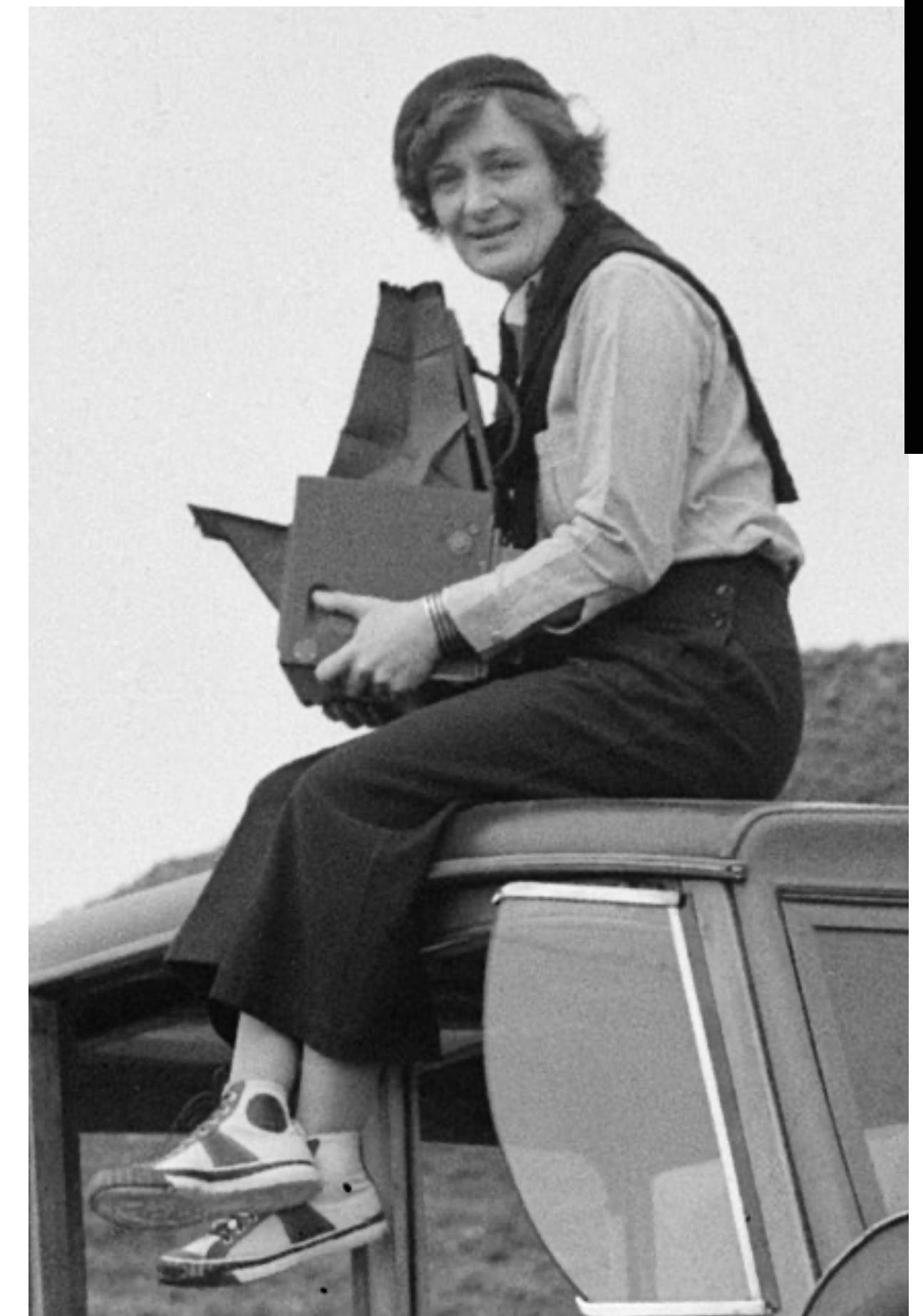


Dow Jones Industrial Average





Migrant mother, Nipomo, California by Dorothea Lange
<https://en.wikipedia.org/wiki/File:Lange-MigrantMother02.jpg>

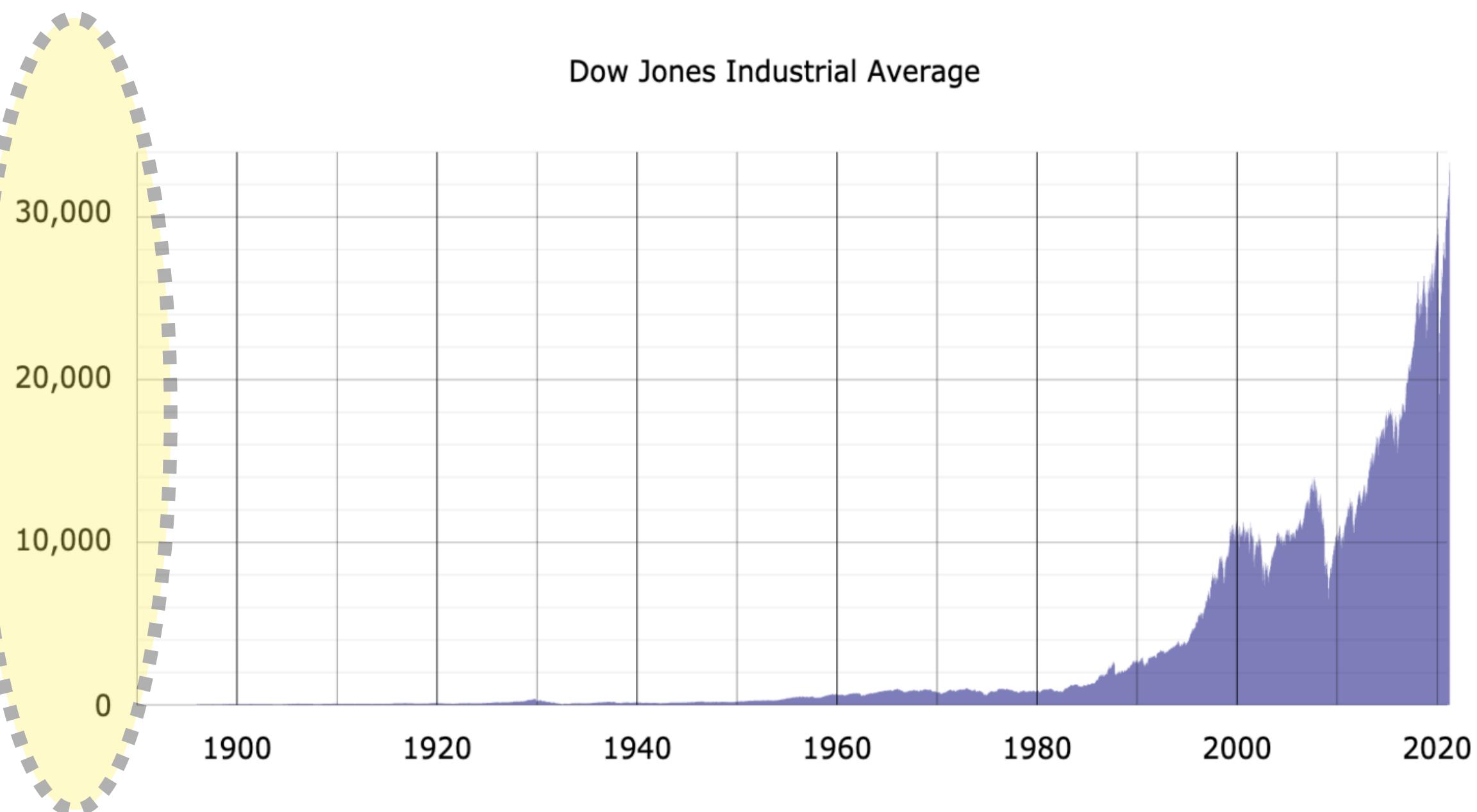


Dorothea Lange (1895-1965),
[Resettlement Administration photographer](#) with a 4x5 camera
Photographer: Rondal Partridge

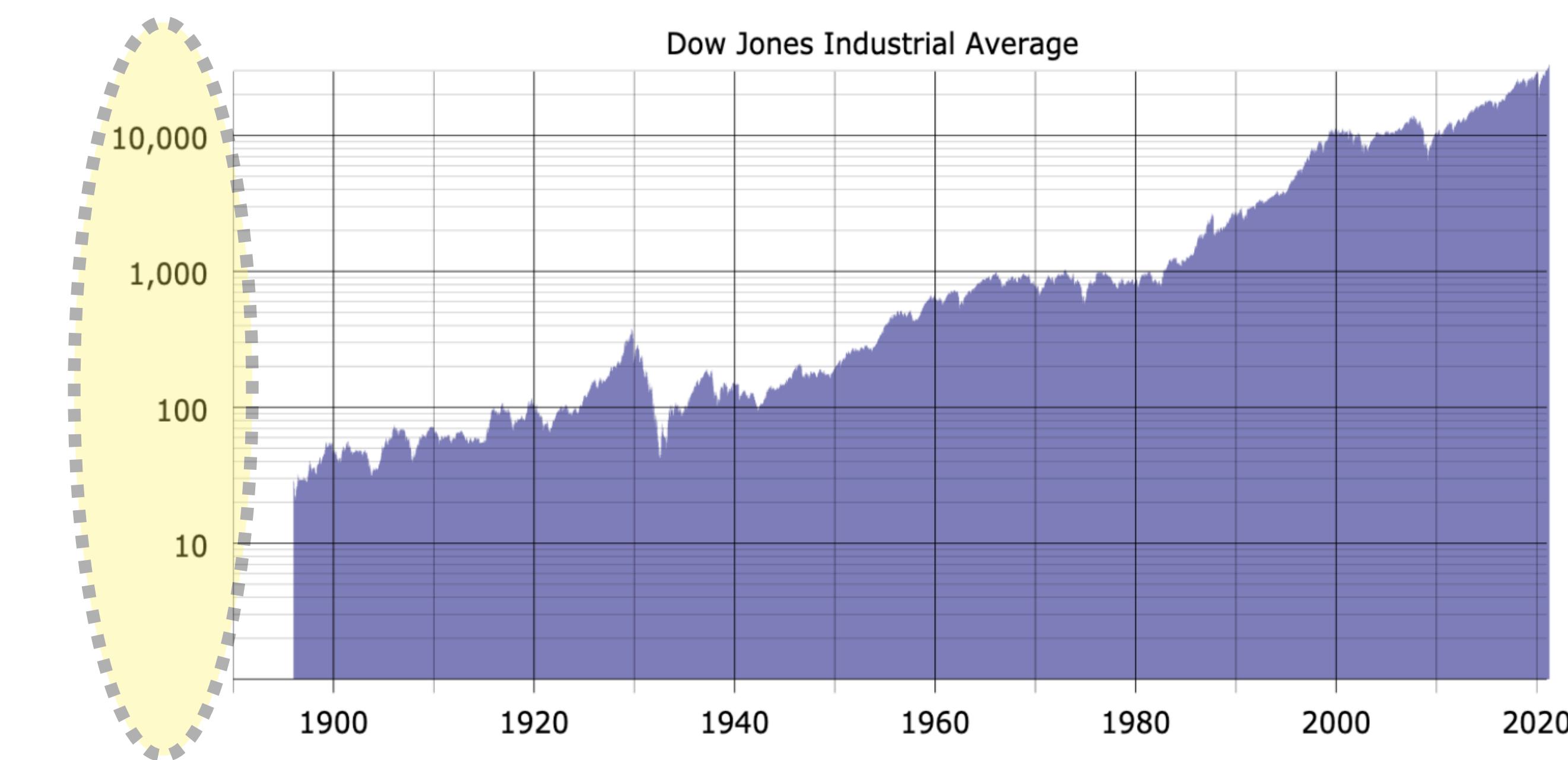


[https://en.wikipedia.org/wiki/Dorothea_Lange#/media/File:Dorothea_Lange_atop_automobile_in_California_\(restored\).jpg](https://en.wikipedia.org/wiki/Dorothea_Lange#/media/File:Dorothea_Lange_atop_automobile_in_California_(restored).jpg)

