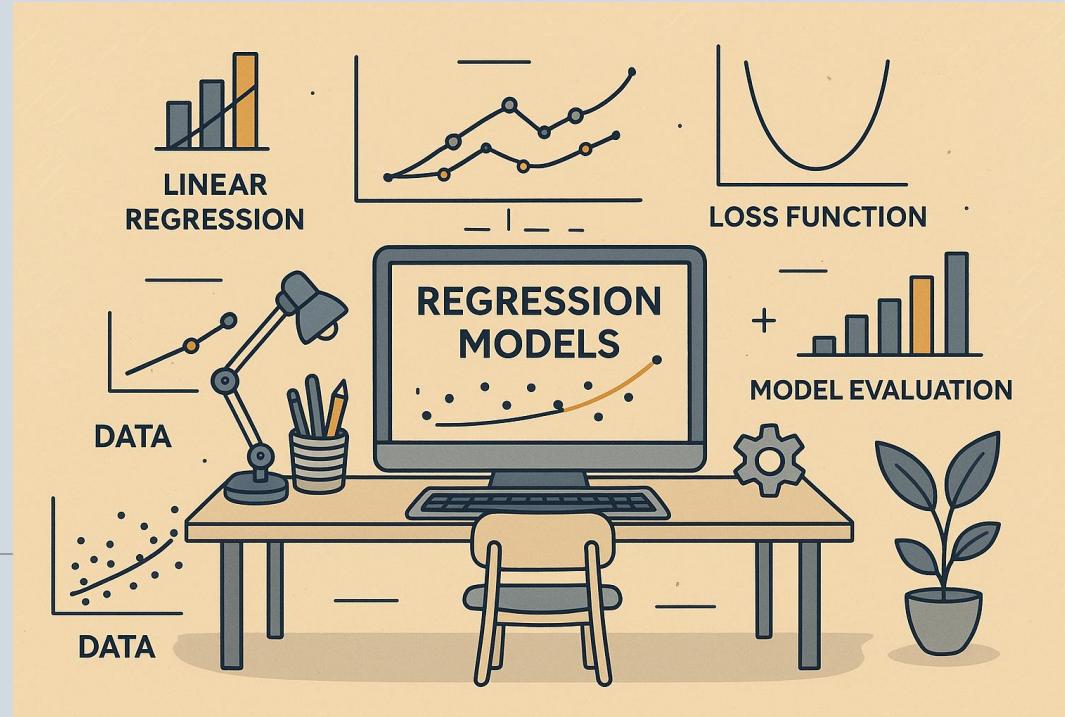
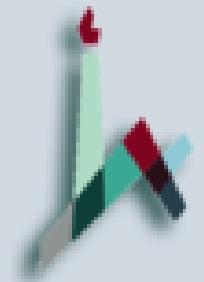


Regression Models



DR. ZVI BEN AMI



Agenda

- Fitting a Model to Data
- Modeling types
- Linear regression
- Curve fitting
- Gradient Descent
- Regularization
- Logistic regression

Fitting a Model to Data

- We build (train) our model using historical data.
- We want to find the “optimal” model parameters given our training data.
- In practice, we need to find the hyperparameters that will predict our target variable as closely as possible, using our available features.
- The process of finding such hyperparameters or patterns in our data is called “fitting”.
- `model.fit(features,target)`

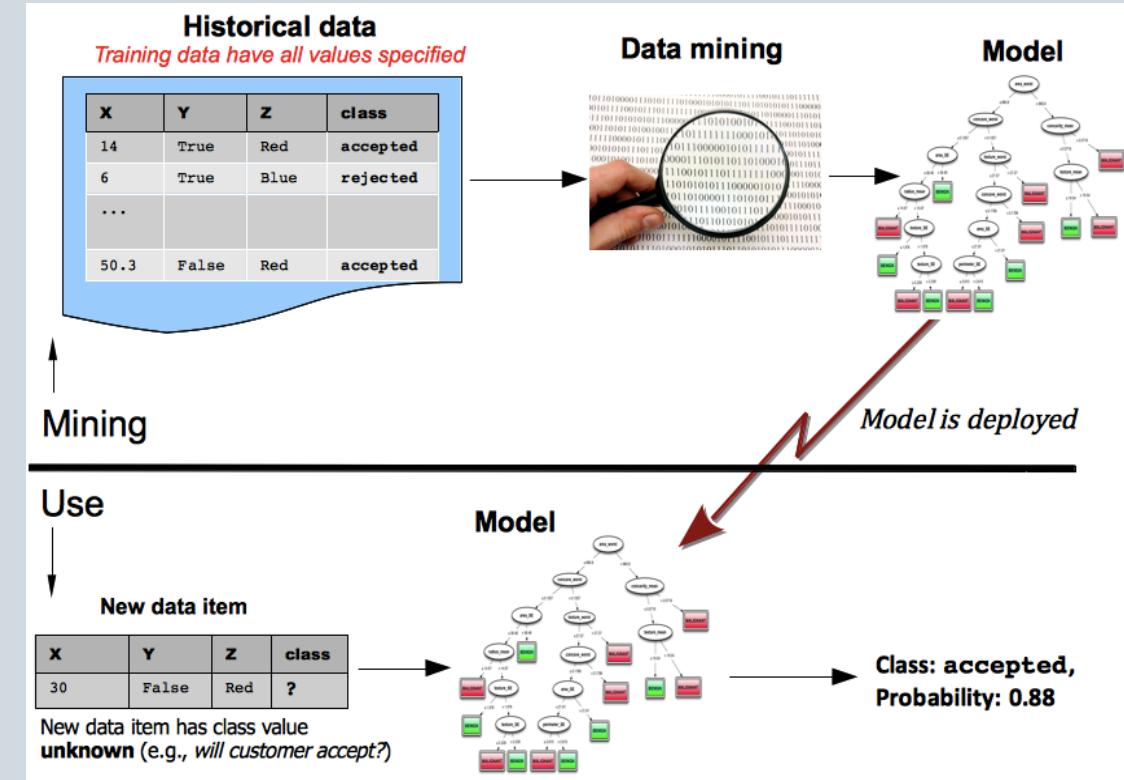


Image from “Data Science for Business”, Provost, Fawcett, 2013

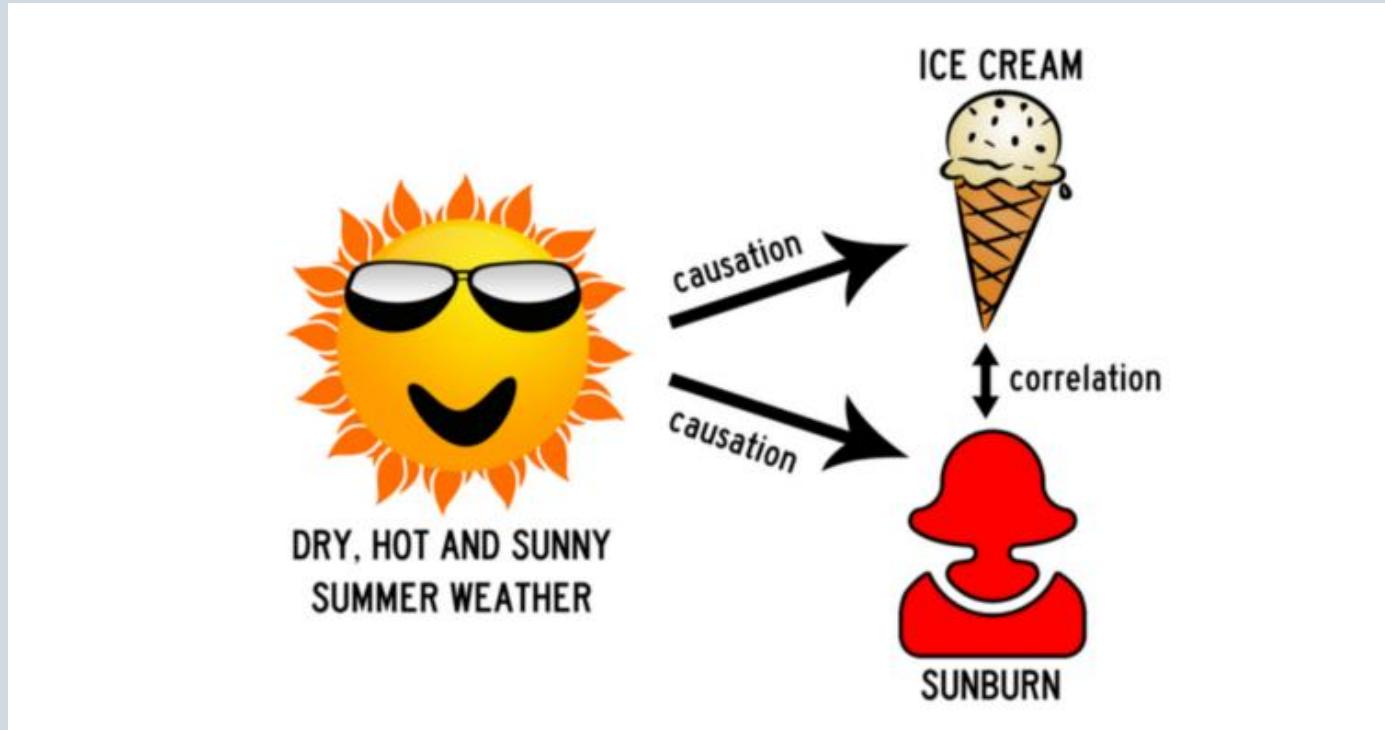
What is the Purpose of our Model?

- Descriptive Modeling:
 - Summarize or represent the data (EDA)
- Explanatory Modeling:
 - Test causal hypotheses
- Predictive Modeling:
 - Predict new/future observations

Some references:

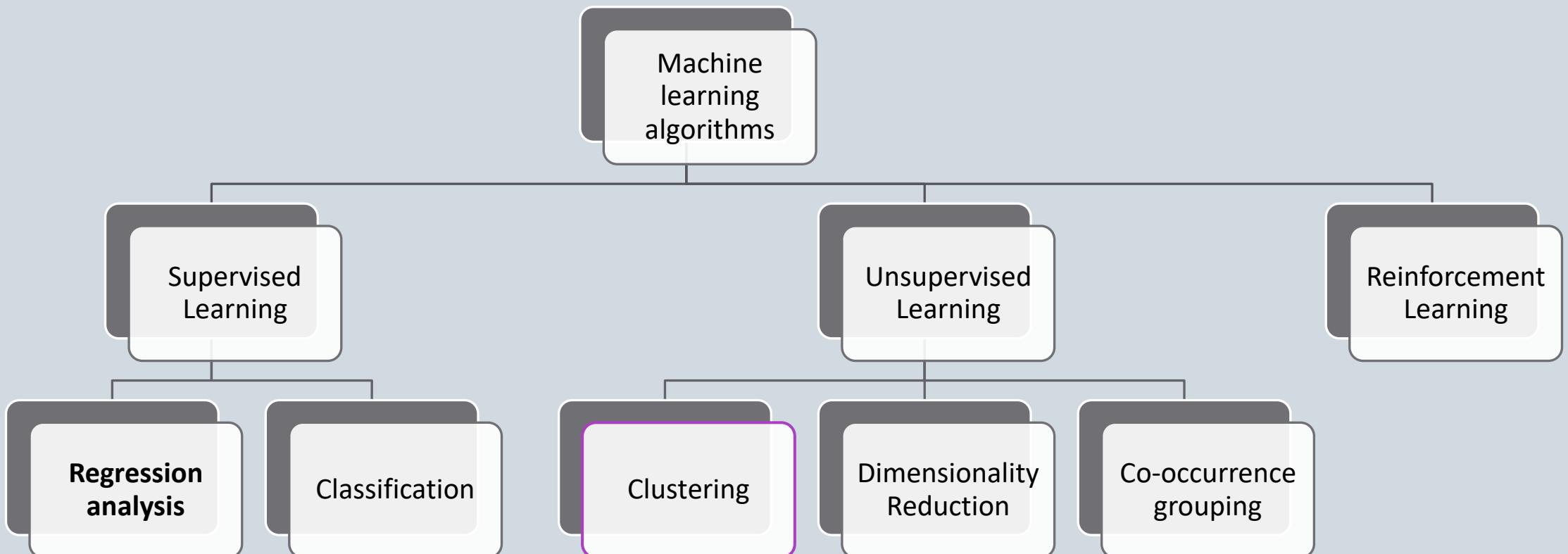
- <https://repub.eur.nl/pub/25837/63604908.pdf>
- <https://www.stat.berkeley.edu/~aldous/157/Papers/shmueli.pdf>
- <https://onlinelibrary.wiley.com/doi/abs/10.1016/j.pmrj.2014.08.941>

