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Semiconductor Devices: Exam questions

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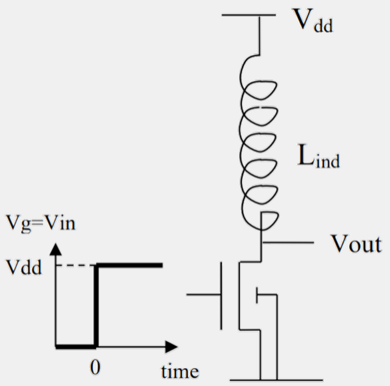
Contents

Question 1



$V_{dd} = 5V$
$V_T = 1V$
$\varepsilon_{ox} = 3.5\ 10^{-13}\ Fcm^{-1}$
$t_{ox} = 70\ nm$
$\mu = 1000\ cm^2V^{-1}s^{-1}$
$L_{ind} = 5\ mH$

An n-MOS transistor with parameters given above is used to switch on an inductive load Lind (e.g. an electrical engine). At time $t = 0$ s the gate voltage of the transistor is switched from $V_{in} = 0$ V to $V_{in} = V_{dd}$. All capacitances in the circuit can be neglected.

a) What is the output voltage before switching on the transistor?

b) What is the output voltage V_{out} at $t = \infty$?

c) How big should we design W/L of the transistor in order that the transistor continues to operate in the purely linear region until $t = 100$ µs, i.e. $V_{out} < V_{ds,sat}/10$ for $t < 100$ µs (with $V_{ds,sat}$ the value of V_{ds} for which the transistor goes from the linear to the saturation region) *(Solution: 1200)*

d) How big is the current in the transistor after 100 µs for the calculated value of W/L? Draw the evolution of the current and the output voltage as a function of time.
(Answer: 96mA)

a, b)

As the voltage is constant, the inductor is a short, so $V_{out} = V_{dd} = 5$ V for both $t < 0$ and $t \to \infty$.

c)

The current in the conductor cannot change instantaneously, so the current through the transistor just after and just before $t = 0$ s is 0. $I(0^+) = I(0^-) = 0$, thereby $V_{DS}(0^+) = 0$ V. The transistor shall start in the linear region. The equations are then:

$$\begin{cases} I_{DS} = \beta\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS} \approx \beta\left(V_{GS} - V_T\right)V_{DS} \\ L_{ind}\frac{dI_L}{dt} = V_L \end{cases}$$

$$\begin{cases} I = \beta\left(V_{dd} - V_T\right)V_{out} \\ L_{ind}\frac{dI}{dt} = V_{dd} - V_{out} \end{cases}$$

$$\Rightarrow \quad L_{ind}\beta\left(V_{dd} - V_T\right)\frac{dV_{out}}{dt} = V_{dd} - V_{out}$$

This differential equation can be solved:

$$V_{out} = V_{dd} - Ae^{\frac{-t}{L_{ind}\beta(V_{dd}-V_T)}}$$

Using the starting condition: $V_{out}(0^+) = V_{DS}(0^+) = 0$ V, gives

$$V_{out} = V_{dd}\left(1 - e^{\frac{-t}{L_{ind}\beta(V_{dd}-V_T)}}\right)$$

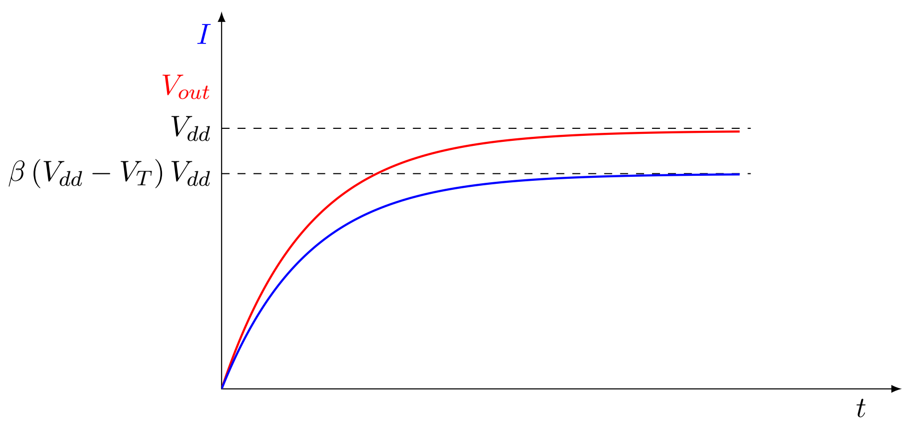
We have to determine W/L (follows from β) if $V_{out} = V_{ds,sat}/10$ at $t = 100$ µs.

$$\begin{aligned}
\frac{V_{ds,sat}}{10} &= V_{dd}\left(1 - e^{\frac{-t}{L_{ind}\beta(V_{dd}-V_T)}}\right) \\
\beta &= \frac{t}{L_{ind}(V_{dd}-V_T)\ln\left(\frac{V_{dd}}{V_{dd}-\frac{V_{ds,sat}}{10}}\right)} = \frac{W}{L}\mu C_{ox} \\
\frac{W}{L} &= \frac{t}{\mu C_{ox} L_{ind}(V_{dd}-V_T)\ln\left(\frac{V_{dd}}{V_{dd}-\frac{V_{GS}-V_T}{10}}\right)} \\
&= \frac{t t_{ox}}{\mu\epsilon_{ox} L_{ind}(V_{dd}-V_T)\ln\left(\frac{V_{dd}}{V_{dd}-\frac{V_{dd}-V_T}{10}}\right)} \\
&= \frac{100\ 10^{-6}\ \text{s}\cdot 70\ 10^{-7}\ \text{cm}}{1000\ \text{cm}^2/\text{Vs}\cdot 3.5\ 10^{-13}\ \text{C/Vcm}\cdot 5\ 10^{-3}\ \text{Vs}^2/\text{C}\cdot(5\ \text{V}-1\ \text{V})\cdot\ln\left(\frac{5\ \text{V}}{5\ \text{V}-\frac{4\ \text{V}}{10}}\right)} \\
&= 1199
\end{aligned}$$

d)

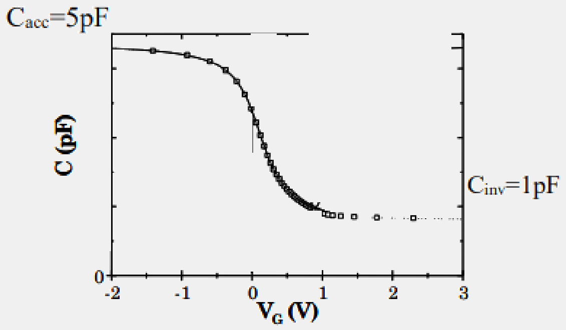
The first equation in part c gave:

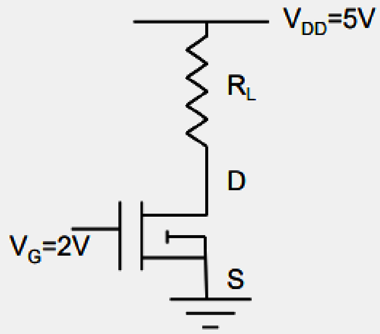
$$\begin{aligned}
I &= \beta\left(V_{dd}-V_T\right)V_{out} \\
I(100\ \mu\text{s}) &= \frac{W}{L}\mu C_{ox}\left(V_{dd}-V_T\right)\frac{V_{ds,sat}}{10} \\
&= 1199\cdot 1000\ \text{cm}^2/\text{Vs}\cdot\frac{3.5\ 10^{-13}\ \text{C/Vcm}}{70\ 10^{-7}\ \text{cm}}\cdot(5\ \text{V}-1\ \text{V})\cdot\frac{4\ \text{V}}{10} \\
&= 96\ \text{mA}
\end{aligned}$$



Question 2

On a MOS-capacitor with an area of 100x100 μm^2 a high frequency C-V measurement is done with results as shown below left:





Use the following values for the parameters:
Flatband voltage V_{fb} = -0.5V
Channel mobility μ = 500 cm^2/(Vs)
R_L=10 kΩ
$\varepsilon_{Si} = 10^{-12}$ F/cm
$\varepsilon_{ox} = 3.5\ 10^{-13}$ F/cm
$n_i = 10^{10}$ cm^{-3}
$q = 1.6\ 10^{-19}$ C
Vt =kT/q = 0.026V

a) Extract from this measurement the oxide thickness and substrate doping concentration of the MOS
(Solution: 70nm, 1.19 10^{15} cm^{-3})

b) In the same technology we design a MOS transistor that is used in the circuit shown in the right figure. Determine W/L of the transistor such that it operates at the transition between linear and saturation region. Determine the transconductance for this operating point.
(Solution: 10.8, 0.43mA/V)

Oxide thickness from the accumulation capacitance:

$$
\begin{aligned}
C_{acc} &= AC_{ox} = \frac{\epsilon_{ox}A}{t_{ox}} \\
t_{ox} &= \frac{\epsilon_{ox}A}{C_{acc}} \\
&= 70 \text{ nm}
\end{aligned}
$$

Doping concentration from the inversion capacitance:

$$
\begin{aligned}
\frac{A}{C_{inv}} &= \frac{dV_{GB}}{dQ} \\
&= \frac{d}{dQ}\left(\Phi_{ms} + \frac{qN_A z_{D,th}^2}{2\epsilon_s} + \frac{qN_A z_{D,th}}{C_{ox}}\right) \\
&= \frac{z_{D,th}}{\epsilon_s} + \frac{1}{C_{ox}} \\
z_{D,th} &= \epsilon_s\left(\frac{A}{C_{inv}} - \frac{A}{C_{acc}}\right)
\end{aligned}
$$

With

$$z_{D,th} = \sqrt{\frac{2\epsilon_s|2\Phi_f|}{qN_A}}$$

This becomes

$$\begin{aligned}
z_{D,th}^2 = \frac{4\epsilon_s|\Phi_f|}{qN_A} &= \epsilon_s^2 A^2 \left(\frac{1}{C_{inv}} - \frac{1}{C_{acc}}\right)^2 \\
N_A &= \frac{4|\Phi_f|}{q\epsilon_s A^2 \left(\frac{1}{C_{inv}} - \frac{1}{C_{acc}}\right)^2} \\
&= \frac{4V_t \ln \frac{N_A}{n_i}}{q\epsilon_s A^2 \left(\frac{1}{C_{inv}} - \frac{1}{C_{acc}}\right)^2}
\end{aligned}$$

This can be calculated iteratively:

$$N_A^{(n+1)} = \frac{4V_t \ln \frac{N_A^{(n)}}{n_i}}{q\epsilon_s A^2 \left(\frac{1}{C_{inv}} - \frac{1}{C_{acc}}\right)^2}$$

Which converges to $N_A = 1.18\ 10^{15}\ \text{cm}^{-3}$

b)

The condition of being on the border between the linear and saturation region tells us that $V_{DS} = V_{DS,sat} = V_{GS} - V_T$. First we calculate V_T:

$$\begin{aligned}
V_T &= 2\Phi_f + V_{fb} + K_2\sqrt{|2\Phi_f - V_{BS}|} \\
&= 2\frac{kT}{q}\ln\frac{N_A}{n_i} + V_{fb} + \frac{\sqrt{2qN_A\epsilon_s}}{C_{ox}}\sqrt{2\frac{kT}{q}\ln\frac{N_A}{n_i}} \\
&= 0.411\ \text{V}
\end{aligned}$$

So $V_{DS} = V_{DS,sat} = V_{GS} - V_T = 1.589$ V and $I = \frac{V_R}{R_L} = \frac{3.411\ \text{V}}{10000\ \Omega} = 0.341$ mA. With this information we can calculate β and later W/L:

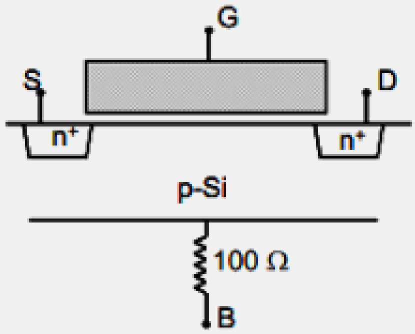
$$\begin{aligned}
I &= \beta\frac{(V_{GS} - V_T)^2}{2} \\
\beta = \frac{W}{L}\mu C_{ox} &= \frac{2I}{(V_{GS} - V_T)^2} \\
\frac{W}{L} &= \frac{2It_{ox}}{\mu\epsilon_{ox}(V_{GS} - V_T)^2} \\
&= 10.8
\end{aligned}$$

The transductance is given by:

$$
\begin{aligned}
g_m &= \frac{dI_{DS}}{dV_{GS}} \\
&= \beta V_{DS,sat} = \beta(V_{GS} - V_T) \\
&= \frac{2I}{(V_{GS} - V_T)^2}(V_{GS} - V_T) \\
&= \frac{2I}{V_{GS} - V_T} \\
&= 0.429\ \frac{\mathrm{mA}}{\mathrm{V}}
\end{aligned}
$$

Question 3

We consider a MOSFET as shown on the figure below:



The parameters of the MOSFET are:
Flatband voltage V_{FB} = -0.5V
Oxide thickness t_{ox}=10nm
Mobility μ = 500 $cm^2/(Vs)$
Channel width W=10 µm
Channel length L = 1 µm
Substrate doping concentration $N_A = 10^{17}$ cm^{-3}
Source/drain doping concentration $N_{S/D} = 10^{20}$ cm^{-3}

1) Calculate for each of the 4 cases below the current-voltage characteristic I_{DS}-V_{DS} and sketch this current –voltage characteristic on a I_{DS}-V_{DS} diagram
   a. V_G = 5V, V_S = 0V, V_B = 0V, V_{DS} from 0 to 5V
   b. V_G = 5V, V_S = 0V, V_B = -5V, V_{DS} from 0 to 5V
   c. V_G = 5V, V_D = 0V, V_B = -5V, V_{DS} from 0 to 5V
   d. V_G = 5V, V_S = 0V, V_B = -5V, V_{DS} from 0 to -5V
2) Calculate the value of the output resistance of the MOSFET around V_{DS}=0V for these 4 cases, as well as the value of the current V_{DS} =5V for cases a, b and c and for V_{DS}=-5V for case d. *(Solution: a. 136 Ohm and 15.4mA b. 167Ohm and 10.3mA c. 167 Ohm and 58.6mA d. 167 Ohm and -58.6mA)*
3) Sketch the band diagram in the channel at the edge of the source as a function of the depth z orthogonal to the gate dielectric for V_{DS} =5V for cases a, b and c and for V_{DS}=-5V for case d. Indicate the location of the quasi-Fermi levels for electrons and holes, the Fermi level of the gate and the band bending in the semiconductor. How much is the total band bending in each of the 4 cases? *(Solution: a. 0.84eV b. 5.84eV c. 0.84eV d. 5.84eV)*
4) Describe in your own words what would change if V_B=0V would be chosen for cases c and d?

Remarks:
- leakage currents of the source and drain diodes can be neglected
- the simplified expressions of the current equations can be used.
- use following values of the material constants:

ε_{ox} = 3.5 10^{-13} F/cm
ε_{Si} = 10^{-12} F/cm
n_i = $10^{10}cm^{-3}$
Vt =kT/q = 0.026V
q = 1.6 10^{-19} C

1.

$$\begin{aligned}\beta &= \frac{W}{L}\mu C_{ox} \\ &= \frac{W}{L}\mu\frac{\epsilon_{ox}}{t_{ox}} \\ &= 1.75\ \frac{\mathrm{mA}}{\mathrm{V}^2}\end{aligned}$$

The threshold voltage is

$$\begin{aligned}V_T &= 2\Phi_f + V_{FB} - \frac{1}{C_{ox}}\sqrt{2qN_A\epsilon_s}\sqrt{|2\Phi_f - V_{BS}|} \\ &= 2V_t\ln\frac{N_A}{n_i} + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\sqrt{|-2V_t\ln\frac{N_A}{n_i} - V_{BS}|} \\ &= 0.838\ \mathrm{V} - 0.5\ \mathrm{V} + \begin{cases}0.468\ \mathrm{V} & a \\ 1.235\ \mathrm{V} & b,d \\ 0.511\ \mathrm{V}^{1/2}\cdot\sqrt{5.838\ \mathrm{V} - V_{DS}} & c\end{cases} \\ &= \begin{cases}0.851\ \mathrm{V} & a \\ 1.613\ \mathrm{V} & b,d \\ 0.383\ \mathrm{V} + 0.511\ \mathrm{V}^{1/2}\cdot\sqrt{5.838\ \mathrm{V} - V_{DS}} & c\end{cases}\end{aligned}$$

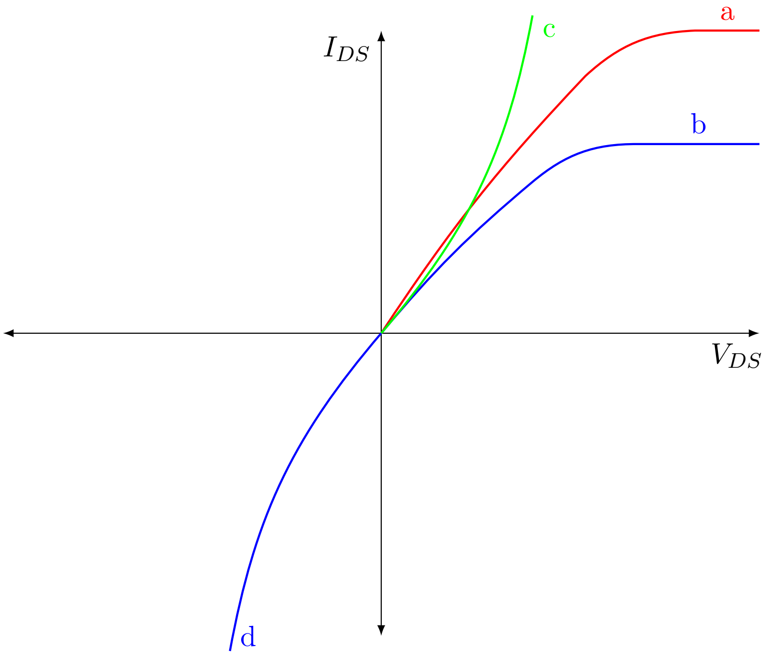
The saturation voltage is:

$$\begin{aligned}V_{DS,sat} &= V_{GS} - V_T \\ &= \begin{cases}4.149\ \mathrm{V} & a \\ 3.387\ \mathrm{V} & b,d \\ 4.617\ \mathrm{V} + V_{DS,sat} - 0.511\ \mathrm{V}^{1/2}\cdot\sqrt{5.838\ \mathrm{V} - V_{DS,sat}} & c \quad (\text{using } V_{GS} = V_{GD} + V_{DS})\end{cases} \\ &= \begin{cases}4.149\ \mathrm{V} & a \\ 3.387\ \mathrm{V} & b,d \\ -76\ \mathrm{V} \implies \text{Always linear} & c \qquad (\text{by solving the equation above})\end{cases}\end{aligned}$$

So the current-voltage characteristics are:

$$I_{DS} = \begin{cases}\beta\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS} & V_{DS} < V_{DS,sat} \\ \beta\frac{(V_{GS}-V_T)^2}{2} & V_{DS} > V_{DS,sat}\end{cases}$$

$$\begin{cases}I_{DS} = \begin{cases}1.75\ \frac{\mathrm{mA}}{\mathrm{V}^2}\left(4.149\ \mathrm{V} - \frac{V_{DS}}{2}\right)V_{DS} & V_{DS} < 4.149\ \mathrm{V} \\ 15.1\ \mathrm{mA} & V_{DS} > 4.149\ \mathrm{V}\end{cases} & a \\ I_{DS} = \begin{cases}1.75\ \frac{\mathrm{mA}}{\mathrm{V}^2}\left(3.387\ \mathrm{V} - \frac{V_{DS}}{2}\right)V_{DS} & V_{DS} > 3.387\ \mathrm{V} \\ 10.0\ \mathrm{mA} & V_{DS} > 3.387\ \mathrm{V}\end{cases} & b \\ I_{DS} = 1.75\ \frac{\mathrm{mA}}{\mathrm{V}^2}\left(4.617\ \mathrm{V} - 0.511\ \mathrm{V}^{1/2}\cdot\sqrt{5.838\ \mathrm{V} - V_{DS}} + \frac{V_{DS}}{2}\right)V_{DS} & c \\ I_{DS} = 1.75\ \frac{\mathrm{mA}}{\mathrm{V}^2}\left(4.617\ \mathrm{V} - 0.511\ \mathrm{V}^{1/2}\cdot\sqrt{5.838\ \mathrm{V} + V_{DS}} - \frac{V_{DS}}{2}\right)V_{DS} & d\end{cases}$$



2.

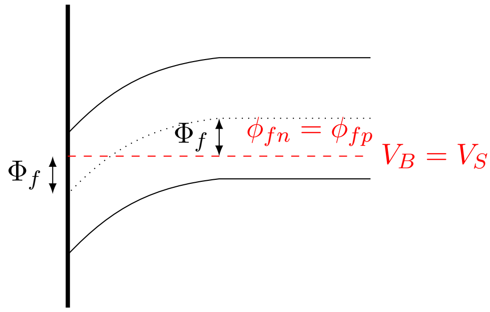
$$
\begin{aligned}
R &= \frac{dV_{DS}}{dI_{DS}} \\
&= \frac{1}{\frac{dI_{DS}}{dV_{DS}}} \\
&= \frac{1}{\begin{cases} 1.75\ \frac{\text{mA}}{\text{V}^2} \cdot 4.149\ \text{V} & a \\ 1.75\ \frac{\text{mA}}{\text{V}^2} \cdot 3.387\ \text{V} & b, c, d \end{cases}} \\
&= \begin{cases} 137.7\ \Omega & a \\ 168.7\ \Omega & b, c, d \end{cases}
\end{aligned}
$$

Just filling in the equation of part 1 gives:

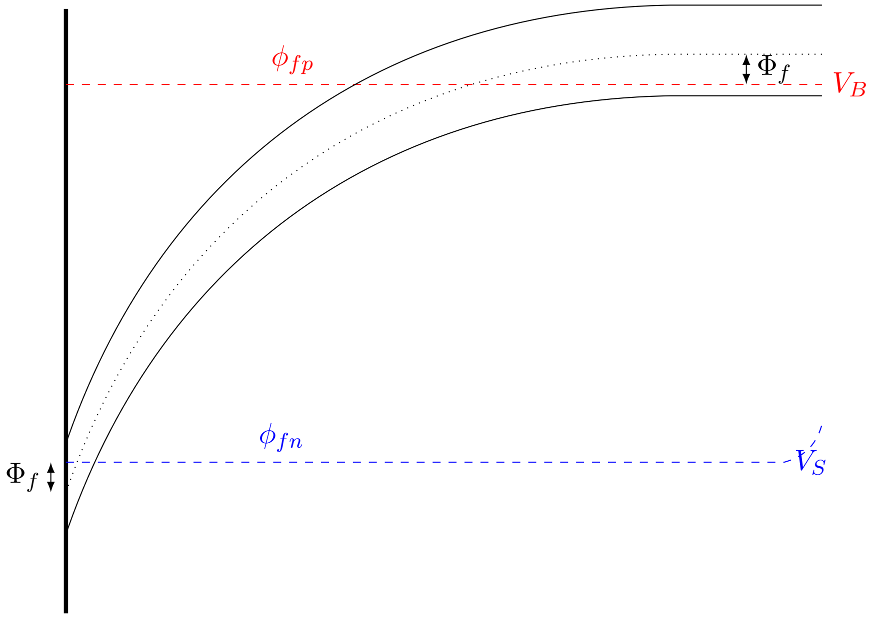
$$
I_{DS}(\pm 5\ \text{V}) = \begin{cases} 15.1\ \text{mA} & a \\ 10\ \text{mA} & b \\ 58.2\ \text{mA} & c \\ -58.2\ \text{mA} & d \end{cases}
$$

3.

a and c: the same, only 5 V displaced. The total band bending is $2\Phi_f = 0.838$ V.



b and d: the same, only 5 V displaced. The total band bending is $2\Phi_f - V_{BS} = 5.838$ V.



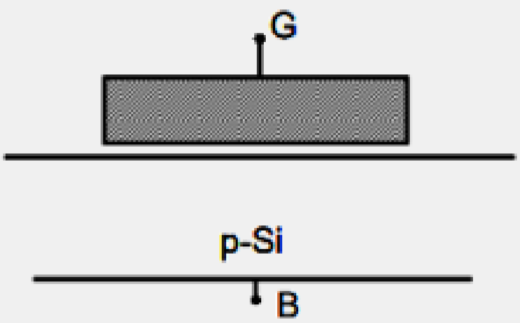
4.

ϕ_{fp} would shift down by 5 V.So, d would look like a. And c would have band bending in the other direction.

Question 4

We consider a MOS capacitor on a p-type substrate with the following parameters:

V_{FB} = -0.5V
t_{ox}=10nm
W=100 µm
L = 100 µm
$N_{sub} = 10^{17}cm^{-3}$



At time t=0 the gate-bulk voltage of the MOS-capacitor is switched from -5V to +5V. The capacitor is measured using a small signal voltage at a frequency of 1 MHz.

1) Calculate the width of the space charge region, the electrostatic potential drop across the silicon and the measured capacitance (in F):
   - before switching *(Solution: 0mm, 0V, 35pF)*
   - immediately after switching
   - at t= ∝

   and this for the 3 following cases:
   a) time constant for minority carrier generation = ∝ (surface recombination/ generation velocity s = 0) – (*Solution: immediately and at t=∝: 0.235 µm, 4.42V, 3.79pF)*
   b) time constant minority carrier generation = 0 (surface recombination/ generation velocity s = ∝) - (*Solution: immediately and at t=∝: 0.102 µm, 0.84V, 35pF)*
   c) time constant minority carrier generation = 100 ms (surface recombination/ generation velocity s = eindig) - (*Solution: immediately: 0.235 µm, 4.42V, 3.79pF, at t=∝: 0.102 µm, 0.84V, 7.64pF)*
2) Draw the band diagram for case c. Indicate the quasi-Fermi levels for electrons and holes, the Fermi level of the gate and the electrostatic potential drop across the silicon:
   - before switching
   - immediately after switching
   - at t=∝

<u>Remarks</u>:
- other generation/recombination mechanisms as well as diffusion can be neglected
- use the following values for the material parameters:

$\varepsilon_{ox} = 3.5\ 10^{-13}$ F/cm
$\varepsilon_{Si} = 10^{-12}$ F/cm
$n_i = 10^{10}cm^{-3}$
Vt =kT/q = 0.026V
$q = 1.6\ 10^{-19}$ C

1)

Before switching the device is in accumulation, so there is no space charge region. $\Delta\Psi_s$ is very small in accumulation, so can be neglected. The capacitance is equal to the oxide capacitance:

$$\begin{aligned} C_{acc} &= AC_{ox} \\ &= WL\frac{\epsilon_{ox}}{t_{ox}} \\ &= 35 \text{ pF} \end{aligned}$$

a)

If there is no generation, we might be in deep depletion. The width of the space charge region is given by:

$$\begin{aligned} \Delta\Psi_{tot} &= V_{GB} - \Phi_{ms} = \frac{qN_A z_D^2}{2\epsilon_s} + \frac{qN_A z_D}{C_{ox}} \\ z_D &= \frac{\epsilon_s}{C_{ox}}\left(\sqrt{1 + \frac{2\epsilon_{ox}^2}{qN_A\epsilon_s t_{ox}^2}(V_{GB} - \Phi_{ms})} - 1\right) \\ &= \frac{\epsilon_s t_{ox}}{\epsilon_{ox}}\left(\sqrt{1 + \frac{2\epsilon_{ox}^2}{qN_A\epsilon_s t_{ox}^2}(V_{GB} - V_{FB})} - 1\right) \\ &= 235 \text{ nm} \end{aligned}$$

The electric potential drop across Si is:

$$\begin{aligned} \Delta\Psi_s &= \frac{qN_A z_D^2}{2\epsilon_s} \\ &= 4.42 \text{ V} \end{aligned}$$

The capacitance is

$$\begin{aligned} \frac{A}{C_{depl}} &= \frac{dV_{GB}}{dQ} \\ &= \frac{d}{qN_A dz_D}\left(\Phi_{ms} + \frac{qN_A z_D^2}{2\epsilon_s} + \frac{qN_A z_D}{C_{ox}}\right) \\ &= \frac{z_D}{\epsilon_s} + \frac{1}{C_{ox}} \\ C_{depl} &= \frac{WL}{\frac{z_D}{\epsilon_s} + \frac{t_{ox}}{\epsilon_{ox}}} \\ &= 3.79 \text{ pF} \end{aligned}$$

b)

If their is very fast generation, we're in inversion. The depletion layer is equal to

$$\begin{aligned} \Delta\Psi_{tot} &= V_{GB} - \Phi_{ms} = \frac{qN_A z_{D,th}^2}{2\epsilon_s} + \frac{qN_A z_{D,th}}{C_{ox}} - \frac{Q_{inv}}{C_{ox}} \\ z_{D,th} &= \sqrt{\frac{2\epsilon_s|2\Psi_f|}{qN_A}} \\ &= \sqrt{\frac{4\epsilon_s V_t \ln\frac{N_A}{n_i}}{qN_A}} \\ &= 102 \text{ nm} \end{aligned}$$

The electric potential drop across Si is:

$$\begin{aligned}\Delta\Psi_s &= \frac{qN_A z_{D,th}^2}{2\epsilon_s} \\ &= 0.84 \text{ V}\end{aligned}$$

The capacitance is

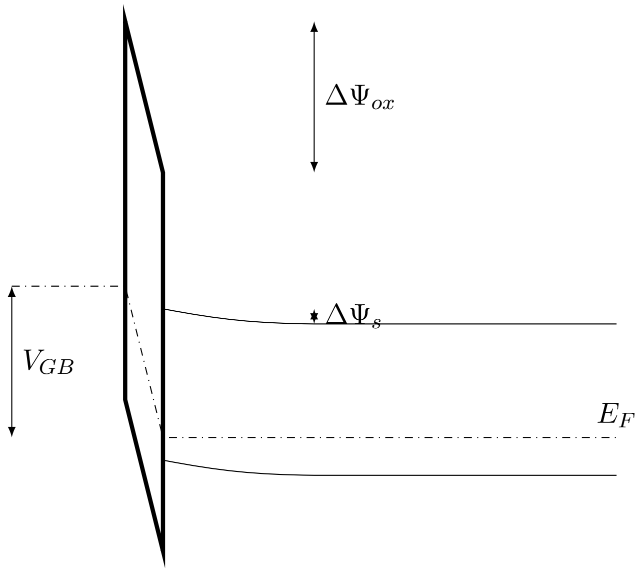
$$\begin{aligned}\frac{A}{C_{inv}} &= \frac{dV_{GB}}{dQ} \\ &= \frac{d}{dQ}\left(\Phi_{ms} + \frac{qN_A z_{D,th}^2}{2\epsilon_s} + \frac{qN_A z_{D,th}}{C_{ox}} - \frac{Q_{inv}}{C_{ox}}\right) \\ &= \frac{1}{C_{ox}} \\ C_{inv} &= AC_{ox} = \frac{WL\epsilon_{ox}}{t_{ox}} \\ &= 35 \text{ pF}\end{aligned}$$

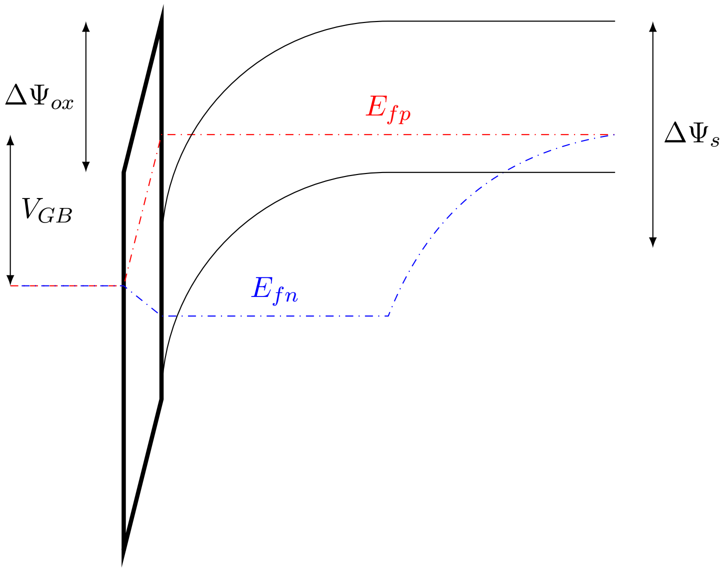
c)

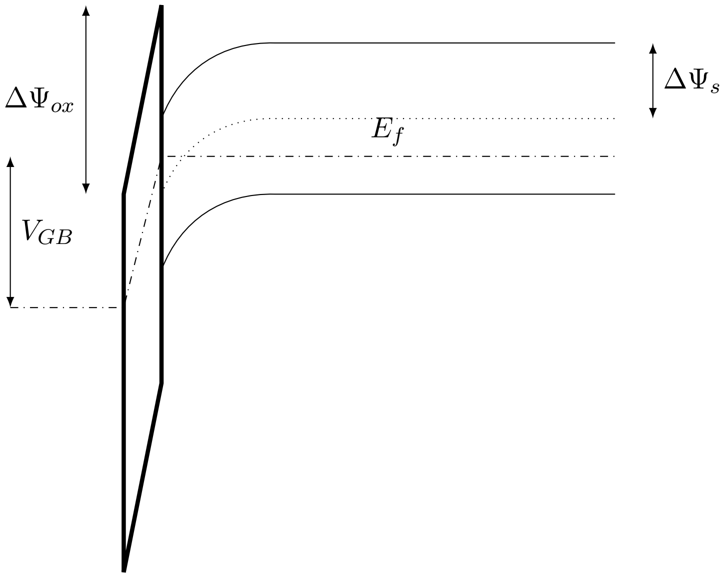
Immediately after switching, no minority carriers can have moved, so the values are the same as in a. After an infinite amount of time, we will reach equilibrium and the space charge region and voltage will go to the values of b. The capacitance can be different, as it is measured for 1 MHz. We need to know whether this is too high for the minority carriers or not. As $f \gg \frac{1}{\tau}$, the minority carriers won't be able to move, so

$$\begin{aligned}C_{high\ freq} &= \frac{WL}{\frac{z_{D,th}}{\epsilon_s} + \frac{t_{ox}}{\epsilon_{ox}}} \\ &= 7.64 \text{ pF}\end{aligned}$$

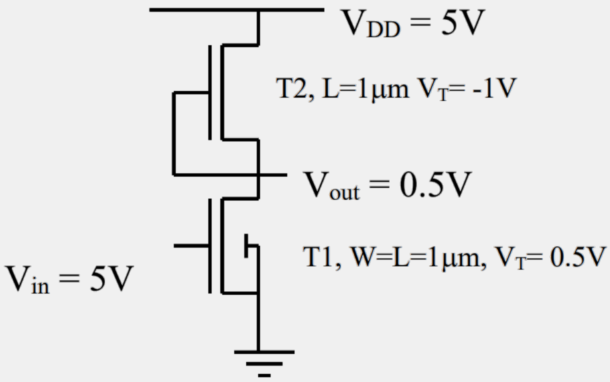
2) (figures not on scale)




$\Delta\Psi_{ox}$
E_{fp}
$\Delta\Psi_s$
V_{GB}
E_{fn}


$\Delta\Psi_{ox}$
$\Delta\Psi_s$
E_f
V_{GB}

Question 5
Consider 2 n-channel MOS transistors connected as shown on the figure



The data for the transistors are as follows: T1: $W_1 = 1$ µm, $L_1 = 1$ µm, $V_{T1} = 0.5$ V, $N_{channel} = 10^{17}$ cm^{-3}, $V_{in} = 5$ V, $t_{ox} = 10$ nm, $\mu = 500$ cm^2/(Vs) (enhancement transistor)
T2 : $L_2 = 1$ µm, $V_{T2} = -1$ V, $t_{ox} = 10$ nm, $\mu = 500$ cm^2/(Vs) (depletion transistor)

a) Calculate the width W_2 of T2 such that $V_{out} = 0.5$ V.
(Answer: 4.25µm)

b) What is the current flowing through both transistors for this condition?
(Answer: 0.372mA)

c) What will be the current through both transistors if we apply a voltage of -5V at the substrate of transistor T1? What will happen with V_{out} in this case?
(Answer: 0.372mA, 0.62V)

d) Draw the band diagram at the source of transistor T1, orthogonal to the gate dielectric, for the two cases: $V_{sub} = 0$ V and $V_{sub} = -5$ V.

Other data are: $\epsilon_{Si} = 10^{-12}$ F/cm, $\epsilon_{ox} = 3.5\ 10^{-13}$ F/cm; $q = 1.6\ 10^{-19}$ C, $n_i = 10^{10}$ cm^{-3}; $V_t = kT/q = 0.026$ V
Note: you can use the simplified expressions for the currents

a., b.

For transistor 1 we have $V_{GS} = 5$ V, $V_{DS} = V_{out} = 0.5$ V $< V_{DS,sat}$ and $\beta = \frac{W}{L}\mu C_{ox} = \frac{W\epsilon_{ox}\mu}{Lt_{ox}} = 0.175\ \frac{\text{mA}}{\text{V}^2}$. The current can be calculated:

$$\begin{aligned} I &= \beta\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS} \\ &= 0.372 \text{ mA} \end{aligned}$$

The same current must run through transistor 2, with $V_{GS} = 0$ V, $V_{DS} = V_{DD} - V_{out} = 4.5$ V $<$ $V_{DS,sat} = 1$ V and $\beta = \frac{W}{L}\mu C_{ox} = \frac{W\epsilon_{ox}\mu}{Lt_{ox}}$ which is unknown.

$$\begin{aligned} I &= \beta\frac{(V_{GS}-V_T)^2}{2} \\ \beta = \frac{W\epsilon_{ox}\mu}{Lt_{ox}} &= \frac{2I}{(V_{GS}-V_T)^2} \\ W &= \frac{2ILt_{ox}}{\epsilon_{ox}\mu(V_{GS}-V_T)^2} \\ &= 4.25\ \mu\text{m} \end{aligned}$$

c.

$K_2 = \frac{1}{C_{ox}}\sqrt{2qN\epsilon_s} = 0.511\ \text{V}^{1/2}$. And $\Phi_f = V_t \ln\frac{N_A}{n_i} = 0.419$ V. So the threshold voltage changes by:

$$\begin{aligned} V_{T1} &= V_{T1,0} + K_2\left(\sqrt{2\Phi_f - V_{BS}} - \sqrt{2\Phi_f}\right) \\ &= 0.5\ \text{V} + 0.768\ \text{V} \\ &= 1.268\ \text{V} \end{aligned}$$

We assume first that the upper transistor remains in saturation, in that case the current remains the same. The voltage is calculated from the current equation of the lower transistor:

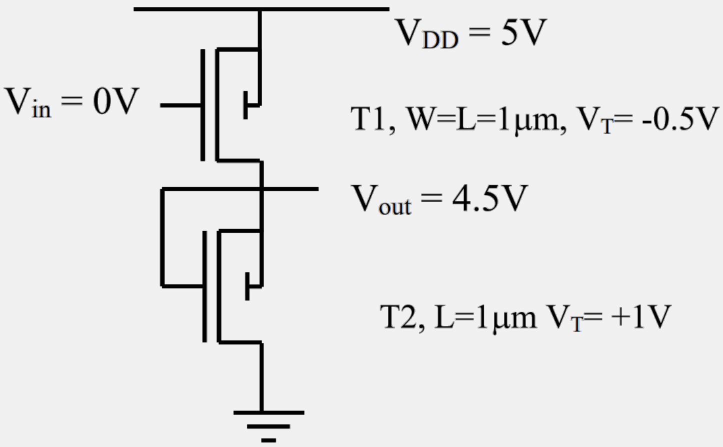
$$\begin{aligned} I &= \beta\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)V_{DS} \\ V_{out} = V_{DS} &= \frac{I}{\beta\left(V_{GS} - V_T - \frac{V_{DS}}{2}\right)} \end{aligned}$$

Because I'm too lazy to solve the quadratic equation, I solve it iteratively. Which gives $V_{out} = 0.621$ V.

d.