Spot the difference

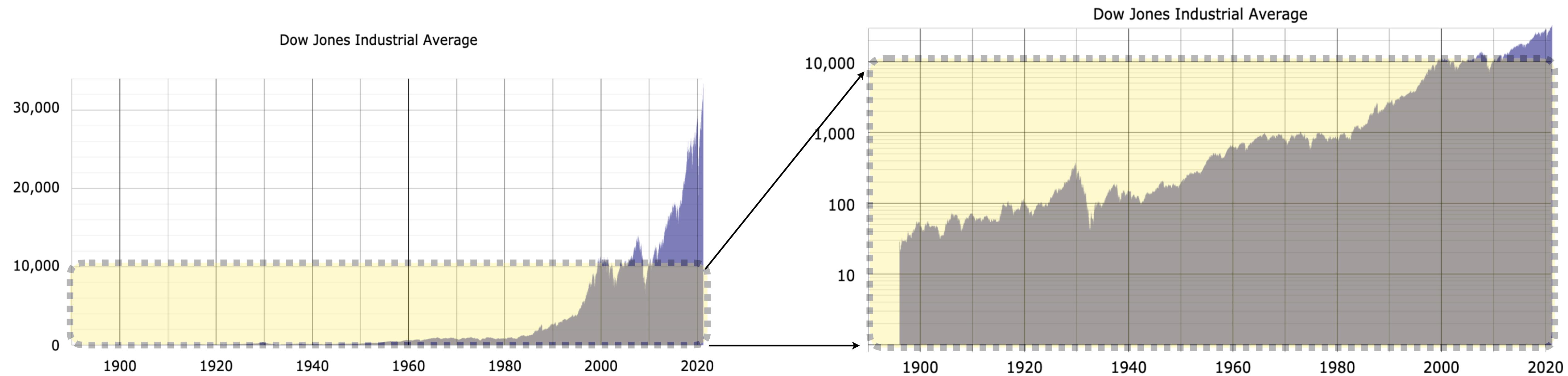
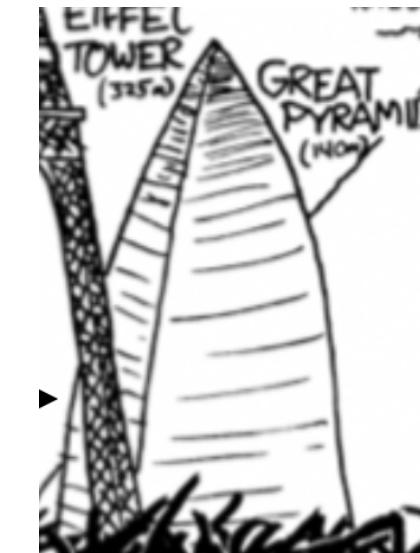


Linear

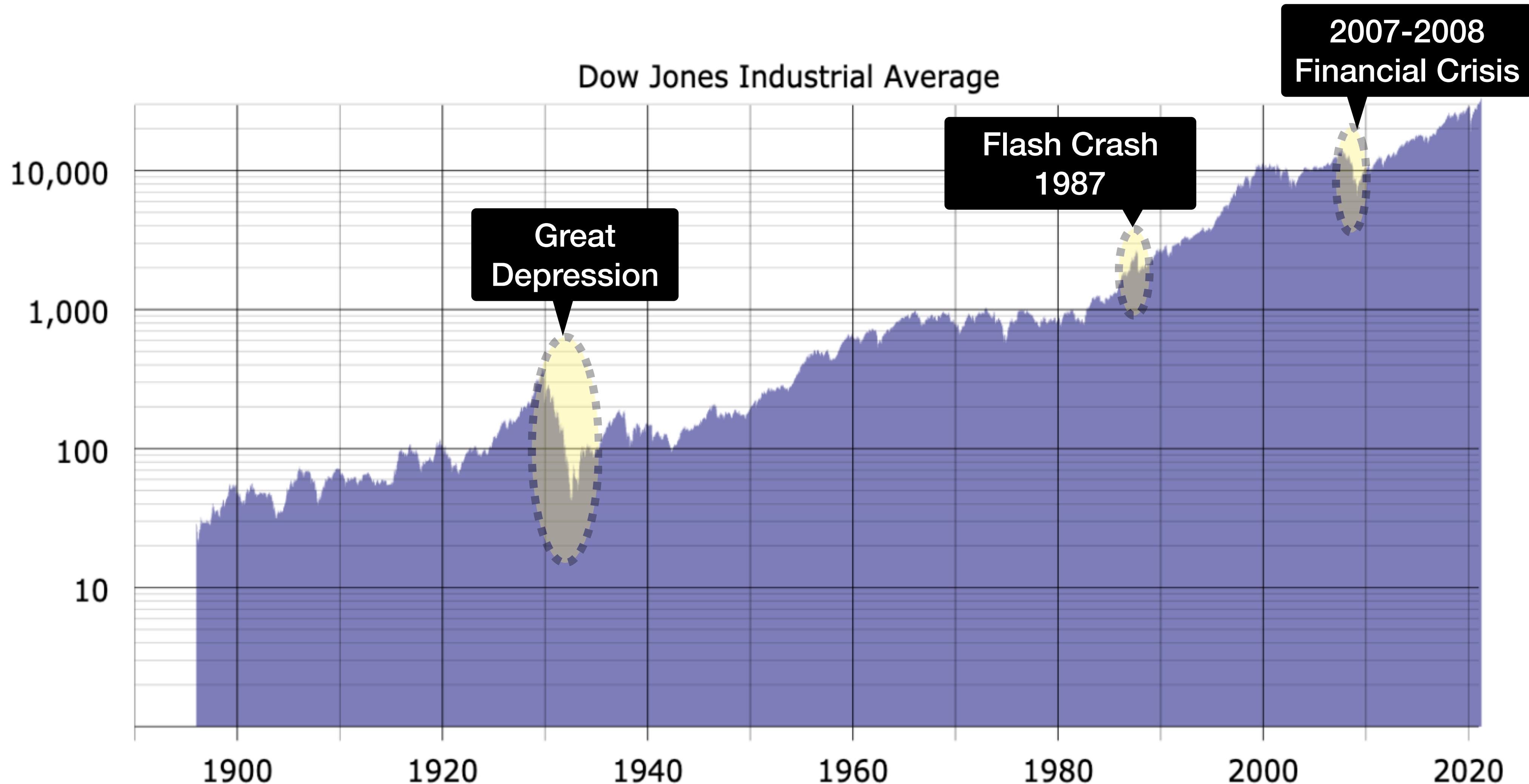


Log scale

Displaying relative change



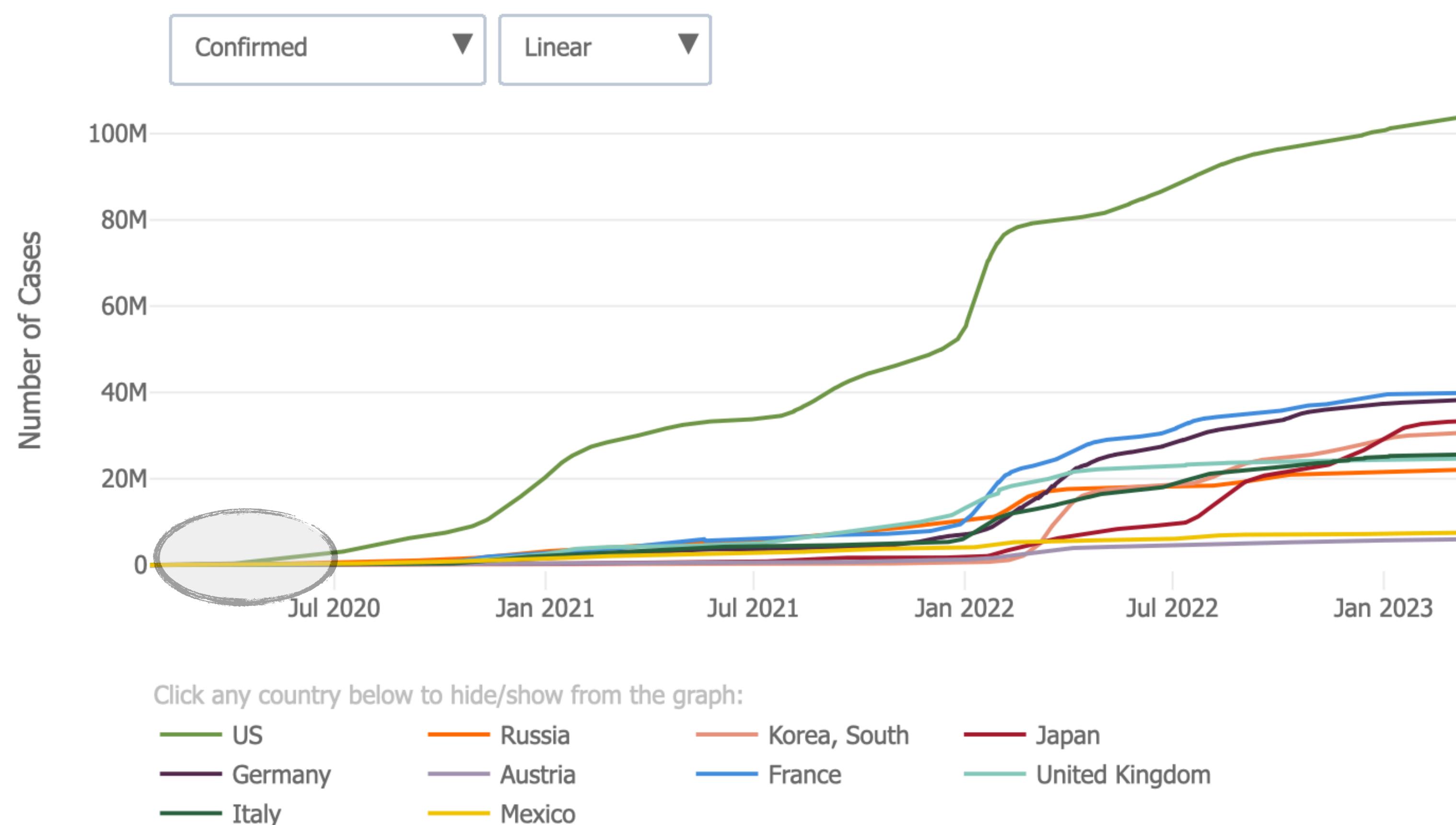
Dow Jones Industrial Average 1900-2021



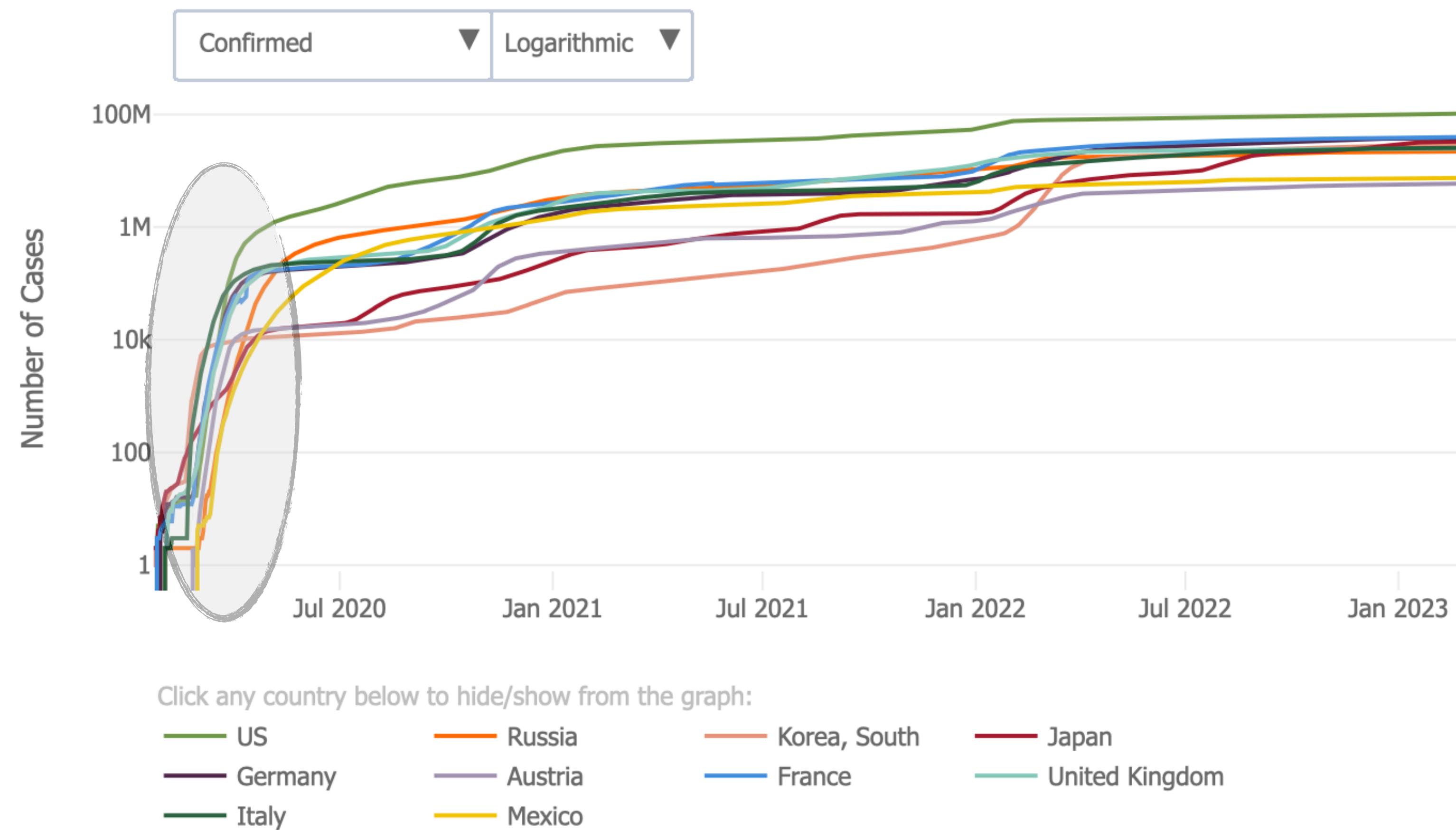
Why logarithmic scale?

