Solutions

STAT 217: Quiz 26

1. To orient themselves with respect to their surroundings, some bats use echolocation. Such a trait has evolved in very few animal species, perhaps because of the high energy costs involved in producing pulses. Zoologists collected data on in-flight energy expenditure (watts) and body mass (g) from 20 energy studies on three types of flying vertebrates: echolocating bats, non-echolocating bats, and non-echolocating birds. Below is the first five rows of data:

```
##    MASS                     TYPE ENERGY
## 1 779.0  non-echolocating bats  43.70
## 2 628.0  non-echolocating bats  34.80
## 3 258.0  non-echolocating bats  23.30
## 4 315.0  non-echolocating bats  22.40
## 5  24.3 non-echolocating birds   2.46
## 6  35.0 non-echolocating birds   3.93
```

(a) What is the response variable and what are the explanatory variables? Label each as quantitative or categorical.

response - energy expenditure (watts)
explanatory ~ mass (g) Quantitative
type of flying vertebrate Categorical

(b) Let's first consider the parallel lines model. Write out the TRUE multiple linear regression ADDITIVE model (with β's). You will need to use indicator variables. Make sure to define each indicator variable.

$$\mu\{energy/mass, type\} = \beta_0 + \beta_1 I_{NEbird} + \beta_2 I_{NEbat} + \beta_3 mass$$

$I_{NEbird} = \begin{cases} 1 \text{ if NEbird} \\ 0 \text{ else} \end{cases}$ $I_{NEbat} = \begin{cases} 1 \text{ if NEbat} \\ 0 \text{ if else} \end{cases}$

(c) Based on the additive model you defined in (b), write out the TRUE simple linear regression model for echolocating bats.

$$\mu\{energy/mass, type = e\,bat\} = \beta_0 + \beta_1 \cdot 0 + \beta_2 \cdot 0 + \beta_3 mass$$
$$= \beta_0 + \beta_3 mass$$

(d) In terms of betas, what is the slope and y-intercept for the SLR line for echolocating bats?

β_3 β_0

(e) Based on the additive model you defined in (b), write out the TRUE simple linear regression model for non-echolocating bats.

$$\mu\{energy \mid mass, type = NEbat\} = \beta_0 + \beta_1 \cdot 0 + \beta_2 \cdot 1 + \beta_3\, mass$$
$$= \beta_0 + \beta_2 + \beta_3\, mass$$

(f) In terms of betas, what is the slope and y-intercept for the SLR line for non-echolocating bats?

β_3 $\qquad$ $\beta_0 + \beta_2$

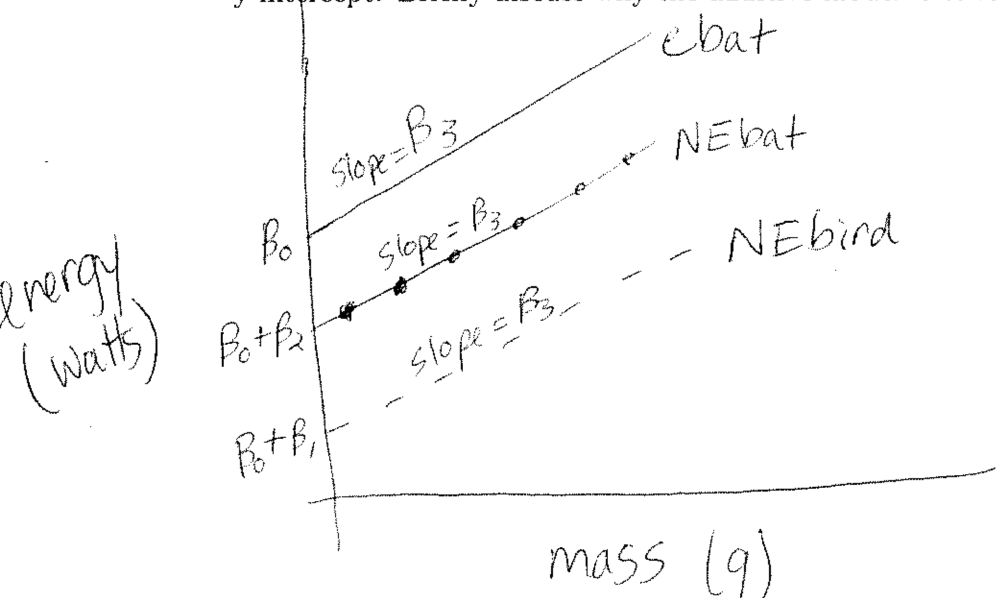
(g) Based on the additive model you defined in (b), write out the TRUE simple linear regression model for non-echolocating birds.

$$\mu\{energy \mid mass, type = NEbirds\} = \beta_0 + \beta_1 \cdot 1 + \beta_2 \cdot 0 + \beta_3\, mass$$
$$= \beta_0 + \beta_1 + \beta_3\, mass$$

(h) In terms of betas, what is the slope and y-intercept for the SLR line for non-echolocating birds?

β_3 $\qquad$ $\beta_0 + \beta_1$

(i) Sketch a plot like the one I showed in class. For each line, label the slope and the y-intercept. Briefly discuss why the additive model is called a "parallel lines model".



each line has the same slope

2. Repeat parts (b) through (i) for an interaction or "separate lines" model. Please do this on a separate sheet of paper.

(2) Interaction Model

$$\mu\{energy|mass, type\} = \beta_0 + \beta_1 I_{NEbird} + \beta_2 I_{NEbat} + \beta_3 mass + \beta_4 mass \cdot I_{NEbird} + \beta_5 mass \cdot I_{NEbat}$$

* indicator variables are defined as they were in (b)

$$\mu\{energy|mass, type=ebat\} = \beta_0 + \beta_1 \cdot 0 + \beta_2 \cdot 0 + \beta_3 mass + \beta_4 mass \cdot 0 + \beta_5 mass \cdot 0$$

$$= \beta_0 + \beta_3 mass$$

y-intercept: β_0 slope: β_3 for ebats

$$\mu\{energy|mass, type=NEbird\} = \beta_0 + \beta_1 1 + \beta_2 \cdot 0 + \beta_3 mass + \beta_4 mass \cdot 1 + \beta_5 mass \cdot 0$$

$$= \beta_0 + \beta_1 + (\beta_3 + \beta_4) mass$$

y-intercept: $\beta_0 + \beta_1$ slope: $\beta_3 + \beta_4$ for NEbird

$$\mu\{energy|mass, type=NEbat\} = \beta_0 + \beta_1 \cdot 0 + \beta_2 \cdot 1 + \beta_3 mass + \beta_4 mass \cdot 0 + \beta_5 mass \cdot 1$$

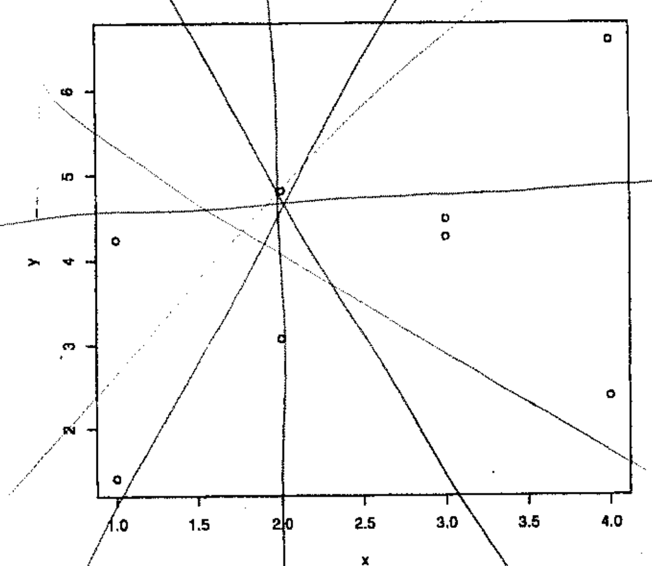
$$= \beta_0 + \beta_2 + (\beta_3 + \beta_5) mass$$

y-intercept: $\beta_0 + \beta_2$ slope: $\beta_3 + \beta_5$ for NEbat

STAT 217: Class exercise 10/20

```
set.seed(105)
x <- c(rep(1:4,2))
y <- rnorm(8,4,2)

plot(y~x)
```



1. Draw a line of best fit through the above data.
2. Find the SSE for your estimated regression line.