Question 6

Consider 2 p-channel MOS transistors that are connected like in the figure below



The data of the p-channel MOSFET transistors are:
T1: $W_1 = 1$ µm, $L_1 = 1$ µm, $V_{T1} = -0.5$ V, $N_{Pchannel} = 10^{17}$ cm^{-3}, $V_{in} = 0$ V, $t_{ox} = 10$ nm, $\mu = 500$ cm2/(Vs) (enhancement transistor)
T2: $L_2 = 1$ µm, $V_{T2} = +1$ V, $t_{ox} = 10$ nm, $\mu = 500$ cm2/(Vs) (depletion transistor)

a) Design the width W2 of transistor T2 to obtain an output voltage $V_{out} = 4.5$ V.
(Solution: $W = 4.25$ µm)

b) What current is flowing through both transistors for this condition?
(Solution: 0.372 mA)

c) What will be the output voltage V_{out} and the current through both transistors if we do not connect the substrate of transistor T1 to the V_{DD} but apply +10 V: $V_{sub}(T1) = 10$ V?
(Solution: 0.372 mA, 4.38 V)

d) Draw the band diagram at the source of transistor T1 as a function of the depth, perpendicular to the channel, and this for the two cases: $V_{sub}(T1) = 5$ V and $V_{sub}(T1) = 10$ V. Indicate the position of the quasi-Fermi levels of holes and electrons and the total electrostatic potential drop in the depletion region.

e) How much will V_{out} be for both cases if we apply $V_{in} = 5$ V? How much current will flow through T1 and T2 for this condition?
(Solution: 0 V, 0 A)

Use the following numerical values for the parameters below: $\epsilon_{Si} = 10^{-12}$ F/cm, $\epsilon_{ox} = 3.5\ 10^{-13}$ F/cm; $q = 1.6\ 10^{-19}$ C, $n_i = 10^{10}$ cm^{-3}; $V_t = kT/q = 0.026$ V
Note: you can use the simplified expressions for the currents (expressions (35) on p. IX.24 and (39) on p. IX.39)

a, b.

$$\begin{aligned}\beta_1 &= \frac{W_1}{L_1}\mu\frac{\epsilon_{ox}}{t_{ox}} \\ &= 0.175\ \frac{\text{mA}}{\text{V}}\end{aligned}$$

For the first transistor we have $V_{DS1} = -0.5$ V and $V_{DS1,sat} = V_{GS1} - V_{T1} = -5\text{ V} + 0.5\text{ V} = -4.5$ V. Thus the transistor is in the linear regime. The current is

$$\begin{aligned} I &= \beta_1 \left(V_{GS1} - V_{T1} - \frac{V_{DS1}}{2} \right) V_{DS1} \\ &= 0.372 \text{ mA} \end{aligned}$$

For the second transistor we have $V_{DS2} = -4.5$ V and $V_{DS2,sat} = V_{GS2} - V_{T2} = 0\text{ V} - 1\text{ V} = -1$ V. Thus the transistor is in the saturation regime. β_2 follows from the current equation:

$$\begin{aligned} I &= \beta_2 \frac{(V_{GS2} - V_{T2})^2}{2} \\ \beta_2 &= \frac{2I}{(V_{GS2} - V_{T2})^2} \\ &= 0.744 \frac{\text{mA}}{\text{V}} \end{aligned}$$

And the width is:

$$\begin{aligned} \beta_2 &= \frac{W_2}{L_2} \mu \frac{\epsilon_{ox}}{t_{ox}} \\ W_2 &= \frac{L_2 \beta_2 t_{ox}}{\mu \epsilon_{ox}} \\ &= 4.25 \ \mu\text{m} \end{aligned}$$

c.

The threshold voltage of the first transistor will change by:

$$\begin{aligned} \Delta V_T &= \frac{t_{ox}}{\epsilon_{ox}} \sqrt{2qN_{Pchannel}\epsilon_s} \left(\sqrt{2\Phi_f} - \sqrt{|2\Phi_f - V_{BS}|} \right) \\ &= \frac{t_{ox}}{\epsilon_{ox}} \sqrt{2qN_{Pchannel}\epsilon_s} \left(\sqrt{2V_t \ln \frac{N_A}{n_i}} - \sqrt{\left| 2V_t \ln \frac{N_A}{n_i} - V_{BS} \right|} \right) \\ &= -0.575 \text{ V} \end{aligned}$$

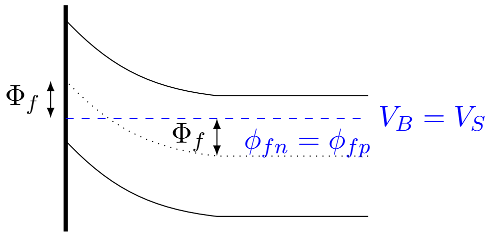
And the new threshold voltage is then $V_{T1} = -1.075$ V.

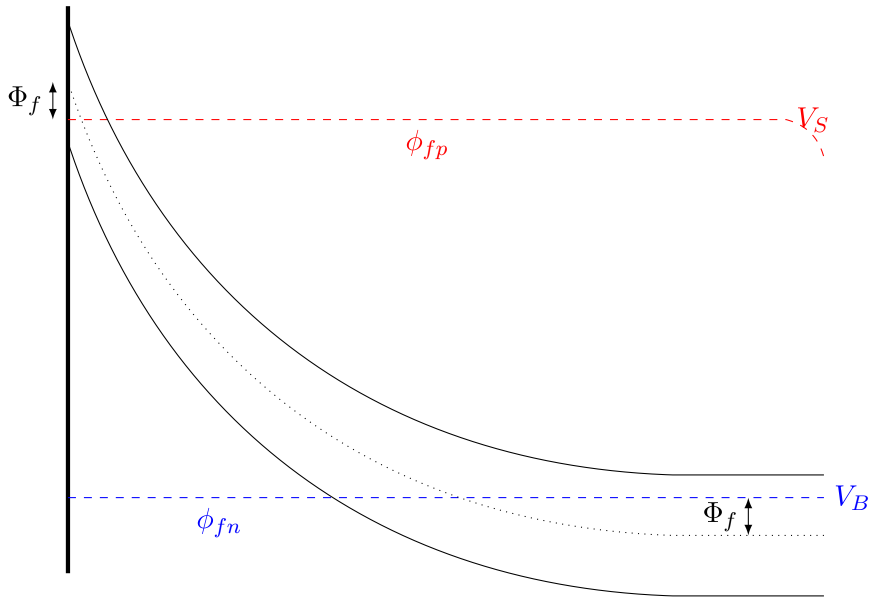
The second transistor will probably still be in saturation.So, the current remains the same. $I = 0.372$ mA. The output voltage then depends on the first transistor. Let's first assume it is in the linear regime:

$$\begin{aligned} I &= \beta_1 \left(V_{GS1} - V_{T1} - \frac{V_{DS1}}{2} \right) V_{DS1} \\ V_{DS1} &= -0.59 \text{ V} \end{aligned}$$

The output voltage is then $V_{out} = V_{DD} + V_{DS1} = 4.41$ V

d.





e.

$V_{GS1} = 0$ V $> V_T$, so there will be no current. The first transistor is an open circuit. And as $I = 0 \implies V_{DS2} = 0$, so $V_{out} = 0$ V.

Question 7

For a MOS-technology the following measurement data were obtained:

- the capacitance measured in accumulation of a MOS-cap on p-substrate with an area of 100x100µm2 was measured to be 17.5pF.
- the threshold voltage of an n-channel MOSFET was measured at 2 values of the bulk voltage, as given in the table below:

V_{BS} [V]	V_T [V]
0	0.45
-5	1.54

a) Extract from these measurement data the following parameters of the MOSFET technology:

- the oxide thickness t_{ox} *(Answer: 20 nm)*
- the substrate doping N_A *(Answer: 4.96 10^{16} cm^{-3})*
- the flatband voltage V_{FB} (Answer: -1V)

b) Calculate which high frequency capacitance one will measure on the MOS-cap at a $V_{GB} = 5$ V *(Answer: 5 pF)*

c) Draw the band diagram in the channel at the source side of the n-MOSFET as a function of the position in the direction perpendicular to the channel of the transistor, and this for $V_{GS} = 5$ V and for $V_{BS} = 0$ V and -5 V. Indicate on this band diagram also the position of the quasi-Fermi levels of holes and electrons, as well as the total electrostatic potential drop in the space charge region.

Use the following numerical values for the constants:
$\epsilon_{Si} = 10^{-12}$ F/cm, $\epsilon_{ox} = 3.5\ 10^{-13}$ F/cm; $q = 1.6\ 10^{-19}$ C, $n_i = 10^{10}$ cm^{-3}; $V_t = kT/q = 0.026$ V

a.

$$
\begin{aligned}
C_{acc} &= A\frac{\epsilon_{ox}}{t_{ox}} \\
t_{ox} &= A\frac{\epsilon_{ox}}{C_{acc}} \\
&= 20 \text{ nm}
\end{aligned}
$$

$$
\begin{aligned}
V_T &= 2V_t \ln\frac{N_A}{n_i} + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\sqrt{2V_t\ln\frac{N_A}{n_i} - V_{BS}} \\
V_{T2} - V_{T1} &= \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\left(\sqrt{2V_t\ln\frac{N_A}{n_i} - V_{BS}} - \sqrt{2V_t\ln\frac{N_A}{n_i}}\right) \\
N_A &= \frac{\epsilon_{ox}^2(V_{T2}-V_{T1})^2}{t_{ox}^2 2q\epsilon_s}\frac{1}{\left(\sqrt{2V_t\ln\frac{N_A}{n_i} - V_{BS}} - \sqrt{2V_t\ln\frac{N_A}{n_i}}\right)^2}
\end{aligned}
$$

This can be solved iteratively, we find $N_A = 4.965\ 10^{16}$ cm^{-3}.

Finally, the flatband voltage can be calculated:

$$\begin{aligned}
V_{T1} &= 2V_t \ln\frac{N_A}{n_i} + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\sqrt{2V_t \ln\frac{N_A}{n_i}} \\
V_{FB} &= V_{T1} - 2V_t \ln\frac{N_A}{n_i} - \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\sqrt{2V_t \ln\frac{N_A}{n_i}} \\
&= -1.00 \text{ V}
\end{aligned}$$

b.

As $V_{GB} = V_{GS} > V_T(V_{BS} = 0)$ we're in inversion.

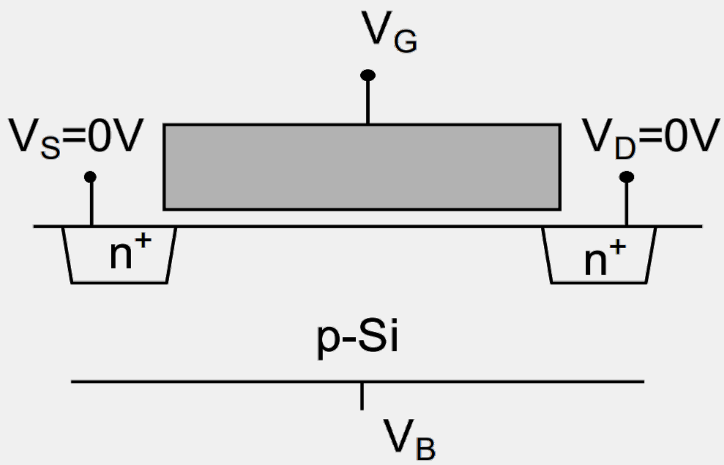
$$C_{min} = \frac{1}{\frac{1}{C_{acc}} + \frac{z_{D,th}}{A\epsilon_s}}$$

With

$$\begin{aligned}
z_{D,th} &= \sqrt{\frac{4\epsilon_s V_t \ln\frac{N_A}{n_i}}{qN_A}} \\
&= 142 \text{ nm}
\end{aligned}$$

The capacitance becomes $C_{min} = 5.02$ pF.

Question 8
Consider a n–channel MOSFET as shown below:



The known parameters of the MOSFET are:
Threshold voltage $V_T = 0.5$ V at $V_{BS} = 0$ V (referred to the source voltage)
Substrate doping concentration $N_A = 2\ 10^{16}$ cm^{-3}
Oxide thickness $t_{ox} = 20$ nm
Mobility $\mu = 500$ cm^2/(Vs)
$W = L = 1$ µm

a) Extract from these known parameters the flatband voltage V_{FB}?
(Solution: $V_{FB} = -0.65$ V)

b) Calculate the width of the space charge region in the substrate and the electrostatic potential drop $\Delta\Psi$ over the space charge region and the resistance of the transistor at $V_{ds} = 0$ V for following cases:

   a. $V_{GS} = 2$ V and $V_{BS} = 0$ V *(Solution: 0.217 µm, 0.754V, 7.62kOhm)*
   b. $V_{GS} = 0$ V and $V_{BS} = 0$ V *(Solution: 0.152 µm, ∞ Ohm)*
   c. $V_{GS} = 2$ V and $V_{BS} = -5$ V *(Solution: 0.6 µm, 5.754V, 14.3kOhm)*
   d. $V_{GS} = 0$ V and $V_{BS} = -5$ V *(Solution: 0.54 µm, 4.67V, ∞Ohm)*

c) Draw the band diagram in the direction perpendicular to the channel for these 4 cases. Indicate on this band diagram the position of the quasi-Fermi levels of holes and electrons, as well as the total electrostatic potential drop in the space charge region.

Use the following numerical values for the constants:
$\epsilon_{Si} = 10^{-12}$ F/cm, $\epsilon_{ox} = 3.5\ 10^{-13}$ F/cm; $q = 1.6\ 10^{-19}$ C, $n_i = 10^{10}$ cm^{-3}; $V_t = kT/q = 0.026$ V

a.

$$
\begin{aligned}
V_T &= 2\Phi_f + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_S}\sqrt{2\Phi_f - V_{BS}} \\
V_{FB} &= V_T - 2V_t\ln\frac{N_A}{n_i} - \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_S}\sqrt{2V_t\ln\frac{N_A}{n_i}} \\
&= 0.5\ \text{V} - 0.754\ \text{V} - 0.397\ \text{V} \\
&= -0.652\ \text{V}
\end{aligned}
$$

b.

For a we have $V_{GS} > V_T$, so we're in inversion, while for b $V_{GS} < V_T$, so the MOSFET is in depletion. For c and d we should calculate $V_T first$

$$
\begin{aligned}
V_T &= 2\Phi_f + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_S}\sqrt{2\Phi_f - V_{BS}} \\
&= 0.754\ \text{V} - 0.652\ \text{V} + 1.097\ \text{V} \\
&= 1.200\ \text{V}
\end{aligned}
$$

So for c we also have $V_{GS} > V_T$, so we're in inversion, while for d $V_{GS} < V_T$, so the MOSFET is in depletion.

For the MOSFETs in inversion, the width of the SCR is:

$$
\begin{aligned}
z_{D,th} &= \sqrt{\frac{2\epsilon_s|2\Phi_f - V_{BS}|}{qN_A}} \\
&= \begin{cases} 0.217\ \mu\text{m} & a \\ 0.600\ \mu\text{m} & c \end{cases}
\end{aligned}
$$

For the MOSFETs in depletion, the width of the SCR is:

$$
\begin{aligned}
z_{D,th} &= \frac{\epsilon_s t_{ox}}{\epsilon_{ox}}\left(\sqrt{1 + \frac{2\epsilon_{ox}^2}{qN_A\epsilon_s t_{ox}^2}\left|\Delta\Psi_{tot} + \frac{Q_{ox}+Q_{ss}}{C_{ox}}\right|} - 1\right) \\
&= \frac{\epsilon_s t_{ox}}{\epsilon_{ox}}\left(\sqrt{1 + \frac{2\epsilon_{ox}^2}{qN_A\epsilon_s t_{ox}^2}|V_{GB} - V_{FB}|} - 1\right) \\
&= \frac{\epsilon_s t_{ox}}{\epsilon_{ox}}\left(\sqrt{1 + \frac{2\epsilon_{ox}^2}{qN_A\epsilon_s t_{ox}^2}|V_{GS} - V_{BS} - V_{FB}|} - 1\right) \\
&= \begin{cases} 0.153\ \mu\text{m} & b \\ 0.540\ \mu\text{m} & d \end{cases}
\end{aligned}
$$

The electrostatic potential drop over the SC is:

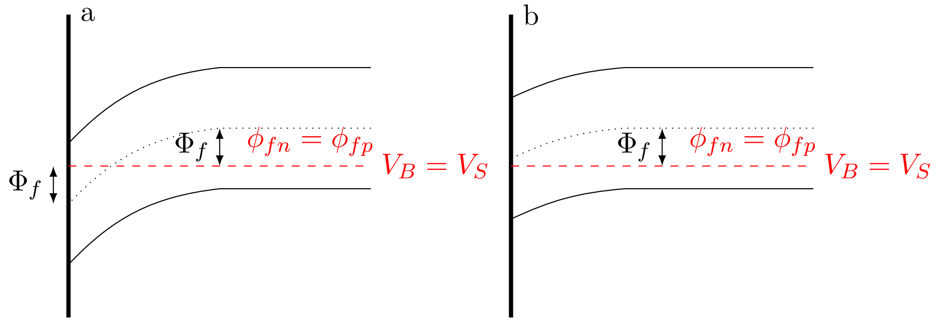
$$
\begin{aligned}
\Delta\Psi_{Si} &= \frac{qN_A z_D^2}{2\epsilon_s} \\
&= \begin{cases} 0.754\ \text{V} & a \\ 0.373\ \text{V} & b \\ 5.754\ \text{V} & c \\ 4.665\ \text{V} & d \end{cases}
\end{aligned}
$$

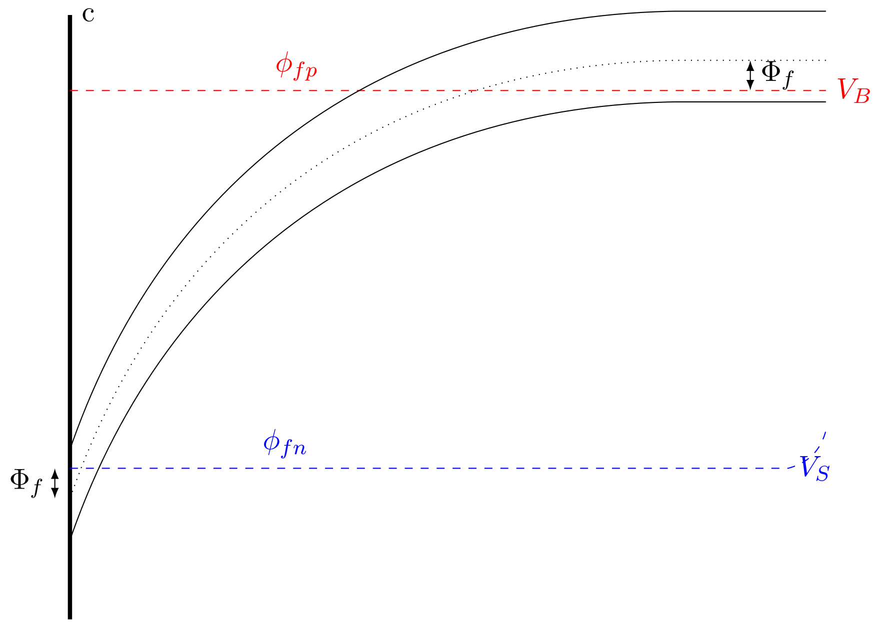
a and c will be in the linear regime (small V_{ds}), while b and d are still in depletion and thus habe an infinte resitance. The resistance for a and c is given by

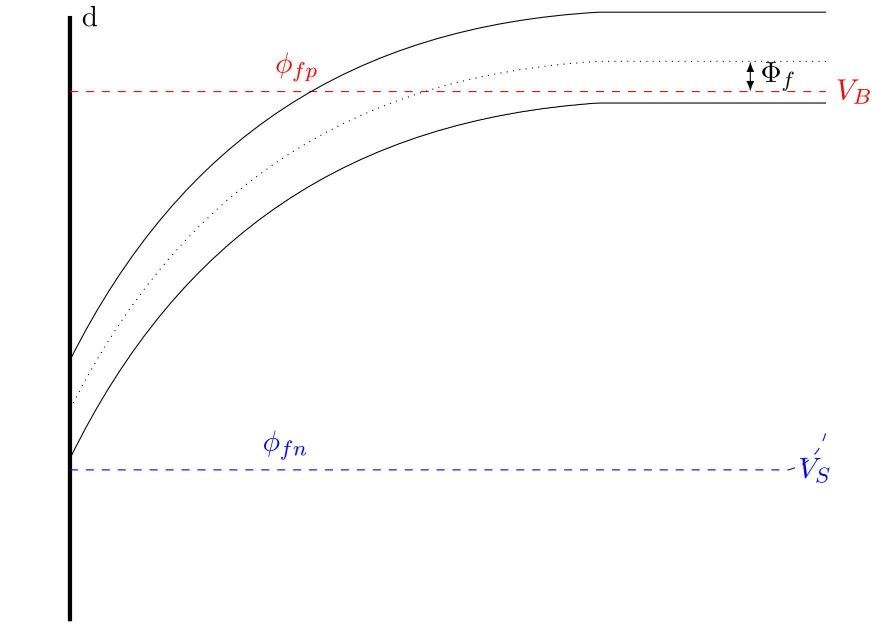
$$\begin{aligned} R &= \frac{1}{\frac{dI_{DS}}{V_{DS}}} \\ &= \frac{1}{\beta(V_{GS} - V_T)} \\ &= \begin{cases} 7619\ \Omega & a \\ 14286\ \Omega & c \end{cases} \end{aligned}$$

With $\beta = \frac{W}{L}\mu\frac{\epsilon_{ox}}{t_{ox}} = 87.5\ \mu\text{A/V}^2$.

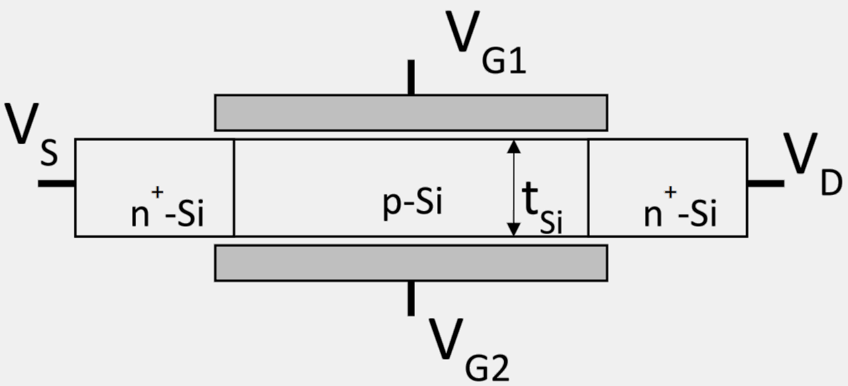
c.




c
ϕ_{fp}
Φ_f
V_B
ϕ_{fn}
V_S
Φ_f


d
ϕ_{fp}
Φ_f
V_B
ϕ_{fn}
V_S

Question 9
Consider a device as shown in the figure below:



The following parameters are known for this device:
Threshold voltage $V_T = 0.5$ V at $V_{BS} = 0$ V (referred to the source voltage), equal at top and back interface.
Substrate doping concentration $N_A = 2\ 10^{16}$ cm^{-3}
Oxide thickness $t_{ox} = 20$ nm (equal at top and back interface)
Thickness of the silicon film $t_{Si} = 2$ µm
Electron mobility in the channel $\mu = 500$ cm^2/(Vs)
$W = L = 1$ µm
$V_{G1} = V_{G2} = 4$ V
$V_S = V_D = 0$ V
At the p-Si body an external voltage $V_B = 0$ V is applied.

a) Calculate the thickness of the silicon film that is not depleted for the applied voltages (the space charge region at the source and drain junctions can be neglected). What is the electrostatic potential drop across the depletion regions in the Si-film?
(Solution: 1.566 µm, 0.754 V)

b) Calculate the resistance between source and drain around $V_{DS} = 0$ V for this situation.
(Solution: 1.63 kOhm)

c) Which voltage V_B do we have to apply at Si-body to make sure that the Si-film is at the edge of full depletion?
(Solution: -15.25 V)

d) Calculate again the resistance between source and drain around $V_{DS} = 0$ V for this new situation.
(Solution: 2.76 kOhm)

e) Sketch the band diagram in the direction perpendicular to the channel for $V_B = 0$ V (case a) and for the condition at the edge of full depletion (case c). Indicate on the band diagram the position of the quasi-Fermi levels for holes and electrons and the total electrostatic potential drop in the depletion regions.

Use the following numerical values for the constants in your calculations:
$\epsilon_{Si} = 10^{-12}$ F/cm, $\epsilon_{ox} = 3.5\ 10^{-13}$ F/cm; $q = 1.6\ 10^{-19}$ C, $n_i = 10^{10}$ cm^{-3}; $V_t = kT/q = 0.026$ V

a.

Before we can calculate the thickness of the depletion layer, we need to know whether we're already in inversion or not. As $V_{GS} > V_T$, this will be the case.

The depletion layer is then:

$$\begin{aligned} z_{D,th} &= \sqrt{\frac{2\epsilon_s|2\Phi_f - V_{BS}|}{qN_A}} \\ &= \sqrt{\frac{4\epsilon_s V_t \ln \frac{N_A}{n_i}}{qN_A}} \\ &= 217 \text{ nm} \end{aligned}$$

So what is left is $t_{Si} - 2z_D = 1.566$ µm. The potential drop over Si is $2\Phi_f = 2V_t \ln \frac{N_A}{n_i} = 0.754$ V (per definition for inversion).

b.

$$\begin{aligned} R &= \frac{1}{\frac{dI_{DS}}{dV_{DS}}} \\ &= \frac{1}{\beta(V_{GS} - V_T)} \end{aligned}$$

With $\beta = 2\frac{W}{L}\mu\frac{\epsilon_{ox}}{t_{ox}} = 0.175$ mA/V^2 (we have 2 channels in parallel):

$$R = 1633\ \Omega$$

c.

We need to make sure that $2z_{D,th} = t_{Si}$

$$\begin{aligned} t_{Si} = 2z_{D,th} &= 2\sqrt{\frac{2\epsilon_s|2\Phi_f - V_{BS}|}{qN_A}} \\ V_{BS} &= 2\Phi_f - \frac{t_{Si}^2 qN_A}{8\epsilon_s} \\ &= -15.25 \text{ V} \end{aligned}$$

d.

The threshold voltage will have changed:

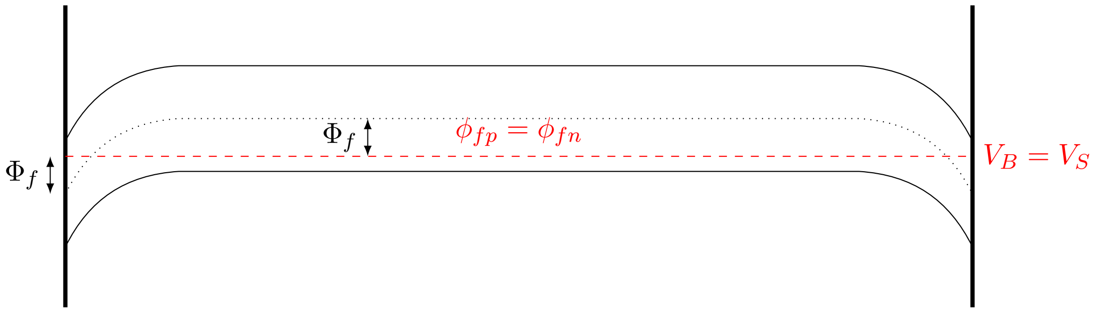
$$\begin{aligned} \Delta V_T &= \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\left(\sqrt{2\Phi_f - V_{BS}} - \sqrt{2\Phi_f}\right) \\ &= \frac{t_{ox}}{\epsilon_{ox}} qN_A \left(z_{D,th2} - z_{D,th1}\right) \\ &= 1.432 \text{ V} \end{aligned}$$

And the new threshold voltage is thus $V_T = 1.932$ V, so the resistance shall be

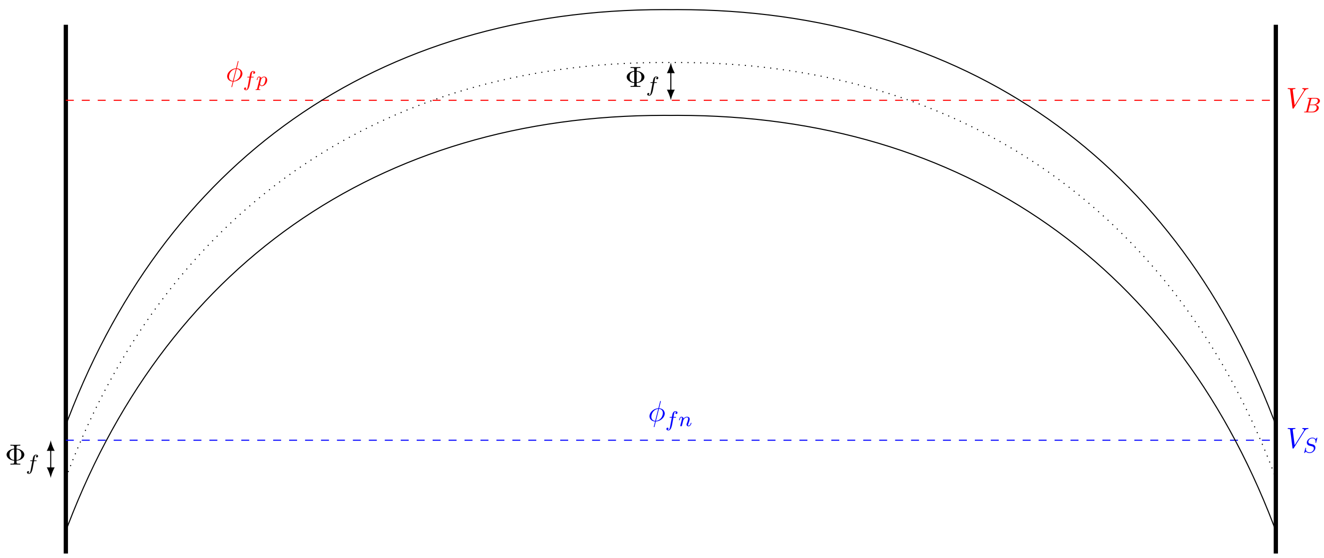
$$\begin{aligned} R &= \frac{1}{\frac{dI_{DS}}{dV_{DS}}} \\ &= \frac{1}{\beta(V_{GS} - V_T)} \\ &= 2763\ \Omega \end{aligned}$$

e.

For $V_{BS} = 0$ V. The potential drop over the SCR is $\Delta\Psi_{Si} = 2\psi_f = 0.754$ V.

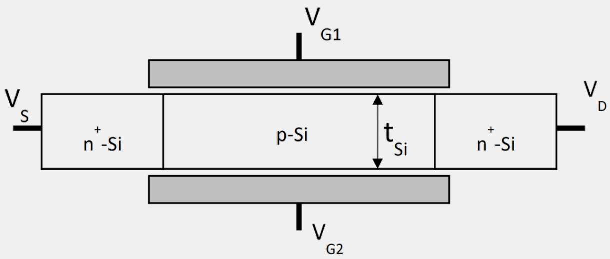


For $V_{BS} = -15.25$ V. The potential drop over the SCR is $\Delta\Psi_{Si} = 2\psi_f - V_{BS} = 16.00$ V. (figure not on scale)



Question 10

We consider a structure like shown in the figure below:



The following parameters are known for this structure:
Threshold voltage $V_T = 0.5$ V at $V_{BS} = 0$ V (referred to the source voltage), identical at top and back interface.
Substrate doping $N_A = 2\ 10^{16}$ cm^{-3}
Oxide thickness $t_{ox} = 20$ nm (identical at top and back interface)
Thickness of the silicon film $t_{Si} = 0.8$ µm
Mobility $\mu = 500$ cm^2/(Vs)
$W = L = 1$ µm
$V_{G1} = V_{G2}$; $V_S = V_D = 0$ V; The p-Si film is connected to an external voltage VB.

a) Extract from these known parameters the flatband voltage V_{FB}? *(Answer: -0.65V)*

b) Calculate the thickness of the silicon film that is not depleted for the following applied voltages and for $V_B = 0$ V (the space charge region of the source and drain can be neglected):
$V_{G1} = V_{G2} = 5$ V *(Answer: 0.366 µm)*
$V_{G1} = V_{G2} = 0$ V *(Answer: 0.496 µm)*
Calculate the electrostatic potential drop across the space charge regions and the resistance between source and drain around $V_{DS} = 0$ V for these 2 conditions?
(Answer: 0.754 V, 1.27 kOhm for $V_G = 5$ V and 0.370 V, infinite resistance for $V_G = 0$ V)

c) Draw the band diagram in the direction perpendicular to the channel for these two conditions. Indicate on this band diagram the position of the quasi-Fermi levels of holes and electrons, as well as the total electrostatic potential drop in the space charge regions.

d) We now apply a bulk voltage $V_B = -5$ V. Calculate the gate voltages $V_{G1} = V_{G2}$ in order to bring the Si-film at the edge of full depletion (i.e the full Si film is depleted)?
(Answer: -2.36 V)

e) Calculate again the electrostatic potential drop across the space charge regions and the resistance between source and drain around $V_{DS} = 0$ V for this condition?
(Answer: 2.56 V, infinite resistance)

f) Draw again the band diagram in the direction perpendicular to the channel for this condition at the edge of full depletion. Indicate on this band diagram the position of the quasi-Fermi levels for holes and electrons as well as the total electrostatic potential drop in the space charge regions

Use the following numerical values for the constants
$\epsilon_{Si} = 10^{-12}$ F/cm, $\epsilon_{ox} = 3.5\ 10^{-13}$ F/cm; $q = 1.6\ 10^{-19}$ C, $n_i = 10^{10}$ cm^{-3}; $V_t = kT/q = 0.026$ V

a.

$$
\begin{aligned}
V_T &= 2\Phi_f + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A}\sqrt{2\Phi_f - V_{BS}} \\
V_{FB} &= V_T - 2V_t \ln\frac{N_A}{n_i} - \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A}\sqrt{2V_t\ln\frac{N_A}{n_i}} \\
&= 0.5\text{ V} - 0.754\text{ V} - 0.397\text{ V} \\
&= -0.652\text{ V}
\end{aligned}
$$

b.

If $V_G = 5\text{ V} > V_T$, the MOSFET is in inversion and if $V_G = 0\text{ V} < V_T$ the MOSFET is in depletion. In the first case

$$
\begin{aligned}
z_{D,th} &= \sqrt{\frac{2\epsilon_s 2\Phi_f}{qN_A}} \\
&= 0.217\ \mu\text{m}
\end{aligned}
$$

So what is left is $t_{Si} - 2z_{D,th} = 0.366\ \mu\text{m}$.

In the second case:

$$
\begin{aligned}
z_D &= \frac{\epsilon_s t_{ox}}{\epsilon_{ox}}\left(\sqrt{1 + \frac{2\epsilon_{ox}^2}{qN_A\epsilon_s t_{ox}^2}\left|\Delta\Psi_{tot} + \frac{Q_{ox}+Q_{ss}}{C_{ox}}\right|} - 1\right) \\
&= \frac{\epsilon_s t_{ox}}{\epsilon_{ox}}\left(\sqrt{1 + \frac{2\epsilon_{ox}^2}{qN_A\epsilon_s t_{ox}^2}\left|V_{GB} - V_{FB}\right|} - 1\right) \\
&= 0.153\ \mu\text{m}
\end{aligned}
$$

So what is left is $t_{Si} - 2z_D = 0.495\ \mu\text{m}$.

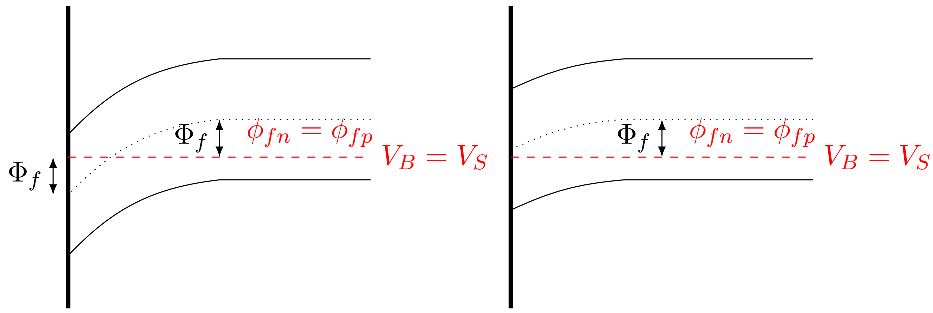
The electrostatic potential over the SC is given by

$$
\begin{aligned}
\Delta\Psi_{Si} &= \frac{qN_A z_D^2}{2\epsilon_s} \\
&= \begin{cases} 0.754\text{ V} & V_{G1} = V_{G2} = 5\text{ V} \\ 0.373\text{ V} & V_{G1} = V_{G2} = 0\text{ V} \end{cases}
\end{aligned}
$$

In the second case $V_G < V_T$ so the resistance is infinite. In the first case $\beta = 2\frac{W}{L}\mu\frac{\epsilon_{ox}}{t_{ox}} = 0.175\text{ mA/V}^2$ (2 separate channels) and he resistance is

$$
\begin{aligned}
R &= \frac{1}{\frac{dI_{DS}}{dV_{DS}}} \\
&= \frac{1}{\beta(V_{GS} - V_T)} \\
&= 1270\ \Omega
\end{aligned}
$$

c. (only half drawn, apply symmetry)



d.

First we check whether it is possible to bring the MOSFET to full depletion, the required $z_D = 0.4$ µm:

$$\begin{aligned} z_{D,th} &= \sqrt{\frac{2\epsilon_s(2\Phi_f - V_{BS})}{qN_A}} \\ &= 0.600 \text{ µm} \end{aligned}$$

So $2z_{D,th} > t_{ox} \implies$ okay!

$$\begin{aligned} z_D &= \frac{\epsilon_s t_{ox}}{\epsilon_{ox}}\left(\sqrt{1 + \frac{2\epsilon_{ox}^2}{qN_A\epsilon_s t_{ox}^2}|V_{GB} - V_{FB}|} - 1\right) \\ V_{GB} &= V_{FB} \pm \frac{qN_A\epsilon_S t_{ox}^2}{2\epsilon_{ox}^2}\left[\left(z_D\frac{\epsilon_{ox}}{\epsilon_S t_{ox}} + 1\right)^2 - 1\right] \\ &= -0.652 \text{ V} + 3.291 \text{ V} \\ &= 2.639 \text{ V} \end{aligned}$$

$V_G = V_{GB} + V_B = -2.36$ V.

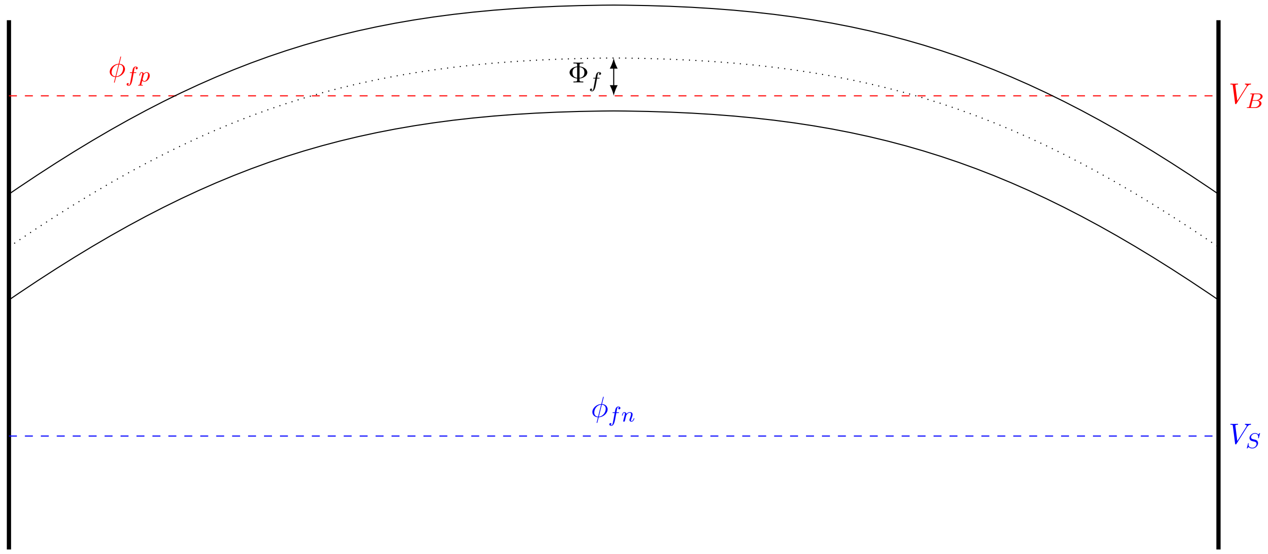
e.

The electrostatic potential over the SCR is:

$$\begin{aligned} \Delta\Psi_{Si} &= \frac{qN_A z_D^2}{2\epsilon_s} \\ &= 2.56 \text{ V} \end{aligned}$$

From this we can immediately see that the MOSFET is not in inversion ($\Delta\Psi < -V_{BS} + 2\Phi_f$). So, the resistance is infinite

f.


ϕ_{fp}
Φ_f
V_B
ϕ_{fn}
V_S

Question 11

A p-n diode has the following properties that we assume independent of temperature:

$$\begin{aligned} N_A &= 10^{19}\ \mathrm{cm}^{-3} \\ N_D &= 10^{16}\ \mathrm{cm}^{-3} \\ N_C &= 1.7\ 10^{19}\ \mathrm{cm}^{-3} \\ N_V &= 1.7\ 10^{19}\ \mathrm{cm}^{-3} \\ E_g &= 1.1\ \mathrm{eV} \\ W_n = W_p &= 1\ \mu\mathrm{m}\ \text{(p and n region are both short regions)} \end{aligned}$$

Furthermore, we assume that the electron mobility and minority carrier lifetimes are temperature **dependent**:

300 K: $\mu_n = 1500$ cm^2/Vs; $\mu_p = 700$ cm^2/Vs; τ_e (lifetime electrons in p-region)= 1 ms; τ_h (lifetime holes in n-region)= 10 ms

200 K: $\mu_n = 3000$ cm^2/Vs; $\mu_p = 1400$ cm^2/Vs; τ_e (lifetime electrons in p-region)= 10 ms; τ_h (lifetime holes in n-region)= 100 ms

Nature constants:
$\epsilon_{Si} = 10^{-12}$ F/cm; $\epsilon_{ox} = 10^{-13}$; $q = 1.6\ 10^{19}$ C; $k = 1.38\ 10^{-23}$ J/K
We operate the diode at 300 K and at 200 K.
Answer the following 5 questions:

1) Calculate the extent of the space charge region width at 300 K and 200 K.
(Answer: 0.333 μm at 300K and 0.347 μm at 200K)

2) What is the inverse of the slope of ln(current) versus voltage at 300 K and at 200 K, and this for forward bias in the region where the diode current is dominated by diffusion?
(Answer: 59mV/dec and 39mV/dec)

3) Calculate the saturation current density (i.e. the prefactor to the exponent of the diffusion current density) at 300 K and at 200K.
(Answer: 2.5 10^{-10} A/cm^2 at 300K and 1.2 10^{-19} A/cm^2 at 200K)

4) Calculate the electron and hole transit times at 300 K and 200K.
(Answer: 0.4ns at 300K and 0.3ns at 200K) ⇒ this answer mistakenly just sums τ_{fn} and τ_{fp} to come to a general number for τ_f. However, for determining the total transit time, this formula should be used: $\tau_f = \frac{J_n}{J_n+J_p}\tau_{fn} + \frac{J_p}{J_n+J_p}\tau_{fp}$.