- Display data over a very wide range of values in a compact way
- Display relative change instead of absolute quantity of change
- Easier to identify trends like exponential growth

Confirmed Covid-19 cases

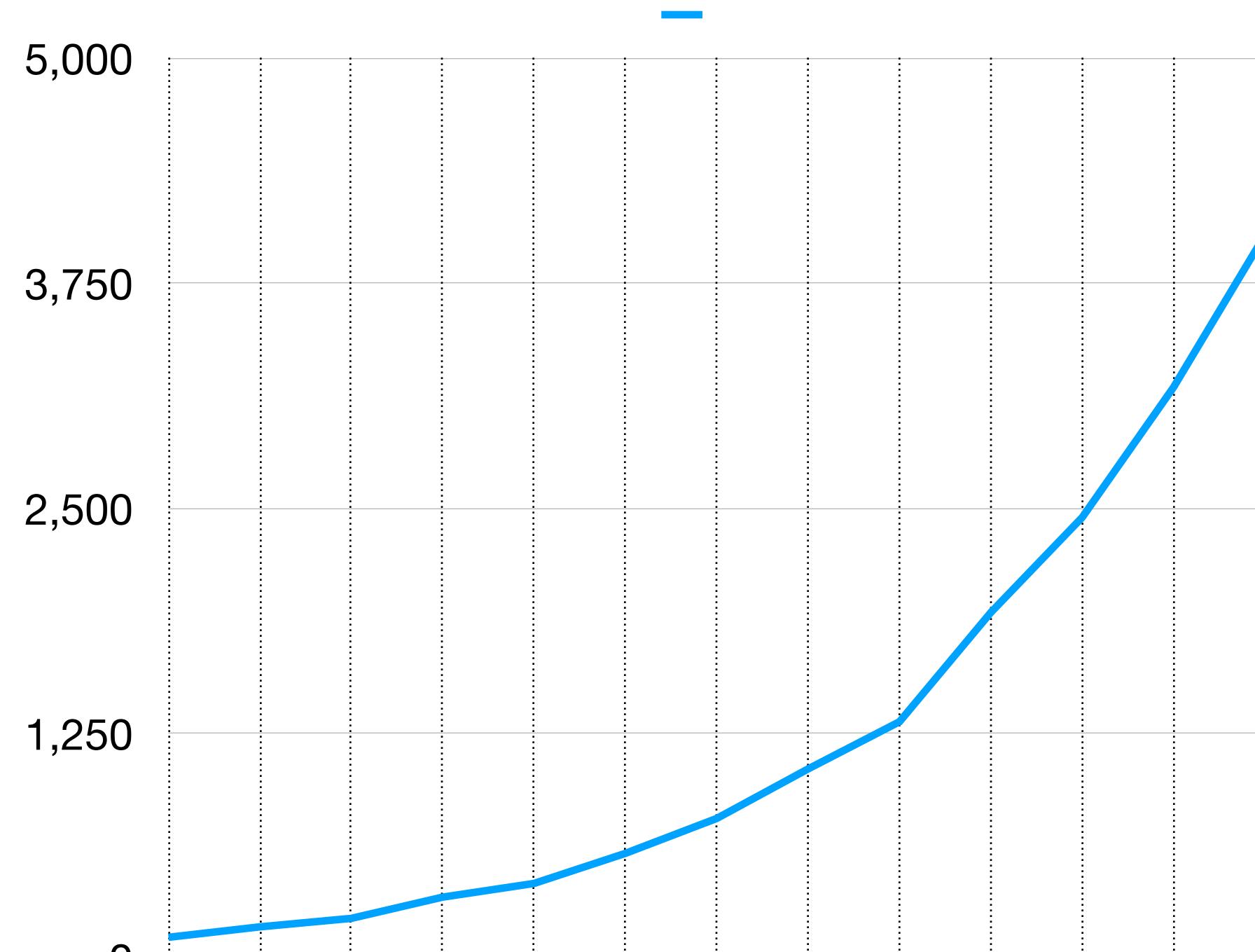


Confirmed Covid-19 cases: logarithmic

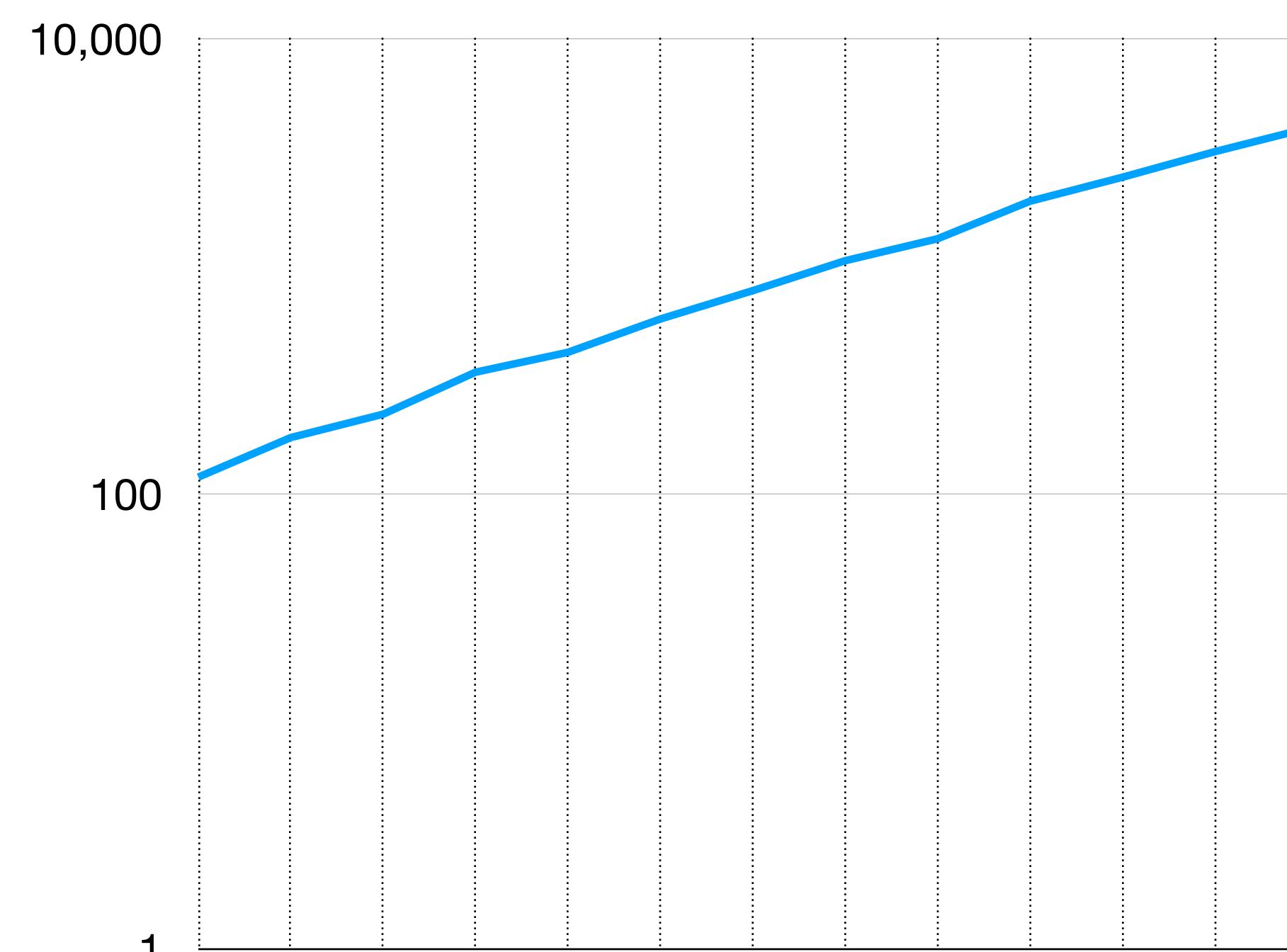


Cases in US: 2020-03-04 to 2020-03-16

Linear



Logarithmic



Date	Cases
2020-03-04	118
2020-03-05	176
2020-03-06	223
2020-03-07	341
2020-03-08	417
2020-03-09	584
2020-03-10	778
2020-03-11	1053
2020-03-12	1315
2020-03-13	1922
2020-03-14	2450
2020-03-15	3173
2020-03-16	4019

Latency (logarithmic heatmap)

Display data over a wide range of values in a compact way

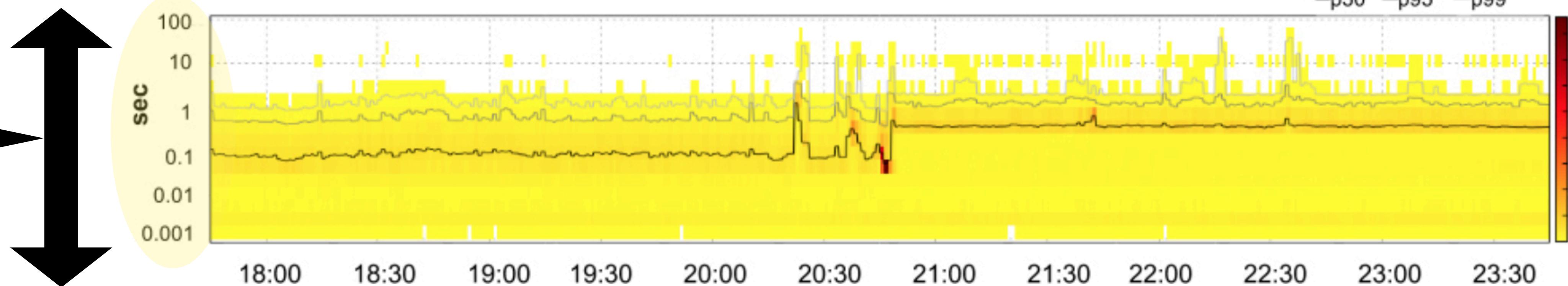


Figure 12-4. Application's latency, showing 50th, 95th, and 99th percentiles (lines) with a heatmap showing how many requests fell into a given latency bucket at any point in time (shade)

How do we describe the shape of data?