Correlation vs. Causality



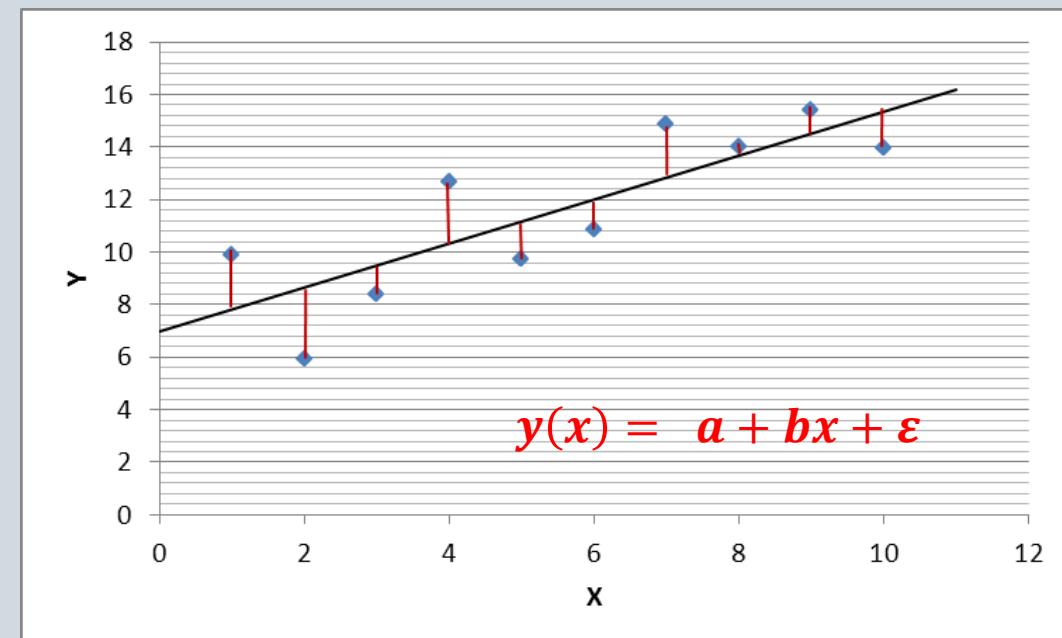
Source : <https://medium.com/p/66b6cfa702f0>

Model Types



Linear Regression

- Model the relation between a numeric target using a regression function that with one or more predicting variables.
- Fit a model to existing (historical) data
- The objective is to find optimal parameters
- Cost Function
 - Mathematical function of the model parameters (α, b_1, \dots) given the (training) data.
 - MSE, RMSE, AMSE, etc.
- Once the cost function is specified, there are various (automated) methods to find the optimal α, b_1, \dots values. For example:
 - Normal equations
 - Gradient descent
- Use fitted model to Predict the numeric target of new data (variable / feature vector)



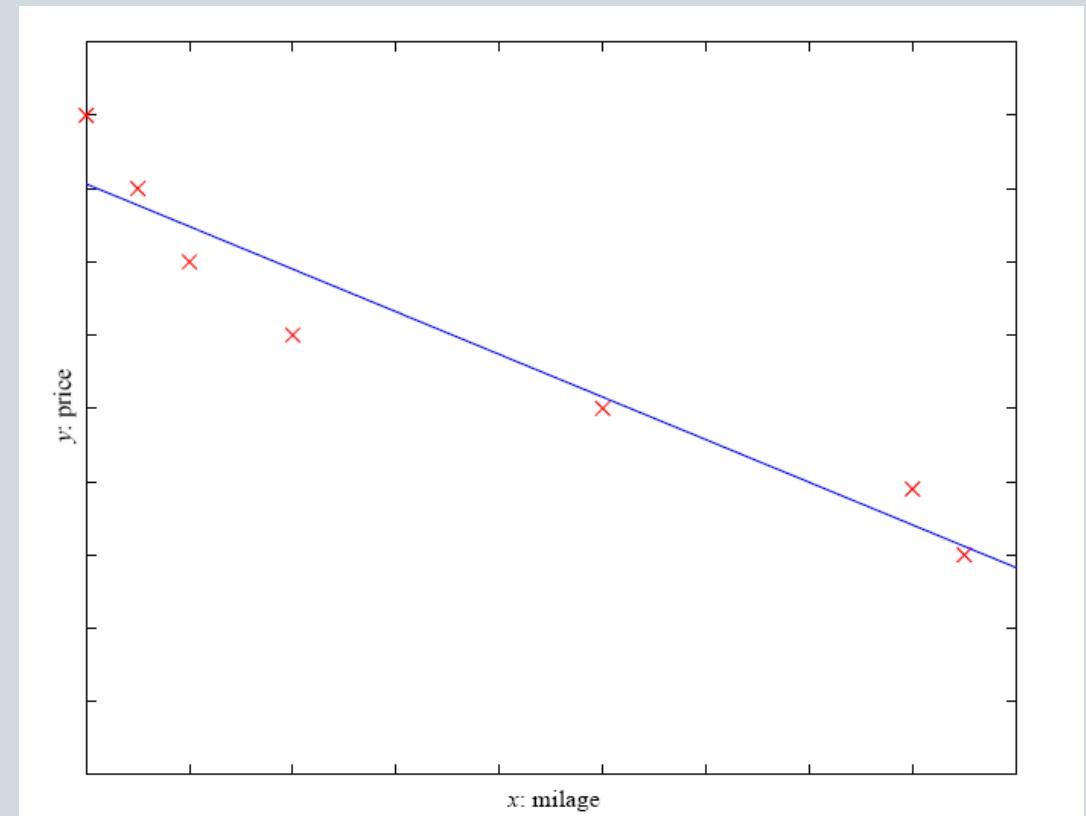
Linear Regression

Regressions typically applied to a subset of the population and can be used to:

- Learn relationship between parameters
 - i.e. “explain” observed outcomes
 - Note: **not causally!**
- Make predictions for the entire population

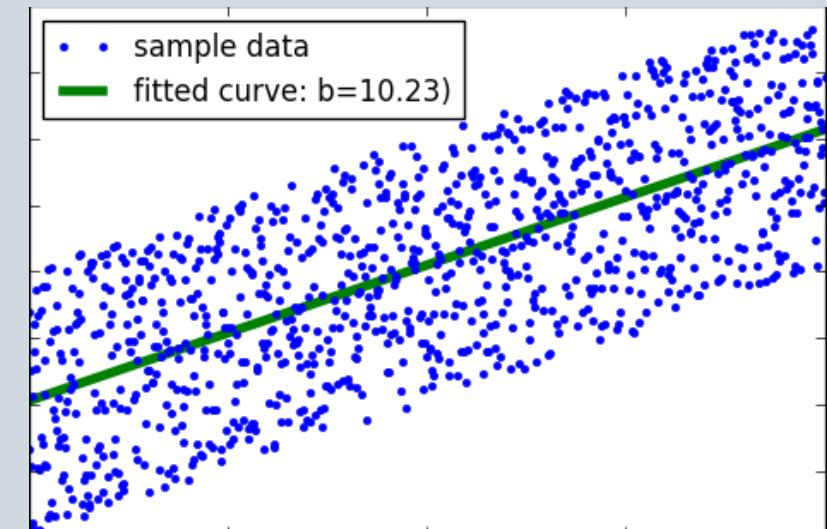
Supervised Learning: Prediction With Regression

- Example: Price of a used car
 - x : car attributes
 - y : price
 - $y = g(x | \beta)$
 - $g()$ model,
 - β parameters



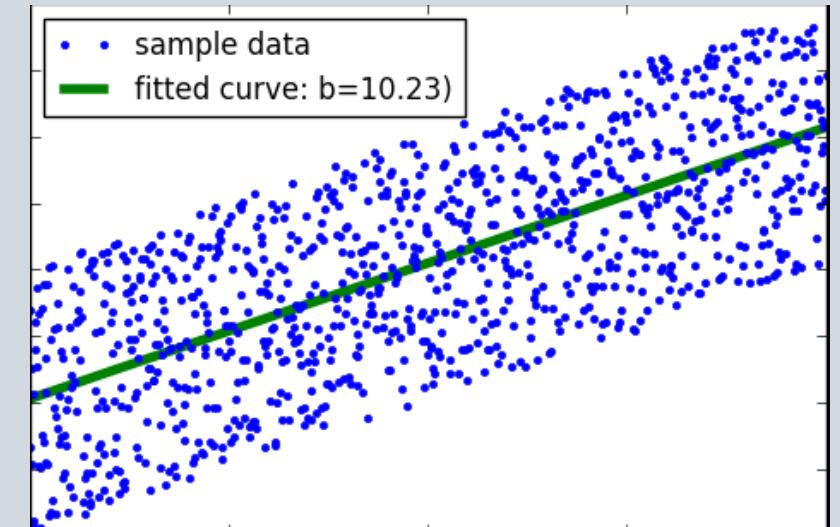
ML: Curve Fitting

- Curve fitting is the simplest case of machine learning.
- Curve fitting can be used to:
 - Confirm that the data is consistent with a certain pattern and validate the underlying mechanism
 - Discover curve parameters. Useful to
 - draw insights (how x relates to y?) & compare different cases
 - make predictions of what if (e.g. what if we could change how x relates to y by certain percentage)?
 - Design more sophisticated models (regressions)
 - Complement the sample (inter/extrapolate) – predict what kind of patterns will be observed



