Una Parte Della Semantica Del Linguaggio

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$$\mathbf{T_{exp}} ::= \{ \mathbf{a'} \mid \mathbf{a'} \in \mathbf{K}\} \quad \forall \mathbf{a'}, \mathbf{b'} \in \mathbf{K} : \mathbf{a'} \neq \mathbf{b'} \Rightarrow \mathbf{a'} \lesseqgtr \mathbf{b'} \wedge \mathbf{typeof}\ \mathbf{a'} = \mathbf{typeof}\ \mathbf{b'}$$

$$\mathbf{T} ::= (\mathbf{typeof}\ \mathbf{a'} = \mathbf{Int}|\mathbf{String})|...$$

$$\mathbf{Var} ::= \mathbf{X_1}|\mathbf{X_2}|... \qquad \mathbf{typeof}\ \mathbf{X_i} = \mathbf{string}$$

$$\mathbf{SExp} ::= ...|\mathbf{T_{exp}}|\mathbf{Var}|\mathbf{Empty}(\mathbf{a'})|\mathbf{Singleton}(\mathbf{a'}, \mathbf{typeof}\ \mathbf{a'})$$
$$|\mathbf{Of}(\mathbf{typeof}\ \mathbf{a'}, (\mathbf{a'} \rightarrow \mathbf{Int})\ \mathbf{op}\ (\mathbf{b'} \rightarrow \mathbf{Int}), \mathbf{succ}(\mathbf{b'}))$$
$$|\mathbf{T_{exp}} \cup \mathbf{T'_{exp}}|\mathbf{T_{exp}} \cap \mathbf{T'_{exp}}$$
$$|\mathbf{T_{exp}} \ominus \mathbf{T'_{exp}}|\mathbf{T_{exp}} \cup \{\mathbf{a'}\}|\mathbf{T_{exp}} \cap \{\mathbf{a'}\}|\mathbf{Max}\{\mathbf{T_{exp}}\}|\mathbf{Min}\{\mathbf{T_{exp}}\}|...$$

$$\mathbf{BExp} ::= \mathbf{True}|\mathbf{False}|\mathbf{Empty}(\mathbf{SExp})|\mathbf{Isin}(\mathbf{a'}, \mathbf{SExp})|\mathbf{SubSet}(\mathbf{SExp}, \mathbf{SExp})$$
$$|\mathbf{Pred}(\mathbf{SExp})|...$$

$$\mathbf{Com} ::= ...|\mathbf{For\_all}(\mathbf{Pred}(\mathbf{SExp}), \mathbf{SExp})$$
$$|\mathbf{Exists}(\mathbf{Pred}(\mathbf{SExp}), \mathbf{SExp})|\mathbf{Filter}(\mathbf{Pred}(\mathbf{SExp}), \mathbf{SExp})$$
$$|\mathbf{Map}(\mathbf{Fun}(\mathbf{i}, \mathbf{c}), \mathbf{SExp})|...$$

$$\mathbf{Op} ::= -|+|*$$

$$(1)$$

1 Axiom

$$\frac{\overline{}}{\mathbf{Empty}(\mathbf{a'}) : \mathbf{T_{exp}}} \qquad \frac{\overline{}}{\mathbf{Singleton}(\mathbf{a'}, \mathbf{typeof}\ \mathbf{a'}) : \mathbf{T_{exp}}}$$

$$\frac{\overline{}}{\mathbf{Of}(\mathbf{typeof}\ \mathbf{a'}, (\mathbf{a'} \rightarrow \mathbf{Int})\ \mathbf{op}\ (\mathbf{b'} \rightarrow \mathbf{Int}), \mathbf{succ}(\mathbf{b'})) : \mathbf{T_{exp}}}$$

$$(2)$$

2 SExp

$$\frac{\mathbf{env} \triangleright \mathbf{Den}(\mathbf{X_0}) \rightarrow \mathbf{T'_{exp}} \quad \mathbf{env} \triangleright \mathbf{Den}(\mathbf{X_1}) \rightarrow \mathbf{T''_{exp}}}{\mathbf{env} \triangleright \mathbf{Den}(\mathbf{X_0}) \cup \mathbf{Den}(\mathbf{X_1}) \rightarrow \mathbf{T_{exp}}}$$