- Center of data

Describe the center of the data

Indices of central tendencies

- 1. Mean (average)**

Mean (average)

$$\$80,000 + \$65,000 + \$135,000 + \$120,000 = \$400,000$$

$$\frac{\text{Sum}}{\text{count}} = \frac{\$400,000}{4} = \$100,000$$

Sum → \$400,000 Mean ← \$100,000
count → 4



Fulcrum by
weight of gnomes

\$80,000

\$65,000

\$135,000

\$120,000

Mean (average)



\$80,000	\$65,000	\$135,000	\$120,000	\$7,000,000,000
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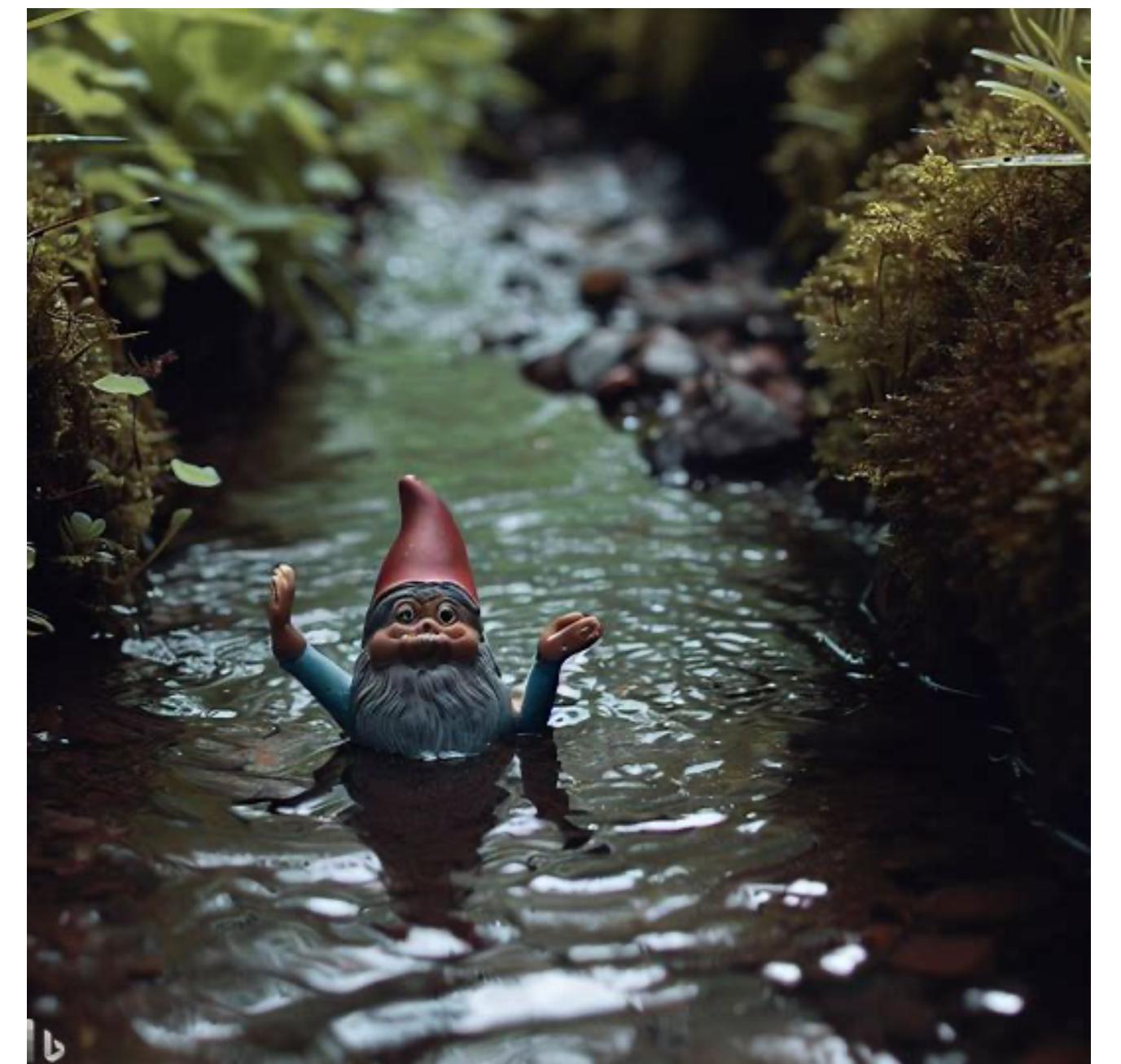


$$\frac{(\$400,000 + \$7,000,000,000)}{5} = \$1,400,080,000$$

Mean

*“Then there is the man who drowned
crossing a stream with an average depth
of six inches.”*

- W. I. E. Gates



Average limitations

- Averages hide the outliers.
- Outliers **skew** averages.
- Averages are not enough alone to describe the data!



Describing the center of the data

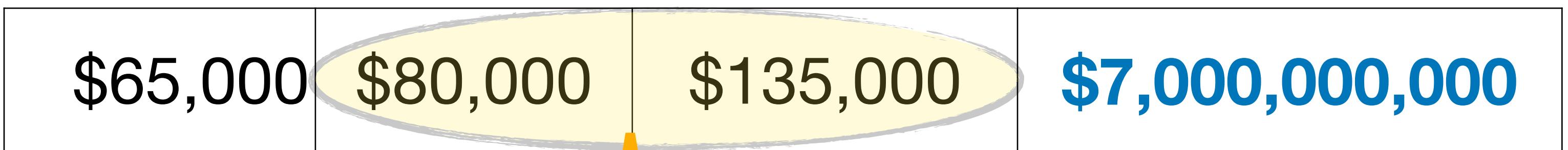
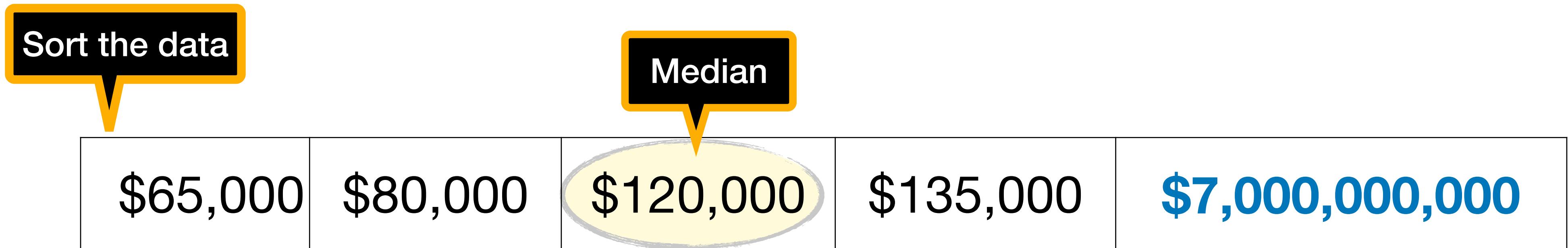
Indices of central tendencies

