Interval-based Robot Localization with Uncertainty Evaluation

Yuehan Jiang[1][a], Second Author Name[1][b] and Bernardo Wagner[2][c]

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Abstract: Being able to provide trustworthy localization for a robot in a map is essential for various tasks with safety related requirements. Considering the usually unknown error model of LiDAR sensor, we propose to solve the localization using interval error bounds. As a result, the interval solution gives an interval uncertainty estimation of the robot pose that is guaranteed to enclose all possible values. To evaluate the quality of interval uncertainty estimation, we compare the result with the uncertainty estimation of a factor graph based probabilistic approach. Experiments validate that our interval approach can provide guaranteed pose uncertainty and show advantage over probabilistic approach.

1 INTRODUCTION

In mobile robotics, the localization task requires the robot to not only accurately determine its position in the environment, but also provide trustworthy result which is critical for safe navigation and operation. In indoor environments where GPS (global positioning system) is not available, the robot usually localize itself by using sensory information (e.g. wheel odometry, 3D point cloud, or camera images) and update its belief of position with respect to a given map. A variety of probabilistic techniques have been proposed to reach the localization goal on a prior map (Gutmann et al., 1998) (Oh et al., 2004) (Wilbers et al., 2019). Nevertheless, most approaches focus on providing accurate point-wise solutions, while the quality of the uncertainty estimation of environmental perception and localization result are not thoroughly investigated.

Probabilistic approaches define Gaussian uncertainty over sensor measurements and robot positions while giving explicit estimates of robot positions (Siegwart, 2011). However, the uncertainty assumption could be inaccurate and unreliable due to non-Gaussian sensor errors and nonlinear robot models. Stochastic distributions can not always truly represent the error model of sensors like LiDARs due to the presence of systematic errors (Voges, 2020). Moreover, due to the nonlinearity of the system, a linearization process is required which can only give an approximation of the position estimate. Due to the above limitations of probabilistic approaches, the resulting pose estimation and its uncertainty might be erroneous and not able to represent the true result. Thus, our goal is to tackle these problems by providing a localization solution which can provide guaranteed uncertainty estimation for robot position that encloses the true values.

In this paper, we propose to give a solution to a 2D landmark-based localization problem using interval analysis (Jaulin et al., 2001). In comparison to classic probabilistic methods that use Gaussian distribution to represent the uncertainty of landmark (i.e. map) positions, measurement model (i.e. range and bearing) as well as robot poses (position and heading), we model them with intervals. More precisely, the robot poses and landmark positions are modelled as interval boxes, with lower and upper bounds, where we assume that the true value can be anywhere within. In particular, we utilize the constraint propagation techniques provided by interval methods which enable us to compute the uncertainty propagation from measurement model to resulting robot pose. The benefit of using interval bounds ensures that all possible values that fulfill the constraints will be retained. As a result, interval method will not lead to an overconfident solution of robot pose uncertainty because there is no approximation in the uncertainty propagation.

The contributions of the paper are:

- an interval-based solution for landmark-based localization problem which provides uncertainty estimation that guarantees to enclose all possible results
- comparison of robot pose uncertainty estimation between proposed interval-based approach and

[a] https://orcid.org/0000-0000-0000-0000
[b] https://orcid.org/0000-0000-0000-0000
[c] https://orcid.org/0000-0000-0000-0000

probabilistic approach

- failure cases which show that probabilistic approach leads to incorrect uncertainty estimation with the presence of systematic error while interval method result still holds true

In the remainder of this paper we first introduce related work in Session 2. In Session 3 we introduce the basics of interval analysis. Subsequently, we formulate our localization problem and establish our interval-based localization solution, as well as the state-of-the-art probabilistic solution in Session 4. In Session 5, we evaluate the uncertainty estimation of the two solutions by four small experiments and analyze the results.

2 RELATED WORK

In map-based localization, the representation of a known map can be a 2D floor plan, an occupancy grid map, a geometrical feature map or a set of landmarks (Siegwart, 2011). In the existing localization literature, landmark-based methods are widely used by different sensor modalities benefiting from the identifiable landmarks in the environments like poles (Brenner, 2010) (Schaefer et al., 2019), visual features (Wilbers et al., 2019) and natural landmarks (Rohou et al., 2020). Our method also use landmarks to present the map as a-priori. Typically the landmarks observed by the sensor are associated with the map in order to track the robot in the map. The data association problem is challenging and false association introduce outliers which need to be dealt with. Since data association is not the focus of this paper, we assume that the association between locally-observed landmarks and the map landmarks is known.