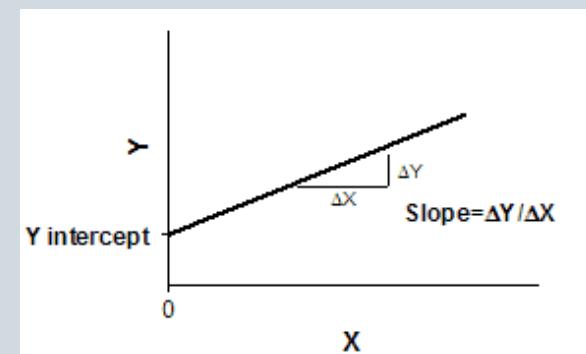
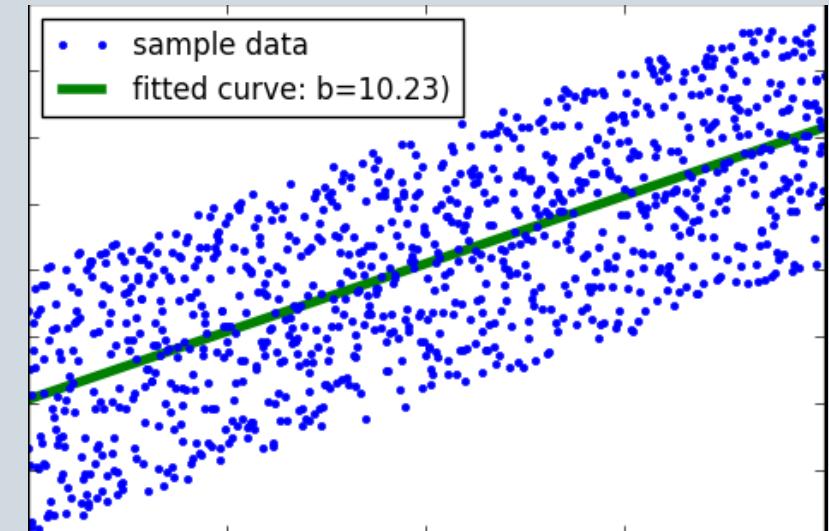
ML: Curve Fitting

- Curve fitting goes well beyond linear curves
 - (e.g. high-degree polynomials; exponents, logs and power laws; trigonometric functions; splines)
- Always start with scatter plot of the raw data
- Consider scaling of x and/or y axis for different datasets.

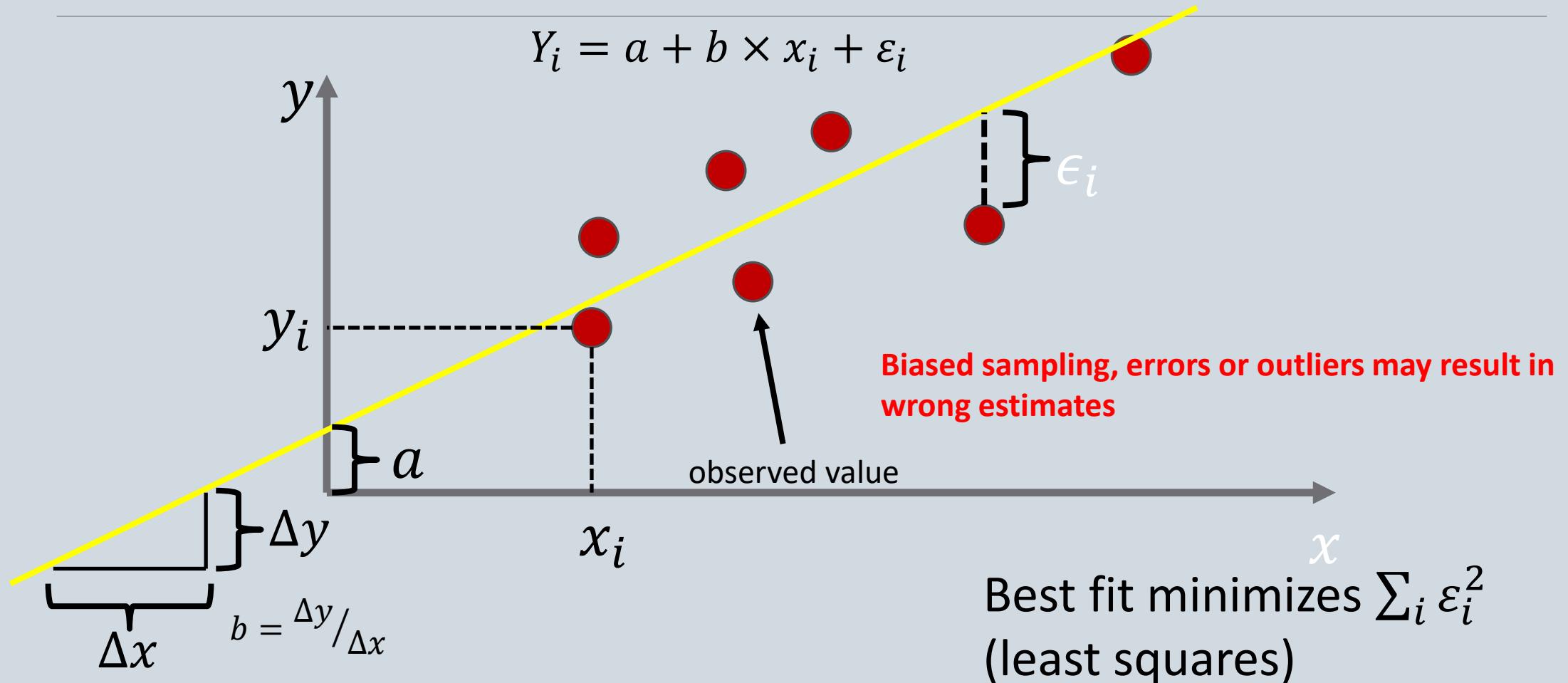


ML - 1st Order Polynomial

- 1st order polynomial fit **assumes** linear relationship between input (independent variable) - x and output (dependent variable) – y .
- Given two vectors, x and y , find a and b (scalars) so that
$$y(x) = bx + a + \varepsilon$$
And ε is minimal (in some sense, eg. $\sum \varepsilon_i^2$ - least square)
 a – intercept. a is the point at which the fitted line hits y axis (at $x = 0$)
 b – slope of the line. Define how fast y changes with x
- Quality of fit depends on the assumption:
data may not be linear



ML - 1st Order Polynomial



Regression Metrics

- y_i – actual value for instance i
- \hat{y}_i – predicted value for instance i
- n – number of instances

Mean Squared Error (MSE)

$$= \frac{1}{n} \sum_i^n (y_i - \hat{y}_i)^2$$

Root Mean Squared Error (RMSE)

$$= \sqrt{\frac{1}{n} \sum_i^n (y_i - \hat{y}_i)^2}$$

Mean Absolute Error (MAE)

$$= \frac{1}{n} \sum_i^n |y_i - \hat{y}_i|$$

Mean Absolute Percentile Error (MAPE)

$$= \frac{1}{n} \sum_i^n \left| \frac{y_i - \hat{y}_i}{y_i} \right|$$

Linear Regression: Supervised Learning

- 1st order polynomial fit is a private case of linear regression fitting.

$$y(x) = a + bx + \varepsilon$$

x, y and ε are vectors, a and b are scalars

The objective is to discover a and b

- General linear regression finds how y depends on several parameters

$$y(x) \sim a + \beta x$$

x is a matrix, $N \times M$

N – number of observations

M – number of parameters

y is a vector, $N \times 1$

β – vector $1 \times M$ – translates how y depends on each parameter in y

- y is a **dependent** variable (it depends) or **target** variable
- x is **independent** or **explanatory** variable (helps explaining y) or **features**
- *Linear regression is a model!*
- It is essential that the data fits the model

[Regression_models.ipynb](#)

Simple linear regression – 1st order polynomial (1 variable)

Multiple linear regression – 2 or more explanatory variables

Ordinary Least Squares (OLS)

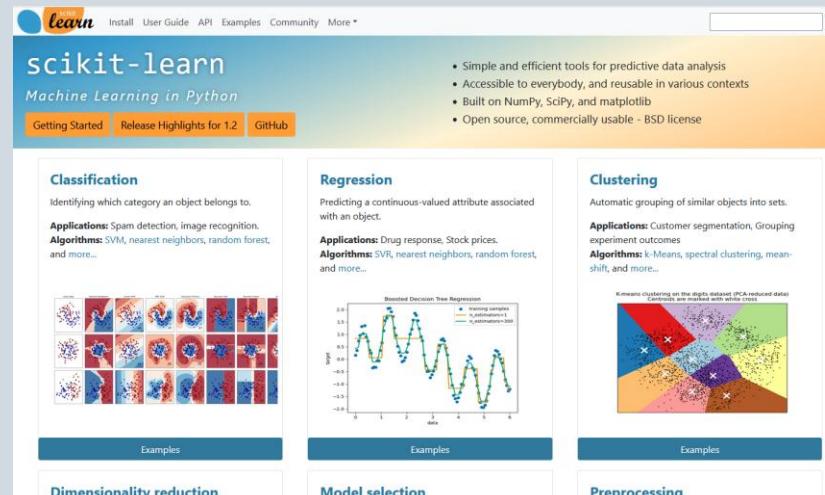
- Ordinary Least Squares:
 - A method to estimate the parameters (β) of a linear regression model.
- Goal:
 - Find the line (or hyperplane) that minimizes the sum of squared residuals: $\min \sum(y_i - \hat{y}_i)^2$
- Key Idea:
 - Residual = difference between actual value (y) and predicted value (\hat{y}).
- Why Squared Errors?
 - Penalizes large errors more strongly.
 - Gives a unique, closed-form solution (Normal Equation).
- Result:
 - Provides the “best fit” line under the assumptions of linear regression.

Interpreting the OLS Output

Target variable	Dep. Variable:	y	R-squared:	0.894	R2- How much of the independent variable is explained by changes in our dependent variables
Model type	Model:	OLS	Adj. R-squared:	0.894	Adj-R2 – important when there are multiple features
Number of observations	Method:	Least Squares	F-statistic:	4194.	F-score of the null hypothesis
Date:	Fri, 01 Apr 2022		Prob (F-statistic):	0.00	Accuracy of the null hypothesis
Time:	16:34:53		Log-Likelihood:	-7742.7	
No. Observations:	1000		AIC:	1.549e+04	
Df Residuals:	997		BIC:	1.551e+04	
Df Model:	2				
Covariance Type:	nonrobust				
<hr/>					
Numbers our predicting variables	coef	std err	t	P> t	[0.025 0.975]
Normalcy distribution of residuals (0=prefect)	intercept	-28.1727	26.494	-1.063	0.288 -80.162 23.817
Probability residuals normally distributed. (1=prefect)	x	9.5522	0.306	31.255	0.000 8.952 10.152
	x2	0.5090	0.006	86.089	0.000 0.497 0.521
<hr/>					
Normalcy distribution of residuals (0=prefect)	Omnibus:	534.067	Durbin-Watson:	1.941	measurement of homoscedasticity (uneven distribution)
Probability residuals normally distributed. (1=prefect)	Prob(Omnibus):	0.000	Jarque-Bera (JB):	55.841	Alternate measure for Omnibus
	Skew:	0.022	Prob(JB):	7.49e-13	
	Kurtosis:	1.843	Cond. No.	6.72e+03	Measurement of sensitivity
<hr/>					
Peakiness of data		Symmetry in data			

Scikit-learn

- Scikit-learn is a Python library for machine learning and data analysis.
- It provides tools for classification, regression, clustering, and dimensionality reduction.
- Scikit-learn includes algorithms for supervised and unsupervised learning, feature extraction, and model selection and evaluation.
- It has a user-friendly interface and is widely used in industry and academia for various applications.
- Scikit-learn is constantly updated and improved with new features and algorithms.