1. Mean

2. Median



Median



$$\text{Median} = (\$80,000 + \$135,000) / 2 = \$107,500$$

NFL players salaries



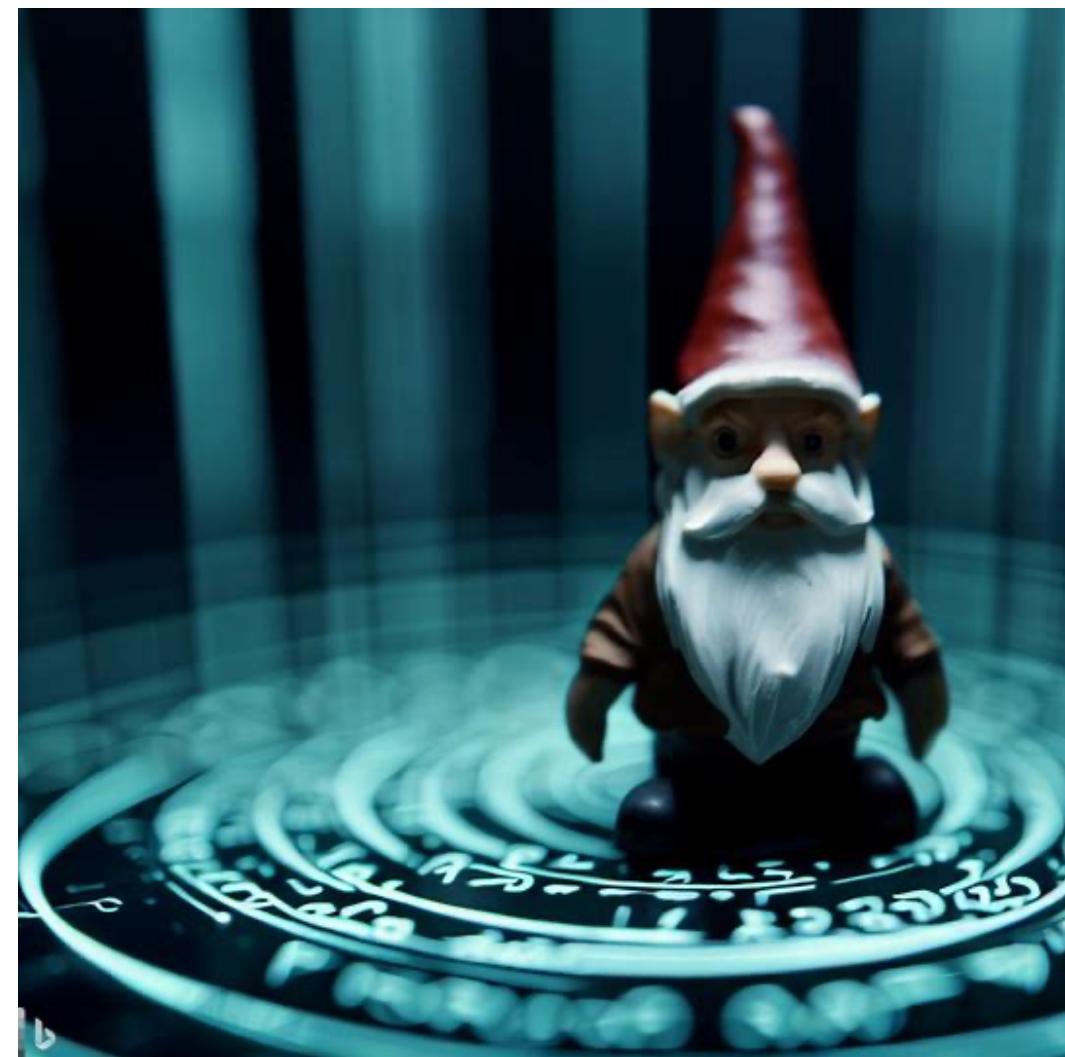
Mean: \$2,000,000

Median: \$860,000

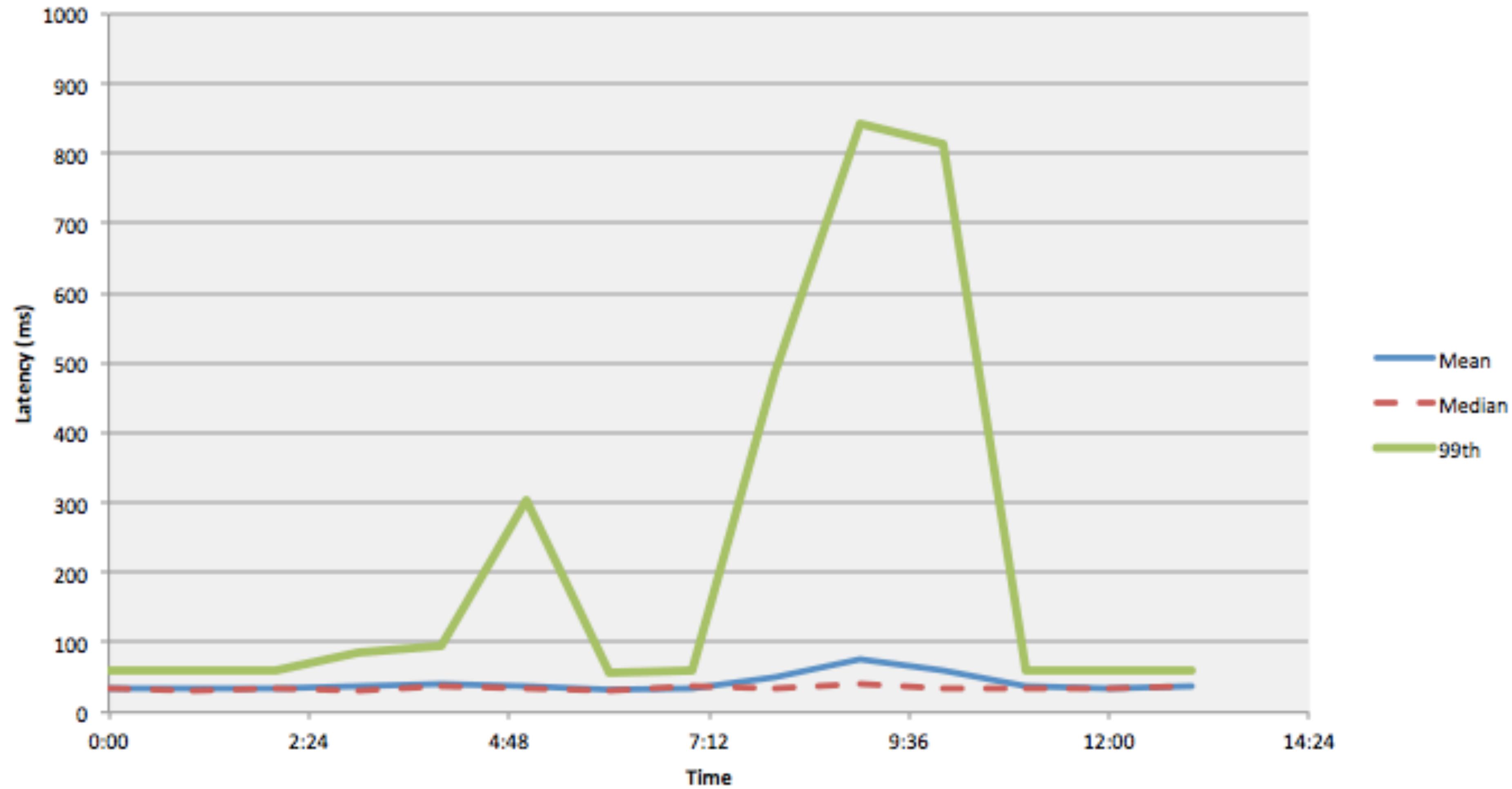
Summarizing the center of the data

Indices of central tendencies

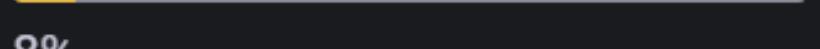
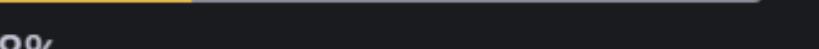
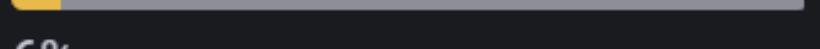
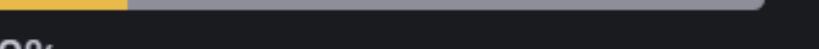
1. Mean or average
2. Median
3. Mode - when it's unique, this is the value that appears the most often in a data set



Mean, median latency



Average CPU usage

Cluster	Provider	Nodes	Average CPU usage past 6h	Max CPU usage (6h)	Average memory usage past 6h	Max memory usage ...
se-demo-cluster	gce	9	 47%	49.65%	 27%	27.82%
opentelemetry-sedemo	gce	3	 8%	32.50%	 28%	32.03%
opentelemetry-demo	gce	3	 6%	7.27%	 20%	21.79%

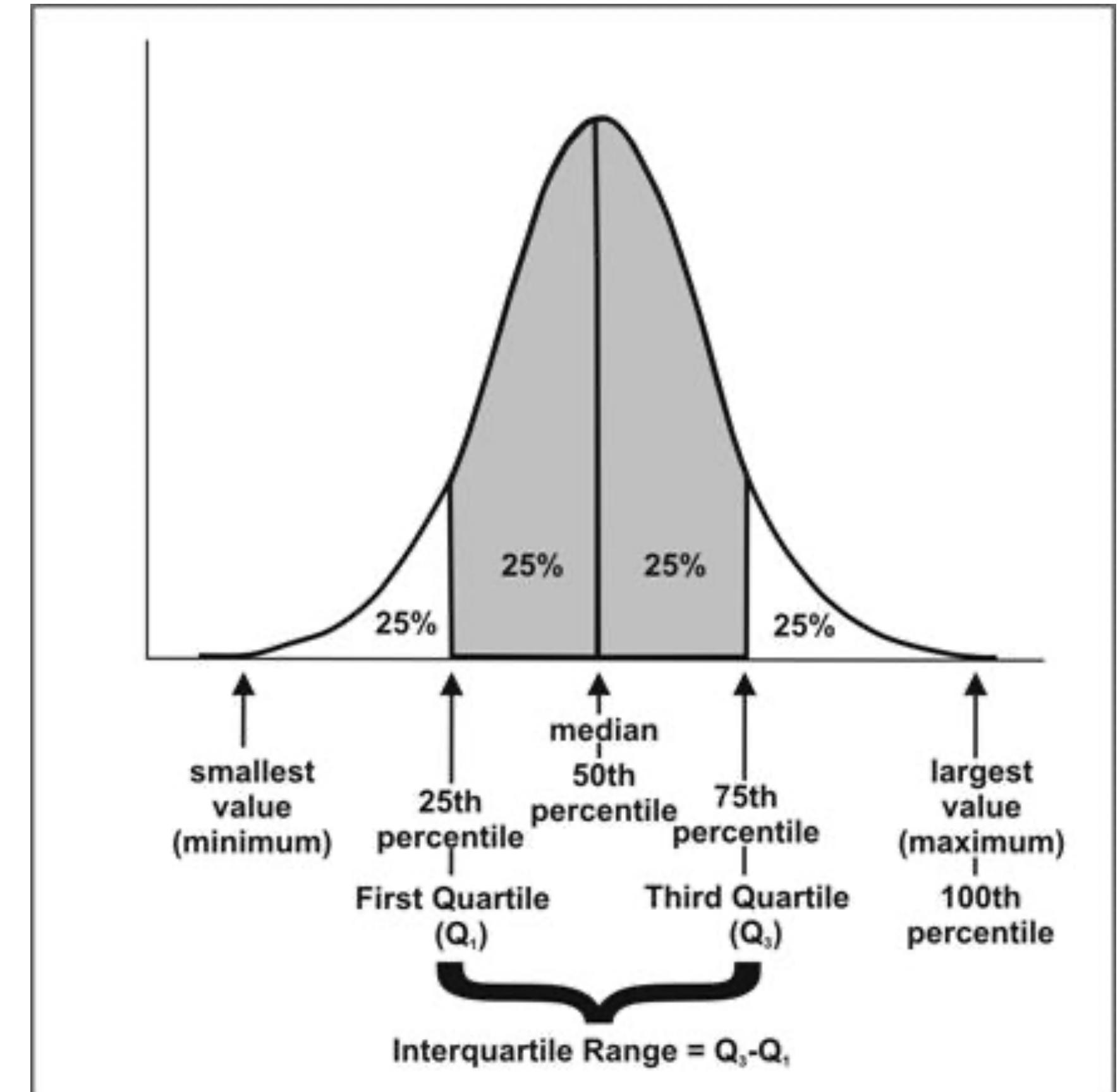
<https://grafana.com/blog/2023/03/03/how-to-optimize-resource-utilization-with-kubernetes-monitoring-for-grafana-cloud/>

How do we measure the spread in data?



How do we measure the spread in data?

1. Range
 - $\max - \min$
2. Inter-quartile range (IQR)
 - Distance between 25th and 75th percentiles
3. Standard deviation



Standard deviation



How concentrated the values are around the mean.