In recent years, interval methods have already shown its potential in robot localization and simultaneous localization and mapping problems. The work in (Rohou et al., 2020) solves the localization problem for an underwater vehicle in a known map but with unreliable data association. By formulating a constraint network with a constellation contractor, the data association and state estimation problems are solved together, of which the results are guaranteed localization sets which contain the actual but unknown values. In an unmanned aerial vehicle (UAV) localization application (Kenmogne et al., 2017) interval has been used to compute an interval pose domain of the 6 DOF pose of UAV by tracking known image features with bounded-error. In domain of autonomous driving, the work in (Voges and Wagner, 2021) firstly propose a guaranteed visual-LiDAR odometry for autonomous vehicles that uses LiDAR and a monocular camera to compute the 6 DOF pose estimate with bounded uncertainty. Evaluation on real data from large-scale experiments shows that the vehicle's real poses are enclosed in a guaranteed way. The odometry in this work is solved by incrementally computing the rigid body transformation using depth-augmented image features re-identified by an image feature detector. Authors of (Ehambram et al., 2021) extend this work (Voges and Wagner, 2021) by fusing information from LiDAR and stereo cameras to solve the dead-reckoning of a vehicle and use interval contraction method for consecutive robot pose estimation. Despite that the above methods show that interval analysis can provide robust uncertainty of robot pose which always enclose the ground truth data, no detailed investigation has been done to compare the interval result with probabilistic methods. Interval has the benefit of giving guaranteed estimation of robot pose, however it can also suffer from pessimistic result (i.e. too large intervals) which could not yield any useful information when the error bounds of perceived landmarks are chosen too big. Thus, we think it's important to validate the use of interval by showing under what cases interval has advantages over probability approaches in terms of uncertainty estimation.

In the probabilistic methods for robot localization, Kalman filter (Chen, 2012) is a common approach. It applies Gaussian error model for the uncertainty and minimize the residuals by formulating a least square problem. Monte Carlo localization methods (Dellaert et al., 1999) (Thrun et al., 2001) (Wen et al., 2021) are also traditional approaches that use a weighted sample set to approximate the posterior of robot state. More recently, graph-based optimization has become the standard solution for SLAM and widely applied for localization applications (Kummerle et al., 2011) (Kaess et al., 2011) (Wu et al., 2017) (Wilbers et al., 2019). The pose graph representation allows us to add robot poses and landmark positions as nodes in the graph, while the constraints between the poses and between the poses and landmarks can be added as edges (or factors) that connect the nodes in the graph. A non-linear least square problem is formulated and the error function is minimized in a bundle-adjustment manner to determine the best robot pose estimate and landmark position solution that fulfill the given constraints. In this paper, we use factor graph (Kaess et al., 2011) to formulate our localization problem since the measurement constraints between robot poses and landmarks, as well as the map prior can be easily represented as "factors". We establish a factor-graph based localization solution as the state-of-the-art probabilistic approach. The uncertainty estimation of this method will be compared with our interval-based uncertainty and evaluated.

3 INTERVAL ANALYSIS

Interval analysis is a powerful numerical tool based on the idea of enclosing real numbers in intervals (Jaulin et al., 2001). Instead of an exact value, unknown variables are defined by an interval that is a subset of $\mathbb{R}$, denoted by $[x] = [\underline{x}, \overline{x}]$. The variable is bounded by a lower bound $\underline{x}$ and an upper bound $\overline{x}$, without any assumption about its probability distribution. By using the simple error bounds, interval provides a guaranteed way to represent the uncertainty of a variable. For example, if we know that the accuracy of a distance measurement is ± 0.3m, then the true value x^* is enclosed by an interval $x^* \in [x]$, with an uncertainty of the radius $r([x]) = (\overline{x} - \underline{x})/2 = 0.3m$. An interval box $[\mathbf{x}]$ is define as a vector of intervals.

Interval computation enables classical real arithmetic operations, $+, -, \times$, and $\div$, as well as elementary functions like $\sin, \cos, \exp \ldots$ on intervals based on set theory. Here is an example of addition operation on intervals:

$$[-2, 4] + [5, 6] = [-2+5, 4+6] = [3, 10] \quad (1)$$

For a 2D robot localization problem, where we consider the set of possible robot pose as an interval domain $\mathbb{X}$, the problem can be characterized as following:

$$\mathbb{X} = \{\mathbf{x} \in \mathbb{R}^3 \mid \mathbf{f}(\mathbf{x}) \in \mathbb{Y}\} = \mathbf{f}^{-1}(\mathbb{Y}), \quad (2)$$

where $\mathbf{f}: \mathbb{R}^3 \to \mathbb{R}^2$ is a non-linear function, which can be the measurement constraint, and $\mathbb{Y}$ the landmark positions. The robot pose $\mathbb{X}$ can be obtained by using Set Inverter Via Interval Analysis (SIVIA) (Jaulin et al., 2001) to calculate a subpaving solution set from an initial search domain.