5) We bias the diode at $V_{pn} = 0.3$ V in forward bias, and apply a small signal voltage on top of this DC bias. Estimate without calculating whether the maximum frequency will be higher at 300 K or at 200 K and explain.

1)

The intrinsic carrier concentration is

$$\begin{aligned} n_i &= \sqrt{N_C N_V e^{-\frac{E_g}{kT}}} \\ &= \begin{cases} 9.98\ 10^{9}\ \mathrm{cm}^{-3} & T = 300\ \mathrm{K} \\ 2.42\ 10^{5}\ \mathrm{cm}^{-3} & T = 200\ \mathrm{K} \end{cases} \end{aligned}$$

The built-in voltage is

$$
\begin{aligned}
V_{bi} &= \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) \\
&= \begin{cases} 0.894\ \text{V} & T = 300\ \text{K} \\ 0.963\ \text{V} & T = 200\ \text{K} \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
W_{SCR} &\approx -X_n \\
&\approx \sqrt{\frac{2\epsilon_s}{qN_D}V_{bi}} \\
&= \begin{cases} 0.334\ \mu\text{m} & T = 300\ \text{K} \\ 0.347\ \mu\text{m} & T = 200\ \text{K} \end{cases}
\end{aligned}
$$

2)

As $I \sim e^{\frac{V_{pn}}{V_t}}$, $\log I \sim \frac{V_{pn}}{V_t}\log e$ and $\frac{V_{pn}}{\log I} \sim \ln(10)V_t$, which is 59.5 meV/dec at 300 K and 39.7 meV/dec at 200 K.

3)

$$
\begin{aligned}
J_S &= qV_t n_i^2\left(\frac{1}{B_p}+\frac{1}{B_n}\right) \\
&= qV_t n_i^2\left(\frac{\mu_p}{N_D(W_n+X_n)}+\frac{\mu_n}{N_A(W_p-X_p)}\right) \\
&\approx \frac{qV_t n_i^2\mu_p}{N_D(W_n+X_n)} \\
&= \begin{cases} \frac{1.60\ 10^{-19}\ \text{C}\cdot 0.0259\ \text{V}\cdot\left(9.98\ 10^{9}\ \text{cm}^{-3}\right)^2\cdot 700\ \text{cm}^2/\text{Vs}}{10^{16}\ \text{cm}^{-3}(1-0.334)\cdot 10^{-4}\ \text{cm}} & T = 300\ \text{K} \\ \frac{1.60\ 10^{-19}\ \text{C}\cdot 0.0172\ \text{V}\cdot\left(2.42\ 10^{5}\ \text{cm}^{-3}\right)^2\cdot 1400\ \text{cm}^2/\text{Vs}}{10^{16}\ \text{cm}^{-3}(1-0.347)\cdot 10^{-4}\ \text{cm}} & T = 200\ \text{K} \end{cases} \\
&= \begin{cases} 4.34\ 10^{-10}\ \text{A}/\text{cm}^2 & T = 300\ \text{K} \\ 3.46\ 10^{-19}\ \text{A}/\text{cm}^2 & T = 200\ \text{K} \end{cases}
\end{aligned}
$$

(Quite large deviation from answer)

Comment: I think that the large deviation from the answer is due to the approximation $J_S \approx \frac{qV_t n_i^2\mu_p}{N_D(W_n+X_n)}$. Even though $X_p \to 0$, $\frac{\mu_n}{N_A(W_p-X_p)}$ does not go to zero, hence the approximation is not correct. Calculating the expression before the approximation is possible, but tedious. I got $J_S = 2.2\cdot 10^{-10}$A/cm^2 for $T = 300$K and $J_S = 1.7\cdot 10^{-19}$A/cm^2 for $T = 200$K, which is unfortunately also not exactly the provided answer.

It is the $N_A \gg N_D$ that allows the approximation. Also if you add an extra term, the result should be larger, right?

4)

The transit time for electrons is:

$$\begin{aligned}
\tau_{fn} &= \frac{Q_n}{J_n} \\
&= \frac{qn_{p0}(W_p - X_p)}{2} \cdot \frac{B_n}{qV_t n_i^2} \\
&= \frac{qn_i^2(W_p - X_p)}{2N_A} \cdot \frac{N_A(W_p - X_p)}{qV_t \mu_n n_i^2} \\
&= \frac{(W_p - X_p)^2 N_A}{2V_t \mu_n N_A} \\
&\approx \frac{W_p^2}{2V_t \mu_n} \\
&= \begin{cases} 0.129 \text{ ns} & T = 300 \text{ K} \\ 0.097 \text{ ns} & T = 200 \text{ K} \end{cases}
\end{aligned}$$

The transit time for holes is:

$$\begin{aligned}
\tau_{fp} &\approx \frac{(W_n + X_n)^2}{2V_t \mu_p} \\
&= \begin{cases} 0.122 \text{ ns} & T = 300 \text{ K} \\ 0.089 \text{ ns} & T = 200 \text{ K} \end{cases}
\end{aligned}$$

0.276 ns and 0.208 ns respectively if we neglect X_n.

The total transit time is:

$$\begin{aligned}
\tau_f &\approx \frac{Q_n}{J_n} \\
&= q\frac{n_{p0}W_p + p_{n0}(W_n + X_n)}{2} \cdot \frac{1}{qV_t n_i^2 \left(\frac{1}{B_n} + \frac{1}{B_p}\right)} \\
&\approx \frac{p_{n0}(W_n + X_n)}{2} \cdot \frac{B_p}{qV_t n_i^2} \\
\tau_f &\approx \tau_{fp}
\end{aligned}$$

5)

The transit time is the smallest at 200 K, so $f_{3dB,max} = \frac{1}{2\pi\tau}$ is the largest at 200 K.

Question 12

Consider a one-sided abrupt silicon p$^+$-n diode, with doping levels $N_A = 10^{19}$ cm^{-3} and $N_D = 10^{15}$ cm^{-3}.

a) At room temperature we measure at a forward bias of 0.15V a current of 0.16 mA. What will be the reverse saturation current if the junction does not contain generation/recombination centers?
(Solution: 500nA)

b) Assume that the n-region contains a concentration $N_{tr} = 10^{15}$ cm^{-3} of generationrecombination centers, positioned at the intrinsic energy level, and with identical electron and hole capture cross sections $\sigma_n = \sigma_p = 10^{-15}$ cm^2. The bulk of the n-region, far away from the junction, is illuminated by light which causes a generation $G = 10^{22}$ cm^{-3}s^{-1} of electron-hole pairs. Calculate the steady-state hole concentration in the n-region (far away from the junction). Draw the band diagram of the n-region far away from the junction and indicate the exact positions of the Fermi levels for electrons and holes.
(Solution: 1.6 10^{15} cm^{-3}, electron quasi-Fermi level 0.324 eV above E_i, hole quasi-Fermi level 0.312 eV below E_i)

c) Consider the n-region of part b) (i.e. with generation-recombination centers) in the dark at the junction with the p$^+$ region. How much is the generation current density at a reverse bias of 10 V?
(Solution: 2.9 μA/cm2)

Use the following numerical values for the constants: $\epsilon_{Si} = 10^{-12}$ F/cm; $q = 1.6\ 10^{-19}$ C, $n_i = 10^{10}$ cm^{-3}; thermal voltage $V_t = kT/q = 0.026$ V, thermal velocity $v_{th} = 10^7$ cm/s

a)

The saturation current is calculated from the following equation

$$\begin{aligned} J &= J_S\left(e^{\frac{V_{pn}}{V_t}} - 1\right) \\ J_S &= \frac{J}{e^{\frac{V_{pn}}{V_t}} - 1} \\ &= \frac{0.16\ 10^{-3}\ \text{A}}{e^{\frac{0.15}{0.0259}} - 1} \\ &= 4.90\ 10^{-7}\ \text{A} \end{aligned}$$

b)

$$\begin{aligned} \tau &= \frac{1}{\sigma v_{th} N_{tr}} \\ &= 100\ \text{ns} \end{aligned}$$

The generation is quite high (in the order of $G \cdot \tau = 10^{15} = N_D$) so the exact formula needs to be used:

$$\begin{aligned} G = U &= \frac{1}{\tau}\frac{pn - n_i^2}{n + p + 2n_i} \\ &\approx \frac{1}{\tau}\frac{pn}{n + p} \end{aligned}$$

With $n \approx p + N_D$

$$\begin{aligned}
G &\approx \frac{1}{\tau}\frac{p(p+N_D)}{2p+N_D} \\
0 &= p^2 + (N_D - 2G\tau)p - N_D G\tau \\
p &= \frac{2G\tau - N_D + \sqrt{(2G\tau - N_D)^2 + 4N_D G\tau}}{2} \\
&= 1.62\ 10^{15}\ \text{cm}^{-3}
\end{aligned}$$

and so $n = p + N_D = 2.62\ 10^{15}\ \text{cm}^{-3}$.

The quasi-fermi levels are

$$\begin{aligned}
E_{fn} &= E_i + kT \ln \frac{n}{n_i} \\
E_{fn} - E_i &= 0.323\ \text{V} \\
E_{fp} - E_i &= -kT \ln \frac{n}{n_i} \\
&= -0.311\ \text{V}
\end{aligned}$$

c)

The lifetime is

$$\begin{aligned}
\tau^* &= \frac{\sigma_n + \sigma_p}{\sigma_n \sigma_p v_{th} N_{rg}} \\
&= \frac{2}{\sigma v_{th} N_{rg}} \\
&= 200\ \text{ns}
\end{aligned}$$

The built-in voltage is given by

$$\begin{aligned}
V_{bi} &= V_t \ln \frac{N_A N_D}{n_i^2} \\
&= 0.835\ \text{V}
\end{aligned}$$

The space charge region width is

$$\begin{aligned}
W_{SCR} &= \sqrt{\frac{2\epsilon_{Si}}{q}\frac{N_A + N_D}{N_A N_D}(V_{bi} - V_{pn})} \\
&\approx \sqrt{\frac{2\epsilon_{Si}}{qN_D}(V_{bi} - V_{pn})} \\
&= 3.68\ \mu\text{m}
\end{aligned}$$

The generation current is then:

$$\begin{aligned}
J_{gen} &= qW_{SCR}\frac{n_i}{\tau^*} \\
&= 2.94\ \mu\text{A/cm}^2
\end{aligned}$$

Question 13

An n-MOS transistor has the following properties that we assume independent of temperature:
Workfunction of the gate metal = 4.1eV
Electron affinity (this is energy difference between the bottom of the conduction band and the vacuum level) = 4.0eV
Oxide thickness tox = 5 nm
Width = 0.1 µm
Length = 0.1 µm
Substrate doping 5x10^{17}cm-3
NC (effective density of states in the conduction band) = 1.7 10^{19} cm-3
NV (effective density of states in the valence band) = 1.7 10^{19} cm-3
Eg (bandgap)=1.1eV
Qox=Qss=0
Furthermore, we assume that the electron mobility is temperature dependent:
μ_n=1500 cm2/Vs @ 300K
μ_n=1000 cm2/Vs @ 400K
Nature constants: $\epsilon_s = 10^{-12}$ F/cm; $\epsilon_{ox} = 3.5\ 10^{-13}$ F/cm; $q = 1.6\ 10^{-19}$ C; $k = 1.38\ 10^{-23}$ J/K
We first operate the transistor at 300K. Then, we heat up the transistor to 400K.
Answer the following 4 questions:

1) Calculate the flatband voltage at 300K and 400K
   (Solution: -0.91V at 300K, -0.88V at 400K)

2) Calculate the threshold voltage (at $V_{BS} = 0$ V) at 300K and 400K
   (Solution: 0.564V at 300K, 0.515V at 400K)

3) We operate the transistor at $V_{DS} = V_{GS} = 1$ V and $V_{BS} = 0$ V. How large is the current at 300K and at 400K? Explain where the difference comes from
   (Solution: 98 µA at 300K and 81 µA at 400K)

4) Does the sensitivity of the threshold voltage to V_{BS} change when going from 300K to 400 K? If so, how much?

1.

$$
\begin{aligned}
V_{FB} &= \phi_{ms} - \frac{Q_{ox} + Q_{ss}}{C_{ox}} \\
&= \phi_{ms} - 0 \\
&= \phi_m - E_{aff} - \frac{E_g}{q} - \frac{kT}{q} \ln \frac{N_A}{N_V} \\
&= 4.1 - 4.0 - 1.1 + \begin{cases} 0.091 & T = 300 \text{ K} \\ 0.122 & T = 400 \text{ K} \end{cases} \\
&= \begin{cases} -0.909 & T = 300 \text{ K} \\ -0.878 & T = 400 \text{ K} \end{cases}
\end{aligned}
$$

2.

$$
\begin{aligned}
V_T &= 2\Phi_f + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\sqrt{|2\Phi_f - V_{BS}|} \\
&= 2\frac{kT}{q}\ln\frac{N_A}{n_i} + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{4kTN_A\epsilon_s \ln\frac{N_A}{n_i}} \\
&= 2\left(\frac{E_g}{2q} + \frac{kT}{q}\ln\frac{N_A}{N_C}\right) + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{4qN_A\epsilon_s\left(\frac{E_g}{2q} + \frac{kT}{q}\ln\frac{N_A}{N_C}\right)} \\
&= \begin{cases} 0.918 - 0.909 + 0.547 & T = 300\text{ K} \\ 0.857 - 0.878 + 0.529 & T = 400\text{ K} \end{cases} \\
&= \begin{cases} 0.556 & T = 300\text{ K} \\ 0.504 & T = 400\text{ K} \end{cases}
\end{aligned}
$$

3.

I use the values of the solution for V_T. We're in the saturation regime:

$$
\begin{aligned}
I &= \beta\frac{(V_{GS} - V_T)^2}{2} \\
&= \frac{W}{L}\mu C_{ox}\frac{(V_{GS} - V_T)^2}{2} \\
&= \begin{cases} 1.05\ \frac{\text{mA}}{\text{V}^2} \cdot 0.095\ \text{V}^2 & T = 300\text{ K} \\ 0.7\ \frac{\text{mA}}{\text{V}^2} \cdot 0.118\ \text{V}^2 & T = 400\text{ K} \end{cases} \\
&= \begin{cases} 99.8\ \mu\text{A} & T = 300\text{ K} \\ 82.3\ \mu\text{A} & T = 400\text{ K} \end{cases}
\end{aligned}
$$

The mobility is lower at high temperatures.

4.

The sensitivity increases a bit for high T. This can be seen from the calculation in the second question. V_{BS} is underneath the square root and the square root is smaller for $T = 300$ K, which makes it more sensitive. We can express this mathematically:

$$
\begin{aligned}
\left.\frac{dV_T}{dV_{BS}}\right|_{V_{BS}=0} &= \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\frac{-1}{2\sqrt{2\Phi_f}} \\
&= \frac{-t_{ox}}{2\epsilon_{ox}}\sqrt{\frac{qN_A\epsilon_s}{\Phi_f}} \\
&= \frac{-t_{ox}}{2\epsilon_{ox}}\sqrt{\frac{qN_A\epsilon_s}{\frac{E_g}{2q} + \frac{kT}{q}\ln\frac{N_A}{N_C}}} \\
&= \begin{cases} -0.298 & T = 300\text{ K} \\ -0.309 & T = 400\text{ K} \end{cases}
\end{aligned}
$$

As $\ln\frac{N_A}{N_C} < 0$, this increases (in absolute value) for higher T. Another way to see is that the semiconductor becomes more intrinsic at higher T, so that Φ_f becomes smaller, which gives a higher $\left|\frac{dV_T}{dV_{BS}}\right|$ for higher T according to the second equation above.

We have an 3.5 % increase in sensitivity for 400 K compared to 300 K.

Question 14

An intrinsic semiconductor with direct bandgap is illuminated with a light source that produces a constant and uniform generation of charge carriers G. Numerical values:

$G = 10^{18}$ cm^{-3}s^{-1}
Radiative recombination constant $B = 10^{-10}$ cm^{3}s^{-1}
Bandgap (that you may assume to be temperature independent) = 1.4 eV
N_c and N_v may be assumed temperature independent.
At room temperature the intrinsic concentration $n_i = 10^8$ cm^{-3}
Boltzmann constant $k = 1.38 \cdot 10^{-23}$ m^2 kg s^{-2} K^{-1}

a. At which temperature will the intrinsic concentration be as large as the generated carrier concentration?
(*Solution: 340C*)

b. Draw the band diagram for the case of the illuminated semiconductor at room temperature. Don't forget to draw the level of the quasi-Fermi levels!
(*Solution: electron quasi-Fermi level 0.356eV above E_i, hole quasi-Fermi level 0.356eV below E_i*)

c. Draw the band diagram for the case of the non-illuminated semiconductor at the temperature of a. Don't forget to draw the level of the quasi-Fermi levels!

a. If the intrinsic concentration is as large as the generated concentration, the total concentration is the double of the intrinsic concentration: $n = n_i$ and $p = n_i$.

$$\begin{aligned} G &= R_r \\ G &= Bnp \\ G &= B(n_i)(n_i) \\ n_i &= \sqrt{\frac{G}{B}} \\ &= 10^{14} \text{ cm}^{-3} \end{aligned}$$

First N_c and N_v are determined by the data at room temperature: Room Temperature =300K

$$\begin{aligned} n_i &= \sqrt{N_c N_v e^{\frac{-E_G}{kT}}} \\ N_c N_v &= \frac{n_i^2}{e^{\frac{-E_G}{kT}}} \\ &= 3.148\ 10^{39} \end{aligned}$$

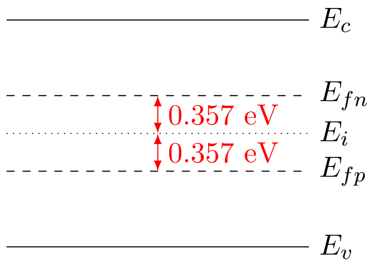
Then T can be determined in function of n_i:

$$\begin{aligned} T &= -\frac{E_G}{k \ln \frac{n_i^2}{N_c N_v}} \\ &= 613 \text{ K} = 340\ ^\circ\text{C} \end{aligned}$$

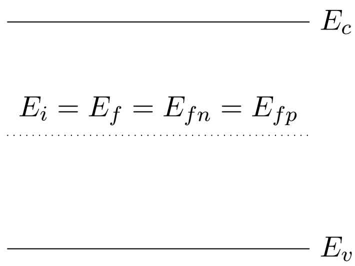
b.

$$
\begin{aligned}
E_{fn} &= E_i + kT \ln \frac{n}{n_i} \\
E_{fn} - E_i &= kT \ln \frac{10^{14}}{10^8} \\
&= 0.357 \text{ eV}
\end{aligned}
$$

The same can be calculated for $E_i - E_{fp}$.



c.



Question 15

A p-type semiconductor with uniform background doping N_A is illuminated such that the generation G of electron-hole pairs is uniform in the material. The lifetime of the electrons τ in the semiconductor is known and constant.

Numerical values:

$$\begin{aligned} N_A &= 10^{17}\ \mathrm{cm}^{-3} \\ \tau &= 1\ \mathrm{ms} \\ n_i &= 10^{10}\ \mathrm{cm}^-3 \end{aligned}$$

a) How large should G be to produce a minority carrier concentration of 1% of the doping concentration. (Solution: 10^{18} cm^{-3})

b) Plot on a LOG-LOG scale the steady-state concentration of minority carriers as a function of (steady-state) G for G between 10^3 cm^{-3}s^{-1} and the value reached in a)

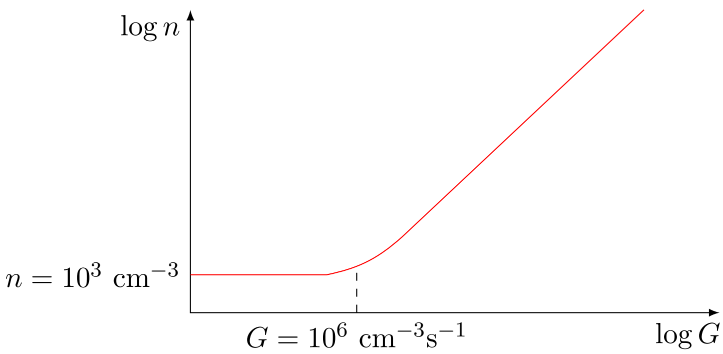
c) Assume the steady-state G of question a) was applied until time $t = t_0$. We switch off the light at $t = t_0$. Plot the evolution of the electron concentration as a function of time for $t > t_0$.

a.

$$\begin{aligned} G = U &= \frac{n - n_{p0}}{\tau} \\ G &= \frac{10^{15}\ \mathrm{cm}^{-3} - 10^{3}\ \mathrm{cm}^{-3}}{0.001\ \mathrm{s}} \\ &= 10^{18}\ \mathrm{cm}^{-3}\mathrm{s}^{-1} \end{aligned}$$

(the original minority carrier concentration is $n_{p0} = \frac{n_i^2}{N_A} = 10^3$ cm^{-3})

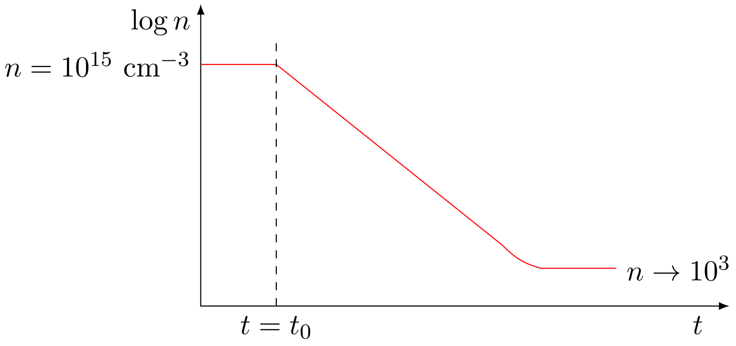
b.



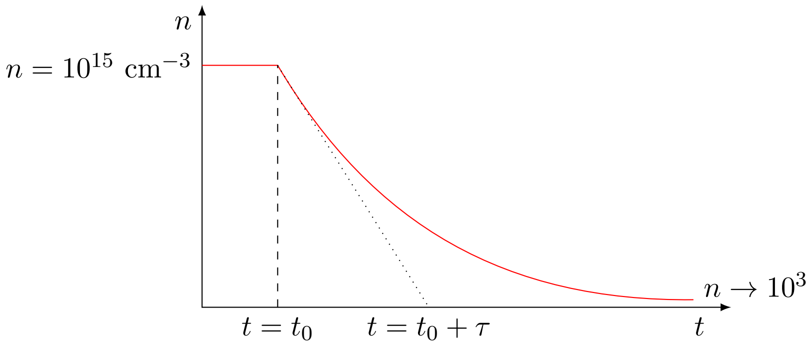
c.

$$n = n_{p0} + (G\tau)e^{\frac{-(t-t_0)}{\tau}}$$

In semilogplot:



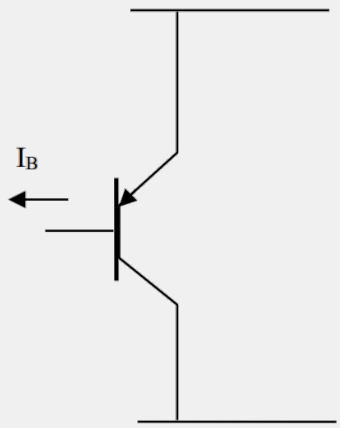
In normal plot:



Question 16

Consider a pnp bipolar transistor, biased at constant V_{EC}. The transistor can be considered to be ideal, with no recombination in the EB space charge region or in the neutral base. We consider that the base-emitter junction is a one-sided abrupt junction, and that the emitter is short. We bias the transistor with $V_{EC} = 3$ V, and a steady-state base current is supplied at the base, as shown in the figure.

$I_s = 10^{-13}$ A
$V_t = kT/q = 0.025$ V
$\beta_F = 1000$
$N_{base} = 10^{17}$ cm^{-3}
$N_{collector} = 10^{15}$ cm^{-3}
$\tau_{F,base} = 2\ 10^{-12}$ s
Area $= 5\ 10^{-5}$ cm^2
$V_{bi,EB} = 0.9$ V
$V_{EC} = 3$ V
$\varepsilon = 11.9 \Rightarrow \varepsilon.\varepsilon_o = 10^{-12}$ F/cm
$n_i = 10^{10}$ cm^{-3}



a) In which **operation regime** is the transistor with these applied bias conditions?

b) How large is the steady-state **base current** needed to make the diffusion capacitance of the base-emitter junction as large as the junction capacitance of that junction? Limit the accuracy of your calculations to one meaningful digit!
(Solution: 0.25mA)

c) How large is the **collector-base** voltage for this condition, and is this forward or reverse?
(Solution: -2.29V)

d) At which V_{EB} does this bipolar transistor go into high injection? *(Solution: 0.8V)*

a)

The operation regime depends on the base voltage. $V_{EB} \approx V_{EB,bi}$, so $V_B = V_{EC} - V_{EB} \approx 2.1$ V. By consequence the BE junction is in forward bias and the BC junction is in reverse bias. Thereby the transistor is in forward operation.

b)

The junction capacitance is equal to

$$
\begin{aligned}
C_j &= \frac{A}{\sqrt{\frac{2}{q\epsilon_s}\left(\frac{1}{N_A}+\frac{1}{N_D}\right)V_{EB,bi}}\sqrt{1-\frac{V_{EB}}{V_{EB,bi}}}} \\
&\approx A\sqrt{\frac{q\epsilon_s N_B}{2(V_{EB,bi}-V_{EB})}} \\
&\approx A\sqrt{\frac{q\epsilon_s N_B}{2\left[V_{EB,bi}-V_t\ln\left(\frac{\beta_F I_B}{I_S}\right)\right]}}
\end{aligned}
$$

The diffusion capacitance is equal to

$$
\begin{aligned}
C_D &= \frac{\tau_f I_E}{V_t} \\
&\approx \frac{\tau_f \beta_F I_B}{V_t}
\end{aligned}
$$

By making these equal, I_B can be calculated:

$$
\begin{aligned}
C_D &= C_j \\
\frac{\tau_f \beta_F I_B}{V_t} &= A \sqrt{\frac{q\epsilon_s N_B}{2\left[V_{EB,bi} - V_t \ln\left(\frac{\beta_F I_B}{I_S}\right)\right]}} \\
I_B^{(n+1)} &= \frac{A V_t}{\tau_f \beta_F} \sqrt{\frac{q\epsilon_s N_B}{2\left[V_{EB,bi} - V_t \ln\left(\frac{\beta_F I_B^{(n)}}{I_S}\right)\right]}}
\end{aligned}
$$

Which can be calculated iteratively. The result is $I_B = 1.24$ mA (and $V_{EB} = 0.696$ V).

Small deflection from answer, could partly be explained if in the answer V_{EB} was calculated first, as $I_B(V_{EB})$ is very ill-conditioned.

c)

As $V_{EB} = 0.696$ V, then $V_{CB} = VEB - V_{EC} = 0.696 \text{ V} - 3 \text{ V} = 2.304$. This explains the deflection in the previous answer, they calculated $V_{EB} = 3 - 2.29 = 0.714$ V, which gives $I_B = 0.25$ mA.

d)

High injection happens when the hole concentration in the base at the emitter-base junction reaches the doping concentration, so if

$$
\begin{aligned}
N_B = p &= \frac{n_i^2}{N_B} e^{\frac{V_{EB}}{V_t}} \\
V_{EB} &= 2V_t \ln \frac{N_B}{n_i} \\
&= 0.81 \text{ V}
\end{aligned}
$$

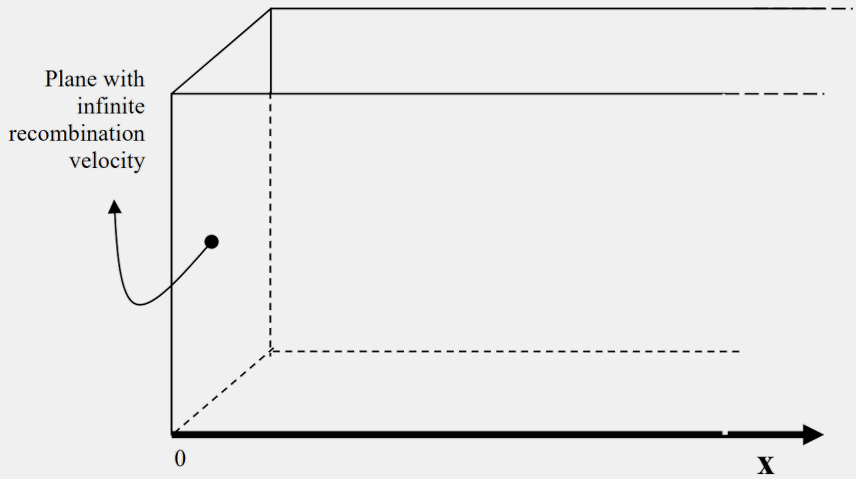
Question 17

Consider a piece of semiconductor with a p-type doping concentration N_A. A lamp is illuminating the semiconductor such that a generation G [$cm^{-3}s^{-1}$] of electron-hole pairs per unit time takes place, which we assume to be completely uniform over the semiconductor and constant as a function of time. One can neglect the thermal generation compared to the optical generation. The lifetime of electrons is τ_n.

a) Derive an expression for the steady-state electron concentration with this illumination?

b) What is the equilibrium electron concentration (n_o) without illumination and what is the steady-state electron concentration under the above-described illumination conditions when we assume the following numerical values: $n_i = 10^{10}$ cm^{-3}, $N_A = 10^{18}$ cm^{-3}, $G = 10^{17}$ $cm^{-3}s^{-1}$, $\tau_n = 10^{-3}$ s, $\mu_n = 10^2$ cm^2/Vs (Solution: 10^2 cm^{-3}, 10^{14} cm^{-3})

c) Consider that the surface recombination velocity of electrons (s in units of cm/s) is infinite at one edge of the semiconductor. Derive a new expression for the electron concentration as a function of position x from the plane of infinite recombination (which we position at $x = 0$, see graph below)



d) Draw the electron concentration AND the hole concentration as a function of position x from the plane of infinite recombination

a.

$$
\begin{aligned}
G = U &= \frac{n - n_0}{\tau_n} \\
n &= n_0 + G\tau_n \approx G\tau_n
\end{aligned}
$$

b.

$$
\begin{aligned}
n_0 &= \frac{n_i^2}{N_A} \\
&= \frac{\left(10^{10}\right)^2}{10^{18}} \\
&= 10^2 \text{ cm}^{-3}
\end{aligned}
$$

$$\begin{aligned} n &= G\tau_n \\ &= 10^{17} \cdot 10^{-3} = 10^{14} \text{ cm}^{-3} \end{aligned}$$

(generation has a negligible effect on $p = 10^{18}$ cm^{-3}, so the approximation is okay)

c.

As the generation does not have much effect on the amount of holes, only the electrons are discussed. Continuity equation:

$$\nabla J_n + q\frac{dn}{dt} + q(U_n - G_n) = 0$$

With no electrical field we have $J_n = qD_n\frac{dn}{dx}$ and in steady-state we get $\frac{dn}{dt} = 0$. The net recombination is given by $U_n - G_n = \frac{n-n_0}{\tau_n} - G_n \approx \frac{n}{\tau_n} - G_n$ as $\frac{n_0}{\tau_n} \lll G_n$. The continuity equation becomes

$$\begin{aligned} qD_n\frac{d^2n}{dx^2} + q\left(\frac{n}{\tau_n} - G_n\right) &= 0 \\ D_n\frac{d^2n}{dx^2} + \frac{1}{\tau_n}n - G_n &= 0 \end{aligned}$$

Solving this differential equation gives us

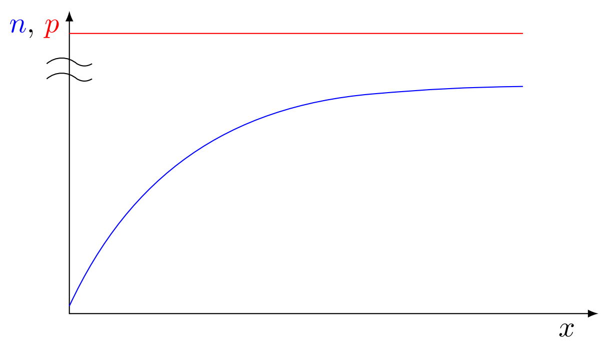
$$n(x) = G_n\tau_n + Ae^{\frac{x}{\sqrt{D_n\tau_n}}} + Be^{-\frac{x}{\sqrt{D_n\tau_n}}}$$

As n cannot go to infinity, $A = 0$. The boundary condition $n(0) = n_0$ gives that $B = -(G_n\tau_n - n_0)$. Then the electron concentration becomes

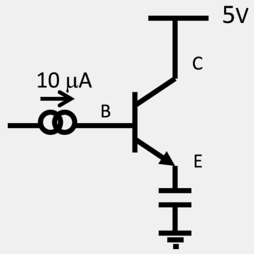
$$n(x) = n_0 + G_n\tau_n\left(1 - e^{-\frac{x}{\sqrt{D_n\tau_n}}}\right)$$

With $D_n = \frac{\mu_n kT}{q} = 2.6\ \frac{\text{cm}^2}{\text{s}}$.

d.



Question 18



Consider a npn transistor with the following parameters:
$\beta_F = 100$
$\beta_R = 0.1$
$I_s = 10^{-15}$ A (collector saturation current)
We force a current of 10 µA into the base.

a) Calculate the internal bipolar currents (I_F, I_{BF}, I_R and I_{BR}) ?
(Solution: 0.989 µA, 9.89 nA, 0.999 µA, 9.99 µA)

b) What will be the collector current, the emitter current, the base voltage and the emitter voltage ?
(Solution: 10 µA, 0, 5.54 V, 5 V)

c) We now remove the capacitor and connect the emitter to ground. Calculate again the internal currents, the collector and emitter current and the base voltage.
(Solution: 1.01 mA, 10 µA, 0, 0, 1 mA, 1.01 mA, 0.72 V)

d) Draw the band diagram of the bipolar transistor for these 2 conditions. Indicate the position of the quasi-Fermi levels of the electrons and holes and the intrinsic level. Voltage axis should be on scale, position axis only qualitative.

a.

We know that $I_E = 0$, so $I_C = -I_B = -10$ µA.

$$\begin{cases} I_E = 0 = \left(1 + \frac{1}{\beta_F}\right) I_F - I_R \\ I_C = I_F - \left(1 + \frac{1}{\beta_R}\right) I_R \end{cases}$$

Which is a system with 2 equations and 2 unknowns and can be solved. Substracting the second equation from $\left(1 + \frac{1}{\beta_R}\right)$ times the first gives

$$\begin{aligned} -I_C &= \left(1 + \frac{1}{\beta_R}\right)\left(1 + \frac{1}{\beta_F}\right) I_F - I_F \\ I_F &= -\frac{I_C}{\frac{1}{\beta_F} + \frac{1}{\beta_R} + \frac{1}{\beta_F \beta_R}} \\ &= -\frac{\beta_F \beta_R I_C}{\beta_F + \beta_R + 1} \\ &= 0.989 \text{ µA} \end{aligned}$$

And I_R is then according to the first equation

$$\begin{aligned}
I_R &= \left(1+\frac{1}{\beta_F}\right) I_F \\
&= 0.999\ \mu\text{A} \\
I_{BF} &= \frac{I_F}{\beta_F} = 9.89\ \text{nA} \\
I_{BR} &= \frac{I_R}{\beta_R} = 9.99\ \mu\text{A}
\end{aligned}$$

b.

Collector current already mentioned $I_C = -I_B = -10\ \mu\text{A}$. The base and emitter voltage follow from the current equations:

$$\begin{aligned}
I_R &= I_s e^{\frac{V_{BC}}{V_t}} \\
V_B &= V_C + V_t \frac{I_R}{I_s} \\
&= 5.54\ \text{V} \qquad (\text{for } V_t = 0.026\ \text{V}) \\
I_F &\approx I_s e^{\frac{V_{BE}}{V_t}} \\
V_E &= V_B - V_t \frac{I_F}{I_s} \\
&= 5.00\ \text{V}
\end{aligned}$$

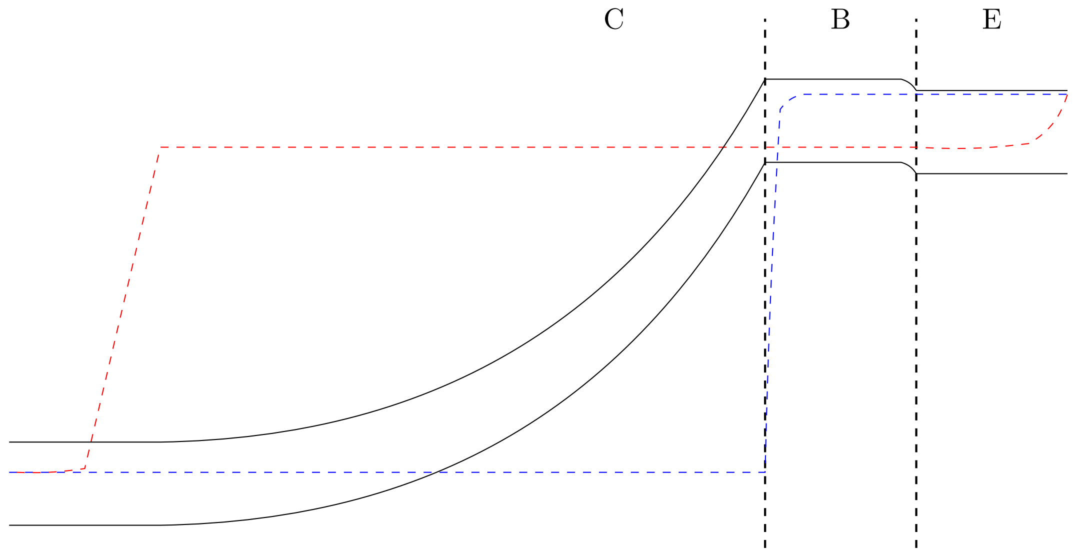
c.

We're in the forward active regime now

$$\begin{aligned}
I_C &= \beta_F I_B \\
&= 1\ \text{mA} \\
I_E &= \left(1+\frac{1}{\beta_F}\right) I_B \\
&= 1.01\ \text{mA} \\
I_F &= I_C = 1\ \text{mA} \\
I_{BF} &= \frac{I_F}{\beta_F} = 10\ \mu\text{A} \\
V_B &= V_E + V_t \frac{I_F}{I_s} \\
&= 0.718\ \text{V}
\end{aligned}$$

d.

Shortcut:


C
B
E

Question 19

The doping concentration in a GaAs pn junction diode at $T = 300$ K are $N_d = 5 \cdot 10^{15}$ cm^{-3} and $N_a = 5 \cdot 10^{16}$ cm^{-3}. The minority carrier concentration at either space charge edge is to be no larger than 10% of the respective majority carrier concentrations. Calculate the maximum forward bias voltage that can be applied to this junction and still meet the required specifications.

$$\begin{aligned} n_i &= 2 \cdot 10^6 \text{ cm}^{-3} \\ V_t &= 26 \text{ mV} \end{aligned}$$

(Solution: 1.06V)

The problem will first arise at the lowest doped region. Thus we need to find the voltage were:

$$\begin{aligned} p_n &= \frac{N_d}{10} \\ \frac{n_i^2}{N_d} e^{\frac{V_{pn}}{V_t}} &= \frac{N_d}{10} \\ V_{pn} &= V_t \ln \frac{N_d^2}{10 n_i^2} \\ &= 1.065 \text{ V} \end{aligned}$$

Question 20
Consider a silicon pn-junction diode at T=300K with the following parameters:
$D_n = 25$ cm^2/s
$D_p = 10$ cm^2/s
$\tau_{p0} = \tau_{n0} = 5\ 10^{-7}$ s
$n_i = 10^{10}$ cm^{-3}
$\epsilon_{Si} = 10^{-12}$ F/cm
$W_n = W_p = 500$ nm

a) Design the doping concentrations of the diode such that the electron and hole current density components in the space charge region are $J_n = 20$ A/cm^2 and $J_p = 1$ A/cm^2, respectively, at $V_{pn} = 0.65$ V.
(Solution: $N_A = 2.9\ 10^{16}\ cm^{-3}$, $N_D = 2.3\ 10^{17}\ cm^{-3}$)

b) If we fix the doping concentrations to these designed values, but lengthen the diode dimensions to $W_n = W_p = 20$ µm how much would the two current density components J_n and J_p in the space charge region become? How much will the ideal reverse saturation current density of the diode be?
(Solution: $J_n = 0.55\ A/cm^2$ and $J_p = 0.03\ A/cm^2$)

c) Determine the diffusion capacitance of the diode for the two cases of 500 nm and 20 µm diodes at the same bias voltage?
(Solution: $W = 500$ nm: $C_{Dn} = 3.8\ 10^{-8}\ F/cm^2$, $C_{Dp} = 4.8\ 10^{-9}\ F/cm^2$, $W = 20$ mm: $C_{Dn} = 1.5\ 10^{-6}\ F/cm^2$, $C_{Dp} = 1.8\ 10^{-7}\ F/cm^2$)

d) Draw/sketch the evolution of $J_n(x)$ and $J_p(x)$ for the two cases of 500 nm and 20 µm diodes.