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

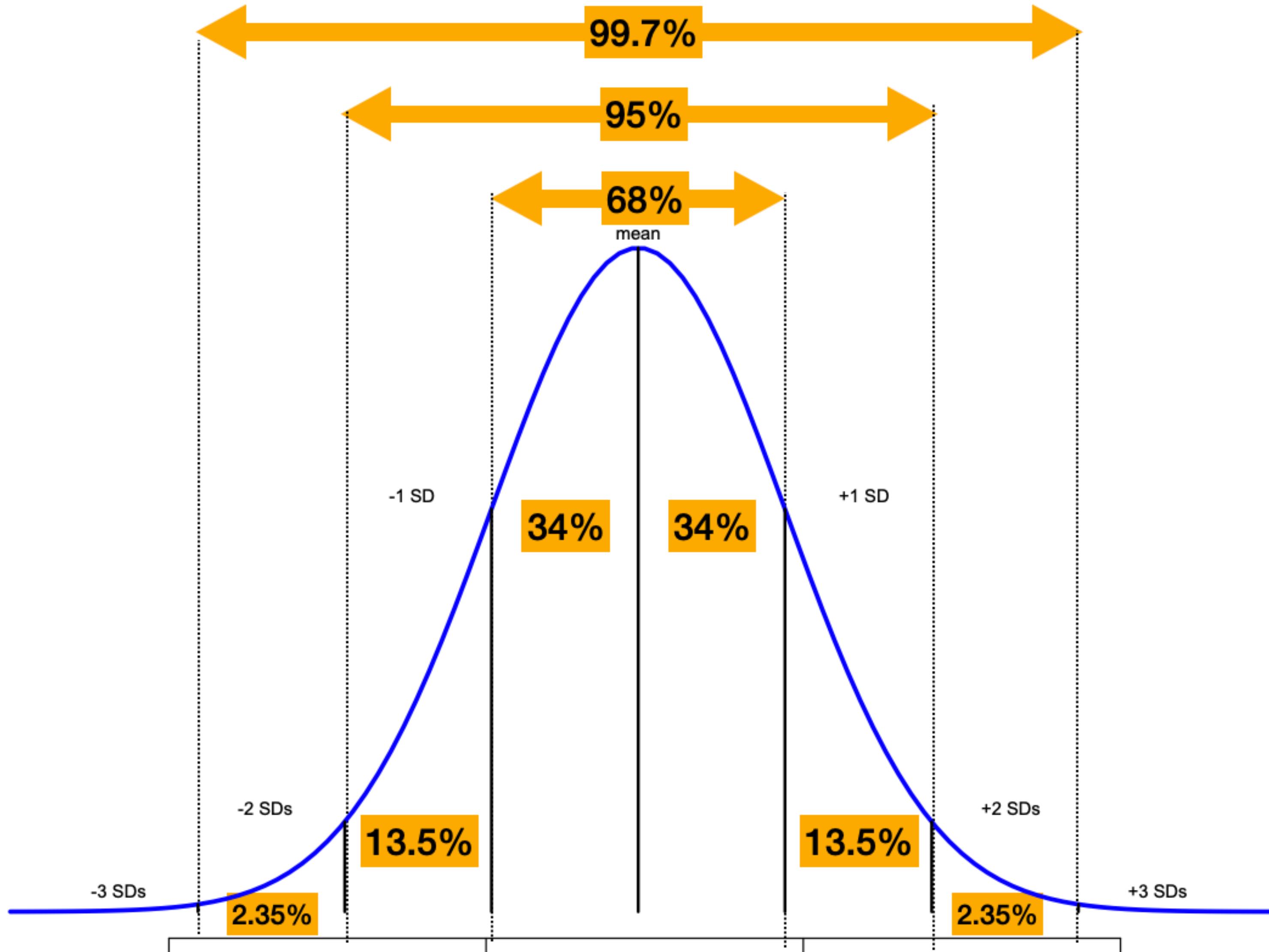
\bar{x} : mean

n: number of values

Variance is the square of standard deviation => s^2

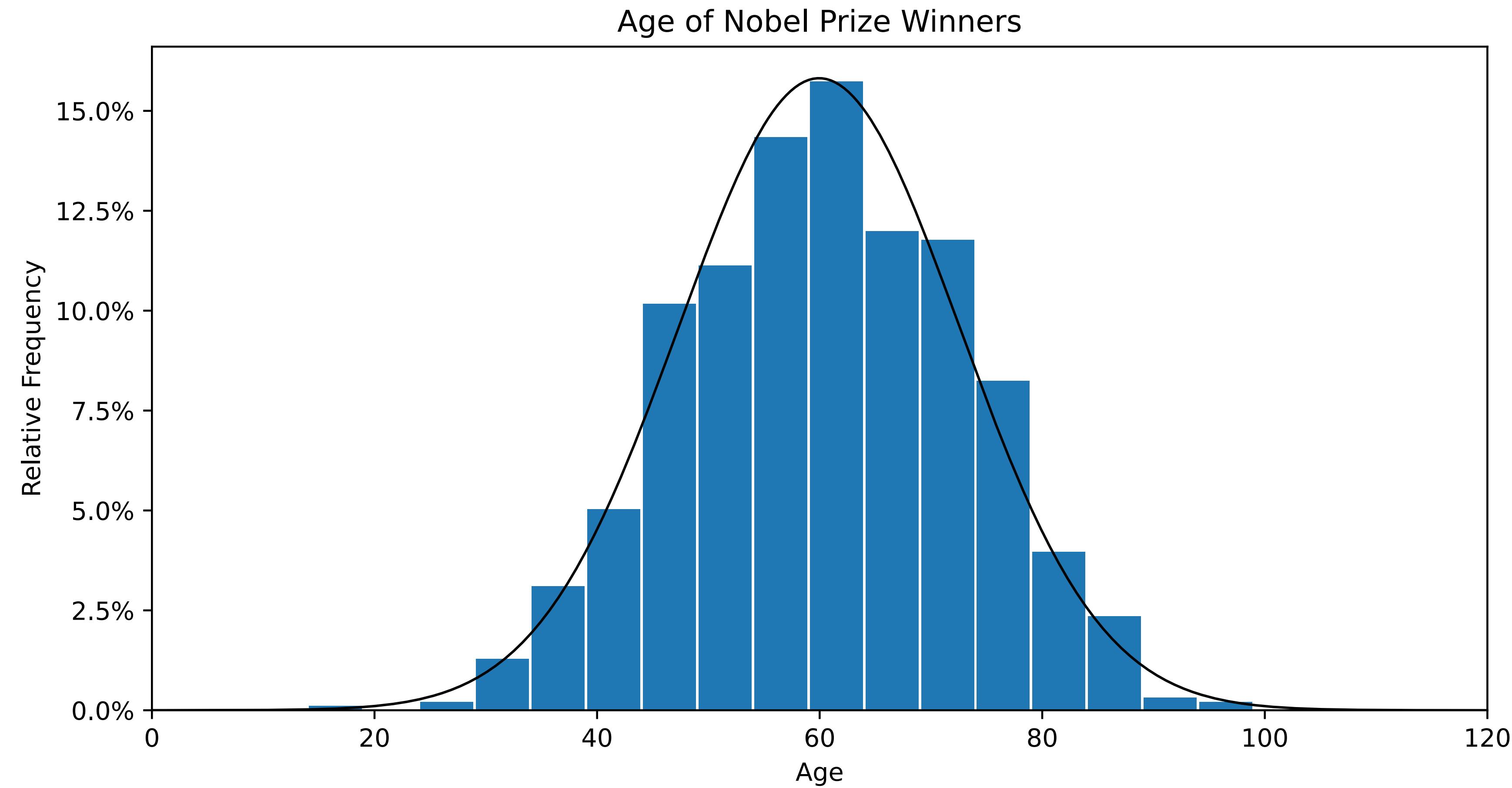
Carl, as a gnome

Empirical rule



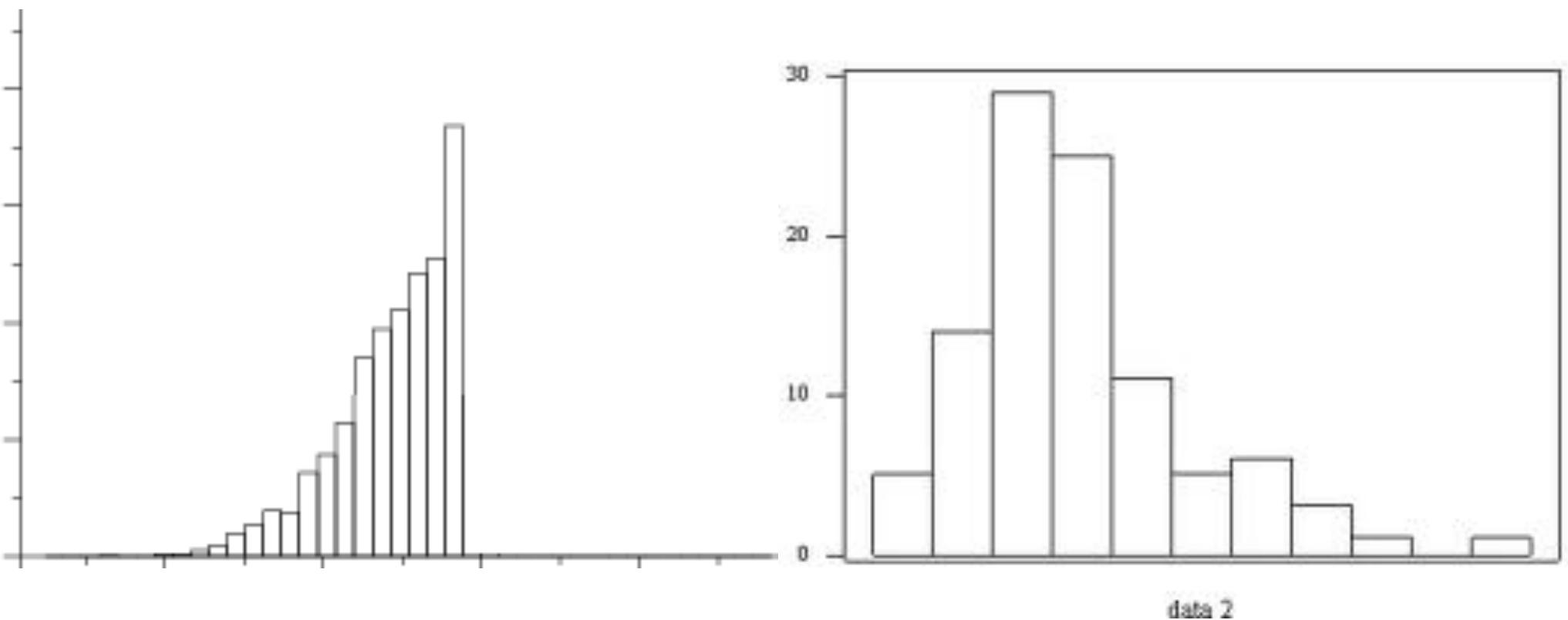
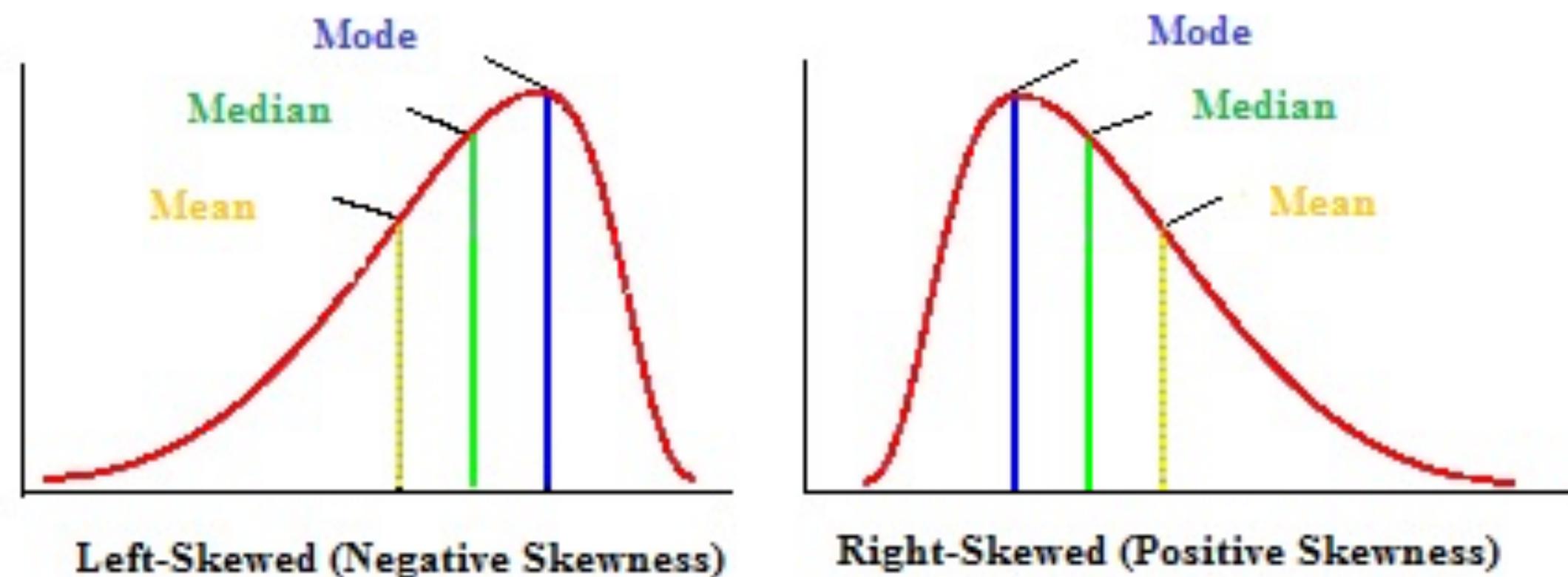
Carl Friedrich Gauss
(1777 - 1855)

Age of Nobel Prize Winners



Histogram of the age of Nobel Prize winners when they won the prize and normal distribution fitted to the data.

Skewed distributions



“We generally prefer to **work with percentiles rather than the mean** (arithmetic average) of a set of values.

