Lars Schultz

#1

$P(d|b, \neg a, j, m)$ is the query

From the graph structure we can first compute:

$$P(d, b, \neg a, j, m) = \prod_{i=1}^{n} P(X_i | \text{Parents}(X_i))$$

$$= \sum_{e,\neg e} P(d|b,E)\, P(b)\, P(\neg a|b,E)\, P(j|d,\neg a)\, P(m|d,\neg a)$$

$$= P(b) \sum_{e,\neg e} P(d|b,E)\, P(\neg a|b,E)\, P(j|d,\neg a)\, P(m|d,\neg a)$$

$$\begin{array}{lllllll} & B & D & A & J & M & \\ = & 0.01 \cdot (& 0.9 & \cdot (1-0.95) & \cdot\ 0.7 & \cdot\ 0.3\ + & e \\ & & 0.8 & \cdot (1-0.9) & \cdot\ 0.7 & \cdot\ 0.3) & \neg e \end{array}$$

$$= 2.625 \cdot 10^{-4}$$

Now compute:

$$P(\neg d, b, \neg a, j, m) = \prod_{i=1}^{n} P(X_i | \text{Parents}(X_i))$$

$$= \sum_{e,\neg e} P(\neg d|b,E)\, P(b)\, P(\neg a|b,E)\, P(j|\neg d,\neg a)\, P(m|\neg d,\neg a)$$

$$= P(b) \sum_{e,\neg e} P(\neg d|b,E)\, P(\neg a|b,E)\, P(j|\neg d,\neg a)\, P(m|\neg d,\neg a)$$

$$\begin{array}{lllllll} & B & D & A & J & M & \\ = & 0.01 \cdot (& (1-0.9) & \cdot (1-0.95) & \cdot\ 0.1 & \cdot\ 0.2\ + & e \\ & & (1-0.8) & \cdot (1-0.9) & \cdot\ 0.1 & 0.2) & \neg e \end{array}$$

$$= 1.4 \cdot 10^{-5}$$

$\longrightarrow$

Dave Schultz

#1 Continued

$$P(d|b,\neg a,j,m) = \frac{P(d,b,\neg a,j,m)}{P(b,\neg a,j,m)} =$$

$$= \frac{P(d,b,\neg a,j,m)}{P(d,b,\neg a,j,m)+P(\neg d,b,\neg a,j,m)} = \frac{2.625\cdot 10^{-4}}{2.625\cdot 10^{-4}+1.4\cdot 10^{-5}}$$

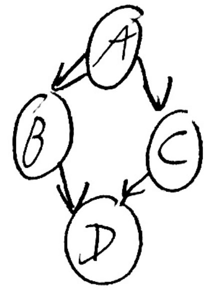
$$= \underline{\underline{0.9493670886}}$$

Lane Schultz

#2 a)

$$P(a) = \frac{\text{True}}{\text{True} + \text{False}} = \frac{270}{270+230} = \frac{270}{500} = 0.54$$

$$P(\neg a) = 1 - P(a) = 1 - 0.54 = 0.46$$



Can use a simmilar methodology for all

P(A)
<0.54, 0.46>

<P(True), 1 − P(True)>

For False values I use 1 − True

A	P(B|A)
T	$\langle \frac{122}{122+148}, \frac{148}{122+148} \rangle$
F	$\langle \frac{115}{115+115}, \frac{115}{115+115} \rangle$

→

A	P(B|A)
T	<0.452, 0.548>
F	<0.5, 0.5>

A	P(C|A)
T	$\langle \frac{138}{138+132}, \frac{132}{138+132} \rangle$
F	$\langle \frac{137}{137+93}, \frac{93}{137+93} \rangle$

→

A	P(C|A)
T	<0.511, 0.489>
F	<0.596, 0.404>

B	C	P(D|B, C)
T	T	$\langle \frac{66}{66+67}, \frac{67}{66+67} \rangle$
T	F	$\langle \frac{66}{66+38}, \frac{38}{66+38} \rangle$
F	T	$\langle \frac{64}{64+78}, \frac{78}{64+78} \rangle$
F	F	$\langle \frac{62}{62+59}, \frac{59}{62+59}$