For these calculations you can neglect the width of the space charge regions compared to the width of the neutral regions as well as the recombination in the space charge region. You can also assume ideal diode behavior (no high injection effects). Use the following constants:
$V_t = 0.026$ V, $q = 1.6\ 10^{-19}$ C

a.

First we check whether the regions are short or long:

$$\begin{aligned} L_n &= \sqrt{D_n \tau_{n0}} \\ &= 35.4\ \mu\text{m} \gg W_p \\ L_p &= \sqrt{D_p \tau_{p0}} \\ &= 22.4\ \mu\text{m} \gg W_n \end{aligned}$$

Both regions are short. The currents are thus (neglecting the width of the SCR)

$$\begin{cases} J_n = \frac{q n_i^2 D_n}{N_A W_p} e^{\frac{V_{pn}}{V_t}} \\ J_p = \frac{q n_i^2 D_p}{N_D W_n} e^{\frac{V_{pn}}{V_t}} \end{cases}$$

$$\begin{cases} N_A = \frac{q n_i^2 D_n}{J_n W_p} e^{\frac{V_{pn}}{V_t}} \\ N_D = \frac{q n_i^2 D_p}{J_p W_n} e^{\frac{V_{pn}}{V_t}} \end{cases}$$

$$\begin{cases} N_A = 2.88\ 10^{16}\ \text{cm}^{-3} \\ N_D = 2.30\ 10^{17}\ \text{cm}^{-3} \end{cases}$$

b.

Both regions are now neither long nor short :(.

$$\begin{cases} J_n = \frac{qn_i^2 D_n}{N_A L_n} \frac{1+e^{-\frac{2W_p}{L_n}}}{1-e^{-\frac{2W_p}{L_n}}} e^{\frac{V_{pn}}{V_t}} \\ J_p = \frac{qn_i^2 D_p}{N_D L_p} \frac{1+e^{-\frac{2W_n}{L_p}}}{1-e^{-\frac{2W_n}{L_p}}} e^{\frac{V_{pn}}{V_t}} \end{cases}$$

$$\begin{cases} J_n = 3.928\ 10^{-12}\ \frac{\mathrm{A}}{\mathrm{cm}^2} \cdot 1.952 \cdot 7.200\ 10^{10} \\ J_p = 3.105\ 10^{-13}\ \frac{\mathrm{A}}{\mathrm{cm}^2} \cdot 1.401 \cdot 7.200\ 10^{10} \end{cases}$$

$$\begin{cases} J_n = 0.552\ \frac{\mathrm{A}}{\mathrm{cm}^2} \\ J_p = 0.031\ \frac{\mathrm{A}}{\mathrm{cm}^2} \end{cases}$$

The ideal reverse saturation current is

$$\begin{aligned} J_{reverse} &= qn_i^2 \left(\frac{D_p}{N_D W_n} + \frac{D_n}{N_A W_p} \right) \\ &= 3.5\ 10^{-12}\ \frac{\mathrm{A}}{\mathrm{cm}^2} \end{aligned}$$

c.

For a short region ($W = 500$ nm):

$$\begin{cases} C_{Dn} = \frac{qn_i^2 W_p}{2N_A V_t} e^{\frac{V_{pn}}{V_t}} \\ C_{Dp} = \frac{qn_i^2 W_n}{2N_D V_t} e^{\frac{V_{pn}}{V_t}} \end{cases}$$

$$\begin{cases} C_{Dn} = 3.85\ 10^{-8}\ \frac{\mathrm{F}}{\mathrm{cm}^2} \\ C_{Dp} = 4.82\ 10^{-9}\ \frac{\mathrm{F}}{\mathrm{cm}^2} \end{cases}$$

For a mixed region ($W = 20$ µm):

$$\begin{cases} C_{Dn} = \frac{qn_i^2 L_n}{N_A V_t} \frac{1-e^{-\frac{W_p}{L_n}}}{1+e^{-\frac{W_p}{L_n}}} e^{\frac{V_{pn}}{V_t}} \\ C_{Dp} = \frac{qn_i^2 W_n}{2N_D V_t} \frac{1-e^{-\frac{W_n}{L_p}}}{1+e^{-\frac{W_n}{L_p}}} e^{\frac{V_{pn}}{V_t}} \end{cases}$$

$$\begin{cases} C_{Dn} = 1.50\ 10^{-6}\ \frac{\mathrm{F}}{\mathrm{cm}^2} \\ C_{Dp} = 1.81\ 10^{-7}\ \frac{\mathrm{F}}{\mathrm{cm}^2} \end{cases}$$

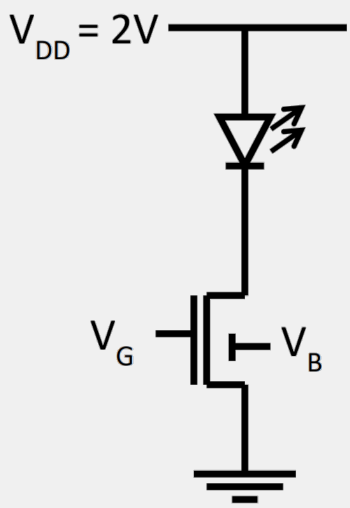
d.

Draw the currents linear for the short regions and something in between linear and exponential for the mixed regions.

Question 21

A light-emitting diode is connected in series with an n-MOSFET as shown on the figure below. The diode has a current-voltage characteristic given by:

$$I_D = I_s \left(e^{\frac{V_{pn}}{V_t}} - 1 \right)$$



The light emission power P is proportional to the current:

$$P = \alpha I_D$$

with $\alpha = 1$ W/A
The parameters of the diode and the MOSFET are given below:
Diode saturation current $I_s = 3.8\ 10^{-12}$ A
MOSFET
$W/L = 1$
Threshold voltage $V_T = 0.5$ V (at $V_{BS} = 0$ V)
Oxide thickness $t_{ox} = 10$ nm
Channel doping $N_A = 10^{17}$ cm^{-3}
Channel mobility $\mu = 3000$ cm^2/Vs
The power supply voltage is constant $V_{DD} = 2$ V.
We control the light emission power of the diode by changing the gate voltage of the MOSFET.

1) Calculate and plot the light emission power P as a function of the gate voltage V_G, for V_G varying between 0 V and V_{DD} and for $V_B = 0$ V.
(Solution: 1.125 mW)

2) How much will the maximal light emission power be at $V_G = 2$ V if we apply a voltage $V_B = -2$ V at the bulk of the transistor.
(Solution: 0.59 mW)

3) Draw the band diagram in the channel at the source side of the n-MOSFET as a function of position in the direction perpendicular to the channel of the transistor, and this for $V_G = 2$ V and for $V_{BS} = 0$ V and -2 V. Indicate on the band diagram the position of the quasi-Fermi levels of holes and electrons, and the electrostatic potential drop in the space charge region.

Use the following constants:
$V_t = 25.8$ mV
$n_i = 10^{10}$ cm^{-3}
$\epsilon_{ox} = 3.33\ 10^{-13}$ F/cm^2
$\epsilon_{Si} = 10^{-12}$ F/cm^2

1.

$\beta = \frac{W}{L} C_{ox} \mu = \frac{W}{L} \frac{\epsilon_{ox}}{t_{ox}} \mu = 1.0$ mA/V^2.

There are 3 regimes:

- $V_G < V_T$: no current, no light emission
- $V_G > V_T$ and $V_G < V_{DS} + V_T$: saturation
- $V_G > V_{DS} + V_T$: linear

The first regime is trivial, the transition from saturation and linear needs to be calculated from the current equations. We first assume that we're in saturation. The current, emission power and drain voltage are then

$$\begin{aligned} I &= \beta\frac{(V_G - V_T)^2}{2} \\ P &= \alpha I = \alpha\beta\frac{(V_G - V_T)^2}{2} \\ V_D &= V_{DD} - V_{pn} = V_{DD} - V_t \ln\frac{I_D}{I_s} \\ &= V_{DD} - V_t \ln\frac{\beta(V_G - V_T)^2}{2I_s} \end{aligned}$$

We are only in saturation if $V_D > V_G - V_T$, thus

$$V_D = V_{DD} - V_t \ln\frac{\beta(V_G - V_T)^2}{2I_s} > V_G - V_T$$
$$V_G < V_{DD} + V_T - V_t \ln\frac{\beta(V_G - V_T)^2}{2I_s}$$

We can solve this iteratively and the solution is $V_G < 1.997$ V. The power is at this point $P = \alpha\beta\frac{(V_G - V_T)^2}{2} = 1.120$ mW. For $V_G > 1.997$ V, the transistor is in the linear regime:

$$\begin{aligned} I = \beta\left(V_G - V_T - \frac{V_D}{2}\right)V_D &= I_s e^{\frac{V_{DD}-V_D}{V_t}} \\ V_D &= V_{DD} - V_t \ln\left(\frac{\beta\left(V_G - V_T - \frac{V_D}{2}\right)V_D}{I_s}\right) \end{aligned}$$

Which gives $V_D = 1.497$ V both for $V_G = 1.997$ V and $V_G = 2$ V. The power is then $P = 1.125$ mW for $V_G = 2$ V.

2.

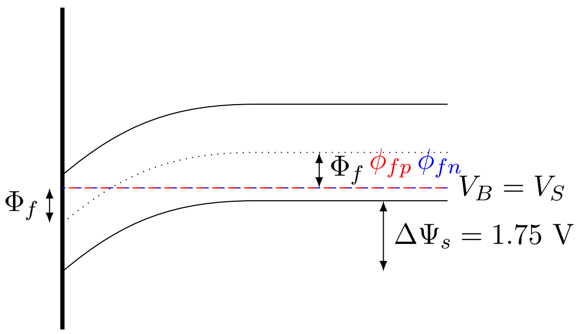
If we apply a voltage $V_B = -2$ V, the threshold voltage V_T will change:

$$\begin{aligned} V_T &= 0.5 \text{ V} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\left(\sqrt{2\Phi_f - V_{BS}} - \sqrt{2\Phi_f}\right) \\ &= 0.5 \text{ V} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\left(\sqrt{2V_t \ln\frac{N_A}{n_i} - V_{BS}} - \sqrt{2V_t \ln\frac{N_A}{n_i}}\right) \\ &= 0.5 \text{ V} + 0.573 \text{ V}^{1/2}(1.683 \text{ V}^{1/2} - 0.912 \text{ V}^{1/2}) \\ &= 0.914 \text{ V} \end{aligned}$$

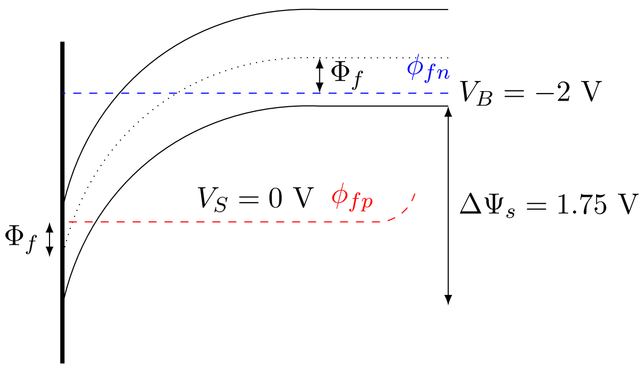
In this case we're definitely in the saturation regime. So, we calculate P as:

$$\begin{aligned} P = \alpha\beta\frac{(V_G - V_T)^2}{2} \\ &= 0.590 \text{ mW} \end{aligned}$$

3.



Φ_f
Φ_f ϕ_{fp} ϕ_{fn}
$V_B = V_S$
$\Delta\Psi_s = 1.75$ V



Φ_f
ϕ_{fn}
$V_B = -2$ V
$V_S = 0$ V ϕ_{fp}
$\Delta\Psi_s = 1.75$ V
Φ_f

Question 22

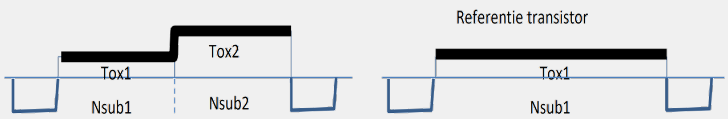
Consider an n-MOS transistor where the thickness of the gate dielectric is doubled in the middle of the channel, as shown on the figure below, in other words t_{ox1} at the source side and t_{ox2} at the drain side are such that $t_{ox2} = 2t_{ox1}$. Other transistor parameters are given below.

The transistor is biased in the **PURE LINEAR** region. We want the current in this linear region (for very small V_{DS}) **for** V_{GS} **= 0.85 V** to be identical to the current of a reference transistor with the same total length L and with the oxide thickness t_{ox1} over the full channel and at the same voltage conditions. In order to reach this someone proposes to change, besides the oxide thickness, also the doping concentration of the substrate at the drain side of the channel, as shown on the figure below.

Calculate how much the doping concentration N_{sub2} should be? The substrate potential is $V_{sub} = 0$ V for both transistors.

(Solution: 2.5 10^{16} cm^{-3})

HINT: you can consider the transistor at the left as a series connection of two transistors



Transistor data:
L=0.1 µm; W=L
Nsub1=10^{17} cm-3
tox1=20 nm
μ_n=1000 cm2/(Vs)
ni=10^{10} cm-3
VFB=-1V
Qox=0 C/cm2
Constants: Vt=kT/q=25 mV; ϵ_{Si}=12; ϵ_{ox}=4; $\epsilon_0 = 8.85\ 10^{-14}$ F/cm

C_{ox} and thereby β is only have as much as in the part on the source side. If we want the current $I = \beta\,(V_{GS} - V_T)\,V_{DS}$ (small V_{DS} assumed) to be equal, $V_{GS} - V_T$ needs to be twice as much. For the left part:

$$\begin{aligned}
V_T &= 2\Phi_f + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s|2\Phi_f - V_{BS}|} \\
&= 2V_t \ln\frac{N_A}{n_i} + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{4qN_A\epsilon_s V_t \ln\frac{N_A}{n_i}} \\
&= 0.806\text{ V} - 1\text{ V} + 0.935\text{ V} \\
&= 0.741\text{ V}
\end{aligned}$$

Which means that V_T needs to drop to $V_T = 0.632$ V if we want that $V_{GS} - V_T$ is twice as large.

$$
\begin{aligned}
\Delta V_T = -0.109 \text{ V} &= 2\Delta\Phi_f + \Delta V_{FB} + \Delta\left(\frac{t_{ox}}{\epsilon_{ox}}\sqrt{4qN_A\epsilon_s\Phi_f}\right) \\
&= 2V_t \ln\frac{N_{A2}}{N_{A1}} - V_t\ln\frac{N_{A2}}{N_{A1}} + \left(\frac{t_{ox2}}{\epsilon_{ox}}\sqrt{4qN_{A2}\epsilon_s V_t \ln\frac{N_{A2}}{n_i}} - 0.935 \text{ V}\right) \\
\frac{t_{ox2}}{\epsilon_{ox}}\sqrt{4qN_{A2}\epsilon_s V_t\ln\frac{N_{A2}}{n_i}} &= 0.935 \text{ V} - V_t\ln\frac{N_{A2}}{N_{A1}} \\
N_{A2} &= \left[\frac{6.789\ 10^8}{\sqrt{\ln\frac{N_{A2}}{n_i}}}\left(0.935 \text{ V} - V_t\ln\frac{N_{A2}}{N_{A1}}\right)\right]^2
\end{aligned}
$$

Which can be calculated iteratively. This gives $N_{A2} = 2.89\ 10^{16}$ cm^{-3}. Interestingly, if we give $N_{A2} = 10^{17}$ cm^{-3} as starting value, the first guess is $N_{A2} = 2.50\ 10^{16}$ cm^{-3}, which is the solution given.

Question 23

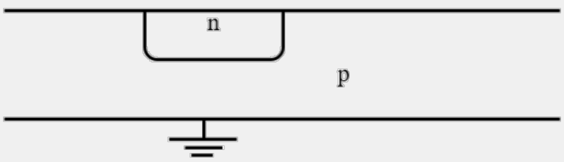
Consider a pn diode as shown below, the parameters are given in the table. The n-region is kept FLOATING, that means, it is not connected electrically to any lead.

1. In some or other way, not of interest for our problem, a positive charge Q_o is present on the n-region at time $t = t_o$. Due to that charge, a voltage $V = +1$ volt is present on the n-region with respect to the grounded p-region. How large is Q_o?
   (Solution: 6.4 10^{-15} C)

2. Inevitably, there is a leakage current in the pn junction. Assume that the leakage current mechanism is diffusion (we neglect generation and recombination in the space charge region). Calculate the leakage current.
   (Solution: 1 10^{-15} A)

3. How long does it take for the leakage current to discharge the floating n-region to one tenth (1/10) of the original charge Q_o?
   (Solution: 5.5 seconds)

Schematic:



Value table:

Doping N_A	1E18 cm^{-3}	Doping N_D	1E16 cm^{-3}
Mobility µn	1000 cm^2/Vs	Mobility µp	1000 cm^2/Vs
Diffusion length Ln	1 micron	Diffusion length Lp	100 nm
ni	1E10 cm^{-3}	Hint: $\int \frac{1}{\sqrt{1+\frac{x}{a}}} = 2a\sqrt{1+\frac{x}{a}}$	
εs	12x8.85E-14 F/cm		
Junction area A	25 μm^2		
kT/q	25.8 mV		

1.

As $V_{pn} = -1$ V, the diffusion capacitance is nearly 0. We need to known the junction capacitance:

$$C_{oj} = \sqrt{\frac{q\epsilon_S N_D}{2V_{bi}}}$$

The built-in potential is: $V_{bi} = V_t \ln \frac{N_A N_D}{n_i^2} = 0.832$ V. Thus:

$$C_{oj} = 7.99 \text{ fF}$$

The charge is thus:

$$\begin{aligned} C &= \frac{dQ}{dV} \\ Q_o &= -\int_0^{-1\ \mathrm{V}} \frac{C_{oj}}{\sqrt{1-\frac{V_{pn}}{V_{bi}}}} dV_{pn} \\ &= 2C_{oj}V_{bi}\left(\sqrt{1+\frac{1\ \mathrm{V}}{V_{bi}}}-1\right) \\ &= 6.43\ 10^{-15}\ \mathrm{C} \end{aligned}$$

2.

This is a reverse current. I'm assuming long regions, as the size of the regions aren't given.

$$\begin{aligned} J_{reverse} &= qV_t n_i^2\left(\frac{\mu_n}{N_A L_n}+\frac{\mu_p}{N_D L_p}\right) \\ &= 4.13\ 10^{-9}\ \frac{\mathrm{A}}{\mathrm{cm}^2} \\ I_{reverse} &= J_{reverse}A = 1.03\ 10^{-15}\ \mathrm{A} \end{aligned}$$

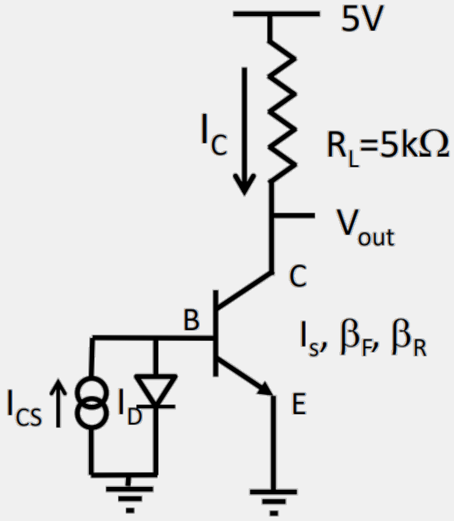
3.

$$\begin{aligned} \Delta t &= \frac{Q}{I} \\ &= 0.9\frac{Q_o}{I} \\ &= 5.58\ \mathrm{s} \end{aligned}$$

As there is still quite some charge left and thus quite some voltage, we can ignore the exponential part.

Question 24

A bipolar transistor is connected to a resistance as shown in the figure below. A current source is sending a constant current I_{CS} into a diode connected at the base and into the base.



The reverse saturation current of the diode is I_D. Generation and recombination, high injection effects in the diode and the series resistance of the diode can be neglected. I_s is the collector saturation current of the npn transistor, and β_F and β_R are the forward and reverse common emitter current gain. The base transport factor of the npn transistor can be assumed to be 1 and recombination in the space charge regions, high injection effects, Early effects, emitter, base and collector resistances can all be neglected.

$$\begin{aligned} I_D &= 10^{-15}\text{ A} \\ I_s &= 10^{-13}\text{ A} \\ \beta_F = \beta_R &= 100 \end{aligned}$$

1. Calculate the base voltage V_B, base current I_B, collector current I_C and collector output voltage V_{out} if the current delivered by the current source $I_{cs} = 1$ µA.
   (Solution: 0.52 V, 0.5 µA , 50 µA, 4.75 V)

2. Calculate the collector current I_C when the current delivered by the current source $I_{cs} = 3$ mA (the current should be within 10% accuracy). How much is now the base voltage V_B and the output voltage V_{out} approximately ? Make also an estimation of the internal bipolar current (I_{BF}, I_{BR}, I_F and I_R).
   (Solution: $I_C = 1$ mA, $V_{BE} = 0.72$ V)

1)

We assume that the transistor is in forward operation. The current equations for the diode and the bipolar transistor give:

$$\begin{cases} V_B = V_t \ln \frac{I_{CS}-I_B}{I_D} & \text{(diode)} \\ V_B = V_t \ln \frac{\beta_F I_B}{I_s} & \text{(bipolar)} \end{cases}$$

These are 2 equations with 2 unknowns, we solve to I_B:

$$\begin{aligned}
V_t \ln \frac{I_{CS} - I_B}{I_D} &= V_t \ln \frac{\beta_F I_B}{I_s} \\
\frac{I_{CS} - I_B}{I_D} &= \frac{\beta_F I_B}{I_s} \\
I_B &= \frac{I_s I_{CS}}{\beta_F I_D + I_s} \\
&= \frac{I_{CS}}{2} \\
&= 0.5 \text{ µA}
\end{aligned}$$

$I_C = \beta_F I_B = 50$ µA, $V_B = V_t \ln \frac{\beta_F I_B}{I_s} = 0.52$ V. $V_{out} = V_{dd} - R_L I_C = 4.75$ V. So, the transistor is indeed in forward operation, the assumption was correct.

2)

Now, we assume the transistor is in saturation. The current equations are:

$$\begin{cases}
V_B = V_t \ln \frac{I_{CS} - I_B}{I_D} & \text{(diode)} \\
I_B = \frac{I_S}{\beta_F} e^{\frac{V_B}{V_t}} + \frac{I_S}{\beta_R} e^{\frac{V_B - V_{out}}{V_t}} & \text{(bipolar)} \\
I_C = I_S e^{\frac{V_B}{V_t}} - \left(1 + \frac{1}{\beta_R}\right) I_S e^{\frac{V_B - V_{out}}{V_t}} = \frac{V_{dd} - V_{out}}{R_L} & \text{(resistor)}
\end{cases}$$

Filling in the first equation in the other two gives:

$$\begin{cases}
I_B = \frac{I_S}{\beta_F} \frac{I_{CS} - I_B}{I_D} + \frac{I_S}{\beta_R} \frac{I_{CS} - I_B}{I_D} e^{\frac{-V_{out}}{V_t}} \\
I_S \frac{I_{CS} - I_B}{I_D} - \left(1 + \frac{1}{\beta_R}\right) I_S \frac{I_{CS} - I_B}{I_D} e^{\frac{-V_{out}}{V_t}} = \frac{V_{dd} - V_{out}}{R_L}
\end{cases}$$

From the first equation we conclude that $I_S \frac{I_{CS} - I_B}{I_D} e^{\frac{-V_{out}}{V_t}} = \beta_R I_B - \frac{\beta_R I_S}{\beta_F} \frac{I_{CS} - I_B}{I_D}$, this can be used to simplify the second equation

$$\begin{cases}
I_B = \frac{I_S}{\beta_F} \frac{I_{CS} - I_B}{I_D} + \frac{I_S}{\beta_R} \frac{I_{CS} - I_B}{I_D} e^{\frac{-V_{out}}{V_t}} \\
I_S \frac{I_{CS} - I_B}{I_D} - \left(1 + \frac{1}{\beta_R}\right) \left(\beta_R I_B - \frac{\beta_R I_S}{\beta_F} \frac{I_{CS} - I_B}{I_D}\right) = \frac{V_{dd} - V_{out}}{R_L}
\end{cases}$$

$$\begin{cases}
I_B = \frac{I_S}{\beta_F} \frac{I_{CS} - I_B}{I_D} + \frac{I_S}{\beta_R} \frac{I_{CS} - I_B}{I_D} e^{\frac{-V_{out}}{V_t}} = I_S \frac{I_{CS} - I_B}{I_D} \left(\frac{1}{\beta_F} + \frac{1}{\beta_R} e^{\frac{-V_{out}}{V_t}}\right) \\
-(\beta_R + 1) I_B + \left(1 + \frac{\beta_R}{\beta_F} + \frac{1}{\beta_F}\right) I_S \frac{I_{CS} - I_B}{I_D} = \frac{V_{dd} - V_{out}}{R_L}
\end{cases}$$

$$\begin{cases}
V_{out} = -V_t \ln \left(\beta_R \left[\frac{I_B I_D}{I_S (I_{CS} - I_B)} - \frac{1}{\beta_F}\right]\right) \\
I_B = \frac{\left(1 + \frac{\beta_R}{\beta_F} + \frac{1}{\beta_F}\right) I_S \frac{I_{CS}}{I_D} - \frac{V_{dd} - V_{out}}{R_L}}{\beta_R + 1 + \left(1 + \frac{\beta_R}{\beta_F} + \frac{1}{\beta_F}\right) \frac{I_S}{I_D}}
\end{cases}$$

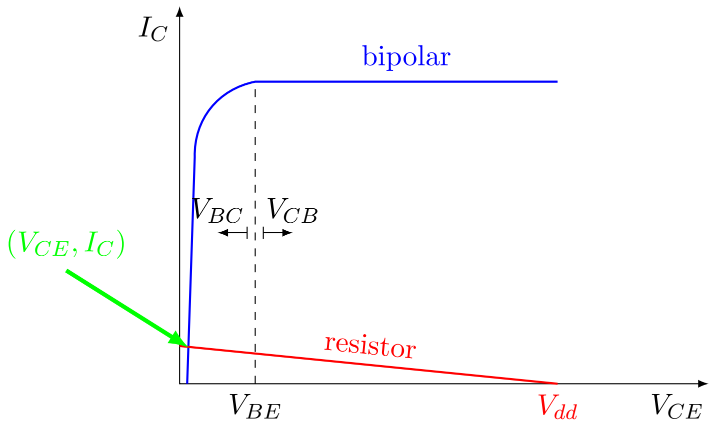
We use that $\beta_F = \beta_R = \frac{I_S}{I_D} = 100$

$$\begin{cases}
V_{out} = -V_t \ln \left(\frac{I_B}{I_{CS} - I_B} - 1\right) \\
I_B = \frac{201 I_{CS} - \frac{V_{dd} - V_{out}}{R_L}}{302}
\end{cases}$$

We can solve to I_B iteratively:

$$I_B^{(n+1)} = \frac{201 I_{CS} - \frac{V_{dd} + V_t \ln\left(\frac{I_B^{(n)}}{I_{CS} - I_B^{(n)}} - 1\right)}{R_L}}{302}$$

Which converges to $I_B = 1.99$ mA, which means $V_{out} = -V_t \ln\left(\frac{I_B}{I_{CS}-I_B} - 1\right) 0.5$ mV, $I_C = \frac{V_{dd} - V_{out}}{R_L} = 1$ mA. $V_B = V_t \ln \frac{I_{CS}-I_B}{I_D} = 0.72$ V, $I_{BF} = \frac{I_S}{\beta_F} e^{\frac{V_B}{V_t}} = \frac{I_S}{\beta_F}\frac{I_{CS}-I_B}{I_D} = 1.01$ mA, $I_F = \beta_F I_{BF} = 101$ mA, $I_{BR} = \frac{I_S}{\beta_R} e^{\frac{V_B - V_{out}}{V_t}} = \frac{I_S}{\beta_R}\frac{I_{CS}-I_B}{I_D} e^{-\frac{V_{out}}{V_t}} = 0.99$ mA, $I_R = \beta_R I_{BR} = 98.7$ mA.
Other possibility: by drawing

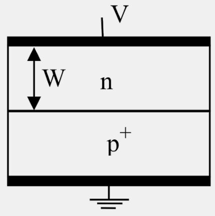


The max current of the bipolar transistor is very high (100-300 mA dependent on how much current the diode uses). And the resistor can maximally take about $\frac{5}{5000} = 1$ mA. So, the bipolar transistor is in deep saturation. With $V_{CE} \approx 0$ V, $I_C \approx \frac{V_{dd}-0}{R_L} = 1$ mA.

Question 25

Consider an abrupt one-sided junction p+/n as shown.
Values to be used:

$N_D = 10^{16}$ cm^{-3}
$N_A = 10^{20}$ cm^{-3}
$\epsilon = 11.9 \Rightarrow \epsilon \cdot \epsilon_0 = 10^{-12}$ F/cm
$n_i = 10^{10}$ cm^{-3}
$V_t = kT/q = 25$ mV
$W = 500$ nm



a. Which voltage should we apply to the n-side such that the Space Charge Region would extend to exactly halfway the n-region?
(Solution: -0.42V)

b. Draw the band diagram through the diode for this case. To draw the quasi-Fermi levels, you may assume the n-region to be short and the p-region to be long.

c. We increase the temperature and keep all the rest constant, including the applied voltage on the n-region. What happens to the width of the space charge region?

a.

The width of the SCR is:

$$\frac{W}{2} = W_{SCR} \approx -X_n \approx \sqrt{\frac{2\epsilon_S}{q}\frac{V_{bi} - V_{pn}}{N_D}}$$
$$V_{pn} = V_{bi} - \frac{q}{2\epsilon_S}\left(\frac{W}{2}\right)^2 N_D$$

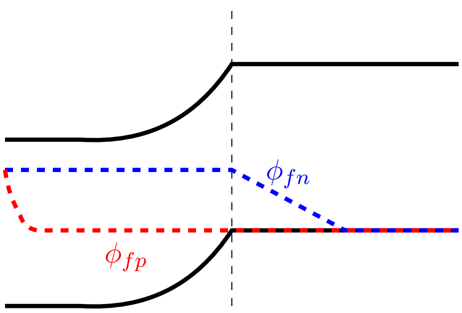
The built in voltage is

$$V_{bi} = V_t \ln \frac{N_D N_A}{n_i^2} = 0.921 \text{ V}$$

So, the voltage is

$$V_{pn} = 0.921 \text{ V} - 0.500 \text{ V} = 0.421 \text{ V}$$
$$V_{np} = V_{pn} = -0.421 \text{ V}$$

b.



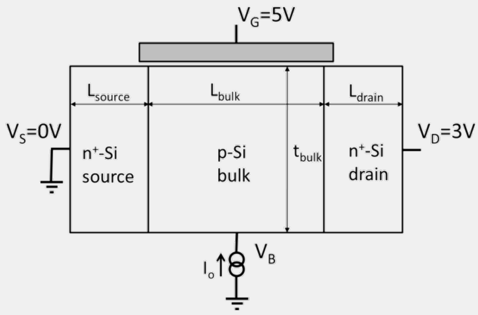
c.

W_{SCR} will change, because the built-in voltage will change. This changes due to the thermal voltage AND the increase in intrinsic charge carriers.

$$
\begin{aligned}
V_{bi} &= V_t \ln\left(\frac{N_D N_A}{n_i^2}\right) \\
&= \frac{kT}{q}\ln\left(\frac{N_D N_A}{N_C N_V e^{-\frac{E_G}{kT}}}\right) \\
&= \frac{kT}{q}\ln\left(\frac{N_D N_A}{N_C N_V}\right) - \frac{kT}{q}\ln\left(e^{-\frac{E_G}{kT}}\right) \\
&= -\frac{kT}{q}\ln\left(\frac{N_C N_V}{N_D N_A}\right) - \frac{kT}{q}\left(-\frac{E_G}{kT}\right) \\
&= -\frac{kT}{q}\ln\left(\frac{N_C N_V}{N_D N_A}\right) + \frac{E_G}{q}
\end{aligned}
$$

As $N_C N_V > N_D N_A$ (in this case), V_{bi} will decrease with rising temperature and so will the width of the space charge region.

Question 26
Consider a MOS-structure as shown on the figure below. The data of the structure are given below:
Source and drain regions:
Doping $N_{source} = N_{drain} = 10^{19}$ cm^{-3}
Length $L_{source} = L_{drain} = 0.5$ µm
Hole diffusion length $D_{psource} = D_{pdrain} = 26$ cm^2/s
Minority carrier lifetime $\tau_p = 5\ 10^{-7}$ s
Bulk region:
Doping $N_{bulk} = 10^{16}$ cm^{-3}
Length $L_{bulk} = 1$ µm
$t_{bulk} = 100$ µm
Electron diffusion length $D_{nbulk\ source} = 13$ cm^2/s
Minority carrier lifetime $\tau_n = 5\ 10^{-5}$ s
Oxide thickness $t_{ox} = 20$ nm
Flatband voltage = -0.5 V



The width of the whole structure is $W = 100$ µm. The mobility of the MOSFET channel is 50 % of that of the bulk. The width of all space charge regions can be neglected versus the geometrical distances, and recombination and generation in the space charge regions and high injection effects can be neglected too.

a) Calculate the threshold voltage of the MOSFET for a bulk voltage $V_B = 0$ V
(Solution: 0.49 V)

b) We force now a current $I_o = 10$ µA into the bulk contact using a constant current source, while keeping the source grounded, the drain at a voltage of 3 V and the gate at 5 V.

1. Calculate the voltage V_B that will appear at the bulk contact!
   (Solution: 0.663 V)
2. Calculate the current at the drain, the source and the bulk for this condition. Explain!
   (Solution: 44.5 mA, 44.51 mA, 10 µA)

Other parameters:
$n_i = 10^{10}$ cm^{-3}
Thermal voltage $V_t = 0.026$ V
$\epsilon_{Si} = 10^{-12}$ F/cm
$\epsilon_{ox} = 3.5\ 10^{-13}$ F/cm
$q = 1.6\ 10^{-19}$ C

a.

$$
\begin{aligned}
V_T &= 2\Phi_f + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\sqrt{2\Phi_f - V_{BS}} \\
&= 2V_t \ln\frac{N_A}{n_i} + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{4qN_A\epsilon_s V_t \ln\frac{N_A}{n_i}} \\
&= 0.718\ \text{V} - 0.5\ \text{V} + 0.274\ \text{V} \\
&= 0.492\ \text{V}
\end{aligned}
$$

b 1.

As $L_n = \sqrt{D_n \tau_n} = 255\ \mu\text{m} \gg L_{bulk}$ the bulk region is short and the device acts as a bipolar transistor. Also the source and drain are short. The device is a superposition of a MOSFET and a bipolar transistor. Let's start with the bipolar part:

$$\begin{aligned}\beta_F &= \frac{E}{B} \\ &= \frac{N_{source} L_{source} V_t}{D_{psource}} \frac{D_{nbulk}}{N_{bulk} L_{bulk} V_t} \\ &= \frac{N_{source}}{N_{bulk}} \frac{L_{source}}{L_{bulk}} \frac{D_{nbulk}}{D_{psource}} \\ &= 250\end{aligned}$$

We assume the transistor works in forward active mode:

$$\begin{aligned}I_{drain} &= \beta_F I_o \\ &= 2.5\ \text{mA.} \\ I_s &= \frac{t_{bulk} W q n_i^2 D_{nbulk}}{N_{bulk} L_{bulk}} \\ &= 2.08\ 10^{-14}\ \text{A}\end{aligned}$$

The voltage can then be calculated as:

$$\begin{aligned}I_{drain} \approx I_F &= I_s e^{\frac{V_B - V_S}{V_t}} \\ V_B &= V_S + V_t \ln \frac{I_{drain}}{I_s} \\ &= 0.663\ \text{V}\end{aligned}$$

b 2.

With this information we can calculate the threshold voltage and thus the current due to the MOSFET.

$$\begin{aligned}V_T &= 2\Phi_f + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}} \sqrt{2qN_A\epsilon_s}\sqrt{2\Phi_f - V_{BS}} \\ &= 0.718\ \text{V} - 0.5\ \text{V} + 0.076\ \text{V} \\ &= 0.294\ \text{V}\end{aligned}$$

$$\begin{aligned}\beta &= \frac{W}{L} \frac{D_{nbulk}}{2V_t} \frac{\epsilon_{ox}}{t_{ox}} \\ &= 4.375\ \frac{\text{mA}}{\text{V}^2}\end{aligned}$$

$V_{DS} = 3\ \text{V} < V_{DS,sat} = V_{GS} - V_T = 4.706\ \text{V}$, so the MOSFET is in the linear regime. The current is then

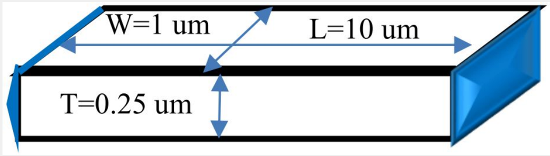
$$\begin{aligned}I_{DS} &= \beta \left(V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} \\ &= 42.1\ \text{mA}\end{aligned}$$

The total current is then

$$\begin{aligned} I_{drain} &= 42.1 \text{ mA} + 2.5 \text{ mA} \\ &= 44.58 \text{ mA} \\ I_{source} &= I_{drain} + I_0 \\ &= 44.59 \text{ mA} \end{aligned}$$

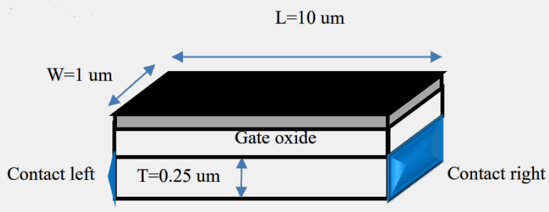
Question 27

Consider a piece of p-doped semiconductor with a doping concentration N_A of 10^{16} cm^{-3} and following dimensions:
Width $W = 1$ µm
Length $L = 10$ µm
Thickness $T = 0.25$ µm
Contacts are processed on both sides of the slab of semiconductor, over the full width and thickness, (see drawing below).



a) Calculate the resistance between the contacts. You may assume the resistance of the contacts to be negligible, and you may assume that the conductivity of the semiconductor is purely due to drift of the majority carriers (negligible contribution of minority carriers). You may assume that the current path is perfectly uniform over the thickness.
(Solution: 250 kOhm)

b) We now process a gate oxide with thickness $t_{ox} = 200$ nm and a metal gate on top of the semiconductor. The flatband voltage of the MOS capacitor that is generated is known to be $V_{FB} = -0.5$ V. Calculate the voltage V_M that needs to be applied at the gate (with respect to the contacts at the left side and right side of the slab that we consider grounded) to fully deplete the semiconductor (in other words: the gate voltage for which the space charge region under the gate will have a width equal to the thickness T of the semiconductor.



(Solution: 2.64V)

c) Draw the band diagram on cross-section perpendicular to the gate, for $V_{gate} = V_M$

d) As we reduce the gate voltage from V_M to V_{FB}, the depletion layer under the gate reduces, and hence the width of the un-depleted semiconductor under the space charge region increases. Calculate and draw the resistance of the undepleted part of the semiconductor as a function of the gate voltage between V_M and V_{FB}.

Constants:
intrinsic concentration $n_i = 10^{10}$ cm^{-3}
hole mobility $\mu_p = 1000$ cm^2/(Vs)
thermal voltage $V_t = 25.8$ mV
permittivity of the semiconductor: $\epsilon_s = 12 \cdot 8.85\ 10^{-14}$ F/cm$= 1.06\ 10^{-12}$ F/cm
permittivity of the oxide $\epsilon_{ox} = 3\ 10^{-13}$ F/cm

a.

$$
\begin{aligned}
p &\approx N_A = 10^{16}\ \mathrm{cm}^{-3} \\
\sigma &= q\mu_p n \\
&= 1.60\ 10^{-19}\ \mathrm{C} \cdot 10^3\ \frac{\mathrm{cm}^2}{\mathrm{Vs}} \cdot 10^{16}\ \mathrm{cm}^{-3} \\
&= 1.60 \frac{\mathrm{C}}{\mathrm{cmVs}} \\
&= 1.60 \cdot 10^{-4} \frac{\mathrm{C}}{\mathrm{\mu mVs}} \\
R &= \frac{L}{WT\sigma} \\
&= 250\ \mathrm{k\Omega}
\end{aligned}
$$

b.

