IDENTIFICATION

Laboratory exercises

Laboratory exercise 1: Identification of a non-parametric model

The exercise consists of two parts. In the first part input-output data are given while in the second the compiled function of the process is available, and students have to prepare appropriate excitation and simulate the process to obtain input-output data. Identification of non-parametric model is then performed.

Part I:

The file `MEASUREx.DAT` (x is a letter – A, B, C etc.) contains input-output measurements that will be used for identification with the empirical transfer function estimation (ETFE). The first column contains measurement times, and the following two hold the measurements of the input and the output, respectively. The last column contains noise sequence that was sampled during the time of no excitation at the input.

Tasks:

1. Load the measurements to the MATLAB workspace by the command `load MEASUREx.DAT`. Plot the signals.
2. Determine the frequency resolution F and the maximum frequency according to the Nyquist-Shannon theorem f_{max}:
$$F = \frac{1}{NT}, \quad f_{\max} = \frac{1}{2T}$$
where N is the length of the signal vectors while T is the sampling time.
3. Calculate Fourier transforms (FT) of the input and the output signal by using the `fft` function (the function returns the samples of FT divided by T; initial frequency: 0, frequency step: F). Plot the amplitude response (absolute value of the FT) where both axes are shown in linear scale.
4. Empirical transfer function estimate (ETFE) calculates frequency response (FR) using:
$$G(j\omega) = \frac{Y(\omega)}{U(\omega)}$$
Absolute value (command `abs`) of the FR has to be transformed to decibels ($20\log|G(j\omega)|$), and phase angle to degrees (commands `angle` and `unwrap`). Amplitude and phase responses should be plotted in the linear-logarithmic scale (command `semilogx`). Note: log → command `log10`!
5. By observing amplitude and phase response, the number of poles (process order n) and zeros should be estimated as well as their character (real, complex) and their mutual position.
6. By using inverse Fourier transform (command `ifft`) impulse response of the identified process has to be calculated, and its real and imaginary (commands `real` and `imag`) parts plotted.
7. Frequency spectrum of the noise will be analysed next. Absolute value of the Fourier transform of the noise $|N(\omega)|$ should be plotted in the linear-linear scale. Determine the character of the noise: white noise (at least in the frequency interval defined by the Nyquist-Shannon theorem) or high-frequent noise. In the latter case, some parameters of the noise model have to be estimated (see Figure 1): low-frequency and high-frequency amplitude of $|N(\omega)|$ is denoted by N_0 and N_∞, respectively. Frequency ω_g also has to be estimated. The model of the colour noise from Figure 1 is given by the following equation:
$$|N(\omega)| = N_\infty + \frac{N_0 - N_\infty}{\sqrt{1 + (\frac{\omega}{\omega_g})^2}}$$