<https://scikit-learn.org/stable/>

Linear Regression Using Scikit-learn

[Regression_models.ipynb](#)

```
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error

data_encoded = pd.get_dummies(data, drop_first=True)
y = data_encoded['price']
X = data_encoded.drop(columns=['price'])

# Fit model
lm = LinearRegression()
lm.fit(X, y)
```

Gradient Descent – Numeric Optimization

- It is not likely that *loss* function (C_{β}) have analytic solution.
- Numeric Analysis solution:
 - Start with arbitrary* solution (β_0, β_1)
 - Iteratively change (β_0, β_1) to improve the solution (minimize the *loss* function C_{β})
 - Continue until you reach a minimum point (or termination condition)

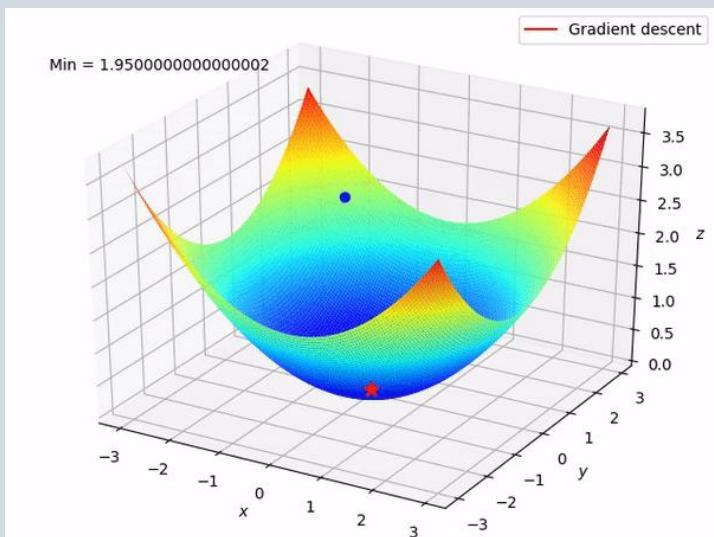
* You may want to use some logic here and assign values in the range you expect the actual β s values to be.

Gradient Descent

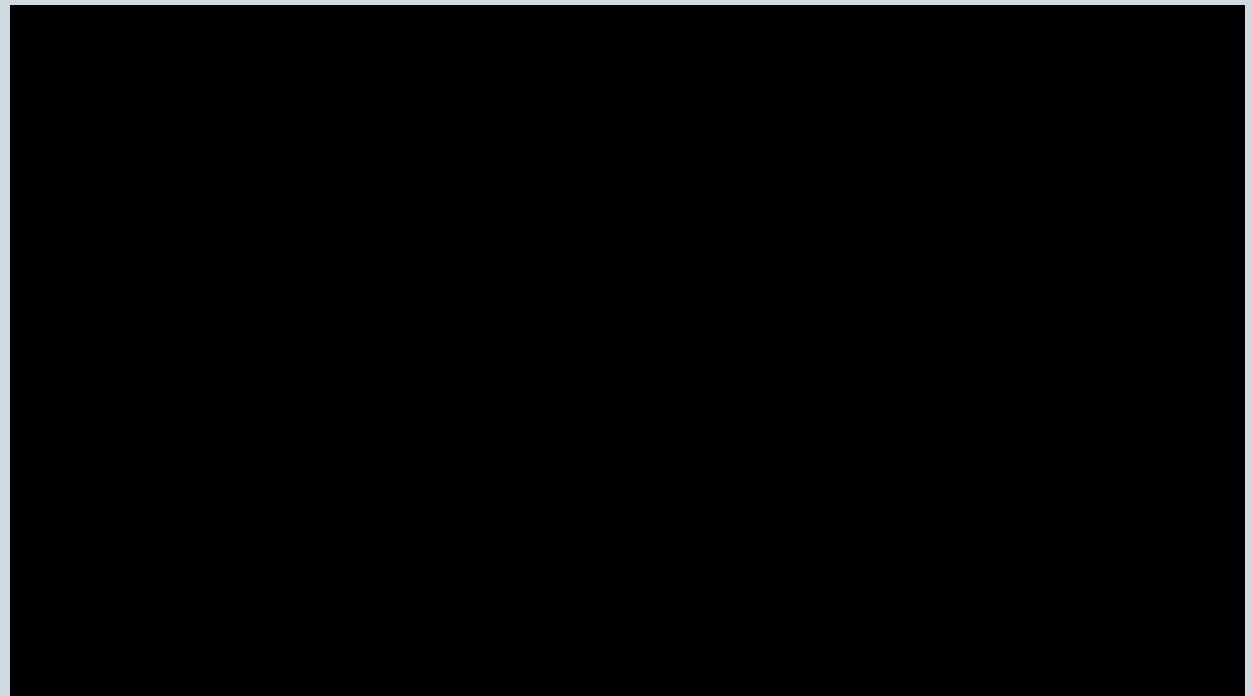
- Cost functions
 - Linear regression: $\min(\text{MSE})$
 - Logistic regression: $\min(-\text{Log Likelihood})$

$$LL = \sum_{i=1}^n \{y_i \ln(p(x_i)) + (1 - y_i) \ln(1 - p(x_i))\}$$

Gradient Descent



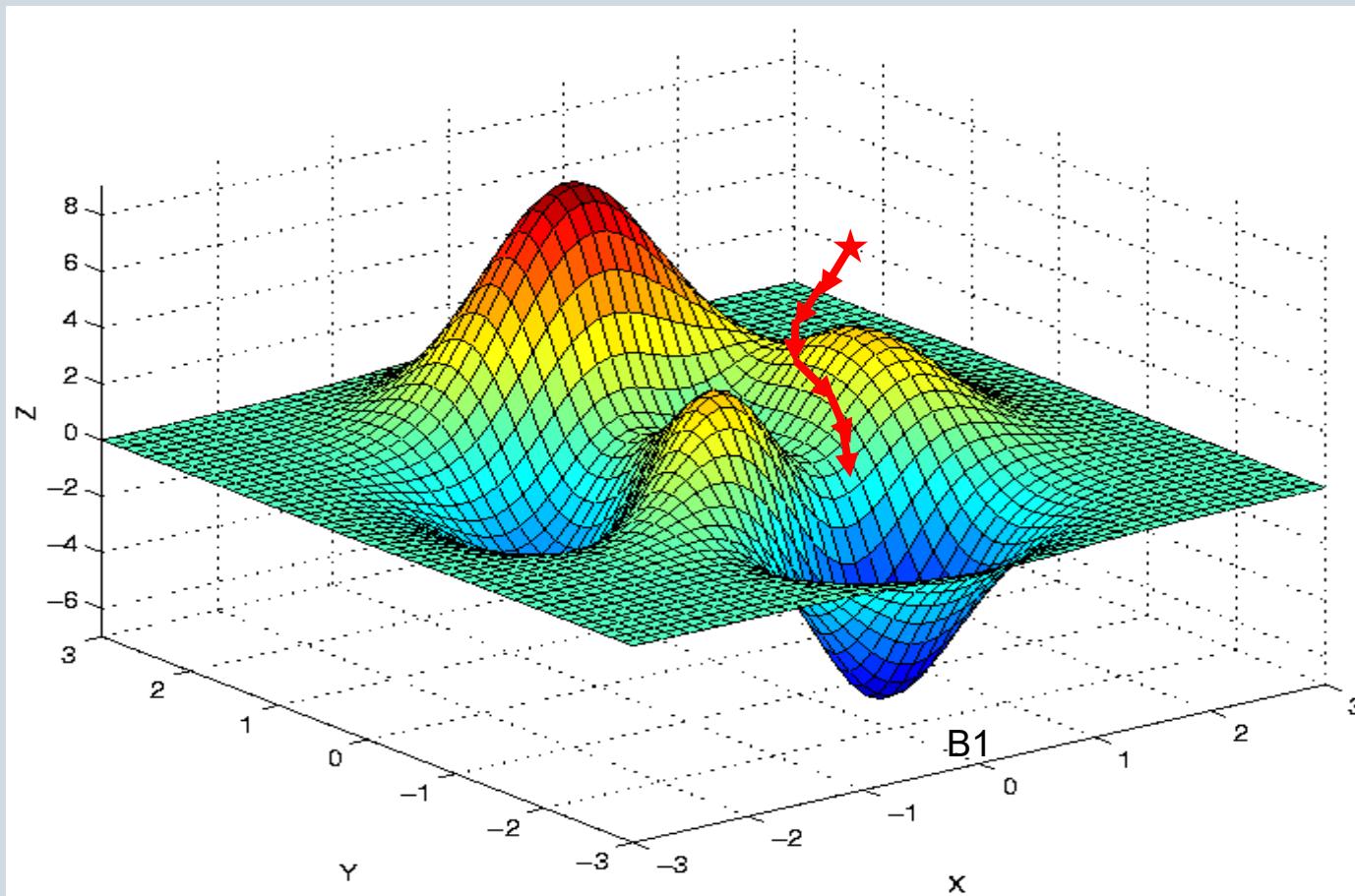
Source: https://jed-ai.github.io/py1_gd_animation/



Source: <https://www.youtube.com/watch?v=vWFjqgb-yIQ&feature=youtu.be>

Linear regression using gradient descent: <https://towardsdatascience.com/linear-regression-using-gradient-descent-97a6c8700931>

Gradient Descent



Gradient Descent

1. Define the “learning rate” γ
2. Select initial solution a_0
3. Calculate the slope of the Loss function at current step (gradient)
 - $\nabla \mathcal{L}(a_i)$
4. Calculate the next step a_{i+1} as
 - $a_{i+1} = a_i - \gamma \nabla \mathcal{L}(a_i)$
5. Termination?
 - If not reached the termination condition go to step 3
 - Else terminate

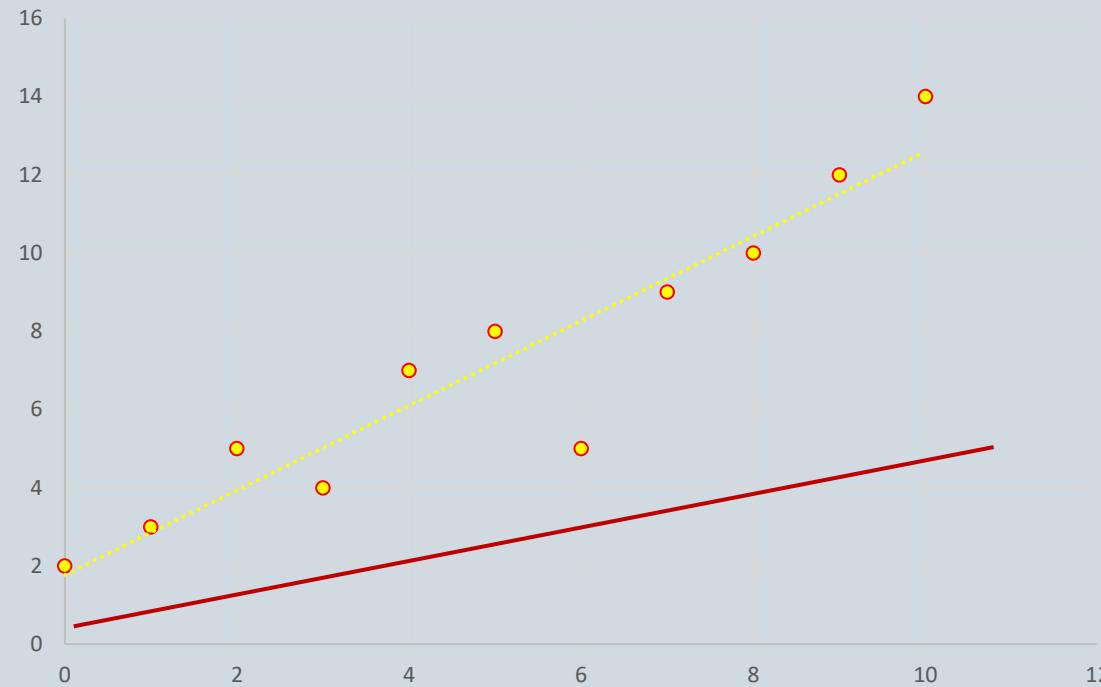
If γ is small enough than $\mathcal{L}(a_1) \geq \mathcal{L}(a_2) \geq \mathcal{L}(a_3) \geq \dots$