$z_{D,th} = 0.31$ µm, so it is possible.

$$
z_D = \frac{\epsilon_S}{C_{ox}} \left(\sqrt{1 + \frac{2C_{ox}^2}{qN_A\epsilon_S} \left| \Delta\Psi_{tot} + \frac{Q_{ox}+Q_{ss}}{C_{ox}} \right|} - 1 \right)
$$

Using $\Delta\Psi_{tot} = V_{GB} - \Phi_{ms}$ and $V_{FB} = \Phi_{ms} - \frac{Q_{ox}+Q_{ss}}{C_{ox}}$:

$$
\begin{aligned}
z_D &= \frac{\epsilon_S}{C_{ox}} \left(\sqrt{1 + \frac{2C_{ox}^2}{qN_A\epsilon_S} \left| V_{GB} - V_{FB} \right|} - 1 \right) \\
V_{GB} &= V_{FB} \pm \frac{qN_A\epsilon_S t_{ox}^2}{2\epsilon_{ox}^2} \left[\left(z_D \frac{\epsilon_{ox}}{\epsilon_S t_{ox}} + 1 \right)^2 - 1 \right] \\
&= -0.5\ \mathrm{V} + 3.138\ \mathrm{V} \\
&= 2.638\ \mathrm{V}
\end{aligned}
$$

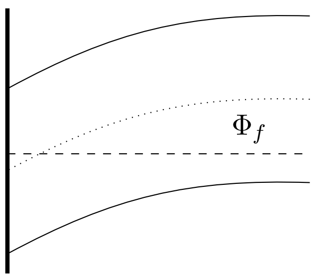
c.

The potential drop over the SC is

$$
\begin{aligned}
\Delta\Psi_{Si} &= -\frac{qN_A z_D^2}{2\epsilon_S} \\
&= -0.472\ \mathrm{V}
\end{aligned}
$$

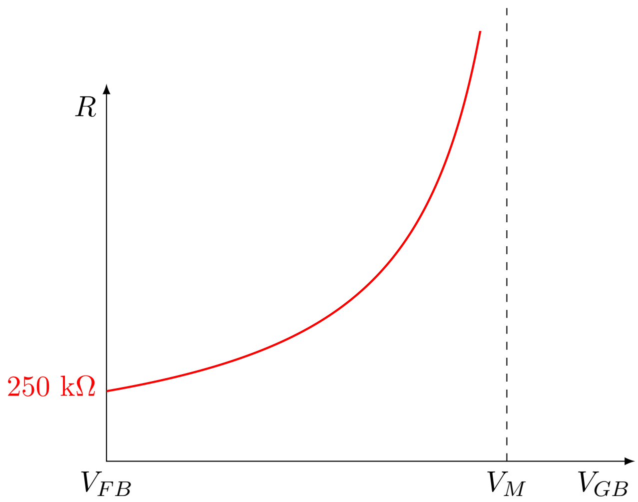
The potential drop over the oxide is unkown as we don't know the oxide charges or the metal work function. If we neglect the oxide charges an approximation is:

$$
\begin{aligned}
\Delta\Psi_{ox} &\approx -\frac{qN_A z_D t_{ox}}{\epsilon_{ox}} \\
&= -2.67\ \mathrm{V}
\end{aligned}
$$



d.

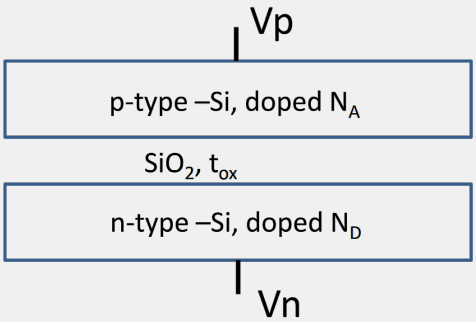
$$
\begin{aligned}
R &= \frac{L}{W(T - z_D)\sigma} \\
&= 250\ \mathrm{k}\Omega \cdot \frac{T}{T - \frac{\epsilon_S}{C_{ox}}\left(\sqrt{1 + \frac{2C_{ox}^2}{qN_A\epsilon_S}\left(V_{GB} - V_{FB}\right)} - 1\right)} \\
&= 250\ \mathrm{k}\Omega \cdot \frac{0.25}{0.25 - 0.70667\left(\sqrt{1 + 0.2653\left(V_{GB} + 0.5\right)} - 1\right)}
\end{aligned}
$$



Question 28

We consider a SOS capacitor instead of a MOS capacitor as drawn on the figure below. The capacitor consists of an oxide dielectric with thickness t_{ox} between two semiconductors, one of which is p-doped and the other is n-doped (S_pOS_n), with equal doping concentrations $N_A(p-region) = N_D(n-region)$. You can assume the fixed charge in the oxide and the surface state density to be 0.

$N_A = N_D = 10^{17}$ cm^{-3}
$t_{ox} = 10$ nm
$\epsilon_{Si} = 10^{-12}$ F/cm
$\epsilon_{ox} = 3.5\ 10^{-13}$ F/cm
$N_c = N_v$
$n_i = 10^{10}$ cm^{-3}
$V_{th} = kT/q = 26$ mV



a. Sketch the band diagram at $V_{pn} = 0$ V across the capacitor. How much is the electrochemical and the electrostatic potential drop over the structure for this condition?

b. Sketch the band diagram at flatband condition. Calculate at which V_{pn} this condition is reached (i.e flatband voltage). How much is the electrochemical and the electrostatic potential drop over the structure for this condition?

c. Sketch the band diagram when the p-region and the n-region go into inversion. Calculate the voltage V_{pn} at which this condition is reached (i.e threshold voltage). How much is the electrochemical and the electrostatic potential drop over the structure for this condition?

d. Sketch the expected $C - V_{pn}$ curve at high frequency and at low frequency and explain the shape of these curves.

a.

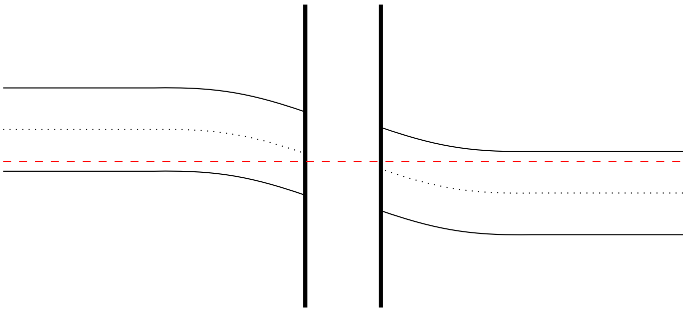
Taking into account the anti-symmetry of the doping, the depletion depth will be the same on both sides. We can calculate it from the total potential difference is.

$$\begin{aligned}
\Delta\Psi_{tot} = V_{GB} - \Phi_{ms} &= \frac{qNz_D^2}{\epsilon_S} + \frac{qNz_Dt_{ox}}{\epsilon_{ox}} = 0.838 \text{ V} \\
2V_t \ln\frac{N}{n_i} &= \frac{qNz_D^2}{\epsilon_S} + \frac{qNz_Dt_{ox}}{\epsilon_{ox}} \\
z_D &= \frac{\epsilon_S}{2qN}\left(-\frac{qNt_{ox}}{\epsilon_{ox}} + \sqrt{\left(\frac{qNt_{ox}}{\epsilon_{ox}}\right)^2 + 8\frac{qN}{\epsilon_S}V_t\ln\frac{N}{n_i}}\right) \\
&= \frac{\epsilon_S t_{ox}}{2\epsilon_{ox}}\left(\sqrt{1 + \frac{8\epsilon_{ox}^2}{qN\epsilon_S t_{ox}^2}V_t\ln\frac{N}{n_i}} - 1\right) \\
&= 59.5 \text{ nm}
\end{aligned}$$

The electrostatic potential drop over the SC and over the oxide is then:

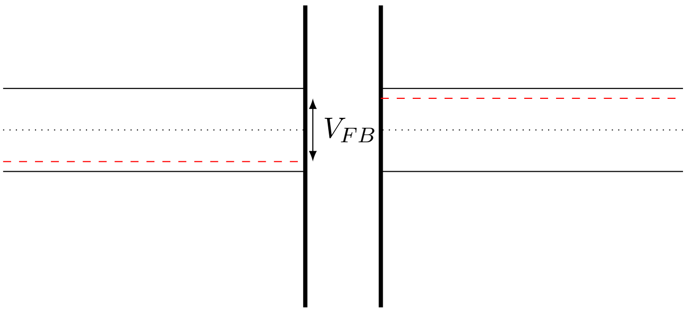
$$\begin{aligned}\Delta\Psi_{Si} &= \frac{qNz_D^2}{2\epsilon_S} \\ &= 0.283 \text{ V} \\ \Delta\Psi_{ox} &= \frac{qNz_D t_{ox}}{\epsilon_{ox}} \\ &= 0.272 \text{ V}\end{aligned}$$

Control: $\Delta\Psi_{tot} = 2\Delta\Psi_{Si} + \Delta\Psi_{ox} = 0.838 \text{ V} \implies$ okay!



b.

Flat-band is at $V_{FB} = \Phi_{ms} = 0.838$ V as in a.



c.

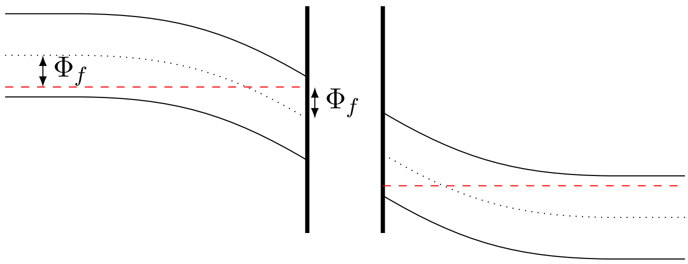
In this case $\Delta\Psi_{Si} = 2\Phi_f = 0.838$ V, thus:

$$\begin{aligned}\Delta\Psi_{Si} &= \frac{qNz_{D,th}^2}{2\epsilon_S} = 2\Phi_f \\ z_{D,th} &= \sqrt{\frac{2\epsilon_S \cdot 2\Phi_f}{qN}} \\ &= 102 \text{ nm}\end{aligned}$$

The electrostatic potential drop over the oxide is

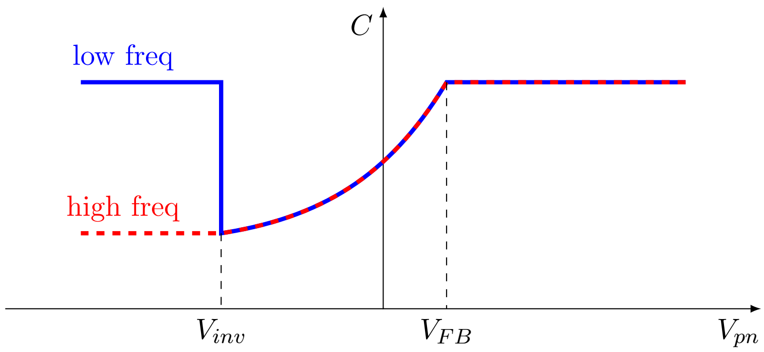
$$\begin{aligned}\Delta\Psi_{ox} &= \frac{qNz_{D,th} t_{ox}}{\epsilon_{ox}} \\ &= 0.468 \text{ V}\end{aligned}$$

The total electrostatic potential drop is thus $\Delta\Psi_{tot} = \Delta\Psi_{Si} + \Delta\Psi_{ox} = 2.144$ V. And the applied voltage is $V_{pn} = -\Delta\Psi_{tot} + \Phi_{ms} = -1.306$ V



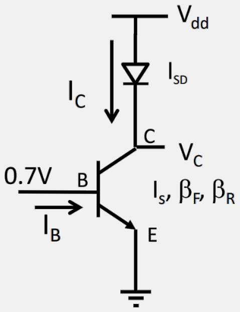
d.

Sketch similar to normal MOS cap.



Question 29

Consider an npn bipolar transistor, loaded by a diode as shown on the figure below.



The following data are known for the components:
npn transistor:
Forward active current gain $\beta_F = 100$
Reverse active current gain $\beta_R = 0.1$
Collector saturation current $I_s = 10^{-13}$ A
diode:
Reverse saturation current $I_{sD} = 10^{-13}$ A

a) Calculate the collector voltage, the base current and the collector current of the npn bipolar transistor when $V_{dd} = 3$ V.
(Solution: 2.3 V, 0.49 mA, 49 mA)

b) Calculate the collector voltage, the base current and the collector current of the npn bipolar transistor when $V_{dd} = 0.7$ V.
(Solution: 0.065 V, 41.59 mA, 3.79 mA)

The thermal voltage is 0.026 V.

a.

We assume the transistor is in forward active mode.

$$\begin{aligned} I_C &= I_s e^{\frac{V_{BE}}{V_t}} \\ &= 49.3 \text{ mA} \\ I_C = I_D &= I_{sD} e^{\frac{V_{DD}-V_C}{V_t}} \\ V_C &= V_{DD} - V_t \ln \frac{I_C}{I_{sD}} \\ &= 2.3 \text{ V} \end{aligned}$$

So BE is forward and BC is reverse, so we're indeed in forward active mode. $I_B = \frac{I_C}{\beta_F} = 0.49$ mA.

b.

In this case, if we would assume that we're in forward active mode, V_C would be 0 V. So, we're in saturation.

$$\begin{cases} I_C = I_s e^{\frac{V_B}{V_t}} - \left(1 + \frac{1}{\beta_R}\right) I_s e^{\frac{V_B - V_C}{V_t}} \\ I_C = I_{sD} e^{\frac{V_{DD}-V_C}{V_t}} \end{cases}$$

$$\begin{aligned} \left[I_s e^{\frac{V_B}{V_t}} - \left(1 + \frac{1}{\beta_R}\right) I_s e^{\frac{V_B - V_C}{V_t}}\right] e^{\frac{V_C - V_B}{V_t}} &= \left[I_{sD} e^{\frac{V_{DD}-V_C}{V_t}}\right] e^{\frac{V_C - V_B}{V_t}} \\ I_s e^{\frac{V_C}{V_t}} - \left(1 + \frac{1}{\beta_R}\right) I_s &= I_{sD} e^{\frac{V_{DD}-V_B}{V_t}} \\ V_C &= V_t \ln \left[\left(1 + \frac{1}{\beta_R}\right) + \frac{I_{sD}}{I_s} e^{\frac{V_{DD}-V_B}{V_t}}\right] \\ &= V_t \ln \left[(1 + 10) + 1\right] \\ &= 0.065 \text{ V} \end{aligned}$$

$$
\begin{aligned}
I_C &= I_{sD} e^{\frac{V_{DD}-V_C}{V_t}} \\
&= 4.11 \text{ mA} \\
I_B &= \frac{I_F}{\beta_F} + \frac{I_R}{\beta_R} \\
&= \frac{I_s}{\beta_F} e^{\frac{V_B}{V_t}} + \frac{I_s}{\beta_R} e^{\frac{V_B-V_C}{V_t}} \\
&= 0.49 \text{ mA} + 41.1 \text{ mA} \\
&= 41.55 \text{ mA}
\end{aligned}
$$

Question 30

Consider a piece of n-type silicon semiconductor. We measure the initial conductivity σ of the semiconductor to be 10 Scm^{-1} at room temperature. When we illuminate the semiconductor with light and wait until steady state is reached, we measure an increased conductivity, and the new value is 12 S cm^{-1} again at room temperature.
Use following values for the semiconductor parameters:
Electron mobility $\mu_n = 1100$ $\text{cm}^2/\text{V.s}$
Hole mobility $\mu_p = 400$ $\text{cm}^2/\text{V.s}$
Intrinsic carrier concentration at room temperature $n_i = 10^{10}$ cm^{-3}
Minority carrier lifetime $\tau = 1$ ms
Thermal voltage $V_{th} = 26$ mV at room temperature.
$q = 1.6 \cdot 10^{-19}$ C
N_c and N_v can be assumed to be equal.

a) What is the doping concentration of the semiconductor if we assume that the donors atoms are fully ionized at room temperature and the conductivity is only determined by drift?
(Solution: 5.7 10^{16}cm^{-3})

b) Calculate the excess concentration generated by the light in steady state (you can assume again that the conductivity is only determined by drift).
(Solution: 8.3 10^{15} cm^{-3})

c) How much is the external generation rate G_{ext} caused by the light illumination? You can assume that the generation/recombination is dominated by Shockley-Read-Hall with equal capture cross sections for hole and electrons and that the recombination centers are located at midgap ($E_{rg} = E_i$)
(Solution: 7.4 10^{21} $\text{cm}^{-3}\text{s}^{-1}$)

d) Draw the band diagram in equilibrium and in steady state after switching on the light. Clearly indicate the position of the Fermi level (in equilibrium) and the quasi-Fermi levels for holes and electrons (in steady state)?

a.

Only the electrons have a significant concentration.

$$
\begin{aligned}
\sigma &= qn\mu_n \\
n &= \frac{\sigma}{q\mu_n} \\
&= 5.7\ 10^{16}\ \text{cm}^{-3} \ggg 10^{10} \\
N_D &= n = 5.7\ 10^{16}\ \text{cm}^{-3}
\end{aligned}
$$

b.

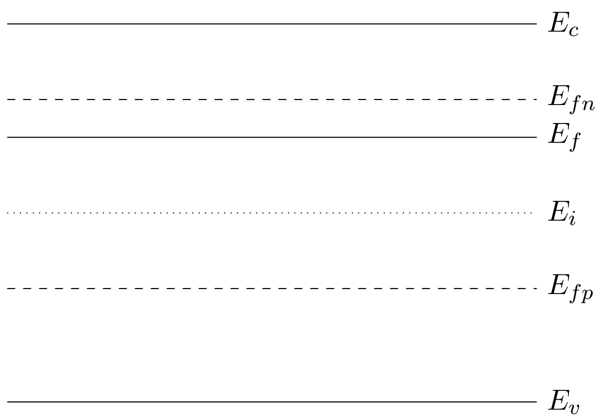
In this case both the electrons and the holes have a significant concentration. We call the excess concentration Δ:

$$
\begin{aligned}
\sigma_2 &= qn\mu_n + qp\mu_p \\
&= q(N_A + \Delta)\mu_n + q\Delta\mu_p \\
&= \sigma_1 + q\Delta\mu_n + q\Delta\mu_p \\
\Delta &= \frac{\sigma_2 - \sigma_1}{q(\mu_n + \mu_p)} \\
&= 8.3\ 10^{15}\ \mathrm{cm}^{-3}
\end{aligned}
$$

c.

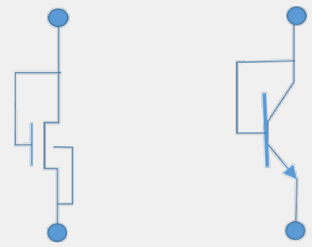
$$
\begin{aligned}
G_{ext} = U_{SRH} &= \frac{1}{\tau}\frac{pn - n_i^2}{n + p + 2n_i \cosh\frac{E_{rg} - E_i}{kT}} \\
&\approx \frac{pn}{\tau(n+p)} \\
&= 7.4\ 10^{18}\ \mathrm{cm}^{-3}\mathrm{s}^{-1}
\end{aligned}
$$

d.



Question 31

Consider an n-MOS transistor with positive threshold voltage ($V_T = 0.5$ V). We short the gate to the drain, as shown in the figure on the left.
Consider an npn bipolar transistor. We short the base to the collector, as shown on the right.
In both cases, the device we make is a two-terminal device.



a) Determine expressions (=formula's) for the quasi-static I-V characteristics for both these devices, for both positive and negative V (in other words, fill in formula in the table below). You can use parameters as you need, but you must define and explain each parameter that you use. You can use simplifications as you deem fit, but you must explain your assumptions and simplifications. You can neglect subthreshold currents for MOSFET's and generation, recombination or high injection effects for bipolar transistors

	I-V expression
nMOS-based device for positive V	I = …
nMOS-based device for negative V	I = ….
npn-based device for positive V	I = …
npn-based device for negative V	I = …

b) How does the temperature affect the quasi-static IV curves? You can assume that work-functions, charge carrier mobility, dielectric constants, bandgaps, effective density of states of conduction band and valence band are independent of temperature. The answers can be qualitative but we expect to see a short argumentation and a description of the function change (examples of descriptions: "no change with temperature in the expression that relates I to V", "linear decrease with increase of temperature", "logarithmic increase with increase of temperature", etc.)

nMOS: $V_{DS} = V_{GS} > V_{GS} - V_T$, so the device is always in saturation. The current is thus

$$I = \begin{cases} \beta\frac{(V-V_T)^2}{2} & V > V_T \\ 0 & V < V_T \end{cases}$$

With $\beta = \frac{W}{L}\mu C_{ox}$ given by the MOSFET parameters. With the assumptions given in the question and additionally neglecting the thermal expansion coefficients, the only temperature-dependent parameter is V_T:

$$\begin{aligned} V_T &= 2\frac{kT}{q}\ln\frac{N_A}{\sqrt{N_C N_V e^{\frac{-E_g}{kT}}}} + V_{FB} + \frac{1}{C_{ox}}\sqrt{2qN_A}\sqrt{\left|2\frac{kT}{q}\ln\frac{N_A}{\sqrt{N_C N_V e^{\frac{-E_g}{kT}}}} - V_{BS}\right|} \\ &= \frac{E_g}{q} - \frac{kT}{q}\ln\frac{N_C N_V}{N_A^2} + V_{FB} + \frac{1}{C_{ox}}\sqrt{2qN_A}\sqrt{\left|\frac{E_g}{q} - \frac{kT}{q}\ln\frac{N_C N_V}{N_A^2} - V_{BS}\right|} \end{aligned}$$

Assuming $N_C N_V > N_A^2$. So, as Φ_f drops of linearly with increasing temperature, there is more or less linear decrease of V_T with increasing temperature (ignoring the square root). As a consequence, there is a linear + quadratic increase in I for increasing temperature.

npn-transistor:

$V_{BC} = 0$ V $\implies I_R = I_{BR} = 0$ A.

$$\begin{aligned} I &= I_C = I_F \\ &= I_S\left(e^{\frac{V}{V_t}} - 1\right) \end{aligned}$$

With $I_S = \frac{AqV_t n_i^2}{B}\left(1 + \frac{W_B^2}{2L_B^2}\right) \approx \frac{AqV_t n_i^2}{B}$. With $B = \frac{N_A W_B}{\mu_n}$ for a short base (almost always the case) with constant doping.

Higher temperature gives linearly increasing V_t, exponentially decreasing n_i and exponentially decreasing exponent. Overall: exponentially increasing I, assuming $\frac{E_g}{q} > V \gg V_t$.

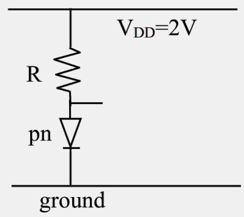
$$\begin{aligned} I &\sim V_t n_i^2\left(e^{\frac{V}{V_t}} - 1\right) \\ &\sim Te^{\frac{-E_g}{kT}}\left(e^{\frac{Vq}{kT}} - 1\right) \\ &\sim T\left(e^{\frac{Vq-E_g}{kT}} - 1\right) \\ &\sim Te^{\frac{Vq-E_g}{kT}} \qquad \text{(for } V \gg V_t\text{)} \end{aligned}$$

Question 32

Consider a p-n diode in series with a resistor, as shown in the sketch below. The supply voltage $V_{DD} = 2$ V. The diode is ideal, so you can neglect effects such as recombination / generation in the space charge region, high injection, etc... The diode is symmetric, i.e. p-region and n-region have identical width, doping and mobility of minority carriers. Both diode regions are short compared to the diffusion lengths of the minority carriers. At room temperature (300K), the saturation current is known (I_{S300}). Other parameters are given in the list.

a) Design the resistor such that the voltage Vpn over the diode at 300K is 0.5V.
(Solution: 575 Ohm)

b) We increase the temperature from 300K to 350K. The resistor can be approximated to be temperature-independent. The mobility of the minority carriers in the diode can be assumed to be inversely proportional to temperature ($\sim 1/T$). You can further assume that work-functions, dielectric constants, bandgaps, effective density of states of conduction band and valence band are temperature-independent.
⇒ What will be the voltage over the diode at 350K?
(Solution: 0.4V)



$$
\begin{aligned}
I_{S300} &= 10^{-11}\ \text{A} \\
\frac{kT}{q}(@300\ \text{K}) &= 25.8\ \text{mV} \\
E_g &= 1.12\ \text{eV}
\end{aligned}
$$

a)

The current is:

$$
\begin{aligned}
I &= I_S e^{\frac{V_{pn}}{V_t}} \\
&= 2.61\ \text{mA}
\end{aligned}
$$

So the resistance must be

$$
\begin{aligned}
R &= \frac{V_R}{I} \\
&= \frac{1.5\ \text{V}}{2.61\ \text{mA}} \\
&= 575\ \Omega
\end{aligned}
$$

b)

We assume W_{SCR} to be negligible to the width of the diode, so we don't need to take its temperature dependency in account

$$
\begin{aligned}
J_S &= qV_t n_i^2 \left(\frac{\mu_p}{W_n N_D} + \frac{\mu_n}{W_p N_A} \right) \\
&\sim V_t n_i^2 \mu \\
&\sim T e^{-\frac{E_g}{kT}} \frac{1}{T} \\
I_{S350} &\sim e^{-\frac{E_g}{kT_2} + \frac{E_g}{kT_1}} I_{S300} \\
I_{S350} &= 493.5 I_{S300} \\
&= 4.9\ 10^{-9}\ \text{A}
\end{aligned}
$$

The voltage over the diode is then determined as follows:

$$
V_{DD} = RI + V_t \ln \frac{I}{I_S}
$$

This equation can be solved iteratively:

$$
I^{(n+1)} = \frac{V_{DD} - V_t \ln \frac{I^{(n)}}{I_S}}{R}
$$

With $V_t = 0.0258\ \text{V} \cdot \frac{350}{300}$. By using a starting value of 2.6 mA (from a), this converges very quickly to 2.785 mA. The voltage can then be determined by:

$$
\begin{aligned}
V_{pn} &= V_{DD} - RI \\
&= 0.40\ \text{V}
\end{aligned}
$$

Or by:

$$
\begin{aligned}
V_{pn} &= V_t \ln \frac{I}{I_S} \\
&= 0.40\ \text{V}
\end{aligned}
$$

Question 33

Consider a p-type silicon semiconductor with a uniform acceptor doping concentration $N_A = 5 \cdot 10^{14}$ cm^{-3} and intrinsic concentration $n_i = 10^{10}$ cm^{-3}. The electron mobility is 1200 cm^2/(Vs) and the hole mobility is 450 cm^2/(Vs). The generation-recombination is determined by Shockley-Read-Hall with electron and hole minority lifetime $\tau_n = \tau_p = 1$ µs and the energy level of the trapping states is at the intrinsic level $E_{rg} = E_i$. N_c and N_v can be assumed to be equal.

a) Calculate the position of the Fermi level in equilibrium referred to the intrinsic level, and indicate this on a band diagram of the semiconductor. You can use $V_t = 26$ mV for the thermal voltage.
(Solution: -0.281eV)

b) We illuminate the semiconductor now with light, by which electron-hole pairs are generated and the quasi-Fermi level of the majority carriers in steady state is shifting over a distance of 0.01 kT.

   i. Calculate the excess electron and hole concentrations generated by the light.
   (Solution: $5 \cdot 10^{12}$ cm^{-3})

   ii. Calculate the position of the quasi-Fermi level of the minority carriers and draw both quasi-Fermi levels on the band diagram for this condition.
   (Solution: 0.162eV, -0.281eV)

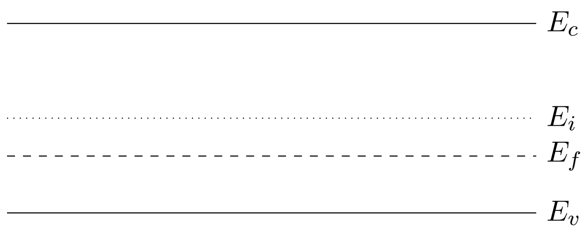
   iii. Calculate the external generation rate G_{ext} for this condition.
   (Solution: $5 \cdot 10^{18}$ cm^{-3}s^{-1})

c) We increase the light intensity now such that the steady-state electron density in the sample increases with a factor 10^{12} compared to the equilibrium value. Calculate now the new external generation rate of electron-hole pairs.
(Solution: 10^{23} cm^{-3}s^{-1}m^{-3})

d) Calculate the conductivity σ of the sample under steady state for the two conditions of light intensity.
(Solution: 0.036 S/cm, 52.8 S/cm)

a.

$$
\begin{aligned}
E_f &= E_i - V_t \ln \frac{N_A}{n_i} \\
E_f - E_i &= -0.281 \text{ V}
\end{aligned}
$$



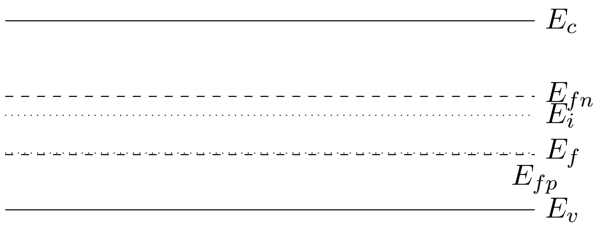
b.

i.

$$
\begin{aligned}
\Delta &= n_i \left(e^{\frac{E_i - E_{fp}}{kT}} - e^{\frac{E_i - E_f}{kT}} \right) \\
&= 10^{10} \left(e^{\frac{0.281}{0.026}} - e^{\frac{0.281}{0.026} - 0.01} \right) \\
&= 5 \cdot 10^{12} \text{ cm}^{-3}
\end{aligned}
$$

ii.

$$
\begin{aligned}
E_{fn} &= E_i + V_t \ln \frac{n}{n_i} \\
E_{fn} - E_i &= 0.026 \text{ V } \ln \frac{5 \cdot 10^{12}}{10^{10}} \\
&= 0.162 \text{ V}
\end{aligned}
$$



iii.

The majority carrier p remains almost constant, so the simplified formula can be used.

c.

In equilibrium the amount of electrons n_0 is

$$
\begin{aligned}
n_0 &= \frac{n_i^2}{N_A} \\
&= 2 \cdot 10^5
\end{aligned}
$$

So the new electron concentration is $n = 2 \cdot 10^{17} = \Delta$. In this case the majority carrier concentration increases to $p = \Delta = 2 \cdot 10^{17}$ as well. And the external generation rate is

$$
\begin{aligned}
G_{ext} = U_{SRH} &= \frac{1}{\tau} \frac{pn - n_i^2}{n + p + 2n_i \cosh\left(\frac{E_{rg} - E_i}{kT}\right)} \\
&\approx \frac{1}{\tau} \frac{pn}{n + p} \\
&\approx \frac{1}{\tau} \frac{\Delta^2}{2\Delta} \\
&= \frac{\Delta}{2\tau} \\
G_{ext} &= 1 \cdot 10^{23} \text{ cm}^{-3}\text{s}^{-1}
\end{aligned}
$$

d.

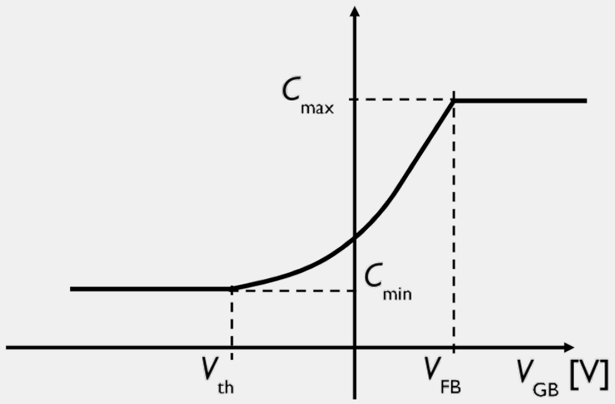
$$
\begin{aligned}
\sigma &= q(\mu_n n + \mu_p p) \\
\sigma_1 &\approx q\mu_p p \\
&= 1.6\ 10^{-19} \cdot 450 \cdot 5\ 10^{14} \\
&= 0.036\ \mathrm{S/cm} \\
\sigma_2 &= q(\mu_n n + \mu_p p) \\
&= q\Delta(\mu_n + \mu_p) \\
&= 1.6\ 10^{-19} \cdot 2\ 10^{17} \cdot 1650 \\
&= 52.9\ \mathrm{S/cm}
\end{aligned}
$$

Question 34

A silicon MOS capacitor with a metal gate, silicon-dioxide gate dielectric and substrate doping concentration of 10^{18} cm^{-3} has a high frequency C-V characteristic as shown below. We want to use this MOS-cap in a voltage controlled oscillator (VCO). In order to tune the frequency of the VCO within the desired frequency range we need to guarantee that $C_{max}/C_{min} = 2$.

a) Calculate the oxide thickness needed to guarantee this condition.
(Solution: 12.1 nm)

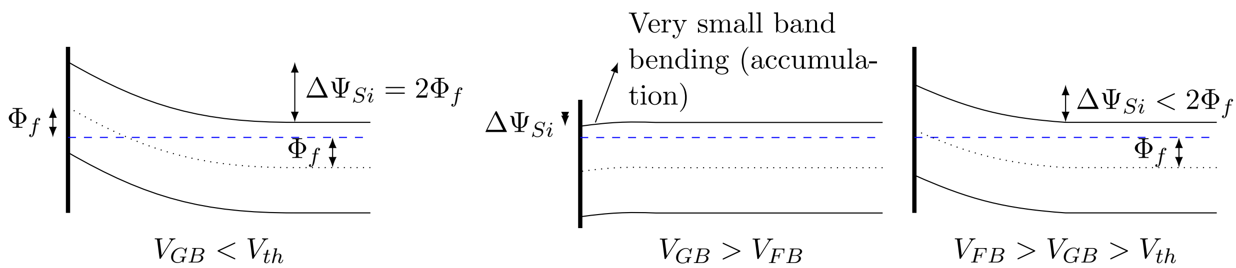
b) Draw the band diagram of the MOS-cap for $V_{GB} < V_{th}$, for $V_{GB} > V_{FB}$ and for $V_{FB} > V_{GB} > V_{th}$. Indicate the position of the Fermi level, the conduction and valence band and the intrinsic level, and the electrostatic potential drops.



a.

$$
\begin{aligned}
C_{max} &= A\frac{\epsilon_{ox}}{t_{ox}} \\
C_{min} &= \frac{A}{\frac{t_{ox}}{\epsilon_{ox}} + \frac{z_{D,th}}{\epsilon_S}} \\
\frac{C_{max}}{C_{min}} = 2 &= 1 + \frac{\epsilon_{ox}}{t_{ox}}\frac{z_{D,th}}{\epsilon_S} \\
t_{ox} &= \frac{\epsilon_{ox} z_{D,th}}{\epsilon_S\left(\frac{C_{max}}{C_{min}} - 1\right)} \\
&= \frac{\epsilon_{ox}}{\epsilon_S\left(\frac{C_{max}}{C_{min}} - 1\right)}\sqrt{\frac{2\epsilon_S \cdot 2\Phi_f}{qN_A}} \\
&= 12.1 \text{ nm}
\end{aligned}
$$

b.



Question 35

Consider a one-sided abrupt n$^+$p diode with following known parameters:
$N_A = 10^{16}$ cm^{-3}; $N_D = 10^{20}$ cm^{-3}; $n_i = 10^{10}$ cm^{-3}; $T = 295$ K; $\mu_e = 1000$ cm^2/(Vs); $\mu_h = 500$ cm^2/(Vs);

1 The temperature rises significantly above 295K.

a) How does the width of the space charge region change and why?

b) How does the slope of the quasi-static forward log(I) versus V_{pn} characteristic change and why?

c) How does the forward bias voltage at which the diode goes into high injection change and why?

2 We take a semiconductor with a smaller bandgap

a) How does the width of the space charge region change and why?

b) How does the slope of the quasi-static forward log(I) versus V_{pn} characteristic change and why?

c) How does the forward bias voltage at which the diode goes into high injection change and why?

3 We increase the doping concentration of the p-doped region with one order of magnitude (but it still remains a one-sided abrupt n$^+$p diode).

a) How does the width of the space charge region change and why?

b) How does the slope of the quasi-static forward log(I) versus V_{pn} characteristic change and why?

c) How does the forward bias voltage at which the diode goes into high injection change and why?

1.a)

$$W_{SCR} \approx \sqrt{\frac{2\epsilon_S}{qN_A}(V_{bi} - V_{pn})}$$

With

$$\begin{aligned}
V_{bi} &= V_t \ln\left(\frac{N_A N_D}{n_i^2}\right) \\
&= V_t \ln\left(\frac{N_A N_D}{N_C N_V e^{-\frac{E_g}{kT}}}\right) \\
&= \frac{kT}{q}\left[\ln\left(\frac{N_A N_D}{N_C N_V}\right) + \frac{E_g}{kT}\right] \\
&= \frac{E_g}{q} - \frac{kT}{q}\ln\left(\frac{N_C N_V}{N_A N_D}\right)
\end{aligned}$$

The built-in potential decreases with increasing temperature (if $N_A N_D < N_C N_V$), so the width of the space charge region as well $W_{SCR} \sim \sqrt{V_{bi}}$. The opposite is true if $N_A N_D > N_C N_V$

1.b)

$$\begin{aligned} I &= I_s e^{\frac{V_{pn} q}{kT}} \\ \log I &= \log I_s + \frac{V_{pn} q}{kT} \end{aligned}$$

The slope decreases.

1.c)

$$\begin{aligned} p = \frac{n_i^2}{N_A} e^{\frac{V_{HI}}{V_t}} &= N_A \\ V_{HI} &= V_t \ln \frac{N_A^2}{n_i^2} \\ &= V_t \ln \frac{N_A^2}{N_C N_V e^{\frac{-E_g}{kT}}} \\ &= \frac{kT}{q} \frac{E_g}{kT} - \frac{kT}{q} \ln \frac{N_C N_V}{N_A^2} \\ &= \frac{E_g}{q} - \frac{kT}{q} \ln \frac{N_C N_V}{N_A^2} \end{aligned}$$

Which decreases for increasing temperature (as long as $N_C N_V > N_A^2$, which is almost always the case).

2.a) From the equations in 1a, we see that V_{bi} and thus W_{SCR} decrease for a smaller bandgap. (assuming N_C, N_V, N_A and N_D remain the same)

2.b) We see from the equations in 1b that the bandgap has no influence on the slope (it has an influence on I_s though).

2.c) From the equations in 1c, we see that V_{HI} decreases for a smaller bandgap. (assuming N_C, N_V, N_A and N_D remain the same)

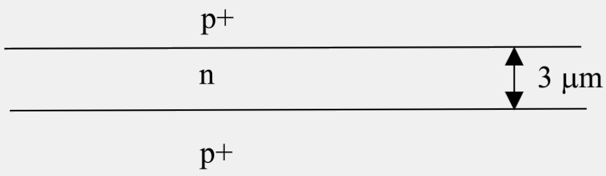
3.a) From the equations in 1a, we see that W_{SCR} decreases for a larger N_A (V_{bi} only becomes a little larger).

3.b) We see from the equations in 1b that the doping has no influence on the slope (it has an influence on I_s though).

3.c) From the equations in 1c, we see that V_{HI} increases for a larger N_A.

Question 36

Consider an n-type Si region sandwiched between two p+ Si regions. The p+ Si regions have a doping concentration $N_A = 10^{19}$ cm^{-3} whereas the n-type region has a doping concentration of $N_D = 10^{15}$ cm^{-3}. In this way two one-sided abrupt p+n junctions are formed.
Constants: $n_i = 10^{10}$ cm^{-3}, $\epsilon_s = 10^{-12}$ F/cm, $q = 1.6\ 10^{-19}$ C, $kT/q = 25$ mV.



a. In a first case the width of the n-region is 3 µm.

1) Calculate the built-in potential V_{bi} between the p+ and the n regions at equilibrium. *(Solution: 0.806 V)*
2) Sketch in a more or less quantitative way below each other the space charge concentration, the electric field and the electrostatic potential through p+/n/p+ structure in equilibrium.
3) Calculate the width of the space charge regions in the n-region in equilibrium. *(Solution:1 µm)*
4) Sketch the band diagram on scale and indicate the position of the Fermi level, the electrostatic potential and V_{bi} on the diagram.

b. We reduce the width of the n-region from 3 µm tot 1 µm.

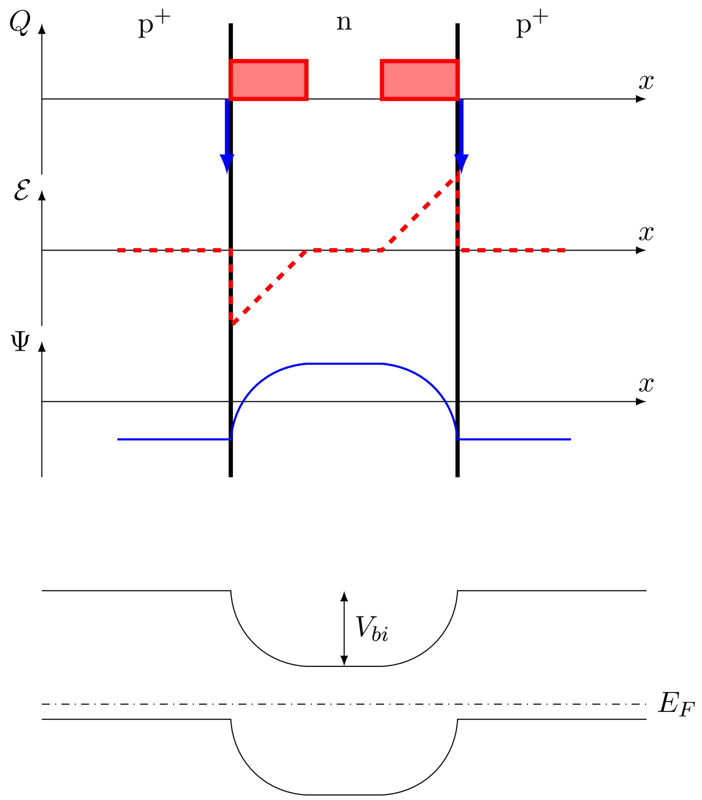
1) Sketch for this case again below each other the space charge concentration, the electric field and the electrostatic potential through p+/n/p+ structure in equilibrium.
2) Sketch the band diagram on scale and indicate the position of the fermi level, the electrostatic potential and V_{bi} on the diagram.
3) Calculate now the built-in electrostatic potential between the p+ region and the middle of the n-region in equilibrium. *(Solution: 0.2 V)*
4) Calculate the concentration of electron and holes in the middle of the n-region at equilibrium. *(Solution: 2.98 10^4 cm^{-3})*

a.