→

B	C	P(D|B, C)
T	T	<0.496, 0.504>
T	F	<0.635, 0.365>
F	T	<0.451, 0.549>
F	F	<0.512, 0.488>

→

Jane Schultz

#2 b)

A	B	C	D
T	F		T
T	F		T
T	F		T
T	F		T
T	F		T
T	F		T
T	F		T
T	F		T
T	F		T
T	F		T

Since all instances have $A=T$, $B=F$, $D=T$ we only need to compute $P(c|a,\neg b,d)$ and $P(\neg c|a,\neg b,d)$ from prior data

E-step

$$P(c|a,\neg b,d) = \frac{P(a,\neg b,c,d)}{P(a,\neg b,c,d)+P(a,\neg b,\neg c,d)}$$

$$= \frac{0.54 \cdot 0.548 \cdot 0.511 \cdot 0.451}{0.54 \cdot 0.548 \cdot 0.511 \cdot 0.451 + 0.54 \cdot 0.548 \cdot 0.489 \cdot 0.51}$$

$$= \underline{0.48}$$

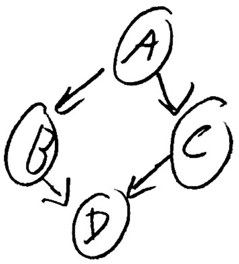
$$P(\neg c|a,\neg b,d) = 1 - P(c|a,\neg b,d) = \underline{0.52}$$

$\underline{\langle 0.48, 0.52 \rangle}$ For the C column rows

The values will most likely be False

$\longrightarrow$

#2 b) continued



M-step

The new count for A is $\langle 280, 230 \rangle$

$P(a) = \frac{280}{280+230} = 0.55$

$P(\neg a) = 1 - P(a) = 0.45$

$P(A)$
$\langle 0.55, 0.45 \rangle$

A	B|A
T	$\langle 122, 158 \rangle$
F	$\langle 115, 115 \rangle$

$\longrightarrow$

A	P(B|A)
T	$\langle 0.44, 0.56 \rangle$
F	$\langle 0.5, 0.5 \rangle$

$P(b|a) = \frac{122}{122+158} = 0.44$

$P(\neg b|a) = 1 - P(b|a) = 0.56$

$P(b|\neg a) = 1/2$

$P(\neg b|\neg a) = 1 - P(b|\neg a) = 1/2$

$\longrightarrow$

[illegible]

#2 b) continued

By maximum likelihood:

$$P(c|a) = \frac{E\#(a \wedge c)}{E\#(c)} = \frac{10 \cdot 0.48 + 138}{10 + 138 + 132} = 0.51$$

$$P(\neg c|a) = 1 - P(c|a) = 0.49$$

$$P(c|\neg a) = \frac{0 \cdot (0.48) + 137}{0 + 137 + 93} = 0.60$$

$$P(\neg c|\neg a) = 1 - P(c|\neg a) = 0.40$$

A	P(C|A)
T	⟨0.51, 0.49⟩
F	⟨0.60, 0.40⟩

→ Unchanged but the problem said use 2 sig figs.

Zane Schultz

#2 b) continued

B	C	P(D|B, C)
T	T	⟨0.50, 0.50⟩
T	F	⟨0.63, 0.37⟩
F	T	⟨0.45, 0.55⟩
F	F	⟨0.51, 0.49⟩

$$P(d|b,c) = \frac{E\#(b \wedge c \wedge d)}{E\#(b \wedge c)}$$

$$= \frac{66 + (0.48)\cdot 0}{66 + 67 + 0}$$

$= 0.50$ → same as before because we only added False B values

$$P(\neg d|b,c) = 1 - P(d|b,c) = 0.50$$

$$P(d|\neg b,c) = \frac{64 + 10(0.48)}{64 + 78 + 10} = 0.45$$

$$P(\neg d|\neg b,c) = 0.55$$

$$P(d|b,\neg c) = \frac{66 + 0\cdot(0.48)}{66 + 38 + 0} = 0.63$$

$$P(\neg d|b,\neg c) = 0.37$$

$$P(d|\neg b,\neg c) = \frac{62 + 0.48\cdot 10}{62 + 59 + 10} = 0.51$$

$$P(\neg d|\neg b,c) = 0.49$$