$$\frac{\mathbf{env} \triangleright \mathbf{Den}(\mathbf{X_0}) \rightarrow \mathbf{T'_{exp}} \quad \mathbf{env} \triangleright \mathbf{Den}(\mathbf{X_1}) \rightarrow \mathbf{T''_{exp}}}{\mathbf{env} \triangleright \mathbf{Den}(\mathbf{X_0}) \cap \mathbf{Den}(\mathbf{X_1}) \rightarrow \mathbf{T_{exp}}}$$

$$(3)$$

$$\frac{env \triangleright Den(X_0) \to T'_{exp} \quad env \triangleright Den(X_1) \to T''_{exp}}{env \triangleright Den(X_0) \ominus Den(X_1) \to T_{exp}}$$

$$\frac{env \triangleright Den(X_0) \to T'_{exp} \quad env \triangleright \{a'\} \to Singleton(a', typeof\ a')}{env \triangleright Den(X_0) \cap \{a'\} \to T_{exp}}$$

$$\frac{env \triangleright Den(X_0) \to T'_{exp} \quad env \triangleright \{a'\} \to Singleton(a', typeof\ a')}{env \triangleright Den(X_0) \cup \{a'\} \to T_{exp}} \tag{4}$$

$$\frac{env \triangleright Den(X) \to T_{exp}}{env \triangleright Max(Den(X)) \to a' \mid a' > \forall\ a'' \in T_{exp}}$$

$$\frac{env \triangleright Den(X) \to T_{exp}}{env \triangleright Min(Den(X)) \to a' \mid a' < \forall\ a'' \in T_{exp}}$$

3 BExp

$$\frac{env \triangleright SExp \to T_{exp}}{env \triangleright Empty(SExp) \to Bool}$$

$$\frac{env \triangleright a' \to Singleton(a', typeof\ a') \quad env \triangleright SExp \to T_{exp}}{env \triangleright Isin(a', SExp) \to Bool}$$

$$\frac{env \triangleright SExp \to T'_{exp} \quad env \triangleright SExp \to T''_{exp}}{env \triangleright SubSet(SExp, SExp) \to Bool}$$

$$\frac{env \triangleright SExp \to Fun(i, c)}{env \triangleright Pred(SExp) \to Bool}$$

4 Com

$$\frac{env \triangleright Pred(SExp) \to Bool \quad env \triangleright SExp \to T_{exp}}{env \triangleright For\_all(Pred(SExp), SExp) \to Bool}$$

$$\frac{\text{env} \triangleright \textbf{Pred(SExp)} \rightarrow \textbf{Bool} \qquad \text{env} \triangleright \textbf{SExp} \rightarrow \textbf{T}_{\text{exp}}}{\text{env} \triangleright \textbf{For\_all(Pred(SExp), SExp)} \rightarrow \textbf{Bool}}$$

$$\frac{\text{env} \triangleright \textbf{Pred(SExp)} \rightarrow \textbf{Bool} \qquad \text{env} \triangleright \textbf{SExp} \rightarrow \textbf{T}_{\text{exp}}}{\text{env} \triangleright \textbf{Exists(Pred(SExp), SExp)} \rightarrow \textbf{Bool}}$$

$$\frac{\text{env} \triangleright \textbf{Pred(SExp)} \rightarrow \textbf{Bool} \qquad \text{env} \triangleright \textbf{SExp} \rightarrow \textbf{T}_{\text{exp}}}{\text{env} \triangleright \textbf{Filter(Pred(SExp), SExp)} \rightarrow \textbf{T}_{\text{exp}}}$$

$$\frac{\text{env} \triangleright \textbf{Fun(i, c)} \rightarrow \textbf{Closure(i, c, amb)} \qquad \text{env} \triangleright \textbf{SExp} \rightarrow \textbf{T}_{\text{exp}}}{\text{env} \triangleright \textbf{Map(Fun(i, c), SExp)} \rightarrow \textbf{T}_{\text{exp}}}$$