Gradient Descent

1. Define the “learning rate” γ
 - Represent the size of each parameter adjustment

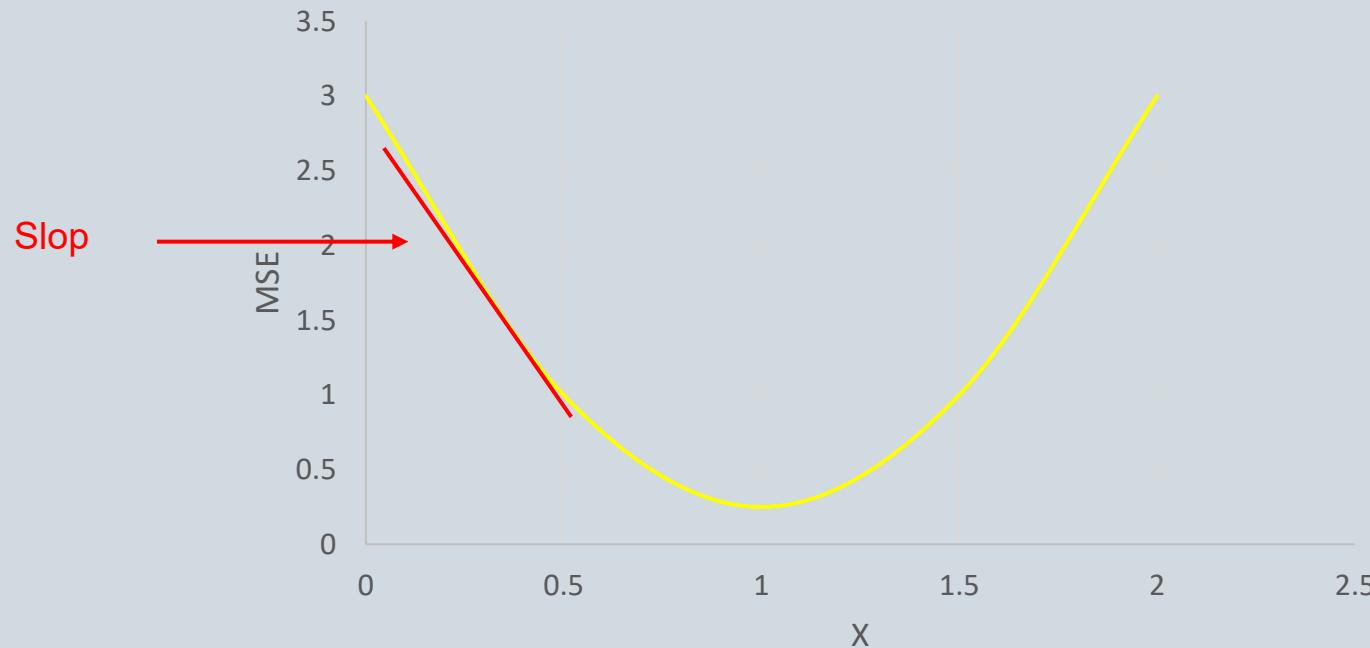
Gradient Descent

2. Select initial solution a_0
 1. Set initial intersection and parameters



Gradient Descent

3. Calculate the slope of the Loss function at current step (gradient) $\nabla \mathcal{L}(a_i)$
 - The lost function could be the MSE or any other lost function



Gradient Descent

4. Calculate the next step a_{i+1} as

- $a_{i+1} = a_i - \gamma \nabla \mathcal{L}(a_i)$
- Basically, we adjust the parameters by the learning rate * the slope of the derivative

Gradient Descent

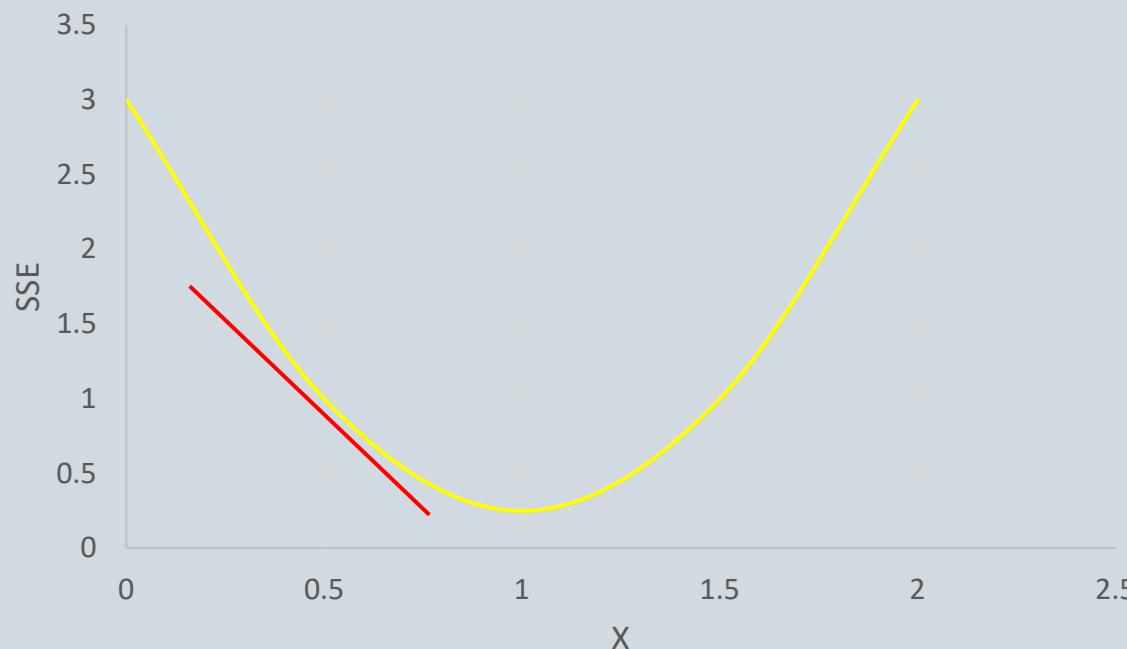
5. Termination?

- Termination condition:
 - The adjusted learning rate is very small (normally close to 0)
 - Or the maximum number of iteration reached.
- Terminate if reached the termination

Gradient Descent

5. Termination?

- If not reached the termination condition go to step 3
- Calculate the slope of the Loss function at current step (gradient) $\nabla \mathcal{L}(a_i)$



Gradient Descent

5. Termination?

- If not reached the termination condition go to step 3
- Calculate the slope of the Loss function at current step (gradient) $\nabla \mathcal{L}(a_i)$



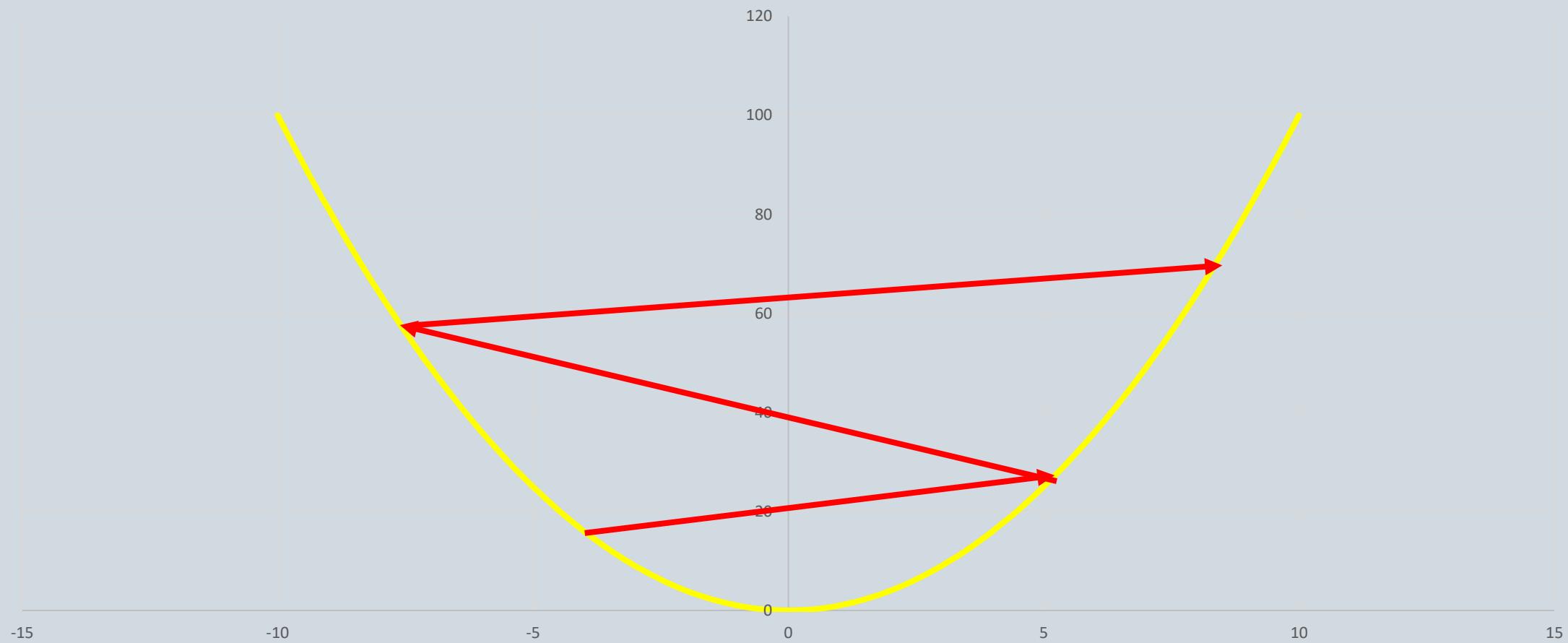
Gradient Descent

5. Termination?

- The adjusted learning rate is very small
- Terminate



Gradient Descent



Gradient Descent – Linear Regression Model

Algorithm:

Start with initial values for $\beta_0, \beta_1 \dots$

Repeat until convergence:

$$\beta_0^{i+1} \leftarrow \beta_0^i - \alpha \frac{\partial \mathcal{L}_{\beta^i}}{\partial \beta_0}$$

$$\beta_1^{i+1} \leftarrow \beta_1^i - \alpha \frac{\partial \mathcal{L}_{\beta^i}}{\partial \beta_1}$$

...