Alternatively, this can also be solved by formulating a Constraint Satisfaction Problem (CSP) as a set of constraints and variables. We give an exemplary CSP as:

$$\mathcal{H}: \begin{cases} \textbf{Variable}: & \mathbf{x}^n \\ \textbf{Constraint}: & \mathbf{g}(\mathbf{x}^n) = \mathbf{0}, \\ \textbf{Domain}: & [\mathbf{x}^n] \end{cases} \quad (3)$$

where $\mathbf{x}^n \in [\mathbf{x}^n]$ is the unknown variable vector, and $\mathbf{g}(\mathbf{x})^{\mathbf{n}}$ is the constraints to fulfill in the CSP $\mathcal{H}$.

Contractors (Jaulin et al., 2001) can be built to contract the variable's initial domain, in order to get a resulting interval box which satisfies all the constraints. Here the contracting means that the initial interval domain is reduced by removing all values that do not satisfy the constraints. The forward-backward contractor is the most convenient contractor to contract the domain of CSP $\mathcal{H}$ where all constraints are decomposed into primitive constraints (i.e. a sequence of functions containing only a single operator) and each of the primitive constraints is contracted until the interval boxes converge towards the smallest possible domain. The benefit of contraction is that all possible solutions are guaranteed to be contained in the contracted domain.

In this work, interval analysis is used for solving a 2D robot localization problem. A CSP is easily formulated and the interval domain of the robot pose is suitable to represent its uncertainty. Contractors are built to propagate the uncertainty of map landmarks and measurement constraints, resulting in a guaranteed robot pose uncertainty, which corresponds to the resulting interval domain of CSP $\mathcal{H}$.

4 PROBLEM FORMULATION

In this session, we formulate a 2D localization problem. Imagine tracking a moving robot in a known map of landmarks $\mathbb{M}$. The robot carries a LiDAR sensor and detects and measures the radial distance r and horizontal angle θ of its surrounding landmarks to the robot itself. For each LiDAR scan a subset of the map landmarks $\mathbf{m}_l$ can be measured, where $l \in \{1, ..., L\}$. L is the number of landmarks the robot observe from its current scan. We assume that a feature extraction algorithm provides measurements of landmarks in current scan and association between the locally observed landmarks $\mathbf{m}_l$ and the map $\mathbb{M}$ is already solved.

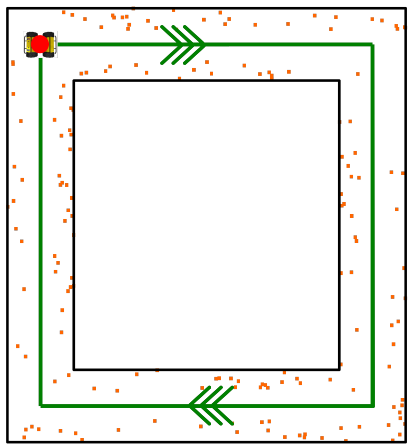

Figure 1: The 2D view of a robot moving in a $10m \times 10m$ square-shape corridor (bounded by two black boxes). The robot starts from the red point (0, 0) and follows the clockwise green trajectory (ground truth). Orange boxes are landmarks to be measured.

In Figure 1 we give an example which will be further investigated in Session 5. This localization problem can be formulated by the state estimation equations:

$$\begin{cases} \textbf{Measurement}: & \mathcal{G}(\mathbf{x}, \mathbf{z}, \mathbf{m}_l) = \mathbf{0} \\ \textbf{Association}: & \mathbf{m}_l \in \mathbb{M} \end{cases}, \tag{4}$$

where we denote $\mathbf{x} = (x, y, \psi)$ as the unknown robot pose, $\mathbf{z} = (r, \theta)$ as the measurement for each $\mathbf{m}_l$ in current scan, and $\mathbf{m}_l = (m_{l_x}, m_{l_y})$ the position of landmark. More specifically, the measurement constraint in Equation 4 can be broken down as follows:

$$\mathcal{G}(\mathbf{x}, \mathbf{z}) = \begin{pmatrix} x + r \cdot \cos(\psi + \theta) - m_{l_x} \\ y + r \cdot \sin(\psi + \theta) - m_{l_y} \end{pmatrix}. \tag{5}$$

We illustrate the above measurement model in Figure 2 where the robot measures its distance and azimuthal angle with respect to a landmark.