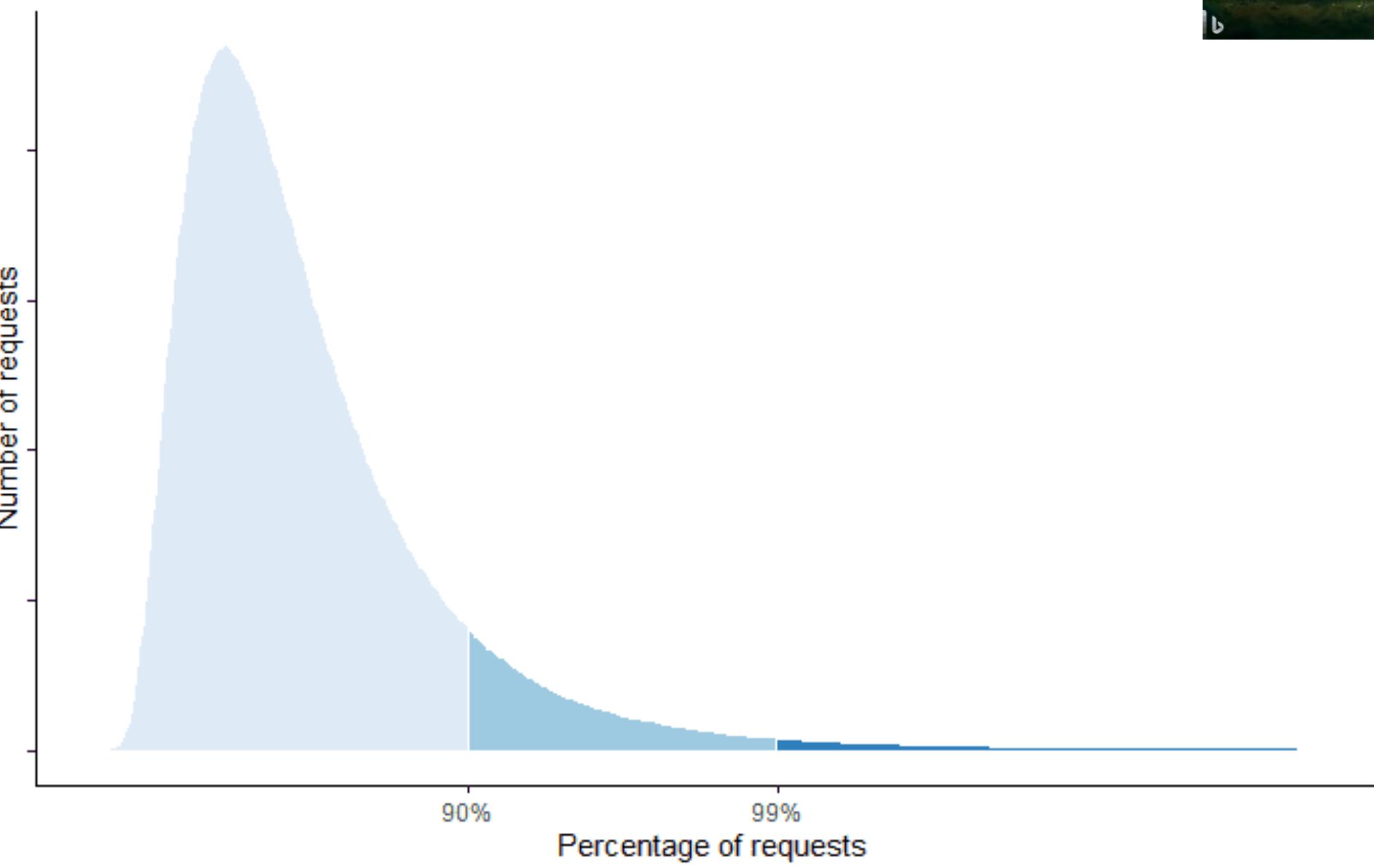
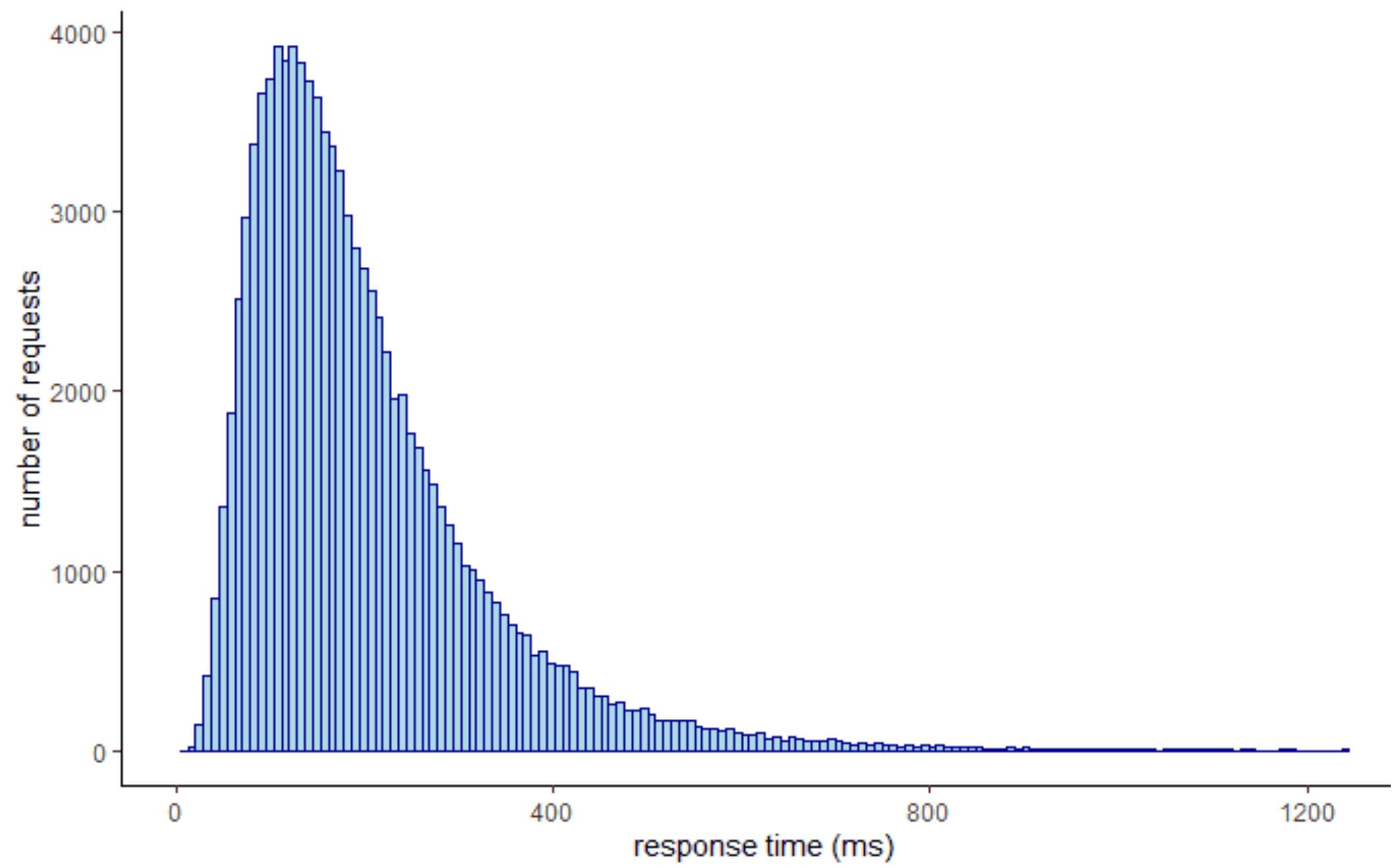
Doing so makes it possible to **consider the long tail of data points**, which **often have significantly different** (and more interesting) **characteristics than the average.**”

<https://sre.google/sre-book/service-level-objectives/>

Stephanie Glen. "Skewed Distribution: Definition, Examples"
From **StatisticsHowTo.com**: Elementary Statistics for the rest
of us!

[https://www.statisticshowto.com/probability-and-statistics/
skewed-distribution/](https://www.statisticshowto.com/probability-and-statistics/skewed-distribution/)

Long tail of latency



<https://robertovitillo.com/why-you-should-measure-tail-latencies/>

How do we measure relative standing



Exam scores => Low mean (hard exam) vs high mean (easy exam)

Percentile analogy

- In a race of 100 people, p99 is the “time” for the 99th person to cross the finish line.

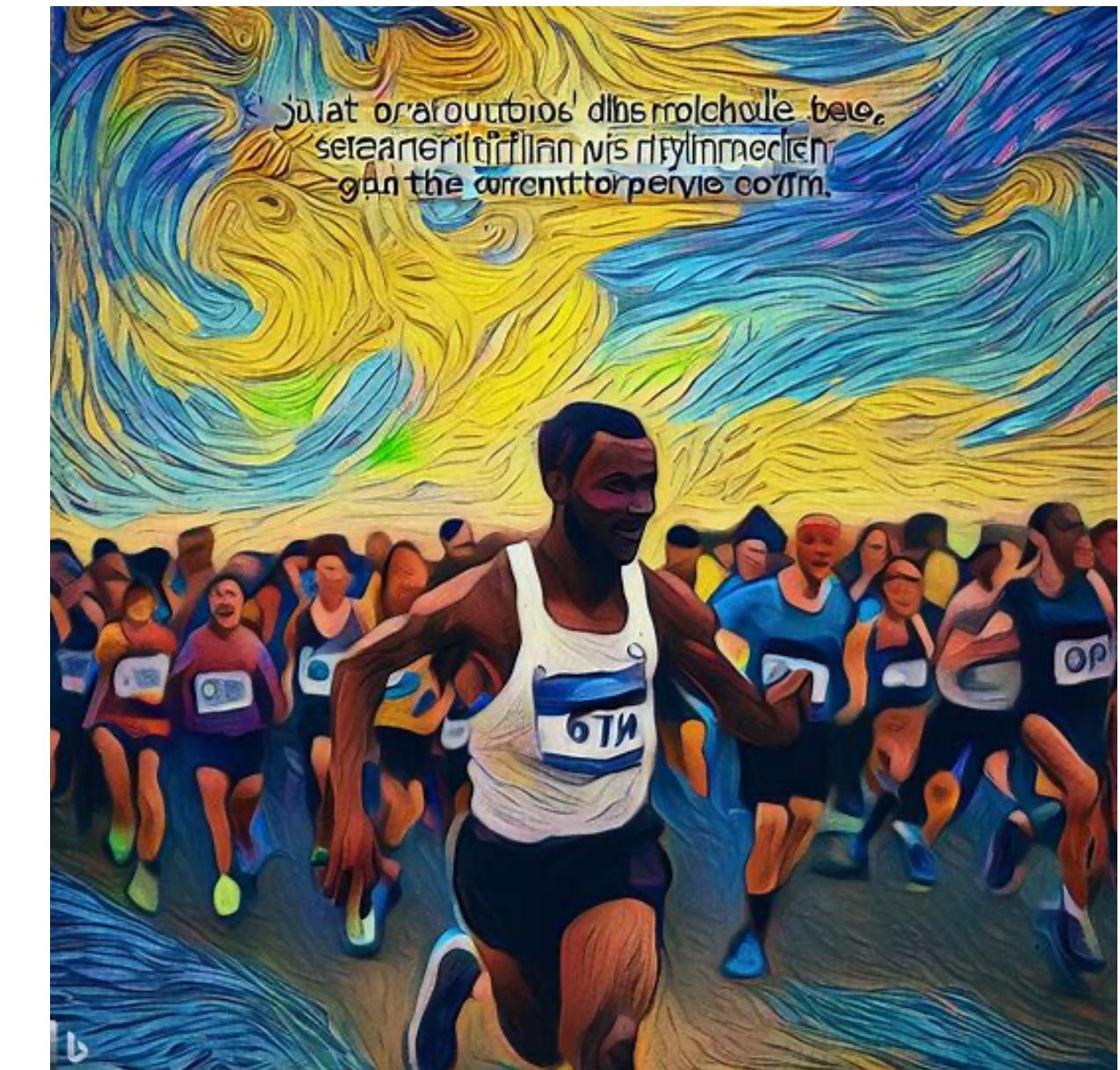
Vishwesh · Aug 26, 2021

p99 = if 100 people running in a race, its time taken for 99th person to cross the finish line.

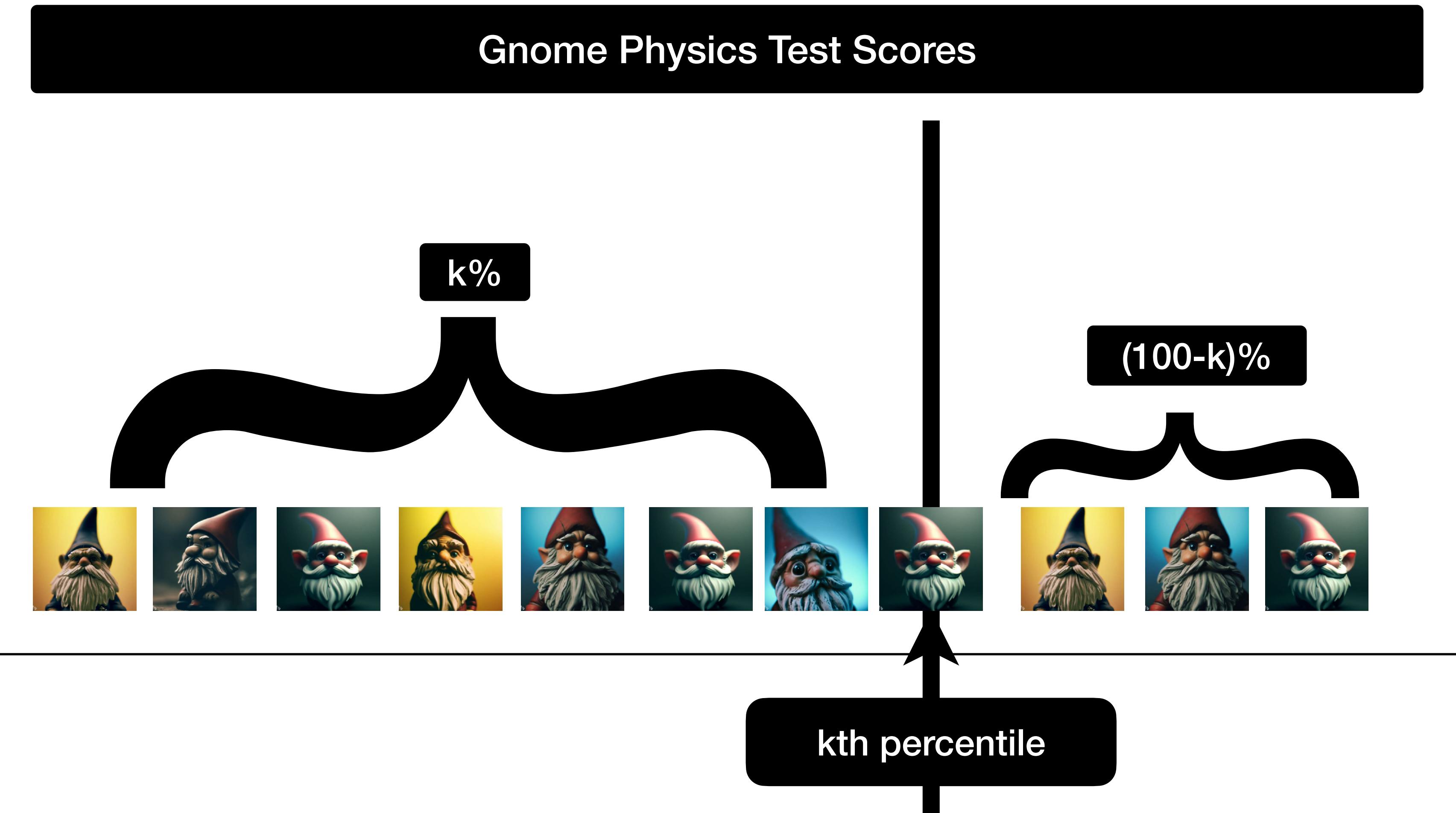
Apply the same thing to all other percentiles!