1)

$$\begin{aligned} V_{bi} &= V_t \ln \frac{N_A N_D}{n_i^2} \\ &= 0.806 \text{ V} \end{aligned}$$

2, 4)

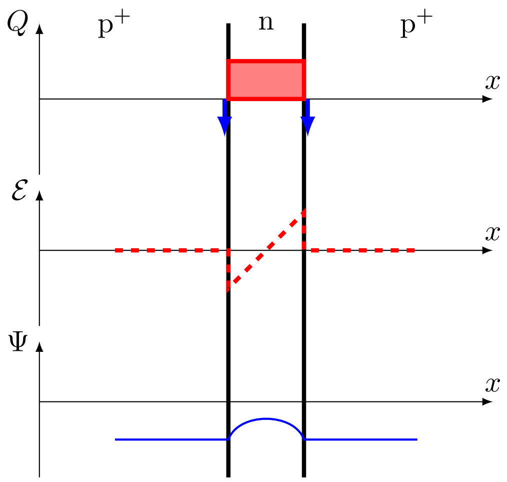


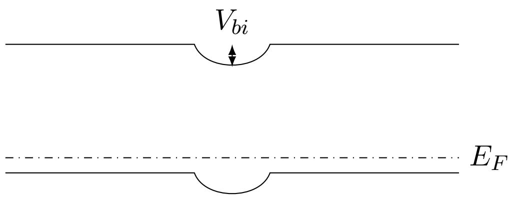
3)

$$\begin{aligned} W_{SCR} \approx -X_n &= \sqrt{\frac{2\epsilon_s}{q}\frac{1}{N_D}V_{bi}} \\ &= 1\ \mu\text{m} \end{aligned}$$

b.

1, 2)





3)

$$
\begin{aligned}
V_{bi} &= \int_{-\frac{W_n}{2}}^{0} dx' \int_{0}^{-dx'} \frac{\rho}{\epsilon_s} dx \\
&= \int_{-\frac{W_n}{2}}^{0} dx' \int_{0}^{-dx'} \frac{qN_D}{\epsilon_s} dx \\
&= -\frac{qN_D}{\epsilon_s} \int_{-\frac{W_n}{2}}^{0} x' dx' \\
&= \frac{qN_D W_n^2}{8\epsilon_s} \\
&= 0.2 \text{ V}
\end{aligned}
$$

4)

$$
\begin{aligned}
n &= n_i e^{\frac{E_F - E_i}{kT}} \\
n(0) &= n_i e^{\frac{E_F(p) + V_{bi} - E_i}{kT}}
\end{aligned}
$$

With $E_F(p)$ the fermi level in the p-region, so $E_F(p) - E_i = -kT \ln \frac{N_A}{n_i}$

$$
\begin{aligned}
n(0) &= n_i e^{\frac{E_F(p)-E_i}{kT}} e^{\frac{V_{bi}}{kT}} \\
&= \frac{n_i^2}{N_A} e^{\frac{V_{bi}}{kT}} \\
&= 29810 \text{ cm}^{-3} \\
p(0) &= \frac{n_i^2}{n(0)} \\
&= N_A e^{-\frac{V_{bi}}{kT}} \\
&= 3.35\ 10^{15} \text{ cm}^{-3}
\end{aligned}
$$

Question 37

A piece of p-doped semiconductor is illuminated with a uniform light source causing a constant and uniform generation G ($\text{cm}^{-3}\text{s}^{-1}$) of electron-hole pairs. The semiconductor has the following known parameters:
$N_A = 10^{17}$ cm^{-3}, $n_i = 10^{10}$ cm^{-3}, $kT/q = 25$ mV, bandgap = 1.1 eV.
The generation/recombination is determined by Shockley-Read-Hall, with lifetime of electrons and holes given by $\tau_n = \tau_p = 1$ ms, whereby the energy level of the trapping centers are at the intrinsic level: $E_{rg} = E_i$. N_c and N_v can be assumed to be equal.
G changes from 100 cm^{-3}s^{-1} up to 10^{22} cm^{-3}s^{-1}.
Calculate and plot the position of the quasi-Fermi levels of electrons and holes as a function of the generation G (G on log-scale). Or with other words: determine the locus of E_{Fn} and E_{Fp} on the diagram below:

In equilibrium $p = 10^{17}$ cm^{-3} and

$$\begin{aligned} n &= \frac{n_i^2}{N_A} \\ &= 10^3 \text{ cm}^{-3} \end{aligned}$$

So

$$\begin{aligned} E_f &= E_i - kT \ln \frac{N_A}{n_i} \\ E_f - E_i &= -0.403 \text{ V} \end{aligned}$$

At very small $G \ll \frac{n_0}{\tau} = 10^6$ cm^{-3}s^{-1}, both the minority n and majority carrier concentration p stays constant, so also their quasi-fermi level remains constant.

At small $10^6 \ll G \ll \frac{N_A}{\tau} = 10^{20}$ cm^{-3}s^{-1}, the majority carrier concentration p says constant and n can be calculated as $n = G\tau$. The quasi-fermi level is then

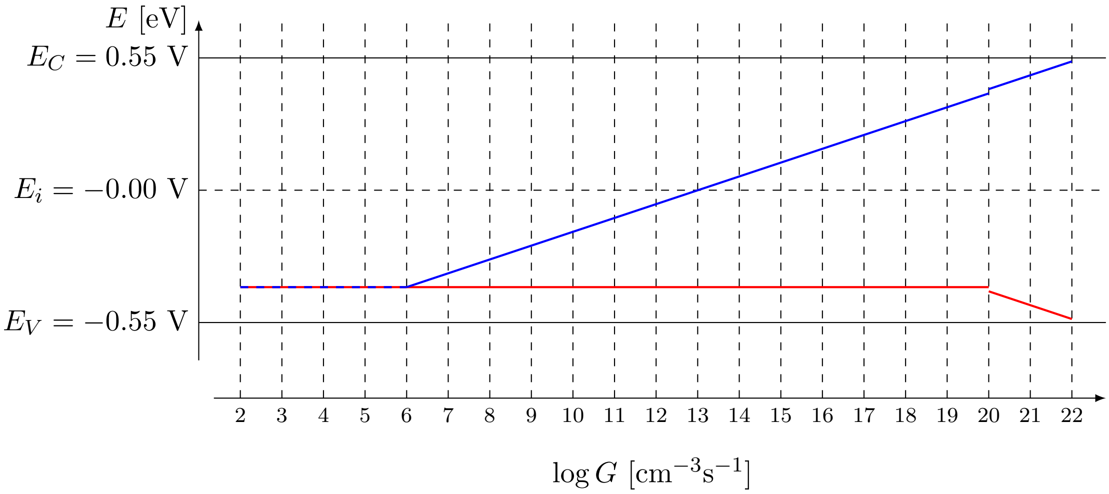
$$\begin{aligned} E_{fn} &= E_i + kT \ln \frac{n}{n_i} \\ E_{fn} - E_i &= kT \ln \frac{G\tau}{n_i} \end{aligned}$$

For large $G \gg \frac{N_A}{\tau} = 10^{20}$ cm^{-3}s^{-1}, the majority carrier concentration p changes and we approximate $n = p = \Delta$, with

$$\begin{aligned} G = U_{SRH} &= \frac{1}{\tau} \frac{pn - n_i^2}{n + p + 2n_i \cosh\left(\frac{E_{rg} - E_i}{kT}\right)} \\ &\approx \frac{1}{\tau} \frac{pn}{n+p} \\ &\approx \frac{1}{\tau} \frac{\Delta^2}{2\Delta} \\ &= \frac{\Delta}{2\tau} \\ n = p &= 2\tau G \end{aligned}$$

So

$$
\begin{aligned}
E_{fn} - E_i &= kT \ln \frac{2G\tau}{n_i} \\
E_{fp} - E_i &= -kT \ln \frac{2G\tau}{n_i}
\end{aligned}
$$



Question 38

Consider a pn-junction diode with constant doping concentration, fabricated in silicon. The acceptor doping of the p-region is known to be $N_A = 2 \cdot 10^{16}$ cm^{-3}. We also know that the two neutral regions are electrically long regions. The other known data of the diode are given below:
electron diffusion constant $D_n = 18$ cm^2/s
hole diffusion constant $D_p = 10$ cm^2/s
minority carrier lifetimes $\tau_{p0} = \tau_{n0} = 10^{-8}$ s
junction cross section area $A = 5\ 10^{-3}$ cm^2
thermal voltage at room temperature $V_t = 26$ mV
intrinsic concentration $n_i = 1.45\ 10^{10}$ cm^{-3}
silicon permitivity $\epsilon_{Si} = 10^{-12}$ F/cm
unit charge $q = 1.6\ 10^{-19}$ C

i) Design the donor concentration of the n-region such that 90 % of the current in the space charge region is carried by electrons when the diode is forward biased. You can assume that recombination in the space charge region can be neglected and that there are no high injection effects.
(Answer: 1.34 10^{17} cm^{-3})

ii) Calculate the built-in potential at equilibrium ($V_{pn} = 0$ V).
(Answer: 0.785 V)

iii) How much will the forward current be at $V_{pn} = 0.5$ V. You can again assume that recombination in the space charge region can be neglected and that there are no high injection effects.
(Answer: 0.089mA)

iv) At which forward bias will the pn-diode start operating in high injection regime?
(Answer: 0.735 V)

v) How much will the reverse current of the diode be at $V_{pn} = -5$ V

a. If there is no generation or recombination in the space charge region?
(Answer: 0.4 pA)

b. If there is thermal SRH- generation in the space charge region with generation time $\tau^* = 2.5\ 10^{-3}$ s
(Answer: 0.7 pA)

i.

$$\frac{J_n}{J_p} = \frac{90\ \%}{10\ \%} = 9 = \frac{D_n N_D L_p}{D_p N_A L_n}$$ Other variables like $q, n_i, e^{\frac{V_{pn}}{V_t}}$ are the same in the current equations

$$= \frac{N_D}{N_A}\sqrt{\frac{D_n}{D_p}}$$ using $L = \sqrt{D\tau}$ and $\tau_{p0} = \tau_{n0}$

$$N_D = 9N_A\sqrt{\frac{D_p}{D_n}} = 1.34\ 10^{17}\ \text{cm}^{-3}$$

ii.

$$\begin{aligned} V_{bi} &= V_t \ln \frac{N_A N_D}{n_i^2} \\ &= 0.785 \text{ V} \end{aligned}$$

iii.

$$\begin{aligned} I &= A \frac{1}{0.9} \frac{q n_i^2}{N_A} \sqrt{\frac{D_n}{\tau_{n0}}} e^{\frac{V_{pn}}{V_t}} \\ &= 89.1 \ \mu\text{A} \end{aligned}$$

iv.

High injection if the minority carriers are equal to the doping concentration, this will first happen on the p-side:

$$\begin{aligned} n_p = N_A &= \frac{n_i^2}{N_A} e^{\frac{V_{HI}}{V_t}} \\ V_{HI} &= V_t \ln \frac{N_A^2}{n_i^2} \\ &= 0.735 \text{ V} \end{aligned}$$

v.

a.

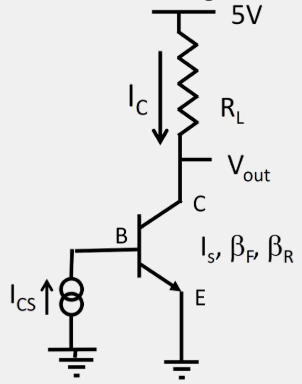
$$\begin{aligned} I_{reverse} &= A \frac{1}{0.9} \frac{q n_i^2}{N_A} \sqrt{\frac{D_n}{\tau_{n0}}} \\ &= 0.40 \text{ pA} \end{aligned}$$

b.

$$\begin{aligned} I_{reverse} &= A \frac{1}{0.9} \frac{q n_i^2}{N_A} \sqrt{\frac{D_n}{\tau_{n0}}} + A q W_{SCR} \frac{n_i}{\tau^*} \\ &= A \frac{1}{0.9} \frac{q n_i^2}{N_A} \sqrt{\frac{D_n}{\tau_{n0}}} + A q \sqrt{\frac{2\epsilon_S}{q} \frac{N_A N_D}{N_A - N_D} (V_{bi} - V_{pn})} \frac{n_i}{\tau^*} \\ &= 0.40 \text{ pA} + 0.30 \text{ pA} \\ &= 0.70 \text{ pA} \end{aligned}$$

Question 39

A bipolar transistor is connected to a load resistance R_L as shown on the figure below. A current source is sending a constant current I_{CS} into the base.



I_s is the collector saturation current of the npn transistor, and β_F and β_R are the forward and reverse common emitter current gain. The base transport factor of the npn transistor can be assumed to be 1 and recombination in the space charge regions, high injection effects, Early effects, emitter, base and collector resistances can all be neglected.
$I_s = 10^{-13}$ A
$\beta_F = \beta_R = 100$
The thermal voltage $V_{th} = 0.026$ V.

1. Design the load resistance R_L such that the output voltage V_{out} is at 4.5 V when the current delivered by the current source $I_{cs} = 1$ µA.
   (Answer: 5 kOhm)
2. Calculate the base voltage V_B and the collector current I_C for this condition.
   (Answer: 0.54 V, 100 µA)
3. Calculate the collector current I_C when the current delivered by the current source $I_{cs} = 2$ mA (the current should be within 10% accuracy). How much is now the base voltage V_B approximately? Make also an estimation of the internal bipolar current (I_{BF}, I_{BR}, I_F and I_R).
   (Answer: 1mA, 0.72V, 1mA, 1mA, 0.1A, 0.1A)

1/2.

$V_C = V_{out} = 4.5$ V. Let's first assume the transistor is in the forward regime. In that case $I_C = \beta_F I_{cs} = 100$ µA. We check the linear regime by calculating V_B:

$$\begin{aligned} I_C &\approx I_s e^{\frac{V_{BE}}{V_t}} \\ V_B &= V_E + V_t \ln \frac{I_C}{I_s} \\ &= 0.539 \text{ V} \end{aligned}$$

Which means that BE junction is in forward and the CB junction in reverse, so the transistor is indeed in the forward regime.

The resistor is then $R_L = \frac{V_{DD}-V_{out}}{I_C} = 5$ kΩ.

3.

I_B is extremely high, so it is impossible that $I_C = \beta_F I_B$. This means the transistor is in saturation. So $V_{CE} \approx 0$, so

$$\begin{aligned} V_C &= 0 \text{ V} \\ I_C &= \frac{V_{DD} - V_C}{R_L} = 1 \text{ mA} \\ I_B &= \frac{I_F}{\beta_F} + \frac{I_R}{\beta_R} = \frac{I_s}{\beta_F} e^{\frac{V_B}{V_t}} + \frac{I_s}{\beta_R} e^{\frac{V_B - V_C}{V_t}} \end{aligned}$$

From which follows $I_F = I_R = \frac{\beta I_B}{2} = 100$ mA.

$$\begin{aligned} I_{BF} &= \frac{I_F}{\beta_F} = 1 \text{ mA} \\ I_{BF} &= \frac{I_F}{\beta_F} = 1 \text{ mA} \end{aligned}$$

$V_B = V_t \ln \frac{I_F}{I_s} = 0.718$ V, which confirms that the transistor is in saturation. We still need to check whether $V_C \approx 0$

$$\begin{aligned} I_B &= \frac{I_F}{\beta_F} + \frac{I_R}{\beta_R} = \frac{I_s}{\beta_F} e^{\frac{V_B}{V_t}} + \frac{I_s}{\beta_R} e^{\frac{V_B - V_C}{V_t}} \\ V_C &= V_B - V_t \ln\left(\beta_R \frac{I_B}{I_s} - \frac{\beta_R}{\beta_F} e^{\frac{V_B}{V_t}}\right) = 0 \end{aligned}$$

If we want to calculate everything exactly, we use the current equations:

$$\begin{cases} I_B = \frac{I_F}{\beta_F} + \frac{I_R}{\beta_R} \\ I_C = I_F - I_R - I_{BR} = I_F - \left(1 + \frac{1}{\beta_R}\right) I_R \\ I_C = \frac{V_{DD} - V_C}{R_L} \end{cases}$$

$$\begin{cases} I_B = \frac{I_s}{\beta_F} e^{\frac{V_B}{V_t}} + \frac{I_s}{\beta_R} e^{\frac{V_B - V_C}{V_t}} \\ \frac{V_{DD} - V_C}{R_L} = I_s e^{\frac{V_B}{V_t}} - \left(1 + \frac{1}{\beta_R}\right) I_s e^{\frac{V_B - V_C}{V_t}} \end{cases}$$

From the first equation we get

$$V_C = V_B - V_t \ln\left(\beta_R \frac{I_B}{I_s} - \frac{\beta_R}{\beta_F} e^{\frac{V_B}{V_t}}\right)$$

Filling this into the second one gives

$$\begin{aligned} \frac{V_{DD} - V_B + V_t \ln\left(\beta_R \frac{I_B}{I_s} - \frac{\beta_R}{\beta_F} e^{\frac{V_B}{V_t}}\right)}{R_L} &= I_s e^{\frac{V_B}{V_t}} - \left(1 + \frac{1}{\beta_R}\right) I_s e^{\frac{V_B - V_B + V_t \ln\left(\beta_R \frac{I_B}{I_s} - \frac{\beta_R}{\beta_F} e^{\frac{V_B}{V_t}}\right)}{V_t}} \\ \frac{V_{DD} - V_B + V_t \ln\left(\beta_R \frac{I_B}{I_s} - \frac{\beta_R}{\beta_F} e^{\frac{V_B}{V_t}}\right)}{R_L} &= I_s e^{\frac{V_B}{V_t}} - \left(1 + \frac{1}{\beta_R}\right) I_s \left(\beta_R \frac{I_B}{I_s} - \frac{\beta_R}{\beta_F} e^{\frac{V_B}{V_t}}\right) \end{aligned}$$

Using $\beta = \beta_F = \beta_R$ gives

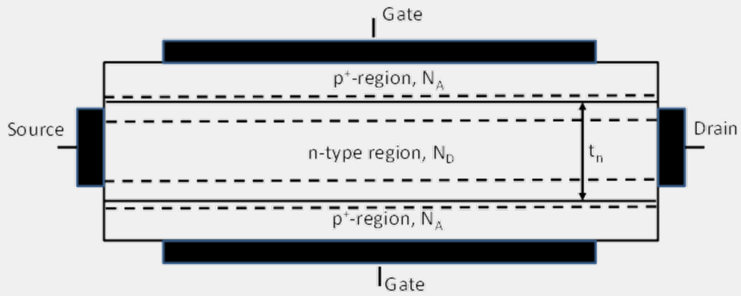
$$\begin{aligned} \frac{V_{DD} - V_B + V_t \ln\left(\beta \frac{I_B}{I_s} - e^{\frac{V_B}{V_t}}\right)}{R_L} &= I_s e^{\frac{V_B}{V_t}} - \left(1 + \frac{1}{\beta}\right) I_s \left(\beta \frac{I_B}{I_s} - e^{\frac{V_B}{V_t}}\right) \\ \frac{V_{DD} - V_B + V_t \ln\left(\beta \frac{I_B}{I_s} - e^{\frac{V_B}{V_t}}\right)}{R_L} &= \left(2 + \frac{1}{\beta}\right) I_s e^{\frac{V_B}{V_t}} - (\beta + 1) I_B \\ V_B &= V_t \ln\left(\frac{\frac{V_{DD} - V_B + V_t \ln\left(\beta \frac{I_B}{I_s} - e^{\frac{V_B}{V_t}}\right)}{R_L} + (\beta + 1) I_B}{\left(2 + \frac{1}{\beta}\right) I_s}\right) \end{aligned}$$

Which converges to $V_B = 0.719$ V. So,

$$\begin{aligned}
V_C &= V_B - V_t \ln\left(\beta_R \frac{I_B}{I_s} - \frac{\beta_R}{\beta_F} e^{\frac{V_B}{V_t}}\right) = 0.517 \text{ mV} \\
I_C &= \frac{V_{DD} - V_C}{R_L} = 0.9999 \text{ mA} \\
I_F &= I_s e^{\frac{V_B}{V_t}} = 101 \text{ mA} \\
I_{BF} &= \frac{I_F}{\beta_F} = 1.01 \text{ mA} \\
I_R &= I_s e^{\frac{V_B - V_C}{V_t}} = 99 \text{ mA} \\
I_{BF} &= \frac{I_F}{\beta_F} = 0.99 \text{ mA}
\end{aligned}$$

Question 40

A Junction Field Effect Transistor (JFET) consist of an n-type semiconductor region sandwiched between two highly doped p-type regions, as shown on the figure below. The n-region forms a conducting channel in between the two space charge regions of the two p$^+$-n junctions. The length of the channel is called L, the width of the channel is W and the thickness of the n-region between the two metallurgical junctions is called t_n (see fig).
The left side of this conductive region is contacted and is called the source (V_S), the right side is contacted and acts as the drain of the transistor (V_D). The two p$^+$-regions are contacted together and act as the gate of the JFET (V_G).



You can assume that current in the n-channel region is only conducted by majority carriers, and that there is no current flowing in the space charge regions. The V_{DS} can be assumed to be very small (10 mV).

- When the gate voltage is changed the space charge regions around the two junctions are becoming wider or narrower (depending on the polarity of V_G), in this way modulating the current and thus the conductance of the n-region.
- When the two space charge regions are touching each other the n-region is fully depleted and the transistor goes into cut off and conduct no current anymore. The V_{GS} where this happens is called the cut off voltage V_{CO}.

The data for the JFET structure are as follows:
p$^+$-doping concentration $N_A = 10^{19}$ cm^{-3}
n-doping concentration $N_D = 5\ 10^{15}$ cm^{-3}
thickness of the n-region from junction to junction: $t_n = 2$ µm
mobility of electrons = 1000 cm^2/Vs
channel length L = 5 µm
channel width W = 20 µm
Constants: $n_i = 10^{10}$ cm^{-3}, $\epsilon_s = 10^{-12}$ F/cm, $q = 1.6\ 10^{-19}$ C, $kT/q = 25$ mV

1. Calculate the effective thickness of the n-channel region at $V_{GS} = 0$ V?
   (Answer: 1.08 µm)
2. Based on this calculate the conductance G_{ch} (in Siemens=Ω^{-1} !) at $V_{GS} = 0$ V and around $V_{DS} = 0$ V.
   (Answer: 3.5 10^{-4} A/V)
3. Derive an expression for the cutoff voltage V_{CO}, i.e the V_{GS} voltage for which the n-region is fully depleted. At which gate voltage will the JFET be in cut-off?
   (Answer: -3.15 V)
4. Write an expression for the conductance around $V_{DS} = 0$ V as a function of the gate voltage V_{GS} for V_{GS} between V_{CO} and 0 V ? Sketch this conductance as a function of V_{GS}.
5. Draw the band diagram of the JFET in the vertical direction from the top p$^+$-region to the bottom p$^+$-region, and this for $V_{GS} = 0$ V and for $V_{GS} = V_{CO}$)?

1.

The SCR thickness on the n-side is

$$|X_n| \approx \sqrt{\frac{2\epsilon_s}{q}\frac{1}{N_D}(V_{bi} - V_{GS})}$$

With $V_{bi} = V_t \ln \frac{N_A N_D}{n_i^2} = 0.846$ V, this becomes $|X_n| = 460$ nm. So the effective thickness is $t_{eff} = t_n - 2|X_n| = 1.08$ µm.

2.

$$\begin{aligned} G_{ch} &= q\mu_n n \frac{W t_{eff}}{L} \\ &= q\mu_n N_D \frac{W t_{eff}}{L} \\ &= 3.46\ 10^{-4}\ \mathrm{S} \end{aligned}$$

3.

The n-region is completely depleted if

$$\begin{aligned} t_n &= 2|X_n| \\ &= 2\sqrt{\frac{2\epsilon_s}{q}\frac{1}{N_D}(V_{bi} - V_{GS})} \\ V_{GS} &= V_{bi} - t_n^2 \frac{qN_D}{8\epsilon_s} \\ &= 0.846\ \mathrm{V} - 4\ \mathrm{V} \\ &= -3.15\ \mathrm{V} \end{aligned}$$

4.

$$\begin{aligned} G_{ch} &= q\mu_n n \frac{W t_{eff}}{L} \\ &= q\mu_n N_D \frac{W(t_n - 2|X_n|)}{L} \\ &= q\mu_n N_D \frac{W}{L}\left(t_n - 2\sqrt{\frac{2\epsilon_s}{q}\frac{1}{N_D}(V_{bi} - V_{GS})}\right) \end{aligned}$$

Question 41

A p-type semiconductor with uniform background doping $N_A = 10^{16}$ cm^{-3} and intrinsic concentration $n_i = 10^{10}$ cm^{-3} is illuminated such that the generation G_{ext} of electron-hole pairs is uniform in the material. The generation/recombination is determined by Shockley-Read- Hall, with a lifetime of electrons and holes $\tau_n = \tau_p = 0.1$ ms, for which the energy level of the trap centers is at the intrinsic energy level: $E_{rg} = E_i$. N_c and N_v can be assumed to be equal.

a) How large should G be to produce a minority carrier concentration of 1% of the doping concentration.
(Answer: 10^{18} $cm^{-3}s^{-1}$)

b) How large should G be to produce a minority carrier concentration of 10 times the doping concentration.
(Answer: $5 \cdot 10^{20}$ $cm^{-3}s^{-1}$)

c) Draw the band diagram of the semiconductor in equilibrium and for the conditions of a) and b). Indicate the positions of the quasi-Fermi levels of electrons and holes.

d) Assume the steady-state G of question a) was applied until a time $t = t_0$. We switch off the light at $t = t_0$. Plot the evolution of the electron concentration as a function of time for $t > t_0$.

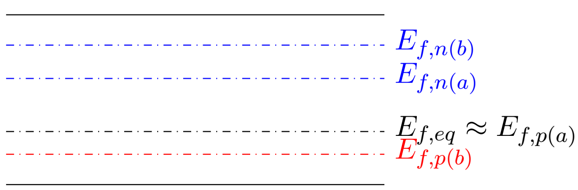
a) If $n \approx \Delta n = 0.01N_A$, then $\Delta p \ll p$ and G can be approximated as:

$$\begin{aligned} G &= \frac{n - n_{po}}{\tau_n} \\ &= \frac{\Delta n}{\tau_n} \\ &= \frac{0.01N_A}{\tau_n} \\ &= 10^{18} \text{ cm}^{-3}\text{s}^{-1} \end{aligned}$$

b) If $n = 10N_A$, then $p \approx n = 10N_A \gg n_i$ and G can be calculated as:

$$\begin{aligned} G &= \frac{1}{\tau} \frac{pn - n_i^2}{n + p + 2n_i \cosh\left(\frac{E_{rg}-E_i}{kT}\right)} \\ &\approx \frac{1}{\tau} \frac{(10N_A)^2}{20N_A} \\ &= \frac{5N_A}{\tau_n} \\ &= 5 \cdot 10^{20} \text{ cm}^{-3}\text{s}^{-1} \end{aligned}$$

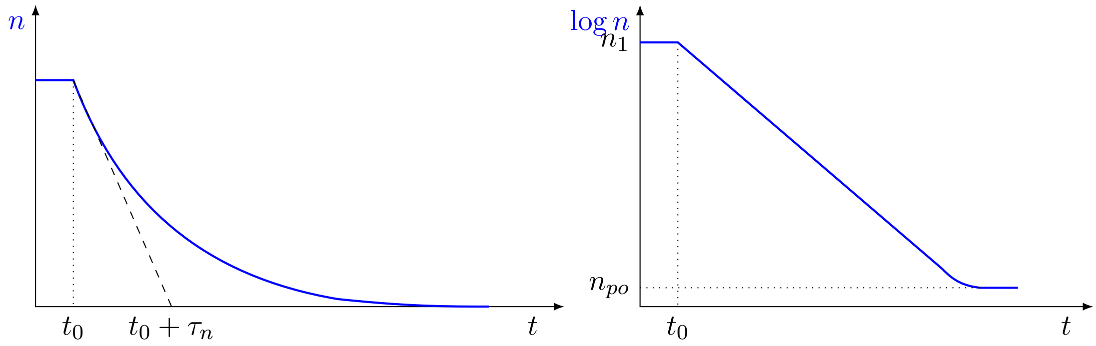
c)



d)

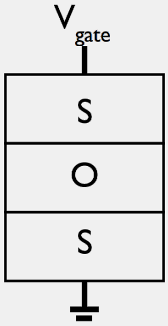
$$\frac{dn}{dt} = U = \frac{n - n_{po}}{\tau_n}$$
$$n = n_1 - (n_1 - n_{po})e^{\frac{-t}{\tau_n}}$$

With $n_1 = 10^{14}$ cm^{-3} and $n_{po} = 10^4$ cm^{-3}.



Question 42

A MOS-cap is fabricated, but instead of a metal gate, a semiconductor material 'S' is used. The two S-regions are p-type doped silicon with identical doping levels and thickness of 1 micron. You can assume that the fixed charge in the oxide and the surface state density are 0.



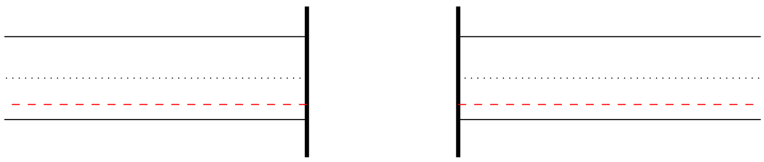
$N_A = 10^{18}$ at/cm^3
$n_i = 10^{10}$ at/cm^3
$E_g = 1.12$ eV
E_i: midgap
$kT/q = 25.8$ mV
$q = 1.6\ 10^{-19}$ C
$\epsilon_{Si} = 10^{-12}$ F/cm
$\epsilon_{ox} = 3.5\ 10^{-13}$ F/cm
$t_{ox} = 10$ nm

a. For $V_{gate} = 0$ V: draw the band diagram (including (quasi)-Fermi levels, and intrinsic level) + charge diagram (= position-dependent Q across the device).

b. For $V_{gate} = -6$ V, draw the band diagram (including (quasi)-Fermi levels) + charge diagram (= position-dependent Q across the device). Indicate gate-side and groundside.

a.

As both materials are exactly the same $\Phi_{ms} = 0$ V and thus $V_{FB} = 0$ V.



There are no charges.

b.

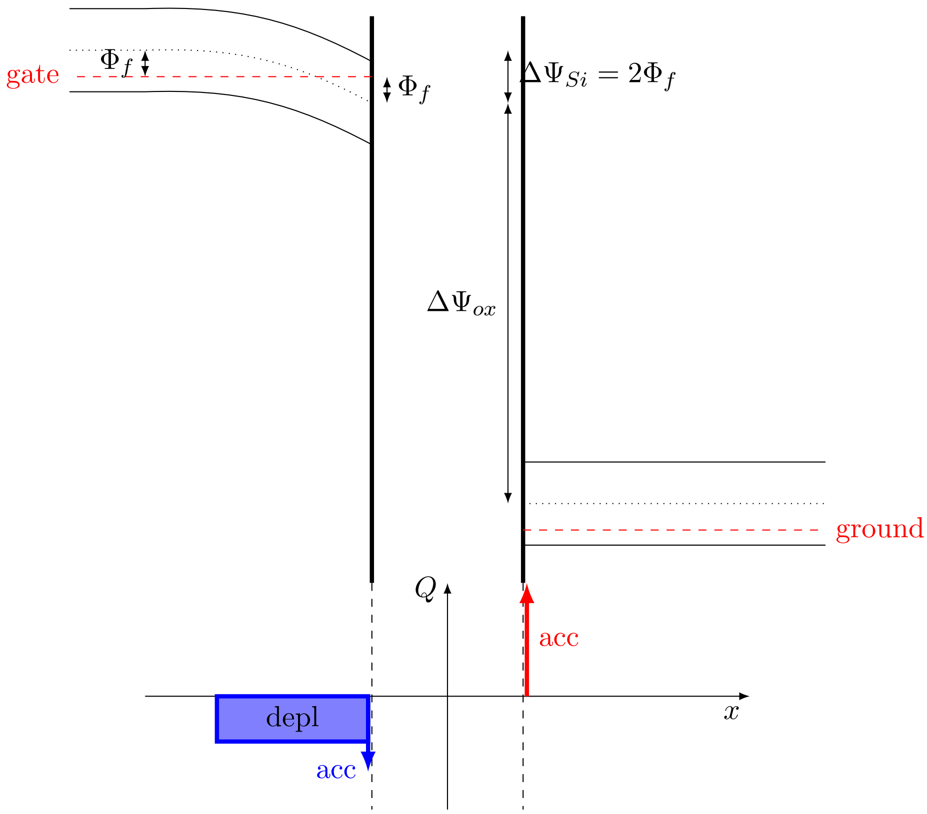
The top side will be either in depletion or in inversion and the bottom side in accumulation. So, we can assume the bottom acts like a metal.

Let's first assume the top side is in depletion. We need to calculate the electrostatic potential drop over the top SC and over the oxide:

$$\begin{aligned}
\Delta\Psi_{tot} = V_G - \Psi_{ms} &= \frac{qN_A z_D^2}{2\epsilon_S} + \frac{qN_A z_D t_{ox}}{\epsilon_{ox}} \qquad \text{with} \Psi_{ms} = 0 \\
z_D &= \frac{\epsilon_S t_{ox}}{\epsilon_{ox}}\left(\sqrt{1 + \frac{2\epsilon_{ox}^2}{qN_A\epsilon_S t_{ox}^2}|V_G|} - 1\right) \\
&= 62.6 \text{ nm} \\
\Delta\Psi_{Si} &= \frac{qN_A z_D^2}{2\epsilon_S} \\
&= 3.137 \text{ V} > 2\Phi_f
\end{aligned}$$

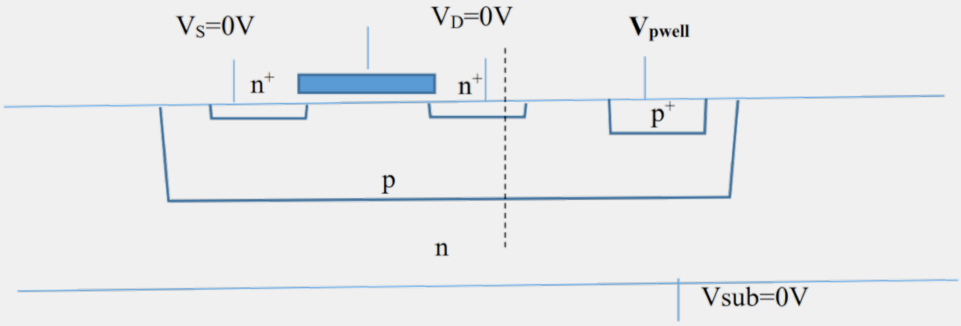
This means we're in inversion, our assumption was wrong. We can calculate the depletion depth again:

$$
\begin{aligned}
\Delta\Psi_{Si} &= 2V_t \ln \frac{N_A}{n_i} \\
&= 0.951 \text{ V} \\
z_{D,th} &= \frac{4\epsilon_S \Phi_f}{qN_A} \\
&= 34.5 \text{ nm} \\
\Delta\Psi_{ox} &= \Delta\Psi_{tot} - \Delta\Psi_{Si} \\
&= 5.05 \text{ V}
\end{aligned}
$$



Question 43

Consider the figure below. An n-MOS transistor is made in a p-well, the p-well is a doped p-region in an n-type silicon substrate. The transistor has a long channel length $L = 10$ micron. The p-well is 1 micron deep, its doping is $N_A = 10^{16}$ cm^{-3}. It is biased at a voltage V_{pwell}. The n-substrate is doped at the level $N_D = 10^{15}$ cm^{-3}. It is biased at a voltage $V_{sub} = 0$ V. The depth of the source and drain regions is 100 nm. The doping of these regions is $N_{SD} = 10^{18}$ cm^{-3}. The bias at source and drain is 0 V ($V_S = V_D = 0$ V). The threshold voltage of the transistor for zero applied bias at the p-well ($V_{pwell} = 0$ V) is 0.8 V (for $V_S = V_D = 0$ V).



a) Calculate the flatband voltage of this transistor.
(Answer: -1.5 V)

b) What is the p-well voltage that we can apply when the transistor is biased in inversion, before reaching punch-through of the p-well (punch-through means that the p-well is fully depleted by too wide space charge regions)? Accuracy of 1 mV is demanded.
(Answer: -2.812 V)

c) Draw the band diagram through the drain at the indicated position by the dashed line, p-well voltage of b).

$n_i = 10^{10}$ cm^{-3}
$t_{ox} = 100$ nm
$\epsilon_{ox} = 3.5$, $\epsilon_{Si} = 12$, $\epsilon_0 = 8.85\ 10^{-14}$ F/cm
$kT/q = 25.8$ mV
$L = 10$ micron
depth p-well = 1 micron; depth substrate > 100 micron
depth source and drain junctions = 100 nm
N_D (substrate)= 10^{15} cm^{-3}
N_A (well)= 10^{16} cm^{-3}
N_D (source/drain)= 10^{18} cm^{-3}
$V_S = V_D = 0$ V
$V_{sub} = 0$ V

a.

$$\begin{aligned}
V_{FB} &= V_T - 2\Phi_f - \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\sqrt{2\Phi_f} \\
&= V_T - 2V_t \ln\frac{N_A}{n_i} - \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_s}\sqrt{2V_t \ln\frac{N_A}{n_i}} \\
&= 0.8\text{ V} - 0.713\text{ V} - 1.589\text{ V} \\
&= -1.502\text{ V}
\end{aligned}$$

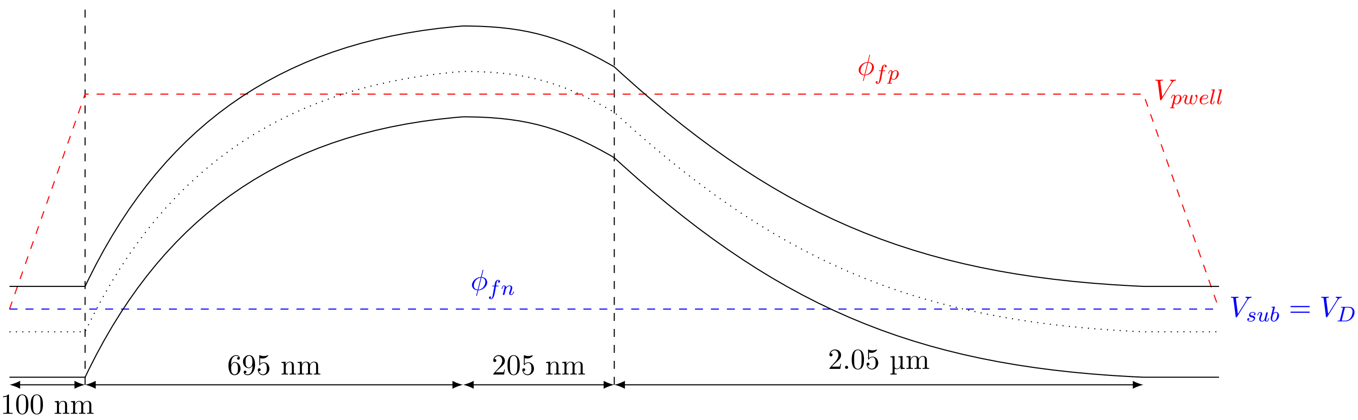
b.

$$X_{p1} + X_{p2} = t_{pwell} - t_s = 0.9 \text{ µm} \quad = \quad \sqrt{\frac{2\epsilon_s}{q}\frac{1}{N_A\left(1+\frac{N_A}{N_{SD}}\right)}(V_{bi}-V_{pn})} + \sqrt{\frac{2\epsilon_s}{q}\frac{1}{N_A\left(1+\frac{N_A}{N_D}\right)}(V_{bi}-V_{pn})}$$

Which has as solution $V_{pn} = -2.840$ V. If we assume $\frac{N_A}{N_{SD}} \ll 1$, we find the solution $V_{pn} - 2.812$ V. Ironically, they ask to solve with an accuracy of 1 mV, but they make approximations that give an error of 30 mV.

c.

The SCR in the p-region at the source/drain is $X_{p1} = 695$ nm and at the substrate is $X_{p2} = 205$ nm. The SCR in the substrate is $X_n = 2.05$ µm. Part in substrate not perfectly on scale.