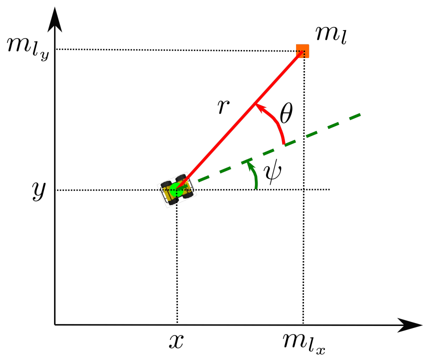

Figure 2: The robot measurement model.

Next, we will introduce our interval-based solution and factor graph-based probabilistic solution for the aforementioned localization problem, respectively.

4.1 Interval-based Localization

We use interval boxes to represent the known and unknowns in the localization model in Equation 4. This allows us to formulate a CSP $\mathcal{L}$:

$$\mathcal{L}: \begin{cases} \textbf{Variable}: \mathbf{x}, \mathbf{z}, \mathbf{m}_l, \mathbb{M} \\ \textbf{Constraint}: \begin{array}{c} \mathcal{G}(\mathbf{x}, \mathbf{z}, \mathbf{m}_l) = \mathbf{0} \\ \mathbf{m}_l \in \mathbb{M} \end{array} \\ \textbf{Domain}: [\mathbf{x}], [\mathbf{z}], [\mathbf{m}_l], [\mathbb{M}] \end{cases}, \tag{6}$$

where $[\mathbf{x}]$ is the interval domain of robot pose that we want to solve and $[\mathbf{z}] = ([r], [\theta])$ is our measurement of landmarks in each LiDAR scan. Interval boxes are suitable to represent the uncertainty of LiDAR measurements because we usually only know the accuracy of the distance and beam divergence. Besides the measurement uncertainty, the uncertainty of known map landmarks $\mathbb{M}$ is also specified. The uncertainty of map landmarks can be provided by the accuracy of give map or feature extraction algorithm. In this CSP we assume no information about the prior robot position, thus its initial domain is set to $[\mathbf{x}] = ([-\infty, \infty], [-\infty, \infty], [-\pi, \pi])^T$.

Then, we build forward-backward contractors for solving each constraint in CSP $\mathcal{L}$ and get the intersection to contract the desired domain. Before using the contractor, we first break down our constraints into simple equations according to Equation 5 and 6:

$$\begin{cases} a = \psi + \theta \\ d_x = r \cdot \cos(a) \\ d_y = r \cdot \sin(a) \\ m_{l_x} = x + d_x \\ m_{l_y} = y + d_y \\ \mathbf{m}_l \in \mathbb{M} \end{cases}, \tag{7}$$

where a and $\mathbf{d} = (d_x, d_y)$ are intermediate variables for decomposing complex constraints into elementary equations. Subsequently, we apply addition contractor $\mathcal{C}_+$ (e.g. $\mathcal{C}_+([a], [\psi], [\theta])$) for addition equations and polar contractor $\mathcal{C}_{polar}([d_x], [d_y], [r], [a])$ for the polar equations in Equation 7. As a result, related interval domains are reduced until smallest domains are reached.

In our localization problem, the CSP $\mathcal{L}$ will be solved for each position that the robot travels, which means that whenever the robot measures landmarks from one LiDAR scan, the contractors will be applied to contract the initial domain $[\mathbf{x}]$ and get the contracted interval box for the robot pose. The resulted interval box represents the uncertainty of robot pose.

4.2 Factor Graph based Localization

The idea of factor graph is to include unknown variables and factors which represent constraints between variables in one graph and solve a maximum a posteriori (MAP) estimate problem. Here we illustrate the factor graph of our localization problem in Figure 3:

As a probabilistic approach, all quantities and edges involved are labeled with a probability distribution. Given that all measurements are affected by Gaussian noise, the goal of our localization problem is to estimate a Gaussian approximation of the posterior of the robot poses, which is the uncertainty of the robot poses represented by Gaussian distribution. In the following we formulate our localization problem as an optimization problem that determines robot poses and landmark positions based on a set of measurements.