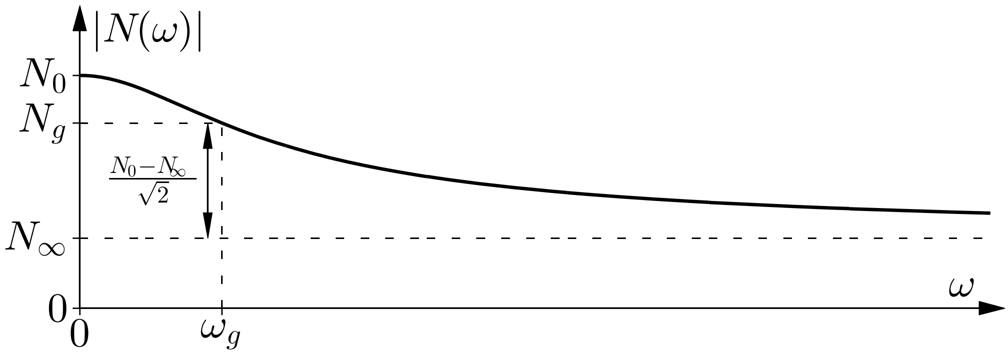

Figure 1. Model of the colour noise

8. Determine the standard deviation of the error of the amplitude-response estimate due to noise:
$$\left|\Delta G_n(j\omega)\right| = \frac{|N(\omega)|}{|U(\omega)|}$$
In the above equation two approximations of $|N(\omega)|$ should be used:
   a. The measured one – obtained by FT of the measured noise:
   $$\left|\Delta G_n(j\omega)\right| = \frac{|N(\omega)|}{|U(\omega)|}$$
   b. The theoretical one – obtained from the noise model (the parameters were estimated in task 7).
9. The following graphs should be plotted:
   a. Standard deviation of the absolute error of the frequency response $\left|\Delta G_n(j\omega)\right|$. Both versions obtained in task 8 should be shown in linear-linear scale.
   b. Absolute value of the empirical transfer function estimate $\left|\hat{G}(j\omega)\right|$ should be plotted together with its upper bound $\left|\hat{G}(j\omega)\right| + \left|\Delta G_n(j\omega)\right|$ and its lower bound $\left|\hat{G}(j\omega)\right| - \left|\Delta G_n(j\omega)\right|$. The theoretical noise model from 8a should be used. The amplitude response in decibels should be plotted versus frequency and the latter is shown in logarithmic scale.

Part II:

Unknown process will be identified with non-parametric identification methods in this part. The process is given in the form of the compiled MATLAB function `procesx.mexw32` (x is a letter – A, B, C etc.). The parameters of the function `y=procesx(u)` are:

`u` – input signal (given as a column vector),
`y` – output signal.

Input signal is an arbitrary appropriate signal chosen by the students. Note that sampling time is fixed to $T = 0.1$. This means that input and output sequences can be regarded as discrete signals. The length of the sampling time should be taken into account when analysing system properties in frequency domain.

Tasks:

1. Determine frequency response of the process by means of ETFE.
2. Plot the corresponding amplitude and phase response.
3. Frequency response should be validated at five distinct frequencies. The process should be excited by harmonic excitation and the frequency response obtained using orthogonal correlation. The results should be depicted on the same plot as the results of the ETFE.
4. Additional task (not mandatory): non-parametric models should also be identified by correlation analysis and spectral analysis.

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Laboratory exercise 2: Identification of a parametric model

The exercise consists of two parts. In the first part input-output data are given while in the second the compiled function of the process is available, and students have to prepare appropriate excitation and simulate the process to obtain input-output data. Identification of parametric model is then performed.

Part I:

The file `MEASUREx.DAT` (x is a letter – A, B, C etc.) contains input-output measurements that will be used for identification with least square parameter estimation. The first column contains measurement times, and the following two hold the measurements of the input and the output, respectively. The last column contains noise sequence that was sampled during the time of no excitation at the input.

A parametric model of the process will be identified in the form of a transfer function:

$$G(z)=\frac{Y(z)}{U(z)}=\frac{b_1 z^{-1}+\ldots+b_n z^{-n}}{1+a_1 z^{-1}+\ldots+a_n z^{-n}}z^{-d}=\frac{b_1 z^{n-1}+\ldots+b_n}{z^n+a_1 z^{n-1}+\ldots+a_n}z^{-d}$$

The parameters of the model are delay d, system order n, and parameters of the numerator (b_1, b_2,…b_n) and the denominator (a_1, a_2,…a_n). The parameters that define the structure (n and d) will first be determined and then the other parameters will be obtained by least square parameter estimation. The above transfer function can be transformed into a difference-equation form:

$$y(k)=-a_1 y(k-1)-a_2 y(k-2)-\ldots-a_n y(k-n)+b_1 u(k-1-d)+b_2 u(k-2-d)+\ldots+b_n u(k-n-d)$$

This description of the system is suitable for parameter estimation.

Tasks:

1. By analysing the measured input and output, pure delay should be determined. Based on the frequency response analysis (from Lab exercise 1) process order should be estimated. Note that the orders of the continuous system and its discrete equivalent are the same.
2. Now that the structure is chosen (in the form of d and n) the system of linear equations can be written down. If signals are given in the time interval $k\in[0,N-1]$, the difference equation of the system can be rewritten for (N-n-d) distinct value of k ($k=n+d,n+d+1,\ldots,N-1$):

   $$\begin{aligned}&y(n+d)=-a_1 y(n+d-1)-a_2 y(n+d-2)-\ldots-a_n y(d)+b_1 u(n-1)+b_2 u(n-2)+\ldots+b_n u(0)\\&\vdots\\&y(N-1)=-a_1 y(N-2)-\ldots-a_n y(N-n-1)+b_1 u(N-2-d)+\ldots+b_n u(N-1-n-d)\end{aligned}$$