Q ⤵ 4 ⓘ ⬆

<https://last9.io/blog/your-percentiles-are-incorrect-p99-of-the-times/>

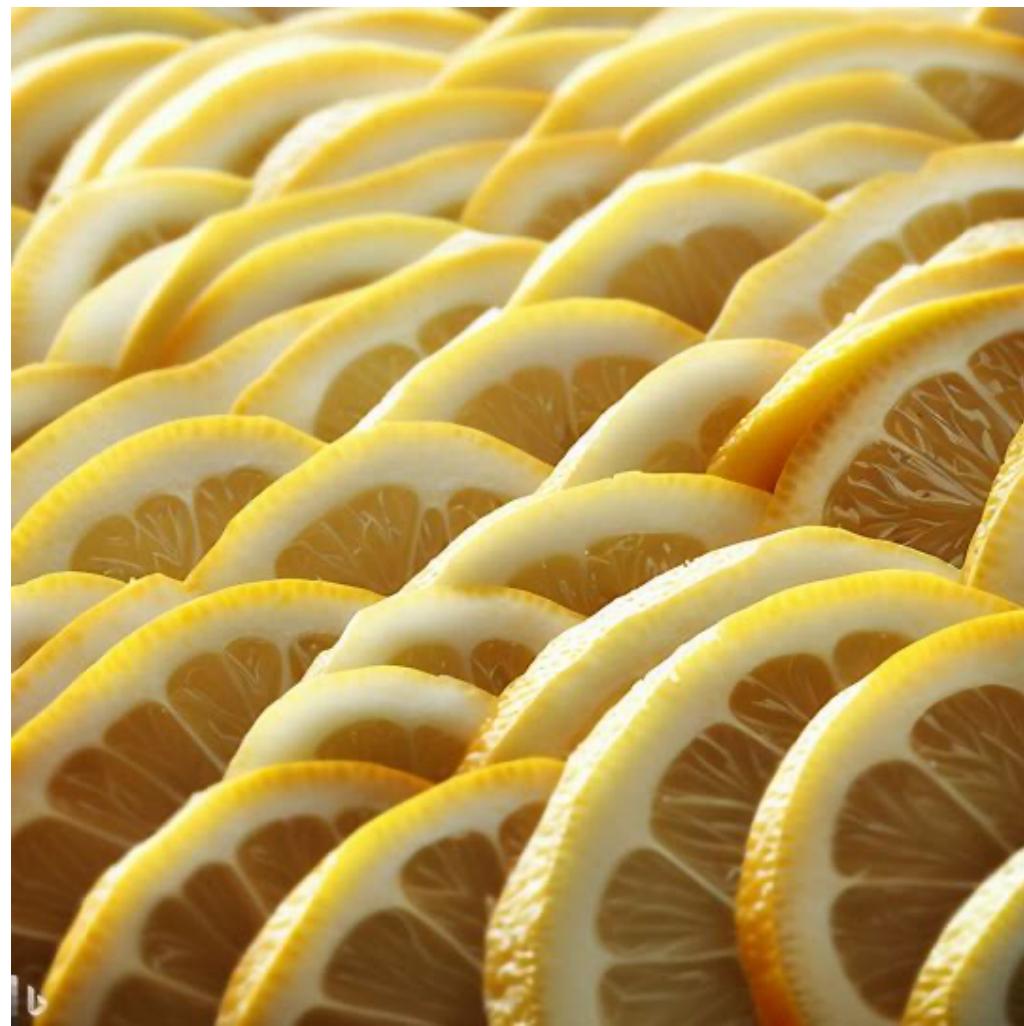


Percentile



Quantile

- 2-quantile: median (p50)
- 4-quantiles: quartiles
 - interquartile range, midspread or middle fifty → $IQR = Q_3 - Q_1$.
- 100-quantiles: percentiles

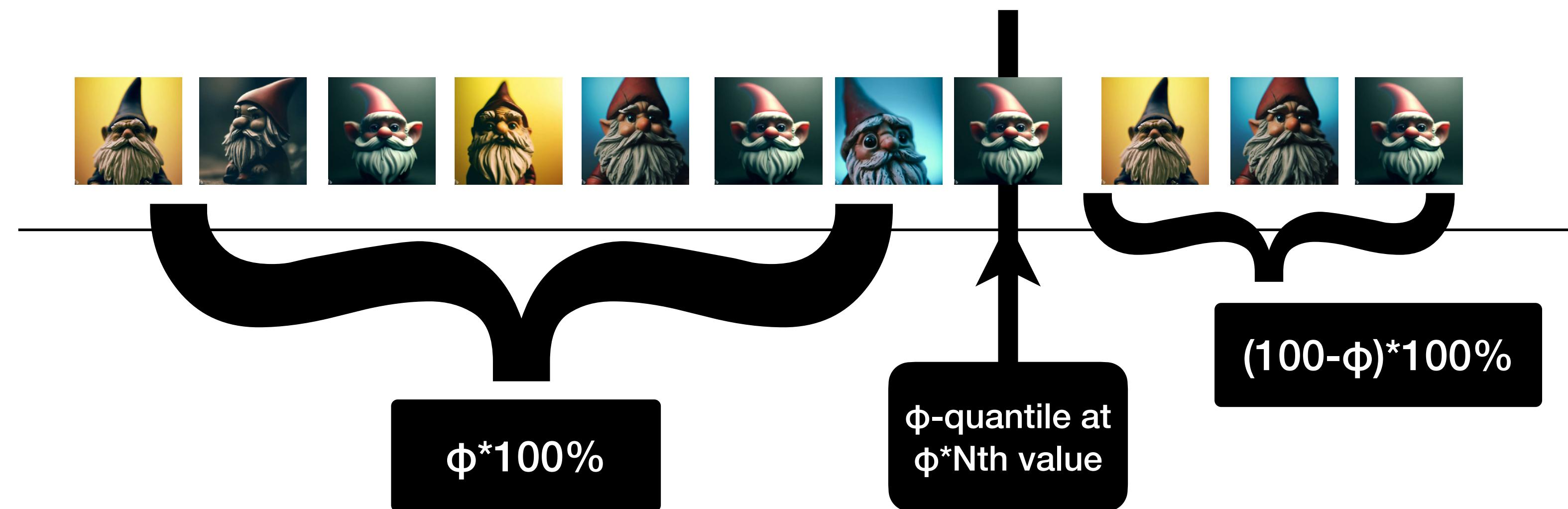


<https://en.wikipedia.org/wiki/Quantile>

Stephanie Glen. "Quantile: Definition and How to Find Them in Easy Steps" From **StatisticsHowTo.com**: Elementary Statistics for the rest of us!
<https://www.statisticshowto.com/quantile-definition-find-easy-steps/>

ϕ -quantile

Gnome Physics Test Scores



How to calculate percentiles?

Naive implementation to find p90

$$0.9 * 13 = 11.7 \Rightarrow 12$$

1	2	3	4	5	6	7	8	9	10	11	12	13
20	34	34	35	40	40	41	54	55	71	73	77	79

10% of scores are higher than 77.

P90
(0.9-quantile)

90% of scores are less than 77.

U.S. household income dispersion



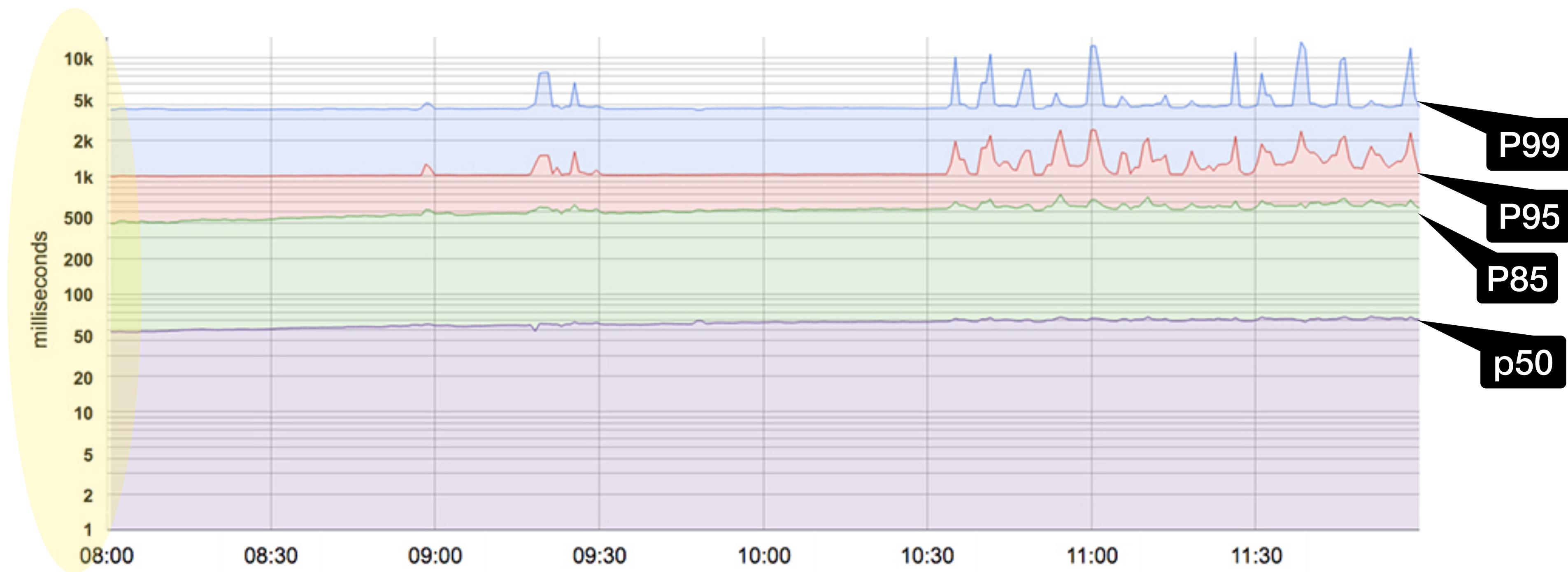
Year	Measures of income dispersion												
	Household income at selected percentiles										Household income ratios at selected percentiles		
	10th percentile limit	20th percentile limit	30th percentile limit	40th percentile limit	50th (median)	60th percentile limit	70th percentile limit	80th percentile limit	90th percentile limit	95th percentile limit	90th/10th	90th/50th	50th/10th
2021.....	15,660	28,007	40,524	55,000	70,784	89,744	113,210	149,131	211,956	286,304	13.53	2.99	4.52
2020 ¹	16,386	28,544	41,763	55,044	71,186	89,534	113,519	148,620	211,438	287,841	12.90	2.97	4.34
2019.....	16,984	29,762	42,815	56,700	72,808	91,657	116,267	151,017	213,171	286,138	12.55	2.93	4.29
2018.....	15,784	27,621	39,924	53,948	68,168	85,823	108,071	140,265	198,844	268,368	12.60	2.92	4.32

Table A-4a at <https://www.census.gov/content/dam/Census/library/publications/2022/demo/p60-276.pdf>

Percentiles have universal interpretation



p50, p85, p95, p99 latency



50th, 85th, 95th, and 99th percentile latencies <https://sre.google/sre-book/service-level-objectives/>, (CC BY-NC-ND 4.0)

How do we calculate percentiles?



To calculate percentiles
across potentially billions of
values

approximate percentiles
are calculated

Approximate percentiles from a stream



Compressed data structures:
t-digest, Greenwald-Khanna,
HDRHistogram, KLL

Approximate
kth percentile
at time t



Key take aways

- Logarithmic scale graphs display wide range of data in a compact way
- Observe the long tail of latency with percentiles
- Averages get skewed by outliers
- Percentiles are most of the time approximated

Thank You

Lerna Ekmekcioglu
<https://www.linkedin.com/in/lerna/>

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