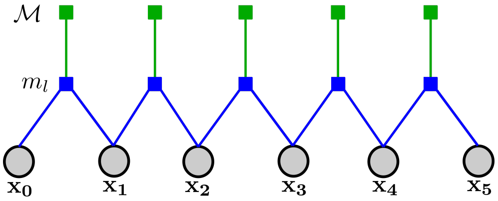

Figure 3: Factor graph representation of the localization problem. We have robot poses $\mathbf{x}$ (circles), observed landmarks $\mathbf{m}_l$ (blue squares) as variables and the map $\mathbb{M}$ (green squares) as the prior. Blue edges are the measurement factors between poses and observed landmarks $\mathbf{m}_l$. Green edges represent the map factor between the observed landmarks and map $\mathbb{M}$.

Let $\mathbf{v} = (\mathbf{x}, \mathbf{m}_l)$ be the state vector that consists of robot poses $\mathbf{x} = (x_1, ...x_T)^T$ and landmarks $\mathbf{m}_l = (m_{l_1}, ...m_{l_K})^T$ where T, K are the numbers of all poses and landmarks, respectively. Here we define the robot poses as $\mathbf{x}_t \in SE(2)$ and landmarks as $\mathbf{m}_{l_k} \in R^2$. The optimization problem can be represented as a MAP estimate according to Bayes' law:

$$\mathbf{v}^* = \arg\max_{\mathbf{v}} p(\mathbf{v}|\mathbf{z}) = \arg\max_{\mathbf{v}} p(\mathbf{z}|\mathbf{v})\, p(\mathbf{v}) \quad (8)$$

where $\mathbf{z}$ is the measurements gathered by the robot. The term $p(\mathbf{v})$ is the prior over all states which requires to be given. With assumption of independent Gaussian distribution on the measurements, Equation 8 can be written as:

$$\mathbf{v}^* = \arg\min_{\mathbf{v}} \sum_i \mathbf{e}_i(\mathbf{v}, \mathbf{z}_i)^T \Omega_i\, \mathbf{e}_i(\mathbf{v}, \mathbf{z}_i) + \mathcal{F}^{map}(\mathbf{m}_l) \quad (9)$$

where $\mathbf{e}_i(\mathbf{v}, \mathbf{z}_i)$ is the error function that measures how well the state vector $\mathbf{v}$ satisfy the constraint $\mathbf{z}_i$, Ω_i is the information matrix of measurement $\mathbf{z}_i$) and $\mathcal{F}^{map}$ is the prior knowledge of the map landmarks. In our localization problem, apart from defining measurement factors which can be described by the left part on the right side of Equation 9, we also define the map factors, which is a Gaussian prior about the landmark positions in the known map (similarly, in the interval approach we define the prior of map landmarks as interval boxes). To solve Equation 9 and compute the Gaussian approximation of the robot poses as well as landmark positions, we use standard least-square optimization method Levenberg-Marquardt algorithm using the GTSAM solver for factor graph (Dellaert, 2012).

5 EXPERIMENTAL VALIDATION

Our experiments are designed to support the claim that our interval-based localization method can ensure a guaranteed uncertainty estimation without assuming any probability distribution over the measurement. To confirm the reliability of our uncertainty estimation, we compare our approach against the probabilistic (factor graph based) approach in several small experiments where the measurement model does not follow non-zero Gaussian distribution. We use simulated data for our experiments to leave out the influence of inaccurate data association and outliers, so that we can investigate how the measurement model and uncertainty propagation strategy can affect the final uncertainty estimation. Firstly, we introduce important parameters that we use for the experiments.

We assume the robot moving in a $10m \times 10m$ square-shape indoor corridor with an initial position at (0, 0) as illustrated in Figure 1. The LiDAR on the robot has frequency of 10Hz. In order to calculate comparable uncertainty estimation for the interval and factor graph methods, we carefully choose the uncertainty of map landmarks and measurements. For the factor graph based method, we define the standard deviations for prior map landmarks as $\sigma_{\mathbb{M}_x} = \sigma_{\mathbb{M}_y} = 0.1m$, and for the angle and distance measurements as $\sigma_\theta = 0.01°$, $\sigma_r = 0.1m$, respectively. For interval-based method, we intend to define the interval box region of the map and measurements as such, that it will always include the 95% confidence region of Gaussian error ellipse. According to the χ^2 distribution, we choose 3σ as the radius of the intervals. Specifically, for map landmarks the uncertainty is $[\Delta_{\mathbb{M}_x}] = [\Delta_{\mathbb{M}_y}] = [-0.3, 0.3]m$, and for the measurements the uncertainties are $[\Delta_\theta] = [-0.03, 0.03]°$, $[\Delta_r] = [-0.3, 0.3]m$.