Gradient Descent – Linear Regression Model

Algorithm:

$$\mathcal{L} = MSE = \frac{SSE}{n}$$

Initialize β_0, β_1

Repeat until convergence:

$$\circ SSE' = -2X^T Y + 2X^T X\beta$$

$$\circ \beta_0^{i+1} = \beta_0^i - \gamma \frac{1}{n} \sum_{j=1}^n (\beta_0^{i+1} + \beta_1^{i+1} x_j - y_j)$$

$$\circ \beta_1^{i+1} = \beta_1^i - \gamma \frac{1}{n} \sum_{j=1}^n (\beta_0^{i+1} + \beta_1^{i+1} x_j - y_j) x_j$$

Main Linear Regression Assumptions

- Linearity: $E(\varepsilon_i) = 0$
 - response variable is a linear combination of the predictors.
- Constant variance: $\text{Var}(\varepsilon_i) = 0$
 - variance does not depend on the value of the predictors
- Independence of error: $\text{Cov}(\varepsilon_i \varepsilon_j) = 0$
 - Errors are not correlated
- Normal Distribution of errors: $\varepsilon \sim \text{Norm}(0, \sigma)$

Regularization

- Motivation for Regularization
 - Instead of eliminating variables – we reduce their effect by lowering their coefficient values
 - Smaller coefficient values results in “simpler hypotheses”
 - Which in turn reduces the chances of over-fitting (to-be discussed)
- Ridge, Lasso Regression and Elastic Net
 - Popular techniques in machine learning used for **regularizing linear models to avoid overfitting and improve predictive performance.**
 - Add a **penalty** term to the model’s cost function to constrain the coefficients, but they differ in how they apply this penalty.

Ridge Regression

- Ridge Regression
 - A.K.A. L2 regularization
 - Add penalty equivalent to the square of coefficients
 - Minimize $\text{MSE} + \lambda * \text{sum of square of coefficients}$
 - λ = regularization strength
 - If $\lambda=0$ than we deal with normal linear regression
- Example:
 - Suppose we want to predict a student's exam score using two features:
 - Study hours
 - Sleep hours
 - The model gives coefficients:
 - $\beta_1 = 8$ (effect of study hours)
 - $\beta_2 = -2$ (effect of sleep hours)
 - Regularization parameter: $\lambda = 0.05$
 - Step 1: Penalty Term
 - $\lambda(\beta_1^2 + \beta_2^2) = 0.05 \cdot (8^2 + (-2)^2) = 3.4$
 - Step 2: Ridge Loss
 - The **prediction error term** $\sum(y_i - \hat{y}_i)^2$ depends on the dataset.
 - The **regularization penalty** we just calculated is **3.4**.
 - So the Ridge Loss = prediction error + **3.4**.

$$\text{RidgeLoss} = \sum_{i=1}^m (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^m \beta_j^2$$

Lasso Regression

- **Lasso Regression**
 - A.K.A. L1 regularization
 - Add penalty equivalent to the absolute square of coefficients
 - Minimize $\text{MSE} + \lambda * \text{sum of absolute values of coefficients}$
 - λ = regularization strength
 - If $\lambda=0$ than we deal with normal linear regression
- **Example:**
 - Suppose we want to predict a student's exam score using two features:
 - Study hours
 - Sleep hours
 - The model gives coefficients:
 - $\beta_1 = 8$ (effect of study hours)
 - $\beta_2 = -2$ (effect of sleep hours)
 - Regularization parameter: $\lambda = 0.05$
 - Step 1: Penalty Term
 - $\lambda(|\beta_1| + |\beta_2|) = 0.05 \cdot (8 + 2) = 0.5$
 - Step 2: Lasso Loss
 - The **prediction error term** $\sum(y_i - \hat{y}_i)^2$ depends on the dataset.
 - The **regularization penalty** we just calculated is **0.5**.
 - So the Ridge Loss = prediction error + **0.5**.

$$\text{LassoLoss} = \sum_{i=1}^m (y_i - \hat{y}_i)^2 + \lambda \sum_{i=1}^m |\beta_i|$$

Effect of Regularization on Coefficients

- Ridge never zeros coefficients while Lasso can
- Ridge:
 - Ridge penalty is proportional to the square of the coefficient.
 - The shrinkage is smooth and continuous — the red line (solution) just gets pulled closer to zero.
 - **Result:** coefficients are reduced in size but never exactly zero.
- Lasso:
 - Lasso penalty is proportional to the absolute value of the coefficient.
 - At small values, the penalty is “flat” (seen as the horizontal red line at the origin).
 - This means that if the benefit of including a predictor is smaller than the penalty λ , the coefficient jumps to exactly zero.
 - **Result:** Lasso can completely remove predictors.

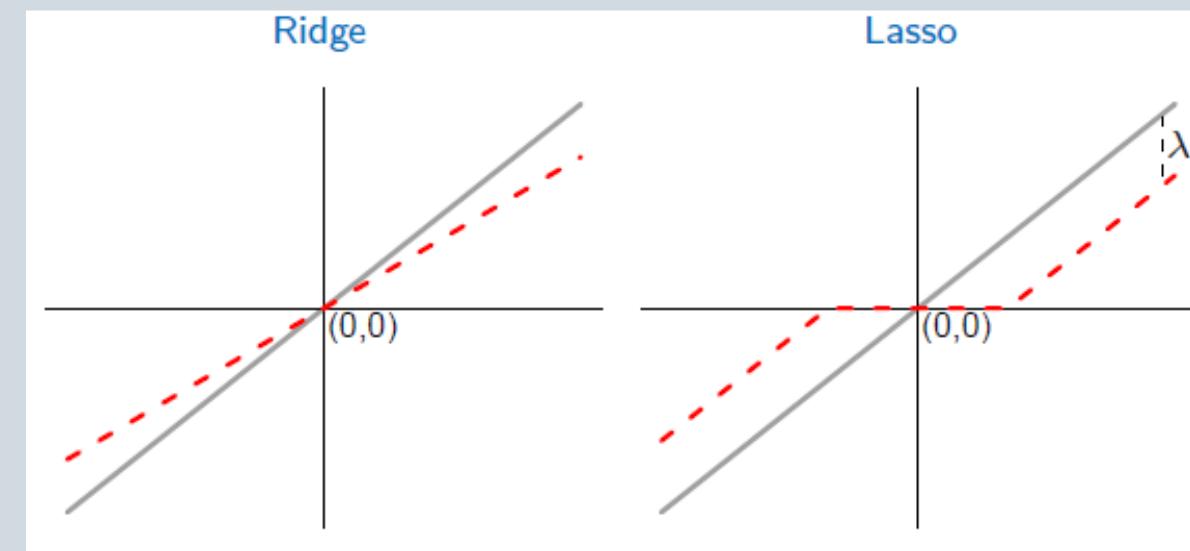


Figure from “An Introduction to Statistical Learning”, James et al., 2013

Elastic Net Regression

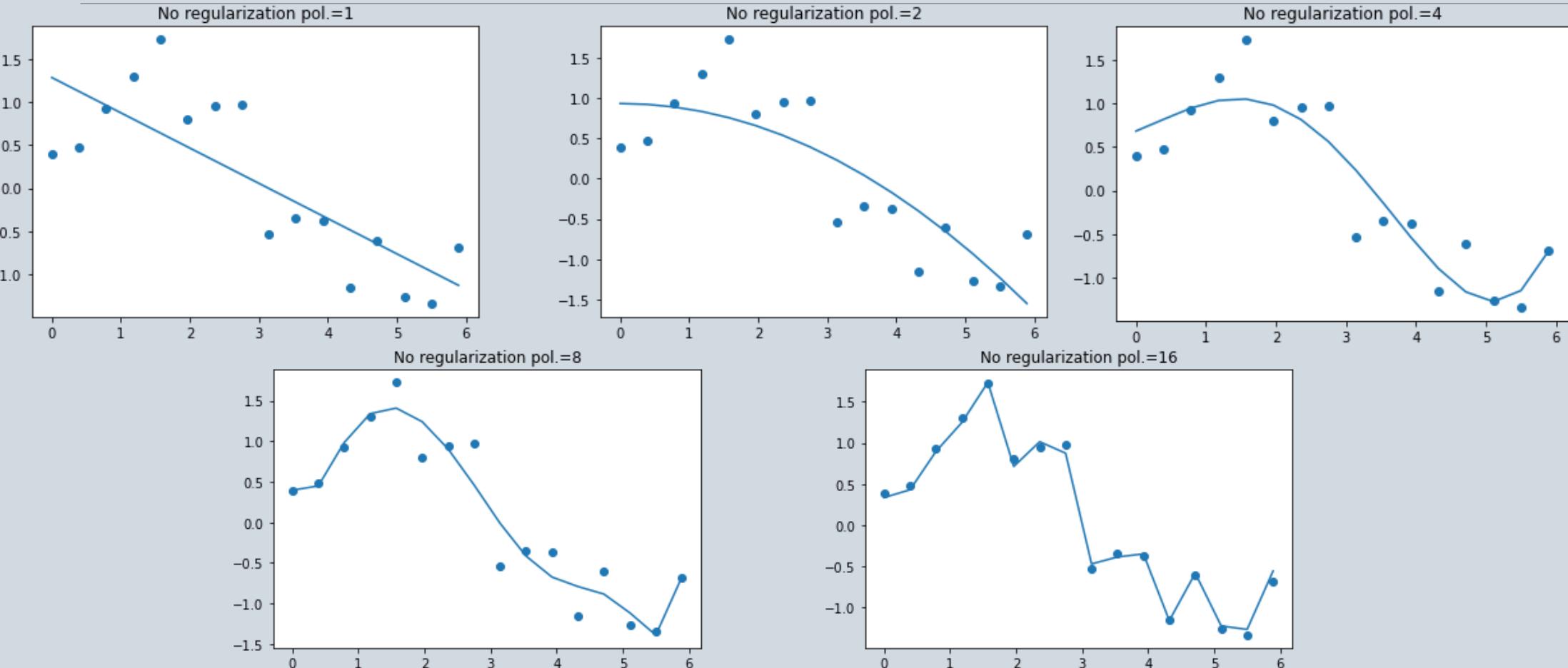
- Elastic Net Regression
 - A.K.A. L1+L2 regularization
 - Combines both L1 (Lasso) and L2 (Ridge) penalties.
 - Performs feature selection while also controlling the size of coefficients.
 - Helps handle multicollinearity by distributing weights across correlated features.
 - Unlike Lasso, which might arbitrarily select one feature and drop the rest, Elastic Net provides a more balanced solution when many predictors are correlated.
- Example:
 - Suppose we want to predict a student's exam score using two features:
 - Study hours
 - Sleep hours
 - The model gives coefficients:
 - $\beta_1 = 8$ (effect of study hours)
 - $\beta_2 = -2$ (effect of sleep hours)
 - Regularization parameter: $\lambda = 0.05$
 - Mix between L1 and L2:
 - $1_ratio=p=0.6$
 - ($p=1, \rho=1 \rightarrow$ pure Lasso; $p=0, \rho=0 \rightarrow$ pure Ridge)
 - Penalty Term
 - $Loss = \sum(y_i - \hat{y}_i)^2 + \lambda[p \sum |\beta_i| + (1-p) \sum \beta_j^2]$
 - $Loss = 0.05 \cdot [0.6 \cdot (|8| + |-1|) + 0.4 \cdot (8^2 + (-2)^2)] = 1.66$
 - So the **Elastic Net loss = prediction error + 1.66**.