Question 44
We want to design a pn-diode with a one-sided abrupt p$^+$-n junction with constant doping concentrations that has to be able to block a maximum voltage of 80V. The p-doping concentration is $N_A = 10^{19}$ cm^{-3}. You can use the following numerical values for the semiconductor constants: $n_i = 1\ 10^{10}$ cm^{-3} , $\epsilon_{Si} = 10^{-12}$ F/cm, $V_{th} = 0.026$ V, $D_p = D_n = 20$ cm^2/s, $q = 1.6\ 10^{-19}$ C.

a) Design the donor concentration at the n-side of the junction such that the breakdown voltage of the diode is higher than 100V (so that we have a safety margin of 20V), if you know that breakdown will happen as soon as the maximum field in the space charge region is $E_{max} = 4\ 10^5$ V/cm.
(Solution: 4.96 10^{15} cm^{-3})

b) At which voltage will this diode enter then the high injection region?
(Solution: 0.68V)

c) Design the length of the n-region W_n so that the maximum 3db-frequency of the diode is higher than 10 MHz. You can assume that the n-region is electrically short and that the contribution of the junction capacitance, of the diffusion capacitance of the p-region and the length of the space charge region compared to W_n can be neglected.
(Solution: 8 μm)

a)

$$
\begin{aligned}
E_{max} &= \int_0^{X_p} \frac{\rho}{\epsilon_{Si}} \\
&= \frac{qN_A X_p}{\epsilon_{Si}} \\
X_p^2 &= \frac{\epsilon_{Si}^2 E_{max}^2}{q^2 N_A^2} = \frac{2\epsilon_{Si}}{q} \frac{1}{N_A\left(1+\frac{N_A}{N_D}\right)}(V_{bi} - V_{pn}) \\
\frac{\epsilon_{Si}^2 E_{max}^2}{q^2 N_A^2} &\approx \frac{2\epsilon_{Si}}{q} \frac{1}{N_A\left(1+\frac{N_A}{N_D}\right)}(-V_{pn}) \\
1 + \frac{N_A}{N_D} &= \frac{2qN_A(-V_{pn})}{\epsilon_{Si} E_{max}^2} \\
N_D &= \frac{N_A}{\frac{2qN_A(-V_{pn})}{\epsilon_{Si} E_{max}^2} - 1} \\
&\approx \frac{N_A}{\frac{2qN_A(-V_{pn})}{\epsilon_{Si} E_{max}^2}} \\
&= \frac{\epsilon_{Si} E_{max}^2}{2q(-V_{pn})} \\
&= 5\ 10^{15}\ \text{cm}^{-3}
\end{aligned}
$$

If needed we can increase the accuracy by calculating the built-in potential and calculating N_D again iteratively: $V_{bi} = V_t \ln \frac{N_A N_D}{n_i^2} = 0.88$ V $\rightarrow N_D = \frac{\epsilon_{Si} E_{max}^2}{2q(V_{bi} - V_{pn})} = 4.96$ cm^{-3}

b)

High-injection: the minority carrier concentration is equal to the doping concentration. This will first be the case in the lowly-doped region (n).

$$
\begin{aligned}
p = p_{n0} e^{\frac{V_{HI}}{V_t}} &= N_D \\
V_{HI} &= V_t \ln \frac{N_D}{p_{n0}} \\
&= V_t \ln \frac{N_D^2}{n_i^2} \\
&= 0.682 \text{ V}
\end{aligned}
$$

c)

$$
\begin{aligned}
2\pi f_{3dB,max} &= \frac{1}{\tau_f} \approx \frac{1}{\tau_{fn}} \\
&= \frac{J_n}{Q_{Dn}} \\
&= \frac{2D_n}{(W_n)^2} \\
W_n &= \sqrt{\frac{D_n}{\pi f_{3dB,max}}} \\
&= 8.0 \text{ µm}
\end{aligned}
$$

Question 45

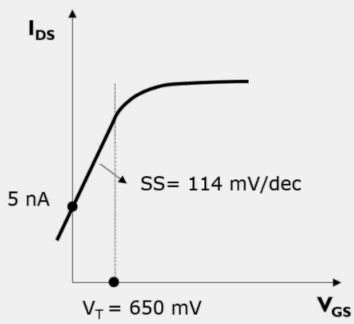
The $I_d - V_{gs}$ characteristic of an n-channel MOSFET measured at $V_{BS} = 0$ V and $V_{DS} = 2$ V is as shown on the figure below (note the current axis is a logarithmic axis!). The off current of the MOSFET at $V_{gs} = 0$ V is measured to be 5 nA, the threshold voltage is 0.65 V and the subthreshold swing of the MOSFET $SS = 114$ mV/dec.
We want to reduce the off-current of the MOSFET to a value of 5 pA by applying a bulk bias V_{BS}.

a) Determine an analytical expression for V_{BS} that we have to apply as a function of the known parameters of the MOSFET (SS, N_A, V_t, n_i, V_{FB}, t_{ox} , ϵ_{ox}, ϵ_{Si}, q, T, ...) and calculate this voltage V_{BS} (both value and polarity!). You can assume that the subthreshold slope does not change when applying a bulk bias.
(Solution: -1V)

b) Draw the band diagram in the channel of the MOSFET close to the source as a function of the position perpendicular to the channel of the transistor, and this for $V_{GS} = 2$ V and for the two cases of V_{BS}. Indicate the position of the quasi-Fermi level of holes and electrons and the total electrostatic potential drop over the depletion region.

The parameters of the MOSFET are as follows

- $N_A = 10^{17}$ cm^{-3}
- $V_t = 26$ mV
- $n_i = 1.45 \cdot 10^{10}$ cm^{-3}
- $V_{FB} = -0.86$ V
- $t_{ox} = 15$ nm
- $\epsilon_{ox} = 3.5 \cdot 10^{-13}$ F/cm
- $\epsilon_{Si} = 10^{-12}$ F/cm
- $q = 1.6 \cdot 10^{-19}$ C
- $Q_{ox} = Q_{ss} = 0$
- $T = 300$ K



a.

To decrease the off-current we need to increase V_T:

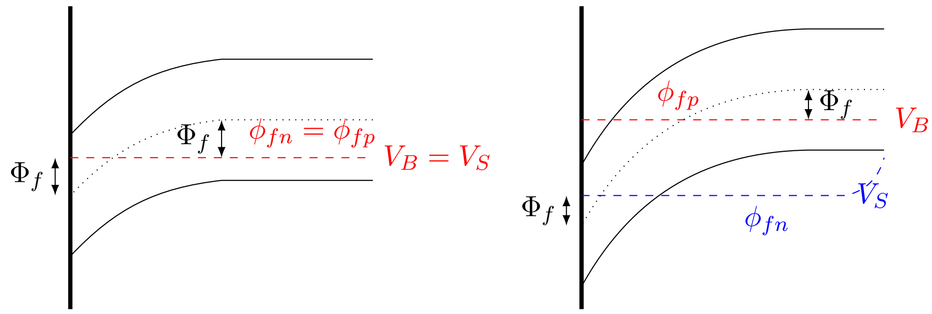
$$V_T = V_{T0} - SS \log\left(\frac{I_{off,required}}{I_{off,0}}\right)$$

This decrease is controlled by V_{BS}:

$$\begin{aligned}
V_T &= 2\Phi_f + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_A\epsilon_S}\sqrt{(2\Phi_f - V_{BS})} \\
V_{BS} &= 2V_t \ln\frac{N_A}{n_i} - \frac{1}{2qN_A\epsilon_S}\left[\frac{\epsilon_{ox}}{t_{ox}}\left(V_T - V_{FB} - 2V_t\ln\frac{N_A}{n_i}\right)\right]^2 \\
&= 2V_t \ln\frac{N_A}{n_i} - \frac{1}{2qN_A\epsilon_S}\left[\frac{\epsilon_{ox}}{t_{ox}}\left(V_{T0} - SS\log\left(\frac{I_{off,required}}{I_{off,0}}\right) - V_{FB} - 2V_t\ln\frac{N_A}{n_i}\right)\right]^2 \\
&= 0.819 \text{ V} - 1.816 \text{ V} \\
&= -0.997 \text{ V}
\end{aligned}$$

b.

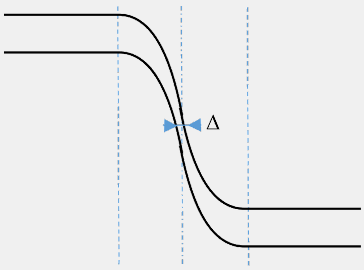
$V_{GS} > V_T$ in both cases, so we're in inversion.



Question 46

Consider a symmetrical pn-diode with constant doping concentrations across which we apply a reverse voltage V. At the location of the maximum electrical field in the space charge region Zener tunneling of electrons from the valence band to the conduction band can occur, if the distance Δ through which tunneling can occur through the forbidden energy region is smaller than 5 nm. So Δ is the horizontal distance between the valence band and the conduction band at the location of maximum electrical field.

As a hint we sketch below the shape of the band diagram for the considered diode.



a) Calculate the distance Δ as a function of the applied reverse voltage and write this as an expression (with symbols) as a function of the parameters of the diode and the applied reverse voltage. The diode is completely symmetrical and the two regions have equal constant doping concentrations $N_D = N_A = N$, $N_c = N_v$; the dielectric constant of the semiconductor is ϵ_s; bandgap E_G; built-in voltage V_{bi}. You can use engineering approximations where you think they are justified!

b) Calculate the reverse voltage that need to be applied to reach $\Delta = 5$ nm for a diode with the following parameters: $N_D = N_A = 5\ 10^{18}$ cm^{-3}; $N_c = N_v$; $\epsilon_s = 10^{-12}$ F/cm; $E_G = 1.1$ eV; V_{bi} =1 V. (Solution: 5.05V)

a)

The distance is given by:

$$\begin{aligned}
\Delta &= \frac{E_g}{\frac{dE}{dx}} \\
&= \frac{E_g}{q\mathcal{E}} \\
&= \frac{E_g}{q\int_0^{W_p} \frac{\rho}{\epsilon_s}} \\
&\approx \frac{E_g}{\frac{q^2 N_A X_p}{\epsilon_s}} \qquad \text{(Shockley approx.)} \\
&= \frac{E_g \epsilon_s}{q^2 N_A X_p} \\
\Delta &= \frac{E_g \epsilon_s}{q^2 N_A \sqrt{\frac{2\epsilon_s}{q}\frac{1}{N_A\left(1+\frac{N_A}{N_D}\right)}(V_{bi}-V_{pn})}}
\end{aligned}$$

b)

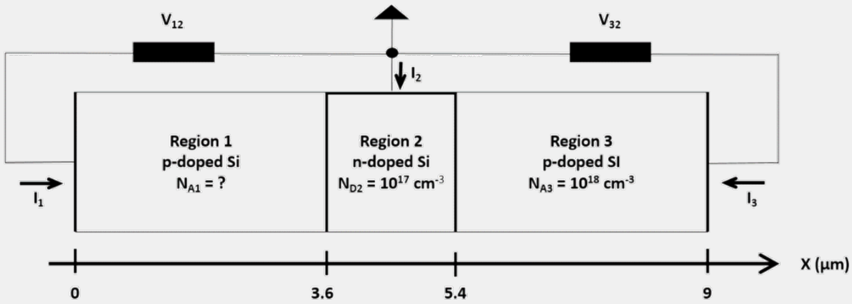
Getting V_{pn} out of the above equation and using $N_A = N_D$ gives:

$$\begin{aligned} V_{pn} &= V_{bi} - \frac{E_g^2 \epsilon_s}{q^3 N_A \Delta^2} \\ &= 5.05 \text{ V} \end{aligned}$$

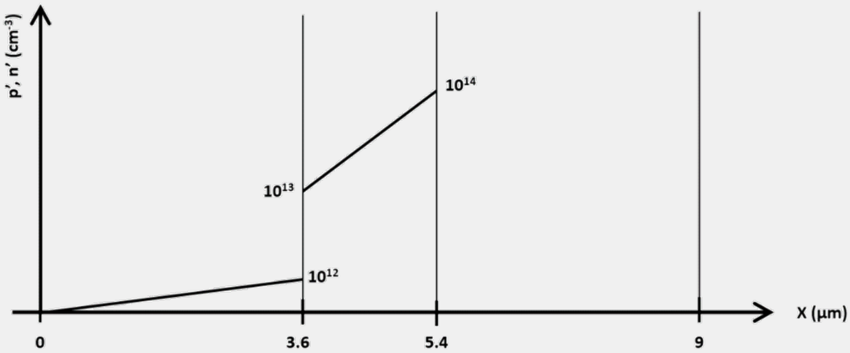
Question 47

Consider the following pnp structure:

- Throughout the structure, $D_e = 40$ cm^2/s, $D_h = 15$ cm^2/s, $\tau_{min} \approx \infty$.
- The cross-section has an area of 10^{-4} cm^2
- $kT/q = 25$ mV and $n_i = 1.5 \cdot 10^{10}$ cm^{-3}



For a certain bias, the excess minority carrier concentration in Region 1 and Region 2 could be measured to be as follows:



Neglect the expansion of the space charge regions at the junctions.
Questions:

a. Determine $J_{e,1-2}$, the electron current density at the junction between Region 1 and 2.

b. Determine $J_{h,1-2}$, de hole current density at the junction between Region 1 and 2.

c. Determine I_1?

d. Determine the doping concentration in Region 1, N_{A1}?

e. How large is the excess electron concentration at 5.4+ micron (this is at the edge of the space charge region in the neutral p Region 3)?

f. How large is the bias voltage V_{32}?

a, b.

$$\begin{aligned}
J_{h,1-2} &= -qD_h \frac{dp}{dx} \\
&= -qD_h \frac{10^{14} - 10^{13}}{5.4 - 3.6}\ 10^4 \text{ cm}^{-4} \\
&= -1.2 \ \frac{\text{A}}{\text{cm}^2} \\
J_{e,1-2} &= qD_e \frac{dn}{dx} \\
&= qD_e \frac{10^{12}}{3.6}\ 10^4 \text{ cm}^{-4} \\
&= 17.8 \ \frac{\text{mA}}{\text{cm}^2}
\end{aligned}$$

c.

$$\begin{aligned}
I_1 &= A(J_{h,1-2} + J_{e,1-2}) \\
&= -0.118 \text{ mA}
\end{aligned}$$

d.

First we need to determine the voltage:

$$\begin{aligned}
p_2(0) &= \frac{n_i^2}{N_{D2}} e^{\frac{V_{12}}{V_t}} \\
V_{12} &= V_t \ln \frac{p_2(0) N_{D2}}{n_i^2} \\
&= 0.578 \text{ V}
\end{aligned}$$

Now we can determine the doping in the Region 1:

$$\begin{aligned}
n_1(0) &= \frac{n_i^2}{N_{A1}} e^{\frac{V_{12}}{V_t}} \\
N_{A1} &= \frac{n_i^2}{n_2(0)} e^{\frac{V_{12}}{V_t}} \\
&= \frac{n_i^2}{n_1(0)} \frac{p_B(0) N_{D2}}{n_i^2} \\
&= \frac{p_2(0) N_{D2}}{n_1(0)} \\
&= 10^{18} \text{ cm}^{-3}
\end{aligned}$$

f.

It is easier to determine the voltage first

$$\begin{aligned}
p_2(W_2) &= \frac{n_i^2}{N_{D2}} e^{\frac{V_{32}}{V_t}} \\
V_{32} &= V_t \ln \frac{p_2(W_2) N_{D2}}{n_i^2} \\
&= 0.637 \text{ V}
\end{aligned}$$

e.

$$
\begin{aligned}
n_3(W_2) &= \frac{n_i^2}{N_{A3}} e^{\frac{V_{32}}{V_t}} \\
&= \frac{n_i^2}{N_{A3}} \frac{p_2(W_2) N_{D2}}{n_i^2} \\
&= \frac{p_2(W_2) N_{D2}}{N_{A3}} \\
&= 10^{13}\ \mathrm{cm}^{-3}
\end{aligned}
$$

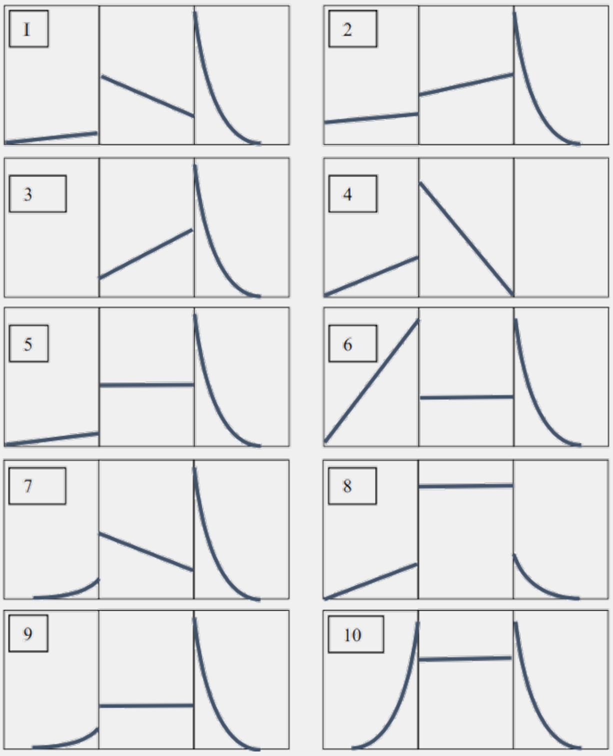
Question 48

Consider an npn bipolar transistor in saturation.

a. Suppose:

- that the transistor in saturation has $V_{EC} = 0$ V;
- that the emitter is short and the collector is long;
- that emitter, collector and base have constant doping concentrations $N_E > N_B > N_C$.

Which of the following sketches of minority carriers has the proper shape? Explain why?



b. Sketch the band diagram of a bipolar transistor in saturation for $V_{EC} = 0$ V, assuming that the emitter is short and the collector is long. Draw all energy levels (vacuum, conduction, valence) and quasi-Fermi levels. Indicate where one can read:

- the applied voltages V_{BE} and V_{BC}
- the doping levels N_E, N_B and N_C
- the minority carrier diffusion length (L_C) in the collector
- the electron affinity χ of the semiconductor

c. Using the numerical values below, draw the Gummel Plot for the REVERSE operation, including recombination in space charge regions and high inversion (you can neglect r_b and assume that the base transport factor is 1):

$N_E = 10^{19}$ cm^{-3}; $N_B = 10^{16}$ cm^{-3}; $N_C = 10^{15}$ cm^{-3}; $n_i = 10^{10}$ cm^{-3}; $\epsilon_{Semiconductor} = 10^{-12}$ F/cm; $q = 1.6\ 10^{-19}$ C; $kT/q = 25$ mV; $\mu_e = 1000$ cm^2/(Vs); $\mu_h = 500$ cm^2/(Vs); $W_E = 100$ nm; $W_B = 100$ nm; $L_C = 10$ micron; Area $A = 10^{-4}$ cm^2; $\tau_{rec\ SCR} = 1$ µs

a.

$$
\begin{aligned}
V_{EC} &= 0 \\
V_{BE} &= V_{BC} \\
p_E(0) &= \frac{n_i^2}{N_E} e^{\frac{V_{BE}}{V_t}} \\
n_B(0) &= \frac{n_i^2}{N_B} e^{\frac{V_{BE}}{V_t}} \\
n_B(W_B) &= \frac{n_i^2}{N_B} e^{\frac{V_{BC}}{V_t}} = n_B(0) \\
p_C(W_B) &= \frac{n_i^2}{N_C} e^{\frac{V_{BC}}{V_t}}
\end{aligned}
$$

2 makes no sense, as p_E should be p_0 at the left side, moreover, as $V_{EC} = 0$, the ratio $p_E(0)/p_E(W_E)$ should be the same as $p_C(W_B)/p_C(W_C + W_B)$.

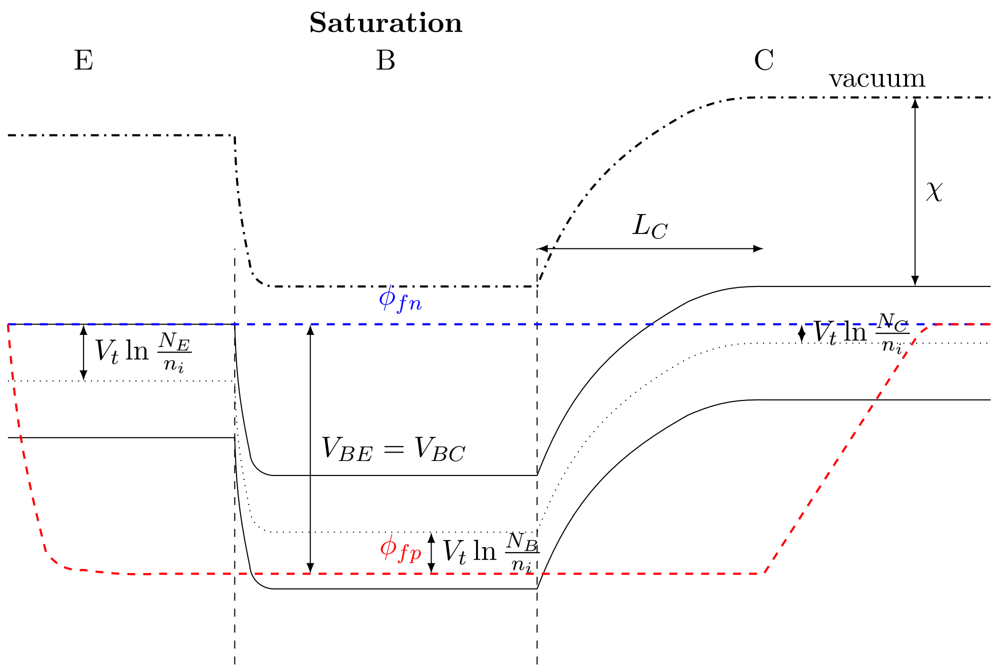
Saturation: 3 and 4 can be excluded.

Emitter short and collector long: 7, 9 and 10 can be excluded.

From $V_{EC} = 0$ (and the base being short), we now that the minority carrier concentration in the base should be flat (equal on both side), this means the solutions is either 5, 6 or 8.

$N_E > N_B > N_C$ excludes 6 and 8, so the correct solution is 5.

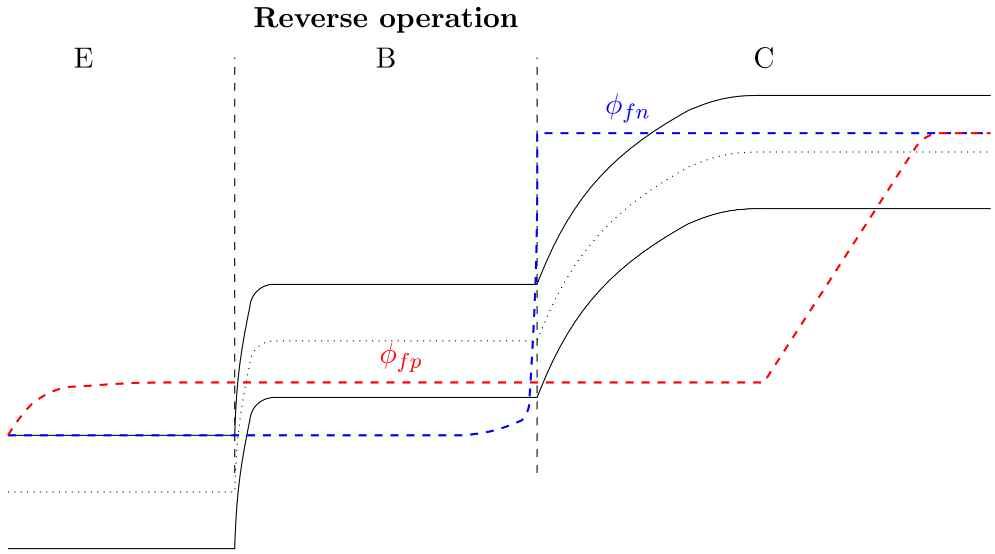
b.



c. The ideal characteristics in reverse operation:

$$\begin{aligned}
I_S &= \frac{AqV_t n_i^2}{B} \\
&= \frac{AqV_t n_i^2 \mu_e}{N_B W_B} \\
&= 4\ 10^{-12}\ \text{A} \\
\beta_R &= \frac{C}{B} \\
&= \frac{N_C L_C \mu_e}{N_B W_B \mu_h} \\
&= 20
\end{aligned}$$

I ignored the width of the SCR in B here. If you calculate it $X_{p,BE} = 0.35\ \mu\text{m}/\text{V}^{1/2} \cdot \sqrt{0.863\ \text{V} - V_{pn}} \geq 0.33\ \mu\text{m} > W_B$. This is a problem, the entire base will be SCR. Let's ignore this fact.



Recombination current:

$$\begin{aligned}
I_{rec,BC} &= \frac{1}{2} AqW_{BC} \frac{n_i}{\tau} \left(e^{\frac{V_{BC}}{2V_t}} - 1 \right) \\
&= \frac{1}{2} Aq \sqrt{\frac{2\epsilon_S}{q} \frac{N_A + N_D}{N_A N_D} (V_{bi} - V_{pn})} \frac{n_i}{\tau} \left(e^{\frac{V_{BC}}{2V_t}} - 1 \right) \\
&\approx \frac{1}{2} Aq \sqrt{\frac{2\epsilon_S}{q} \frac{N_B + N_C}{N_B N_C} V_t \ln\left(\frac{N_B N_C}{n_i^2} \right)} \frac{n_i}{\tau} \left(e^{\frac{V_{BC}}{2V_t}} - 1 \right) \\
&= 7.46\ 10^{-12}\ \text{A} \cdot \left(e^{\frac{V_{BC}}{2V_t}} - 1 \right)
\end{aligned}$$

We'll call $I_{rec,0} = 7.46\ 10^{-12}$ A. Important to know is where the recombination current and ideal current are equal:

$$\begin{aligned}
I_{rec,BC} &= I_R \\
V_{BC} &= 2V_t \ln \frac{I_{rec,0}}{I_S} \\
&= 0.031\ \text{V}
\end{aligned}$$

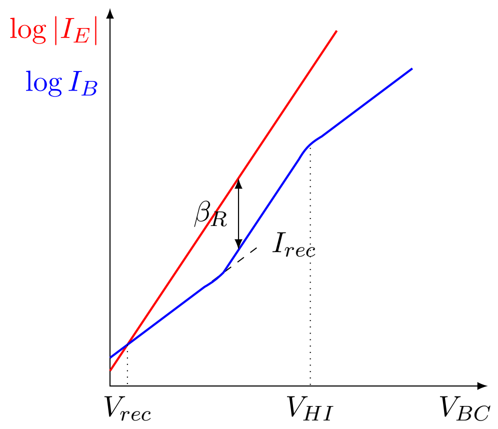
High injection:

$$\begin{aligned} p_C = N_C &= \frac{n_i^2}{N_C} e^{\frac{V_{HI}}{V_t}} \\ V_{HI} &= V_t \ln \frac{N_C^2}{n_i^2} \\ &= 0.576 \text{ V} \end{aligned}$$

NOTE: the carriers in the collector go in high injection (HI). The carrier concentration in the collector determines the base current, so it is the base current that goes in HI. While in forward bias, the base carriers go in HI, so then it is the emitter current that goes in HI.

The currents are finally:

$$\begin{aligned} I_B &= \frac{I_R}{\beta_R} + I_{rec,BC} \\ &= \frac{I_S}{\beta_R} e^{\frac{V_{BC}}{V_t}} + I_{rec,0} e^{\frac{V_{BC}}{2V_t}} \\ I_E &= -I_R = -I_S e^{\frac{V_{BC}}{V_t}} \\ I_C &= -\left(1 + \frac{1}{\beta_R}\right) I_R - I_{rec,BC} \\ &= -\left(1 + \frac{1}{\beta_R}\right) I_S e^{\frac{V_{BC}}{V_t}} - I_{rec,0} e^{\frac{V_{BC}}{2V_t}} \end{aligned}$$

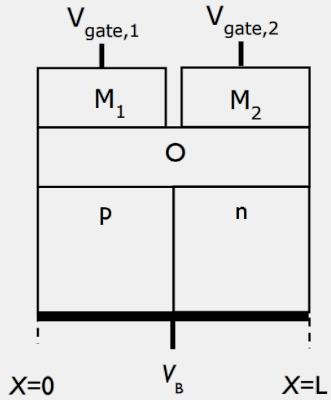


Question 49

A device as shown below is provided. The gate electrodes are grounded ($V_{gate,1} = V_{gate,2} = 0$ V). An external voltage source V_B is applied to the semiconductor contact: V_B is 0 V at time $t = 0$ s, and decreases steadily with 500 mV/s beyond $t = 0$ s until $V_B = -4$ V is reached. The following parameters are known:

$N_A = 10^{17}$ at/cm^3
$N_D = 10^{17}$ at/cm^3
$n_i = 10^{10}$ at/cm^{-3}
$E_{aff,semic} = 4$ eV
$E_g = 1.12$ eV
$N_c = N_v$
$\epsilon_{semic}/\epsilon_0 = 11.9$

$t_{ox} = 5$ nm
$Q_{ox} = 0$ C/cm^2
$\epsilon_{ox}/\epsilon_0 = 3.9$
$WF_{gate,1} = WF_{gate,2} = 5$ eV
Area_electrode1=10^{-5}cm^2
$\epsilon_0 = 8.85 \cdot 10^{-14}$ F/cm
$T = 300$ K



a) At $t = 0$ s, what bias condition (accumulation, flatband, depletion, inversion threshold, inversion) is present underneath gate electrode 1 and what bias condition underneath gate electrode 2?

b) At which time is inversion threshold reached underneath the gate electrode 1?

c) For $t = 8$ s, draw a band diagram in the semiconductor right underneath the oxide from $x = 0$ until the end of gate electrode 1 and from the start of gate electrode 2 until $x = L$ (= you can neglect the impact of the p-n space-charge region). Include E_{vac}, E_c, E_v and the quasi-Fermi levels, as well as the energetic distances between them.

d) For $t = 8$s, what is the value of the capacitance, measured by applying a small-signal to $V_{gate,1}$.?

a.

Let's start with calculating the flat-band potentials:

$$
\begin{aligned}
V_{FB,1} &= \Phi_{ms1} - \frac{Q_{ox}t_{ox}}{\epsilon_{ox}} \\
&= \frac{WF}{q} - \left(\frac{E_{aff}}{q} + \frac{E_g}{2q} + V_t \ln\frac{N_A}{n_i}\right) \\
&= 0.024 \text{ V} \\
V_{FB,2} &= \Phi_{ms2} - \frac{Q_{ox}t_{ox}}{\epsilon_{ox}} \\
&= \frac{WF}{q} - \left(\frac{E_{aff}}{q} + \frac{E_g}{2q} - V_t \ln\frac{N_D}{n_i}\right) \\
&= 0.856 \text{ V}
\end{aligned}
$$

From this we already see that electrode 1 is in accumulation (very close to FB) $V_{gate,1} < V_{FB,1}$, so more holes are attracted towards the interface. Whether electrode 2 is in depletion or inversion (threshold), will have to be determined from $V_{T,2}$.

$$\begin{aligned} V_{T2} &= 2\Phi_f + V_{FB} - \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_D\epsilon_S}\sqrt{|2\Phi_f|} \\ &= -2V_t \ln\frac{N_D}{n_i} + V_{FB} - \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_D\epsilon_S}\sqrt{|2\Phi_f|} \\ &= -0.832\text{ V} + 0.856\text{ V} - 0.139\text{ V} \\ &= -0.115\text{ V} \end{aligned}$$

As $V_{G2} - V_B > V_{T2}$, so we're still in depletion. We don't need V_{T1} for question a, but we'll need it for question b. In the same way we can calculate V_{T1} for electrode 1:

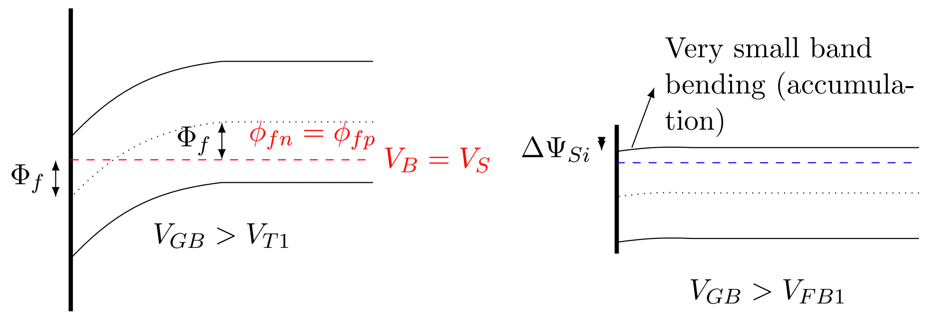
$$\begin{aligned} V_{T1} &= 2\Phi_f + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_D\epsilon_S}\sqrt{|2\Phi_f|} \\ &= 2V_t \ln\frac{N_D}{n_i} + V_{FB} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_D\epsilon_S}\sqrt{|2\Phi_f|} \\ &= 0.832\text{ V} + 0.856\text{ V} + 0.139\text{ V} \\ &= 1.826\text{ V} \end{aligned}$$

b.

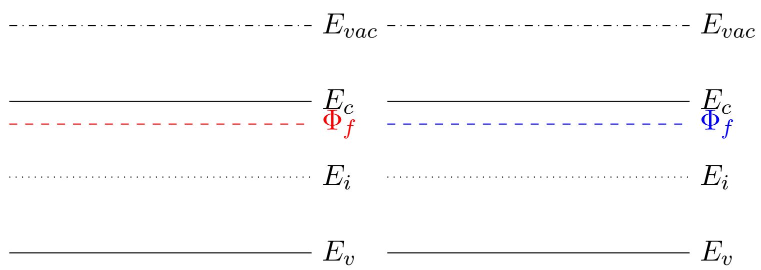
The time we need is $t = \frac{V_{T1}}{\frac{dV_{GB}}{dT}} = \frac{1.826\text{ V}}{0.5\text{V/s}} = 3.653$ s.

c.

Electrode 1 will be in inversion, while electrode 2 will be in accumulation. If we neglect the SCR, how is the band diagram then postion dependent? Isn't it constant under the capacitor? I'll just draw it in the z-direction:



In the x-direction we would get something like this.



d.

Electrode 1 is in inversion. Are we applying a low or high frequency?

For low frequency:

$$
\begin{aligned}
C &= \frac{A\epsilon_{ox}}{t_{ox}} \\
&= 6.9 \text{ pF}
\end{aligned}
$$

For high frequency:

$$
C = \frac{A}{\frac{t_{ox}}{\epsilon_{ox}} + \frac{z_{D,th}}{\epsilon_S}}
$$

With

$$
\begin{aligned}
z_{D,th} &= \sqrt{\frac{2\epsilon_s 2\Phi_f}{qN_A}} \\
&= 0.1 \text{ µm} \\
C &= 0.88 \text{ pF}
\end{aligned}
$$

Question 50

Question 51

Question 52

Question 53

You are asked to design an n-type MOS capacitor such that the voltage difference between flatband and inversion threshold ($|V_{FB} - V_{th}|$) is as large as possible.

The following data are known about the used semiconductor:
intrinsic concentration $n_i = 10^{10}$ cm^{-3}, band gap $E_g = 1.12$ eV, $E_{aff} = 4.05$ eV, intrinsic level E_i: midgap, $\varepsilon_{Si} = 10^{-12}$ F/cm, $Q_{ox} = Q_{ss} = 0$ C/cm^2, $\varepsilon_{ox} = 3.5\ 10^{-13}$ F/cm.
Constants: thermal voltage $kT/q = 26$ mV, $q = 1.6\ 10^{-19}$ C.

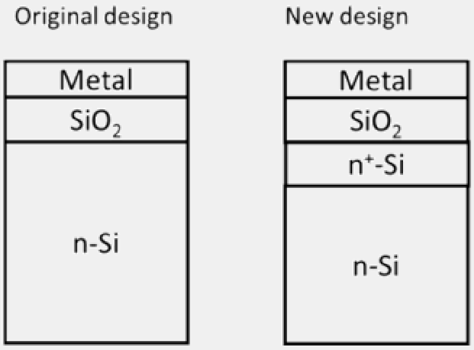
1) Reflect on the parameters that determine this voltage difference ($|V_{FB} - V_{th}|$) and fill out the table below. Motivate your answer with physical reasoning and/or mathematical equations.
   a. Will you design the oxide thickness to be as thick as possible, as thin as possible or does it not matter?
   b. Will you design the doping concentration to be as high as possible, as low as possible or does it not matter?
   c. Will you design the workfunction (WF_{gate}) of the metal to be larger than, smaller than or independent of the WF of the bulk semiconductor WF_{semic}?
   d. If fixed oxide charge (positive charge) would be present, will this increase, decrease or not affect the difference $|V_{FB} - V_{th}|$?

t_{ox}	as thick as possible	as thin as possible	$\neq f(t_{ox})$
N_D	as high as possible	as low as possible	$\neq f(N_D)$
WF_{gate}	$> WF_{semic}$	$< WF_{semic}$	$\neq f(WF_{semic})$
$Q_{ox} > 0$	$\lvert V_{FB} - V_{th}\rvert \nearrow$	$\lvert V_{FB} - V_{th}\rvert \searrow$	$\lvert V_{FB} - V_{th}\rvert$ unaffected

2) In the original design of the capacitor we have $N_D = 10^{17}$ at/cm^3, $t_{ox} = 5$nm and $WF_{gate} = 4.5$eV. We want to change this design by inserting a thin layer with thickness t_d=7 nm of highly n-doped Si ($N_{D2}=10^{19}$ at/cm^3) between the existing n-doped bulk semiconductor and the oxide (see 'new design' below). Find out whether $|V_{FB} - V_{th}|$ will increase, decrease or is not affected based on the subquestions below.

   Note: you can neglect the thickness of the SCR between the n-doped and the n$^+$-doped region. This implies that both regions are neutral at flatband. The flatband voltage will therefore be determined by the thin top layer of 10^{19} at/cm^3 doping.

Question 53



a. Determine the flatband voltage for the original design (10^{17} at/cm^3 doping only).and for the new design (10^{17} at/cm^3 plus top layer of 7 nm thickness and 10^{19} at/cm^3 doping). Determine how much (sign and absolute value) the new design affects $|V_{FB} - V_{th}|$ when making the assumption for now that V_{th} is not affected by the design change.
b. Before determining V_{th}, determine first how much band bending is achieved in the semiconductor (in [eV]), when only the 7 nm layer of 10^{19} at/cm^3 doping is depleted. Is this band bending sufficient for reaching inversion threshold?
c. Calculate and draw the space charge density ρ and the electric field in the semiconductor for the new design, assuming depletion up to 5nm into the lower doped bulk semiconductor.
d. Inversion threshold represents the voltage at which the minority carrier concentration at the oxide/semiconductor interface equals the majority carrier concentration at $z_{D,th}$. Based on this definition determine V_{th} for the new design. Is $|V_{FB} - V_{th}|$ for the new design larger than, smaller than or unaffected compared to its value for the original design?
e. Draw the band diagram in the semiconductor at flatband voltage and at inversion threshold voltage. Indicate the position of the quasi-Fermi level of holes and electrons, conduction band energy, valence band energy, intrinsic energy and the total electrostatic potential drop in the semiconductor.

1.

$$\begin{aligned}
V_{FB} &= \Phi_{ms} - \frac{Q_{ox} + Q_{ss}}{C_{ox}} \\
&= \frac{WF_{gate}}{q} - \frac{E_{aff}}{q} - \frac{E_g}{2q} + V_t \ln\frac{N_D}{n_i} - \frac{Q_{ox} + Q_{ss}}{C_{ox}} \\
V_T &= 2\Phi_f + V_{FB} - \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_D\epsilon_s}\sqrt{|2\Phi_f|} \\
|V_T - V_{FB}| &= 2V_t \ln\frac{N_D}{n_i} + \frac{t_{ox}}{\epsilon_{ox}}\sqrt{2qN_D\epsilon_s}\sqrt{2V_t \ln\frac{N_D}{n_i}}
\end{aligned}$$

We see that t_{ox} and N_D have an influence: they should both be as large as possible. A thick oxide does not change the flat band, but it means a larger voltage will be needed to create to same amount of charge. N_D increases the needed electrostatic potential drop over the semiconductor to reach inversion and furthermore makes the SC act more as a metal, so it is harder to create an electrostatic potential drop.

WF_{gate} and Q_{ox} don't have an influence on the difference between the threshold and the flat band voltage.

2.

a.

$$\begin{aligned}V_{FB1} &= \Phi_{ms} - \frac{Q_{ox}+Q_{ss}}{C_{ox}}\\ &= \frac{WF_{gate}}{q} - \frac{E_{aff}}{q} - \frac{E_g}{2q} + V_t \ln \frac{N_{D1}}{n_i}\\ &= 0.309 \text{ V}\end{aligned}$$

Similarly, for the second design, we can just fill in $N_D = 10^{19}$ cm^{-3}, thus $V_{FB2} = 0.429$ V. If V_{th} does not change, then $|V_{FB} - V_{th}|$ increases by +0.12 V ($V_{FB} > V_{th}$ for n-type).

b.

The maximum amount of band bending is:

$$\begin{aligned}\Delta\Psi_{Si} &= \frac{qN_{D2}t_d^2}{2\epsilon_S}\\ &= 0.392 \text{ V} < -2\Phi_f = 1.078 \text{ V}\end{aligned}$$

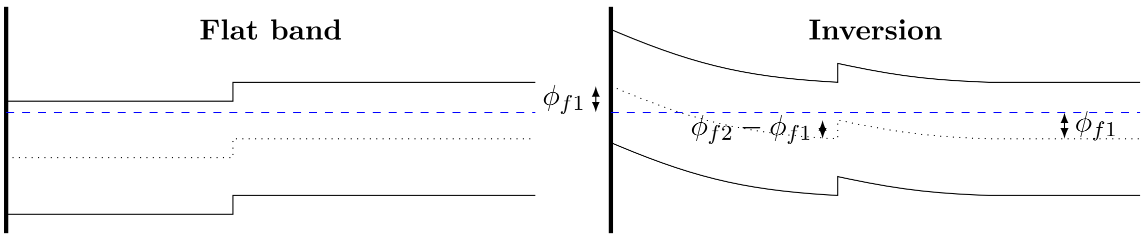
It is not sufficient.

c.