In the following sessions, we design four small experiments to show the comparison of uncertainty estimation results of the two methods. We change the conditions by manipulating the Gaussian distribution assumption for the measurements of probabilistic method. For interval method, the measurements always follow the interval uncertainty. First we compare the interval uncertainty with probabilistic uncertainty when the measurements follows Gaussian distribution. The non-zero Gaussian measurement is just a perfect assumption, while in real cases, the LiDAR measurements could contain systematic errors that are hard to identify. Thus in the second and third experiment we simulate systematic measurement errors for both methods and show how the two approaches handle the uncertainty propagation. In the last experiment, we use uniformly distributed measurements for the probabilistic approach.

5.1 Comparison of Uncertainty Estimation

Our first experiment is designed to show the comparison of robot pose uncertainty estimation using our interval-based localization algorithm and factor graph based optimization algorithm. The measurements in factor graph are generated using Gaussian distribution. We show in Figure 4 the result of both methods.

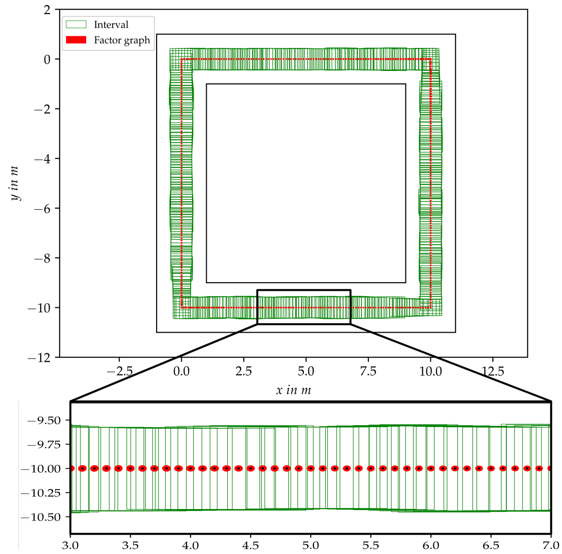

Figure 4: Uncertainty of robot poses given by interval method and factor graph method. The green boxes represent the interval pose uncertainty. Red ellipses represent the probabilistic pose uncertainty which is the 95% confidence region. The black dots inside the red ellipses are the ground truth value.

As can be seen, the probabilistic uncertainty throughout the trajectory are inside of the interval pose uncertainty boxes. Moreover, 100% ground truth poses are enclosed by the probabilistic pose uncertainty. The result shows that both methods can give guaranteed uncertainty while the probabilistic uncertainty can provide more accurate estimation than interval method.

5.2 Impact of 1-σ Systematic Error

In the second experiment we add a positive 1-σ_r systematic error to the distance measurements of both methods. This is to resemble possible systematic error from the range measurements from LiDAR. The results are shown in Figure 5.

Noted that the size of interval uncertainty slightly reduces with respect to the first experiment result, and it still encloses the 95% probabilistic uncertainty and the ground truth. The interval boxes shrink because

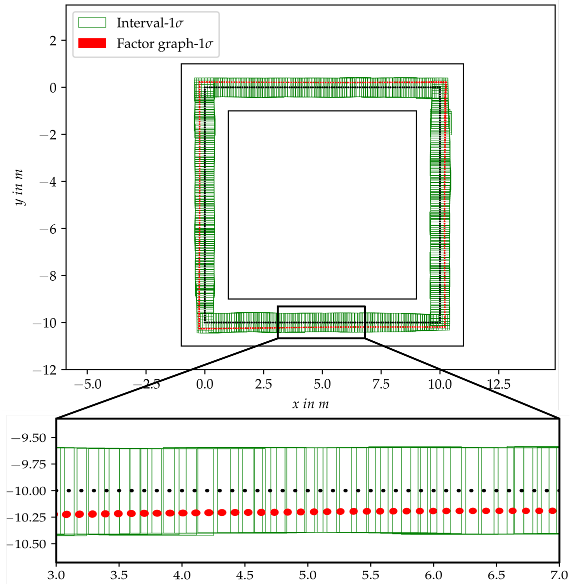

Figure 5: Green boxes represent the interval pose uncertainty. Red ellipses represent the 95% confident region of probabilistic uncertainty. Black dots are the ground truth value.

solutions that is not consistent with the new measurements are removed. The probabilistic uncertainty (red ellipse) shows an obvious shift to the outer bound of interval box, and no longer includes the ground truth value. The results implies that the probabilistic method underestimates the uncertainty and give wrong estimate, while interval method is still able to enclose the ground truth.

5.3 Impact of 2-σ Systematic Error

In the third experiment we decide to further investigate the influence of systematic error of measurements. We add a positive 2-σ_r systematic error to the distance measurements of both methods. The results are shown in Figure 6.