$$ElasticNetLoss = \sum_{i=1}^m (y_i - \hat{y}_i)^2 + \lambda \sum_{i=1}^m |\beta_i| + \lambda \sum_{i=1}^m \beta_j^2$$

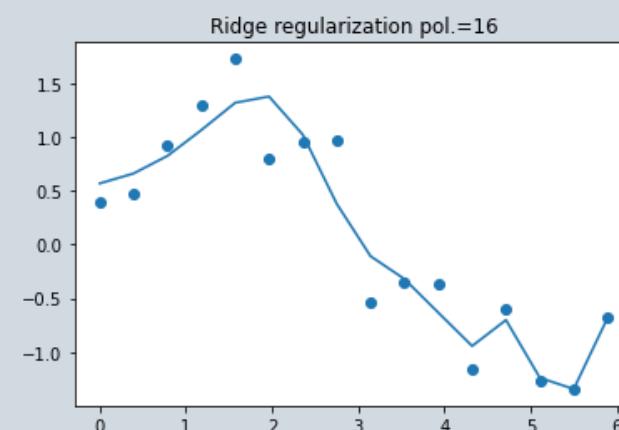
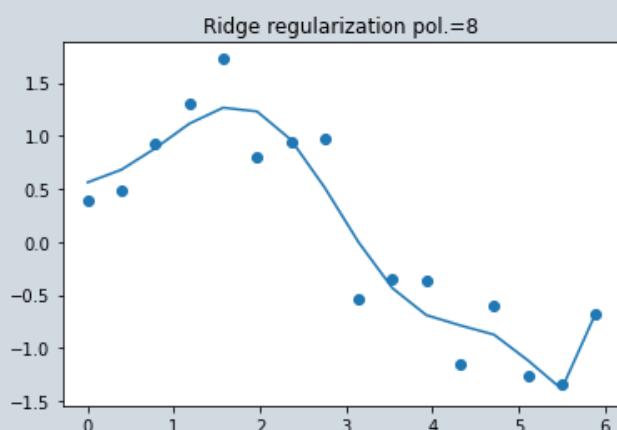
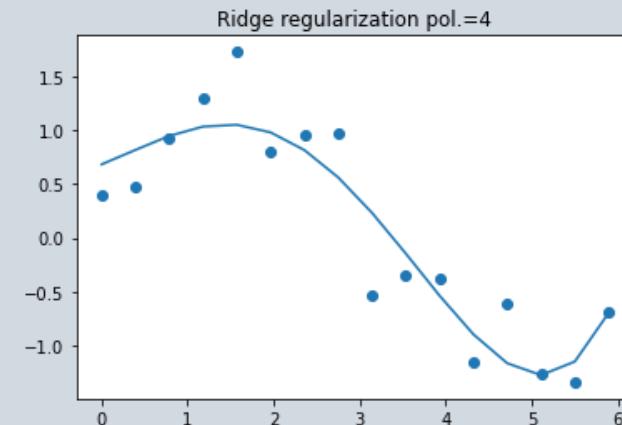
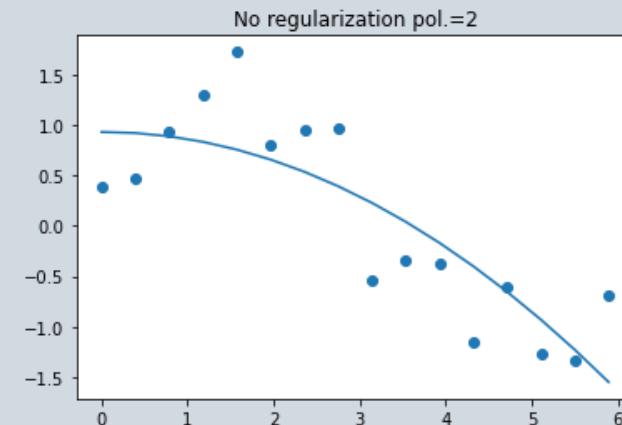
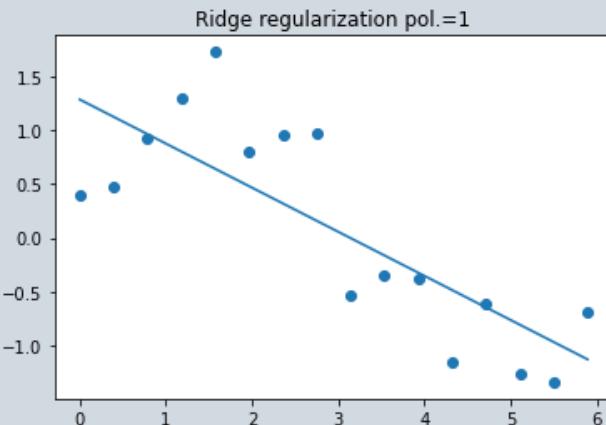
When to Use What?

- **Ridge Regression (L2)**
 - Use when all predictors are potentially relevant.
 - Goal: reduce overfitting without removing features.
 - Works well when predictors are moderately correlated.
 - Example: House price prediction where size, location, rooms, and year built all matter.
- **Lasso Regression (L1)**
 - Use when only a subset of predictors is important.
 - Goal: perform automatic feature selection.
 - Best when you want a simpler model with fewer features.
 - Example: Genetic studies where only a few out of thousands of genes matter.
- **Elastic Net Regression (L1 + L2)**
 - Use when predictors are many and highly correlated.
 - Goal: balance shrinkage (Ridge) and feature selection (Lasso).
 - Avoids instability of Lasso dropping correlated predictors.
 - Example: Text or genomics data where features overlap heavily.

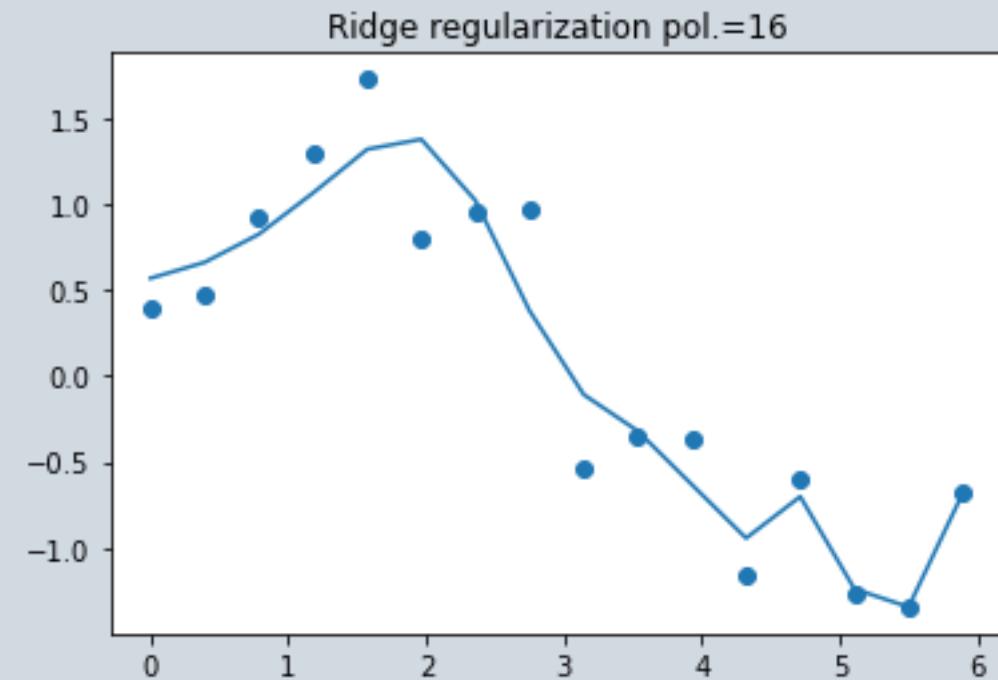
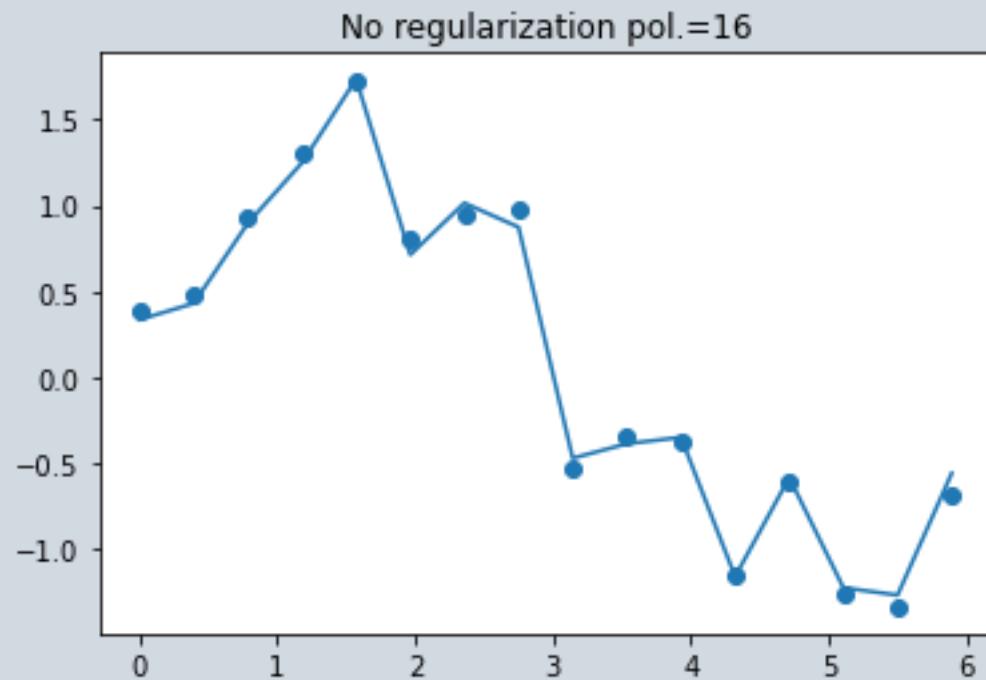
Polynomial Regression



Polynomial Regression – Ridge $\alpha = 1.5$



Polynomial Regression – Before and After



Lasso Regression in Python

```
>>> from sklearn import linear_model
>>> clf = linear_model.Lasso(alpha=0.1)
>>> clf.fit([[0,0], [1, 1], [2, 2]], [0, 1, 2])
Lasso(alpha=0.1)
>>> print(clf.coef_)
[0.85 0.  ]
>>> print(clf.intercept_)
0.15...
```

https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.Lasso.html#sklearn.linear_model.Lasso

Regression_models.ipynb

Ridge Regression in Python

```
>>> from sklearn.linear_model import Ridge
>>> import numpy as np
>>> n_samples, n_features = 10, 5
>>> rng = np.random.RandomState(0)
>>> y = rng.randn(n_samples)
>>> X = rng.randn(n_samples, n_features)
>>> clf = Ridge(alpha=1.0)
>>> clf.fit(X, y)
Ridge()
```

https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.Ridge.html#sklearn.linear_model.Ridge

Regression_models.ipynb

Elastic Net Regression in Python

```
>>> from sklearn.linear_model import ElasticNet  
>>> from sklearn.datasets import make_regression
```

```
>>> X, y = make_regression(n_features=2, random_state=0)  
>>> regr = ElasticNet(random_state=0)  
>>> regr.fit(X, y)  
ElasticNet(random_state=0)  
>>> print(regr.coef_)  
[18.83816048 64.55968825]  
>>> print(regr.intercept_)  
1.451  
>>> print(regr.predict([[0, 0]]))  
[1.451]
```

https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.ElasticNet.html

Regression_models.ipynb

Discrete Models: Logistic Regression

- Linear regression is useful for problems in which outcomes change gradually with input
However, there are many cases in which the dependent variable is discrete (binary):
 - Voting (for/against), survival/mortality, adoption of a product, gender
- Rather than linear dependence on the input, we need a function that would switch quickly from one output (0) to another (+1)
- Essentially, we want to predict probability of an outcome: $P(0 | X)$ and $P(1 | X)$ – probability of an outcome given some set of independent variables.