We'll call $t_D + 5$ nm $= z_D$

$$\begin{aligned}\rho &= \begin{cases} qN_{D2} & z < t_d\\ qN_{D1} & z > t_d\end{cases}\\ \mathcal{E} &= -\int_{z_D}^{0} \frac{\rho}{\epsilon_S} dz\\ &= \begin{cases} \frac{qN_{D1}}{\epsilon_S}(z_D - z) & z > t_d\\ \frac{qN_{D1}}{\epsilon_S}\cdot(z_D - t_d) + \frac{qN_{D2}}{\epsilon_S}(t_d - z) & z > t_d\end{cases}\end{aligned}$$

e.



d.

From the figure above we see that inversion is reached if[1]

$$\begin{aligned}\Delta\Psi_{Si} &= \phi_{f1} + (\phi_{f2} - \phi_{f1}) + \phi_{f1} = \phi_{f1} + \phi_{f2}\\ -\int_0^{z_{D,th}} \mathcal{E} dz &= \phi_{f1} + \phi_{f2}\\ \frac{qN_{D1}(z_{D,th} - t_d)^2}{2\epsilon_S} + \frac{qN_{D1}(z_{D,th} - t_d)t_d}{\epsilon_S} + \frac{qN_{D2}t_d^2}{2\epsilon_S} &= V_t \ln \frac{N_{D1}N_{D2}}{n_i^2}\end{aligned}$$

[1]The total electrostatic potential drop is actually $2\phi_{f1}$, however there is a delta electric field at the junction between the 2 n-regions, that we're not taking into account at the left side which is equal to $-(\phi_{f2} - \phi_{f1})$. So, to be correct we should move the $(\phi_{f2} - \phi_{f1})$ to the other side of the equation, but this way it is easier and the result is the same.

We can solve this equation to $z_{D,th}$:

$$\begin{aligned}
z_{D,th} &= t_d + \frac{-qN_{D1}t_d + \sqrt{(qN_{D1}t_d)^2 + qN_{D1}\left(2V_t\epsilon_S \ln\frac{N_{D1}N_{D2}}{n_i^2} - qN_{D2}t_d^2\right)}}{qN_{D1}} \\
&= \sqrt{t_d^2 + \left(\frac{2V_t}{qN_{D1}}\epsilon_S \ln\frac{N_{D1}N_{D2}}{n_i^2} - \frac{N_{D2}}{N_{D1}}t_d^2\right)} \\
&= \sqrt{\left(1 - \frac{N_{D2}}{N_{D1}}\right)t_d^2 + \frac{2V_t}{qN_{D1}}\epsilon_S \ln\frac{N_{D1}N_{D2}}{n_i^2}} \\
&= 84.4 \text{ nm}
\end{aligned}$$

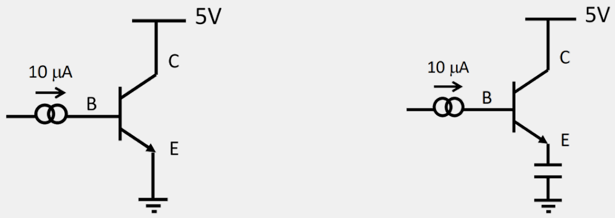
Which can be used to determine the inversion threshold:

$$\begin{aligned}
V_{th} &= \Delta\Psi_{tot} + \Phi_{ms} \\
&= \Delta\Psi_{Si} + \Delta\Psi_{ox} + V_{FB} \\
&= \phi_{f1} + \phi_{f2} - \frac{qN_{D1}(z_{D,th} - t_d) + qN_{D2}t_d}{C_{ox}} \\
|V_{FB} - V_{th}| &= V_t \ln\frac{N_{D1}N_{D2}}{n_i^2} + \frac{qN_{D1}(z_{D,th} - t_d) + qN_{D2}t_d}{\epsilon_{ox}}t_{ox} \\
&= 2.735 \text{ V}
\end{aligned}$$

While for design 1 $|V_{FB1} - V_{th1}| = 1.072$ V. So, we do see that it increases.

Question 54

Consider a npn transistor that is connected as in the left figure, with the following + parameters:
$\beta_F = 100$
$\beta_R = 0.1$
$I_s = 10^{-15}$ A (collector saturation current)
Constants: thermal voltage $kT/q = 26$ mV, $q = 1.6\ 10^{-19}$ C



We force a current of 10 µA into the base.

a) Calculate the internal bipolar currents (I_F, I_{BF}, I_R and I_{BR})?
(Solution: 1.01 mA, 10 µA, 0, 0)

b) What will be the collector current, the emitter current and the base voltage?
(Solution: 1 mA, 1.01 mA, 0.72 V)

c) We now disconnect the emitter from ground and leave it electrically floating (which is equivalent in DC to inserting a capacitor, as shown on the right fgure). Calculate again the internal currents, the collector and emitter current, the base voltage and now also the emitter voltage.
(Solution: 0.989 µA, 9.89 nA, 0.999 µA, 9.99 µA, 10 µA, 0, 5.54 V, 5 V)

a, b.

We'll assume we're in the forward active regime, thus $I_R = I_{BR} = 0$

$$\begin{aligned} I_C &= \beta_F I_B \\ &= 1 \text{ mA} \\ I_E &= \left(1 + \frac{1}{\beta_F}\right) I_F \\ &= 1.01 \text{ mA} \\ I_F &= I_C = 1 \text{ mA} \\ I_{BF} &= \frac{I_F}{\beta_F} = 10\ \mu\text{A} \\ V_B &= V_E + V_t \frac{I_F}{I_s} \\ &= 0.718 \text{ V} \end{aligned}$$

As $V_C > V_B > V_E$, we're indeed in forward active operation.

c.

We know that $I_E = 0$, so $I_C = -I_B = -10$ µA.

$$\begin{cases} I_E = 0 = \left(1 + \frac{1}{\beta_F}\right) I_F - I_R \\ I_C = I_F - \left(1 + \frac{1}{\beta_R}\right) I_R \end{cases}$$

Which is a system with 2 equations and 2 unknowns and can be solved. Substracting the second equation from $\left(1+\frac{1}{\beta_R}\right)$ times the first gives

$$\begin{aligned}
-I_C &= \left(1+\frac{1}{\beta_R}\right)\left(1+\frac{1}{\beta_F}\right)I_F - I_F \\
I_F &= -\frac{I_C}{\frac{1}{\beta_F}+\frac{1}{\beta_R}+\frac{1}{\beta_F\beta_R}} \\
&= -\frac{\beta_F\beta_R I_C}{\beta_F+\beta_R+1} \\
&= 0.989\ \mu\text{A}
\end{aligned}$$

And I_R is then according to the first equation

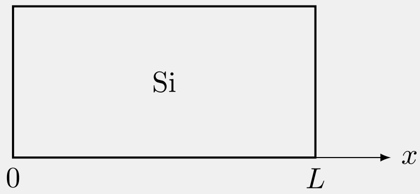
$$\begin{aligned}
I_R &= \left(1+\frac{1}{\beta_F}\right)I_F \\
&= 0.999\ \mu\text{A} \\
I_{BF} &= \frac{I_F}{\beta_F} = 9.89\ \text{nA} \\
I_{BR} &= \frac{I_R}{\beta_R} = 9.99\ \mu\text{A}
\end{aligned}$$

The base and emitter voltage follow from the current equations:

$$\begin{aligned}
I_R &= I_s e^{\frac{V_{BC}}{V_t}} \\
V_B &= V_C + V_t\frac{I_R}{I_s} \\
&= 5.54\ \text{V} \qquad (\text{for } V_t = 0.026\ \text{V}) \\
I_F &\approx I_s e^{\frac{V_{BE}}{V_t}} \\
V_E &= V_B - V_t\frac{I_F}{I_s} \\
&= 5.00\ \text{V}
\end{aligned}$$

Question 55

A piece of n-doped silicon has a doping concentration that is linearly varying from $N_D(0) = 10^{15}$ cm^{-3} to $N_D(L) = 10^{17}$ cm^{-3}. The following parameters are known: $n_i = 10^{10}$ cm^{-3}, $\mu_n = 1400$ $\text{cm}^2/(\text{V.s})$, $\mu_p = 500$ $\text{cm}^2/(\text{V.s})$, $T = 300$ K, $L = 500$ µm. The semiconductor is not contacted or under an applied voltage and is thus in thermal equilibrium.



a) How much is the diffusion current of electrons and holes in the semiconductor? Give formula and values, take care also to add the correct sign!

b) How much is the total current and the drift current of electrons in the semiconductor? Give formula and values, take care also to add the correct sign!

c) How much is the electrical field in the semiconductor for $x = 0$ and for $x = L$? Give formula and values, take care also to add the correct sign! Sketch the evolution of the field as a function of the position between 0 and L.

d) Summarize in words what is happening inside the semiconductor!

a)

The diffusion current of electrons is:

$$\begin{aligned} J_{diff,n} &= qD_n \nabla n \\ &= \mu_n kT \frac{dn}{dx} \\ &\approx \mu_n kT \frac{N_D(L)}{L} \\ &= 1500 \ \frac{\text{cm}^2}{\text{Vs}} \cdot 1.38 \ 10^{-23} \ \frac{\text{J}}{\text{K}} \cdot 300 \ \text{K} \cdot \frac{10^{17} \ \text{cm}^{-3}}{0.05 \ \text{cm}} \\ &= 12.4 \ \frac{\text{A}}{\text{cm}^2} \end{aligned}$$

(beware of units (with J = V C and A = C/s)).

The diffusion current of holes is:

$$\begin{aligned} J_{diff,p} &= -qD_p \nabla p \\ &= -\mu_p kT \frac{dp}{dx} \\ &\approx \mu_p kT \frac{n_i^2}{N_D(0)L} \\ &= 4.14 \ 10^{-12} \ \frac{\text{A}}{\text{cm}^2} \lll J_{diff,n} \end{aligned}$$

The total diffusion current is then $J_{diff} = J_{diff,n} + J_{diff,p} \approx J_{diff,n} = 12.4 \ \frac{\text{A}}{\text{cm}^2}$.

b)

As the semiconductor is not connected, the total current must be zero. Thus the drift current needs to cancel the diffusion current: $J_{drift,n} \approx J_{drift} = -J_{diff} = -12.4\ \frac{\text{A}}{\text{cm}^2}$.

c)

The electric field can be calculated in 2 different ways.

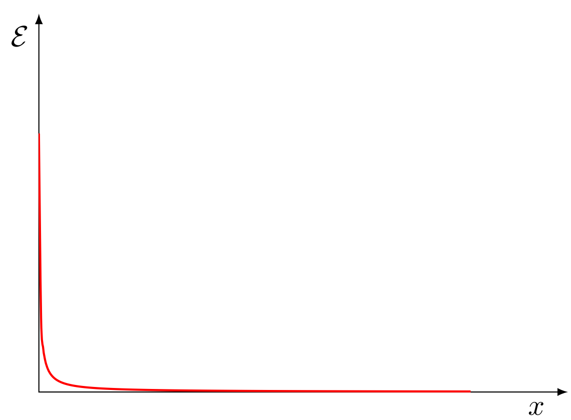
1) the Fermi-level needs to be flat in equilibrium

$$\begin{aligned}
\nabla\Phi_f &= 0 \\
\frac{d}{dx}\left(-\frac{kT}{q}\ln\frac{N_D(x)}{n_i}\right) - \mathcal{E} &= 0 \\
\mathcal{E} &= -\frac{kT}{qN_D(x)}\frac{dN_D(x)}{dx} \\
&\approx -\frac{kT}{qN_D(x)}\frac{N_D(L)}{L} \\
&= -\frac{0.0259\ \text{V}}{0.0005\ \text{m}}\cdot\frac{1}{0.01+0.99\frac{x}{L}} \\
&= -51.7\ \frac{\text{V}}{\text{m}}\cdot\frac{1}{0.01+0.99\frac{x}{L}}
\end{aligned}$$

2) the total current must be zero everywhere:

$$\begin{aligned}
J_{drift,n} \approx J_{drift} &= -J_{diff} \\
q\mu_n n\mathcal{E} &= -qD_n\frac{dN_D}{dx} \\
\mathcal{E} &= -\frac{D_n}{\mu_n}\frac{N_D(L)}{N_D(x)L} \\
\mathcal{E} &= -\frac{kT}{q}\frac{N_D(L)}{N_D(x)L}
\end{aligned}$$

Which gives rise to the same formula as in the previous method. So, $\mathcal{E}(L) = -51.7\ \frac{\text{V}}{\text{m}}$ and $\mathcal{E}(0) = -5170\ \frac{\text{V}}{\text{m}}$



d)

As there is an concentration gradient inside the semiconductor, the electrons will move from high concentration to low concentration. This means one side becomes a little positive and the other side a little negative, whereby an electric field is created. Electrons keep moving from the high concentration to the low concentration until the electric field creates a drift current that completely cancels the diffusion current.

In all formulas above it was assumed that the movement of the electrons does not change the concentration of the electrons significantly so the concentration can be assumed equal to N_D. This needs to be checked:

$$
\begin{aligned}
\Delta\Psi = \frac{d^2\Psi}{dx^2} &= -\frac{\rho}{\epsilon} \\
\rho &= -\epsilon\frac{d^2\Psi}{dx^2} \\
&= \epsilon\frac{d\mathcal{E}}{dx} \\
&= -\frac{kT\epsilon}{q}\frac{d}{dx}\left(\frac{\frac{dN_D(x)}{dx}}{N_D(x)}\right) \\
&= -\frac{kT\epsilon}{q}\frac{N_D(x)\frac{d^2N_D(x)}{dx^2} - \left(\frac{dN_D(x)}{dx}\right)^2}{\left(N_D(x)\right)^2} \\
&= \frac{kT\epsilon}{q}\frac{\left(\frac{dN_D(x)}{dx}\right)^2}{\left(N_D(x)\right)^2} \\
&= \frac{kT\epsilon}{q}\frac{N_D(L)^2}{L^2\left(N_D(x)\right)^2} \\
&= O\left(100\frac{kT\epsilon}{qL^2}\right) \\
&= O\left(5170\ \frac{\mathrm{V}}{\mathrm{m}}\frac{1}{0.0005\ \mathrm{m}}\cdot 11.9\cdot 8.85\ 10^{-12}\ \frac{\mathrm{F}}{\mathrm{m}}\right) \\
&= O\left(1\ 10^{-3}\ \frac{\mathrm{C}}{\mathrm{m}^3}\right) \\
&= O\left(1\ 10^{-9}\ \frac{\mathrm{C}}{\mathrm{cm}^3}\right)
\end{aligned}
$$

This means the change in concentration is about $\Delta n = \frac{\rho}{q} = O(7\ 10^9\ \mathrm{cm}^{-3})$, which is indeed negligible compared to the total electron concentration n, so $n \approx N_D$ is a valid approximation.

Willem: Why is the charge calculated here always positive, you can't create charge. Anyone an idea?

Question 56

A piece of p-doped semiconductor is illuminated with a uniform light intensity so that a constant generation G_{ext} of electron-hole pairs is created. The generation and recombination are determined by the Shockley-Read-Hall mechanism, with a minority carrier lifetime of electrons and holes $\tau_n = \tau_p = 0.1$ ms. The energy level of the trapping states is at the intrinsic level (midgap) $E_{rg} = E_i$. The parameters of the semiconductor are: $N_A = 10^{15}$ cm^{-3}, $n_i = 10^{10}$ cm^{-3}, $\mu_n = 1400$ cm^2/(V.s), $\mu_p = 450$ cm^2/(V.s), $T = 300$ K, $E_g = 1.1$ eV.

a) Which excess concentrations $\Delta n = \Delta p$ of electrons and holes have to be generated if we want to reduce the resistance of the semiconductor in steady-state with two orders of magnitude? Give formulas and values!

b) How big should G_{ext} be to generate these excess concentrations in steady-state?

c) Calculate the position of the Fermi level (in equilibrium, before illumination) and of the quasi-Fermi levels of holes and electrons (in steady state, during illumination) and indicate these levels on a band diagram.

a.

$$\sigma = \mu_n e n + \mu_p e p$$

Without external generation:

$$\sigma_0 = \mu_p e N_A$$

With external generation, n and p will have to increase a lot. Such that $n \approx p \approx \Delta n \approx \Delta p$:

$$\begin{aligned}
\frac{\sigma}{\sigma_0} &= \frac{\mu_n e n + \mu_p e p}{\mu_p e N_A} \\
&\approx \frac{\mu_n e \Delta n + \mu_p e \Delta n}{\mu_p e N_A} \\
\Delta n &= \frac{\sigma}{\sigma_0} \frac{\mu_p e N_A}{\mu_n e + \mu_p e} \\
&= \frac{\sigma}{\sigma_0} \frac{N_A}{\frac{\mu_n}{\mu_p} + 1} \\
&= 100 \cdot \frac{10^{15}}{3.111 + 1} \\
&= 2.43\ 10^{16}\ \text{cm}^{-3}
\end{aligned}$$

A better approximation be calculated using $n \approx \Delta n \approx \Delta p = p - N_A$

$$\begin{aligned}
\frac{\sigma}{\sigma_0} &= \frac{\mu_n e \Delta n + \mu_p e (\Delta n + N_A)}{\mu_p e N_A} \\
\Delta n &= \left(\frac{\sigma}{\sigma_0} - 1\right) \frac{\mu_p e N_A}{\mu_n e + \mu_p e} \\
&= \left(\frac{\sigma}{\sigma_0} - 1\right) \frac{N_A}{\frac{\mu_n}{\mu_p} + 1} \\
&= 99 \cdot \frac{10^{15}}{3.111 + 1} \\
&= 2.41\ 10^{16}\ \text{cm}^{-3}
\end{aligned}$$

b.

$$\begin{aligned}G_{ext} &= \frac{1}{\tau}\frac{pn-n_i^2}{p+n+2n_i\cosh\left(\frac{E_{rg}-E_i}{kT}\right)}\\ &\approx \frac{1}{\tau}\frac{\Delta n^2}{2\Delta n}\\ &= \frac{\Delta n}{2\tau}\\ &= 1.20\ 10^{20}\ \text{cm}^{-3}\text{s}^{-1}\end{aligned}$$

c.

In equilibrium:

$$\begin{aligned}E_F &= E_i - kT\ln\frac{N_A}{n_i}\\ E_F - E_i &= -0.297\ \text{eV}\end{aligned}$$

During illumination:

$$\begin{aligned}E_{Fp} &= E_i - kT\ln\frac{p}{n_i}\\ &= E_i - kT\ln\frac{N_A+\delta n}{n_i}\\ E_{Fp}-E_i &= -0.380\ \text{eV}\\ E_{Fn} &= E_i + kT\ln\frac{n}{n_i}\\ &= E_i + kT\ln\frac{\delta n}{n_i}\\ E_{Fn}-E_i &= 0.379\ \text{eV}\end{aligned}$$

Question 57

Consider a one-sided abrupt n$^+$p Si junction at room temperature. The doping concentration of the n$^+$-region is $N_D = 10^{19}$ cm^{-3}. The diffusion constant of the holes is $D_p = 10$ cm^2/s, the one of the electrons is $D_n = 25$ cm^2/s. The length of the neutral n-region is $W_n = 10$ µm, that of the neutral p-region is $W_p = 5$ µm. Both regions can be considered as electrical short regions. The diode goes into breakdown when the maximum electrical field in the space charge region $E_{max} = 5\ 10^5$ V/cm, and you can consider this value to be independent of temperature.
Following data of the diode are known: intrinsic concentration $n_i = 10^{10}$ cm^{-3}, permittivity $\epsilon = 10^{-12}$ F/cm, thermal voltage $V_t = 0.025$ V, unit charge $q = 1.6\ 10^{-19}$ C.
We want to design the doping of the lowest doped p-side N_A such that the diode goes into high injection at voltages higher than 0.6 V, and that no breakdown happens at a voltage below 100 V. Thus we are looking to determine the range in which N_A should be chosen. In order to do so follow the following steps.

a. Derive an expression for the range of N_A such that the high injection voltage $V_{HI} > 0.6$ V.

b. Derive an expression for the range of NA such that the breakdown voltage is higher than 100V.

c. Which value of N_A will you choose if we want to make the diffusion capacitance at a given voltage as small as possible? Give the value and the motivation using a formula!

d. Which value of NA will you choose if we want to make the diffusion capacitance at a given current as small as possible? Give the value and the motivation using a formula!

a.

High-injection: the minority carrier concentration is equal to the doping concentration. This will first be the case in the lowly-doped region (p)

$$\begin{aligned} n_p &= \frac{n_i^2}{N_A} e^{\frac{V_{HI}}{V_t}} = N_A \\ N_A &= n_i e^{\frac{V_{HI}}{2V_t}} \\ N_A &> 1.63\ 10^{15}\ \text{cm}^{-3} \end{aligned}$$

b.

We need to make sure that $\mathcal{E}(0) < 5\ 10^5$ V/cm for $V = -100$ V.

$$\begin{aligned} \mathcal{E}(0) &= \int_0^{X_p} \frac{\rho}{\epsilon} dx \\ &= \int_0^{\sqrt{\frac{2\epsilon}{q}\frac{1}{N_A}(V_{bi}-V_{pn})}} \frac{N_A q}{\epsilon} dx \\ &= \frac{N_A q}{\epsilon}\sqrt{\frac{2\epsilon}{q}\frac{1}{N_A}(V_{bi}-V_{pn})} \\ N_A &= \mathcal{E}(0)^2 \frac{q}{2\epsilon(V_{bi}-V_{pn})} \\ &\approx \mathcal{E}(0)^2 \frac{\epsilon}{2qV_{bd}} \\ N_A &< 7.81\ 10^{15}\ \text{cm}^{-2} \end{aligned}$$

When taking into account V_{pn} and calculating N_A iteratively, we get $N_A < 7.81\ 10^{15}$ cm^{-2} (only a small deflection as expected). So we have to pick $1.63\ 10^{15}$ cm^{-3} $< N_A < 7.75\ 10^{15}$ cm^{-2}.

c.

The junction capacitance is:

$$C_j = \frac{C_{oj}}{1 - \left(\frac{V_{pn}}{V_{bi}}\right)^{1/n}}$$

With

$$\begin{aligned} C_{oj} &= \frac{1}{\sqrt{\frac{2}{\epsilon q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right) V_{bi}}} \\ &\approx \sqrt{\frac{\epsilon q N_A}{2 V_t \ln\left(\frac{N_A N_D}{n_i^2}\right)}} \end{aligned}$$

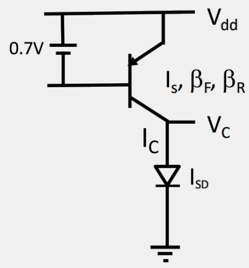
Which is smallest for small N_A, so we can pick $N_A = 1.63\ 10^{15}$ cm^{-3}.

d.

The diffusion capacitance is $C_d \approx C_{dn} \sim \frac{1}{n_{p0}} = \frac{n_i^2}{N_A}$, which is the smallest for large N_A, so we need to pick $N_A = 7.75\ 10^{15}$ cm^{-2}.

Question 58

Question 59
Consider a bipolar transistor loaded connected to a diode as shown on the figure below.



The following data of the devices are known:
bipolar transistor:
Forward current gain $\beta_F = 100$
Reverse current gain $\beta_R = 1$
Collector saturation current $I_s = 10^{-13}$ A
Between the base and the power supply voltage we have voltage source of 0.7 V applied.
All non-idealities such as recombination in the base, in the base-emitter junction, Early effect and high injection can be neglected.
diode:
Reverse saturation current $I_{sD} = 10^{-11}$ A.
The thermal voltage 0.026 V.

a) Calculate the collector voltage, the base current and the collector current of the bipolar transistor when $V_{dd} = 3$ V.

b) Calculate the collector voltage, the base current and the collector current of the bipolar transistor when $V_{dd} = 0.7$ V.

Draw for both cases first the Ebers-Moll model for the bipolar transistor. Voltages should be calculated within 1 mV accuracy.

a. The voltage over the diode is going to be about 0.5-0.8 V, so we can assume the transistor is in forward operation.

$$\begin{aligned} I_C &= I_s e^{\frac{V_{EB}}{V_t}} \\ &= 49.3 \text{ mA} \end{aligned}$$

The voltage can then be calculated from the diode:

$$\begin{aligned} I_C &= I_{sD} e^{\frac{V_C}{V_t}} \\ V_C &= V_t \ln \frac{I_C}{I_{sD}} \\ &= V_{EB} + V_t \ln \frac{I_s}{I_{sD}} \\ &= 0.580 \text{ V} \end{aligned}$$

We're indeed in forward operation. The base current is

$$\begin{aligned} I_B &= \frac{I_C}{\beta_F} \\ &= 0.49 \text{ mA} \end{aligned}$$

b.

If we would assume that we're in forward operation, we would get the same I_C and V_C, but then $V_{BC} < 0$, which is a contradiction. So, we're in saturation. We have 2 current equations:

$$\begin{aligned} I_C &= I_s \left[e^{\frac{V_{EB}}{V_t}} - \left(1 + \frac{1}{\beta_R}\right) e^{\frac{V_{CB}}{V_t}} \right] \\ I_C &= I_{sD} e^{\frac{V_C}{V_t}} \end{aligned}$$

These are 2 equations, with 2 unknowns (I_C and V_C). Dividing the first equation by the second equation gives:

$$\begin{aligned} 1 &= \frac{I_s}{I_{sD}} \left[e^{\frac{V_{EB}}{V_t}} e^{\frac{-V_C}{V_t}} - \left(1 + \frac{1}{\beta_R}\right) e^{\frac{V_B}{V_t}} \right] \\ V_C &= V_t \ln \left[\frac{e^{\frac{V_{EB}}{V_t}}}{\frac{I_{sD}}{I_s} + \left(1 + \frac{1}{\beta_R}\right) e^{\frac{V_B}{V_t}}} \right] \\ &= V_{EB} - V_t \ln \left[\frac{I_{sD}}{I_s} + \left(1 + \frac{1}{\beta_R}\right) e^{\frac{V_B}{V_t}} \right] \\ &= V_{EB} - V_t \ln [102] \\ &= 0.580 \text{ V} \end{aligned}$$

With $V_B = V_{dd} - V_{EB} = 0$ V. As $V_{BC} < 0$, we're indeed in saturation.

The collector current is then:

$$\begin{aligned} I_C &= I_{sD} e^{\frac{V_C}{V_t}} \\ &= I_{sD} \frac{1}{\frac{I_{sD}}{I_s} + \left(1 + \frac{1}{\beta_R}\right) e^{\frac{V_B}{V_t}}} e^{\frac{V_{EB}}{V_t}} \\ &= I_{sD} \frac{1}{102} e^{\frac{V_{EB}}{V_t}} \\ &= 48.3 \text{ mA} \end{aligned}$$

Verification: $I_C = I_s \left[e^{\frac{V_{EB}}{V_t}} - \left(1 + \frac{1}{\beta_R}\right) e^{\frac{V_{CB}}{V_t}} \right] = 48.3 \text{ mA} \implies$ okay!

The base current is:

$$\begin{aligned} I_B &= \frac{I_F}{\beta_F} + \frac{I_R}{\beta_R} \\ &= I_s \left[\frac{1}{\beta_F} e^{\frac{V_{EB}}{V_t}} + \frac{1}{\beta_R} e^{\frac{V_{CB}}{V_t}} \right] \\ &= 0.98 \text{ mA} \end{aligned}$$

Question 60

Question 61

Question 62

Consider a p-channel MOSFET as in the figure below, connected in series with a diode. The power supply voltage is $V_{DD} = 3$ V.

Data of the MOSFET are:

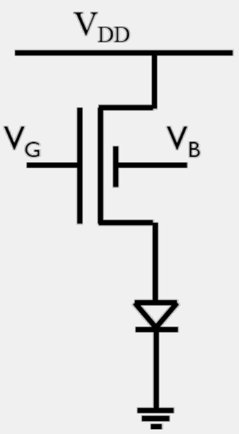
$W/L = 1$, $C_{ox} = 10^{-7}$ F/cm, $\mu = 1000$ cm2/Vs, $V_T = -0.7$ V for $V_{BS} = 0$ V, $N_{bulk} = 2\ 10^{16}$ at/cm^3, $\epsilon_{bulk} = 10^{-12}$ F/cm, $n_i = 10^{10}$ cm^{-3}, $\epsilon_{ox} = 3.5\ 10^{-13}$ F/cm

$V_G = 1$ V, $V_B = 4$ V

Data of the diode are:

- Area $= 10^{-2}$ cm^2
- $I_s = 10^{-12}$ A

Constants: $V_t = 0.026$ V, $q = 1.6\ 10^{-19}$ C.



a. Indicate source and drain of the MOSFET.

b. How much is V_{FB} and V_T for the applied voltages?

c. Calculate the electric field in the oxide at the source side of the MOSFET for the applied voltages.

d. Calculate the voltage that appears over the diode.

e. Draw the band diagram of the MOSFET for this operating point at the source side, along a cross section from the bulk to the gate (perpendicular to the oxide), only in the semiconductor. Indicate quantitatively the position of the quasi-Fermi levels, intrinsic level, conduction and valence band and the electrostatic potential drop.

I assume $C_{ox} = 10^{-7}$ F/cm^2 instead of $C_{ox} = 10^{-7}$ F/cm as given.

a. Source = where the carriers come from, p-type → holes are positively charged so they come from the top. Drain is then the bottom.

b.

$V_T = -0.7$ V for $V_{BS} = 0$, but in this case $V_{BS} = V_B - V_S = 1$ V.

$$\begin{aligned}
V_T &= -0.7\text{ V} - \frac{1}{C_{ox}}\sqrt{2qN_D\epsilon_s}\left(\sqrt{|2\Phi_f - V_{BS}|} - \sqrt{|2\Phi_f|}\right) \\
&= -0.7\text{ V} - \frac{1}{C_{ox}}\sqrt{2qN_D\epsilon_s}\left(\sqrt{\left|-2V_t\ln\left(\frac{N_D}{n_i}\right) - V_{BS}\right|} - \sqrt{2V_t\ln\left(\frac{N_D}{n_i}\right)}\right) \\
&= -0.7\text{ V} - 0.8\text{ V}^{1/2}\left(1.324\text{ V}^{1/2} - 0.869\text{ V}^{1/2}\right) \\
&= -1.065\text{ V}
\end{aligned}$$

V_{FB} can be calculated for $V_T = -0.7$ V and $V_{BS} = 0$.

$$\begin{aligned}
V_T &= 2\phi_f + V_{FB} - \frac{1}{C_{ox}}\sqrt{2qN_D\epsilon_s}\sqrt{|2\Phi_f - V_{BS}|} \\
V_{FB} &= V_T + 2V_t\ln\frac{N_D}{n_i} + \frac{1}{C_{ox}}\sqrt{4qN_D\epsilon_s V_t\ln\frac{N_A}{n_i}} \\
&= -0.7\text{ V} + 0.754\text{ V} + 0.411\text{ V} \\
&= 0.465\text{ V}
\end{aligned}$$

c.

$$\begin{aligned}
\Delta\Psi_{tot} = V_{GB} - \Phi_{ms} &= -\frac{qN_D z_{D,th}^2}{2\epsilon_s} - \frac{qN_D z_{D,th}}{C_{ox}} - \frac{Q_{ox}}{C_{ox}} - \frac{Q_{inv}}{C_{ox}} \\
V_{GB} - \Phi_{ms} + \frac{Q_{ox}}{C_{ox}} &= -\frac{qN_D z_{D,th}^2}{2\epsilon_s} - \frac{qN_D z_{D,th}}{C_{ox}} - \frac{Q_{inv}}{C_{ox}} \\
V_{GB} - V_{FB} &= -\frac{qN_D z_{D,th}^2}{2\epsilon_s} + \Psi_{ox}
\end{aligned}$$

With $z_{D,th} = \sqrt{\frac{2\epsilon_s|2\Phi_f - V_{BS}|}{qN_D}} = 331$ nm, we get

$$\begin{aligned}
\Psi_{ox} = \mathcal{E}t_{ox} &= -\frac{qN_D z_{D,th}^2}{2\epsilon_s} - V_{GB} + V_{FB} \\
\mathcal{E} &= -\frac{qN_D z_{D,th}^2}{2\epsilon_s} - V_{GB} + V_{FB} \\
\mathcal{E} &= \frac{C_{ox}}{\epsilon_{ox}}\left(-\frac{qN_D z_{D,th}^2}{2\epsilon_s} - V_{GB} + V_{FB}\right) \\
&= \frac{1}{35\ \text{nm}}\left(-1.754\ \text{V} + 3\ \text{V} + 0.465\ \text{V}\right) \\
&= 4.89\ 10^5\ \frac{\text{V}}{\text{cm}}
\end{aligned}$$

d.

$V_{DS,sat} = V_{GS} - V_T = -2\ \text{V} + 1.065\ \text{V} = -0.935$ V. And $\beta = \frac{W}{L}\mu C_{ox} = 10^{-4}\ \text{A/V}^2$. We assume that the MOSFET is in saturation, the current is then

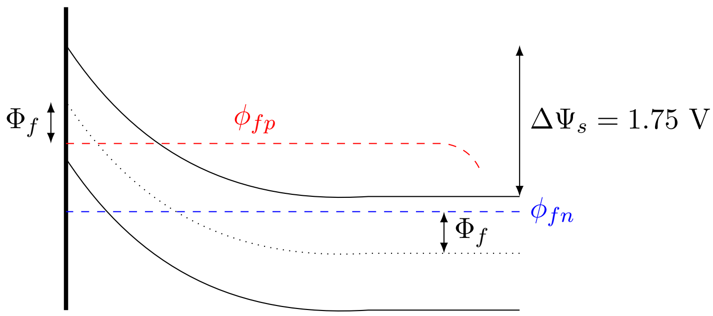
$$\begin{aligned}
I &= \beta\frac{(V_{GS} - V_T)^2}{2} \\
&= 43.7\ \mu\text{A}
\end{aligned}$$

And the diode voltage is

$$\begin{aligned}
I &= I_s e^{\frac{V_D}{V_t}} \\
V_D &= V_t \ln\frac{I}{I_s} \\
&= 0.457\ \text{V}
\end{aligned}$$

So $V_{DS} = V_D - V_{DD} = -2.543$ V and $|V_{DS}| > |V_{DS,sat}|$, which means that the assumption that the MOSFET is in saturation is correct.

e.



Question 63

Consider an n-MOSFET connected to a load resistance R_L. We power the circuit with a voltage source V_{DD}. The following parameters are given:
$V_{DD} = 5$ V, $R_L = 1$ MΩ
MOSFET:

- Threshold voltage $V_T = 1$ V for $V_{BS} = 0$ V
- Subthreshold swing SS is 80mV/decade
- Subthreshold current at $V_{GS} = V_T - 0.7$ V is $I_{DS} = 1$ pA

Constants: thermal voltage $kT/q = 26$ mV, $q = 1.6\ 10^{-19}$ C.
You can assume that the MOSFET subthreshold current is independent of V_{DS}.

1. We bias the MOSFET in the subthreshold region at $V_{GS} = V_T - 0.3$ V and $V_{BS} = 0$.
   a) Make a sketch of the circuit with R_L and the n-MOSFET between power and ground. Don't forget the bulk-source connection on this sketch. Also indicate where the drain electrode of the MOSFET is located.
   b) Calculate the subthreshold current I_{DS} for the applied bias $V_{GS} = V_T - 0.3$ V
   c) Determine the voltage at the drain of the MOSFET for the applied bias $V_{GS} = V_T - 0.3$ V
2. We now apply $V_{BS} = -0.3$ V and a gate voltage sufficient to turn the MOSFET on. Draw the band diagram in a cross section orthogonal to the gate electrode, in the semiconductor only: indicate the position of the quasi-Fermi levels, the intrinsic level, conduction and valence band and the electrostatic potential drop in the semiconductor.

1.

a.

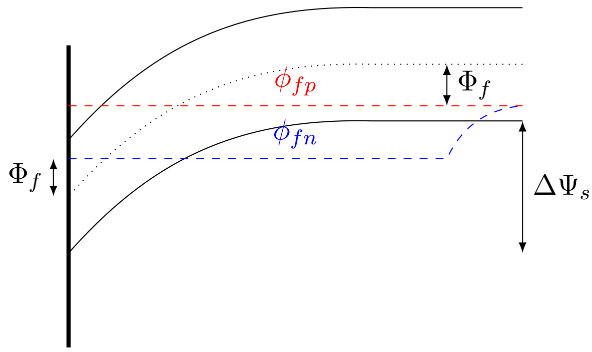
[Include sketch with source of MOSFET to ground, drain to resistor and resistor to power, bulk to source and gate to voltage source]

b.

$$\begin{aligned} I_{DS} &= 1 \text{ pA} \cdot 10^{\frac{V_{GS}-(V_T-0.7 \text{ V}}{0.08 \text{ V}}} \\ &= 0.1 \text{ µA} \end{aligned}$$

c.

$$\begin{aligned} V_D &= V_{DD} - R_L I \\ &= 4.9 \text{ V} \end{aligned}$$


ϕ_{fp}
Φ_f
ϕ_{fn}
Φ_f
$\Delta\Psi_s$

Question 64

Question 65

In a p-n diode under bias the minority carrier concentration profile in the p-region could be measured as shown in the figure below (the y-axis is on a linear scale!) Following parameters are known as well:

- $\mu_e = 1000\ \mathrm{cm}^2/(\mathrm{Vs})$, $\mu_h = 500\ \mathrm{cm}^2/(\mathrm{Vs})$
- Length of the p-region is 50 µm, length of the n-region is 1 mm
- $N_D = 10^{17}\ \mathrm{cm}^{-3}$
- $\tau_e = \tau_p = 100\ \mu\mathrm{s}$
- Cross section area is $10^{-5}\ \mathrm{cm}^2$
- You can neglect recombination and generation in the space charge region, as well as the width of the space charge region compared to the diffusion lengths.
- $kT/q = 25\ \mathrm{mV}$
- $n_i = 1\ 10^{10}\ \mathrm{cm}^{-3}$

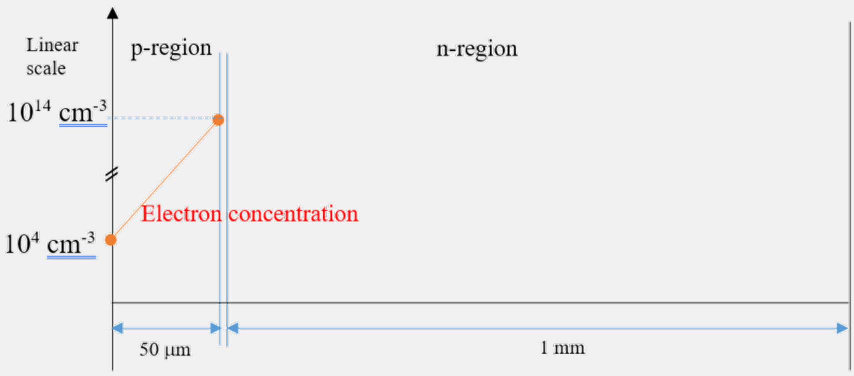
Determine for this diode:

a) the doping concentration of the p-region

b) the applied bias

c) sketch quantitatively (both x- and y-axis) the hole concentration profile in the n-region

d) calculate the current through the diode!



a.

$$
\begin{aligned}
n_{p0} &= \frac{n_i^2}{N_A} \\
N_A &= \frac{n_i^2}{n_{p0}} \\
&= 10^{16}\ \mathrm{cm}^{-3}
\end{aligned}
$$

b.

$$
\begin{aligned}
n_p &= n_{p0} e^{\frac{V_{pn}}{V_t}} \\
V_{pn} &= V_t \ln\left(\frac{n_p}{n_{p0}}\right) \\
&= 0.576 \text{ V}
\end{aligned}
$$

c.

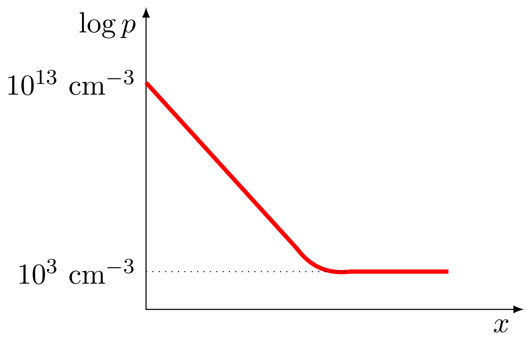
First we calculate the hole concentration in the 2 ends:

$$
\begin{aligned}
p_{n0} &= \frac{n_i^2}{N_D} \\
&= 10^3 \text{ cm}^{-3} \\
p_n &= p_{n0} e^{\frac{V_{pn}}{V_t}} \\
&= 10^{13} \text{ cm}^{-3}
\end{aligned}
$$

As the n-region is (most likely) a long region, we must calculated the diffusion length:

$$
\begin{aligned}
L_p &= \sqrt{D_p \tau_p} \\
&= \sqrt{\mu_p V_t \tau_p} \\
&= 354 \text{ µm}
\end{aligned}
$$

This is on the edge of long region and intermediate.



d.

We neglect recombination in the SCR.

$$
\begin{aligned}
I &= I_n + I_p \\
&= AqV_t \left(\mu_n \frac{n_p - n_{p0}}{W_p} + \mu_p \frac{p_n - p_{n0}}{L_p}\right) \\
&= 0.80 \text{ µA} + 5.7 \text{ nA} \\
&= 0.80 \text{ µA}
\end{aligned}
$$