We can clearly see that interval boxes are still enclosing the ground truth, however most of the probabilistic error ellipses go even further away from the true value and out of the box bounds. We believe that in this case probabilistic method gives very wrong uncertainty estimation that almost all of the means of error ellipses are outside of the guaranteed interval uncertainty.

To conclude, we find that our interval method is robust to systematic errors of measurements, while probabilistic method becomes significantly unreliable. The results of the second and third experiment show that the probabilistic method tend to underestimate the uncertainty of pose estimation and can even give totally wrong uncertainty estimation in the pres-

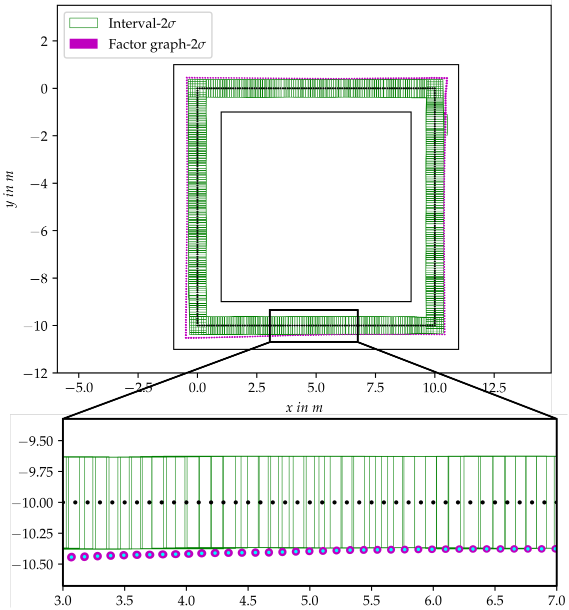

Figure 6: Green boxes represent the interval pose uncertainty. Purple ellipses represent the 95% confident region of probabilistic uncertainty. Cyan dots are the mean of probabilistic pose estimation. Black dots are the ground truth value.

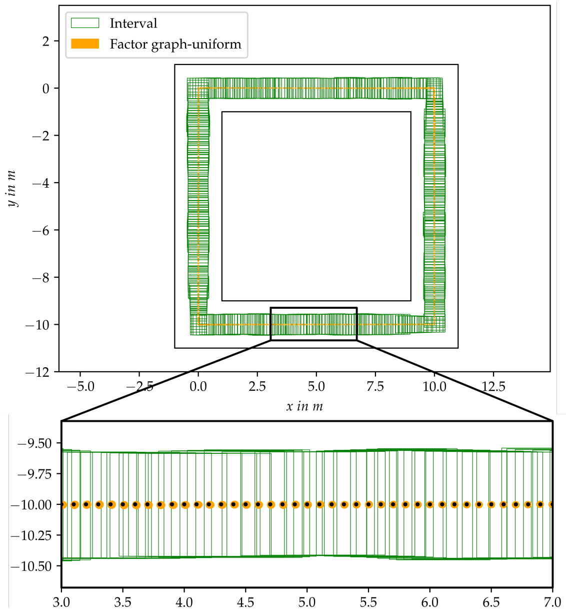

Figure 7: Green boxes represent the interval pose uncertainty. Orange ellipses represent the 95% confident region of probabilistic uncertainty. Black dots are the ground truth value.

ence of large systematic error. Interval method on the contrary can guarantee to provide trustworthy uncertainty estimate and not underestimate the pose uncertainty.

5.4 Impact of Uniformly Distributed Measurement

We assume our measurements are uniformly distributed throughout the interval uncertainty domain. In other words, we create measurements that follow a uniform distribution inside the interval region $[\Delta_\theta] = [-0.03, 0.03]^\circ$, $[\Delta_r] = [-0.3, 0.3]\,m$ and compare the robot pose uncertainty of the two methods. Results are displayed in Figure 7.

As seen from the result, interval uncertainty encloses the probabilistic uncertainty and the ground truth poses. Also noted that the orange probabilistic error ellipses enclose the ground truth throughout the trajectory. This means that despite the actual measurements do not have Gaussian distribution, but uniform distribution, the factor graph method is still able to provide correct pose uncertainty estimation.