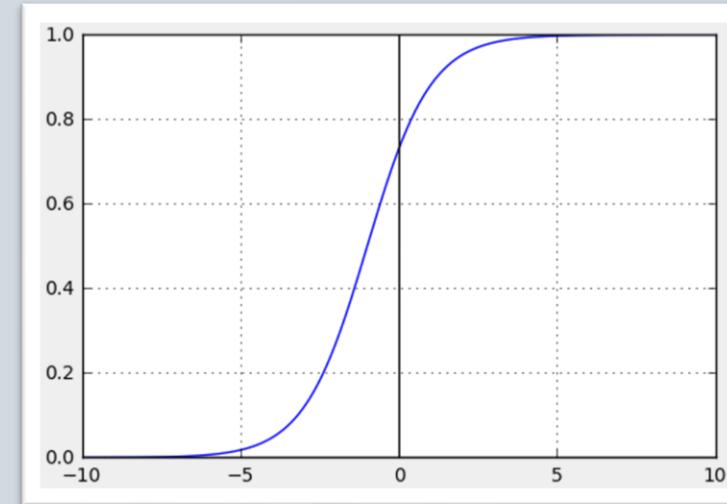
Logit

- The logistic distribution constrains the estimated probabilities to lie between 0 and 1.

$$p(x) = \frac{1}{1+exp(-\alpha-\beta \cdot x)}$$

Note:

- $\alpha + \beta \cdot x = 0 \Rightarrow p(x) = 0.5$
- $\alpha + \beta \cdot x \rightarrow \infty \Rightarrow p(x) = 1$
- $\alpha + \beta \cdot x = -\infty \Rightarrow p(x) = 0$



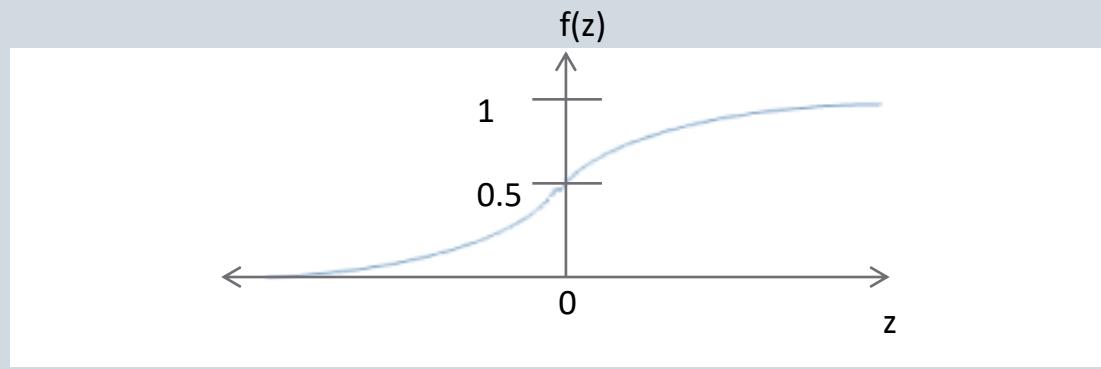
```
import numpy
import matplotlib.pyplot as plt

alpha = 1
beta = 1
x = numpy.linspace(-10,10,10000)
p_x = 1/(1+numpy.exp(-alpha-beta*x))

fig, ax = plt.subplots()
plt.plot(x,p_x,'-')
plt.grid()
```

Logistic Regression

- Binary Dependent Variable $y=0$ “no” ; $y=1$ “yes”
- We want to estimate $h_{\beta}(x)$, where $h_{\beta}(x)$ is our hypothesis (guess) for the probability that $y=1$ given a feature vector x and parameters $\beta=\beta_0, \beta_1, \beta_2, \dots$
 - $P(y = 1|x, \beta)$
- We can predict “ $y=1$ ” if $h_{\beta}(x) > 0.5$
- We define $h_{\beta}(x) = f(\beta_0 + \beta_1 X_1 + \beta_2 X_2 \dots)$
- Where f is the sigmoid (logistic) function: $f(z) = \frac{1}{1+e^{-z}}$
- Inserting $z = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \dots$
- We can predict “ $y=1$ ” if $\frac{1}{1+e^{-(\beta_0 + \beta_1 X_1 + \beta_2 X_2 \dots)}} > 0.5$

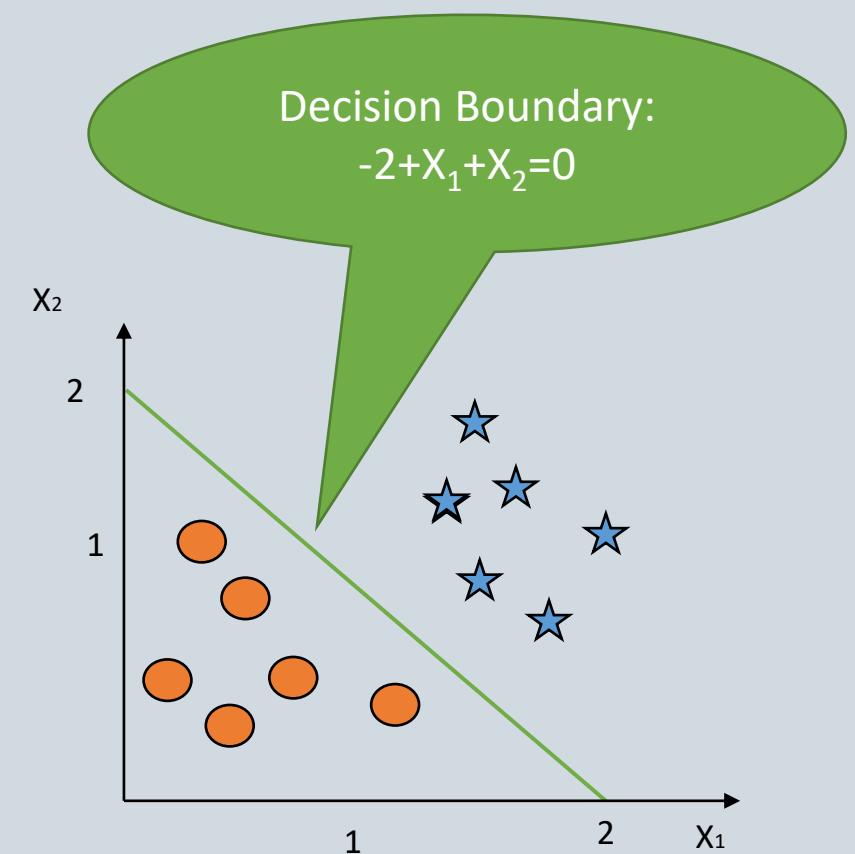


$$z = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \dots$$

Decision Boundary

- Classify “y=1” if $Pr(y \text{ is } 1|X) > 0.5$
- Or $\frac{1}{1+e^{-(\beta_0+\beta_1x_1+\beta_2x_2\dots)}} > 0.5$
- Or $(\beta_0+\beta_1x_1+\beta_2x_2\dots) > 0$
- Example:
 - Predict “y=1” if $-2+x_1+x_2 > 0$
 - parameter values: $\beta_0 = -2$, $\beta_1=1$, $\beta_2=1$

[Regression_models.ipynb](#)



Logistic Regression Using Scikit-learn

Regression_models.ipynb

Logit using sklearn

```
: from sklearn.linear_model import LogisticRegression
X=data[['age','yrs_married','children']].values
y=data['had_affairs'].values
log_reg = LogisticRegression().fit(X,y)
```

executed in 79ms, finished 22:52:16 2023-03-26

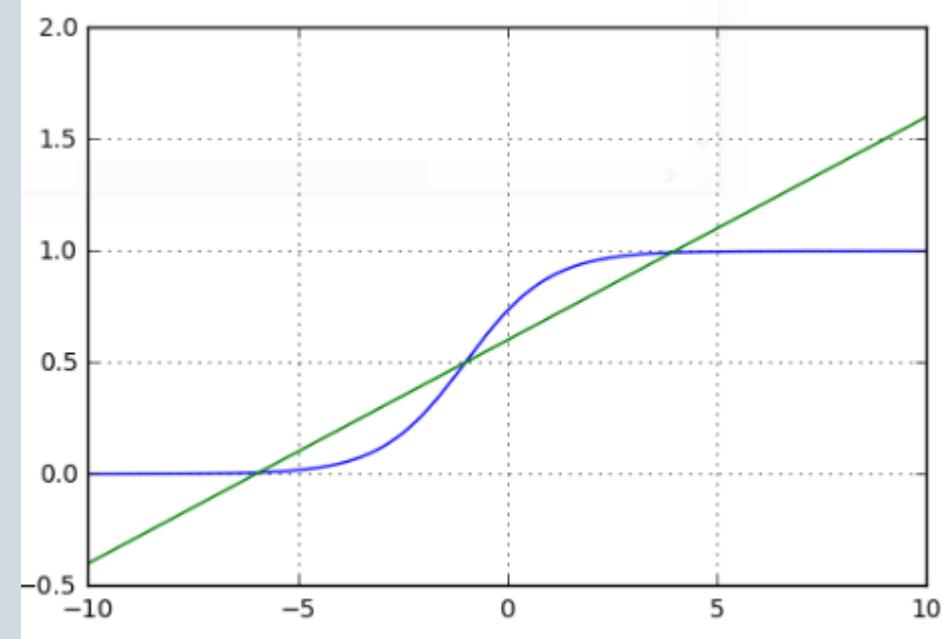
```
: log_reg.predict(np.array([28,4,2]).reshape(1, -1))
```

executed in 15ms, finished 22:52:16 2023-03-26

Logit vs Linear Model

```
%matplotlib inline
import numpy
import matplotlib.pyplot as plt

alpha = 1
beta = 1
x = numpy.linspace(-10,10,10000)
p_x = 1/(1+numpy.exp(-alpha-beta*x))
p_x1 = 0.6 + 0.1*x
fig, ax = plt.subplots()
plt.plot(x,p_x, '-')
plt.hold(True)
plt.plot(x,p_x1, 'g-')
plt.grid()
```



See: <http://nbviewer.jupyter.org/gist/justmarkham/6d5c061ca5aee67c4316471f8c2ae976>

THANK YOU FOR LISTENING

ZVI.BENAMI@MAIL.HUJI.AC.IL