   The above overdetermined system of linear algebraic equations can be rearranged in the matrix-vector form:

   $$\mathbf{y}=\boldsymbol{\psi}\boldsymbol{\theta}$$

   where matrix $\boldsymbol{\psi}$ contains measured signal samples, and vector $\boldsymbol{\theta}$ the unknown parameters:

   $$\boldsymbol{\theta}^T=\begin{bmatrix}a_1 & \cdots & a_n & b_1 & \cdots & b_n\end{bmatrix}$$

   Find the least square parameter estimate $\hat{\boldsymbol{\theta}}$ (see remark at the end of the document). Discrete transfer function of the system is to be composed of the estimated parameters.
3. Plot the frequency response of the identified system (function `bode`). Amplitude and phase response should be compared to the ones in task 1. If the results of the comparison are not satisfactory, parameter estimation has to be repeated with a higher or a lower system order.

4. Verify the resulting discrete model by simulating its output to the measured input signal (function `lsim`). The measured and the simulated output should be shown in the same plot. Calculate the covariance matrix and standard deviations of estimated parameters:

$$\mathrm{cov}[\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}]=\frac{1}{(N-n-d)-1}\sum_{k=n+d}^{N-1}(y(k)-\hat{y}(k))^2(\boldsymbol{\psi}^T\boldsymbol{\psi})^{-1}$$

5. Find the continuous equivalent of the estimated discrete model (function `d2c`).

Part II:

Unknown process will be identified with parametric identification methods in this part. The process is given in the form of the compiled MATLAB function `procesx.mexw32` (x is a letter – A, B, C etc.). The parameters of the function `y=procesx(u)` are:

`u` – input signal (given as a column vector),
`y` – output signal.

Input signal is an arbitrary appropriate signal chosen by the students. Note that sampling time is fixed to $T = 0.1$. This means that input and output sequences can be regarded as discrete signals. The length of the sampling time should be taken into account when dealing with continuous models.

Tasks:

1. Determine the structure of the system (order, delay).
2. Determine transfer function parameters by using least square parameter estimation. Calculate the covariance matrix and standard deviations of estimated parameters.
3. The identified discrete model should be validated by comparing the responses of the actual and the identified system to an arbitrary input signal (that needs to be different from the one used in identification). Both responses should be plotted in the same figure.

Remark:

The system of linear equations given by

$$\mathbf{Ax}=\mathbf{b}$$

where **x** is a vector of unknowns, can be solved in MATLAB using a command:

```
x=A\b
```

If the equations are linearly independent and their number equals the number of unknowns, then `x` is the exact solution. If the system is overdetermined, then the solution in the least-square sense is provided by the above command.

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Laboratory exercise 3

Working with a real process and measurement of the static curve

Static curve or static characteristic of a chosen laboratory pilot plant should be determined. The measurements are taken by a computer and appropriate converters. The acquisition takes place within the Matlab/Simulink framework.

Static curve defines the process behaviour in stationary state across the whole operating region. It is measured by repeatedly exciting the process with a constant or a step, and measuring the stationary state of the output. The inputs have to cover the whole operating region of the process. Static curve is depicted in the input/output plot.

Next, select an operating point where linear model of the process will be identified. The operating point should be chosen such that the linear approximation covers as large operating region as possible.

Identification of a linear model of the laboratory pilot plant

The laboratory pilot plant (or its subprocess) will be identified in the operating point selected in the first part of the laboratory exercise.

The identified model should be validated as follows:

- In case of parametric identification the responses of the actual and the identified system to an arbitrary input signal (that needs to be different from the one used in identification) should be compared. Standard deviations of the identified parameters should also be given.
- In case of non-parametric identification the process should be excited by harmonic excitation (in case of fast plants at least 10 frequencies are analysed, otherwise only 5). The frequency response should be estimated using orthogonal correlation. Standard deviation of the absolute error of the frequency response estimate (by ETFE) should also be estimated.

The report should include:

- static curve of the process,
- the selected operating point and sampling time,
- input signal for identification,
- in case of parametric identification: discrete and continuous transfer function, standard deviations of the parameters of the discrete transfer function,
- in case of non-parametric identification: the estimated order, the number of poles and zeros, standard deviation of the absolute error of the frequency response estimate (by ETFE), the results of the orthogonal correlation,
- graphical illustration of the validation results,
- any comments about the model.

The report should fit two pages (one sheet of paper)!