6 CONCLUSIONS

We present an interval-based approach for a 2D localization problem that can estimate the robot pose uncertainty in a guaranteed way. To further evaluate the reliability of the uncertainty we design four comparison experiments with the state-of-the-art probabilistic approach. From the results of our experiments we find out that the systematic error in the measurement model can significantly impede the performance of probabilistic approach. Not only does the uncertainty estimation of probabilistic method underestimate the actual pose uncertainty, but wrongly estimated solution may be given. In comparison, our interval method is not sensitive to systematic measurement error and robustly provide guaranteed uncertainty estimation. This "failure" case reflects the unreliability of the approximated result of probabilistic method as well as its problem in uncertainty propagation. It also shows the benefits of using intervals for solving robot localization problem, that is, no prior information of the robot pose is required, and able to provide all possible solutions with guarantee.

In the further work, we aim to apply the interval-based method to real data. We plan to tackle the landmark extraction and association problem which can introduce more sources of error that will influence the uncertainty estimation. Besides, we would like to investigate again the uncertainty estimation performance of interval method and probabilistic method with noisy and distribution-unknown real measurement data.

REFERENCES

Brenner, C. (2010). Vehicle localization using landmarks obtained by a lidar mobile mapping system.

Chen, S. Y. (2012). Kalman filter for robot vision: A survey. *IEEE Transactions on Industrial Electronics*, 59(11):4409–4420.

Dellaert, F. (2012). Factor graphs and gtsam: A hands-on introduction. Technical Report GT-RIM-CP&R-2012-002, GT RIM.

Dellaert, F., Fox, D., Burgard, W., and Thrun, S. (1999). Monte carlo localization for mobile robots. In *Proceedings 1999 IEEE International Conference on Robotics and Automation (Cat. No.99CH36288C)*. IEEE.

Ehambram, A., Voges, R., and Wagner, B. (2021). Stereo-visual-LiDAR sensor fusion using set-membership methods. In *2021 IEEE 17th International Conference on Automation Science and Engineering (CASE)*. IEEE.

Gutmann, J.-S., Burgard, W., Fox, D., and Konolige, K. (1998). An experimental comparison of localization methods. In *Proceedings. 1998 IEEE/RSJ International Conference on Intelligent Robots and Systems. Innovations in Theory, Practice and Applications (Cat. No.98CH36190)*. IEEE.

Jaulin, L., Kieffer, M., Didrit, O., and Walter, É. (2001). Interval analysis. In *Applied Interval Analysis*, pages 11–43. Springer London.

Kaess, M., Johannsson, H., Roberts, R., Ila, V., Leonard, J., and Dellaert, F. (2011). iSAM2: Incremental smoothing and mapping with fluid relinearization and incremental variable reordering. In *2011 IEEE International Conference on Robotics and Automation*. IEEE.

Kenmogne, I.-F., Drevelle, V., and Marchand, E. (2017). Image-based UAV localization using interval methods. In *2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE.

Kummerle, R., Grisetti, G., Strasdat, H., Konolige, K., and Burgard, W. (2011). g2o: A general framework for graph optimization. In *2011 IEEE International Conference on Robotics and Automation*. IEEE.

Oh, S. M., Tariq, S., Walker, B., and Dellaort, F. (2004). Map-based priors for localization. In *2004 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS) (IEEE Cat. No.04CH37566)*. IEEE.

Rohou, S., Desrochers, B., and Jaulin, L. (2020). Set-membership state estimation by solving data association. In *2020 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE.

Schaefer, A., Buscher, D., Vertens, J., Luft, L., and Burgard, W. (2019). Long-term urban vehicle localization using pole landmarks extracted from 3-d lidar scans. In *2019 European Conference on Mobile Robots (ECMR)*. IEEE.

Siegwart, R. (2011). *Introduction to autonomous mobile robots*. MIT Press, Cambridge, Mass.

Thrun, S., Fox, D., Burgard, W., and Dellaert, F. (2001). Robust monte carlo localization for mobile robots. *Artificial Intelligence*, 128(1-2):99–141.

Voges, R. (2020). Bounded-error visual-lidar odometry on mobile robots under consideration of spatiotemporal uncertainties.

Voges, R. and Wagner, B. (2021). Interval-based visual-LiDAR sensor fusion. *IEEE Robotics and Automation Letters*, 6(2):1304–1311.

Wen, H., Chen, X., Papagiannis, G., Hu, C., and Li, Y. (2021). End-to-end semi-supervised learning for differentiable particle filters. In *2021 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE.

Wilbers, D., Merfels, C., and Stachniss, C. (2019). Localization with sliding window factor graphs on third-party maps for automated driving. In *2019 International Conference on Robotics and Automation (ICRA)*. IEEE.

Wu, C., Huang, T. A., Muffert, M., Schwarz, T., and Grater, J. (2017). Precise pose graph localization with sparse point and lane features. In *2017 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE.