

¹ PyUncertainNumber for uncertainty propagation: ² more than just probability arithmetic

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⁶ Summary

⁷ The forces on stars, galaxies, and dark matter under external gravitational fields lead to the
⁸ dynamical evolution of structures in the universe. The orbits of these bodies are therefore key
⁹ to understanding the formation, history, and future state of galaxies. The field of “galactic
¹⁰ dynamics,” which aims to model the gravitating components of galaxies to study their structure
¹¹ and evolution, is now well-established, commonly taught, and frequently used in astronomy.
¹² Aside from toy problems and demonstrations, the majority of problems require efficient
¹³ numerical tools, many of which require the same base code (e.g., for performing numerical
¹⁴ orbit integration).

¹⁵ Statement of need

¹⁶ Gala is an Astropy-affiliated Python package for galactic dynamics. Python enables wrapping
low-level languages (e.g., C) for speed without losing flexibility or ease-of-use in the
user-interface. The API for Gala was designed to provide a class-based and user-friendly
¹⁹ interface to fast (C or Cython-optimized) implementations of common operations such as
²⁰ gravitational potential and force evaluation, orbit integration, dynamical transformations, and
²¹ chaos indicators for nonlinear dynamics. Gala also relies heavily on and interfaces well with
²² the implementations of physical units and astronomical coordinate systems in the Astropy
²³ package ([Astropy Collaboration, 2013](#)) (`astropy.units` and `astropy.coordinates`).

²⁴ Gala was designed to be used by both astronomical researchers and by students in courses
on gravitational dynamics or astronomy. It has already been used in a number of scientific
publications ([Pearson et al., 2017](#)) and has also been used in graduate courses on Galactic
dynamics to, e.g., provide interactive visualizations of textbook material ([Binney & Tremaine,
2008](#)). The combination of speed, design, and support for Astropy functionality in Gala will
enable exciting scientific explorations of forthcoming data releases from the *Gaia* mission ([Gaia
Collaboration, 2016](#)) by students and experts alike.

³¹ `pyuncertainnumber` enables rigorous uncertainty analysis for real-world situations of mixed
uncertainties and partial knowledge. Aleatoric and epistemic uncertainties are recognised and
treated appropriately in characterisation and propagation.

³⁴ Uncertainty arithmetic is underpinned by probability bounds analysis. While it has the potential
to automatically compile a non-deterministic subroutines via primitives such as intervals or
uncertain numbers, its usages face several challenges.

³⁷ Besides the issues of xx such as dependency problems, one notable challenge is that code
accessibility is often not guaranteed. Also, the lack of capability one the main reasons restricting
the adoption of xxx in practice.

⁴⁰ pyuncertainnumber addresses that by enabling non-intrusive capability. How to work with
⁴¹ black-box models? This capability significantly boost its versatility for scientific computations
⁴² by interfacing with many engineering softwares.

⁴³ Interval propagation in a non-intrusive manner

⁴⁴ Interval analysis has the advantages of providing rigorous enclosures of the solutions to problems,
⁴⁵ especially for engineering problems subject to epistemic uncertainty, such as modelling system
⁴⁶ paramters due to lack-of-knowledge or characterising measurement incertitude. It is evident
⁴⁷ that computational tasks requiring complex numerical solutions of intervals are non-intrusive
⁴⁸ (i.e. the source code is not accessible). Besides, it shoule be noted even for crystal boxes
⁴⁹ (i.e. source code is accessible), naive interval arithmetic still faces challenges such as the
⁵⁰ infamous interval dependency issue. Though it may be mitigated through mathematical
⁵¹ rearrangements in some cases, it will be challenging for most of the cases.

⁵² Generally, the interval propagation problem can be cast as an optimisation problem where the
⁵³ minimum and maximum are sought via a function mapping. The functio, for example g in
⁵⁴ Eq.(xx), is not necessarily monotonic or linear and may well be a black-box model. Hence, for
⁵⁵ black box models the optimisation can only be solved via gradient-free optimisation techniques.

$$Y = g(I_{x1}, I_{x2}, \dots, I_{xn}) \quad (1)$$

$$Y_m \text{in}, Y_m \text{ax} \quad (2)$$

⁵⁶ where $I_{x1}, I_{x2}, \dots, I_{xn}$ are intervals.

⁵⁷ pyuncertainnumber provides a series of non-intrusive methodologies of varying applicability.
⁵⁸ It should be noted that there is generally a trade-off between applicability and efficiency. But
⁵⁹ with more knowledge about the characteristics of the underlying function, one can accordingly
⁶⁰ dispatch an efficient method. For example, when monotonicity is known one can use vertex
⁶¹ methods which 2_n .

Table 1: Several methods for interval propagation

Method	End-points	Subinterval reconstitution	Cauthy-Deviate method	Bayesian optimisation	Genetic algorithm
As-sumption Result	monotonicity	heavy computation	linearity and gradient required	No	No

⁶² As shown in ??, tabulation of xxx given a black box model.

⁶³ Mixed uncertainty propagation for black-box models

⁶⁴ Most realistic situation bla bla. Imprecise world bla bla. After faithful characterisation, the
⁶⁵ ability to propagate is the key in many critical engineering applications.

⁶⁶ Dependency structures bla bla. It has been echoed in the engineering applications and also the
⁶⁷ NASA challenge.

⁶⁸ Sampling methods play a significant role in xxx

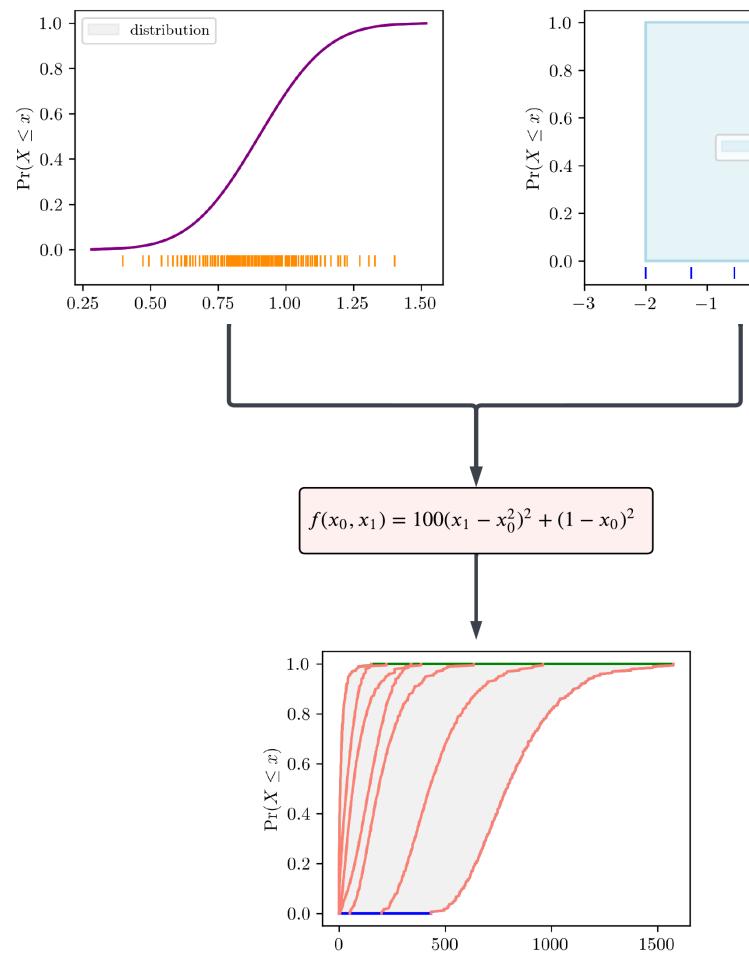
- ⁶⁹ Double Monte Carlo
⁷⁰ Interval Monte Carlo...

⁷¹ Propagation of p-boxes via surrogate models

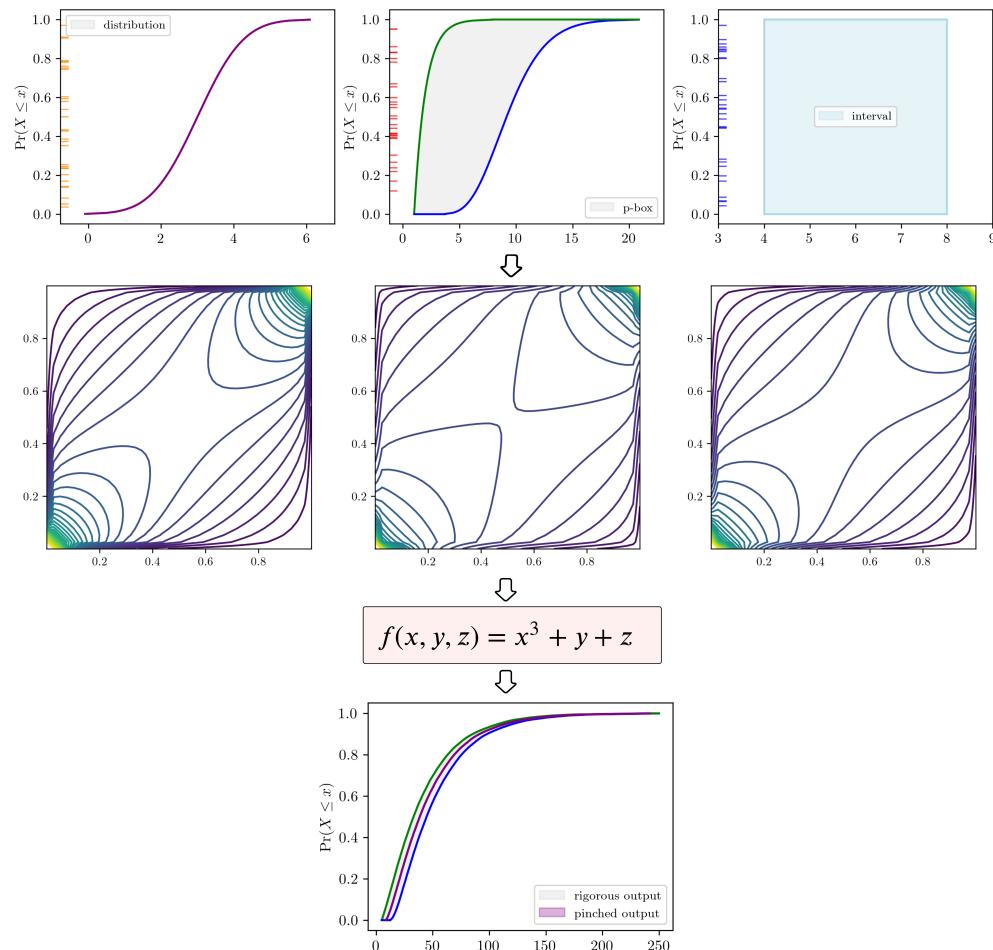
⁷² Citations

- ⁷³ Citations to entries in paper.bib should be in [rMarkdown](#) format.
⁷⁴ If you want to cite a software repository URL (e.g. something on GitHub without a preferred citation) then you can do it with the example BibTeX entry below for Smith et al. ([2020](#)).
⁷⁵ For a quick reference, the following citation commands can be used: - @author:2001 -> “Author et al. (2001)” - [@author:2001] -> “(Author et al., 2001)” - [@author1:2001;
⁷⁶ @author2:2001] -> “(Author1 et al., 2001; Author2 et al., 2002)”

⁷⁹ Figures



- ⁸⁰ Figures can be included like this:
⁸¹ ?? illustrates the *nested Monte Carlo* method.



82
83 ?? illustrates the *interval Monte Carlo* method.

Conclusion

84
85 Significance: this provides compatibility as interfacing with many engineering applications.
86 boost its usage.

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87
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