Computación

Científica

Marcos Iturbe

Ejercicio 1

$p=8$

$f(x) = x\,(1-\cos x)$

$\alpha = 0$ $\qquad f'(x) = (1-\cos x) + x\,(\operatorname{sen} x)$

$f(\alpha) = 0$ $\qquad f'(\alpha) = 0$

$f''(x) = \operatorname{sen} x + \operatorname{sen} x + x \cos x$

$f''(\alpha) = 0$

$$x_n \to x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

$x_0 \to 1$

$$x_1 \to 1 - \frac{0{,}45964769}{1{,}30116868} = 0{,}646704$$

$$x_2 \to 0{,}646704 - \frac{0{,}13058625}{0{,}59177669} = 0{,}42603588$$

$$x_3 \to 0{,}42603588 - \frac{0{,}38082866 \times 10^{-1}}{0{,}26545428} = 0{,}28257289$$

$$x_4 \to 0{,}28257289 - \frac{0{,}11206495 \times 10^{-1}}{0{,}11844784} = 0{,}18798607$$

$$x_5 \to 0{,}18798607 - \frac{3{,}3118273 \times 10^{-3}}{0{,}52748300 \times 10^{-1}} = 0{,}12520059$$

$$x_6 \to 0{,}12520059 - \frac{9{,}7991024 \times 10^{-4}}{2{,}3461629 \times 10^{-2}} = 8{,}3430675 \times 10^{-2}$$

$$x_7 \to 8{,}3430675 \times 10^{-2} - \frac{2{,}9019862 \times 10^{-4}}{1{,}04309\,26 \times 10^{-2}} = 5{,}5609692 \times 10^{-2}$$

$$\varepsilon_6/\varepsilon_5 = \frac{-4{,}1709915 \times 10^{-2}}{-6{,}278548 \times 10^{-2}} = 0{,}66432422$$

$$\varepsilon_7/\varepsilon_6 = \frac{-2{,}7820983 \times 10^{-2}}{-4{,}1709915 \times 10^{-2}} = 0{,}66701126 1$$

$cte \approx 0{,}66$

$$\frac{\varepsilon_{k+1}}{\varepsilon_k} \approx \frac{m-1}{m} \approx 0{,}66 \implies m = 3$$

* Newton modificado:

$$x_{n+1} = x_n - m \frac{f(x)}{f'(x)}$$

$x_0 \leftarrow 1$

$$x_1 \leftarrow 1 - 3 \frac{0{,}45969769}{1{,}3011686 8} = -0{,}059888$$

$$x_2 \leftarrow -0{,}05988 - 3 \frac{-1{,}0736423 \times 10^{-4}}{5{,}3757436 \times 10^{-3}} = \boxed{3{,}5932374 \times 10^{-5}}$$

Ejercicio 2

Distancia recorrida entre el s = 11 y s = 16

t (s)	v (ms^{-1})
0	0
10	227,04
15	362,78
20	517,35
22,5	602,97
30	901,67

$$\int_{11}^{16} v(t)\, dt$$

$$v(t) \approx P_2(t) = 12,05 + 17,733t + 0,3766t^2$$

$$v(t) \approx P_3(t) = -4,381 + 21,289t + 0,13065t^2 + 0,005466t^3$$

$$\int_{11}^{16} P_2(t)\, dt = 12,05t + \frac{17,733t^2}{2} + \frac{0,3766t^3}{3} \Big|_{11}^{16} =$$

$$\boxed{1604,327167 \text{ m}}$$

$$\int_{11}^{16} P_3(t)\, dt = -4,381t + \frac{21,289t^2}{2} + \frac{0,13065t^3}{3} + \frac{0,005466t^4}{4} \Big|_{11}^{16}$$

$$= \boxed{1605,066268 \text{ m}}$$

aceleración en $t = 16$

$$a = \frac{dv}{dt} \qquad v(t) \approx f_{2,3}(t)$$

$$f_2'(t) = 17,733 + (0,3766 t \times 2)$$

$$f_2'(16) = \boxed{29,7842 \text{ m/s}^2}$$

$$f_3'(t) = 21,289 + 2 \cdot 0,13065t + 3 \cdot 0,005466t^2$$

$$f_3'(16) = \boxed{29,667688 \text{ m/s}^2}$$

Ejercicio 3

$$\sum_{i=1}^{n} i^3 = \boxed{\varphi(n) \equiv 1 + 7(n-1) + 6(n-1)(n-2) + (n-1)(n-2)(n-3)}$$

n	$\varphi(n)$				
1	1				
2	8	7			
3	27	19	6		
4	64	37	9	1	
5	125	61	12	1	0
6	216	91	15	1	0

Ejercicio 4

$$I(f) = \int_0^1 \sqrt{1+2t^2}\, dt \qquad h = \tfrac{1}{2}$$

$$S_1 = \frac{h}{3}\left(f(0) + f(1) + 4f(\tfrac{1}{2})\right) = \frac{h}{3}\left[1 + \sqrt{3} + 4\sqrt{\tfrac{3}{2}}\right] = 1,271838382$$

$h = \frac{1}{4}$

$$S_2 = \frac{h}{3}\left(f(0)+f(1) + 4\left[f(\tfrac{1}{4})+f(\tfrac{3}{4})\right] + 2f(\tfrac{1}{2})\right) =$$

$$\frac{h}{3}\left[\sqrt{3}+1 + 4\left(\frac{3\sqrt{2}}{4} + \frac{\sqrt{34}}{4}\right) + 2\sqrt{\tfrac{3}{2}}\right] = 1,271261094$$

$h = \frac{1}{8}$

$$S_4 = \frac{h}{3}\left[f(0)+f(1) + 4\left(f(\tfrac{1}{8})+f(\tfrac{3}{8})+f(\tfrac{5}{8})+f(\tfrac{7}{8})\right) + 2\left(f(\tfrac{1}{2}) + f(\tfrac{1}{4}) + f(\tfrac{3}{4})\right)\right] = \frac{h}{3}\left[1+\sqrt{3} + 4\left(\frac{\sqrt{66}}{8} + \frac{\sqrt{82}}{8} + \frac{\sqrt{114}}{8} + \frac{9\sqrt{2}}{8}\right) + 2\left(\sqrt{\tfrac{3}{2}} + \frac{3\sqrt{2}}{4} + \frac{\sqrt{34}}{4}\right)\right] = 1,271272865$$

$0 \quad \frac{1}{8} \quad \frac{1}{4} \quad \frac{3}{8} \quad \frac{1}{2} \quad \frac{5}{8} \quad \frac{3}{4} \quad \frac{7}{8} \quad 1$

$S_1 = 1,271838382$

$S_2 = 1,271261094$

$S_4 = 1,271272865$

$$S_{n1} = S_2 + \frac{S_2 - S_1}{4^2 - 1} = 1,271222608$$

$-3,84 \times 10^{-5}$

$S_{n2} = 1,27127364$

$7,8411 \times 10^{-7}$

$$S_{n3} = S_{n2} + \frac{S_{n2} - S_{n1}}{4^3 - 1} =$$

$\boxed{\varepsilon \simeq 8,1 \times 10^{-7}}$

$\boxed{1,27127445}$

Ejercicio 5

$$\int_1^3 \frac{e^x}{x}\,dx \simeq 8{,}038733067$$

$h = 1$

$$S_1 = \frac{h}{3}\left[f(1) + f(3) + 4f(2)\right] = \frac{1}{3}\left[2{,}718281828 + 6{,}695178174\right.$$

$$\left. + 4 \times (3{,}694528049)\right] = 8{,}063857667$$

$$S_2 = 8{,}040946349$$

$$S_n = S_2 + \frac{S_2 - S_1}{4^2 - 1} = 8{,}039418928$$

$-1{,}5274 \times 10^{-3} \longrightarrow$ tendra 4 cifra significativas

Ejercicio 6

$$I(f) = \int_0^2 (x-1)\, e^{-x}\, dx = -\int_0^1 (x-1)\, e^{-x}\, dx + \int_1^2 (x-1) e^{-x}\, dx$$

$$h = \frac{1}{2}$$

$$I(f) \simeq -\frac{1}{6}\left(-1 + 0 + 4 \times \left(-0,5\, e^{-0,5}\right)\right) + \frac{1}{6}\left(0 + e^{-2} + 4\left(0,5 e^{-1,5}\right)\right)$$

$$= \boxed{0,4657761538}$$